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Constraining flavour symmetries at the EW scale I: the A_4 Higgs potential

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Abstract: We consider an extension of the Standard Model in which the symmetry is enlarged by a global flavour factor A_4 and the scalar sector accounts for three copies of the Standard Model Higgs, transforming as a triplet of A_4. In this context, we study the most general scalar potential and its minima, performing for each of them a model independent analysis on the related phenomenology. We study the scalar spectrum, the new contributions to the oblique corrections, the decays of the Z and $W^{\pm}$, the new sources of CP and flavour violation, which all are affected by the introduction of multiple Higgses transforming under A_4. We find that this model independent approach discriminates the different minima allowed by the scalar potential.

Keywords: Spontaneous Symmetry Breaking, Beyond Standard Model, Discrete and Finite Symmetries, Global Symmetries

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1 Introduction

The current data on neutrino oscillations seem to point at one small and two large angles in the neutrino mixing matrix [1–6]. The data are consistent with various mixing patterns, where in particular the agreement with the tri-bimaximal [7, 8] mixing pattern is striking [9].

The use of non-Abelian discrete flavour symmetries has been proposed in different models (for a review see [9]) to generate both the mentioned lepton mixing patterns and the quark ones. In general, in those models, one introduces so called flavons, scalar fields charged in the flavour space, usually very heavy. Once the flavons develop specific vacuum expectation values (vevs), this translates to structures in the masses and mixings of the fermions. However, imposing the correct symmetry breaking patterns on the flavons is highly non-trivial. This holds in particular if two or more flavons are used, breaking in different directions in flavour space. So far, only a few techniques have been developed, all of which need a supersymmetric context or the existence of extra dimensions [9].

Alternatively, one can look at models that require only one flavour symmetry breaking direction. In this case the scalar potential that implements the breaking can be non supersymmetric and does not require extra dimensions. Of particular interest is the possibility that one set of fields simultaneously takes the role of the flavons and the Standard Model (SM) Higgs fields, identifying the breaking scales of the electroweak and the flavour symmetries.

In this paper, we will consider the discrete flavour symmetry A_4 and we will assume that there are three copies of the Standard Model Higgs field, that transform among each other as a triplet of A_4 [10–16]. The presence of this extended Higgs sector has an deep impact on the high energy phenomenology: indeed new contributions to the oblique corrections as well as new sources of CP and flavour violation usually appear in this context. We will analyse the constraints coming from these observables for all the vacuum configurations allowed by the scalar potential and will discuss on the viability of each of them.

The structure of the paper is as follows. In section 2, we will introduce the scalar potential invariant under A_4 and under the gauge group of the Standard Model. In section 3 we will introduce the various physical Higgs fields that are present in the model.

In the subsequent two sections we will present the different minima allowed by the potential and discuss the corresponding Higgs spectrum. These minima may or may not violate CP symmetry, depending on the question whether all vacuum expectation values of the Higgses are real or not. In section 4 we will discuss the cases that do not violate the CP symmetry, while in section 5 we discuss CP breaking minima.

Section 6 we will discuss bounds on the allowed parameters using respectively unitarity constraints, decays of the Z and $W^{\pm}$ bosons and constraints by oblique corrections. We note that all these bounds are rather model independent, meaning that they depend on the flavour symmetry assignment of the relevant Higgs fields, but not on those of the fermions in the theory. Further bounds can be derived from fermion decays and meson oscillations, but these bounds are always model dependent. We will present some of these in an accompanying paper [17].

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Finally, in section 7 we present the results of our analysis and in section 8 we conclude. In the appendix A we report useful formulae for the analysis of the T, S and U parameters.

2 The A_4 scalar potential

We consider the Standard Model gauge group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ with the addition of a global flavour symmetry A_4 [18, 19]. We consider three copies Φ_a, $a = 1, 2, 3$, of the conventional SM Higgs field (i.e. a singlet of $\mathrm{SU}(3)_c$, doublet of $\mathrm{SU}(2)_L$ and with hypercharge $Y = 1/2$) such that the three Higgses are in a triplet of the flavour group A_4. Once the flavour structure of the quarks and leptons is specified, each Φ_a will couple to the three fermion families according to the group theory rules in a model dependent way. We will study these couplings in more detail in [17].

Below, we will write down the most general scalar potential for the three Higgses that is invariant under the flavour and gauge symmetries of the model. After the fields occupy one of the minima of the potential, electroweak symmetry gets broken (while electromagnetism is conserved) and we can develop the fields around their vacuum expectation values as

$$\Phi_a = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathrm{Re}\ \Phi_a^1 + i\,\mathrm{Im}\ \Phi_a^1 \\ \mathrm{Re}\ \Phi_a^0 + i\,\mathrm{Im}\ \Phi_a^0 \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} \mathrm{Re}\ \phi_a^1 + i\,\mathrm{Im}\ \phi_a^1 \\ v_a e^{i\omega_a} + \mathrm{Re}\ \phi_a^0 + i\ \mathrm{Im}\, \phi_a^0 \end{pmatrix}. \tag{2.1}$$

Here $v_a e^{i\omega_a}$ is the vacuum expectation value of the a^{th} Higgs field. One or two of the v_a can be zero, implying that the corresponding Higgs field does not develop a vev. Furthermore, if all vevs are real (so if all ω_a are zero) CP is conserved, while if one or more ω_as are nonzero, CP is broken. Note that in general, there is the freedom to put one of the phases to zero by a global rotation.

We will use the A_4 basis as developed by Ma and Rajasekaran (MR) [10]. The analysis could also be done in a different A_4 basis, for instance the one of Altarelli and Feruglio [20]. The results would then look different, but would obviously be equivalent. In the MR basis, the most general potential $V[\Phi_a]$ can be written as [10, 21].

$$\begin{aligned} V[\Phi_a] = {} & \mu^2(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 + \Phi_3^\dagger\Phi_3) + \lambda_1(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 + \Phi_3^\dagger\Phi_3)^2 + \\ & + \lambda_3(\Phi_1^\dagger\Phi_1\Phi_2^\dagger\Phi_2 + \Phi_1^\dagger\Phi_1\Phi_3^\dagger\Phi_3 + \Phi_2^\dagger\Phi_2\Phi_3^\dagger\Phi_3) + \\ & + \lambda_4(\Phi_1^\dagger\Phi_2\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_3\Phi_3^\dagger\Phi_1 + \Phi_2^\dagger\Phi_3\Phi_3^\dagger\Phi_2) + \\ & + \frac{\lambda_5}{2}\left[e^{i\epsilon}\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_3)^2 + (\Phi_3^\dagger\Phi_1)^2\right] + e^{-i\epsilon}\left[(\Phi_2^\dagger\Phi_1)^2 + (\Phi_3^\dagger\Phi_2)^2 + (\Phi_1^\dagger\Phi_3)^2\right]\right], \end{aligned} \tag{2.2}$$

in agreement with the usual notation adopted in the two Higgs Doublet Models (2HDM). The parameter μ^2 is typically negative in order to have a stable minimum away from the origin. All the other parameters, λ_i, are real parameters which must undergo to the condition for a potential bounded from below: this forces λ_1 and the combination $\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 \cos\epsilon$ to be positive.

It is interesting to notice that, contrary to other multi Higgs (MH) scenarios, here we can not recover the SM limit, with one light scalar and all the others decoupled and very heavy. The flavour symmetry constrains the potential parameters in such a way that the

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scalar masses are never independent from each other. This can be easily understood by a parameter counting: the scalar potential in eq. (2.2) presents 6 independent parameters and the number of the physical quantities is 8, i.e. the electroweak (EW) vev and the seven masses for the massive scalar fields.

We will study the minima of the potential in eq. (2.2) under electromagnetism conserving vevs as specified in eq. (2.1) by studying the first derivative system

$$\frac{\partial V[\Phi]}{\partial \Phi_{\mathcal{I}}} = 0\,, \tag{2.3}$$

where $\Phi_{\mathcal{I}}$ is of the fields Re Φ_a^1, Re Φ_a^0, Im Φ_a^1 or Im Φ_a^0 and by requiring that the Hessian

$$\frac{\partial^2 V[\Phi]}{\partial \Phi_{\mathcal{I}} \partial \Phi_{\mathcal{J}}} \tag{2.4}$$

has non negative eigenvalues, or in other words that all the physical masses are positive except those ones corresponding to the Goldstone bosons (GBs) that vanish.

In sections 4 and 5 we will verify that this potential presents a number of solutions. Some of them are natural in the sense that they do not require *ad hoc* values of the potential parameters; these are only constrained by requiring the boundness at infinity and the positivity of all the physical scalar masses. The only potential parameter constrained is the bare mass term μ^2 which is related to the physical Electroweak (EW) vev, $v_w^2 = v_1^2 + v_2^2 + v_3^2$. Others require specific relations between the adimensional scalar potential parameters and may have extra Goldstone bosons.

3 The physical Higgs fields

The symmetry breaking of the Higgs fields of equation eq. (2.1) leads to a large number of charged and neutral Higgs bosons as well as the known Goldstone bosons of the Standard Model.

In the most general case, where CP is not conserved, the neutral real and imaginary components of eq. (2.1) mix to five CP non-definite states and a GB:

$$\begin{aligned} h_\alpha &= U_{\alpha a} \mathrm{Re}\, \phi_a^0 + U_{\alpha(a+3)} \mathrm{Im}\, \phi_a^0\,, \\ \pi^0 &= U_{6a} \mathrm{Re}\, \phi_a^0 + U_{6(a+3)} \mathrm{Im}\, \phi_a^0\,. \end{aligned} \tag{3.1}$$

Here $a = 1, 2, 3$ and $\alpha = 1 - 5$, while $\alpha = 6$ represents the GB π^0. In matrixform this reads

$$\begin{pmatrix} h_1 \\ \vdots \\ h_5 \\ \pi^0 \end{pmatrix} = U \begin{pmatrix} \mathrm{Re}\, \phi_1^0 \\ \vdots \\ \mathrm{Re}\, \phi_3^0 \\ \mathrm{Im}\, \phi_1^0 \\ \vdots \\ \mathrm{Im}\, \phi_3^0 \end{pmatrix} \tag{3.2}$$

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Clearly eq. (3.1) holds also in the CP conserved case: in that case the 6 by 6 scalar mass matrix reduces to a block diagonal matrix with two 3 by 3 mass matrices leading to three CP even states and 2 CP odd states and the GB π^0.

The three charged scalars mix into two new charged massive states and a charged GB.

$$\begin{pmatrix} H_1^+ \\ H_2^+ \\ \pi^+ \end{pmatrix} = S \begin{pmatrix} \phi_1^1 \\ \phi_2^1 \\ \phi_3^1 \end{pmatrix} , \tag{3.3}$$

where π^+ is the Goldstone boson eaten by the gauge bosons W^+. In general, the S is a complex unitary matrix. In the special case where CP is conserved, its entries are real (and it is thus an orthogonal matrix).

4 CP conserved solutions

In this section, we will study minima of the potential in eq. (2.2) in which only $\mathrm{Re}\,\phi_a^0$ develops a vev, i.e. the CP symmetry is conserved. In this case we expect having 3 neutral scalar CP-even states, 2 CP-odd states and 2 charged scalars as well as a real and a complex GBs originating from respectively the CP-odd states and the charged states.

In the CP-conserved case all the ω_a vanish and the first derivative system in eq. (2.3) reduces to

$$\begin{aligned}
v_1[2(v_1^2+v_2^2+v_3^2)\lambda_1 + (v_2^2+v_3^2)(\lambda_3+\lambda_4+\lambda_5\cos\epsilon) + 2\mu^2] &= 0\,, \\
v_2[2(v_1^2+v_2^2+v_3^2)\lambda_1 + (v_1^2+v_3^2)(\lambda_3+\lambda_4+\lambda_5\cos\epsilon) + 2\mu^2] &= 0\,, \\
v_3[2(v_1^2+v_2^2+v_3^2)\lambda_1 + (v_1^2+v_2^2)(\lambda_3+\lambda_4+\lambda_5\cos\epsilon) + 2\mu^2] &= 0\,, \\
v_1(v_2^2-v_3^2)\lambda_5\sin\epsilon &= 0\,, \\
v_2(v_1^2-v_3^2)\lambda_5\sin\epsilon &= 0\,, \\
v_3(v_2^2-v_1^2)\lambda_5\sin\epsilon &= 0\,,
\end{aligned} \tag{4.1}$$

where the first three derivatives refer to the real components Φ_a^0 and the second ones to the imaginary parts. In the most general case, when neither ϵ nor λ_5 is zero, the last three equations allow two different solutions

1) $v_1 = v_2 = v_3 = v = v_w/\sqrt{3}$;

2) $v_1 \neq 0$ and $v_2 = v_3 = 0$ (and permutations of the indices); in this case $v_1 = v_w$.

Both these solutions are solutions of the first three equations as well, provided that

$$\begin{cases} \mu^2 = -(3\lambda_1+\lambda_3+\lambda_4+\lambda_5\cos\epsilon)v_w^2/3 & \text{for the first case} \\ \mu^2 = -\lambda_1 v_w^2 & \text{for the second case.} \end{cases} \tag{4.2}$$

In this cases λ_5 can be chosen positive, as a sign can be absorbed in a redefinition of ϵ.

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Next, we consider the case where $\sin\epsilon$ is 0. This implies $\epsilon = 0$ or π. We may however absorb the minus sign corresponding to the second case in a redefinition of λ_5 that is now allowed to span over both positive and negative values.

Assuming $v_1 \neq 0$, we may solve the first equation in eq. (4.1) with respect to μ^2. Then by substituting μ^2 in the other two equations we get

$$\begin{aligned} v_2(v_1^2 - v_2^2)(\lambda_3 + \lambda_4 + \lambda_5) &= 0\,, \\ v_3(v_1^2 - v_3^2)(\lambda_3 + \lambda_4 + \lambda_5) &= 0\,. \end{aligned} \tag{4.3}$$

Next to the two solutions present in the general case, this system has two further possible solutions

3) $v_3 = 0, v_2 = v_1 = v_w/\sqrt{2}$ and permutations. This requires

$$\mu^2 = -\left(4\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5\right) v_w^2/4\,. \tag{4.4}$$

4) $(\lambda_3 + \lambda_4 + \lambda_5) = 0$. This condition implies that in the real neutral direction there is a enlarged–$O(3)$ accidental symmetry that is spontaneously broken by the vacuum configuration, thus we xpect extra GBs. Indeed in this case v_1, v_2 and v_3 are only restricted to satisfy $v_1^2 + v_2^2 + v_3^2 = v_w^2$ and the parameter μ^2 is given by $\mu^2 = -\lambda_1 v_w^2$.

Finally, the case $\lambda_5 = 0$ allows special cases of the solutions 1) to 4), but does not give rise to new solutions. For this reason, we will discuss only the general cases and the case $\epsilon = 0$ in the remainder of this section and comment what happens for $\lambda_5 = 0$.

4.1 $\epsilon \neq 0$: the alignment (v, v, v)

In the basis chosen, the vacuum alignment (v, v, v) preserves the Z_3 subgroup of A_4.[1] It is convenient to perform a basis transformation into the Z_3 eigenstate basis, $1, 1' \sim \omega, 1'' \sim \omega^2$ according to

$$\begin{aligned} \varphi &= (\Phi_1 + \Phi_2 + \Phi_3)/\sqrt{3} \sim 1 \\ \varphi' &= (\Phi_1 + \omega\Phi_2 + \omega^2\Phi_3)/\sqrt{3} \sim \omega \\ \varphi'' &= (\Phi_1 + \omega^2\Phi_2 + \omega\Phi_3)/\sqrt{3} \sim \omega^2\,. \end{aligned} \tag{4.5}$$

When A_4 is broken to Z_3 in the Z_3 eigenstate basis, $\varphi \sim 1$ behaves like the standard Higgs doublets: its neutral real component develops a vacuum expectation values $\langle \varphi^{0R} \rangle = v_w$ and all its other components correspond to the GBs eaten by the corresponding gauge bosons. The physical real scalar gets a mass given by

$$m_{h_1}^2 = \frac{2}{3} v_w^2 (3\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 \cos\epsilon). \tag{4.6}$$

[1] In the special case where $\epsilon = 0$, the symmetry of the vacuum is enlarged to S_3 even if S_3 is not a subgroup of A_4. The reason is that setting $\epsilon = 0$ effectively enlarges the symmetry of the potential to S_4 (once also SU(2) × U(1) gauge invariance is required), which does have S_3 as a subgroup.

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The neutral components of the other two doublets φ' and φ'' mix into two complex neutral states and their masses are given by

$$m_n^{\prime,\prime\prime\,2} = \frac{v_w^2}{6}\left(-\lambda_3-\lambda_4-4\lambda_5\cos\epsilon \pm \sqrt{(\lambda_3+\lambda_4)^2+4\lambda_5^2(1+2\sin^2\epsilon)-4(\lambda_3+\lambda_4)\lambda_5\cos\epsilon}\right). \tag{4.7}$$

The charged components of φ', φ'' do not mix, their masses being

$$m_{ch}^{\prime,\prime\prime\,2} = -\frac{v_w^2}{6}\left(3\lambda_4+3\lambda_5\cos\epsilon \pm \sqrt{3}\lambda_5\sin\epsilon\right). \tag{4.8}$$

4.2 $\epsilon \neq 0$: the alignment $(v,0,0)$

In the chosen A_4 basis, the vacuum alignments $(v,0,0)$ preserves the Z_2 subgroup of A_4. As we did with the vacuum alignment that conserved the Z_3 subgroup, in this case it is useful to rewrite the scalar potential by performing the following Z_2 conserving basis transformation

$$\begin{aligned}\Phi_1 &\to \Phi_1\,,\\ \Phi_2 &\to e^{-i\epsilon/2}\Phi_2\,,\\ \Phi_3 &\to e^{i\epsilon/2}\Phi_3\,.\end{aligned} \tag{4.9}$$

Φ_1 is even under Z_2 and behaves like the standard Higgs doublet, while Φ_2 and Φ_3 are odd. For what concerns the neutral states, the 6×6 mass matrix is diagonal in this basis and with some degenerated entries: using a notation similar to the 2DHM, we have

$$\begin{aligned} m_{h_1}^2 &\equiv 2\lambda_1 v_w^2\,, & m_{h_2}^2 &= m_{h_3}^2 = \frac{1}{2}(\lambda_3+\lambda_4-\lambda_5)v_w^2\,,\\ m_{h_4}^2 &= m_{h_5}^2 = \frac{1}{2}(\lambda_3+\lambda_4+\lambda_5)v_w^2\,, & m_{\pi^0}^2 &= 0\,, \end{aligned} \tag{4.10}$$

where the last state corresponds to the GB. The charged scalar mass matrix is also diagonal with

$$m_{C_1}^2 = m_{C_2}^2 = \frac{1}{2}\lambda_3 v_w^2\,, \qquad m_{\pi^+}^2 = 0\,, \tag{4.11}$$

where the last state corresponds to the GB. The degeneracy in the mass matrices are imposed by the residual Z_2 symmetry. Contrary to the previous case the neutral scalar mass eigenstates are real and not complex.

4.3 $\epsilon = 0$: the alignment $(v,v,0)$

This vacuum alignment does not preserve any subgroup of A_4 and it holds that $v = v_w/\sqrt{2}$. From the minimum equations we have that

$$\mu^2 = -\frac{1}{4}v_w^2(4\lambda_1+\lambda_3+\lambda_4+\lambda_5)\,. \tag{4.12}$$

The scalar and pseudoscalar mass eigenvalues are given by

$$\begin{aligned} m_{h_1}^2 &= -\frac{v_w^2}{2}(\lambda_3+\lambda_4+\lambda_5)\,, & m_{h_2}^2 &= \frac{v_w^2}{2}(4\lambda_1+\lambda_3+\lambda_4+\lambda_5)\,,\\ m_{h_3}^2 &= \frac{v_w^2}{4}(\lambda_3+\lambda_4+\lambda_5)\,, & m_{h_4}^2 &= -\lambda_5 v_w^2\,,\\ m_{h_5}^2 &= \frac{v_w^2}{4}(\lambda_3+\lambda_4-3\lambda_5)\,, & m_{\pi^0}^2 &= 0\,. \end{aligned} \tag{4.13}$$

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For the charged sector we have

$$m^2_{C_1} = \frac{v_w^2}{4}(\lambda_3 - \lambda_4 - \lambda_5)\,, \qquad m^2_{C_2} = -\frac{v_w^2}{2}(\lambda_4 + \lambda_5) \qquad m^2_{C_3} = 0\,. \tag{4.14}$$

For $\lambda_5 \neq 0$ the alignment $(v, v, 0)$ has the correct number of GBs, while for $\lambda_5 = 0$ we have an extra massless pesudoscalar. However in both cases, $\lambda_5 \neq 0$ or $\lambda_5 = 0$, the conditions $m^2_{h_1} > 0$ and $m^2_{h_3} > 0$ can not be simultaneously satisfied. This alignment is therefore a saddle point of the A_4 scalar potential we are studying.

4.4 $\epsilon = 0$: the alignment (v_1, v_2, v_3)

This vacuum alignment, as the previous one, does not preserve any subgroup of A_4. A part from the condition $\epsilon = 0$, we recall that in this case there is the further constraint $\lambda_3 + \lambda_4 + \lambda_5 = 0$ and λ_5 may assume both positive and negative values since we have reabsorbed in the λ_5 sign the case $\epsilon = \pi$.

Let us define $v_w^2 = v_1^2 + v_2^2 + v_3^2 = (1 + s^2 + r^2)v_1^2$ with $s = v_2/v_1$ and $r = v_3/v_1$ respectively. The mass matrix for the neutral scalar states presents two null eigenvalues–as we expected since the condition $\lambda_3 + \lambda_4 + \lambda_5 = 0$ enlarges the potential symmetry– and a massive one

$$m{h_1}^2 = 2\lambda_1 v_w^2\,. \tag{4.15}$$

At the same time the mass matrix for the CP-odd states has one null eigenvalue–the GB π^0 and two degenerate eigenvalues of mass

$$m^2_{h_2} = m^2_{h_3} = (\lambda_3 + \lambda_4)v_w^2\,. \tag{4.16}$$

Notice that for the special case $\lambda_5 = 0$ we have the constraint $\lambda_3 = -\lambda_4$ that implies two extra massless pseudoscalars. Finally for the charged scalars we have

$$m^2_{C_1} = m^2_{C_2} = \frac{1}{2}\lambda_3 v_w^2\,, \qquad m^2_{C3} = 0 \tag{4.17}$$

The total amount of GBs is 5 (7) for the case $\lambda_5 \neq 0$ ($\lambda_5 = 0$), so we have 2 (4) extra unwanted GBs: this situation is really problematic. We note that the introduction of terms in the potential that softly break A_4 can ameliorate the situation with the Goldstone bosons. We will analyse soft A_4 breaking terms in more detail in [17].

5 CP non-conserved solutions

In this subsection, we consider vacua that exhibit spontaneous CP violation. This occurs if the vev of at least one of the Higgses is inherently complex. A global rotation can always absorb one of the three phases of the vevs.

We note that that the two natural vacua of the previous section (v, v, v) and $(v, 0, 0)$ do not have CP violating analogues as they have only one phase that can be reabsorbed to make all vevs real.

5.1 The alignment $(v_1 e^{i\omega_1}, v_2, 0)$

In this case the third doublet is inert and therefore we are left only with two doublets that develop a complex vev and after the redefinition, there is only one phase ω_1. Taking the generic solution $(v_1 e^{i\omega_1}, v_2, 0)$ the minimum equations are given by

$$\begin{aligned}
v_1\left[\cos\omega_1[2\mu^2+2\lambda_1(v_1^2+v_2^2)+(\lambda_3+\lambda_4)v_2^2]+\lambda_5 v_2^2\cos(\epsilon+\omega_1)\right] &= 0\,,\\
v_2\left[(2\mu^2+2\lambda_1(v_1^2+v_2^2)+(\lambda_3+\lambda_4)v_1^2+\lambda_5 v_1^2\cos(\epsilon+2\omega_1)\right] &= 0\,,\\
v_1\left[\sin\omega_1[2\mu^2+2\lambda_1(v_1^2+v_2^2)+(\lambda_3+\lambda_4)v_2^2]-\lambda_5 v_2^2\sin(\epsilon+\omega_1)\right] &= 0\,,\\
v_2 v_1^2\sin(\epsilon+2\omega_1) &= 0\,.
\end{aligned} \tag{5.1}$$

The last equation can be solved by $\epsilon=-2\omega_1$ or $\epsilon=-2\omega_1+\pi$. Like in section 4, we can absorb the second case by a redefinition of λ_5. The other three equations reduce to

$$\begin{aligned}
v_1\cos\omega_1\left[2\mu^2+2\lambda_1(v_1^2+v_2^2)+(\lambda_3+\lambda_4)v_2^2+\lambda_5 v_2^2\right] &= 0\,,\\
v_2\left[2\mu^2+2\lambda_1(v_1^2+v_2^2)+(\lambda_3+\lambda_4)v_1^2+\lambda_5 v_1^2\right] &= 0\,,\\
v_1\sin\omega_1\left[2\mu^2+2\lambda_1(v_1^2+v_2^2)+(\lambda_3+\lambda_4)v_2^2+\lambda_5 v_2^2\right] &= 0\,.
\end{aligned} \tag{5.2}$$

that are simultaneously solved for $v_1=v_2=v_w/\sqrt{2}$ and

$$\mu^2=-\frac{v_w^2}{4}(4\lambda_1+\lambda_3+\lambda_4+\lambda_5)\,. \tag{5.3}$$

The neutral and charged 6×6 mass matrices are quite simple and it is possible having analytical expression for the mass eigenvalues. For the neutral sector we have

$$\begin{aligned}
m_{h_1}^2&=\frac{1}{2}v_w^2(-\lambda_3-\lambda_4-\lambda_5)\,, & m_{h_2}^2&=\frac{1}{2}v_w^2(4\lambda_1+\lambda_3+\lambda_4+\lambda_5)\,,\\
m_{h_3}^2&=\frac{1}{4}v_w^2(\lambda_3+\lambda_4-\lambda_5+2\lambda_5\cos 3\omega_1)\,, & m_{h_4}^2&=-\lambda_5 v_w^2\,,\\
m_{h_5}^2&=\frac{1}{4}v_w^2(\lambda_3+\lambda_4-\lambda_5-2\lambda_5\cos 3\omega_1)\,, & m_{\pi^0}^2&=0\,,
\end{aligned} \tag{5.4}$$

and for the charged one we have

$$m_{C_1}^2=\frac{v_w^2}{4}(\lambda_3-\lambda_4-\lambda_5)\,, \qquad m_{C_2}^2=\frac{v_w^2}{2}(-\lambda_4-\lambda_5)\,, \qquad m_{C_3}^2=0\,. \tag{5.5}$$

We see that the mass of the fourth neutral boson selects negative values for λ_5, i.e. the second solution $\epsilon=-2\omega_1+\pi$. It is interesting to see that in the CP conserved limits $\omega_1\to 0$ (or π), it is not possible to have both $m_{h_1}^2$ and $m_{h_3}^2$ (respectively $m_{h_5}^2$) positive, but that in the general case, there are points in parameter space where indeed all masses are positive. This is in particular clear in the region around $\cos 3\omega_1=0$.

Finally, as for the CP conserved case, for $\lambda_5=0$ we have two problems: an extra GB and we cannot have all positive massive eigenstates.

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5.2 The alignment $(v_1 e^{i\omega_1}, v_2 e^{i\omega_2}, v_3)$

In this case all the doublets develop a vev $v_i \neq 0$, so we may have two physical CP violating phases. We have the freedom to take $\omega_3 = 0$. In this case the first derivatives system is given by

$$\begin{aligned}
v_1 \{\cos\omega_1[2\mu^2 + 2\lambda_1(v_1^2+v_2^2+v_3^2) + (\lambda_3+\lambda_4)(v_2^2+v_3^2)] + \\
+\lambda_5[v_3^2\cos(\epsilon-\omega_1) + v_2^2\cos(\epsilon+\omega_1-2\omega_2)]\} &= 0\,, \\
v_2 \{\cos\omega_2(2\mu^2 + 2\lambda_1(v_1^2+v_2^2+v_3^2) + (\lambda_3+\lambda_4)(v_1^2+v_3^2) + \\
+\lambda_5[v_3^2\cos(\epsilon+\omega_2) + v_1^2\cos(\epsilon-\omega_2+2\omega_1)]\} &= 0\,, \\
v_3 \{2\mu^2 + 2\lambda_1(v_1^2+v_2^2+v_3^2) + (\lambda_3+\lambda_4)(v_1^2+v_2^2) + \\
+\lambda_5[v_1^2\cos(\epsilon-2\omega_1) + v_2^2\cos(\epsilon+2\omega_2)]\} &= 0\,, \\
v_1 \{\sin\omega_1[2\mu^2 + 2\lambda_1(v_1^2+v_2^2+v_3^2) + (\lambda_3+\lambda_4)(v_2^2+v_3^2)] + \\
+\lambda_5[v_3^2\sin(\epsilon-\omega_1) - v_2^2\sin(\epsilon+\omega_1-2\omega_2)]\} &= 0\,, \\
v_2 \{\sin\omega_2(2\mu^2 + 2\lambda_1(v_1^2+v_2^2+v_3^2) + (\lambda_3+\lambda_4)(v_1^2+v_3^2)) + \\
+\lambda_5[-v_3^2\sin(\epsilon+\omega_2) + v_1^2\sin(\epsilon-\omega_2+2\omega_1)]\} &= 0\,, \\
v_3 \left[\lambda_5(-v_1^2\sin(\epsilon-2\omega_1) + v_2^2\sin(\epsilon+2\omega_2))\right] &= 0\,.
\end{aligned} \tag{5.6}$$

The last equation is solved for $\omega_2 = -\omega_1$ and $v_2 = v_1 = v$. Defining $v_3 = rv$ and $v_1^2 + v_2^2 + v_3^2 = v_w^2$ the previous system reduces to the three equations

$$\begin{aligned}
\mu^2 + \frac{v_w^2}{2(2+r^2)}\left[(4+2r^2)\lambda_1 + (1+r^2)(\lambda_3+\lambda_4) + \frac{\lambda_5}{\cos\omega_1}(r^2\cos(\epsilon-\omega_1) + \cos(\epsilon+3\omega_1))\right] = 0\,, \\
\mu^2 + \frac{v_w^2}{2(2+r^2)}\left[(4+2r^2)\lambda_1 + (1+r^2)(\lambda_3+\lambda_4) + \frac{\lambda_5}{\sin\omega_1}(r^2\sin(\epsilon-\omega_1) + \sin(\epsilon+3\omega_1))\right] = 0\,, \\
\mu^2 + \frac{v_w^2}{(2+r^2)}\left[(2+r^2)\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5\cos(\epsilon-2\omega_1)\right] = 0\,.
\end{aligned} \tag{5.7}$$

We can solve the third equation in eq. (5.7) in terms of μ^2 and then the second equation in terms of λ_5, giving

$$\begin{aligned}
\mu^2 &= -\tfrac{v_w^2}{2+r^2}[(2+r^2)\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5\cos(\epsilon-2\omega_1)]\,, \\
\lambda_5 &= \frac{(r^2-1)(\lambda_3+\lambda_4)\sin\omega_1}{(r^2-1)\sin(\epsilon-\omega_1) - 2\cos\epsilon\sin(3\omega_1)}\,.
\end{aligned} \tag{5.8}$$

Then the first equation in eq. (5.7) has two possible solutions, for λ_4 and ϵ respectively

$$i) \qquad \lambda_4 = -\lambda_3\,, \tag{5.9}$$

$$ii) \qquad \tan\epsilon = \frac{r^2\sin 2\omega_1 + \sin 4\omega_1}{r^2\cos 2\omega_1 - \cos 4\omega_1}\,. \tag{5.10}$$

To test the validity of the solution so far sketched it is necessary to check to be in a true minimum of the potential and not to have extra GBs a part from three corresponding to the GBs eaten by the gauge bosons. However the relations given in eq. (5.8) and eq. (5.10)

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do not allow to get analytical solutions for the scalar masses in case *ii*). For this reason we will consider only three special limits in this case: $r \sim 0$, $r \sim 1$ and r very large. We think that these limit situations could be the most interesting ones in the model building realizations. Indeed models present in literature [11, 12] fall in the third case, r very large.

5.2.1 Case *i*)

In this case the constraints $\lambda_4 = -\lambda_3$ puts λ_5 to zero and enlarge substantially the symmetries of the potential: we have an accidental $O(3)$ in the neutral real direction and two accidental U(1)s due to $\lambda_5 = 0$. For this reason the neutral spectrum has 5 massless particles, the GB π^0 and 4 other GBs, and only one massive state

$$m_{h_1}^2 = 2\lambda_1 v_w^2 \,. \tag{5.11}$$

The charged scalars are

$$m_{C_1}^2 = m_{C_2}^2 = \frac{1}{2}\lambda_3 v_w^2 \,, \qquad\qquad m_{C3}^2 = 0 \tag{5.12}$$

The massive states are degenerate as in the CP conserving minima studied in section (4.3) for the case $\lambda_5 = 0$.

5.2.2 Case *ii*)

As it is not possible to find analytical solutions, here we will study three special limits of case ii.

- $r \sim 0$

  In this case we will neglect terms of order r^2. From eq. (5.10) we have that for $r \sim 0$

  $$\epsilon \sim -4\omega_1 + N\pi \,, \tag{5.13}$$

  thus from eq. (5.8) we have

  $$\begin{aligned} \mu^2 &= -\lambda_1 v_w^2 - (\lambda_3 + \lambda_4)\frac{1 - \cos 6\omega_1}{2 - 4\cos 6\omega_1} \,, \\ \lambda_5 &= \frac{\lambda_3 + \lambda_4}{1 - 2\cos 6\omega_1} \,. \end{aligned} \tag{5.14}$$

  Under these approximations the 6 x 6 neutral scalar mass matrix gives one null mass state, $m_{\pi^0}^2 = 0$, corresponding to the GB and the following five eigenvalues at leading order, given by

  $$\begin{aligned} m_{h_1}^2 &\sim f[\lambda_i]\mathcal{O}(r^2)v_w^2 \\ m_{h_2}^2 &\sim -(\lambda_3 + \lambda_4)/(1 - 2\cos 6\omega_1)v_w^2 \\ m_{h_3}^2 &\sim \left[-2\lambda_1 + (4\lambda_1 + \lambda_3 + \lambda_4)(1 - \cos 6\omega_1)/(1 - 2\cos 6\omega_1)\right] v_w^2 \\ m_{h_4}^2 &\sim -\left[(\lambda_3 + \lambda_4)\cos 6\omega_1 v_w^2/(1 - 2\cos 6\omega_1)\right] v_w^2 \,, \\ m_{h_5}^2 &\sim -\left[2(\lambda_3 + \lambda_4)\sin^2 3\omega_1/(1 - 2\cos 6\omega_1)\right] v_w^2 \,, \end{aligned} \tag{5.15}$$

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where $f[\lambda_i]$ stays for a linear combination of the adimensional λ parameters of the potential. The previous neutral spectrum present a lightest state that may be too light to be phenomenologically acceptable. Assuming that the λ's potential parameters run in the 'natural' range $0.1 \div 10$ or, somewhat optimistically, $10^{-2} \div 10^2$. For what concerns r we are in the limit of $r^2 \sim 0$, so as reference value we may take $r^2 \sim 10^{-3} \div 10^{-2}$. By combining these two ranges we find upper bounds

$$\begin{aligned} m_{h_1}^2 &\leq 200\,\mathrm{GeV} \quad \text{for} \quad \lambda_i \sim 100, r^2 \sim 10^{-2}\,, \\ m_{h_1}^2 &\leq 25\,\mathrm{GeV} \quad \text{for} \quad \lambda_i \sim 10, r^2 \sim 10^{-3}\,. \end{aligned} \tag{5.16}$$

Since $f[\lambda_i] \sim 100$ may be obtained only for very peculiar combinations of the potential parameters, the previous estimates indicate that for relative tiny value of r the spectrum may present very light neutral states.

On the contrary, in the charged sector we have the two GBs eaten by the corresponding gauge bosons, $m_{C_3}^2 = 0$, and two complex massive states with masses

$$\begin{aligned} m_{C_1}^2 &\sim -[\lambda_4 + (\lambda_3 + \lambda_4 \cos 6\omega_1)/(1 - 2\cos 6\omega_1)]v_w^2/2 \\ m_{C_2}^2 &\sim -[2\lambda_4 + (\lambda_3 + 2\lambda_4 \cos 6\omega_1)/(1 - 2\cos 6\omega_1)]v_w^2/2\,. \end{aligned} \tag{5.17}$$

- $r \sim 1$

In this limit we may write $r \sim 1 + \delta$ and make an expansion in terms of δ neglecting terms of order δ^2. Thus we have

$$\epsilon \sim \pi/2 - \omega_1 - \delta \cot 3\omega_1 + N\pi\,, \tag{5.18}$$

and then

$$\begin{aligned} \mu^2 &= -(3\lambda_1 + \lambda_3 + \lambda_4)/3v_w^2 - \delta/9(\lambda_3 + \lambda_4)v_w^2\,, \\ \lambda_5 &= \delta(\lambda_3 + \lambda_4)\csc 3\omega_1\,. \end{aligned} \tag{5.19}$$

Under these approximations the 6 x 6 neutral scalar mass matrix gives the usual null mass state, $m_{\pi^0}^2$, corresponding to the GB and the following five eigenvalues

$$\begin{aligned} m_{h_1}^2 &\sim m_{h_2}^2 \sim f[\lambda_i]\mathcal{O}(\delta^2)v_w^2\,, \\ m_{h_3}^2 &\sim m_{h_4}^2 \sim -(\lambda_3 + \lambda_4)/3v_w^2\,, \\ m_{h_5}^2 &\sim 2(3\lambda_1 + \lambda_3 + \lambda_4)/3v_w^2\,, \end{aligned} \tag{5.20}$$

where again $f[\lambda_i]$ stays for a linear combination of the λ's potential parameters. A analysis similar to the one for the case with $r \sim 0$ shows that the neutral spectrum may present very light states.

In the charged sector we have the GBs eaten by the gauge bosons and two degenerate massive state

$$m_{C_1}^2 \sim m_{C_2}^2 \sim -\lambda_4/2v_w^2\,. \tag{5.21}$$

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- $r \gg 1$

In this case we may perform an expansion in term of $1/r$ and neglect terms of order $1/r^2$. From eq. (5.10) we have that

$$\epsilon \sim 2\omega_1 + N\pi\,, \tag{5.22}$$

and then eq. (5.8) reduces to

$$\begin{aligned} \mu^2 &\sim -\lambda_1 v_w^2\,, \\ \lambda_5 &\sim -(\lambda_3+\lambda_4)\,, \end{aligned} \tag{5.23}$$

Under these approximations we find a massless neutral scalar state, $m^2_{\pi^0} = 0$, and the other 5 neutral masses are given at leading order by

$$\begin{aligned} m^2_{h_1} &\sim m^2_{h_2} \sim f[\lambda_i]\mathcal{O}(1/r^2)v_w^2\,, \\ m^2_{h_3} &\sim 2\lambda_1 v_w^2\,, \\ m^2_{h_4} &\sim m^2_{h_5} \sim (\lambda_3+\lambda_4)v_w^2\,, \end{aligned} \tag{5.24}$$

where once more $f[\lambda_i]$ stays for a linear combination of the λ's potential parameters. The charged scalar mass matrix is diagonal up to terms of order $\mathcal{O}(1/r^2)$ with two massive degenerate states

$$m^2_{C_1} = m^2_{C_2} = \lambda_3 v_w^2/2\,, \tag{5.25}$$

and the correct number of GBs.

If we consider now eq. (5.24) we see that as for $r \sim 0$ and $r \sim 1$ the expressions for $m^2_{h_{1,2}}$ say that we may have two very light neutral scalars. Taking as reference values for r the range $50 \div 200$ we find

$$m^2_{h_{1,2}} \sim \sqrt{f[\lambda_i]}\,5\,\text{GeV}(1\,\text{GeV})\,, \tag{5.26}$$

giving

$$\begin{aligned} m^2_{1,2} &\leq 50^2 \quad \text{GeV}^2 \quad \text{for} \quad r \sim 50\,, \\ m^2_{1,2} &\leq 10^2 \quad \text{GeV}^2 \quad \text{for} \quad r \sim 200\,, \end{aligned} \tag{5.27}$$

where $50(10)\,\text{GeV}$ may be obtained only for very peculiar combination of the potential parameters. In other words we expect that also in the majority of the cases for r in the range $50 - 200$ we will have $m^2_{1,2}$ very light.

In conclusion, taking into account the SM context and the potential given in eq. (2.2), the solution $(e^{i\omega_1}, e^{-i\omega_1}, r)v_w/\sqrt{2+r^2}$ with r small, close to 1 or large give rise to very light states. Of course this does not mean that these states will be light for any value of r but it is a quite strong hint that it is possible that this could be what indeed happens. As mentioned before, the addition of soft A_4 breaking terms to the potential may help in the cases of Goldstone bosons or very light bosons. We will discuss these terms in more detail in [17].

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6 Bounds from the Higgs phenomenology

In this section we analyse the phenomenology corresponding to the different vacua discussed above: unitarity, Z and $W^{\pm}$ decays and oblique parameters. In this way we manage to constrain the parameter space and, in some cases, to rule out the studied vacuum configuration.

6.1 Unitarity

In this section we account for the tree level unitarity constraints coming from the additional scalars present in the theory. We examine the partial wave unitarity for the neutral two-particle amplitudes for $s \gg M_W^2, M_Z^2$. We can use the equivalence theorem, so that we can compute the amplitudes using only the scalar potential described in eq. (2.2). In the regime of large energies, the only relevant contributions are the quartic couplings in the scalar potential [22–25] and then we can write the $J = 0$ partial wave amplitude a_0 in terms of the tree level amplitude T as

$$a_0(s) \equiv \frac{1}{32\pi} \int_{-1}^{1} \mathrm{d}\cos\theta \, T(s) = \frac{1}{16\pi} F[\lambda_i] \,, \tag{6.1}$$

where F represents a function of the λ_i couplings. Using for simplicity the notation

$$\Phi_a = \begin{pmatrix} w_a^+ \\ \dfrac{v_a e^{i\omega_a} + h_a^0 + i z_a}{\sqrt{2}} \end{pmatrix} \,, \tag{6.2}$$

we can write the 30 neutral two-particle channels as follows:

$$w_a^+ w_b^- \,, \; \frac{z_a z_b}{\sqrt{2}} \,, \; \frac{h_a^0 h_b^0}{\sqrt{2}} \,, \; h_a^0 z_b \,. \tag{6.3}$$

Once written down the full scattering matrix a_0, we find a block diagonal structure. The first 12×12 block concerns the channels

$$w_1^+ w_1^- \,, \; w_2^+ w_2^- \,, \; w_3^+ w_3^- \,, \; \frac{z_1 z_1}{\sqrt{2}} \,, \; \frac{z_2 z_2}{\sqrt{2}} \,, \; \frac{z_3 z_3}{\sqrt{2}} \,, \; \frac{h_1^0 h_1^0}{\sqrt{2}} \,, ; \frac{h_2^0, h_2^0}{\sqrt{2}} \,, \; \frac{h_3^0, h_3^0}{\sqrt{2}} \,, \; h_1^0 z_1 \,, \; h_2^0 z_2 \,, \; h_3^0 z_3 \,,$$

while the other three 6×6 blocks are related to the channels

$$w_a^+ w_b^- \,, \; w_b^+ w_a^- \,, \; h_a^0 z_b \,, \; h_b^0 z_a \,, \; z_a z_b \,, \; h_a^0 h_b^0 \,,$$

once we specify the labels (a, b) as (1, 2), (1, 3) and (2, 3). Notice that up this point the analysis is completely general and is valid for all the vacua presented. We specify the vacuum configuration, expressing the quartic couplings λ_i in terms of the masses of the scalars. Afterwards, putting the constraint that the largest eigenvalues of the scattering matrix a_0 is in modulus less than 1, we find upper bounds on the scalar masses which we use in our numerical analysis.

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6.2 Z and $W^{\pm}$ decays

From an experimental point of view gauge bosons decays into scalar particles are detected by looking at fermionic channels, such as for example $Z \to hA \to 4f$ in the 2HDM, or Z decays into partial or total missing energy in a generic new physics scenario. From this point of view gauge bosons decays bound the Higgs sector in an extremely model dependent way. However since in the SM the Z and the $W^{\pm}$ decays into 2 fermions, 4 fermions or *all* have been precisely been calculated and measured, we may focus on the decays $Z, W^{\pm} \to all$. Doing this we overestimate the allowed regions in the parameter space, but we have a first and model independent cut arising by the gauge bosons decay. Once we will pass to a model dependent analysis the region may only be restricted, not enlarged. Furthermore, defining the contribution from new physics as $\Delta\Gamma$, since

$$\Delta\Gamma^{2f}_{Z,W^{\pm}} \sim \Delta\Gamma^{4f}_{Z,W^{\pm}} \sim \Delta\Gamma^{all}_{Z,W^{\pm}} \ll \Gamma_{Z,W^{\pm}}\,, \tag{6.4}$$

we expect the error we commit being quite small.

From LEP data we have

$$\Gamma^{\text{exp}}_{Z,W^{\pm}} = \Gamma^{\text{SM}}_{Z,W^{\pm}} + \Delta\Gamma_{Z,W^{\pm}} \tag{6.5}$$

with $\Delta\Gamma_Z \sim 0.0023\,\text{GeV}$ and $\Delta\Gamma_{W^{\pm}} \sim 0.042\,\text{GeV}$ [26]. Therefore we may calculate the width

$$\begin{aligned} Z &\to h_i h_j\,, \\ W^+ &\to H^+_i h_j\,. \end{aligned} \tag{6.6}$$

for the different multi Higgs (MH) vacuum configuration studied and select the points that satisfy

$$\Gamma^{MH}_{Z,W^{\pm}} \leq \Delta\Gamma_{Z,W^{\pm}}\,. \tag{6.7}$$

Here we have indicated the generic $Z \to h_i h_j$ referring to our notation introduced in section 2. Clearly when CP is conserved the h_i have defined CP and only couplings to CP odd states are allowed. Of course this is not true for the configuration when CP is spontaneously broken.

In the vacuum analysis we did we have seen that in few situations we have extra massless or very light particles. For those cases the gauge bosons decays put strong bounds. For what concerns the Z decays we have

$$\begin{cases} k_Z \leq \Delta\Gamma_Z \dfrac{16\pi}{m_Z}\dfrac{4c_W^2}{g^2} & \text{if both particles } h_i \text{ and } h_j \text{ are massless}\,, \\ k_Z \left(1 - \dfrac{m^2_{h_i}}{m^2_Z}\right)^3 \leq \Delta\Gamma_Z \dfrac{16\pi}{m_Z}\dfrac{4c_W^2}{g^2} & \text{if } h_j \text{ is masslees and } 0 < m^2_{h_i} < m^2_Z\,, \\ k_Z \left(1 - \dfrac{m^2_{h_i} + m^2_{h_j}}{m^2_Z}\right)^3 \leq \Delta\Gamma_Z \dfrac{16\pi}{m_Z}\dfrac{4c_W^2}{g^2} & \text{if } h_i, h_j \neq 0 \text{ and } 0 < m^2_{h_i} + m^2_{h_j} < m^2_Z\,. \end{cases} \tag{6.8}$$

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where g is the SU(2) gauge coupling, c_W the cosine of the Weinberg angle θ_W and the parameter k_Z is given by

$$k_Z = \left(-U^T_{ab}U^T_{(a+3)c} + U^T_{(a+3)b}U^T_{ac}\right)^2 , \tag{6.9}$$

with U defined in eq. (3.2).

Similarly for the $W^\pm$ decays we have

$$k_W \left(1 - \frac{m^2_{C_i}}{m^2_W}\right)^3 \leq \Delta\Gamma_W \frac{16\pi}{m_W}\frac{4c}{g^2} \quad \text{if } h_j \text{ is massless and } m^2_{C_i} < m^2_W \tag{6.10}$$

where, in analogy to the Z decay, the parameter k_W is given by

$$k_W = \left|S^\dagger_{ab}U^T_{ac}\right|^2 + \left|S^\dagger_{(a+3)b}U^T_{(a+3)c}\right|^2 , \tag{6.11}$$

with S defined in eq. (3.3).

6.3 Large mass Higgs decay

Electroweak data analysis considering the data from LEP2 [27] and Tevatron [28] put an upper bound on the mass of the SM Higgs of 194 GeV at 99% CL [26]. In a MH scenario this bound may be roughly translated in the upper bound for the lightest scalar mass, m_{h_1}. For large values of the SM Higgs mass, $m_h \geq 2m_W$, the main channel decay is $h \to W^+W^-$ and the upper bound is completely model independent. Let us indicate as $\Gamma^{SM}_{WW}(194)$ the branching ratio of the SM Higgs into two $W^\pm$ at a mass of 194 GeV.

In a MH model the lightest Higgs boson couples to the gauge bosons with a coupling that is

$$\begin{aligned} g_{h_1ZZ} &= \beta\, g^{SM}_{hZZ} , \\ g_{h_1WW} &= \beta\, g^{SM}_{hWW} , \end{aligned} \tag{6.12}$$

with $\beta \leq 1$. In our case for example β is given by

$$f_a(\cos\omega_a\, U^T_{a1} + \sin\omega_j\, U^T_{(a+3)1}) , \tag{6.13}$$

with $f_a = v_a/v_w$ and ω_a the corresponding CP phase. Taking into account that h_1 is less produced then the SM Higgs and that its $\Gamma^{MH}_{WW}(m_{h_1})$ is reduced with respect to the SM one,

$$\Gamma^{MH}_{WW}(m_{h_1}) \sim |\beta|^4 \Gamma^{SM}_{WW}(m_{h_1}) \leq \Gamma^{SM}_{WW}(194) , \tag{6.14}$$

we can roughly constrain the upper bound for masses $m_{h_1} \geq 194$ GeV.

6.4 Constraints by oblique corrections

The consistence of a MH model has to be checked also by means of the oblique corrections. These corrections can be classified [29–33] by means of three parameters, namely TSU,

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that maybe written in terms of the physical gauge boson vacuum polarizations as [34]

$$
\begin{aligned}
T &= \frac{4\pi}{e^2 c_W^2 m_Z^2}\left[A_{WW}(0) - c_W^2 A_{ZZ}(0)\right] , \\
S &= 16\pi \frac{s_W^2 c_W^2}{e^2}\left[\frac{A_{ZZ}(m_Z^2) - A_{ZZ}(0)}{m_Z^2} - A'_{\gamma\gamma}(0) - \frac{(c_W^2 - s_W^2)}{c_W s_W} A'_{\gamma Z}(0)\right] , \\
U &= -16\pi \frac{s_W^2}{e^2}\left[\frac{A_{WW}(m_W^2) - A_{WW}(0)}{m_W^2} - c_W^2 \frac{A_{ZZ}(m_Z^2) - A_{ZZ}(0)}{m_Z^2} - s_W^2 A'_{\gamma\gamma}(0)\right. \\
&\qquad \left. -2 s_W c_W A'_{\gamma Z}(0)\right] ,
\end{aligned}
\tag{6.15}
$$

where s_W, c_W are sine and cosine of θ_W and e is the electric charge. EW precision measurements severely constrain the possible values of the three parameters T, S and U. In the SM assuming $m_h^2 > m_Z^2$ we have

$$
\begin{aligned}
T_h^{SM} &\sim -\frac{3}{16\pi c_W^2} \log \frac{m_h^2}{m_Z^2} , \\
S_h^{SM} &\sim \frac{1}{12\pi} \log \frac{m_h^2}{m_Z^2} , \\
U_h^{SM} &\sim 0 .
\end{aligned}
\tag{6.16}
$$

For a Higgs boson mass of $m_h = 117\,\text{GeV}$ (and in brackets the difference assuming instead $m_h = 300\,\text{GeV}$), the data allow [26]

$$
\begin{aligned}
S^{\text{exp}} &= 0.10 \pm 0.10(-0.08) \\
T^{\text{exp}} &= 0.03 \pm 0.11(+0.09) \\
U^{\text{exp}} &= 0.06 \pm 0.10(+0.01) .
\end{aligned}
\tag{6.17}
$$

The constraints in eq. (6.17) must be rescaled not only for the different values of the Higgs boson mass but also for a different scalar or fermion field content: for example, if we assume to have a MH scenario this gives a contribution T^{MH} to the T-parameter and we need

$$
T^{\text{NSS}} - T_h^{\text{SM}} = T^{\text{exp}} . \tag{6.18}
$$

A detailed analysis on the TSU in a MH model has been presented in [35, 36] where all the details are carefully explained. However the resulting formulae are valid only for scalar masses larger or comparable to m_Z. Since this is not the case for a generic MH model and particularly for the configurations studied so far, where we have a redundant number of massless or extremely light particles, we improved their results, getting full formulae valid for any value of the scalar masses (see the appendix A for details).

7 Results

We have performed a numerical analysis for all vacuum configurations considered, neglecting the alignment $(v, v, 0)$ since in this case there are tachyonic states. Our aim was to find a region in the parameter space where all the Higgs constraints were satisfied for each configuration considered. We have analysed the points generated through subsequent constraints, from the weaker one to the stronger according to

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- points Y: true minima — all the squared masses positive — (yellow points in the figures);
- points B: unitarity bound (blue points);
- points G: Z and $W^{\pm}$ decays (green points);
- points R: TSU parameters (red points).

The ratios B/Y, G/B, R/G may be used to indicate which is the stronger constraint for each allowed minima. For almost each case we have compared the masses of the two lightest neutral states –except for the alignment studied in section 5.2.1 where we have only one massive neutral state– and the mass of the lightest neutral scalar versus the mass of the lightest charged one. Then we have plotted the TS oblique parameters for all the green points to check that T is the most constrained one –for this reason we have not inserted the plots concerning U.

On the contrary for the CP breaking alignment $(ve^{i\omega_1}, ve^{-i\omega_1}, rv)$ we have personalized the plots for reasons that will be clear in the following.

Notice that in all the following discussion, we refer as m_1 (m_2) to the (next-to-the-) lightest neutral state and as m_{ch_1} as the lightest charged mass state.

7.1 CP conserved solutions

7.1.1 The alignment (v, v, v)

In section 4.1 we have redefined the initial 3 doublets in term of the Z_3 surviving symmetry representation: $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$. One combination corresponds to a Z_3 singlet doublet, that behaves like the SM Higgs: it develops a non-vanishing vev, gives rise to a CP even state which we call h_1 and to the three GBs eaten by the gauge bosons. The others two doublets, φ' and φ'', are inert. From these informations we may already figure out what we expect by the numerical scan:

1) when m_{h_1} is the smallest mass, h_1 is the lightest state and corresponds to the SM-like Higgs. As a result, the usual SM mass upper bound applies. On the contrary as long as we do not consider its coupling with the fermions we do not have a model independent lower mass bound. This is due to a combined effect of the CP and Z_3 symmetries: h_1 is CP even and singlet under Z_3, but couplings like $Zh_1\varphi'^0$, $Zh_1\varphi''^0$, $W^-h_1\varphi'^1$ or $W^-h_1\varphi''^1$ are forbidden because of Z_3 and then gauge boson decays cannot constrain the lower mass of h_1.

2) When φ'^0 (φ''^0) is the lightest state, we do not have an upper bound on this state because the couplings $\varphi'^0W^+W^-$ ($\varphi''^0W^+W^-$) is absent. On the contrary we may have a lower bound because couplings like $Z\varphi'^0\varphi''^0$ and $W^-\varphi'^0\varphi''^1$ are allowed.

Combining the two situations sketched in points 1) and 2), we expect neither lower nor upper bounds for the lightest Higgs mass: according to which of the two cases is most favored, we may expect a denser vertical region around $m_1 \sim m_Z/\sqrt{2}$ when the Z decay

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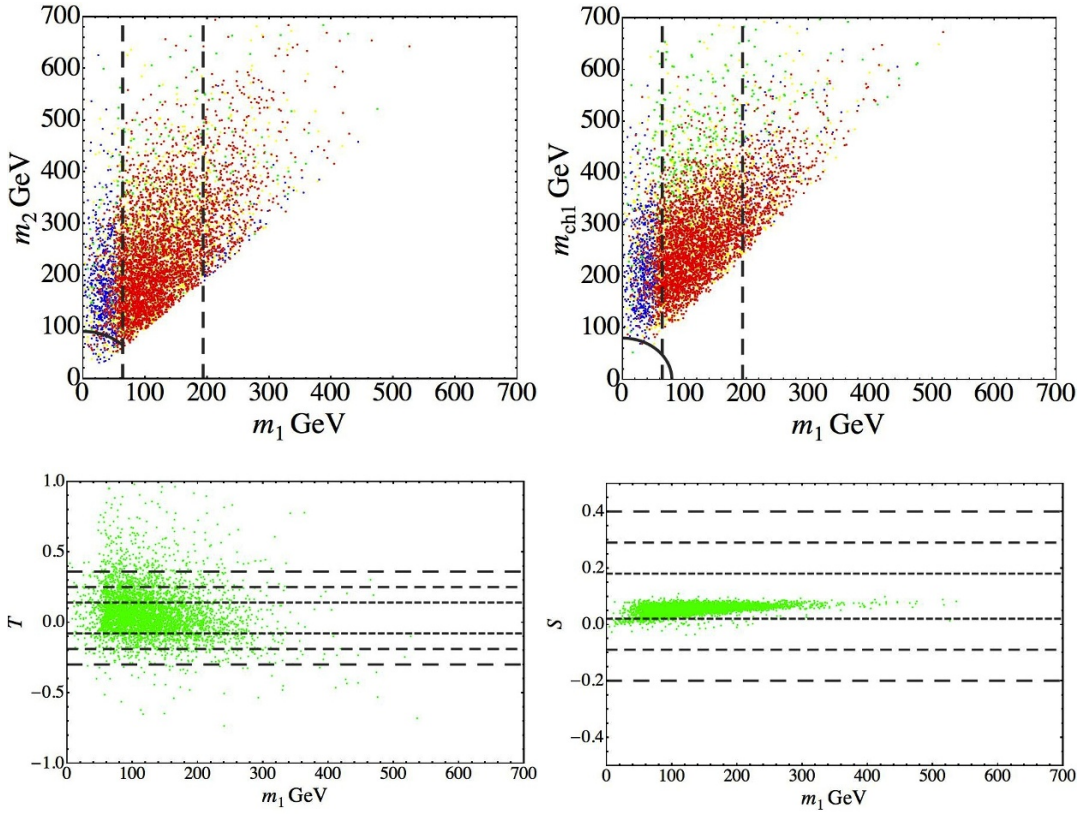

Figure 1. CP conserving alignment (v, v, v): the upper panels show the lightest neutral mass m_1 versus the second lightest neutral mass m_2 and the lightest charged one m_{ch_1} respectively. The gray arc delimits the region below which the Z (W) decay channel opens. On the left plot the arc is only of 45° because $m_2 \geq m_1$. For points below the arc the Z (W) decay may happens. The points allowed stretch in the region close to the border because of the conditions of eq. (6.8). The dashed vertical lines indicates the approximated cuts that occur at $m_1 \sim m_Z/\sqrt{2}$ and $m_1 \sim 194\,\text{GeV}$ according to case 2) and case 1) respectively as explained in the text. The down panels show the contributions to T and S for the G points. The gray dashed lines indicate the experimental values at 3, 2, 1σ level –long,normal,short dashing respectively. The T parameter turns out to be the most constraining one.

channel closes according to eq. (6.8) –case 2) more favored– or a denser vertical line around $m_1 \sim 194\,\text{GeV}$, if the large Higgs mass decay constrain applies –case 1) more favored. Indeed by looking at figure 1 we see that we may find R (allowed) points for very tiny m_1 masses and up to $\sim 500\,\text{GeV}$ when the unitarity bound starts to show its effect. However by looking at the crowded points in figure 1 it seems that case 2) is slightly preferred with respect to case 1). Finally for the G points –those that satisfy the minimum, unitarity and decays conditions– we have compared the contributions to the oblique parameters T and S to see which of the two is more constraining. It turns out to be T, while we have not reported U because its behavior is very similar to S.

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7.1.2 The alignment $(v, 0, 0)$

For what concerns the second natural A_4 minimum, the Z_2 preserving one, things slightly change with respect to the Z_3 surviving case. By section 4.2 we know that as for the Z_3 case we have a SM-like doublet, Z_2 even, that develops the vev, gives rise to a CP even neutral state, h_1, and to the GBs eaten by the gauge bosons. However contrary to the Z_3 case, in the Z_2 minima we have 4 Z_2 odd states, 2 CP even labelled $h_{2,3}$ and 2 CP odd labelled $h_{4,5}$. Moreover the 2 CP even (odd) are degenerate. As done in section 7.1.1 we may sketch what we expect from the numerical analysis:

1) when h_1, the Z_2 even SM-like Higgs, is the lightest we expect the SM Higgs upper bound but no lower bound because the interactions $Zh_1h_{4,5}$ are forbidden by the Z_2 symmetry;

2) when the two lightest are the Z_2 odd degenerate states $h_{2,3}$ –CP even– or $h_{4,5}$ –CP odd– we expect no upper bound. Moreover since they are degenerate we do not expect lower bound too. On the contrary we expect that Z and W decays constrain the third lightest neutral Higgs mass and that of the charged ones.

By looking at figure 2 we see that indeed we have a large amount of points for which $m_1 = m_2$ for values from 0 up to 700 GeV, thus reflecting case 2). Then the points corresponding to case 1) have a sharp cut at $m_1 = 194$ GeV, that rejects many blue points, i.e. those satisfying the unitarity constrain but not the decays one. We have reported also m_1 versus m_3 to check that indeed, when $m_1 \to 0$, m_3 is bounded by m_Z as we expected. Our intuitions are also confirmed by the plot $m_1 - m_{ch_1}$. As for the Z_3 preserving case the most constraining oblique parameter is T.

7.1.3 The alignment (v_1, v_2, v_3) with $\epsilon = 0$, $\lambda_3 + \lambda_4 + \lambda_5 = 0$

In this case we do not have any surviving symmetry which forbid some couplings. However from section 4.3 we know that the conditions $\epsilon = 0$, $\lambda_3 + \lambda_4 + \lambda_5 = 0$ give rise to two extra massless CP even particles. Therefore we expect that

1) when the lightest massive state is CP odd, then its mass is bounded by the Z decay through eq. (6.8);

2) when the lightest massive state is CP even, then its mass could reach smaller values since the Z decay bound would constrain the combination of its mass with the lightest CP odd state mass.

Moreover in both cases we expect the mass of the lightest charged scalar bounded by W decay, according to eq. (6.10), due to its coupling with W and the massless particles.

By figure 3 we see that it seems that case 2) happens very rarely because the cut at $m_1 \sim m_Z$ is in evidence. As for the Z_3 and Z_2 preserving minima the T parameter is the most constraining one.

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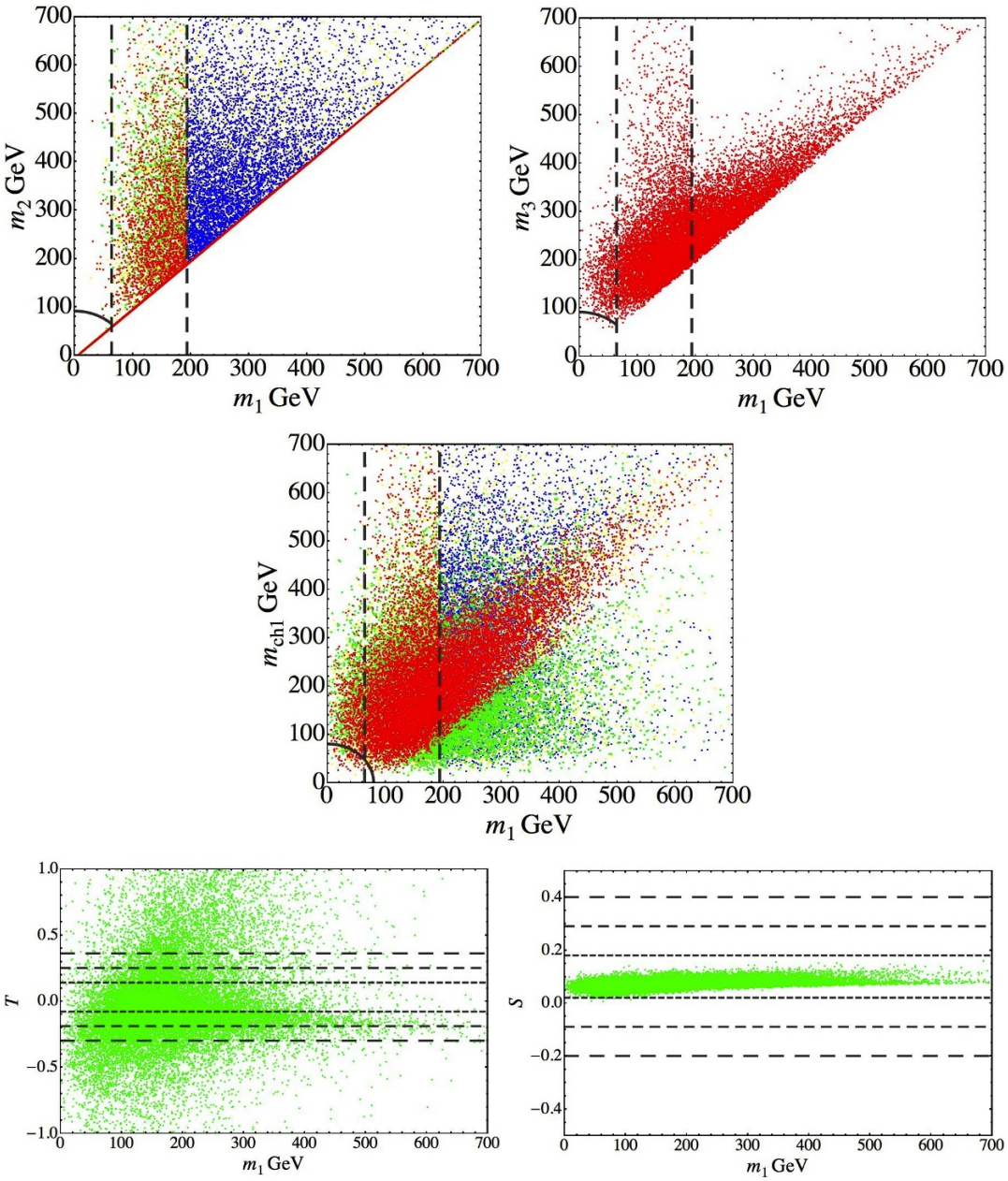

Figure 2. CP conserving alignment $(v, 0, 0)$: the upper panels show m_1 versus m_2 (on the left) and third lightest m_3 (on the right). For the latter we reported only the R points. The central panel shows m_1 versus m_{ch_1}. The gray arc delimits the region below which the Z (W) decay channel opens while the second dashed vertical one the SM-Higgs mass upper bound at 194 GeV. The first dashed vertical line at $m_1 = m_Z/\sqrt{2}$ is reported to help a comparison with the Z_3 preserving case. On the first two plots the arc is only of 45 degrees because $m_{2,3} \geq m_1$. The down panels show the contributions to T and S for the G points. The T parameter turns out to be the most constraining one.

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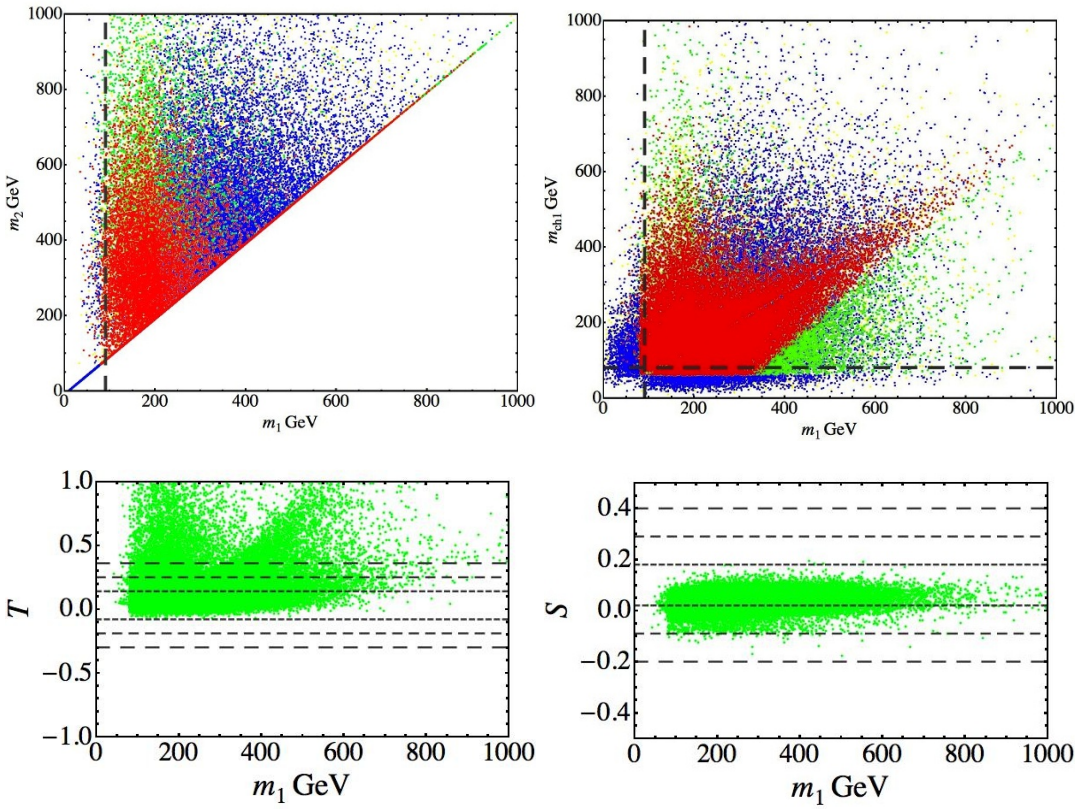

Figure 3. CP conserving alignment (v_1, v_2, v_3): the upper panels show m_1 versus m_2 and m_{ch_1} respectively. The dashed lines at $m_1 = m_Z$ (vertical) and $m_{ch1} = m_W$ (horizontal) delimit the region below which the Z and W decay channels open respectively. The allowed points concentrate close to the borders according to eqs. (6.8)–(6.10). The down panels show the contributions to T and S for the G points. The T parameter turns out to be the most constraining one.

7.2 CP non-conserved solutions

7.2.1 The alignment $(ve^{i\omega_1}, v, 0)$

As for the vacuum alignment (v_1, v_2, v_3) commented in section 7.1.3 the alignment $(ve^{i\omega_1}, v, 0)$ does not preserve any A_4 subgroup. Moreover since even CP is broken any symmetry cannot help us in sketching the behavior we expect. In general any state, having a CP even and a CP odd component, may couple to Z and to another neutral state. However we expect limit situations in which for example CP is almost conserved and the 2 lightest states have almost the same CP parity. Thus for those cases we do not expect any lower bound on m_1 and m_2. On the contrary the coupling between the W with the lightest neutral and the lightest charged scalars does not go to zero when CP is almost restored. Then we expect that the quantity $m_1^2 + m_{ch_1}^2$ is bounded by the W decay (figure 4). For what concerns the upper bound on the lighetst neutral mass state we do not expect any clear cut because we may not identify a SM-like Higgs.

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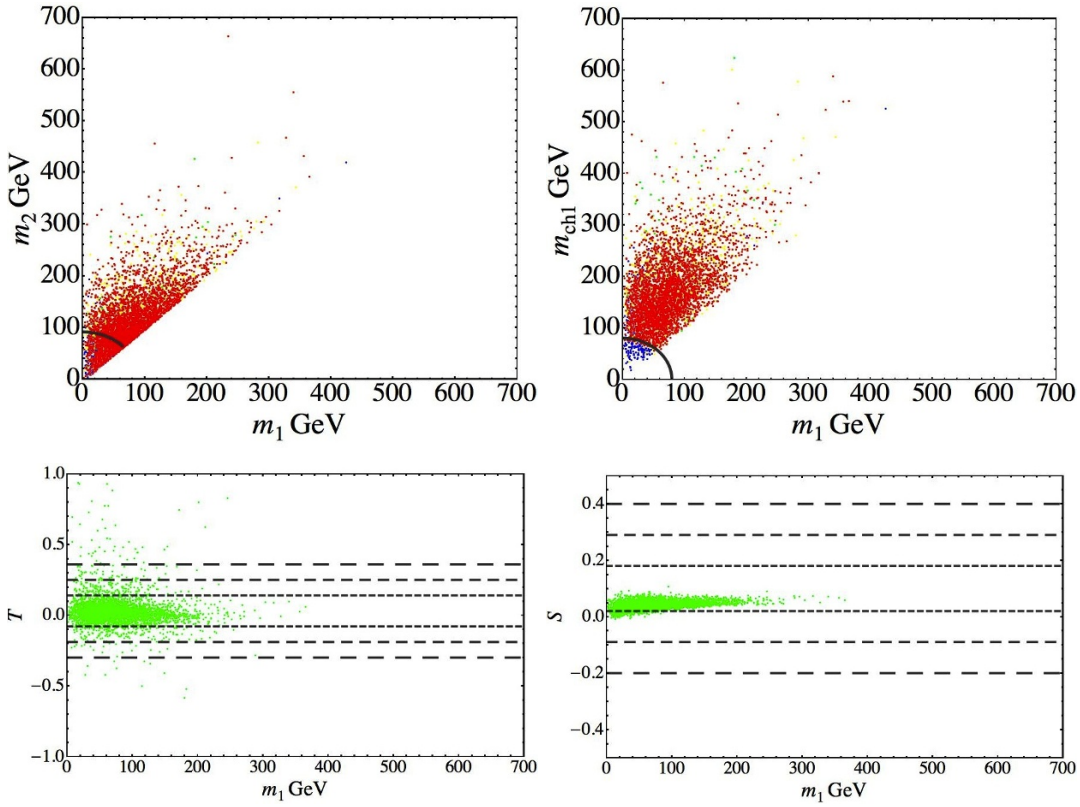

Figure 4. CP no conserving alignment $(ve^{i\omega_1}, v, 0)$: as in the previous figure the upper panels show m_1 versus m_2 and m_{ch_1} respectively. In the plot on the right, the effect of the W decay constraint on $m_1^2 + m_{ch_1}^2$ is clear by looking at the B points. The down panels show the contributions to T and S for the G points. The T parameter turns out to be the most constraining one.

7.2.2 The alignment $(ve^{i\omega_1}, ve^{-i\omega_1}, rv)$ case i)

In section 5.2.1 we have seen that the alignment $(ve^{i\omega_1}, ve^{-i\omega_1}, rv)$ with the constrains $\lambda_5 = 0$, $\lambda_4 = -\lambda_3$, gives rise to 4 extra GBs and only to one neutral state. The simplicity of the analytical expressions for the three no vanishing masses ensures that the boundness constrain $\lambda_1 > 0$ in addition to $\lambda_3 > 0$ give positive masses. Thus in this case the Y points are superfluous. As in the previous cases, we expect the B points to be similar to the Y ones, because we choose our parameters centered in 1 in order not to have problems with unitarity. In conclusion, for this case only the G and R points are interesting. Moreover we expect that the most stringent bound is given by the decay constrains and not by TSU: massless particles give a small contribution to the oblique parameters and due to the limited number of new particles (2 charged degenerate scalars) TSU should not deviate too much by the SM values. Indeed in figure 5 it is shown that the oblique parameters at 3 σ level do not constrain at all the G points. For this reason we reported only the R points in the upper panel of figure 5. By looking at the plot $m_1 - m_{ch_1}$ in figure 5 we see that with respect to the minima so far analyzed we have much less points and that as expected there are cuts in correspondence of m_Z and m_W.

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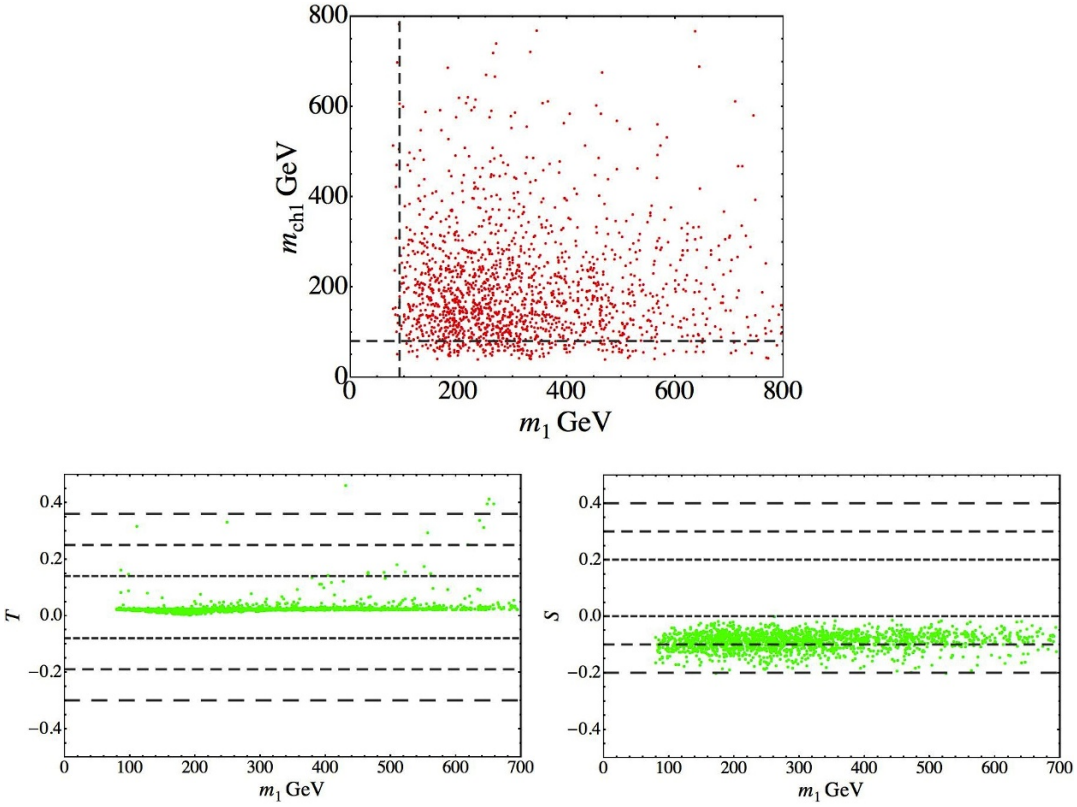

Figure 5. CP no conserving alignment $(ve^{i\omega_1}, ve^{-i\omega_1}, rv)$ case i): the upper panel show m_1 versus m_{ch_1}. Only the R points are reported. The down panels show the contributions to T and S for the G points. For this specific case the TSU oblique parameter constrain is irrelevant compared to the decay one.

In conclusion, the solutions for the alignment $(ve^{i\omega_1}, ve^{-i\omega_1}, rv)$ with $\lambda_5 = 0$, $\lambda_4 = -\lambda_3$ are not easy to find, but the Higgs phenomenology does not completely rule out this vacuum configuration. We could introduce a weight to estimate how much a solution is stable or fine-tuned but this goes over the purposes of this work. We expect that this situation with 4 extra massless particles could be very problematic when considering the model dependent constraints [17].

7.2.3 $(ve^{i\omega_1}, ve^{-i\omega_1}, rv)$ case ii)

In the analytical discussion done in section 5.2.2 we have seen that at least in the special limit $r \sim 0$ ($r \sim 1$ and $r >> 1$) we expect the presence of one (two) very light particles. From all the numerical scans we performed we found out that solutions for the vacuum alignment $(ve^{i\omega_1}, ve^{-i\omega_1}, rv)$ with the constraints of case ii) are very difficult to be found. Moreover from figure 6 we see that for any value of r the two lightest states are always very light, thus confirming our rough analytical approximations. Indeed both m_1 and m_2 are lighter then we expected –especially m_2 for $r \sim 0$– thus indicating that some cancellations have to occur to give all the masses greater then 0. This supports the difficulty to find solutions, difficulty that cannot to be ascribed to any constrain we imposed, because even

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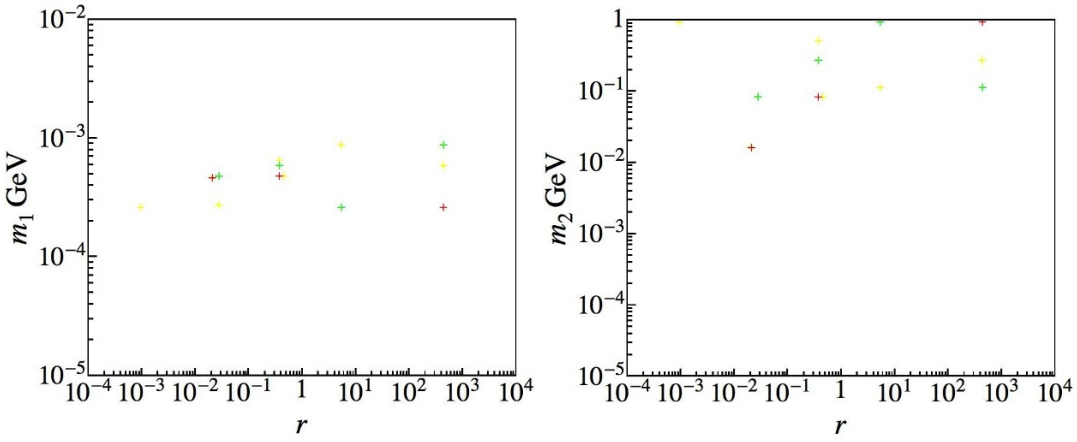

Figure 6. CP no conserving alignment $(ve^{i\omega_1}, ve^{-i\omega_1}, rv)$, case *ii*): the panels show m_1 (on the left) and m_2 (on the right) versus r. The number of points is small, but the interesting information is the order of magnitude of the masses.

in presence of 4 additional GBs as in section 7.2.2 we found out a significant larger number of solutions.

The presence of a single R point in figure 6 is not statistically relevant, but more interesting is the order of magnitude of $m_{1,2}$: even in case *ii*) we expect that the alignment $(ve^{i\omega_1}, ve^{-i\omega_1}, rv)$ may present serious problems once we add model dependent constraints [17].

8 Conclusions

Flavour models based on non-Abelian discrete symmetries under which the SM scalar doublet (and its replicants) transforms non trivially are quite appealing for many reasons. First of all there are no new physics scales, since the flavour and the EW symmetries are simultaneously broken. Furthermore this kind of models are typically more minimal with respect to the ones in which the flavour scale is higher than the EW one: in particular the vacuum configuration is simpler and the number of parameters is lower. We then expect an high predictive power and clear phenomenological signatures in processes involving both fermions and scalars.

Due to the restricted number of parameters and the abundance of sensitive observables in these models, there are many constraints to analyze: the most stringent ones arise by FCNC and LFV processes [17] but even Higgs phenomenology put several constraints on this class of models. The impact of the symmetry breaking in cosmology has been studied in [37].

In this paper we focussed on the A_4 discrete group, but this analysis can be safely generalized for any non-Abelian discrete symmetry. We consider three copies of the SM Higgs fields, that transform as a triplet of A_4. This setting has already been chosen in several papers [10–13] due to the simple vacuum alignment mechanism.

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We have considered all the possible vacuum configurations allowed by the $A_4 \times SM$ scalar potential. These configurations can either conserve or violate CP. For all of them we have considered only model independent constraints, related to the Higgs-gauge boson Lagrangian, and postponing the model dependent analysis to an accompanying paper [17]. The first model independent constraint comes from the partial wave unitarity for the neutral two-particle amplitudes, which puts upper bounds on the scalar masses. Then we have explained how the light scalar mass region can be constrained considering the gauge boson decays. Moreover we have seen how to put an upper bound on the lightest neutral state mass considering the Higgs decay channel $h \to W^+W^-$. Finally the most stringent bounds arise by the oblique parameters TSU.

We have shown that the Higgs-gauge boson model independent analysis can be used to study the parameter space of the difference vacuum configurations. Among the possible solutions which minimize the scalar potential, only one is ruled out due to the presence of tachyonic states. Furthermore, some other configurations may be obtained only by tuning the potential parameters, giving rise to scalar spectrums characterized by very light or even massless particles. Finally, for the remaining ones, we find that they may share common features and this increases the difficulty in discriminating among them. Nevertheless, the model independent approach restricts in a non trivial way the parameter space. In conclusion, we underline that more constraining results can be found considering specific realizations which adopt the different vacuum configurations: we present this analysis in [17].

Note added. While completing this paper we received ref. [38], where the scalar potential with three copies of the SM Higgs doublet transforming as a triplet of A_4 is also studied. We stress the differences between this work an ours. Firstly, in [38], it is assumed that no new CP phases appear in the Higgs vevs, while we take this important possibility into account. Secondly, ref. [38] discusses three interesting, but rather arbitrary vacua, where our analysis exhausts all possible vacua configurations. Lastly, a complete phenomenological study is missing in [38].

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A Analytical formulae for TSU parameters

In this appendix we provide a sort of *translator* from the papers [30, 33] to our notations and furnish the formulae we have used when different from their.

Reminding their notation we are in the case in which $n_d = 3$ and $n_n, n_c = 0$ so we do not have the matrices $\mathcal{T}$ and $\mathcal{R}$. Then we have

$$\begin{aligned} \mathcal{U} &\to S \\ \mathrm{Re}\mathcal{V}_{ki} &\to U_{ki}\,, \\ \mathrm{Im}\mathcal{V}_{ki} &\to U_{k+3i}\,, \\ \omega_k &\to f_k e^{i\omega_k}\,. \end{aligned} \tag{A.1}$$

Moreover they put the GBs as first mass eigenstates while we put them as the last ones and contrary to them we use the standard definition for the photon.

We have rewritten they expression for

$$\frac{A(I,J,Q)-A(I,J,0)}{Q} = \begin{cases} dA(I,J) & \text{for } I\neq 0 \text{ and/or } J\neq 0\,, \\ \dfrac{QF(Q)}{Q} \sim \dfrac{1}{48\pi^2}\log Q & \text{for } I=J=0 \text{ since } A(0,0,0)=0\,. \end{cases} \tag{A.2}$$

For the first row of eq. (A.2) we have used

$$A(I,J,Q) \simeq A(I,J,0) + Q\frac{\partial A(I,J,Q)}{\partial Q}\bigg|_{Q=0} = A(I,J,0) + Q\,dA(I,J) \tag{A.3}$$

with

$$dA(I,J) = \begin{cases} \dfrac{1}{288(I-J)^3\pi^2}\left[I^3+9JI^2+6(I-3J)\log(I)I^2-9J^2I-J^3+6(3I-J)J^2\log(J)\right] & \\ \qquad\qquad \text{for } I,J\neq 0, I\neq J\,, & \\ \dfrac{1}{288\pi^2}(1+6Log[I]) \qquad \text{for } J=0\,, & \\ \dfrac{1}{48\pi^2}(1+\log[I]) \qquad \text{for } I=J\,. & \end{cases} \tag{A.4}$$

The function $\bar{A}(I,J,Q)$ enters only in the loops in which a gauge boson and a scalar run, so we have always $J=Q$ when computing the quantity

$$\frac{\bar{A}(I,J,Q)-\bar{A}(I,J,0)}{Q} = \bar{dA}(I,J)\,. \tag{A.5}$$

As a result, for this function, it does not make sense considering the case $I=J=0$ being $J=Q=m_V^2$ the gauge boson mass. We found

$$\bar{dA}(I,Q) = \begin{cases} \dfrac{1}{8(I-Q)^3\pi^2}\left[Q\left(-I^2+2Q\log(I)I-2Q\log(Q)I+Q^2\right)\right] & \text{for } I\neq Q, I\neq 0\,, \\ \sim 0 \qquad \text{for } I=0\,, & \\ \sim 0 \qquad \text{for } I=Q\,. & \end{cases} \tag{A.6}$$

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References

[1] G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A.M. Rotunno, *Hints of $\theta_{13} > 0$ from global neutrino data analysis*, *Phys. Rev. Lett.* **101** (2008) 141801 [arXiv:0806.2649] [SPIRES].

[2] T. Schwetz, M.A. Tortola and J.W.F. Valle, *Three-flavour neutrino oscillation update*, *New J. Phys.* **10** (2008) 113011 [arXiv:0808.2016] [SPIRES].

[3] M. Maltoni and T. Schwetz, *Three-flavour neutrino oscillation update and comments on possible hints for a non-zero θ_{13}*, PoS(IDM2008)072 [arXiv:0812.3161] [SPIRES].

[4] G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A.M. Rotunno, *What we (would like to) know about the neutrino mass*, in the proceedings of *4th International Workshop on Neutrino Oscillations in Venice: Ten Years after the Neutrino Oscillations*, Venice Italy, 15-18 Apr 2008, pp. 21–28 [arXiv:0809.2936] [SPIRES].

[5] M.C. Gonzalez-Garcia, M. Maltoni and J. Salvado, *Updated global fit to three neutrino mixing: status of the hints of $\theta_{13} > 0$*, *JHEP* **04** (2010) 056 [arXiv:1001.4524] [SPIRES].

[6] A. Gando et al., *Constraints on θ_{13} from A Three-Flavor Oscillation Analysis of Reactor Antineutrinos at KamLAND*, arXiv:1009.4771 [SPIRES].

[7] P.F. Harrison, D.H. Perkins and W.G. Scott, *Tri-bimaximal mixing and the neutrino oscillation data*, *Phys. Lett.* **B 530** (2002) 167 [hep-ph/0202074] [SPIRES].

[8] Z.-z. Xing, *Nearly tri-bimaximal neutrino mixing and CP-violation*, *Phys. Lett.* **B 533** (2002) 85 [hep-ph/0204049] [SPIRES].

[9] G. Altarelli and F. Feruglio, *Discrete Flavor Symmetries and Models of Neutrino Mixing*, *Rev. Mod. Phys.* **82** (2010) 2701 [arXiv:1002.0211] [SPIRES].

[10] E. Ma and G. Rajasekaran, *Softly broken A_4 symmetry for nearly degenerate neutrino masses*, *Phys. Rev.* **D 64** (2001) 113012 [hep-ph/0106291] [SPIRES].

[11] L. Lavoura and H. Kuhbock, *A_4 model for the quark mass matrices*, *Eur. Phys. J.* **C 55** (2008) 303 [arXiv:0711.0670] [SPIRES].

[12] S. Morisi and E. Peinado, *An A_4 model for lepton masses and mixings*, *Phys. Rev.* **D 80** (2009) 113011 [arXiv:0910.4389] [SPIRES].

[13] E. Ma, *Quark and Lepton Flavor Triality*, *Phys. Rev.* **D 82** (2010) 037301 [arXiv:1006.3524] [SPIRES].

[14] E. Ma, *Neutrino Tribimaximal Mixing from A_4 Alone*, *Mod. Phys. Lett.* **A 25** (2010) 2215 [arXiv:0908.3165] [SPIRES].

[15] M. Hirsch, S. Morisi, E. Peinado and J.W.F. Valle, *Discrete dark matter*, *Phys. Rev.* **D 82** (2010) 116003 [arXiv:1007.0871] [SPIRES].

[16] D. Meloni, S. Morisi and E. Peinado, *Neutrino phenomenology and stable dark matter with A_4*, *Phys. Lett.* **B 697** (2011) 339 [arXiv:1011.1371] [SPIRES].

[17] R.d.A. Toorop, F. Bazzocchi, L. Merlo and A. Paris, *Constraining Flavour Symmetries At The EW Scale II: The Fermion Processes*, arXiv:1012.2091 [SPIRES].

JHEP03(2011)035

[18] M. Hamermesh, *Group Theory and Its Application to Physical Problems*, Addison-Wesley Publishing Company, Inc., Reading MA U.S.A. (1962).

[19] J.F. Cornwell, *Group Theory in Physics: An Introduction*, Academic Press, San Diego CA U.S.A. (1997).

[20] G. Altarelli and F. Feruglio, *Tri-Bimaximal Neutrino Mixing, A_4 and the Modular Symmetry*, *Nucl. Phys.* **B 741** (2006) 215 [hep-ph/0512103] [SPIRES].

[21] J.F. Gunion and H.E. Haber, *Conditions for CP-violation in the general two-Higgs-doublet model*, *Phys. Rev.* **D 72** (2005) 095002 [hep-ph/0506227] [SPIRES].

[22] M.J.G. Veltman, *Second Threshold in Weak Interactions*, *Acta Phys. Polon.* **B 8** (1977) 475 [SPIRES].

[23] B.W. Lee, C. Quigg and H.B. Thacker, *Weak Interactions at Very High-Energies: The Role of the Higgs Boson Mass*, *Phys. Rev.* **D 16** (1977) 1519 [SPIRES].

[24] B.W. Lee, C. Quigg and H.B. Thacker, *The Strength of Weak Interactions at Very High-Energies and the Higgs Boson Mass*, *Phys. Rev. Lett.* **38** (1977) 883 [SPIRES].

[25] R. Casalbuoni, D. Dominici, F. Feruglio and R. Gatto, *Tree-level unitarity violation for large scalar mass in multi-Higgs extensions of the standard model*, *Nucl. Phys.* **B 299** (1988) 117 [SPIRES].

[26] Particle Data Group collaboration, K. Nakamura et al., *Review of particle physics*, *J. Phys.* **G 37** (2010) 075021 [SPIRES].

[27] LEP Working Group for Higgs boson searches collaboration, R. Barate et al., *Search for the standard model Higgs boson at LEP*, *Phys. Lett.* **B 565** (2003) 61 [hep-ex/0306033] [SPIRES].

[28] CDF and D0 collaboration, *Combined CDF and DZero Upper Limits on Standard Model Higgs-Boson Production with up to 4.2 fb^{-1} of Data*, arXiv:0903.4001 [SPIRES].

[29] B. Holdom and J. Terning, *Large corrections to electroweak parameters in technicolor theories*, *Phys. Lett.* **B 247** (1990) 88 [SPIRES].

[30] M.E. Peskin and T. Takeuchi, *A New constraint on a strongly interacting Higgs sector*, *Phys. Rev. Lett.* **65** (1990) 964 [SPIRES].

[31] M. Golden and L. Randall, *Radiative corrections to electroweak parameters in technicolor theories*, *Nucl. Phys.* **B 361** (1991) 3 [SPIRES].

[32] A. Dobado, D. Espriu and M.J. Herrero, *Chiral Lagrangians as a tool to probe the symmetry breaking sector of the SM at LEP*, *Phys. Lett.* **B 255** (1991) 405 [SPIRES].

[33] M.E. Peskin and T. Takeuchi, *Estimation of oblique electroweak corrections*, *Phys. Rev.* **D 46** (1992) 381 [SPIRES].

[34] I. Maksymyk, C.P. Burgess and D. London, *Beyond S, T and U*, *Phys. Rev.* **D 50** (1994) 529 [hep-ph/9306267] [SPIRES].

[35] W. Grimus, L. Lavoura, O.M. Ogreid and P. Osland, *The oblique parameters in multi-Higgs-doublet models*, *Nucl. Phys.* **B 801** (2008) 81 [arXiv:0802.4353] [SPIRES].

[36] W. Grimus, L. Lavoura, O.M. Ogreid and P. Osland, *A precision constraint on multi-Higgs-doublet models*, *J. Phys.* **G 35** (2008) 075001 [arXiv:0711.4022] [SPIRES].

[37] M.Y. Khlopov, *Cosmoparticle physics*, World Scientific, Singapore (1999).

[38] A.C.B. Machado, J.C. Montero and V. Pleitez, *Three-Higgs-doublet model with A_4 symmetry*, *Phys. Lett.* **B 697** (2011) 318 [arXiv:1011.5855] [SPIRES].

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