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(Received 24 June 2003; published 30 September 2003)

The simultaneous exchange of two pseudoscalars between fermions leads to a spin-independent force between macroscopic objects. Previous work has demonstrated that one can combine this interaction with tests of the weak equivalence principle, gravitational inverse square law, and studies of laser beam propagation in magnetic fields, to set significant new constraints on the Yukawa couplings of massless pseudoscalars to nucleons. Here we extend these results to massive pseudoscalars, and derive new constraints which relate the strengths of these couplings to the pseudoscalar mass.

DOI: 10.1103/PhysRevD.68.062002 PACS number(s): 04.80.Cc, 14.80.−j

I. INTRODUCTION

In a recent series of papers [1–3], it was shown that gravity experiments testing the validity of the weak equivalence principle (WEP) [3,4], or the inverse square law (ISL) [5], can be used to constrain the Yukawa couplings g_p^2 and g_n^2 of light pseudoscalars to nucleons. Such constraints come about because the simultaneous exchange of two light pseudoscalars leads to a long-range spin-independent potential which, if present in nature, would give rise to apparent deviations from the predictions of Newtonian gravity. From a practical point of view a ‘‘light’’ pseudoscalar is one whose mass m is sufficiently small that its Compton wavelength $\hbar/mc$ is larger than the characteristic size of the experimental apparatus.

Motivated by Refs. [1] and [2], Massó [6] noted that the limits on g_p^2 and g_n^2 could be improved by combining the analysis from Refs. [1,2] with a limit on g_p^2 obtained from an experiment [7] studying laser beam propagation in a magnetic field. The principle behind this experiment is that if a pseudoscalar field ϕ existed, then the coupling of ϕ to two photons would induce effects such as an optical rotation in the laser beam. Assuming that the mass of the light pseudoscalar satisfies $m<10^{-3}$ eV ($\lambda>0.02$ cm), the absence of such effects leads to a bound on g_p^2 given by

$$\frac{g_p^2}{4\pi}<1.7\times 10^{-9}, \tag{1}$$

which is more stringent than limits obtained from current WEP and ISL experiments [3]. Massó then combined Eq. (1) with the constraints on g_p^2 and g_n^2 obtained from the test of the WEP given in Ref. [1] to obtain the bound on the pseudoscalar coupling to neutrons,

$$\frac{g_n^2}{4\pi}<6.8\times 10^{-8}. \tag{2}$$

However, unlike the proton constraint given by Eq. (1), Eq. (2) is only valid for massless pseudoscalar exchange since the analysis of Ref. [1] (and subsequent work [2,3]) assumed $m=0$. The object of the present paper is to show how the results of Refs. [1–3] can be extended to the case of massive pseudoscalars. We will then combine these limits with Eq. (1) to generalize the results from Ref. [6] to obtain new limits on the coupling of massive pseudoscalars to neutrons.

II. CONSTRAINTS FROM GRAVITY EXPERIMENTS

A. Phenomenology

We begin by assuming that the pseudoscalar couples to fermions via the Lagrangian density

$$\mathcal{L}(x)=ig\overline{\psi}(x)\gamma_5\psi(x)\phi(x), \tag{3}$$

where $\phi(x)$ is the field operator for a pseudoscalar of mass m, and $\psi(x)$ denotes either a proton (p), electron (e), or neutron (n) of mass M_p, M_e, or M_n respectively. (We assume that $\hbar=c=1$.) With this coupling, the exchange of a single pseudoscalar between two fermions leads to the familiar spin-dependent potential [8],

$$V^{(2)}(\vec{r};\vec{\sigma}_1,\vec{\sigma}_2)=\frac{g^2}{16\pi M^2}\left\{(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r})\left[\frac{m^2}{r}+\frac{3m}{r^2}+\frac{3}{r^3}\right]-(\vec{\sigma}_1\cdot\vec{\sigma}_2)\left[\frac{m}{r^2}+\frac{1}{r^3}\right]\right\}e^{-mr}. \tag{4}$$

Here $r=|\vec{r}|=|\vec{r}_1-\vec{r}_2|$ is the distance between fermions 1 and 2, M is the fermion mass (M_p, M_e, or M_n), $(1/2)\vec{\sigma}_{1,2}$ are the fermion spins, and we have dropped a term proportional to $\delta^3(r)$. A number of careful experiments incorporating polarized test masses have used $V^{(2)}$ to set stringent limits on the couplings of light pseudoscalars to electrons, $g_e^2/4\pi\lesssim 10^{-16}$ [9]. However, constraints on the couplings to nucleons from the same experiments are many orders of magnitude weaker [1] and require model-dependent calculations.

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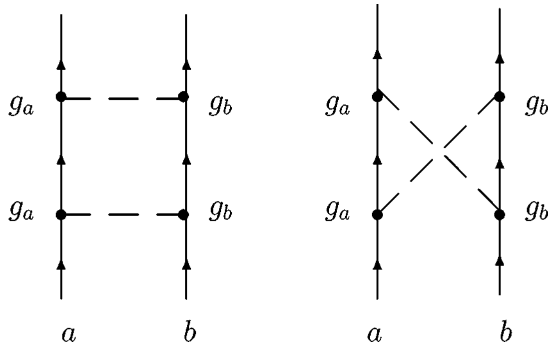

FIG. 1. Contributions to the spin-independent long-range interaction of fermions *a* and *b* arising from two-pseudoscalar-exchange. The solid lines are fermions and the dashed lines denote the pseudoscalars.

In recent years, gravity experiments testing the WEP and gravitational ISL have obtained remarkable sensitivity using unpolarized matter. The leading-order pseudoscalar interaction between unpolarized bodies arises in $O(g^4)$ from the simultaneous exchange of two pseudoscalars between two fermions (Fig. 1). The resulting spin-independent potential is [10,11]

$$V_{ab}^{(4)}(r)=\frac{-g_a^2 g_b^2}{32\pi^3 M_a M_b}\left(\frac{m}{r^2}\right)K_1(2mr), \tag{5}$$

where a and b may each denote p, e, or n, and $K_1(x)$ is the modified Bessel function [12]. (Here we assume $r \gg 1/M_{a,b}$.) In the limit $m\to 0$, Eq. (5) reduces to [13],

$$V_{ab}^{(4)}(r)=\frac{-g_a^2 g_b^2}{64\pi^3 M_a M_b}\frac{1}{r^3}, \tag{6}$$

where we have used $K_1(x)\simeq 1/x$ when $x\ll 1$ [12]. Let us now generalize Eq. (5) to the case of two macroscopic objects 1 and 2 of masses M_1 and M_2 containing Z_i (N_i) [$i=1,2$] protons (neutrons) respectively. If we include only nucleon-nucleon interactions [14], the potential energy W_{12} between the two objects arising from Eq. (5) can be written as

$$W_{12}=\frac{-M_1M_2}{32\pi^3M^2m_H^2\mathcal{V}_1\mathcal{V}_2\lambda}\left(g_p^2\frac{Z_1}{\mu_1}+g_n^2\frac{N_1}{\mu_1}\right)\left(g_p^2\frac{Z_2}{\mu_2}+g_n^2\frac{N_2}{\mu_2}\right)\int d^3r_1\int d^3r_2\frac{K_1(2m|\vec{r}_1-\vec{r}_2|)}{|\vec{r}_1-\vec{r}_2|^2}. \tag{7}$$

Here $\mathcal{V}_i$ is the volume of body i, $\mu_i\equiv m_i/m_H$, $m_H=m({}_1\mathrm{H}^1)$ is the mass of atomic hydrogen [15], $M=(M_p+M_n)/2$, and g_p (g_n) is the proton (neutron) pseudoscalar coupling constant. The values of Z/μ and N/μ for the first 92 elements can be obtained from Table 2.1 of Ref. [15]. If object 2 is small relative to object 1 (as is often the case in tests of the WEP), Eq. (7) reduces to

$$W_{12}=\frac{-M_1M_2}{32\pi^3M^2m_H^2\mathcal{V}_1\lambda}\left(g_p^2\frac{Z_1}{\mu_1}+g_n^2\frac{N_1}{\mu_1}\right)\left(g_p^2\frac{Z_2}{\mu_2}+g_n^2\frac{N_2}{\mu_2}\right)\int d^3r_1\frac{K_1(2m|\vec{r}_1-\vec{r}_2|)}{|\vec{r}_1-\vec{r}_2|^2}. \tag{8}$$

The force on object 2 is then

$$\vec{F}_2=-\vec{\nabla}_2W_{12}=\vec{\mathcal{F}}(\vec{r}_2,\lambda)\left(\frac{M_1M_2}{m_H^2}\right)\left(g_p^2\frac{Z_1}{\mu_1}+g_n^2\frac{N_1}{\mu_1}\right)\times\left(g_p^2\frac{Z_2}{\mu_2}+g_n^2\frac{N_2}{\mu_2}\right), \tag{9}$$

where

$$\vec{\mathcal{F}}(\vec{r}_2,\lambda)=\frac{3}{32\pi^3M^2\mathcal{V}_1\lambda}\int d^3r_1\left[K_1(2|\vec{r}_1-\vec{r}_2|/\lambda)+\frac{2}{3}\left(\frac{|\vec{r}_1-\vec{r}_2|}{\lambda}\right)K_0(2|\vec{r}_1-\vec{r}_2|/\lambda)\right]\frac{(\vec{r}_1-\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|^4}, \tag{10}$$

and $\lambda=1/m$ is the range of the one-pseudoscalar exchange interaction. If, on the other hand, object 2 cannot be considered small, then Eq. (9) generalizes to

$$\vec{F}_2=\frac{3}{32\pi^3M^2\mathcal{V}_1\mathcal{V}_2\lambda}\left(\frac{M_1M_2}{m_H^2}\right)\left(g_p^2\frac{Z_1}{\mu_1}+g_n^2\frac{N_1}{\mu_1}\right)\left(g_p^2\frac{Z_2}{\mu_2}+g_n^2\frac{N_2}{\mu_2}\right)\int d^3r_1\int d^3r_2\left[K_1(2|\vec{r}_1-\vec{r}_2|/\lambda)+\frac{2}{3}\left(\frac{|\vec{r}_1-\vec{r}_2|}{\lambda}\right)K_0(2|\vec{r}_1-\vec{r}_2|/\lambda)\right]\frac{(\vec{r}_1-\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|^4}. \tag{11}$$

In a typical WEP experiment, object 1 is an extended source toward which the relative accelerations of two small samples 2 and 2′ (with masses M_2 and $M_{2'}$) are measured. It then follows from Eq. (9) that the acceleration difference $\Delta\vec{a}_{2-2'}=\vec{a}_2-\vec{a}_{2'}$ arising from the two-pseudoscalar exchange potential is

$$\Delta\vec{a}_{2-2'}=\vec{\mathcal{F}}(\vec{r}_2,\lambda)\left(\frac{M_1}{m_H^2}\right)\left(g_p^2\frac{Z_1}{\mu_1}+g_n^2\frac{N_1}{\mu_1}\right)\left[g_p^2\Delta\left(\frac{Z}{\mu}\right)_{2-2'}+g_n^2\Delta\left(\frac{N}{\mu}\right)_{2-2'}\right], \tag{12}$$

where $\Delta(Z/\mu)_{2-2'}=Z_2/\mu_2-Z_{2'}/\mu_{2'}$ and $\Delta(N/\mu)_{2-2'}=N_2/\mu_2-N_{2'}/\mu_{2'}$. The limits on $\Delta\vec{a}_{2-2'}$ obtained in a typical WEP experiment can be combined with an evaluation of $\vec{\mathcal{F}}(\vec{r}_2,\lambda)(M_1/m_H^2)$ to constrain the couplings $g_p^2(\lambda)$ and $g_n^2(\lambda)$ using

TABLE I. Geometric factor $\Delta a_{2-2'}/\mathcal{A}(\lambda)$, and limits on pseudoscalar couplings, from the WEP experiment by Smith *et al.* [3] for the special cases $g_p^2 = g_n^2$, $g_p^2 \gg g_n^2$, and $g_n^2 \gg g_p^2$ obtained from Eqs. (19), (21), and (23). The final column labeled $g_n^2/4\pi$ (Massó) represents the limit on the coupling to neutrons obtained when combining the Smith results with Massó's constraint on protons given by Eq. (25).

λ (cm)	$\Delta a_{2-2'}/\mathcal{A}(\lambda)$	$\frac{g_{p,n}^2}{4\pi}$ $(g_p^2=g_n^2)$	$\frac{g_p^2}{4\pi}$ $(g_p^2 \gg g_n^2)$	$\frac{g_n^2}{4\pi}$ $(g_n^2 \gg g_p^2)$	$\frac{g_n^2}{4\pi}$ (Massó)
2	1.16×10^{-10}	2.80×10^{-5}	5.65×10^{-6}	4.52×10^{-6}	4.52×10^{-6}
3	7.02×10^{-12}	6.89×10^{-6}	1.39×10^{-6}	1.11×10^{-6}	1.11×10^{-6}
4	1.61×10^{-12}	3.30×10^{-6}	6.66×10^{-7}	5.33×10^{-7}	5.33×10^{-7}
5	6.47×10^{-13}	2.09×10^{-6}	4.22×10^{-7}	3.38×10^{-7}	3.38×10^{-7}
6	3.46×10^{-13}	1.53×10^{-6}	3.09×10^{-7}	2.47×10^{-7}	2.47×10^{-7}
7	2.19×10^{-13}	1.22×10^{-6}	2.46×10^{-7}	1.97×10^{-7}	1.97×10^{-7}
10	9.35×10^{-14}	7.95×10^{-7}	1.61×10^{-7}	1.28×10^{-7}	1.29×10^{-7}
15	4.71×10^{-14}	5.64×10^{-7}	1.14×10^{-7}	9.12×10^{-8}	9.15×10^{-8}
20	3.33×10^{-14}	4.74×10^{-7}	9.58×10^{-8}	7.66×10^{-8}	7.70×10^{-8}
50	1.83×10^{-14}	3.52×10^{-7}	7.10×10^{-8}	5.68×10^{-8}	5.72×10^{-8}
100	1.56×10^{-14}	3.25×10^{-7}	6.56×10^{-8}	5.25×10^{-8}	5.28×10^{-8}
200	1.48×10^{-14}	3.16×10^{-7}	6.39×10^{-8}	5.11×10^{-8}	5.15×10^{-8}
500	1.45×10^{-14}	3.13×10^{-7}	6.32×10^{-8}	5.06×10^{-8}	5.09×10^{-8}
1000	1.44×10^{-14}	3.12×10^{-7}	6.30×10^{-8}	5.04×10^{-8}	5.08×10^{-8}
2000	1.44×10^{-14}	3.12×10^{-7}	6.30×10^{-8}	5.04×10^{-8}	5.08×10^{-8}

$$\left(g_p^2\frac{Z_1}{\mu_1}+g_n^2\frac{N_1}{\mu_1}\right)\left|g_p^2\Delta\left(\frac{Z}{\mu}\right)_{2-2'}+g_n^2\Delta\left(\frac{N}{\mu}\right)_{2-2'}\right|\leq\frac{\Delta a_{2-2'}}{\mathcal{A}(\lambda)},\tag{13}$$

where $\Delta a_{2-2'}=|\Delta\vec{a}_{2-2'}|$, $\vec{\mathcal{A}}(\lambda)\equiv\vec{\mathcal{F}}(\vec{r}_2,\lambda)(M_1/m_H^2)$, and $\mathcal{A}(\lambda)=|\vec{\mathcal{A}}(\lambda)|$. This generalizes the results of Ref. [1] which assumed $\lambda=\infty$.

B. Limits from tests of the WEP and ISL

Presently, the best limits on pseudoscalars based on a test of the WEP come from the experiment by Smith *et al.* [3] which uses a 3 ton ^{238}U source ($Z_1/\mu_1=0.3881050$, $N_1/\mu_1=0.6159057$), and test masses consisting of Cu ($Z_2/\mu_2=0.4599360$, $N_2/\mu_2=0.5490145$) and a Pb alloy ($Z_{2'}/\mu_{2'}=0.4006829$, $N_{2'}/\mu_{2'}=0.6073130$) to obtain $|\Delta a_{2-2'}|\leq2.9\times10^{-13}$ cm/s^2. [Here μ is the mass measured in units of m_H, while in Ref. [3] μ is measured in atomic mass units (u). Therefore, values of Z_i/μ_i and N_i/μ_i given in Ref. [3] have been multiplied by $(m_H/u)=1.00782519$.]

Inserting the values of Z_i/μ_i and N_i/μ_i for the samples used, Eq. (13) becomes

$$(0.388g_p^2+0.616g_n^2)|0.05925g_p^2-0.05830g_n^2|\leq\frac{\Delta a_{2-2'}}{\mathcal{A}(\lambda)}.\tag{14}$$

In Table I we have tabulated $\Delta a_{2-2'}/\mathcal{A}(\lambda)$ for a range of values of λ which were obtained by integrating over the mass distribution of the source in the apparatus. When these results are combined with Eq. (14), limits on $g_p^2(\lambda)$ and $g_n^2(\lambda)$ are obtained, and have been plotted for $\lambda=2$ cm and $\lambda=2000$ cm in Figs. 2 and 3, respectively. We find that the size of the apparatus is sufficiently small that the limits obtained for $\lambda>1000$ cm approximate the $\lambda\to\infty$ constraints found previously in Ref. [3].

As noted in Ref. [1], when Eq. (14) is plotted for the materials used by Smith *et al.* [3] and Gundlach *et al.* [4], the resulting limit curves are parts of two hyperbolas sharing a common asymptote, so such limits are referred to as "hyperbolic." This precludes setting absolute limits on g_p^2 or g_n^2 from this experiment alone, since the term in square brackets

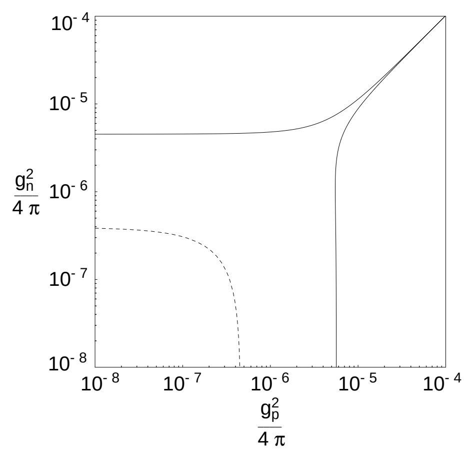

FIG. 2. Constraints on $g_p^2/4\pi$ and $g_n^2/4\pi$ for $\lambda=2$ cm obtained from Eqs. (14) and (18) and Tables I and II. The solid line is the limit from the WEP Experiment of Smith *et al.* [3], and the dashed line is obtained from the ISL experiment of Spero *et al.* [5]. The regions above and to the right of each curve are excluded by the corresponding experiment at the 1σ limit.

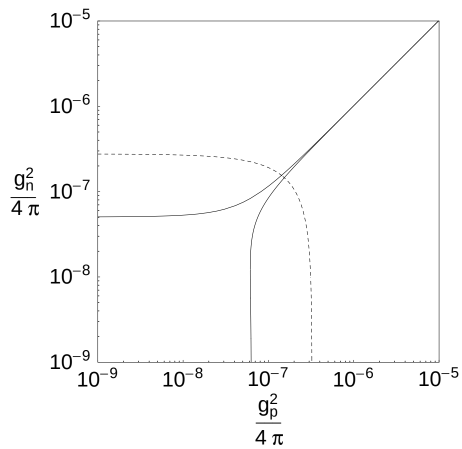

FIG. 3. Constraints on $g_p^2/4\pi$ and $g_n^2/4\pi$ for $\lambda=2000$ cm obtained from Eqs. (14) and (18) and Tables I and II. The solid line is the limit from the WEP Experiment of Smith *et al.* [3], and the dashed line is obtained from the ISL experiment of Spero *et al.* [5]. The regions above and to the right of each curve are excluded by the corresponding experiment at the 1σ limit.

in Eq. (14) vanishes when $g_p^2 \simeq g_n^2$, irrespective of how large each of these constants is. However, by combining "hyperbolic" constraints from a WEP experiment with an ISL test, absolute bounds on both g_p^2 and g_n^2 can be inferred from existing data [2]. To see this, let us return to Eq. (11) which may be rewritten as

$$\vec{F}_2=\vec{\mathcal{H}}(\lambda)\left(g_p^2\frac{Z_1}{\mu_1}+g_n^2\frac{N_1}{\mu_1}\right)\left(g_p^2\frac{Z_2}{\mu_2}+g_n^2\frac{N_2}{\mu_2}\right), \quad (15)$$

where

$$\vec{\mathcal{H}}(\lambda)\equiv\frac{3}{32\pi^3M^2\mathcal{V}_1\mathcal{V}_2\lambda}\left(\frac{M_1M_2}{m_H^2}\right)\int d^3r_1\int d^3r_2 \times\left\{\left[K_1(2|\vec{r}_1-\vec{r}_2|/\lambda)+\frac{2}{3}\left(\frac{|\vec{r}_1-\vec{r}_2|}{\lambda}\right)\times K_0(2|\vec{r}_1-\vec{r}_2|/\lambda)\right]\frac{(\vec{r}_1-\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|^4}\right\}. \quad (16)$$

Combined with an evaluation of $\vec{\mathcal{H}}$, an ISL experiment constrains $\vec{F}_2$, from which one can extract limits on $g_p^2(\lambda)$ and $g_n^2(\lambda)$ by rewriting Eq. (15) as

$$\left(g_p^2\frac{Z_1}{\mu_1}+g_n^2\frac{N_1}{\mu_1}\right)\left(g_p^2\frac{Z_2}{\mu_2}+g_n^2\frac{N_2}{\mu_2}\right)\leqslant\frac{F_2}{\mathcal{H}(\lambda)}, \quad (17)$$

where $F_2=|\vec{F}_2|$ and $\mathcal{H}(\lambda)=|\vec{\mathcal{H}}(\lambda)|$. In the experiment of Spero *et al.* [5], which uses Cu and stainless steel test masses, one finds [2]

$$(0.469g_p^2+0.540g_n^2)(0.460g_p^2+0.549g_n^2)\leqslant\frac{F_2}{\mathcal{H}(\lambda)}. \quad (18)$$

A plot of Eq. (18) in the g_p^2-g_n^2 plane shows that the ISL experiment of Spero *et al.* yields limits that are finite for all physical values of the ratio g_n^2/g_p^2, so we refer to these as "bounded" limits, in contrast to the "hyperbolic" limits yielded by the WEP experiment of Smith *et al.* In Table II we have tabulated $F_2/\mathcal{H}(\lambda)$ from this experiment as a function of λ, and have plotted the limits on g_p^2 and g_n^2 obtained from Eq. (18) for $\lambda=2$ cm and $\lambda=2000$ cm in Figs. 2 and 3, respectively. We find that the scale of the apparatus is such that the constraints for $\lambda>100$ cm approximate the $\lambda\to\infty$ limits obtained in Ref. [2].

As shown in Ref. [2], when "hyperbolic" constraints obtained from Gundlach *et al.* [4] were combined with the "bounded" constraints from the ISL experiment of Spero *et al.* [5], the allowed region in the g_p^2-g_n^2 plane was substantially reduced. This is a consequence of the fact that the Smith [3] or Gundlach [4] limits for the special cases $g_p^2 \ll g_n^2$ and $g_p^2 \gg g_n^2$ were much better than those obtained from the Spero experiment for sufficiently large λ (see Fig. 3). On the other hand, the Spero experiment leads to a significant limit when $g_p^2\simeq g_n^2$ which could not be obtained from the Smith or Gundlach experiments alone. In this way, the "hyperbolic" WEP experiment and the "bounded" ISL experiment complement each other.

It is useful to consider bounds on g_p^2 and g_n^2 for several important special cases. If the light pseudoscalar couples universally to baryon number, then $g_p^2=g_n^2\equiv g_{p,n}^2$. In this case, the limits from Smith *et al.* [3], Eq. (14), and Spero *et al.* [5], Eq. (18), give

$$\frac{g_{p,n}^2}{4\pi}\leqslant 2.6\sqrt{\frac{\Delta a_{2-2'}}{\mathcal{A}(\lambda)}} \quad (\text{Smith [3]}), \quad (19)$$

$$\frac{g_{p,n}^2}{4\pi}\leqslant 0.0789\sqrt{\frac{F_2}{\mathcal{H}(\lambda)}} \quad (\text{Spero [5]}). \quad (20)$$

Numerical results are presented in Tables I and II. Similarly, for $g_p^2 \gg g_n^2$ we find

$$\frac{g_p^2}{4\pi}\leqslant 0.525\sqrt{\frac{\Delta a_{2-2'}}{\mathcal{A}(\lambda)}} \quad (g_p^2\gg g_n^2, \quad \text{Smith [3]}), \quad (21)$$

$$\frac{g_p^2}{4\pi}\leqslant 0.171\sqrt{\frac{F_2}{\mathcal{H}(\lambda)}} \quad (g_p^2\gg g_n^2, \quad \text{Spero [5]}), \quad (22)$$

while for $g_n^2 \gg g_p^2$ we obtain

$$\frac{g_n^2}{4\pi}\leqslant 0.420\sqrt{\frac{\Delta a_{2-2'}}{\mathcal{A}(\lambda)}} \quad (g_n^2\gg g_p^2, \quad \text{Smith [3]}), \quad (23)$$

$$\frac{g_n^2}{4\pi}\leqslant 0.146\sqrt{\frac{F_2}{\mathcal{H}(\lambda)}} \quad (g_n^2\gg g_p^2, \quad \text{Spero [5]}). \quad (24)$$

Numerical results obtained from Eqs. (20)–(23) are presented in Tables I and II. It is important to note that, *irrespective of the value of* g_n^2 (g_p^2), the limit on g_p^2 (g_n^2) from

TABLE II. Geometric factor $F_2/\mathcal{H}(\lambda)$, and limits on pseudoscalar couplings from the ISL experiment by Spero *et al.* [5] for the special cases $g_p^2 = g_n^2$, $g_p^2 \gg g_n^2$, and $g_n^2 \gg g_p^2$ obtained from Eqs. (20), (22), and (24). Note that, irrespective of the value of g_n^2 (g_p^2), the limit on $g_p^2/4\pi$ ($g_n^2/4\pi$) is obtained from the Spero results by setting $g_n^2 = 0$ ($g_p^2 = 0$).

λ (cm)	$F_2/\mathcal{H}(\lambda)$	$\frac{g_{p,n}^2}{4\pi}$ ($g_p^2 = g_n^2$)	$\frac{g_p^2}{4\pi}$ ($g_p^2 \gg g_n^2$)	$\frac{g_n^2}{4\pi}$ ($g_n^2 \gg g_p^2$)
0.5	2.71×10^{-10}	1.30×10^{-6}	2.82×10^{-6}	2.40×10^{-6}
1	2.35×10^{-11}	3.82×10^{-7}	8.29×10^{-7}	7.08×10^{-7}
1.5	1.05×10^{-11}	2.56×10^{-7}	5.54×10^{-7}	4.73×10^{-7}
2	7.20×10^{-12}	2.12×10^{-7}	4.59×10^{-7}	3.92×10^{-7}
3	5.13×10^{-12}	1.79×10^{-7}	3.87×10^{-7}	3.31×10^{-7}
4	4.45×10^{-12}	1.66×10^{-7}	3.61×10^{-7}	3.08×10^{-7}
5	4.13×10^{-12}	1.60×10^{-7}	3.48×10^{-7}	2.97×10^{-7}
6	3.97×10^{-12}	1.57×10^{-7}	3.41×10^{-7}	2.91×10^{-7}
7	3.86×10^{-12}	1.55×10^{-7}	3.36×10^{-7}	2.87×10^{-7}
10	3.72×10^{-12}	1.52×10^{-7}	3.30×10^{-7}	2.82×10^{-7}
15	3.64×10^{-12}	1.51×10^{-7}	3.26×10^{-7}	2.79×10^{-7}
20	3.62×10^{-12}	1.50×10^{-7}	3.25×10^{-7}	2.78×10^{-7}
50	3.59×10^{-12}	1.49×10^{-7}	3.24×10^{-7}	2.77×10^{-7}
100	3.58×10^{-12}	1.49×10^{-7}	3.24×10^{-7}	2.76×10^{-7}
200	3.58×10^{-12}	1.49×10^{-7}	3.24×10^{-7}	2.76×10^{-7}
500	3.58×10^{-12}	1.49×10^{-7}	3.24×10^{-7}	2.76×10^{-7}
1000	3.58×10^{-12}	1.49×10^{-7}	3.24×10^{-7}	2.76×10^{-7}
2000	3.58×10^{-12}	1.49×10^{-7}	3.24×10^{-7}	2.76×10^{-7}

the experiment by Spero *et al.* is obtained from the limiting case given by Eq. (22) [Eq. (24)].

III. COMBINING GRAVITY AND OPTICAL CONSTRAINTS

As demonstrated in the previous section, gravity experiments alone are sufficient to constrain pseudoscalar couplings to nucleons. On the other hand, Massó has noted that optical experiments can provide a more stringent constraint on Yukawa couplings of pseudoscalars to protons [6] than those obtained from the current generation of WEP and ISL experiments. However, the optical experiments cannot limit the coupling to neutrons since they are electrically neutral, so Eq. (1) must be combined with other limits to constrain couplings to neutrons.

Combining the general constraint from the WEP experiment by Smith *et al.* given by Eq. (14) with Eq. (1) gives

$$\frac{g_n^2}{4\pi} < 3.29\times10^{-10} + 0.420\sqrt{\frac{\Delta a_{2-2'}}{\mathcal{A}(\lambda)} + 1.11\times10^{-17}}, \tag{25}$$

for the range of values of $\Delta a_{2-2'}/\mathcal{A}(\lambda)$ found here. Numerical results are given in Table I and plotted in Fig. 4. For the values of λ considered here, one finds that $\Delta a_{2-2'}/\mathcal{A}$ dominates Eq. (25) which can then be approximated by

$$\frac{g_n^2}{4\pi} \lesssim 0.420\sqrt{\frac{\Delta a_{2-2'}}{\mathcal{A}(\lambda)}}. \tag{26}$$

This agrees with Eq. (23) which assumes that $g_n^2 \gg g_p^2$.

As noted earlier, combining the general constraint from the ISL experiment by Spero *et al.* given by Eq. (18) with Eq. (1), does not improve the limits on g_n^2 obtained from the ISL experiment by Spero *et al.* alone. The largest value of g_n^2 which is consistent with both the ISL experiment and Massó's limit is given by Eq. (24).

IV. CONCLUSION

To summarize, we have shown how to obtain constraints on the Yukawa couplings of massive pseudoscalars to nucle-

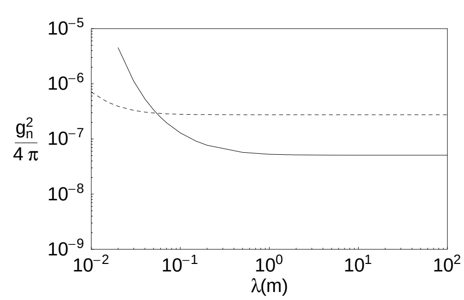

FIG. 4. Constraints on $g_n^2/4\pi$ for massive pseudoscalars. The solid line is the constraint obtained when the results of the WEP experiment of Smith *et al.* [3] are combined with the limit on $g_p^2/4\pi$ from optical experiments [6] [Eq. (25) and Table I]. The dashed line is the limit from the ISL experiment of Spero *et al.* [5] obtained by setting $g_p^2 = 0$ [Eq. (24), Table II]. The region above each line is excluded by the corresponding experiment at the 1σ limit.

ons from gravitational WEP and ISL experiments, thus generalizing earlier results obtained for the special case $m = 1/\lambda = 0$. When these results are combined with the earlier constraint by Massó [6], we obtain the most stringent laboratory limits on the coupling to neutrons. Although these laboratory constraints are less restrictive than those obtained from astrophysical arguments [16], they are more model independent and allow g_p^2 and g_n^2 to be determined separately. Finally, we have shown that that WEP, ISL, and optical experiments play complementary roles in setting such constraints, and so we strongly encourage efforts to improve these types of experiments.

ACKNOWLEDGMENTS

The work of E.G.A. was supported primarily by National Science Foundation Grant PHY-997097. The support for E.F. comes from the U.S. Department of Energy under Contract No. DE-AC 02-76ER01428. The work of R.D.N. was supported in part by National Science Foundation Grant PHY-0108937.

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