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# Constraining the gauge and scalar sectors of the doublet left-right symmetric model

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**ABSTRACT:** We consider a left-right symmetric extension of the Standard Model where the spontaneous breakdown of the left-right symmetry is triggered by doublets. The electroweak  $\rho$  parameter is protected from large corrections in this Doublet Left-Right Model (DLRM), contrary to the triplet case. This allows in principle for more diverse patterns of symmetry breaking. We consider several constraints on the gauge and scalar sectors of DLRM: the unitarity of scattering processes involving gauge bosons with longitudinal polarisations, the radiative corrections to the muon  $\Delta r$  parameter and the electroweak precision observables measured at the  $Z$  pole and at low energies. Combining these constraints within the frequentist CKMfitter approach, we see that the fit pushes the scale of left-right symmetry breaking up to a few TeV, while favouring an electroweak symmetry breaking triggered not only by the  $SU(2)_L \times SU(2)_R$  bi-doublet, which is the case most commonly considered in the literature, but also by the  $SU(2)_L$  doublet.

**KEYWORDS:** Beyond Standard Model, GUT

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## 1 Introduction

Left-Right (LR) symmetric models constitute a category of extensions of the SM that explains the left-handed structure of the SM through the existence of a larger gauge group  $SU(3)_C \otimes SU(2)_L \times SU(2)_R \times U(1)_X$ . This group is broken first at a high-energy scale  $\mu_R$  (of the order of the TeV or higher), inducing a difference between left and right sectors, followed by an electroweak symmetry breaking occurring at a lower scale  $\mu_W$  [1–5]. This extension yields heavy spin-1  $W'$  and  $Z'$  bosons, the former predominantly coupling to right-handed fermions, introducing a new “CKM-like” matrix for right-handed quarks (and similarly for leptons). LR models also lead to new charged and neutral scalar bosons with an interesting pattern of flavour-changing currents [6, 7]. Such a framework has been revived in the recent years for its potential collider implications when parity restoration in the LHC energy reach is considered [8, 9]. Interestingly, recent studies of anomalies in rare  $b$ -decays suggest also the interest of having right-handed currents in order to provide a consistent explanation of all the measurements [10–14].

Stringent constraints come from electroweak precision observables [15] and from direct searches at LHC [16–21], pushing the scale for LR models to several TeV. Studies in the framework of flavour physics suggest also that the structure for the right-handed “CKM-like” matrix should be quite different from the left-handed one [22–26]. A particularly important indirect constraint comes from kaon-meson mixing, pushing again the mass scale for the new scalar particles up to a few TeV or beyond [27–32], for which we reassessed short-distance QCD corrections in order to reach the accuracy now requested for such processes [33].

Various mechanisms can be invoked to trigger the breakdown of the left-right symmetry. Historically, LR models (LRM) were first considered with doublets in order to break the left-right symmetry spontaneously. Later the focus was set on triplet models (Triplet Left-Right Model, or TLRM), due to their ability to generate both Dirac and Majorana masses for neutrinos and thus introduce a see-saw mechanism [34, 35]. We would like to reassess the possibility of a left-right symmetry breaking due to doublets (Doublet Left-Right Model, or DLRM). On one hand, it prevents us from providing a see-saw mechanism for neutrinos, for which there is however no experimental evidence yet, but on the other hand, it protects the electroweak  $\rho$  parameter from large contributions already at the tree level and thus allows in principle a low left-right symmetry breaking scale  $\mu_R$  (which might or might not be in contradiction with other phenomenological constraints).

In this article, we will discuss three classes of constraints on the DLRM related to the presence of heavy gauge bosons and scalars that affect the dynamics of the light gauge bosons  $W$  and  $Z$ : the unitarity of the processes involving the scattering of two gauge bosons, the radiative corrections to the muon  $\Delta r$  parameter, and the electroweak precision observables measured at the  $Z$ -pole and at low energies. We will combine these three constraints into a global fit using the frequentist approach of the CKMfitter collaboration [36–38] in order to constrain the parameters of the model. We leave the discussion of flavour and the combination of all constraints for future work, due to the large number of additional parameters involved.

In section 2, we discuss the basic features of our model. In section 3, we consider the constraints coming from the preservation of unitarity for the scattering of gauge bosons. In section 4, we discuss the breakdown of the custodial symmetry from the  $W$  and  $Z$  self-energies induced in DLRM. In section 5, we discuss the status of electroweak precision observables in these models. In section 6, we perform a fit taking into account all these constraints and discuss the outcome for the parameters of the DLRM, before concluding in section 7. Appendices are devoted to the expressions for EW precision observables, the spectrum, the Feynman rules associated with the DLRM and some aspects of the renormalisation.

## 2 The doublet left-right model

### 2.1 Gauge structure and symmetry breaking

Let us start with the gauge structure of the doublet left-right model (DRLM) and the pattern of its symmetry breaking. We consider the group  $SU(3)_C \otimes SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  with gauge couplings  $g_C, g_L, g_R$  and  $g_X$ , where  $B - L$  is the difference between the baryon and the lepton numbers.<sup>1</sup> At a scale of a few TeV (or higher), a symmetry breaking  $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$  with  $Y$  the hypercharge is triggered by a doublet  $\chi_R$  (1, 1, 2, 1/2). One has in terms of the component fields of the doublet:

$$\chi_{L,R} = \begin{pmatrix} \chi_{L,R}^+ \\ \chi_{L,R}^0/\sqrt{2} \end{pmatrix} \quad \langle \chi_L \rangle = \begin{pmatrix} 0 \\ v_L e^{i\theta_L}/\sqrt{2} \end{pmatrix} \quad \langle \chi_R \rangle = \begin{pmatrix} 0 \\ v_R/\sqrt{2} \end{pmatrix} \quad (2.1)$$

with the expected scale  $v_R = \mathcal{O}(1 \text{ TeV})$  and where one has introduced a doublet  $\chi_L$  (1, 2, 1, 1/2) to preserve a left-right symmetric structure with  $v_L$  at most a few hundred GeV. We will denote the real and imaginary parts of these fields as  $\chi_L^0 = (v_L e^{i\theta_L} + \chi_{L,r}^0 + i\chi_{L,i}^0)$  and  $\chi_R^0 = (v_R + \chi_{R,r}^0 + i\chi_{R,i}^0)$ . Note that performing an obvious extension of the definition of the electric charge of right-handed fields in terms of the right weak isospin and the  $B - L$  quantum numbers the hypercharge acquires a simple meaning in LR models.

At a lower scale, the spontaneous breaking of  $SU(3)_C \otimes SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \otimes U(1)_Q$  leads to the standard electroweak symmetry breaking. It is triggered on one hand by the doublet  $\chi_L$  and on the other hand by the bidoublet  $\phi$  (1, 2, 2, 0) whose presence is mandatory to provide a mass to the fermions, see section 2.4,

$$\phi = \begin{pmatrix} \phi_1^0/\sqrt{2} & \phi_2^+ \\ \phi_1^- & \phi_2^0/\sqrt{2} \end{pmatrix} \quad \langle \phi \rangle = \begin{pmatrix} \kappa_1/\sqrt{2} & 0 \\ 0 & \kappa_2 e^{i\alpha}/\sqrt{2} \end{pmatrix} \quad (2.2)$$

with the conjugate bidoublet  $\tilde{\phi} = \tau_2 \phi^* \tau_2$  transforming similarly.

It proves useful to introduce the real ratios

$$\epsilon = \frac{\kappa_1}{v_R} \quad r = \frac{\kappa_2}{\kappa_1} \quad w = \frac{v_L}{\kappa_1} \quad (2.3)$$

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<sup>1</sup>Thus, contrary to the SM, baryon and lepton numbers are not accidental symmetries in LR models.

as well as the definitions:

$$k^2 = 1 + r^2 + w^2 \quad v^2 = v_R^2 k^2 \epsilon^2 = v_L^2 + \kappa_1^2 + \kappa_2^2 \quad (2.4)$$

Due to the hierarchy of scales involved  $\epsilon$  is a small quantity. We will thus perform in the following an expansion in this parameter. As we will see below, at Leading Order (LO), namely,  $\epsilon = 0$ , the right and the left gauge and scalar fields decouple except for the neutral gauge bosons; mixing starts at Next-to-Leading Order (NLO)  $\epsilon = 1$  or Next-to-Next-to-Leading Order (NNLO)  $\epsilon = 2$  depending whether one considers scalars or charged gauge bosons respectively. In the following we will neglect terms of order  $\epsilon^3$  and higher unless specified. We will also consider  $r < 1$ .<sup>2</sup>

In the case of left-right symmetry breaking triggered by triplets (TLRM), the equivalent of the ratio  $w$  is taken as very small and neglected, on the basis of the breaking of the custodial symmetry which already occurs at LO. As we will see below, this is not the case anymore when the left-right symmetry breaking is triggered by doublets, and we will thus leave open the possibility that  $w$  is of order 1 or larger, letting data constrain its value. Furthermore, for simplicity, we will work under the assumption that

$$\alpha = \theta_L = 0 \quad (2.5)$$

i.e., no additional sources of CP violation come from the breaking of the gauge symmetries.

## 2.2 Spin-0 sector

Scalar self-interactions are described by the following potential

$$\begin{aligned} V = & -\mu_1^2 \langle \phi^\dagger \cdot \phi \rangle - \mu_2^2 \langle \tilde{\phi} \cdot \phi^\dagger + \tilde{\phi}^\dagger \cdot \phi \rangle - \mu_3^2 (\chi_L^\dagger \cdot \chi_L + \chi_R^\dagger \cdot \chi_R) \\ & + \mu'_1 (\chi_L^\dagger \cdot \phi \cdot \chi_R + \chi_R^\dagger \cdot \phi^\dagger \cdot \chi_L) + \mu'_2 (\chi_L^\dagger \cdot \tilde{\phi} \cdot \chi_R + \chi_R^\dagger \cdot \tilde{\phi}^\dagger \cdot \chi_L) \\ & + \lambda_1 \langle \phi^\dagger \cdot \phi \rangle^2 + \lambda_2 (\langle \tilde{\phi} \cdot \phi^\dagger \rangle^2 + \langle \tilde{\phi}^\dagger \cdot \phi \rangle^2) + \lambda_3 \langle \phi^\dagger \cdot \tilde{\phi} \rangle \langle \tilde{\phi}^\dagger \cdot \phi \rangle + \lambda_4 \langle \phi^\dagger \cdot \phi \rangle \langle \tilde{\phi} \cdot \phi^\dagger + \phi \cdot \tilde{\phi}^\dagger \rangle \\ & + \rho_1 [(\chi_L^\dagger \cdot \chi_L)^2 + (\chi_R^\dagger \cdot \chi_R)^2] + \rho_3 [(\chi_L^\dagger \cdot \chi_L)(\chi_R^\dagger \cdot \chi_R)] \\ & + \alpha_1 \langle \phi^\dagger \cdot \phi \rangle [(\chi_L^\dagger \cdot \chi_L) + (\chi_R^\dagger \cdot \chi_R)] \\ & + \frac{\alpha_2}{2} e^{i\delta_2} [\langle \tilde{\phi} \cdot \phi^\dagger \rangle (\chi_L^\dagger \cdot \chi_L) + \langle \phi \cdot \tilde{\phi}^\dagger \rangle (\chi_R^\dagger \cdot \chi_R)] \\ & + \frac{\alpha_2}{2} e^{-i\delta_2} [\langle \tilde{\phi}^\dagger \cdot \phi \rangle (\chi_L^\dagger \cdot \chi_L) + \langle \phi^\dagger \cdot \tilde{\phi} \rangle (\chi_R^\dagger \cdot \chi_R)] \\ & + \alpha_3 (\chi_L^\dagger \cdot \phi \cdot \phi^\dagger \cdot \chi_L + \chi_R^\dagger \cdot \phi^\dagger \cdot \phi \cdot \chi_R) + \alpha_4 (\chi_L^\dagger \cdot \tilde{\phi} \cdot \tilde{\phi}^\dagger \cdot \chi_L + \chi_R^\dagger \cdot \tilde{\phi}^\dagger \cdot \tilde{\phi} \cdot \chi_R) \end{aligned} \quad (2.6)$$

where  $\langle \rangle$  denotes the trace. For simplicity, we have imposed invariance under the discrete left-right symmetry<sup>3</sup> in the scalar sector of the theory

$$\chi_L \leftrightarrow \chi_R, \quad \phi \leftrightarrow \phi^\dagger$$

<sup>2</sup>There is some subtlety in solving the stability equations discussed in the next section in the limit  $r = 1$ . However as the hierarchy  $m_b \ll m_t$  demands  $r$  to be smaller than one, see section 2.4 below, we will not enter into such detail in the following.

<sup>3</sup>See refs. [8, 39, 40] for other discrete symmetries in the context of LR Models.

The scalar potential for doublet fields has a slightly different structure compared to the triplet case [35], and in particular trilinear terms are allowed. In addition to eq. (2.5) we will set for simplicity  $\delta_2 = 0$ , i.e. there are no new sources of CP violation from the scalar sector.

One can minimize the potential with respect to the parameters  $v_R, v_L, \kappa_1, \kappa_2$ , giving four conditions for the stability of the vacuum state. These four equations provide relations among the vacuum expectation values  $\kappa_{1,2}$  and  $v_{L,R}$ , and the underlying parameters of the potential,  $\mu_{1,2,3}^2, \alpha_{1,2,3,4}, \mu'_{1,2}, \rho_{1,3}$  and  $\lambda_{1,2,3,4}$ :

$$0 = \kappa_2 (B + 4\lambda_4 \kappa_1^2) + \kappa_1 [A + \alpha_4 (v_R^2 + v_L^2) + 4(\lambda_3 + 2\lambda_2) \kappa_2^2] + \sqrt{2} \mu'_2 v_L v_R + \alpha_2 \kappa_2 (v_R^2 + v_L^2), \quad (2.7)$$

$$0 = \kappa_1 (B + 4\lambda_4 \kappa_2^2) + \kappa_2 [A + \alpha_3 (v_R^2 + v_L^2) + 4(\lambda_3 + 2\lambda_2) \kappa_1^2] + \sqrt{2} \mu'_1 v_L v_R + \alpha_2 \kappa_1 (v_R^2 + v_L^2), \quad (2.8)$$

$$0 = C + 2\rho v_L^2 + \sqrt{2} \frac{v_L}{v_R} (\mu'_1 \kappa_2 + \mu'_2 \kappa_1) + 2\alpha_2 r \kappa_1^2, \quad (2.9)$$

$$0 = \frac{v_L}{v_R} (C + 2\rho v_R^2) + \sqrt{2} (\mu'_1 \kappa_2 + \mu'_2 \kappa_1) + 2\alpha_2 r w \kappa_1^2, \quad (2.10)$$

where  $\rho \equiv \rho_3/2 - \rho_1$ , and

$$A \equiv -2\mu_1^2 + \alpha_1 (v_R^2 + v_L^2) + 2\lambda_1 (\kappa_1^2 + \kappa_2^2), \quad (2.11)$$

$$B \equiv -4\mu_2^2 + 2\lambda_4 (\kappa_1^2 + \kappa_2^2), \quad (2.12)$$

$$C \equiv -2\mu_3^2 + 2\rho_1 (v_R^2 + v_L^2) + \alpha_1 (\kappa_1^2 + \kappa_2^2) + \alpha_4 \kappa_1^2 + \alpha_3 \kappa_2^2. \quad (2.13)$$

It is useful to further define the combinations of parameters

$$A' \equiv -2\mu_1^2 + \alpha_1 v_R^2, \quad B' \equiv -4\mu_2^2, \quad C' \equiv -2\mu_3^2 + 2\rho_1 v_R^2. \quad (2.14)$$

The minimisation conditions at leading order in  $\epsilon \ll 1$  yield:

$$\mathcal{O}(\epsilon^2) = r \frac{B'}{v_R^2} + \left( \frac{A'}{v_R^2} + \alpha_4 \right) + \sqrt{2} \frac{\mu'_2}{v_R} w + \alpha_2 r, \quad (2.15)$$

$$\mathcal{O}(\epsilon^2) = \frac{B'}{v_R^2} + r \left( \frac{A'}{v_R^2} + \alpha_3 \right) + \sqrt{2} \frac{\mu'_1}{v_R} w + \alpha_2, \quad (2.16)$$

$$\mathcal{O}(\epsilon^2) = \frac{C'}{v_R^2}, \quad (2.17)$$

$$\mathcal{O}(\epsilon^2) = w 2\rho + \sqrt{2} \left( r \frac{\mu'_1}{v_R} + \frac{\mu'_2}{v_R} \right). \quad (2.18)$$

We have four equalities (up to higher orders in  $\epsilon$ ) and three parameters  $\{r, w, v_R\}$  related to the vacuum expectation values of the scalar fields, that we will exploit in order to eliminate the explicit dependence on some parameters of the scalar potential.<sup>4</sup>

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<sup>4</sup>Note that in principle one could solve the stability equations for the parameters  $\{r, w, v_R\}$  using three out of the four equations, and plugging the solutions into the fourth. In other words a combination of parameters in the scalar potential ( $\mu_{1,2}^2/\mu_3^2, \alpha_{1,2,3,4}, \rho_{1,3}, \mu'_1/\mu'_2$ ) is  $\mathcal{O}(\epsilon^2)$  and thus vanishes at leading order in  $\epsilon$ . Whether this resulting combination is stable under radiative corrections, thus requiring or not a certain amount of tuning, remains to be verified.

As far as scalar states are concerned, the theory contains 2 neutral and 2 charged Goldstone bosons (linked to the massive  $W, Z, W', Z'$  gauge bosons, see appendix B), 5 neutral extra scalar bosons (3 CP-even  $H_{1,2,3}^0$  and 2 CP-odd  $A_{1,2}^0$ ), and 2 charged scalar bosons ( $H_{1,2}^\pm$ ),<sup>5</sup> as well as a neutral light scalar  $h^0$  (corresponding to the light SM-like Higgs boson) of mass  $\mathcal{O}(\epsilon^2)$  given as

$$M_h^2 = \frac{v_R^2 \epsilon^2}{2k^2} \left( 4(\lambda_1 (r^2 + 1)^2 + 4r (\lambda_4(1 + r^2) + r\lambda_{23}^+) + w^2 (\alpha_{124} + r^2(\alpha_1 + \alpha_3) + \alpha_2 r) + \rho_1 w^4) - \frac{1}{\rho_1} (\alpha_{124} + r^2(\alpha_1 + \alpha_3) + \alpha_2 r + 2\rho_1 w^2)^2 \right) \quad (2.19)$$

with

$$\alpha_{124} \equiv \alpha_1 + r\alpha_2 + \alpha_4, \quad \lambda_{23}^+ \equiv 2\lambda_2 + \lambda_3 \quad (2.20)$$

In the limit  $r, w \rightarrow 0^+$ , one gets the simplified formula  $M_h^2 = v_R^2 \epsilon^2 (2\lambda_1 - (\alpha_1 + \alpha_4)^2 / (2\rho_1))$ . The expressions for the masses of the new scalar bosons are given in appendix B at leading order in  $\epsilon$ . Note that at that order the CP-even, CP-odd and charged scalars have equal masses  $M_i$  for each value of  $i$ . One can express some of the parameters of the scalar potential in terms of these  $M_i$

$$\mu'_2 = \frac{w(-1 \pm X)}{\sqrt{2}v_R(1+rx)(1+\beta(x)w^2)} (M_1^2 + M_2^2) \quad (2.21)$$

$$\alpha_{34} \equiv \alpha_3 - \alpha_4 = -\frac{1}{(r^2 + 1)v_R^2 w} \left( 2(r^2 - 1)(M_1^2 + M_2^2)w + \sqrt{2}\mu'_2 v_R (r^3 x + r^2 + r(2w^2 - 1)x - 2w^2 - 1) \right) \quad (2.22)$$

with

$$X = \sqrt{1 - \frac{4\delta^2}{(1+\delta^2)^2} \frac{(1+r^2)(1+\beta(x)w^2)}{k^2}} \quad (2.23)$$

and

$$\beta(x) = (1+x^2)/(1+rx)^2, \quad \delta = M_1/M_2, \quad x = \mu'_1/\mu'_2 \quad (2.24)$$

The neutral and charged scalar physical fields decompose as follows up to  $\mathcal{O}(\epsilon^2)$

$$\begin{aligned} h^0 &= \left( \frac{1}{k} + \epsilon^2 c_{\phi_1}^{h^0} \right) \phi_{1,r}^0 + \left( \frac{r}{k} + \epsilon^2 c_{\phi_2}^{h^0} \right) \phi_{2,r}^0 + \left( \frac{w}{k} + \epsilon^2 c_{\chi_{L,r}}^{h^0} \right) \chi_{L,r}^0 + \epsilon c_{\chi_{R,r}}^{h^0} \chi_{R,r}^0 \\ H_i^0 &= \left( -\frac{r+t_i w}{u_i} + \epsilon^2 c_{\phi_1}^{H_i} \right) \phi_{1,r}^0 + \left( \frac{1}{u_i} + \epsilon^2 c_{\phi_2}^{H_i} \right) \phi_{2,r}^0 + \left( \frac{t_i}{u_i} + \epsilon^2 c_{\chi_{L,r}}^{H_i} \right) \chi_{L,r}^0 + \epsilon c_{\chi_{R,r}}^{H_i} \chi_{R,r}^0 \\ A_i^0 &= \left( \frac{r+t_i w}{u_i} + \epsilon^2 c_{\phi_1}^{A_i} \right) \phi_{1,i}^0 + \left( \frac{1}{u_i} + \epsilon^2 c_{\phi_2}^{A_i} \right) \phi_{2,i}^0 + \left( \frac{t_i}{u_i} + \epsilon^2 c_{\chi_{L,r}}^{A_i} \right) \chi_{L,i}^0 + \epsilon c_{\chi_{R,r}}^{A_i} \chi_{R,i}^0 \\ H_3^0 &= \epsilon (c_{\phi_1}^{H_3} \phi_{1,r}^0 + c_{\phi_2}^{H_3} \phi_{2,r}^0 + c_{\chi_{L,r}}^{H_3} \chi_{L,r}^0) + (1 + \epsilon^2 c_{\chi_{R,r}}^{H_3}) \chi_{R,r}^0 \\ H_i^\pm &= \left( \frac{r+t_i w}{u_i} + \epsilon^2 d_{\phi_1}^{H_i} \right) \phi_{1,r}^0 + \left( \frac{1}{u_i} + \epsilon^2 d_{\phi_2}^{H_i} \right) \phi_{2,r}^0 + \left( \frac{t_i}{u_i} + \epsilon^2 d_{\chi_{L,r}}^{H_i} \right) \chi_{L,r}^0 + \epsilon d_{\chi_{R,r}}^{H_i} \chi_{R,r}^0 \end{aligned} \quad (2.25)$$

<sup>5</sup>Contrary to the TLRM, the DLRM has no doubly charged scalars.

where the various coefficients of the  $\epsilon$  and  $\epsilon^2$  terms are combinations of the parameters of the scalar potential. The scalar  $H_3^0$  is the analogue of the SM Higgs boson in the right sector of the theory at LO in  $\epsilon$ . The quantities  $t_i$  and  $u_i$  arising from the determination of the mass eigenstates are defined as

$$t_1 \equiv p, \quad t_2 \equiv q, \quad u_i = \sqrt{1 + t_i^2 + (r + wt_i)^2} \quad (2.26)$$

where

$$p = -\frac{k^2(1+rx)(1-\delta^2 + (1+\delta^2)X) + 2w^2r(r-x)}{2w(k^2(r-x\delta^2) - r^2(r-x))} \quad (2.27)$$

The parameter  $q$  is related to  $p$  via the following relation:

$$q = -\frac{1+r^2+prw}{rw+p(1+w^2)} \quad (2.28)$$

and can be obtained from eq. (2.27) through the replacement  $\delta \rightarrow 1/\delta$ . Here and in the following we assume  $r < 1$  and  $\delta > 1$ .<sup>6</sup> In the limit  $w \rightarrow 0^+$  one has  $\delta = \mathcal{O}(1/\sqrt{w})$ ,  $p = \mathcal{O}(w^2)$  and  $q = -(1+r^2)/(rw)$ , with  $\rho_3 = \mathcal{O}(1/w)$ , see appendix B, leading to similar expressions as in ref. [30] where the TRLM is considered.

As shown in appendix B, some coefficients in the above expansion can be expressed in terms of the functions  $F_{i=1,2}(r, w, p)$  given by:

$$F_i(r, w, p) = \frac{-1+r^2+rw t_i}{k u_i} \quad (2.29)$$

These functions also occur in the couplings of the new Higgs bosons to the quarks, see appendix C.1. Similarly, one can determine the couplings of the extra Higgs bosons to the gauge bosons, see appendix C.2, which involve another combination of interest:

$$S_i = t_i/u_i \quad (2.30)$$

### 2.3 Spin-1 sector

For the gauge sector (as can be seen for instance in ref. [15]), one can express the light  $W$  and heavy  $W'$  bosons in terms of left and right gauge bosons up to terms of  $\mathcal{O}(\epsilon^2)$

$$W_\mu^\pm = W_{L,\mu}^\pm + \frac{2c_W s_R r}{s_W} \epsilon^2 W_{R,\mu}^\pm \quad W'_\mu^\pm = W_{R,\mu}^\pm - \frac{2c_W s_R r}{s_W} \epsilon^2 W_{L,\mu}^\pm, \quad (2.31)$$

whereas the physical neutral gauge bosons (massless  $A$ , light  $Z$  and heavy  $Z'$ ) identify as

$$A = s_W W_L^3 + s_{RCW} W_R^3 + c_{RCW} B \quad (2.32)$$

$$Z = c_W W_L^3 - s_{RSW} \left(1 - \frac{c_R^2 k z_h}{s_W^2} \epsilon^2\right) W_R^3 - c_{RSW} \left(1 + \frac{s_R^2 k z_h}{s_W^2} \epsilon^2\right) B \quad (2.33)$$

$$Z' = -\frac{s_{RCW} k z_h}{s_W} \epsilon^2 W_L^3 + c_R \left(1 + s_R^2 k z_h \epsilon^2\right) W_R^3 - s_R \left(1 - c_R^2 k z_h \epsilon^2\right) B \quad (2.34)$$

---

<sup>6</sup>If  $\delta < 1$  or  $r > 1$   $M_1$  and  $M_2$  have to be swapped and consequently also the expressions for  $p$  and  $q$ .



with the (sines of the) leading-order mixing angles

$$s_R = \frac{g_X}{\sqrt{g_X^2 + g_R^2}} \quad s_W = \frac{s_R}{\sqrt{(g_L/g_R)^2 + s_R^2}} \quad (2.35)$$

and  $c_A = \sqrt{1 - s_A^2}$ , with an obvious notation for sines and co-sines. We also have

$$\frac{e}{g_L} = s_W \quad \frac{g_L}{g_R} = \frac{c_W s_R}{s_W} \quad (2.36)$$

It is well known that the SM possesses an accidental (global) symmetry called the custodial symmetry. Indeed before the breaking of the  $SU(2)_L \times U(1)_Y$  gauge symmetry, the Higgs potential has a global  $SU(2)_L \times SU(2)_R$  symmetry which reduces to  $SU(2)_V$  when the gauge symmetry is broken, see for example ref. [41] for a detailed review. This residual custodial symmetry can most easily be seen by rewriting the Higgs field as a bidoublet under this global symmetry. Under the assumption that the hypercharge gauge coupling vanishes,  $g' = 0$ , the kinetic part of the Lagrangian is also invariant under the custodial symmetry and the gauge bosons  $W^\pm$  and  $Z$  form a degenerate multiplet. Indeed, in that limit  $s_W = 0$  and:

$$\rho \rightarrow M_W^2/M_Z^2 = 1 \quad (2.37)$$

When  $g' \neq 0$  it is easy to see that in the SM the mass matrix for the gauge bosons can be obtained by replacing  $W_\mu^3 \rightarrow W_\mu^3 - g'/g B_\mu = Z_\mu/\cos\theta_W$  so that the  $\rho$  parameter becomes:

$$\rho \equiv M_W^2/(M_Z^2 \cos^2\theta_W) = 1 \quad (2.38)$$

Small deviations from  $\rho = 1$  arise when including radiative corrections. Note that  $\sin\theta_W^2$  is renormalisation scheme dependent and various definitions of this parameter exist in the literature with slightly different numerical values. Correspondingly, there are various definitions of the  $\rho$  parameter [42]. In the on-shell renormalisation scheme that will be discussed below, the above equation is promoted to a definition of the renormalised  $s_W^2$  at all orders in perturbation theory.

In the DLRM, one can also illustrate this custodial symmetry by writing the two Higgs doublet fields as two bi-doublets under the  $SU(2)_L \times SU(2)_R$  symmetry:

$$\phi_{L,R} = \begin{pmatrix} \chi_{L,R}^{0*} & \chi_{L,R}^+ \\ -\chi_{L,R}^- & \chi_{L,R}^0 \end{pmatrix} \quad (2.39)$$

with the following transformation properties:

$$\phi_{L,R} \rightarrow U_{R,L} \phi_{L,R} U_{L,R}^\dagger \quad (2.40)$$

with  $U_{L,R} \in SU(2)_{L,R}$ .

Diagonalising the mass matrix for the gauge bosons, one finds

$$M_{W,W'}^2 = \frac{1}{8} \left( g_L^2 v^2 + g_R^2 V^2 \mp \sqrt{4g_L^2 g_R^2 \kappa^4 + (g_L^2 v^2 - g_R^2 V^2)^2 - 4g_L^2 g_R^2 (\kappa_1^2 - \kappa_2^2)^2} \right) \quad (2.41)$$

$$M_{Z,Z'}^2 = \frac{1}{8} \left( g_L^2 v^2 + g_R^2 V^2 + \frac{g_R^2 s_R^2}{c_R^2} (-2\kappa^2 + V^2 + v^2) \mp \sqrt{4g_L^2 g_R^2 \kappa^4 + (g_L^2 v^2 - g_R^2 V^2)^2 + \frac{g_R^2 s_R^2}{c_R^2} \Delta} \right)$$

$$\Delta = 2g_L^2 (\kappa^4 + (v^2 - \kappa^2)^2 - V^2 v^2) + 2g_R^2 (\kappa^4 + (V^2 - \kappa^2)^2 - V^2 v^2) + \frac{g_R^2 s_R^2}{c_R^2} (-2\kappa^2 + V^2 + v^2)^2$$

where the plus (minus) signs are for the heavy (light) gauge bosons,  $v$  is defined in eq. (2.4),

$$V^2 = \kappa^2 + v_R^2 \quad \text{and} \quad \kappa^2 = \kappa_1^2 + \kappa_2^2. \quad (2.42)$$

In the limit  $g_X \rightarrow 0$ , we get  $s_W, s_R \rightarrow 0$  with a fixed ratio  $s_R/s_W = g_L/g_R$ . Clearly the only difference between the two equations in that limit comes from the last term in the square root in the first equation which cancels when  $r \rightarrow 1$ , leading to degenerate neutral and charge gauge bosons, so that  $M_{W'} = M_{Z'}$  and  $M_W = M_Z$ . Indeed if  $\kappa_1 = \kappa_2$  ( $r = 1$ ), the kinetic energy Lagrangian is invariant under the custodial symmetry, similarly to the SM.

Expanding the masses, eq. (2.42) in  $\epsilon$ , the first two terms read:

$$M_W = M_W^0 \left(1 - \epsilon^2 \frac{w_h^2}{2}\right) \quad M_Z = M_Z^0 \left(1 - \epsilon^2 \frac{z_h^2}{2}\right) \quad (2.43)$$

where

$$\begin{aligned} M_{W'}^0 &= \frac{1}{2} g_L v, & M_Z^0 &= \frac{M_W^0}{c_W} \\ w_h &= \frac{2r}{k}, & z_h &= k \left( c_R^2 - \frac{w^2}{k^2} \right) = w_h + \frac{(1-r)^2 - s_R^2 w^2}{k} \end{aligned} \quad (2.44)$$

The equation for  $z_h$  in terms of  $w_h$  illustrates that  $M_W = M_Z$  for  $r = 1$  and  $s_W = s_R = 0$ . This leads to the relation:

$$\rho \equiv \frac{M_W^2}{M_Z^2 c_W^2} = 1 + \epsilon^2 f(1-r, s_R, w) + \mathcal{O}(\epsilon^4) \quad (2.45)$$

with  $f$  a function which can be determined from the previous equations, so that one recovers the SM relation at LO in  $\epsilon$ , contrary to the TLRM for which the relation is violated already at LO in  $\epsilon$  if  $w \neq 0$ . Typically in the latter one has  $\rho = (1+r^2+2w^2)/(1+r^2+4w^2) + \mathcal{O}(\epsilon^2)$ , see for example [43], so that  $w$  has to be much smaller than  $1+r^2$  and is usually neglected. Thus the DLRM, which does not trigger a breaking of the custodial symmetry for  $v_L \neq 0$  at  $\epsilon = 0$  contrarily to TRLM, allows an easier fulfilment of electroweak-precision tests for a non-vanishing  $v_L$ .

For the heavy gauge bosons one has

$$M_{W'} = M_{W'}^0 \left(1 + \epsilon^2 \frac{1+r^2}{2}\right), \quad M_{Z'} = M_{Z'}^0 \left(1 + \epsilon^2 \frac{c_R^4(1+r^2) + s_R^4 w^2}{2}\right), \quad (2.46)$$

with expressions at LO in  $\epsilon$  similar to the SM ones up to the replacement  $L \rightarrow R$

$$M_{W'}^0 = \frac{1}{2} g_R v_R, \quad M_{Z'}^0 = \frac{M_{W'}^0}{c_R} \quad (2.47)$$

so that one can define an equivalent relation to the SM case

$$\rho_R \equiv \frac{M_{W'}^2}{M_{Z'}^2 c_R^2} = 1 + \mathcal{O}(\epsilon^2) \quad (2.48)$$

## 2.4 Spin-1/2 sector

We have focused on the gauge and scalar sectors of the DLRM which are the main focus of our work here. For completeness, we discuss briefly the fermion sector, although further detail would be needed to account for flavour constraints properly. The Yukawa interactions are given by

$$\mathcal{L}_Y = -\bar{Q}_{L,i}(y_{ij}\phi + \tilde{y}_{ij}\tilde{\phi})Q_{R,j} + h.c. \quad (2.49)$$

in the interaction basis. After electroweak symmetry breaking, the diagonalisation of the Yukawa matrices yields the mass matrices  $M_u$  and  $M_d$  and two unitary “CKM-like” matrices  $V_L$  and  $V_R$  connecting mass and interaction bases. One gets the following structure for the Yukawa matrices

$$y = \frac{\sqrt{2}}{(1-r^2)v_R\epsilon}(M_u - rV_L M_d V_R^\dagger), \quad \tilde{y} = \frac{\sqrt{2}}{(1-r^2)v_R\epsilon}(V_L M_d V_R^\dagger - rM_u). \quad (2.50)$$

As can be seen the limit  $r = 1$  is not allowed, as it creates additional degeneracies among quark flavours at tree level. Here the usual choice is made to assign all the redefinition from interaction to mass states to the down-type fermions (of left and right chiralities). The same “CKM-like” matrices arise in the couplings of the gauge bosons and scalars once fermions are expressed in terms of mass eigenstates. Note that in the case discussed here and contrary to other realizations of LR models in which discrete symmetries are imposed at the energy scales of spontaneous breaking of the LR gauge group [44, 45], the two matrices  $V_L$  and  $V_R$  are not simply related, thus resulting in a large number of new parameters in the flavour sector. We will thus leave the important constraints from this sector to a future work, however we note that the overall good consistency of studies of CP violation with the Kobayashi-Maskawa mechanism embedded into the Standard Model [46] is expected to imply important constraints on deviations of  $V_L$  with respect to the SM picture, and on the structure of  $V_R$ .

A similar discussion could hold for the lepton part, but in the following we are going to neglect neutrino masses, meaning that no mixing matrix is then needed for the lepton part. Let us however stress that there is no possibility to generate a Majorana mass term for the neutrinos using the doublets, so that the neutrinos are Dirac particles.

Finally let us just mention that the couplings of the fermions to the SM-like scalar and the gauge bosons  $W$  and  $Z$  have the same form as in the SM up to corrections of order  $\mathcal{O}(\epsilon^2)$ , see appendix C.1.

## 2.5 Parameters

In the DLRM, one has the following parameters in the Lagrangian:

- the parameters having an immediate equivalent in the SM, namely the fermion masses (9, corresponding to quarks and charged leptons, since we neglect the neutrino masses) and the CKM-like matrix  $V_L$  (depending on 4 parameters).
- the analogue of the CKM matrix in the right sector  $V_R$  leading to 3 moduli and 6 phases.

- the gauge couplings  $g_C$ ,  $g_R$ ,  $g_L$  and  $g_X$ . Here, we will allow  $g_R$  and  $g_L$  to be independent of one another, i.e., we will not restrict ourselves to the fully left-right symmetric case.
- the symmetry breaking  $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$  scale  $v_R$ .
- the electroweak breaking scale involving the three parameters  $\epsilon$ ,  $r$ , and  $w$ .
- the 15 parameters of the scalar potential. At the order at which we work and after exploiting the stability conditions, the only ones that are needed for our present study are  $\mu'_1$ ,  $\mu'_2$ ,  $\rho_1$ ,  $\lambda_1$ ,  $\lambda_4$ ,  $\alpha_2$ , the combinations  $\alpha_{34}$  as well as  $\alpha_{124}$  and  $\lambda_{23}^+$  defined respectively in eqs. (2.22) and (2.20), and

$$\lambda_{23}^- \equiv 2\lambda_2 - \lambda_3 \tag{2.51}$$

In principle, one could extract constraints on these parameters directly from the data. But it turns out more interesting to re-express some of these parameters in terms of observables. This has been the method used in the Standard Model where the choice of the input scheme was depending on the observables to be determined, see for instance [47]. One may for example trade  $g_L$ ,  $g_Y$ ,  $\epsilon v_R$  and one of the parameters of the Higgs potential for the  $Z$ -boson mass  $m_Z$ , the electromagnetic constant  $\alpha$ , the Fermi constant  $G_F$  and the light scalar mass  $M_h$  as done in the SM. Instead of using  $g_R$  and  $v_R$ , we may use the co-sine of the mixing angle  $c_R$  and the mass of the heavy gauge boson  $W'$ . Our final set of parameters will be given in section 6 after having discussed our strategy to perform the fits.

We aim at constraining some of these parameters from the phenomenology of the weak gauge bosons. Before considering electroweak precision observables, it is interesting to discuss the constraints coming from general requirements, namely, the unitarity of processes involving these gauge bosons.

### 3 Constraints from tree-level unitarity

Assuming a weakly coupled theory, bounds on the parameters of the left-right models and more specifically on the masses of additional scalar bosons can be obtained from unitarity arguments on tree-level scattering amplitudes. Ref. [48] investigated such bounds on the mass of the scalar boson in the SM from the scatterings of the longitudinally polarized gauge bosons  $Z$  and  $W$ . For instance, expanding the scattering amplitude  $T(s, \cos\theta)$  in partial waves

$$T(s, \cos\theta) = 16\pi \sum_J a_J(s) P_J(\cos\theta) \tag{3.1}$$

Ref. [48] found at large  $s$  and at tree level that in the presence of a scalar  $h$ , the coefficient associated with the  $J = 0$  partial wave amplitude of the  $ZZ \rightarrow ZZ$  scattering amplitude is given by

$$a_0 \rightarrow -3\sqrt{2}G_F M_h^2 / (16\pi) = -3M_h^2 / (16\pi v^2), \tag{3.2}$$

where in the second equality one has used the relation between the Fermi constant  $G_F$  and the electroweak symmetry breaking scale  $v$ . Due to unitarity, one must have  $|a_0| < 1$  implying a bound on the SM Higgs mass  $M_h^2 < 8\pi\sqrt{2}/(3G_F)$ .

The effects of multiple scalar bosons have been studied some years later in ref. [49], and they have been considered extensively in the literature for various scenarios of new physics, e.g., refs. [50, 51] where radiative corrections have been considered in some cases. Note that such perturbative bounds have also been studied for the TLRM, for instance in refs. [52–55]. The scattering processes for scalar bosons were also discussed, for example in ref. [53] while scattering processes involving both gauge and scalar bosons were considered in ref. [48].

In the DLRM of interest here, we will focus on the scattering of longitudinally polarized gauge bosons. We will work in the unitarity gauge and in the limit where  $s$  is larger than the masses of all the particles involved. The behaviour of the  $T$ -matrix at large  $E$  (where  $E$  denotes the general large energy scale considered,  $s \simeq t \simeq E^2$ , where  $s, t, u$  are the usual Mandelstam variables) allows for some checks of the calculation. Indeed the particular structure of some of the couplings of the DLRM is required to prevent the presence of  $\mathcal{O}(s^2)$  terms in various scattering amplitudes.<sup>7</sup>

### 3.1 Constraints from the scattering of light gauge bosons

We consider first the scattering of the light gauge bosons and their modification compared to the SM results.

- $ZZ \rightarrow ZZ$

It is straightforward to generalise the expression obtained in the SM to the DLRM case. One has in the large  $s$  limit

$$a_0 = -\frac{3\sqrt{2}G_F}{16\pi} \left( M_h^2 + \epsilon^2 \frac{(c^{H_3})^2}{k^2} M_{H_3}^2 + \mathcal{O}(\epsilon^4) \right) \quad (3.4)$$

We recover the SM expression supplemented by a contribution from the analogue of the light Higgs boson in the right sector, namely  $H_3^0$ . The other scalars are further suppressed by  $\epsilon^2$  compared to the latter. Note that the two terms in eq. (3.4) are in fact of the same order in  $\epsilon$  since  $M_h^2 = \mathcal{O}(\epsilon^2)$ . Using the experimental values of  $G_F$  and  $M_h$ , we see that the first term is very small,  $3M_h^2\sqrt{2}G_F/(16\pi) \sim 0.015$ , and the condition  $|a_0| < 1$  essentially gives a constraint on the product  $\epsilon^2(c^{H_3})^2 M_{H_3}^2/k^2$ .

- $WW \rightarrow WW$

Additional diagrams involving the exchange of  $Z'$  or  $H_3^0$  are present in the DLRM compared to the SM ones. The computation of this scattering process is a bit more

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<sup>7</sup>For instance, the couplings  $c^{H_3}$  of  $H_3^0$  to  $W$  and  $Z$ , and the coupling  $c_{\chi_{R,r}}^{h^0}$  of  $h^0$  to  $W'$  and  $Z'$  must be equal, see eqs. (2.25) and (B.9). Furthermore the following relation must be obeyed

$$\sum_{i=1}^2 S_i^2 = \sum_{i=1}^2 \frac{t_i^2}{1 + t_i^2 + (r + wt_i)^2} = \frac{1 + r^2}{k^2} \quad (3.3)$$

where the  $S_i$  are defined in eq. (2.30).

involved since one has to determine the rotation matrix to the physical gauge fields up to  $\mathcal{O}(\epsilon^4)$  in order to check that the  $T$ -matrix does not grow faster than expected with the energy. One gets the following modified expressions compared to the ones in section 2.3:

$$\begin{aligned} Z_\mu &= c_W \left( 1 - \frac{c_R^2 s_R^2}{2s_W^2} k^2 z_h^2 \epsilon^4 \right) W_{L,\mu}^3 + \dots \\ W_\mu &= \left( 1 - \frac{2c_W^2 s_R^2}{s_W^2} r^2 \epsilon^4 \right) W_{L,\mu}^\pm + \dots \end{aligned} \quad (3.5)$$

where we only show the terms actually needed for our purpose.

$E^4$  contributions to the  $T$  matrix come from diagrams with the exchange of gauge bosons and the contact interaction. They involve new terms compared to the SM proportional to  $\epsilon^4/M_W^4$  and thus formally of the same order than the SM ones. However the sum of these contributions cancel and we are left in the  $u$  channel with:

$$T_{\text{vector}} = -\frac{g_L^2 u}{4M_W^2} (1 - 3\epsilon^2 w_h^2) \quad (3.6)$$

The contributions from the Higgs sector in the  $s$  and  $t$  channels are given by:

$$\begin{aligned} T_{\text{scalar}} &= -\frac{g_L^2}{4M_W^2} \left[ \left( 1 - \epsilon^2 \left( 3w_h^2 - \frac{2}{k} c^{h^0} \right) \right) \left( \frac{s^2}{s - M_h^2} + \frac{t^2}{t - M_h^2} \right) \right. \\ &\quad \left. - \epsilon^2 \frac{(c^{H_3})^2}{k^2} \left( \frac{s^2}{s - M_{H_3}^2} + \frac{t^2}{t - M_{H_3}^2} \right) \right] \end{aligned} \quad (3.7)$$

It is easy to check that the  $E^2$  growth cancels when summing up these two types of contributions using the relation eq. (B.13) between  $c^{h^0}$  and  $c^{H_3}$ , so that one finally gets in the large  $s$  limit

$$a_0 = -\frac{2\sqrt{2}G_F}{16\pi} \left( M_h^2 + \epsilon^2 \frac{(c^{H_3})^2}{k^2} M_{H_3}^2 + \mathcal{O}(\epsilon^4) \right) \quad (3.8)$$

which leads to a weaker bound than  $ZZ$  scattering.

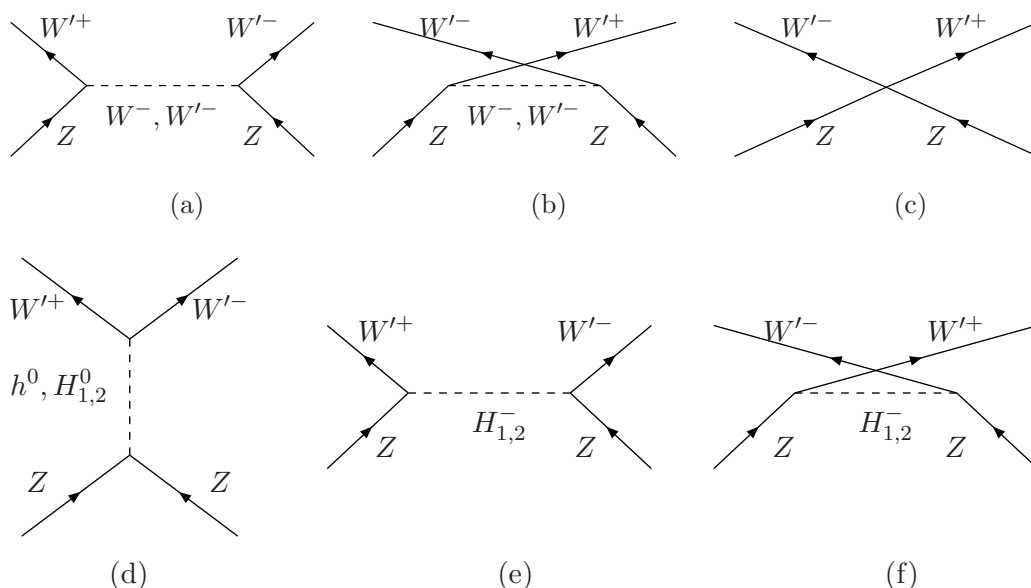
### 3.2 Constraints from the scattering of heavy gauge bosons

Let us consider now the scatterings of heavy gauge bosons with longitudinal polarisations.

- $Z'Z' \rightarrow Z'Z'$

This is the analogue of the  $ZZ$  scattering in the SM, so that this process is expected to constrain the mass of  $H_3^0$ . Like in the case of the SM, no exchanges of gauge bosons are possible and only the three neutral scalar exchange diagrams (d, e, f) shown in figure 1 contribute. The  $E^2$  terms cancel among the diagrams due to the relation between the Mandelstam variables  $s + t + u = 4M_{Z'}^2$ . The partial wave amplitude in the large  $s$  limit up to order  $\mathcal{O}(\epsilon^2)$  reads

$$a_0^{Z'Z'} = -\frac{3}{16\pi v_R^2} \left( M_{H_3^0}^2 + \frac{\epsilon^2}{k^2} \left( (c_R^2 k z_h + s_R^2 w^2 + k c_{\chi_{R,r}}^{h^0})^2 M_{h^0}^2 + \sum_{i=1}^2 ((-c_R^2 + s_R^2) w S_i + c_{\chi_{R,r}}^{H_i})^2 M_{H_i^0}^2 \right) \right) \quad (3.9)$$



**Figure 1.** Diagrams contributing to  $ZZ \rightarrow W'W'$ . The other scatterings discussed in the text can be obtained from these diagrams changing the labelling of the lines accordingly. No contact term is present for  $Z'Z' \rightarrow WW$  and the first line is absent when only  $Z$  or  $Z'$  gauge bosons scatter.

As noted previously one gets at LO in  $\epsilon$  a similar relation to the SM case, eq. (3.2), but in the right sector, with  $H_3^0$  being the equivalent of the Standard Model Higgs boson. One can also rewrite  $M_{H_3^0}$  at LO in  $\epsilon$  in terms of the  $\rho_1$  parameter of the scalar potential, eq. (B.3), leading to the following range:

$$0 < \rho_1 < \frac{8\pi}{3}, \quad (3.10)$$

where the lower bound comes from its relation to  $M_{H_3}$ .

- $W'W' \rightarrow W'W'$

This scattering is the right-sector analogue of  $WW \rightarrow WW$  scattering. There are two  $Z$ -, two  $Z'$ - and two  $\gamma$ -exchange diagrams together with a contact diagram. The triple and quadruple gauge couplings involving the  $W'$  gauge boson are proportional to:

$$i \frac{e^2}{c_W^2 s_R^2} \quad (3.11)$$

All these diagrams grow like  $E^4$  at high energy but their sum yields

$$T_{\text{vector}} = \frac{g_R^2}{4M_{W'}^2} u \quad (3.12)$$

Furthermore, at lowest order in  $\epsilon$ , only  $H_3^0$  contributes. Its  $E^2$  growth cancels that from the gauge bosons so that one finally gets

$$T_{\text{scalar}} = -\frac{M_{H_3^0}^2}{v_R^2} \left( \frac{s}{s - M_{H_3^0}^2} + \frac{t}{t - M_{H_3^0}^2} \right) \quad (3.13)$$

As expected, this scattering essentially gives a bound on the mass of  $H_3^0$  which is somewhat weaker than in the previous case.

### 3.3 Constraints from scatterings involving both light and heavy gauge bosons

Finally we discuss the cases coming from the scattering of heavy and light gauge bosons, both with longitudinal polarisations.

- $W'W' \rightarrow WW$

This scattering gets contributions from  $Z$ ,  $Z'$ ,  $\gamma$  and scalars in the  $s$  and  $t$  channels, but there is no contact term. Summing all the gauge bosons diagrams using the couplings given in appendix C yields:

$$T_{\text{vector}} = \frac{s(1+r^2)}{k^2 v_R^2} + 2t \frac{(1+r^4 + (1+r^2)w^2)}{k^4 v_R^2} \quad (3.14)$$

On the other hand the scalar exchange leads to:

$$T_{\text{scalar}} = -\frac{1}{k^2 v_R^2} \left( (1+r^2 + kc_{\chi_{R,r}}^{h^0}) \frac{s^2}{s-M_h^2} + \frac{4r^2}{k^2} \frac{t^2}{t-M_h^2} + \sum_i F_i^2 k^2 t^2 \left( \frac{1}{t-M_{H_i^0}^2} + \frac{1}{t-M_{A_i^0}^2} \right) + c^{H_3} \frac{s^2}{s-M_{H_3^0}^2} \right) \quad (3.15)$$

Using eq. (B.9) and the fact that

$$\sum_i F_i^2 k^2 = 1 + r^2 \left( 1 - \frac{4}{k^2} \right) \quad (3.16)$$

it is easy to verify that the leading terms in  $s$  and  $t$  cancel exactly the ones in eq. (3.14), so that one gets in the large  $s$  limit

$$a_0^{W'W} = -\frac{1}{16\pi k^2 v_R^2} \left( \frac{k^2(1+r^2 + kc_{\chi_{R,r}}^{h^0}) + 4r^2}{k^2} M_h^2 + \sum_{i=1}^2 F_i^2 k^2 (M_{H_i^0}^2 + M_{A_i^0}^2) + c^{H_3} M_{H_3^0}^2 \right) \quad (3.17)$$

When  $w = 0$ , only one of the coupling  $F_i$  does not vanish and thus the sum is limited to a single value of  $i$ . Furthermore at LO in  $\epsilon$ , the mass  $M_h$  of the light Higgs boson can be neglected while the masses of the CP-even and CP-odd scalars are equal. This leads to the following bound

$$\frac{M_{H^0}^2}{8\pi v_R^2} \frac{(1-r^2)^2}{(1+r^2)^2} + \frac{1}{16\pi} \frac{(1+r^2)\alpha_1 + 2r\alpha_2 + r^2\alpha_3 + \alpha_4}{1+r^2} < 1 \quad (3.18)$$

which translates into a bound on a specific combination of four parameters of the Higgs potential, see eq. (2.20):

$$\alpha_{124} - r\alpha_2 + \frac{1}{1+r^2} \alpha_{34} < 16\pi \quad (3.19)$$

using the relation between  $M_{H_i^0}^2$  and  $\alpha_{34}$  at LO in  $\epsilon$ .



- $Z'Z \rightarrow Z'Z$

At lowest order there are three contributions to the amplitude

$$T = -\frac{1}{v_R^2 k^2} \left[ z_h^2 \left( \frac{s^2}{s - M_{h^0}^2} + \frac{t^2}{t - M_{h^0}^2} \right) + \sum_{i=1}^2 w^2 S_i^2 \left( \frac{s^2}{s - M_{H_i^0}^2} + \frac{t^2}{t - M_{H_i^0}^2} \right) + (c_R^2 k z_h + s_R^2 w^2 + k c_{\chi_{R,r}}^{h^0}) \frac{u^2}{u - M_{h^0}^2} + c^{H_3} \frac{u^2}{u - M_{H_3^0}^2} \right]$$

It is easy to check that the  $E^4$  terms cancel each other using eq. (3.3), the relations between the coefficients  $c$ , and the fact that

$$-z_h^2 - w^2 \left( 1 - \frac{w^2}{k^2} \right) + (c_R^2 k z_h + s_R^2 w^2) = 0. \quad (3.20)$$

Thus one finally gets in the large  $s$  limit

$$a_0^{Z'Z} = -\frac{1}{16\pi v_R^2 k^2} \left( (2z_h^2 + (c_R^2 k z_h + s_R^2 w^2 + k c_{\chi_{R,r}}^{h^0})) M_{h^0}^2 + \sum_{i=1}^2 2w^2 S_i^2 M_{H_i^0}^2 + c^{H_3} M_{H_3^0}^2 \right) \quad (3.21)$$

Imposing the tree unitarity bound  $|a_0^{Z'Z}| < 1$  constrains the combination of parameters present in the equation above.

- $Z'Z' \rightarrow WW$

The diagrams contributing to this scattering process are shown in figure 1. Contrary to  $ZW \rightarrow ZW$  no contact diagrams are present. At lowest order in  $\epsilon$ , the  $W'$  exchange diagram does not contribute and one gets:

$$T_{\text{vector}} = e^2 \frac{c_R^2 s_R^2 c_W^2}{s_W^4} k^2 z_h^2 \frac{M_{Z'}^2 s}{M_W^4} \epsilon^4 \quad (3.22)$$

Using the expressions for the gauge boson masses, this equation reads

$$T_{\text{vector}} = k^2 z_h^2 \frac{s}{v_R^2 k^2} \quad (3.23)$$

The exchanges of the two neutral Higgs bosons  $h^0$  and  $H_3^0$  in the  $s$  channel and charged scalars  $H_i^\pm$  in the  $t$  and  $u$  channels give

$$T_{\text{scalar}} = -\frac{1}{v_R^2 k^2} \left( (c_R^2 k z_h + s_R^2 w^2 + k c_{\chi_{R,r}}^{h^0}) \frac{s^2}{s - M_{h^0}^2} + c^{H_3} \frac{s^2}{s - M_{H_3^0}^2} + w^2 \sum_i S_i^2 \left( \frac{t^2}{t - M_{H_i^\pm}^2} + \frac{u^2}{u - M_{H_i^\pm}^2} \right) \right) \quad (3.24)$$

The  $E^2$  growth of this amplitude cancels the vector exchange due to the relations, eqs. (3.3) and (3.20). One finally gets for  $a_0^{Z'W}$  in the large  $s$  limit

$$a_0^{Z'W} = -\frac{1}{16\pi v_R^2 k^2} \left( (c_R^2 k z_h + s_R^2 w^2 + k c_{\chi_{R,r}}^{h^0}) M_{h^0}^2 + 2w^2 \sum_i S_i^2 M_{H_i^\pm}^2 + c^{H_3} M_{H_3^0}^2 \right) \quad (3.25)$$

At LO the charged and neutral scalar masses are equal so that neglecting the mass of the light Higgs boson one obtains the same constraint as in eq. (3.21) at that same order.

- $ZZ \rightarrow W'W'$

This case involves a contact diagram and the contribution of the  $W'$  exchange. The latter is similar to the SM one and thus involves the ratio  $M_Z^2 s/M_{W'}^4$ . Consequently the sum of these two contributions is  $\mathcal{O}(\epsilon^2)$ . The  $W$  exchange yields at leading order in  $\epsilon$

$$T_{\text{vector}} = 4e^2 r^2 \frac{s_R^2}{s_W^4} \frac{M_{W'}^2 s}{M_Z^2 M_W^2} \epsilon^4 = 4r^2 \frac{s}{v_R^2 k^4} + \mathcal{O}(\epsilon^2) \quad (3.26)$$

where the expressions of the masses at leading order in  $\epsilon$ , eqs. (2.44) and (2.47) have been used in the second line of eq. (3.26). The exchange of the neutral light Higgs boson  $h^0$  in the  $s$  channel and charged scalars in the  $t$  and  $u$  channels gives

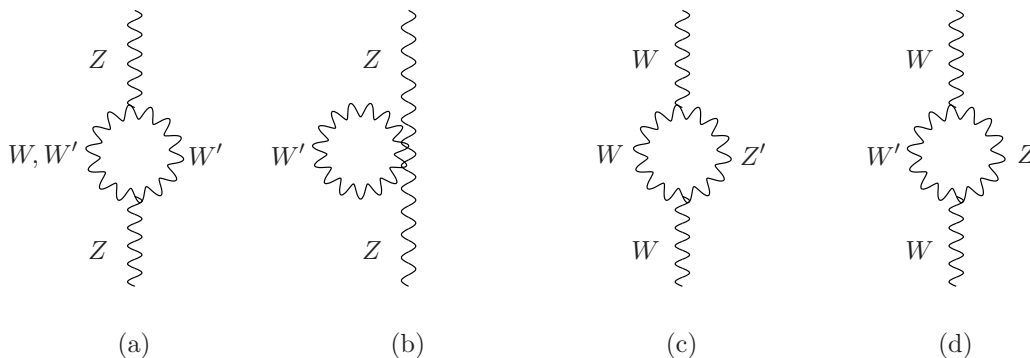
$$T_{\text{scalar}} = -\frac{1}{v_R^2 k^2} \left( (1 + r^2 + kc_{\chi_{R,r}}^{h^0}) \frac{s^2}{s - M_{h^0}^2} + c^{H_3} \frac{s^2}{s - M_{H_3^0}^2} + \sum_i F_i^2 k^2 \left( \frac{t^2}{t - M_{H_i^0}^2} + \frac{u^2}{u - M_{H_i^0}^2} \right) \right) \quad (3.27)$$

In order to determine the  $ZW'$  coupling of the charged scalar boson in table 14 one needs to compute the coefficient of the term proportional to  $\epsilon \chi_R$  in the decomposition of  $H_1^\pm$  given in eq. (2.25). It contributes to the coupling with a multiplicative factor  $\sin^2 \theta_W$  while the LO terms in the decomposition contributes with  $\cos^2 \theta_W$ .

Using eq. (3.16) the  $E^2$  terms cancel in the sum of the vector and scalar contributions and one gets for  $a_0^{ZW'}$  in the large  $s$  limit:

$$a_0^{ZW'} = -\frac{1}{16\pi k^2 v_R^2} \left( (1 + r^2 + kc_{\chi_{R,r}}^{h^0}) M_{h^0}^2 + 2 \sum_i F_i^2 k^2 M_{H_i^0}^2 + c^{H_3} M_{H_3^0}^2 \right) \quad (3.28)$$

Neglecting the mass of the light Higgs boson, one gets the same expression and consequently the same unitarity bounds as for  $W'W' \rightarrow WW$  scattering.



**Figure 2.** Self-energy diagrams for the  $W$  and  $Z$  bosons in Left-Right models: only gauge bosons are considered in the loops.

### 3.4 Summary

Summarizing our results one has altogether four bounds from the unitarity conditions on the masses of the scalar bosons at LO in the  $\epsilon$  expansion:<sup>8</sup>

$$\begin{aligned}
 \frac{(c^{H_3})^2}{k^4} \frac{M_{H_3^0}^2}{v_R^2} &< 16 \frac{\pi}{3} \\
 \frac{M_{H_3^0}^2}{v_R^2} &< 16 \frac{\pi}{3} \\
 2 \sum_i F_i^2 \frac{M_{H_i^\pm}^2}{v_R^2} + \frac{c^{H_3}}{k^2} \frac{M_{H_3^0}^2}{v_R^2} &< 16\pi \\
 2 \frac{w^2}{k^2} \sum_i S_i^2 \frac{M_{H_i^\pm}^2}{v_R^2} + \frac{c^{H_3}}{k^2} \frac{M_{H_3^0}^2}{v_R^2} &< 16\pi
 \end{aligned} \tag{3.29}$$

Note that we have divided all the masses by the characteristic scale  $v_R$  of LR symmetry breaking. Indeed this scale is unknown. A way of probing indirectly such a scale is to use the ElectroWeak Precision Observables (EWPO), as will be discussed in the following. Assuming that these observables will allow for a precise determination of  $v_R$  one can, from the second equation above, extract an upper bound on the mass of  $H_3^0$  at LO in  $\epsilon$  in exactly the same way as the knowledge of  $G_F$  in the SM allowed to put bounds on the mass of the SM Higgs boson. The masses of the other scalar bosons, namely  $M_{H_i^0} \sim M_{A_i^0} \sim M_{H_i^\pm}$  (equal at LO in  $\epsilon$ ) ( $i = 1, 2$ ) involve some extra free parameters among which there are combinations of parameters from the scalar potential, so that in practice the EWPO alone might not be sufficient to extract them. We will come back to the role played by unitarity in probing the scalar sector of the model after introducing the EW precision observables that will be explored in a global analysis.

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<sup>8</sup>We do not distinguish between the LO masses and their expansion up to  $\mathcal{O}(\epsilon^2)$  here.

## 4 $\Delta r$ and the mass of the $W$ at one loop

Another important constraint in any electroweak model comes from the mass of the  $W$  and its connection with the (muon) Fermi decay constant, encoded in  $\Delta r$ . In the SM significant progress has been made in the computation of these quantities as well as of the electroweak precision observables which will be discussed in section 5, leading to reduced theoretical uncertainties. The state of the art for the mass of the  $W$  is a full two-loop electroweak evaluation with higher order QCD corrections and resummation of reducible terms, see ref. [56] for a status report on precision theoretical calculations within the SM (one of the earliest computations can be found in ref. [57]). In theories beyond the Standard Model, some one-loop determinations of the  $W$  mass have also been performed, see for example ref. [58] for the TLRM or more recently in ref. [59] for the Next-to-Minimal Supersymmetric extension of the Standard Model.

There are also computations of the related quantity  $\Delta\rho$  which is defined as the difference of the  $Z$  and  $W$  self-energies at zero momentum transfer, each being weighted by the inverse of the square of the mass of the respective gauge boson, for example in the framework of the two Higgs doublet model [60, 61]. In the SM the loop corrections to  $\Delta\rho$  are finite, but this feature is not necessarily true in models beyond the SM. Here we will perform a one-loop calculation of the muon  $\Delta r$  parameter in the DLRM focusing mainly on the contributions involving ratios of the heavy gauge bosons and the new scalars to the light particles since one may expect contributions proportional to large logarithms of the form  $\log \epsilon^2$ . In order to perform such a computation we first need to discuss the renormalisation of the model.

### 4.1 Renormalisation

In the SM a number of popular renormalisation schemes are used to compute radiative corrections to observables. One of the mostly used when dealing with ElectroWeak Precision Observables (EWPO) is the on-mass shell scheme,<sup>9</sup> in which a set for parameter renormalisation is given in terms of the electric charge and the masses of the various particles, the gauge bosons, the Higgs and the quarks. In this scheme the tree-level formula  $s_W^2 = 1 - M_Z^2/M_W^2$  is promoted to a definition of the renormalised  $s_W$  to all orders in perturbation theory.

We will compute the (renormalised)  $W$  self-energy in the on-mass shell scheme. Following e.g. ref. [47], we associate multiplicative renormalisation constants to each free

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<sup>9</sup>This is in particular the scheme used in the Fortran package Zfitter [62], one of the earliest open source software projects for the computation of fermion pair production and radiative corrections at high energy  $e^+e^-$  colliders, which we will use to perform our fits to electroweak precision observables.

parameter and each multiplet of fields in the Lagrangian:

$$\begin{aligned}
 W_\mu^{L,a} &\rightarrow (Z_2^W)^{1/2} W_\mu^{L,a} & W_\mu^{R,a} &\rightarrow (Z_2^{W'})^{1/2} W_\mu^{R,a} \\
 B_\mu &\rightarrow (Z_2^B)^{1/2} B_\mu & g_X &\rightarrow (Z_1^B)(Z_2^B)^{-3/2} g_X \\
 g_L &\rightarrow (Z_1^W)(Z_2^W)^{-3/2} g_L & g_R &\rightarrow (Z_1^{W'})(Z_2^{W'})^{-3/2} g_R \\
 v_L &\rightarrow (Z^{H_L})^{1/2} v_L (1 - \delta v_L) & v_R &\rightarrow (Z^{H_R})^{1/2} v_R (1 - \delta v_R) \\
 \kappa_1 &\rightarrow (Z^\phi)^{1/2} \kappa_1 (1 - \delta \kappa_1) & \kappa_2 &\rightarrow (Z^\phi)^{1/2} \kappa_2 (1 - \delta \kappa_2)
 \end{aligned} \tag{4.1}$$

Introducing the renormalised constants in the Lagrangian and choosing the eigenmass state basis, we can define new renormalised quantities up to  $\mathcal{O}(\epsilon^2)$ , namely

$$\begin{pmatrix} \delta Z_i^\gamma \\ \delta Z_i^Z \\ \delta Z_i^{Z'} \end{pmatrix} = \begin{pmatrix} s_W^2 & c_W^2 s_R^2 & c_W^2 c_R^2 \\ c_W^2 & s_R^2 (s_W^2 - 2c_R^2 k z_h \epsilon^2) & c_R^2 (s_W^2 + 2s_R^2 k z_h \epsilon^2) \\ 0 & c_R^2 (1 + 2s_R^2 k z_h \epsilon^2) & s_R^2 (1 - 2c_R^2 k z_h \epsilon^2) \end{pmatrix} \begin{pmatrix} \delta Z_i^W \\ \delta Z_i^{W'} \\ \delta Z_i^B \end{pmatrix} \tag{4.2}$$

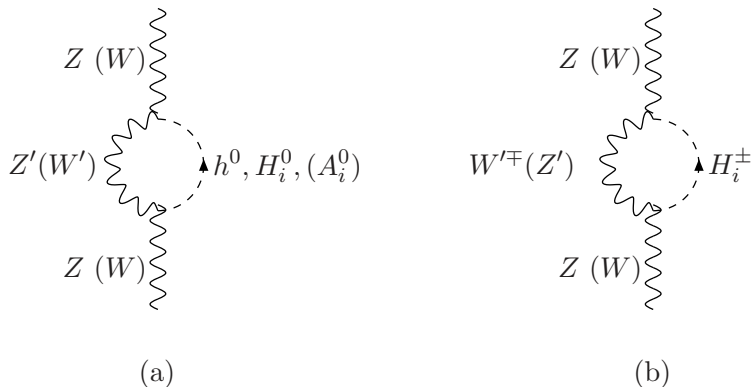
with the standard definition  $\delta Z_i^X = Z_i^X - 1$ . The SM result is recovered at LO in  $\epsilon$  with  $c_R = 1$  for which the right handed fields decouple from the left handed ones, see for example ref. [47].

One can express the self-energies of the gauge bosons in terms of these renormalised quantities, leading with obvious notations:

$$\begin{aligned}
 \hat{\Sigma}^{\gamma\gamma}(q^2) &= \Sigma^{\gamma\gamma}(q^2) + \delta Z_2^\gamma q^2 \\
 \hat{\Sigma}^{ZZ}(q^2) &= \Sigma^{ZZ}(q^2) - \delta M_Z^2 + \delta Z_2^Z (q^2 - M_Z^2) \\
 \hat{\Sigma}^{WW}(q^2) &= \Sigma^{WW}(q^2) - \delta M_W^2 + \delta Z_2^W (q^2 - M_W^2) \\
 \hat{\Sigma}^{Z'Z'}(q^2) &= \Sigma^{Z'Z'}(q^2) - \delta M_{Z'}^2 + \delta Z_2^{Z'} (q^2 - M_{Z'}^2) \\
 \hat{\Sigma}^{W'W'}(q^2) &= \Sigma^{W'W'}(q^2) - \delta M_{W'}^2 + \delta Z_2^{W'} (q^2 - M_{W'}^2) \\
 \hat{\Sigma}^{ZZ'}(q^2) &= \Sigma^{ZZ'}(q^2) - \delta M_{ZZ'}^2 + \delta Z_2^{ZZ'} \left( q^2 - \frac{1}{2}(M_Z^2 + M_{Z'}^2) \right) \\
 \hat{\Sigma}^{WW'}(q^2) &= \Sigma^{WW'}(q^2) - \delta M_{WW'}^2 + \delta Z_2^{WW'} \left( q^2 - \frac{1}{2}(M_W^2 + M_{W'}^2) \right) \\
 \hat{\Sigma}^{\gamma Z}(q^2) &= \Sigma^{\gamma Z}(q^2) - \delta M_{\gamma Z}^2 + \delta Z_2^{\gamma Z} \left( q^2 - \frac{1}{2} M_Z^2 \right) \\
 \hat{\Sigma}^{\gamma Z'}(q^2) &= \Sigma^{\gamma Z'}(q^2) - \delta M_{\gamma Z'}^2 + \delta Z_2^{\gamma Z'} \left( q^2 - \frac{1}{2} M_{Z'}^2 \right)
 \end{aligned} \tag{4.3}$$

where the quantities with a hat define the renormalised self-energies. Furthermore we can use the following on-shell renormalisation conditions:

$$\begin{aligned}
 \text{Re} \hat{\Sigma}^{WW}(M_W^2) &= \text{Re} \hat{\Sigma}^{ZZ}(M_Z^2) = \text{Re} \hat{\Sigma}^{W'W'}(M_{W'}^2) = \text{Re} \hat{\Sigma}^{Z'Z'}(M_{Z'}^2) = 0 \\
 \hat{\Sigma}^{ZZ'}(0) &= \hat{\Sigma}^{WW'}(0) = 0
 \end{aligned} \tag{4.4}$$



**Figure 3.** Same as in figure 2 but with one heavy gauge boson and one scalar boson in the loops.

as well as the QED-like conditions

$$\begin{aligned}
 \hat{\gamma}_\mu^{\gamma ee}(q^2 = 0, \not{q}_1 = \not{q}_2 = m_e) &= ie\gamma_\mu \\
 \hat{\Sigma}^{\gamma Z}(0) &= \hat{\Sigma}^{\gamma Z'}(0) = 0 \\
 \frac{d\hat{\Sigma}^{\gamma\gamma}}{dq^2}(0) &= 0
 \end{aligned}
 \tag{4.5}$$

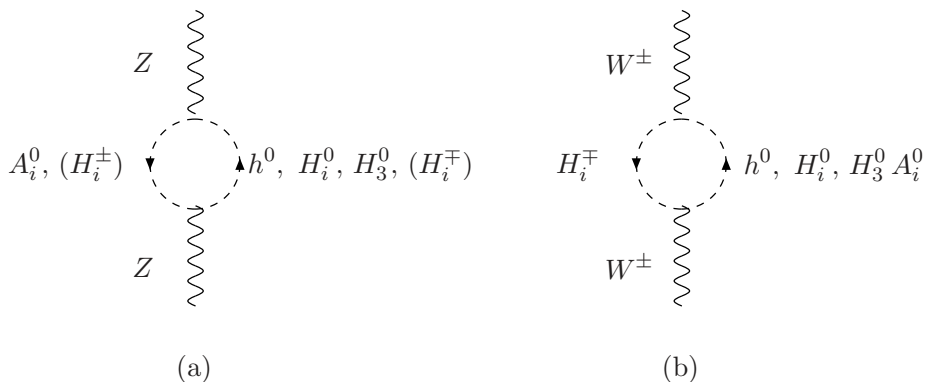
where one has imposed the absence of mixing at  $q^2 = 0$ .

A relation between  $\delta Z_1^\gamma$  and  $\delta Z_2^\gamma$  can be obtained from the renormalisation of the charge discussed in appendix D. Finally the various  $\delta Z_i$  are given up to NLO in  $\epsilon$  by:

$$\begin{aligned}
 \delta Z_2^\gamma &= -\Pi^\gamma(0) \equiv \frac{d\Sigma^{\gamma Z}}{dq^2}(0) \\
 \delta Z_1^\gamma &= -\Pi^\gamma(0) + \frac{s_W}{c_W} \frac{\Sigma^{\gamma Z}(0)}{M_Z^2} \left(1 + \frac{s_R^2 k z_h}{s_W^2} \epsilon^2\right) + \frac{s_R}{c_R c_W} \frac{\Sigma^{\gamma Z'}(0)}{M_Z^2} \left(1 + \frac{s_W^2 z_h}{k s_R^2} (c_R^2 k z_h + s_R^2 w^2) \epsilon^2\right) \\
 \delta Z_2^W &= \delta Z_2^{LO} + \epsilon^2 \delta Z_2^{NLO} \\
 \delta Z_1^W &= \delta Z_1^{LO} + \epsilon^2 \left( \delta Z_2^{NLO} - z_h c_W \left( w^2 \frac{s_R}{k c_R} + \frac{c_R}{s_R} z_h \right) \frac{\Sigma^{\gamma Z'}(0)}{M_Z^2} \right)
 \end{aligned}
 \tag{4.6}$$

with

$$\begin{aligned}
 \delta Z_2^{NLO} &= \frac{c_W}{k^2 s_W^2} \left[ (4r^2 - k^2 z_h^2) \left( \frac{1}{c_W} \delta Z_{ZW\gamma} + c_W \left( \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_{W'}^2}{M_{W'}^2} \right) - c_W \delta Z_{Z'W'\gamma} \right) \right. \\
 &\quad \left. - 2c_R^2 c_W k^3 z_h \left( \frac{\delta M_{Z'}^2}{M_{Z'}^2} - \frac{\delta M_{W'}^2}{M_{W'}^2} \right) \right. \\
 &\quad \left. + s_R \left( \frac{4k^2 r c_W^2}{s_W} \frac{\Sigma^{WW'}(0)}{M_W^2} - 2k^3 z_h \frac{c_R c_W}{s_W} \delta Z_{ZZ'\gamma} - \frac{2}{c_R} k z_h (2w^2 + 3k z_h) \frac{\Sigma^{\gamma Z'}(0)}{M_Z^2} \right) \right]
 \end{aligned}
 \tag{4.7}$$



**Figure 4.** Self-energy diagrams for the  $W$  and  $Z$  boson in Left-Right models involving scalars only.

and one has defined

$$\begin{aligned}
 \delta Z_{ZW\gamma} &= 2 \frac{c_W}{s_W} \frac{\Sigma^{\gamma Z}(0)}{M_Z^2} + \frac{c_W^2}{s_W^2} \left( \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) \\
 \delta Z_{Z'W'\gamma} &= 2 \frac{c_R}{s_R c_W} \frac{\Sigma^{\gamma Z'}(0)}{M_{Z'}^2} + \frac{c_R^2}{s_R^2} \left( \frac{\delta M_{Z'}^2}{M_{Z'}^2} - \frac{\delta M_{W'}^2}{M_{W'}^2} \right) \\
 \delta Z_{ZZ'\gamma} &= \frac{\Sigma^{\gamma Z'}(0)}{M_{Z'}^2} + \frac{c_W}{s_W} \frac{\Sigma^{ZZ'}(0)}{M_{Z'}^2}
 \end{aligned} \tag{4.8}$$

The LO terms are the SM like expressions in the limit  $s_R = 0$ :

$$\begin{aligned}
 \delta Z_2^{LO} &= -\Pi^\gamma(0) + \delta Z_{ZW\gamma} \\
 \delta Z_1^{LO} &= -\Pi^\gamma(0) + \frac{3 - 2s_W^2}{s_W c_W} \frac{\Sigma^{\gamma Z}(0)}{M_Z^2} + \frac{s_R}{c_R c_W} \frac{\Sigma^{\gamma Z'}(0)}{M_{Z'}^2} + \frac{c_W^2}{s_W^2} \left( \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right)
 \end{aligned} \tag{4.9}$$

The renormalisation conditions, eqs. (4.4) and (4.5), have been used to derive these expressions. For completeness the wave-function renormalisation relevant for the heavy gauge bosons is given in appendix D as well as the renormalisation of  $s_W$ . Note that in all these expressions the self-energies and the counterterms are divided by the squared mass of a gauge boson in such a way that the ratio is a quantity of order  $\mathcal{O}(\epsilon^0)$  at LO. In principle  $\Sigma^{\gamma Z'}(0)$  and  $\Sigma^{ZZ'}(0)$  are also quantities of order one, however at LO the former is equal to  $-c_W/s_W \Sigma^{ZZ'}(0)$  so that the combination of these two quantities appearing in  $\delta Z_{ZZ'\gamma}$  turns out to be of order  $\mathcal{O}(\epsilon^2)$ .

Few last remarks are in order. On the right-hand side of the expressions above, the self-energies contain contributions from the heavy particles. Moreover, the heavy particles will only start to contribute at  $\mathcal{O}(\epsilon^2)$  in the SM-like contributions, so that one recovers the SM result at leading order in  $\epsilon$ . Finally one can perform a similar on-shell subtraction for the scalar self-energies and replace eight of the scalar parameters by the eight scalar masses, the remaining parameters being taken as  $\overline{\text{MS}}$  running parameters and thus renormalisation scale dependent. However, the renormalisation of the Higgs sector is not really needed for our purposes here.

## 4.2 Computation of $\Delta r$

Let us now turn to the muon decay amplitude and the determination of  $\Delta r$ . The amplitude can be decomposed as:

$$\mathcal{M} = \mathcal{M}_{LL} + \mathcal{M}_{LR} + \mathcal{M}_{RL} + \mathcal{M}_{RR} \quad (4.10)$$

where each part is proportional to

$$\mathcal{M}_{AB} \propto \mathcal{M}_{AB}^0 = (\bar{u}_{\nu_\mu} \gamma_\mu P_A u_\mu) (\bar{u}_e \gamma_\mu P_B v_{\nu_e}) \quad (4.11)$$

It involves in principle the  $W$  and  $W'$  self-energies. It turns out that at the order  $\epsilon^2$  of interest, only the LL part survives, so that only the propagation of the  $W$  boson will contribute. Therefore, like in the SM case, the matrix element of the loop diagrams can be written as a proportionality factor multiplying the Born matrix element

$$\Delta \mathcal{M} = \Delta r \times \mathcal{M}^{\text{Born}} \quad (4.12)$$

leading to a similar expression to the SM case for the (muon) Fermi constant  $G_F$ :

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8s_W^2 M_W^2} (1 + \Delta r) \quad (4.13)$$

with

$$\Delta r = \frac{\hat{\Sigma}_{WW}(0)}{M_W^2} + \text{vertex corrections} + \text{box contribution} \quad (4.14)$$

There are two kinds of potentially large contributions to  $\Delta r$ . The first one comes from terms involving ratios of a heavy mass and a light mass, and in particular logarithms of these ratios. The second one stems from the terms inversely proportional to  $c_R$  and/or  $s_R$ . Indeed if one of these two becomes very small, these contributions (a priori of order one) would be enhanced. For processes with only external light particles, we can focus on the contributions where at least one particle present in the loop is heavy. Indeed, the  $\mathcal{O}(\epsilon^2)$  corrections from the light particles can be safely neglected as they only involve mass ratios of order one leading to small logarithms and they feature no factors inversely proportional to  $c_R$  and/or  $s_R$ .

Let us start with the self-energies. There are three types of diagrams to compute involving: (i) only intermediate gauge bosons, (ii) one gauge boson and one scalar, (iii) only scalars. These diagrams are shown in figure 2 for the self-energies of the light gauge bosons, with at least one heavy particle in the loop. Similar diagrams can easily be drawn for the other self-energies, either with one light and one heavy gauge bosons or with two heavy gauge bosons. Note that we do not show tadpole diagrams here: as in the SM case, we performed the renormalisation of the scalar vacuum expectation values so that one can omit all tadpole diagrams in the renormalised amplitudes and Green functions [63]. The self-energy contributions to  $\Delta r$  in eq. (4.14) can thus easily be obtained from eqs. (4.3), (4.6), (4.8). It can be decomposed as:

$$\Delta r|_{SE} = \Delta r^{LO}|_{LL} + \epsilon^2 (\Delta r^{NLO}|_{LL} + \Delta r|_{LH} + \Delta r|_{HH} + \dots) \quad (4.15)$$



where the lower indices denote whether the particles on the external lines are light (L) or heavy (H) and the ellipsis denotes neglected contributions to be discussed below. The structure of these various contributions can easily be inferred from the SM calculation of the self-energies, see for example [47] where their expressions are explicitly given. It involves sums of terms which are products of couplings of the internal particles to the external ones, which are summarized for the case of the DLRM in tables 15 with the various coefficients appearing in the case of the gauge bosons to two scalars defined in appendix C.2, and known loop functions, modulo some extra factors. Only the following three loop functions appear

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2}; \quad \int \frac{d^4k}{(2\pi)^4} \frac{1; k_\mu k_\nu}{(k^2 - m^2)((k - q)^2 - m'^2)} \quad (4.16)$$

where  $m$  and  $m'$  are the masses of the internal particles and  $q^2$  is the four-momentum squared of the external particle which is either on or off-shell. The loop function with  $k_\mu k_\nu$  in the numerator can be expanded into two Lorentz covariants times scalar coefficients after integration, and only the coefficient proportional to  $g_{\mu\nu}$  is needed here. In the case of  $\Delta r|_{HH}$  one straightforwardly replaces light masses and couplings with their heavy counterparts.  $\Delta r|_{LL}$  contains contributions which are LO and NLO (i.e.,  $\mathcal{O}(\epsilon^2)$ ) with respect to  $\epsilon$ . It turns out that only SM-like contributions arise at LO, namely contributions from the light particles with exactly the same couplings as in the SM, so that

$$\Delta r^{LO}|_{LL} \equiv \Delta r|_{\text{SM}''} \quad (4.17)$$

We can then consider the vertex corrections and the box contribution. The final result for their sum reads:

$$\Delta_{vb} = \Delta_{vb}|_{\text{SM}''} - \frac{3g_L^2 s_R^2 s_W^2}{2c_W^2} \frac{M_W^2}{M_{W'}^2} \log \frac{M_{Z'}^2}{M_W^2} \quad (4.18)$$

where  $\Delta_{vb}|_{\text{SM}''}$  are SM like contributions and one has again only considered the contributions which involve ratios of light to heavy particles.

Up to now we have focused on the contributions from the gauge bosons and scalars. Other contributions might be numerically relevant, in particular from the top quark which are very important in the SM as its contributions are proportional to its mass squared

$$\Delta r_{SM} \sim -\rho_{SM}^{\text{top}} / \tan^2 \theta_W, \quad \rho_{SM}^{\text{top}} = \frac{3G_F}{8\sqrt{2}\pi^2} m_t^2 \quad (4.19)$$

This explains in particular why this quantity has been used to constrain the mass of the top quark within the SM before more direct measurements, e.g., [64]. However it has been found that in theories where  $\rho \neq 1$  the dependence of  $\rho^{\text{top}}$  on the top quark mass can be very different from the SM (and much weaker): for example in models with an extra  $U(1)'$  symmetry it is logarithmic [65]. In TLRM  $\rho_{SM}^{\text{top}}$  is multiplied by a factor  $M_W^2 / (M_{W'}^2 - M_W^2)$  leading to a decreasing contribution from the top as the mass of the  $W'$  increases [66]. Indeed, in presence of new physics in theories with  $\rho \neq 1$  the entire structure of loop corrections is modified and the Appelquist-Carazzone decoupling was found not to hold. This casts some doubts about the validity of the usual implementation

of new physics corrections, which amounts to combining the loop corrections to the SM with modifications from new physics at tree level in this case [65, 67–69]. But this issue has been later discussed in detail in ref. [70] in the framework of a NP model with an extra  $U(1)'$  gauge symmetry: introducing a renormalisation scheme with manifest decoupling,  $\rho^{\text{top}}$  takes its SM form up to terms vanishing as  $M_{Z'} \rightarrow \infty$ . It has thus been pointed out in that reference that a renormalisation scheme can indeed be chosen in such a way that new physics effects can be treated as corrections to the well established SM results. The main difference between such a renormalisation scheme and the one in [65] for example lies in the way the couplings related to the extended sectors are treated. In the latter they are expressed in terms of some low energy observables leading to uncertainties becoming larger with the mass of the additional gauge boson while in the former they are taken as  $\overline{\text{MS}}$  running parameters, see [70] for more details. The fact that the dependence of  $\rho$  with  $m_t$  differs with the renormalisation scheme is of course due to a different absorption of the  $m_t$  dependence in some of the renormalized couplings. Actually, in our case, the deviation from 1 only appears at order  $\epsilon^2$  contrarily to the TLRM and our renormalisation scheme does fulfil the Appelquist-Carazzone decoupling so that the loops involving the top quark will only give a small  $\mathcal{O}(\epsilon^2)$  correction to the (SM-like) quadratic result. We can thus safely neglect the new physics contributions related to the top quark.

Adding up all contributions described above, our final expression for  $\Delta r$  can be schematically written as

$$\Delta r = \Delta r|_{\text{“SM”}} + \epsilon^2 \Delta r|_{NLO} \quad (4.20)$$

where  $\Delta r|_{\text{“SM”}}$  is identical to the SM contribution, so that one recovers the SM expectations in the limit  $\epsilon = 0$ . The expression for  $\Delta r|_{NLO}$  is quite lengthy and can be provided to the interested reader as a *Mathematica* notebook upon request. Up to now we have only considered coefficients of the fields up to  $\mathcal{O}(\epsilon^2)$ . In principle one would have to compute them up to  $\epsilon^4$  since terms of order  $\epsilon^4$  in the self-energies could in principle contribute to  $\Delta r$  at  $\epsilon^2$ , but we consider that this task goes beyond the scope of this article. Even though  $\Delta r$  itself has no dependence on this scale  $\mu$  in principle, there is however still one in practice since we did not perform a complete calculation. We will come back to this dependence when discussing our results.

## 5 Electroweak precision observables

### 5.1 Computation in left-right models

Apart from the mass of the  $W$  and  $\Delta r$  other important constraints on the SM and its extensions are provided by the precision electroweak measurements at the Z resonance which were performed at LEP and SLC [71], as well as the weak charges. Note that the field of electroweak precision test is very active and will remain so in the future with the advent of new high energy lepton colliders which would make it possible to increase the precision of the electroweak fits by an order of magnitude or more, and hence allow to probe the effect of higher operators in the Standard Model effective field theory at an unprecedented level, see for instance [72].

Here we will consider the following observables  $\mathcal{O}$ :

- the mass of the  $Z$ -boson, its total width as well as the hadronic cross section;
- various ratios of cross sections

$$R(l) = \frac{\Gamma_Z(had)}{\Gamma_Z(l\bar{l})} \text{ for } l = e, \mu, \tau \quad R(q) = \frac{\Gamma_Z(q\bar{q})}{\Gamma_Z(had)} \text{ for } q = c, b; \quad (5.1)$$

- the unpolarized forward backward asymmetries  $A_{FB}(l)$ ,  $l = e, \mu, \tau$ ,  $A_{FB}(q)$ ,  $q = c, b$ , and the final state couplings  $\mathcal{A}_l$ ,  $l = e, \mu, \tau$ ,  $\mathcal{A}_q$ ,  $q = c, b$ ;
- the weak charges  $Q_W$  measured from atomic parity experiments for Caesium and Thallium, as well as for the proton.

In order to determine these observables one needs to know the vector and axial vector couplings of the  $Z$  gauge boson to fermions in the DLRM. They read at tree level

$$\begin{aligned} g_V(f) &= \frac{e}{2s_W c_W} \left( (T_{3L}^f - 2Q^f s_W^2) + kz_h \epsilon^2 (T_{3R}^f c_R^2 - (X_L^f + X_R^f) s_R^2) \right) \\ g_A(f) &= \frac{e}{2s_W c_W} \left( T_{3L}^f - kz_h \epsilon^2 (T_{3R}^f c_R^2 + (X_L^f - X_R^f) s_R^2) \right) \end{aligned} \quad (5.2)$$

where  $T_{3(L,R)}$  are respectively the left/right weak isospin and  $Q_{L,R}^f = T_{3(L,R)}^f + X_{L,R}^f$  the charges of the left and right handed fermions, with  $Q_L^f = Q_R^f = Q^f$ . Detailed expressions of the observables in terms of these couplings are found in appendix A. In the limit where  $\epsilon = 0$  one recovers the SM expressions, and in the case where  $r = w = 0$  these expressions agree with ref. [15]. However,  $Q_W(p)$  and  $Q_W(n)$  lead to an atomic parity violation for  ${}^A\text{Cs}_Z$  different from the one found in [15], which can be traced back to an improper value of  $2A - Z$  used in that reference [73].

As can be seen, most of the observables depend only on the two combinations of DLRM parameters  $s_R^2 \epsilon^2$  and  $kz_h \epsilon^2$ . Only the  $W$  width and the weak charges depend on  $r^2 \epsilon^2$  and  $w^2 \epsilon^2$ .

## 5.2 Parametrization of the observables

Let us consider all the observables we have discussed previously, namely

$$\mathcal{O}^{\text{DLRM}} = \{G_F, M_W, \Gamma_W, \text{EWPO at the } Z \text{ pole, weak charges}\}, \quad (5.3)$$

to which one has added the total width of the  $W$  gauge boson,  $\Gamma_W$ . These will be used as data in a global fit in the next section. They have the general form:

$$\mathcal{O}^{\text{DLRM}} = \mathcal{O}^{\text{[SM]}} + \epsilon^2 \mathcal{O}^{\text{[DLRM]}} \quad (5.4)$$

where  $\mathcal{O}^{\text{[SM]}}$  are the LO contributions in the series expansion in  $\epsilon$ , and  $\mathcal{O}^{\text{[DLRM]}}$  are the corrections at NLO. As we have seen the former contributions are all SM-like. One can take advantage of this fact to use the developed tools in the SM to compute these quantities

and to incorporate the radiative corrections, which are known to be very important in order to reproduce for example the mass of the  $W$  gauge boson in the SM. For the NLO contributions  $\mathcal{O}^{\text{[“DLRM”]}}$  we will assume that their typical size is such that one can keep only their tree-level contributions, as loop corrections would be counted as higher-order contributions compared to the order up to which we are working, cf. the discussion after eq. (4.19) in section 4.2 concerning the validity of this procedure to implement new physics corrections.

For the SM-like contributions, we will use the Zfitter package [62, 74].<sup>10</sup> The input of this package is the set of parameters  $\mathcal{S}^{\text{[“SM”]}}$

$$\mathcal{S}^{\text{[“SM”]}} = \{G_F^0, M_Z^0, M_h^0, m_t^0, \alpha_s(M_Z^0), \Delta\alpha_{\text{had}}^{(5)}(M_Z^0)\} \quad (5.5)$$

where the superscript 0 denotes the combination of parameters of the theory corresponding to the LO expressions ( $\epsilon = 0$ ) of the observables under consideration. Contrarily to the fit done in the SM, they differ from their physical values by order  $\epsilon^2$  corrections. This allows us to determine  $M_W^0$  as:

$$(M_W^0)^2 = (M_Z^0)^2 \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\alpha(M_Z^0)\pi}{\sqrt{2}G_F^0(M_Z^0)^2}(1 + \Delta r|_{LO})} \right) \quad (5.6)$$

as well as  $s_W \equiv \sqrt{1 - (M_W^0/M_Z^0)^2}$  and the EWPO at leading order in  $\epsilon$ .

Calling Zfitter in the course of the global fit is far too much time consuming so that in practice it is advantageous to parametrize the observables  $\mathcal{O}^{\text{[“SM”]}}$ . Varying the input parameters  $\mathcal{S}^{\text{[“SM”]}}$  by 2.5% around their experimental central values, i.e.,

$$\begin{aligned} 122.7 \text{ GeV} &< M_h^0 < 128.7 \text{ GeV} \\ 169 \text{ GeV} &< m_t^0 < 177.4 \text{ GeV} \\ 89 \text{ GeV} &< M_Z^0 < 93.4 \text{ GeV} \\ 0.116 &< \alpha_s(M_Z^0) < 0.121 \\ 0.0269 &< \Delta\alpha_{\text{had}}^{(5)}(M_Z^0) < 0.0282 \\ 1.139 \times 10^{-5} \text{ GeV}^{-2} &< G_F^0 < 1.194 \times 10^{-5} \text{ GeV}^{-2} \end{aligned} \quad (5.7)$$

and allowing the observable to vary by at most 4%,<sup>11</sup> we obtain a rather accurate parametrization of  $\mathcal{O}^{\text{[“SM”]}}$  as:

$$\begin{aligned} \mathcal{O}^{\text{[“SM”]}} = & c_0 + c_1 L_H + c_2 \Delta_t + c_3 \Delta_{\alpha_s} + c_4 \Delta_{\alpha_s}^2 + c_5 \Delta_{\alpha_s} \Delta_t + c_6 \Delta_{\alpha} + c_7 \Delta_Z \\ & + c_8 \Delta_H \Delta_t + c_9 L_H^2 + c_{10} \Delta_t^2 + c_{11} \Delta_{\alpha}^2 + c_{12} \Delta_Z^2 + c_{13} \Delta_Z \Delta_{\alpha_s} + c_{14} \Delta_Z \Delta_t + c_{15} \Delta_G \\ & + c_{16} \Delta_G^2 + c_{17} \Delta_G \Delta_Z + c_{18} \Delta_Z^3 + c_{19} \Delta_G^3 + c_{20} \Delta_G \Delta_Z^2 + c_{21} \Delta_{\alpha} \Delta_Z + c_{22} \Delta_G^2 \Delta_Z \\ & + c_{23} \Delta_G^4 + c_{24} \Delta_G^3 \Delta_Z + c_{25} \Delta_G^2 \Delta_Z^2 + c_{26} \Delta_G \Delta_{\alpha} + c_{27} \Delta_G \Delta_{\alpha}^2 + c_{28} \Delta_G^2 \Delta_{\alpha} \\ & + c_{29} \Delta_Z^4 + c_{30} \Delta_Z \Delta_{\alpha}^2 + c_{31} \Delta_Z^2 \Delta_{\alpha} + c_{32} \Delta_Z^2 \Delta_{\alpha}^2 + c_{33} \Delta_G \Delta_Z^3 + c_{34} \Delta_G^4 \Delta_Z \end{aligned} \quad (5.8)$$

<sup>10</sup>We have used the version Zfitter 6.42. The flag “IALEM” of Zfitter is set to 2 to use  $\Delta\alpha_{\text{had}}^{(5)}(M_Z^0)$  as input. In Zfitter the value of  $G_F^0$  is fixed to its physical value, we thus modified the programs so as to let this parameter free, see also [75]. Otherwise, we use the same flags as in the subroutine DIZET.

<sup>11</sup>We have also considered a variation of the input parameters  $\mathcal{S}^{\text{[“SM”]}}$  by 10% and of the observable by at most 15%, with similar results for our analyses of SM and DLRM.

where

$$\begin{aligned}
 \Delta_\alpha &= \frac{\Delta\alpha(M_Z^0)}{0.059} - 1, & \Delta_{\alpha_s} &= \frac{\alpha_s(M_Z^0)}{0.1184} - 1, \\
 L_H &= \log \frac{M_h^0}{125.7 \text{ GeV}}, & \Delta_H &= \frac{M_h^0}{125.7 \text{ GeV}} - 1, \\
 \Delta_t &= \left( \frac{m_t^0}{173.2 \text{ GeV}} \right)^2 - 1, & \Delta_Z &= \frac{M_Z^0}{91.1876 \text{ GeV}} - 1, \\
 \Delta_G &= \frac{G_F^0}{1.16637 \times 10^{-5} \text{ GeV}^{-2}} - 1
 \end{aligned} \tag{5.9}$$

Note that we have traded here the parameter  $\Delta\alpha_{had}^{(5)}(M_Z)$  for  $\Delta\alpha(M_Z)$ :

$$\Delta\alpha(s) = \Delta\alpha_{had}^{(5)}(s) + \Delta\alpha_{e\mu\tau}(s) + \Delta\alpha_{top}(s), \tag{5.10}$$

where [76, 77]

$$\Delta\alpha_{e\mu\tau}(M_Z) = 0.031497686, \quad \Delta\alpha_{top}(M_Z) = -0.000072, \tag{5.11}$$

with negligible uncertainties.

The coefficients  $c_i$  as well as the maximal difference (in percent) between the Zfitter value and our parametrization are collected in tables 20–22. They have been determined using a grid of 15 points in each direction of the parameter space  $\mathcal{S}^{[SM]}$ . Note that the maximal deviations are of the order or smaller than one percent except for the forward backward asymmetries  $A_{FB}(c)$  and  $A_{FB}(l)$  which are of the order of 10%. We have tested the stability of the results with the number of points. It turns out that the result for  $A_{FB}(b)$  is rather unstable. Thus for the three asymmetries we will use their definitions in terms of  $\mathcal{A}_e$  and  $\mathcal{A}_f$  [42] to parametrize them, namely,  $A_{FB}(f) = 3\mathcal{A}_e\mathcal{A}_f/4$ .

## 6 Global fits

We now have all the ingredients to perform a global fit to the parameters of the DLRM using the information on the EWPO discussed above with further constraints from unitarity and perturbativity.

### 6.1 Method

We want to perform the statistical combination of the various observables and constraints. We will follow the CKMfitter statistical approach used in flavour physics to combine constraints in a frequentist framework [36, 38, 78], building a  $\chi^2$  function from the likelihoods of the various observables. The theoretical uncertainties are treated following the Rfit scheme corresponding to a modification of the likelihood including a bias parameter left free to vary within the quoted range for the theoretical uncertainty [38]. For a parameter of interest, the  $\chi^2$  is considered at different values of this parameter and minimised with respect to the other parameters of the fit. The result is interpreted as a  $p$ -value associated to each possible value of the parameter, which can be used to determine confidence

intervals for the parameter within a particular model. The compatibility of the various measurements with the model considered can also be assessed through the computation of the corresponding pull.

We will consider both the SM and the DLRM, which will allow a direct comparison between the two models. In the SM case, the fit parameters are the ones from  $S^{\text{[“SM”]}}$ . In the DLRM, one adds to these parameters  $c_R, r, w$ , the 5 Higgs potential parameters  $\alpha_{124}, \alpha_2, \lambda_{23}^\pm, \lambda_4$  and the ratio  $x = \mu'_1/\mu'_2$  as well as the three dimensionless quantities  $M_{H_2}/v_R, \delta$  defined in eq. (2.24) and  $\delta_3 = M_{H_3}/M_{W'}$  (though, of course, different choices will not change the physical results).

In the parameters of the DLRM defined above we have discarded  $\epsilon$ . We can rewrite eq. (4.13) to exhibit a structure similar to eq. (5.4)

$$\frac{G_F}{\sqrt{2}} = \frac{M_W^0{}^2}{M_W^2} \frac{G_F^0}{\sqrt{2}} + \epsilon^2 \frac{e^2}{8s_W^2 M_W^2} \Delta r|_{NLO} \tag{6.1}$$

with

$$\frac{G_F^0}{\sqrt{2}} = \frac{e^2}{8s_W^2 M_W^0{}^2} (1 + \Delta r|_{LO}) \tag{6.2}$$

Since  $G_F$  is determined with such a high precision we will fix it to its central value and use eq. (6.1) to determine the parameter  $\epsilon$ . One has to solve an equation of the type  $a + \epsilon^2(c + \log \epsilon^2) = 0$  where the logarithm comes from the contribution of one heavy and one light particle in the loops. Its solution can be expressed as  $\epsilon^2 = -a/W(-ae^c)$  where  $W(x)$  is the Lambert function.<sup>12</sup> We thus obtain  $\epsilon$  in terms of  $G_F$  (and all the other parameters defined above) using our computation of  $\Delta r$ .

Our fit will thus include the following constraints:

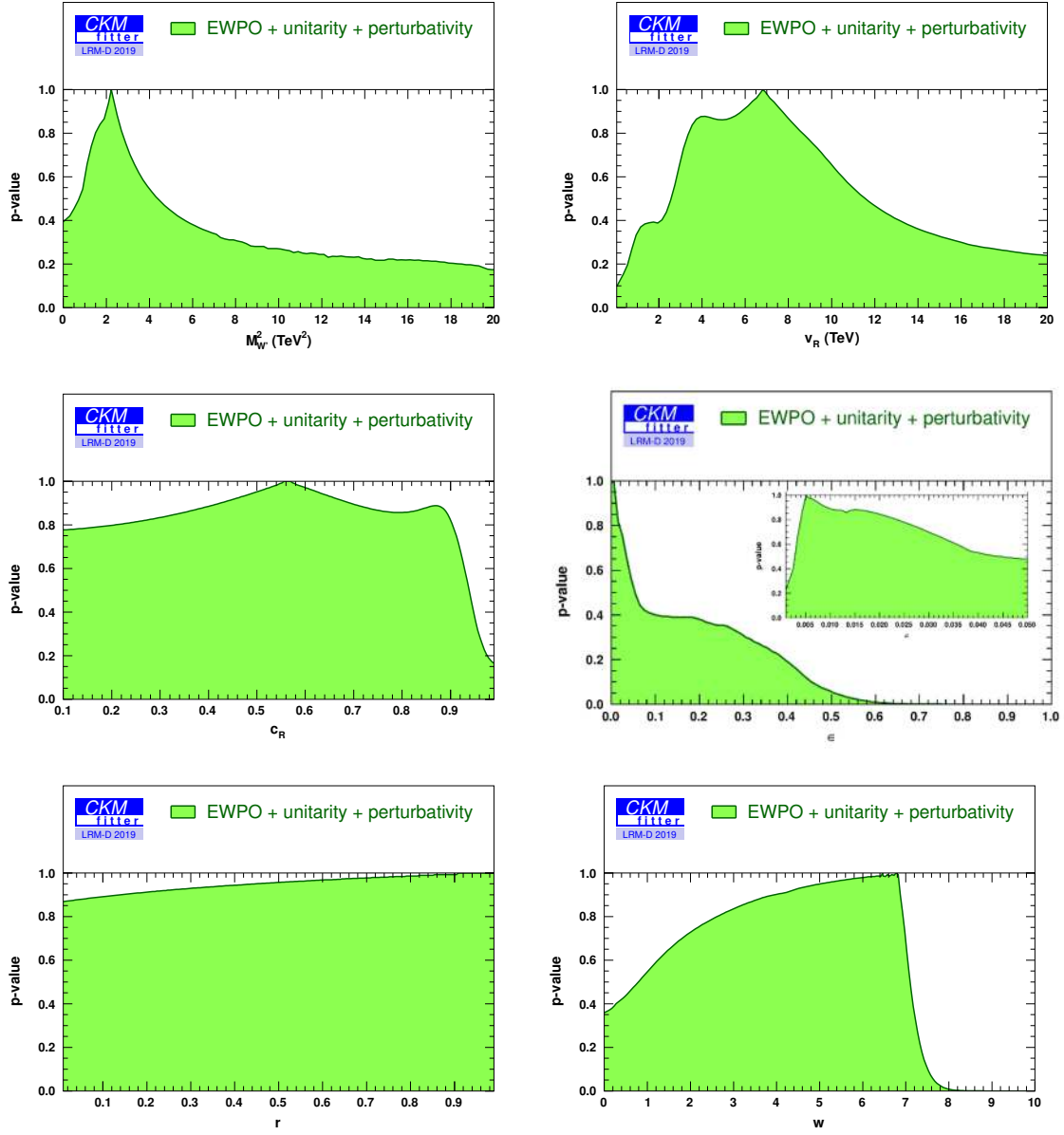
- We can straightforwardly use the above discussion concerning the EWPO expressed in the DLRM  $\mathcal{O}^{\text{[“DLRM”]}}$ , eq. (5.3);
- We include the bounds required in order to satisfy perturbative unitarity at tree level discussed in section 3;
- We also impose perturbativity constraints in the sense that  $\epsilon^2$  corrections to any of the observables discussed are limited to be at most half of the LO ( $\epsilon^0$ ) terms in the same quantity.

## 6.2 Results

We start by discussing the results of our fit assuming the SM, given in table 1. The input for  $M_W$  includes an estimation of  $\pm 4$  MeV for the theoretical error of missing higher-order

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<sup>12</sup>The Lambert function  $W(x)$  is multivalued except at zero. The real-valued  $W$  is defined only for  $x \geq -1/e$  and is double-valued on the interval  $-1/e < x < 0$ . The additional constraint  $W \geq -1$  defines the principal branch  $W_0(x)$  which is single valued, while the lower branch  $W_{-1}(x)$  decreases from  $W_{-1}(-1/e) = -1$  to  $W_{-1}(0) = -\infty$ . We will concentrate here on the principal branch for which the solutions to  $\epsilon$  are the largest. These are indeed the most interesting ones since they should lead to the largest deviation to the SM results.



**Figure 5.** The  $p$ -values for some parameters of the DLRM: the value of the parameter at the best-fit point corresponds to a  $p$ -value of 1, while different Confidence Level (CL) regions are read from different values of  $p$ -value (a  $1\sigma$  CL region corresponds to a  $p$ -value of  $\sim 0.33$ ). The figure for the  $p$ -value of  $\epsilon$  has an insert of a zoomed region.

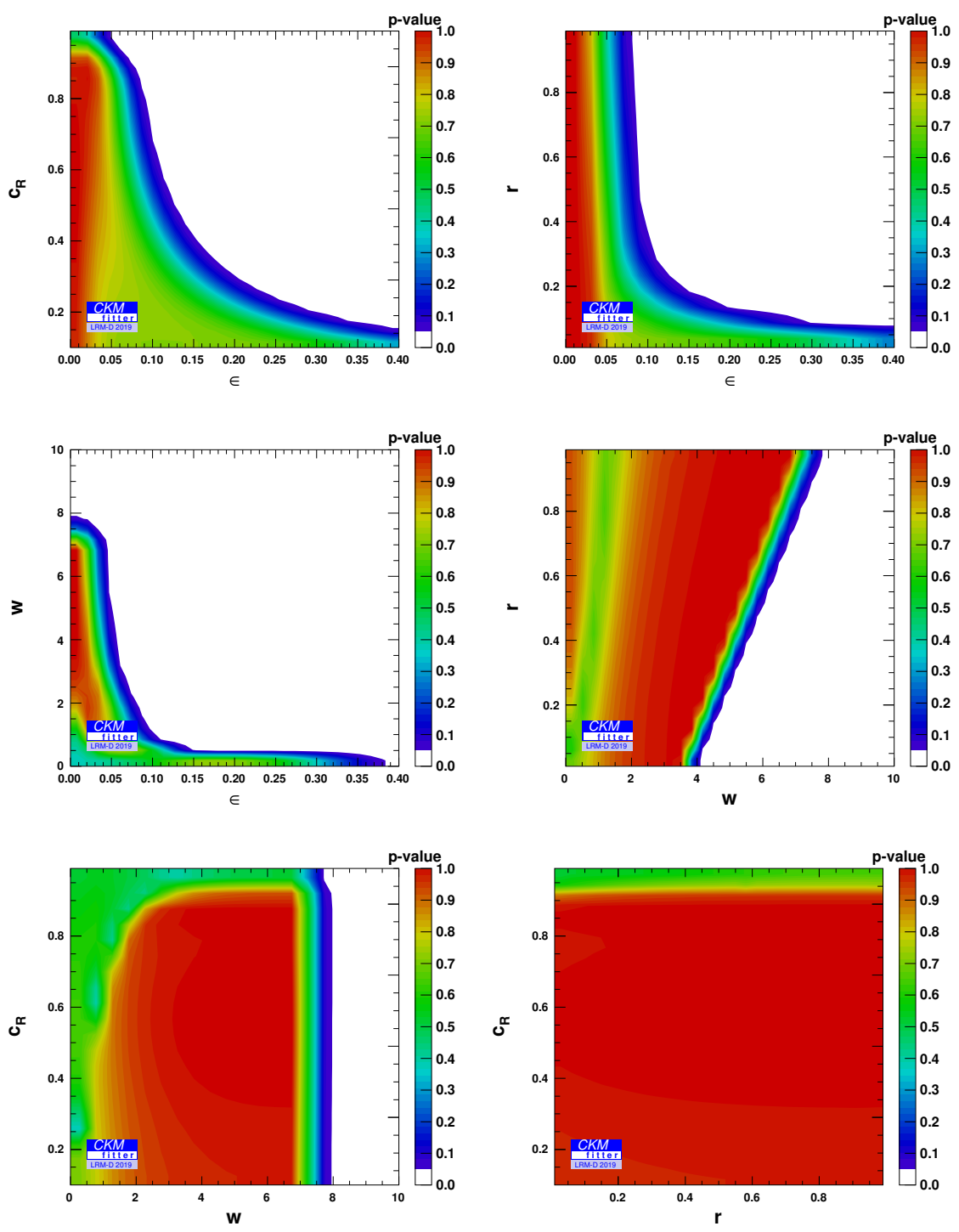


Figure 6. Correlations between the parameters  $\epsilon, r, w$  and  $c_R$  of the DLRM.



perturbative calculations. Note that we have taken for  $\alpha_s(M_Z^2)$  the value of  $0.1184 \pm 0.0012$  [42], to which we will come back later.

The minimum value of the  $\chi^2$  is  $\chi_{\min}^2|_{SM} = 22.4$  with a (naive) number of degrees of freedom equal to 23,<sup>13</sup> thus resulting in a  $p$ -value of  $\sim 0.5$ . The compatibility for a given observable within the model considered can be assessed using the one-dimensional pull<sup>14</sup> defined as

$$\text{pull} = \sqrt{\chi_{\min}^2 - \chi_{\min,!o}^2} \tag{6.3}$$

where ! $o$  means that the  $\chi^2$  is built and minimised without the experimental information on the observable under consideration:  $\chi_{!o}^2$  thus leads to an indirect prediction for this observable. As it is well known, there are a few tensions among the EWPO in the SM, notably  $A_{FB}(b)$  and  $A_e^{SLD}$ , which exhibit an important correlation among their pulls [42]. Note that our results differ slightly for some observables, as for example the mass of the  $W$ , from the ones of the global fit by the Gfitter collaboration [80]. However we did not use exactly the same inputs and the calculations of the observables are not done with the same level of sophistication here.

Let us now turn to the DLRM. The results of the global fit are obtained as follows:

- i) In eq. (2.21) for  $\mu'_2$  the negative sign has been chosen, but the positive one gives essentially the same results;
- ii) No bound on  $M_{W'}$  is considered (we will come back to this point at the end of the section);
- iii) The parameters of the Higgs Lagrangian are restricted within the range  $[-20, 20]$ , in order to avoid non-perturbative regimes related to strong couplings, i.e., we impose  $\alpha_2/(4\pi) \lesssim 1$ , etc. Similarly, we require that  $g_X^2/(4\pi) < 1$  and  $g_R^2/(4\pi) < 1$ , thus implying  $0.1 < c_R < 0.99$ . Together with the conditions based on eq. (2.25) discussed at the end of section 6.1, these requirements are collectively called “perturbativity” in our analysis;
- iv) We exclude the case  $r = 1$  in our analysis, which is not allowed by the hierarchy of the masses of the fermions, by imposing  $0 \leq r \leq 0.99$ . Following the discussion of section 2.4, such a range of values for  $r$  does not guarantee that the hierarchy of masses is respected, but as we will see  $r$  plays a minor role in the fit, so that narrower ranges could be chosen with no impact on the analysis;
- v) As discussed in section 4, there is a residual dependence on the renormalisation scale in the expression of  $\Delta r$ . In principle, we should assign a theoretical uncertainty typically of order one so as to take into account the missing contributions in our computation. However, given the large number of free parameters, for instance the ones of the scalar potential, the fit is insensitive to the presence of this extra uncertainty and the results remain unchanged;

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<sup>13</sup>As discussed in refs. [36, 79], the precise number of degrees of freedom can be difficult to assess in the presence of theoretical uncertainties and constraints depending only on some of the parameters of the fit.

<sup>14</sup>Note that in the context of EWPO a different definition is usually found in the literature.

- vi) We take  $\mu = M_Z$ , which is the natural scale of the problem. This choice is in agreement with the one for  $\alpha_s$ , which we remind the reader is an  $\overline{\text{MS}}$  scheme running parameter. In the DLRM, where  $M_Z^0 \neq M_Z$  in general, we implement the running of  $\alpha_s$  between these two scales at the leading log;
- vii) Given that DLRM corrections to  $\Delta\alpha_{had}^{(5)}$ ,  $m_{top}^{pole}$  have not been computed, we add a 10% uncertainty to their inputs so that they are allowed to receive additional contributions in the DLRM. This could naturally be improved by a computation of these quantities in the DLRM, however at the prize of adding new parameters in our global fit, which lies beyond the scope of this article. We follow the same procedure for  $M_h^0$  since we do not consider the  $\epsilon^2$  corrections to this quantity. The latter involves parameters of the scalar potential which (as we will see below) are very badly constrained in our analysis.

Comparing the results from the DLRM shown in table 2 with table 1 one observes the same tensions as in the SM, so that there is no concrete improvement from the DLRM as far as the EWPO are concerned. The minimum of the  $\chi^2$  is  $\chi_{\min}^2|_{DLRM} = 20.2$ , with a (naive) number of degrees of freedom of 20,<sup>15</sup> leading to a  $p$ -value of  $\sim 0.4$ . One can consider the SM as a limiting case of the DLRM, so that both can be seen as two nested hypotheses [38, 79]. It then follows that the quantity  $\chi_{\min}^2|_{SM} - \chi_{\min}^2|_{DLRM}$  is distributed as following a  $\chi^2$  law with a (naive) number of degrees of freedom of 3. It can be interpreted as a  $0.5\sigma$  deviation, not large enough for preferring the DLRM hypothesis over the SM one.

As seen from table 2, among the parameters specific to DLRM the set of observables in eq. (5.3) constrains  $\epsilon$  and  $w$  while  $c_R$  and  $r$  remain essentially unconstrained. Values of  $\epsilon \lesssim 0.3$  and large values of the parameter  $w \sim \mathcal{O}(1)$  are favoured. Consequently the doublet  $\chi_L$  plays an important role in triggering the spontaneous EW symmetry breaking, whereas it is essentially absent in the TLRM due to the breaking of the custodial symmetry and the bi-doublet  $\phi$  alone triggers this breaking. The role played by  $w$  in the DLRM can further be seen from table 3. The results just discussed are also illustrated in figure 5, from which one reads  $1\sigma$  confidence level regions by taking for each parameter the intervals corresponding to  $p$ -values higher than  $\sim 0.33$ . Also shown in this figure is the LR breaking scale  $v_R$ , which is constrained by the EWPO to be  $v_R = 6.8_{-0.8}^{+8.6}$  TeV at  $1\sigma$ . We note that in the analysis of the EWPO in the framework of the TLRM with  $w = 0$  [30], small values of the ratio of the EW and LR symmetry breaking scales (equivalent to the quantity  $\sqrt{2k}\epsilon$  discussed here) have been favoured with a preference for  $c_R < 0.7$ .

The correlations between the four parameters  $\epsilon, r, w$  and  $c_R$  are shown in figure 6. As expected from the fact that  $r, w$  and  $c_R$  enter our expressions always multiplied by  $\epsilon$  there are strong correlations between the latter, which sets the ratio of EW and LR symmetry breaking scales, and the former three. Much weaker correlations, or no correlations at all, are observed among  $r, w$  and  $c_R$ . In figure 8, we show the allowed values of the combinations

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<sup>15</sup>This number of degrees of freedom is the same as in the SM case minus the parameters that we constrained in the DLRM, namely  $\{\epsilon, w, c_R\}$ . Obviously, the DLRM contains many more parameters, but as they are not constrained by the fit, we do not include them in our counting for the degrees of freedom relevant for a statistical interpretation of the value of  $\chi_{\min}^2$ .

Observable	input	full fit ( $1\sigma$ )	prediction ( $1\sigma$ )	pull
$10^5 \times G_F$ [GeV $^{-2}$ ]	1.1663787(6)	1.1663787(6)	$1.1668 \pm 0.0011$	0.4
$M_Z$ [GeV]	$91.1876 \pm 0.0021$	$91.1875 \pm 0.0021$	$91.186^{+0.018}_{-0.016}$	0.1
$M_h$ [GeV]	$125.1 \pm 0.2$	$125.10 \pm 0.20$	$108^{+45}_{-29}$	0.4
$m_{\text{top}}^{\text{pole}}$ [GeV]	$172.47 \pm 0.46 \pm 0.50$	$173.00^{+0.46}_{-1.36}$	$174.5^{+4.8}_{-3.4}$	0.4
$\Delta\alpha(M_Z^2)$	–	$0.05865^{+0.00040}_{-0.00049}$	$0.05865^{+0.00040}_{-0.00049}$	–
$\alpha_s(M_Z^2)$	$0.1184 \pm 0 \pm 0.0012$	$0.11876^{+0.00085}_{-0.00157}$	$0.1188 \pm 0.0026$	0.0
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$	$2.49566^{+0.00081}_{-0.00115}$	$2.4954^{+0.0011}_{-0.0010}$	0.3
$\sigma_{\text{had}}$ [nb]	$41.541 \pm 0.037$	$41.4763^{+0.0099}_{-0.0068}$	$41.4720^{+0.0111}_{-0.0026}$	1.7
$R_b$	$0.21629 \pm 0.00066$	$0.215835^{+0.000047}_{-0.000017}$	$0.215835^{+0.000046}_{-0.000017}$	0.4
$R_c$	$0.1721 \pm 0.0030$	$0.172242^{+0.000025}_{-0.000044}$	$0.172242^{+0.000025}_{-0.000044}$	0.1
$R_e$	$20.804 \pm 0.050$	$20.7439^{+0.0086}_{-0.0123}$	$20.7384^{+0.0133}_{-0.0074}$	0.8
$R_\mu$	$20.785 \pm 0.033$	$20.7441^{+0.0086}_{-0.0123}$	$20.7335^{+0.0172}_{-0.0025}$	1.2
$R_\tau$	$20.764 \pm 0.045$	$20.7910^{+0.0086}_{-0.0123}$	$20.7966^{+0.0033}_{-0.0170}$	0.9
$A_{FB}(b)$	$0.0992 \pm 0.0016$	$0.10348^{+0.00079}_{-0.00077}$	$0.1048 \pm 0.0010$	2.8
$A_{FB}(c)$	$0.0707 \pm 0.0035$	$0.07397 \pm 0.00060$	$0.07399 \pm 0.00060$	0.6
$A_{FB}(e)$	$0.0145 \pm 0.0025$	$0.01634 \pm 0.00024$	$0.0163 \pm 0.0002$	0.4
$A_{FB}(\mu)$	$0.0169 \pm 0.0013$	$0.01634^{+0.00025}_{-0.00024}$	$0.0163 \pm 0.0002$	0.3
$A_{FB}(\tau)$	$0.0188 \pm 0.0017$	$0.01634^{+0.00025}_{-0.00024}$	$0.0163 \pm 0.0002$	1.4
$\mathcal{A}_b$	$0.923 \pm 0.020$	$0.934717^{+0.000105}_{-0.000092}$	$0.93472^{+0.00010}_{-0.00009}$	0.4
$\mathcal{A}_c$	$0.670 \pm 0.027$	$0.66813^{+0.00049}_{-0.00048}$	$0.66812 \pm 0.00048$	0.2
$\mathcal{A}_e^{SLD}$	$0.1516 \pm 0.0021$	$0.1476 \pm 0.0011$	$0.148 \pm 0.001$	2.3
$\mathcal{A}_e(P_\tau)$	$0.1498 \pm 0.0049$	$0.1476 \pm 0.0011$	$0.148 \pm 0.001$	0.5
$\mathcal{A}_\mu^{SLD}$	$0.142 \pm 0.015$	$0.1476 \pm 0.0011$	$0.148 \pm 0.001$	0.4
$\mathcal{A}_\tau^{SLD}$	$0.136 \pm 0.015$	$0.1476 \pm 0.0011$	$0.148 \pm 0.001$	0.8
$\mathcal{A}_\tau(P_\tau)$	$0.1439 \pm 0.0043$	$0.1476 \pm 0.0011$	$0.148 \pm 0.001$	0.9
$M_W$ [GeV]	$80.379 \pm 0.014 \pm 0.004$	$80.3696^{+0.0079}_{-0.0095}$	$80.3669^{+0.0094}_{-0.0134}$	0.5
$\Gamma_W$ [GeV]	$2.085 \pm 0.042$	$2.09144^{+0.00087}_{-0.00123}$	$2.09144^{+0.00087}_{-0.00122}$	0.2
$Q_W(p)$	$0.0719 \pm 0.0045$	$0.07350 \pm 0.00058$	$0.07352 \pm 0.00058$	0.4
$Q_W(Cs)$	$-72.62 \pm 0.43$	$-72.928^{+0.042}_{-0.036}$	$-72.930^{+0.041}_{-0.036}$	0.7
$Q_W(Tl)$	$-116.4 \pm 3.6$	$-115.423^{+0.063}_{-0.053}$	$-115.423^{+0.062}_{-0.053}$	0.3

**Table 1.** Results of the global fit in the SM. The first row gives the parameters of the fit,  $\{G_F, M_Z, M_h, m_{\text{top}}^{\text{pole}}, \Delta\alpha, \alpha_s(M_Z^2)\}$ . The third column, “full fit”, gives the result from the fit, the fourth one, “prediction”, is the value of the observable predicted in the SM without knowledge of its experimental value, while in the last column the pull is defined as in eq. (6.3). The inputs are taken from: [42] for  $G_F$  and  $\alpha_s$ , [80] for  $m_{\text{top}}^{\text{pole}}$ , [93] for  $M_h$ , [71] for  $M_Z$  and the EWPO in the second row, [80, 94–96] for  $M_W$ , [97] for  $\Gamma_W$ , [98] for  $Q_W(p)$ , [42, 99, 100] for  $Q_W(Cs)$  and [101, 102] for  $Q_W(Tl)$ . When two uncertainties are present, the first is statistical while the second is theoretical, treated in the Rfit scheme of [36, 38, 78].

Observable	input	full fit ( $1\sigma$ )	prediction ( $1\sigma$ )	pull
$10^5 \times G_F^{(0)}$ [GeV $^{-2}$ ]	–	$1.166 \pm 0.001$	–	–
$M_Z^{(0)}$ [GeV]	–	$91.21 \pm 0.03$	–	–
$M_h^{(0)}$ [GeV]	$125.1 \pm 12$	$125 \pm 12$	$25_{-9}^{+38}$	–
$M_W^{(0)}$ [GeV]	–	$80.37 \pm 0.02$	–	–
$m_{\text{top}}^{\text{pole},(0)}$ [GeV]	$172.47 \pm 17$	$172_{-4}^{+7}$	$172_{-4}^{+8}$	–
$\Delta\alpha(M_Z^2)$	$0.05898 \pm 0.0032$	$0.059_{-0.003}^{+0.002}$	$0.058_{-0.001}^{+0.004}$	–
$\alpha_s(M_Z^2)$	$0.1184 \pm 0 \pm 0.0012$	$0.1196_{-0.0018}^{+0.0001}$	$0.130 \pm 0.006$	1.8
$c_R$	[0.1, 0.99]	$0.56_{-0.46}^{+0.39}$	–	–
$\epsilon$	$\geq 0$	$0.01_{-0.01}^{+0.32}$	–	–
$r$	[0, 0.99]	no bound	–	–
$w$	$\geq 0$	$6.9_{-6.8}^{+0.4}$	–	–
$\alpha_{124}, \alpha_2, \lambda_1, \lambda_{23}^{\pm}, \lambda_4$	[-20, 20]	no bound	–	–
$M_{H_1}, M_{H_2}$	$\geq 0$	no bound	–	–
$M_{H_3}$ [TeV]	$\geq 0$	$20_{-20}^{+50}$	–	–
$x = \mu'_1/\mu'_2$	–	no bound	–	–
$10^5 \times G_F$ [GeV $^{-2}$ ]	1.1663787 fixed	1.1663787 fixed	–	–
$M_Z$ [GeV]	$91.1876 \pm 0.0021$	$91.187_{-0.005}^{+0.006}$	$91.17 \pm 0.06$	0.2
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$	$2.496 \pm 0.003$	$2.491_{-0.009}^{+0.016}$	0.3
$\sigma_{\text{had}}$ [nb]	$41.541 \pm 0.037$	$41.51 \pm 0.03$	$41.42 \pm 0.07$	1.4
$R_b$	$0.21629 \pm 0.00066$	$0.2158_{-0.0002}^{+0.0001}$	$0.2158_{-0.0004}^{+0.0001}$	0.5
$R_c$	$0.1721 \pm 0.0030$	$0.17232_{-0.00007}^{+0.00006}$	$0.17232_{-0.00007}^{+0.00006}$	0.1
$R_e$	$20.804 \pm 0.050$	$20.73 \pm 0.01$	$20.73 \pm 0.01$	1.1
$R_\mu$	$20.785 \pm 0.033$	$20.73 \pm 0.01$	$20.73_{-0.02}^{+0.01}$	1.6
$R_\tau$	$20.764 \pm 0.045$	$20.78 \pm 0.01$	$20.78 \pm 0.01$	0.6
$A_{FB}(b)$	$0.0992 \pm 0.0016$	$0.1033 \pm 0.0009$	$0.105 \pm 0.001$	2.8
$A_{FB}(c)$	$0.0707 \pm 0.0035$	$0.0738_{-0.0008}^{+0.0007}$	$0.0739_{-0.0008}^{+0.0007}$	0.6
$A_{FB}(e)$	$0.0145 \pm 0.0025$	$0.0165_{-0.0005}^{+0.0004}$	$0.0165_{-0.0005}^{+0.0004}$	0.3
$A_{FB}(\mu)$	$0.0169 \pm 0.0013$	$0.0165 \pm 0.0005$	$0.0162_{-0.0003}^{+0.0007}$	0.4
$A_{FB}(\tau)$	$0.0188 \pm 0.0017$	$0.0165 \pm 0.0005$	$0.0162_{-0.0003}^{+0.0005}$	1.5
$\mathcal{A}_b$	$0.923 \pm 0.020$	$0.9342 \pm 0.0004$	$0.9342_{-0.0003}^{+0.0004}$	0.4
$\mathcal{A}_c$	$0.670 \pm 0.027$	$0.6674_{-0.0007}^{+0.0008}$	$0.6674_{-0.0007}^{+0.0008}$	0.2
$\mathcal{A}_e^{\text{SLD}}$	$0.1516 \pm 0.0021$	$0.148 \pm 0.001$	$0.145 \pm 0.002$	2.5
$\mathcal{A}_e(P_\tau)$	$0.1498 \pm 0.0049$	$0.148 \pm 0.001$	$0.147 \pm 0.001$	0.5
$\mathcal{A}_\mu^{\text{SLD}}$	$0.142 \pm 0.015$	$0.148 \pm 0.001$	$0.148 \pm 0.001$	0.4
$\mathcal{A}_\tau^{\text{SLD}}$	$0.136 \pm 0.015$	$0.148 \pm 0.001$	$0.148 \pm 0.001$	0.8
$\mathcal{A}_\tau(P_\tau)$	$0.1439 \pm 0.0043$	$0.148 \pm 0.001$	$0.148 \pm 0.001$	0.9
$M_W$ [GeV]	$80.379 \pm 0.014 \pm 0.004$	$80.37_{-0.01}^{+0.02}$	$80.2 \pm 0.2$	0.8
$\Gamma_W$ [GeV]	$2.085 \pm 0.042$	$2.091_{-0.003}^{+0.002}$	$2.091_{-0.003}^{+0.002}$	0.1
$Q_W(p)$	$0.0719 \pm 0.0045$	$0.074_{-0.006}^{+0.002}$	$0.074_{-0.011}^{+0.003}$	0.5
$Q_W(Cs)$	$-72.62 \pm 0.43$	$-72.8_{-0.2}^{+0.6}$	$-72.9_{-0.1}^{+1.4}$	0.5
$Q_W(Tl)$	$-116.4 \pm 3.6$	$-115.2_{-0.2}^{+0.9}$	$-115.0_{-0.5}^{+0.6}$	0.4
$M_{W'}^2$ [TeV $^2$ ]	–	$2_{-2}^{+6}$	–	–
$M_{Z'}^2$ [TeV $^2$ ]	–	$7_{-7}^{+730}$	–	–

**Table 2.** Same as in the table 1 but in the DLRM except for  $m_{\text{top}}^{\text{pole},(0)}$  and  $M_h^{(0)}$  for which we have increased the error bars of the input value, see the discussion of the results in section 6.

$w$	$\chi_{\min}^2$	$M_{W'}^2$ [TeV <sup>2</sup> ]	$\chi_{\min}^2$
free	20.2	> 0	20.2
1	20.5	> 20	21.7
0.01	22.2	> 50	22.0

**Table 3.** Impact on the  $\chi^2$  from (left)  $w$ , and (right)  $M_{W'}$ .

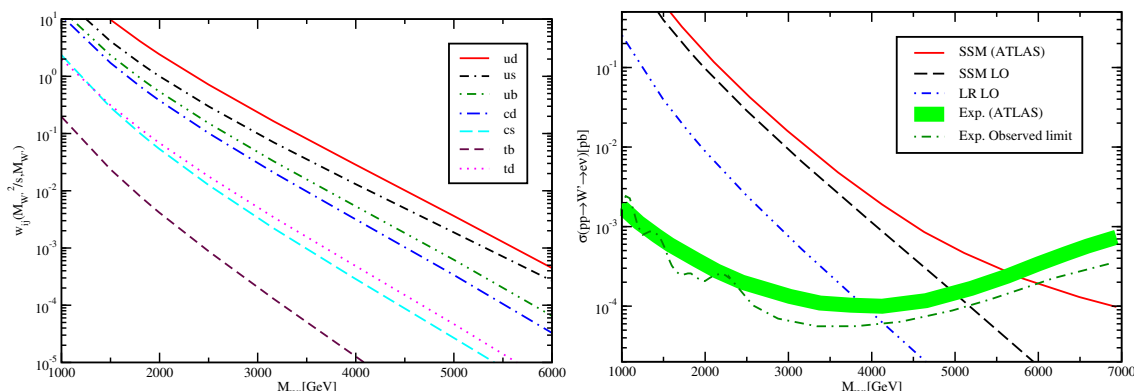
$kz_h\epsilon^2$ ,  $w_h^2\epsilon^2$ ,  $w^2\epsilon^2$  and  $s_R^2\epsilon^2$  which are the natural quantities appearing in the EWPO, see appendix A with  $z_h$  and  $w_h$  defined in eq. (2.44). Their correlations are given in figure 9.

The set of observables in eq. (5.3) alone is not sufficient to set bounds on the Higgs sector of the theory specific to the DLRM. Indeed, the  $p$ -values obtained for the Higgs masses  $M_{H_{1,2,3}}$  and the parameters of the Higgs Lagrangian are very flat over a wide range of values, so that no stringent confidence interval can be deduced. However, using the bound on  $v_R$  given previously the unitarity relation from the analysis of  $Z'Z' \rightarrow Z'Z'$  scattering sets the bound  $M_{H_3} \lesssim 63$  TeV at  $1\sigma$ . This is analogous to the bound resulting from eq. (3.2) in the SM framework, where the knowledge of the EW scale  $v$  sets the bound  $M_h \lesssim 4v$  on the mass of the light scalar. More impact from the unitarity relations relies on constraining the parameters  $F_i, S_i w/k, c^{H_3}/k^2$  ( $i = 1, 2$ ) of eq. (3.29). Note that  $F_i, S_i w/k$  ( $i = 1, 2$ ) depend on the parameters  $r, w, M_{H_1}/M_{H_2}, \mu'_1/\mu'_2$ , while  $c^{H_3}/k^2$  depends also on additional parameters of the scalar potential. The fit leads to:

$$-1.1 < F_1 < 0.2, \quad -0.9 < F_2 < 0.5, \quad -0.6 < S_1 w/k < 0.3, \quad |S_2 w/k| < 0.7, \quad |c^{H_3}/k^2| < 250 \quad (6.4)$$

at  $1\sigma$ . Apart from  $c^{H_3}/k^2$  they are rather small and compatible with zero thus limiting the sensitivity to the scalar spectrum. If more information on the possible allowed values of the parameters of the scalar potential was included, together with other indirect or direct bounds on part of the scalar spectrum, the remaining scalar masses would be probed more accurately by the unitarity constraints.

Let us come back to the  $W'$  and  $Z'$  masses and first discuss briefly what is known about them. These heavy spin-1 charged and neutral particles (including sequential  $W$  and  $Z$  particles having the same couplings to fermions as the  $W$  and  $Z$ , excited Kaluza-Klein modes, etc., which are also generically called  $W'$  and  $Z'$ ) can be looked for at hadron colliders as well as in measurements of processes at energies much below their masses. Active searches have been pursued by LEP II, ATLAS and CMS and some bounds have been obtained, however we should stress that these bounds depend on the specific models considered, including different possible realizations of LR models. Focusing on the latter a lower bound of  $M_{W'} > 715$  GeV has been given at 90% CL from an electroweak fit [81], while in ref. [82] the constraint  $M_{W'} > 3.25$  TeV is found at 95% CL from a search for vector-boson resonances decaying to a top quark and bottom quark assuming a  $W'tb$  coupling similar to the SM  $Wtb$  one: for a  $W'tb$  coupling  $\sim 10$  times smaller, the analysis [82] is not sensitive to this decay channel of  $W'$ . On the theoretical side, ref. [83] discusses contributions of the  $W'$  to the SM-like Higgs decaying into two photons within a LRM realized with doublets, reaching sensitivities to  $M_{W'}$  of  $\mathcal{O}(200)$  GeV for  $g_L = g_R$ . Also, a thorough analysis of flavour constraints has been performed in ref. [45] in the case of



**Figure 7.** (Left) The quantity  $w_{ij}(M_{W'}^2/s, M_{W'})$ , eq. (6.6) for up and down type quarks of various generations as a function of the  $W'$  mass. (Right)  $pp \rightarrow W' \rightarrow e\nu$  cross section in the Sequential Standard Model (“SSM LO”) and in the LR model (“LR LO”) at LO. Also shown are the SSM result from ATLAS [16] as well as their observed and expected upper limits at 95% CL.

TLRM with an additional discrete symmetry. Together with the requirement of model perturbativity [53] and constraints on CP violation [32], the  $W'$  scale is pushed to at least 7 TeV. This value is somewhat larger than the ones quoted before and is similar to what is generally found in other models. For example, a lower 6 TeV limit has been obtained by ATLAS [16] from the extraction of the cross section for  $pp \rightarrow W' \rightarrow l\nu$  using a data sample of  $139 \text{ fb}^{-1}$  of proton-proton collisions at  $\sqrt{s} = 13 \text{ TeV}$  in the context of the Sequential Standard Model (SSM) where the couplings of the  $W'$  are assumed to be identical to the SM  $W$  boson ones, whereas the couplings to the SM bosons are set to zero. Concerning the  $Z'$ , assuming  $g_L = g_R$  two lower bounds are quoted in [42]: one from  $p\bar{p}$  direct search  $M_{Z'} > 630 \text{ GeV}$  [84] and the other from an electroweak fit  $M_{Z'} > 1162 \text{ GeV}$  [85], both at 95% CL.

In view of the above discussion we first leave  $M_{W'}$  free in the fit. It favours masses below  $\sim 3 \text{ TeV}$ , see table 2, as well as a value of  $c_R$  at the best-fit point of  $c_R \sim 0.5$ , meaning that the  $Z'$  mass is roughly twice as large as the  $W'$  one (however, the uncertainty on the latter result does not allow at that point to differentiate between a value of the  $Z'$  mass comparable or much higher than the  $W'$  one). While the value of  $M_{W'}$  is in agreement with some of the bounds given above it points towards a rather large suppression of the  $W'$  production in the direct search from ATLAS discussed in the previous paragraph [16], and thus poses the question of the possible existence of couplings of the  $W'$  to the quarks which would allow it. In order to answer it we calculate the production cross section at leading-order. It is given by, see for example [86, 87]:

$$\sigma(pp \rightarrow W'X) = \frac{\pi}{6s} g_R^2 \sum_{i,j} |V_{ij}^R|^2 w_{ij}(M_{W'}^2/s, M_{W'}) \quad (6.5)$$

where  $V_R$  is the analogue of the CKM matrix in the right-handed sector, and the functions  $w_{ij}$  encode the information about the proton structure

$$w_{ij}(z, \mu) = \int_z^1 \frac{dx}{x} \left( u_i(x, \mu) \bar{d}_j \left( \frac{z}{x}, \mu \right) + \bar{u}_i(x, \mu) d_j \left( \frac{z}{x}, \mu \right) \right) \quad (6.6)$$



with  $u_i(x, \mu)$  and  $d_i(x, \mu)$  the parton distribution functions (PDF) for the up and down type quarks of the  $i$ th and  $j$ th generations in the proton, respectively, at the factorization scale  $\mu$  and parton momentum fraction  $x$ . In the following the results have been obtained using the NNPDF23LO PDF set [88] and a constant branching fraction for  $W'$  boson decays into leptons of one generation of 8.2%, see [16]. The quantity  $w_{ij}(M_{W'}^2/s, M_{W'})$  is shown on the left-panel of figure 7 for various combinations of  $i$  and  $j$ , while the LO cross section, eq. (6.5) within the DLRM and SSM,<sup>16</sup> and the results from ATLAS are displayed on the right-panel of figure 7. The curve from ATLAS labelled “SSM” on the figure has been obtained using the PYTHIA v8.183 event generator [89] to produce the Monte-Carlo events in the SSM at LO, as well as the A14 set of tuned parameters [90] for the parton showering and hadronization process. Clearly, in order to decrease the bound on the  $W'$  mass one needs to suppress the largest contributions to the process under consideration, namely the ones from the PDF of the up and down quarks of the first generation. This can be achieved partially by taking an anti-diagonal matrix  $V_R$ . The result is displayed on the figure for  $g_R = g_L/\sqrt{3}$ , which corresponds approximately to the minimal possible value of  $g_R$  in the LR model, see eq. (2.36). A more refined calculation would be needed, which however goes beyond the scope of the paper, but it seems difficult to lower the limit of the  $W'$  mass much more than 4 TeV.

We have thus studied how the quality of the fit would deteriorate if we would increase the lower bound on  $M_{W'}$ . The result is shown on table 3 where the minimum of the  $\chi^2$  gets increasingly close to the SM limit (equivalent to  $\epsilon = 0$ ). As an example, we show in table 4 the parameters of the DLRM when the mass of the  $W'$  is constrained to be larger than 10 TeV. As expected from table 3, this reduces the quality of the fit, increasing the  $\chi_{\min}^2$  to 22.0. Moreover, since  $M_{W'}$  helps probing the scale of spontaneous breaking of the LR symmetry, eq. (2.47), the ratio  $\epsilon$  is much smaller than in the case of table 2, its constraint at  $1\sigma$  being now  $\epsilon < 0.02$ . Since the NP contributions scale with the size of  $\epsilon$ , the extracted central values and ranges of the various quantities get in general closer to the results extracted from the SM fit of table 1, and in particular the values of the observables at LO ( $M_Z^{(0)}$ , etc.) are closer to their experimental values (respectively,  $M_Z$ , etc.). The constraints on the spontaneous LR and EW breaking parameters  $c_R, r, w$  are still similar to those obtained with  $M_{W'}$  left free, which is consistent with figure 6 for small values of  $\epsilon$ , while many additional parameters of the DLRM remain unconstrained. Also note that, following eq. (2.47), the mass of the  $Z'$  gauge boson is necessarily larger or equal to 10 TeV and, following eq. (3.29), the mass of the scalar  $H_3$ , the heavier analogue of the SM-like Higgs boson, is less constrained since the DLRM scale  $v_R$  is pushed to higher values.

Finally, the value for  $\alpha_s(M_Z^2)$  used here corresponds to the world average value from PDG [42]. As illustrated by their review on QCD, obtaining a world average is not a trivial exercise. There are various ways of determining  $\alpha_s(M_Z^2)$  which can be grouped into certain sub-categories as  $e^+e^-$  into hadronic states, deep inelastic scattering (DIS), hadronic  $\tau$  decay, lattice QCD, heavy quarkonia decays and hadron collider data. Actually the FLAG lattice average [91] dominates the world average. Some of the non-lattice determinations

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<sup>16</sup>Note that at LO there is a rather large dependence of the result on the chosen factorization scale  $\mu$ .

Observable	input	full fit ( $1\sigma$ )	prediction ( $1\sigma$ )	pull
$10^5 \times G_F^{(0)}$ [GeV $^{-2}$ ]	–	$1.167 \pm 0.001$	–	–
$M_Z^{(0)}$ [GeV]	–	$91.188^{+0.014}_{-0.003}$	–	–
$M_h^{(0)}$ [GeV]	$125.1 \pm 12$	$125 \pm 12$	$90^{+160}_{-50}$	–
$M_W^{(0)}$ [GeV]	–	$80.37^{+0.02}_{-0.01}$	–	–
$m_{\text{top}}^{\text{pole},(0)}$ [GeV]	$172.47 \pm 17$	$174^{+5}_{-4}$	$174^{+5}_{-4}$	–
$\Delta\alpha(M_Z^2)$	$0.05898 \pm 0.0032$	$0.059 \pm 0.001$	$0.059 \pm 0.001$	–
$\alpha_s(M_Z^2)$	$0.1184 \pm 0 \pm 0.0012$	$0.1187^{+0.0009}_{-0.0015}$	$0.119 \pm 0.003$	0.0
$c_R$	[0.1, 0.99]	no bound	–	–
$\epsilon$	$\geq 0$	$0.001^{+0.018}_{-0.001}$	–	–
$r$	[0, 0.99]	no bound	–	–
$w$	$\geq 0$	$5.7^{+1.5}_{-5.7}$	–	–
$\alpha_{124}, \alpha_2, \lambda_1, \lambda_{23}^\pm, \lambda_4$	[-20, 20]	no bound	–	–
$M_{H_1}, M_{H_2}$	$\geq 0$	no bound	–	–
$M_{H_3}$ [TeV]	$\geq 0$	$50^{+70}_{-50}$	–	–
$x = \mu'_1/\mu'_2$	–	no bound	–	–
$M_Z$ [GeV]	$91.1876 \pm 0.0021$	$91.187 \pm 0.002$	$91.16 \pm 0.03$	1.0
$M_{Z'}^2$ [TeV $^2$ ]	–	$> 10^2$	–	–

**Table 4.** Same scenario of table 2, but with the constraint  $M_{W'} > 10$  TeV imposed.

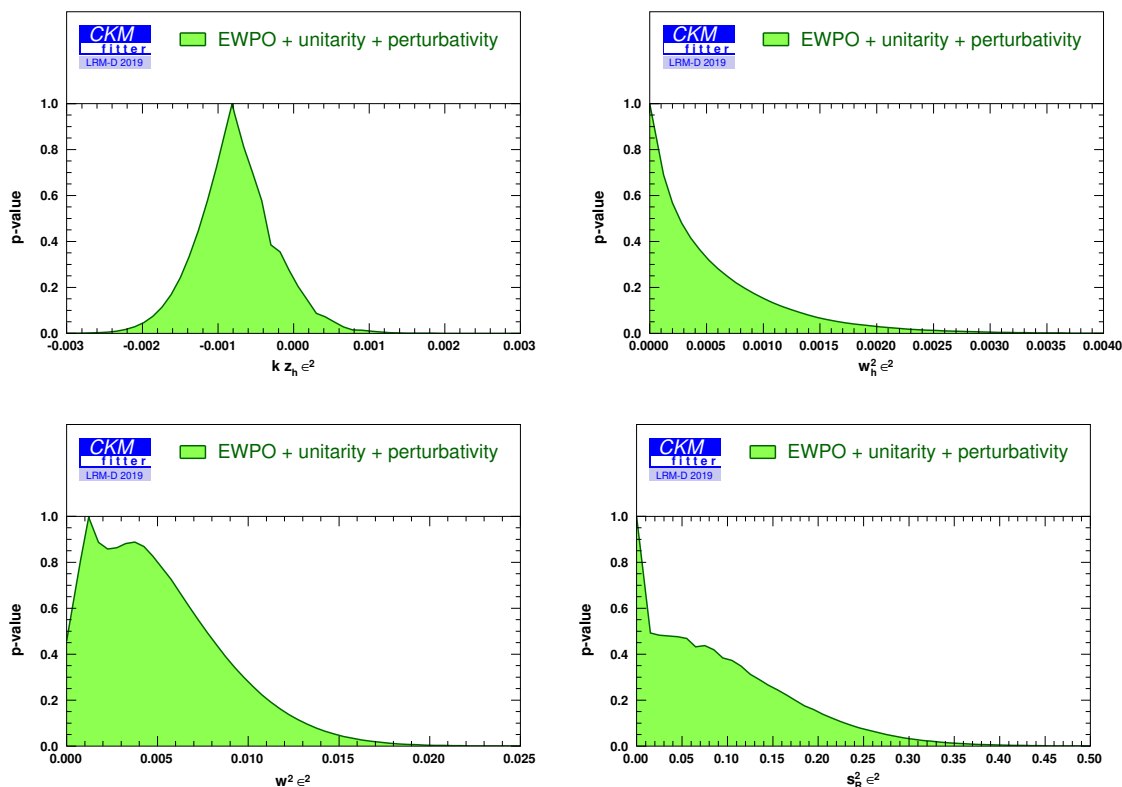
are in good agreement with FLAG, but some are quite a bit lower. The EWPO are also used to determine the strong coupling which results into central values slightly larger than the world average one, but compatible with it, the latest update of the global fit to electroweak precision data by Gfitter [92] giving  $\alpha_s(M_Z^2) = 0.1196 \pm 0.0030$ . Considering a more conservative interval of  $0.117 \pm 0.005$  in our global fit in the DLRM clearly improves the  $\chi^2$  of the fit  $\chi_{\text{min}}^2|_{DLRM} = 18.9$  (the SM  $\chi^2$ , however, remains  $\chi_{\text{min}}^2|_{SM} = 22$ ). This results mainly from a better agreement of the DLRM predictions both of  $\sigma_{had}$  with its experimental value and of  $\alpha_s(M_Z^2)$  with its more conservative input. The pulls for these two quantities decrease respectively to 0.9 and 1.3, while the ones of some other observables get only slightly smaller.

Note that if we disregard in the fit the information on  $\alpha_s$  (column labelled “prediction”) the fit goes towards larger values of  $\alpha_s$ .

## 7 Conclusions

In this article, we have considered a left-right symmetric extension of the Standard Model where the spontaneous breakdown of the left-right symmetry is triggered by doublets. The  $\rho$  parameter is then protected at tree level from large corrections in this Doublet Left-Right Model (DLRM), contrary to the case where triplets are considered. This allows in principle for more diverse patterns of symmetry breaking. There are, however, possibly large radiative corrections coming from the new scalar and vector particles to the  $W$  and  $Z$  self-energies that we investigated. The new scalars can also be probed by unitarity con-



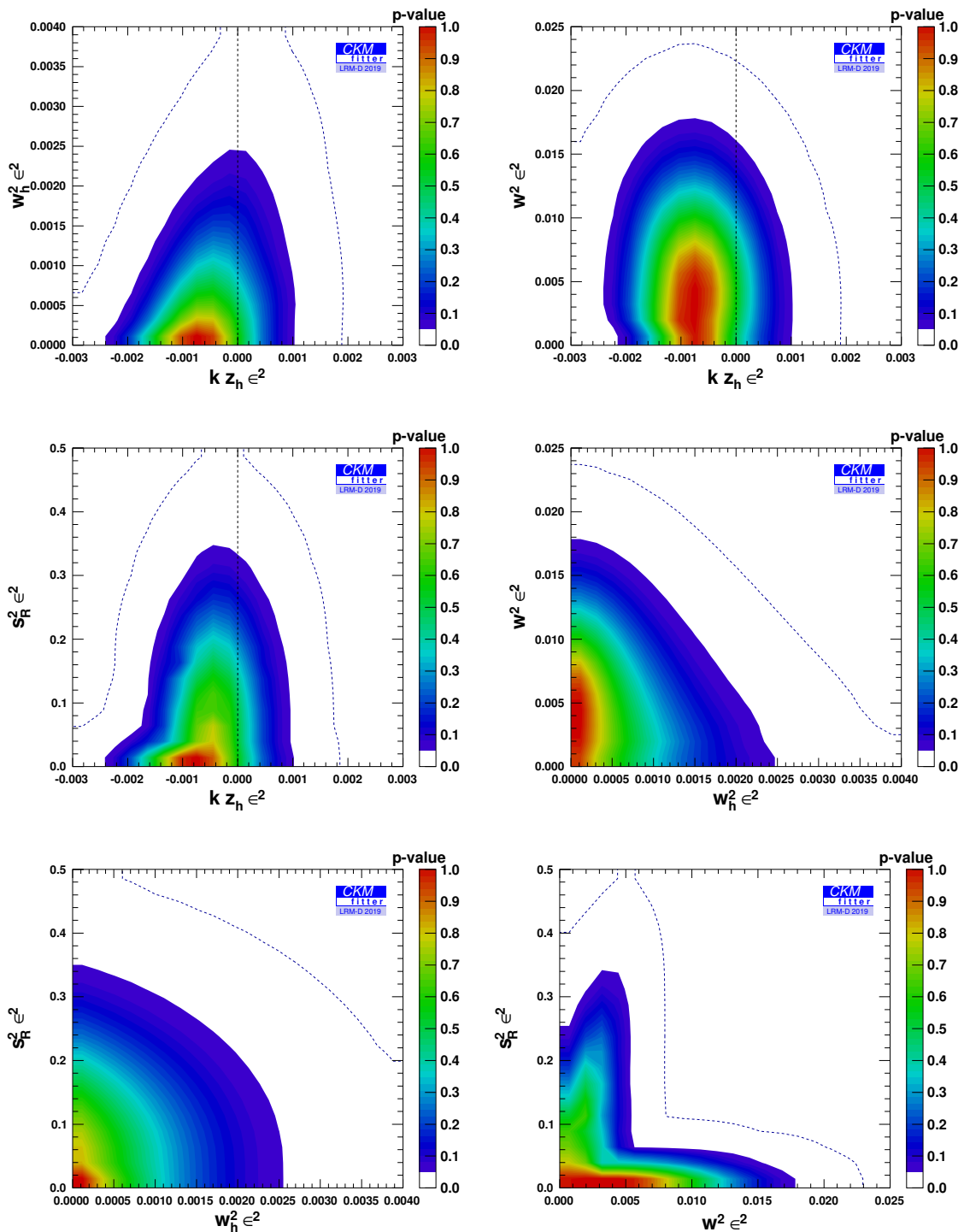


**Figure 8.** The  $p$ -values for some parameters of the DLRM.

straints, exactly as considerations of unitarity in the scattering of longitudinally polarized gauge bosons helped setting theoretical bounds on the mass of the SM Higgs boson much before its direct discovery at LHC.

Combining unitarity, electroweak precision observables and the radiative corrections to the muon  $\Delta r$  parameter within a frequentist (CKMfitter) approach, we see that the model is only mildly constrained: the fit bounds DLRM corrections to remain small, pushing the LR scale to be of the order of a few TeV, thus limiting the sensitivity to the new fundamental parameters. Nonetheless, a new qualitative feature, favoured by the data, emerges from our analysis of DLRM, which is the possibility of having spontaneous EW symmetry breaking triggered also by a doublet under  $SU(2)_L$ , as opposed to the case mostly studied in which EWSB is triggered only by the bi-doublet under  $SU(2)_L \times SU(2)_R$ . This possibility has not received much attention in the literature as it is not allowed in the triplet scenario, in which a triplet under  $SU(2)_L$  is considered.

The favoured masses of the new  $W'$  and  $Z'$  gauge bosons are found in the range  $\mathcal{O}(1 - 3)$  TeV. On the other hand, the large number of new possible scalar self-interactions limits the determination of the masses in the scalar sector, that could be much lighter or much heavier than the LR scale. The requirement of tree-level unitarity, establishing relations between the scalar masses and the LR scale, could be of much help once further constraints sensitive to the scalar sector are added to our analysis. In that respect, our



**Figure 9.** Correlations between the combinations  $kz_h \epsilon^2$ ,  $w_h^2 \epsilon^2$ ,  $w^2 \epsilon^2$  and  $s_R^2 \epsilon^2$ , where  $z_h$  and  $w_h$  are defined in eq. (2.44). The dotted lines set the  $3\sigma$  region.

study can be improved by the consideration of the flavour sector of DLRM. Indeed, a large set of clean flavour observables is available and they are known to set important bounds on generic new interactions changing quark flavours, that in the DLRM are encoded in the CKM-like matrix and its right-handed sector analogue. While these two matrices are closely related in LR models with an additional discrete symmetry (e.g., parity), it is not the case here, leading to a certain number of new parameters. Direct collider bounds on the  $W'$  gauge boson which exploit quark channel decays, or production mechanisms, depending on the same right-handed matrix can be used to determine these parameters as we have briefly discussed in the results section. Indeed we have seen that in order to be in agreement with the results from ATLAS [16] on the production of the  $W'$ , an anti-diagonal matrix would probably be necessary with a bound on the  $W'$  mass of roughly 4 TeV, somewhat higher than what the EW fit gives at the  $1\sigma$  level. However it would pass the EWP test at roughly the same level as the SM. These additional flavour and collider constraints will clearly help in assessing the range of parameters allowed for the DLRM and the viability of the novel pattern of EW symmetry breaking that this model may embed, being the subject of future work (a preliminary version of a full EW fit together with CP violation in kaon meson-mixing is given at [103]).

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## A Corrections from DLRM to EW precision observables

Hereafter, we provide the expressions of the DLRM contributions to the EW precision observables that we have considered:

$$A_\ell = A_\ell^0 + kz_h \varepsilon^2 \frac{8s_W^2 (2s_W^2 - 1) (1 - 2c_R^2 c_W^2)}{(8s_W^4 - 4s_W^2 + 1)^2} \tag{A.1}$$

$$A_c = A_c^0 + kz_h \varepsilon^2 \frac{48s_W^2 (4s_W^2 - 3) (1 - 4c_R^2 c_W^2)}{(32s_W^4 - 24s_W^2 + 9)^2} \tag{A.2}$$

$$A_b = A_b^0 + kz_h \varepsilon^2 \frac{24s_W^2 (2s_W^2 - 3) (-2c_R^2 c_W^2 - 1)}{(8s_W^4 - 12s_W^2 + 9)^2} \tag{A.3}$$

$$A_{FB}(\ell) = A_{FB}^0(\ell) - kz_h \varepsilon^2 \frac{12s_W^2 (8s_W^4 - 6s_W^2 + 1) (1 - 2c_R^2 c_W^2)}{(8s_W^4 - 4s_W^2 + 1)^3} \tag{A.4}$$

$$\begin{aligned}
A_{FB}(c) &= A_{FB}^0(c) + kz_h \varepsilon^2 \frac{18s_W^2}{(8s_W^4 - 4s_W^2 + 1)^2 (32s_W^4 - 24s_W^2 + 9)^2} \\
&\times \left( -2c_R^2 (s_W^2 - 1) (1024s_W^8 - 1600s_W^6 + 992s_W^4 - 310s_W^2 + 39) \right. \\
&\left. + 254s_W^2 - 16 (48s_W^4 - 76s_W^2 + 49) s_W^4 - 33 \right) \tag{A.5}
\end{aligned}$$

$$\begin{aligned}
A_{FB}(b) &= A_{FB}^0(b) + kz_h \varepsilon^2 \frac{72s_W^2}{(8s_W^4 - 12s_W^2 + 9)^2 (8s_W^4 - 4s_W^2 + 1)^2} \\
&\times \left( c_R^2 (-91s_W^2 + 4 (-16s_W^6 + 56s_W^4 - 78s_W^2 + 57) s_W^4 + 15) \right. \\
&\left. + 25s_W^2 + 8 (s_W^2 - 4) s_W^4 - 6 \right) \tag{A.6}
\end{aligned}$$

$$\begin{aligned}
\Gamma_Z &= \Gamma_Z^0 - \frac{\alpha(M_Z^0)M_Z^0}{144k^2 c_W^2 s_W^2} kz_h \varepsilon^2 \left( kz_h (8(\xi + 19)s_W^4 - 12(\xi + 9)s_W^2 + 9(\xi + 6)) \right. \\
&\left. + 4k^2 (-2s_W^2 ((\xi + 49)s_R^2 - 3(\xi + 9)) - 3\xi s_R^2) \right) \tag{A.7}
\end{aligned}$$

$$R_\ell = R_\ell^0 + kz_h \varepsilon^2 \frac{8 (c_W^2 s_R^2 (8(\xi + 4)s_W^4 - 4(4\xi + 13)s_W^2 + 3(\xi + 3)) - 8(\xi + 4)s_W^6 + 3(\xi + 3)s_W^2)}{3(8s_W^4 - 4s_W^2 + 1)^2} \tag{A.8}$$

$$R_u = R_u^0 + kz_h \varepsilon^2 \frac{12(\xi + 2) (s_R^2 (16s_W^6 - 52s_W^4 + 45s_W^2 - 9) + 16s_W^6 - 9s_W^2)}{(8(\xi + 10)s_W^4 - 12(\xi + 6)s_W^2 + 9(\xi + 4))^2} \tag{A.9}$$

$$R_d = R_d^0 + kz_h \varepsilon^2 \frac{24\xi (s_R^2 (-16s_W^6 + 52s_W^4 - 45s_W^2 + 9) - 16s_W^6 + 9s_W^2)}{(8(\xi + 10)s_W^4 - 12(\xi + 6)s_W^2 + 9(\xi + 4))^2} \tag{A.10}$$

$$\begin{aligned}
\sigma_{had} &= \sigma_{had}^0 + kz_h \varepsilon^2 \frac{36\pi}{k^2 (M_Z^0)^2 (8(\xi + 19)s_W^4 - 12(\xi + 9)s_W^2 + 9(\xi + 6))^2} k^2 \\
&\left( \frac{8k^2}{(8(\xi + 19)s_W^4 - 12(\xi + 9)s_W^2 + 9(\xi + 6))} (-96s_W^8 (\xi^2 + (\xi(3\xi + 19) + 49)s_R^2 + 7\xi + 12) \right. \\
&+ 8s_W^6 (6\xi^2 + (\xi(64\xi + 467) + 1285)s_R^2 + 33\xi - 27) + 64(\xi + 1)(\xi + 4) (s_R^2 + 1) s_W^{10} \\
&- 36s_W^4 (\xi(13\xi + 101)s_R^2 - \xi(\xi + 9) + 251s_R^2 - 27) \\
&+ 9s_W^2 ((\xi(23\xi + 180) + 410)s_R^2 - 3(\xi(\xi + 8) + 18)) - 27(\xi(\xi + 8) + 18)s_R^2) \\
&\left. + kz_h (8s_W^4 - 4s_W^2 + 1) (8(\xi + 10)s_W^4 - 12(\xi + 6)s_W^2 + 9(\xi + 4)) \right) \tag{A.11}
\end{aligned}$$

$$\Gamma_W = \Gamma_W^0 - w_h^2 \varepsilon^2 \frac{3\alpha(M_Z^0)M_W^0}{8s_W^2} \tag{A.12}$$

$$\begin{aligned}
Q_W(p) &= Q_W^0(p) + \varepsilon^2 \frac{1}{k^2} \left( (2r^2 (2s_W^2 (c_R^2 w^2 + 4) + (1 - 2c_R^2) w^2 - 2) \right. \\
&\left. + w^2 (4c_R^2 (s_W^2 - 1) (w^2 + 1) + (3 - 4s_W^2) w^2 + 2) \right) \tag{A.13}
\end{aligned}$$

$$Q_W(n) = Q_W^0(n) + \varepsilon^2 \frac{(2 + w^2)(2r^2 + w^2)}{k^2} \tag{A.14}$$

where  $\xi = 0.989$  is a kinematic correction for the channel  $Z \rightarrow b\bar{b}$ , see e.g. [104].

## B Scalar sector

### B.1 Scalar mass eigenstates

The neutral scalar sector exhibits two Goldstone bosons,  $G_Z^0$  and  $G_{Z'}^0$ ,

$$\begin{aligned} G_Z^0 &= \frac{1}{k}(-\phi_{1,i}^0 + r\phi_{2,i}^0 + w\chi_{L,i}^0) + \frac{\epsilon}{k}(1 + r^2 - s_R^2 k^2)\chi_{R,i}^0 + \mathcal{O}(\epsilon^2) \\ G_{Z'}^0 &= \chi_{R,i}^0 + \epsilon[c_R^2(\phi_{1,i}^0 - r\phi_{2,i}^0) + ws_R^2\chi_{L,i}^0] + \mathcal{O}(\epsilon^2) \end{aligned} \quad (\text{B.1})$$

a light Higgs boson of mass  $M_h \sim \mathcal{O}(\epsilon^2)$ , 3 CP-even heavy scalar bosons of masses  $M_i$  at LO defined in eq. (2.25) and two CP-odd heavy scalar bosons

$$A_i^0 = \frac{1}{u_i} \left( (r + t_i w)\phi_{1,i}^0 + \phi_{2,i}^0 + t_i \chi_{0,i}^L \right) + \mathcal{O}(\epsilon^2) \quad M_{A_i^0}^2 = M_i^2 + \mathcal{O}(\epsilon^2) \quad (\text{B.2})$$

with at LO in  $\epsilon$

$$\begin{aligned} M_1^2 + M_2^2 &= -\frac{v_R}{2(r^2 - 1)w} \left( (r^2 + 1)v_R w \alpha_{34} + \sqrt{2}(\mu_1' r^3 + \mu_2' r^2 + \mu_1' r(2w^2 - 1) - \mu_2'(2w^2 + 1)) \right) \\ (M_1^2 - M_2^2)^2 &= (M_1^2 + M_2^2)^2 - \frac{v_R^2 k^2}{(r^2 - 1)w} \left( \sqrt{2}(r\mu_1' + \mu_2')v_R \alpha_{34} + 2w((\mu_1')^2 - (\mu_2')^2) \right) \\ M_3^2 &= 2\rho_1 v_R^2 \end{aligned} \quad (\text{B.3})$$

and  $\alpha_{34}$  is defined in eq. (2.20). The following equalities hold:

$$\begin{aligned} bq(w - rp) &= \frac{M_2^2}{M_1^2} dp(w - rq) \\ &= 2M_2^2 w \left( \sqrt{2}(M_1^2 + M_2^2)rw + v_R(\mu_2' r^3 + \mu_1' r^2 + \mu_2' r(w^2 + 2) + \mu_1' w^2) \right) \\ b &= v_{RW} \left( 2M_2^2 rw(\mu_1' - \mu_2' r) + 2M_1^2 w(\mu_2' + \mu_1' r) + \sqrt{2}v_R((r\mu_1' + \mu_2')^2 + w^2((\mu_1')^2 + (\mu_2')^2)) \right) \\ d &= b(M_1^2 \leftrightarrow M_2^2) \end{aligned} \quad (\text{B.4})$$

We have also the relation (which can be explicitly checked, but comes from the orthogonality of the various eigenvectors)

$$r^2 + 1 + rw(p + q) + pq(w^2 + 1) = 0 \quad (\text{B.5})$$

In the charged sector, one has also 2 Goldstone bosons

$$\begin{aligned} G_W^\pm &= \frac{1}{k} \left[ (-\phi_{1,i}^\pm + r\phi_{2,i}^\pm + w\chi_{L,i}^\pm) + 2r\epsilon\chi_{R,i}^0 \right. \\ &\quad \left. + \frac{2r\epsilon^2}{k^2}(r^3\phi_1^\pm - (1 + w^2)\phi_2^\pm + rw(w\phi_1^\pm + \chi_L^\pm)) \right] + \mathcal{O}(\epsilon^3) \\ G_{W'}^\pm &= \chi_{R,i}^\pm + \epsilon(r\phi_{1,i}^\pm - \phi_{2,i}^\pm) + \mathcal{O}(\epsilon^2) \end{aligned} \quad (\text{B.6})$$

Note that there is a mass degeneracy among the charged and neutral scalars at LO in  $\epsilon$ .

The limit of small  $w$  brings significant simplifications in the expressions, which we provide for illustration. We have<sup>17</sup>

$$\begin{aligned} M_2^2 &= \frac{1}{2} \frac{1+r^2}{1-r^2} v_R^2 \alpha_{34} & M_1^2 &= \frac{1}{2} v_R^2 (\rho_3 - 2\rho_1) \\ b &= \mathcal{O}(w^2) & d &= \sqrt{2} \mu'_2 (r^2 + 1) v_R^2 w (r\mu'_1 + \mu'_2) \end{aligned} \quad (\text{B.7})$$

## B.2 Some useful relations

The spin-0 states are linear combination of the various scalar field of the theory, given in eq. (2.25). Their coefficients obey useful relations:

$$\begin{aligned} d_{\chi_{R,r}}^{H_i} &= \frac{k}{\sqrt{2}} F_i \\ -\frac{1}{u_i} d_{\chi_{R,r}}^{H_i} &= (r+t_i w) d_{\phi_1}^{H_i} + d_{\phi_2}^{H_i} + t_i d_{\chi_{L,r}}^{H_i} \end{aligned} \quad (\text{B.8})$$

$$\begin{aligned} c_{\chi_{R,r}}^{H_i} &= -w S_i + \frac{1}{2(M_{H_i}^2 - M_{H_3}^2)} \left( 2M_{H_2}^2 w S_i + \left( 2\frac{r}{u_i} + w S_i \right) v_R^2 \alpha_{34} - \left( \alpha S_i + \beta \frac{w}{u_i} \right) v_R \mu'_2 \right) \\ -k c_{\chi_{R,r}}^{h^0} &= c^{H_3} = \frac{1}{2\rho_1} \left( (1+r^2)\alpha_1 + 2r\alpha_2 + r^2\alpha_3 + \alpha_4 + 2w^2\rho_1 \right) \end{aligned} \quad (\text{B.9})$$

and

$$c^{\mathcal{H}_i} \equiv \pm c_{\phi_1}^{\mathcal{H}_i} + r c_{\phi_2}^{\mathcal{H}_i} + w c_{\chi_{L,r}}^{\mathcal{H}_i}, \quad i = 1, \dots, 3 \quad (\text{B.10})$$

with  $\mathcal{H}$  stands for either a CP odd or a CP even neutral scalar boson and the  $+/-$  corresponds to the CP even/odd ones respectively. One finds

$$c^{H_i} = -S_i w^3 + \frac{v_R}{\sqrt{2} \delta^2 M_{H_1}^2} \left( \alpha S_i + \beta \frac{w}{u_i} \right) w^2 \mu'_2 \quad (\text{B.11})$$

and

$$\alpha = 1 + w^2 + r x, \quad \beta = r - x \quad (\text{B.12})$$

A similar definition holds for  $c^{h^0}$  and the charged scalar boson  $d_{H_i^\pm}$  with again a nice relation between  $c^{h^0}$  and  $c^{H_3}$  necessary for unitarity to be fulfilled.

$$(c^{H_3})^2 = -2k c^{h^0} \quad (\text{B.13})$$

Two additional relations between the  $c_{\phi_1, \phi_2, \chi_{L,r}}^{H_i}$  which turn out to be useful for the calculation of the  $W'$  self-energy are

$$\begin{aligned} r c_{\phi_1}^{H_i} - c_{\phi_2}^{H_i} &= \frac{t_i^2 w (1+r^2 + t_i r w) (1+\delta^2)}{2u_i^3 (-1+\delta^2)} \left( -w \frac{3k^2 - w^2}{1+r^2} \right. \\ &\quad \left. + \sqrt{2} \frac{\mu'_2 v_R}{M_{H_1}^2 (1+\delta^2)} \left( (r-x) \left( 2 - \frac{u_i}{1+r^2 + t_i r w} \right) - w \frac{k^2}{1+r^2} \left( 1 - \frac{2k^2}{1-r^2} \right) \right. \right. \\ &\quad \left. \left. + (1+w^2 + r x) \left( 1 + \frac{w^2}{1+r^2} \left( 1 - \frac{2k^2}{1-r^2} \right) \right) \right) \right) + \dots \\ c^{H_i}|_{i=1,2} &= -S_i w^3 + \frac{w^2}{\sqrt{2}} \frac{\mu'_2 v_R}{M_{H_1}^2 \delta^2} \left( (1+w^2 + r x) S_i + (r-x) \frac{w}{u_i} \right) + \dots \end{aligned} \quad (\text{B.14})$$

where the dots refer to a lengthy expression which involves terms of a similar type.

<sup>17</sup>We have minimised the potential with respect to  $\mu_1^2, \mu_2^2, \mu_3^2, \rho_3$ . In the limit where  $w$  is small, it proves however more natural to express the masses in terms of  $\rho_3 - 2\rho_1 = -\sqrt{2}(r\mu'_1 + \mu'_2)/(v_R w)$ .

## C Feynman rules

### C.1 Couplings to fermions

The couplings are given in terms of the functions  $F_i$  defined in eqs. (2.29), as can be seen in tables 5–19. It is easy to show that  $F_1 = 0$  and  $F_2 = (-1 + r^2)/(1 + r^2)$  in the limiting case  $w = 0$ , for  $\delta = \mathcal{O}(1/\sqrt{w})$ , thus recovering the formulae given in [52] in the case  $g_R = g_L$  and  $r = 0$ . These two quantities have the very important property that

$$\sum_{i=1,2} F_i(r, w, p) G_i(r, w, p) = 1/k \tag{C.1}$$

where the  $G_i$  are similar functions appearing for similar couplings

$$G_i(r, w, p) = \frac{1}{(-1 + r^2)} \frac{1 + r^2 + rwt_i}{\sqrt{1 + t_i^2 + (r + wt_i)^2}} \tag{C.2}$$

This property is required for example when showing the gauge invariance of the computation of meson mixing within the DLRM, however this goes beyond the scope of our article. In the limiting case  $w = 0$ ,  $G_1 = \sqrt{1 + r^2}/(-1 + r^2)$  and  $G_2 = 0$  so that  $kF_1G_1 = 1$  independently of the value of  $r$ . Interestingly, the quantity

$$\mathcal{F}^r = -2k^2 F_1 G_1 F_2 G_2 \tag{C.3}$$

is proportional to  $rw^3$  in the limit  $w \rightarrow 0$ . One has in terms of the parameters of the model:

$$kF_1G_1 = \frac{1}{2(1 - r^2)(1 + \beta(x)w^2)(1 - \delta^2)} \left( (-k^2 + (k^2 - 2(1 + \nu(x)))X)(1 + \delta^2) + 2(1 + \nu(x) + (r^2 - \beta(x)w^2(1 - r^2) + \nu(x))\delta^2) \right) \tag{C.4}$$

with

$$\nu(x) = w^2/(1 + rx) \tag{C.5}$$

$F_2G_2$  is obtained from this equation changing  $\delta \rightarrow 1/\delta$ .

Most of the couplings of the gauge bosons with the heavy scalars are proportional to  $w$  so that they vanish in the limit  $w = 0$  which is the case of the triplet left right models extensively used in the literature.

### C.2 Couplings to gauge bosons

The couplings to gauge bosons involve

$$u_i = \sqrt{1 + t_i^2 + (r + wt_i)^2} \tag{C.6}$$

and  $w_h$  and  $z_h$  are the quantities which appear in the masses of the light gauge bosons at order  $\epsilon^4$ , see eqs. (2.43). The various coefficients in the coupling of the gauge bosons to

two scalars, table 19 and 13 are defined as follows:

$$\begin{aligned}
 \tilde{w}_h^2 &= w_h^2 - \frac{c_{h^0}}{k} & \tilde{z}_h^2 &= z_h^2 - \frac{c_{h^0}}{k} \\
 w_{\mathcal{H}_i} &= \frac{8r(r+t_i w)}{u_i^2} \pm 2b_{\mathcal{H}_i} & z_{\mathcal{H}_i} &= 2kz_h(S_i^2 - c_R^2) \quad \tilde{z}_{\mathcal{H}_i} = z_{\mathcal{H}_i} + 2b_{\mathcal{H}_i} \\
 c_{H_i^\pm} &= \frac{1}{c_W^2 - s_W^2} \left( z_{\mathcal{H}_i} + \frac{4c_W^2 - 3}{c_W^2 - s_W^2} F_i^2 k^2 \right) \\
 b_{\mathcal{H}_i} &\equiv \frac{1}{u_i} \left( \mp c_{\phi_1}^{\mathcal{H}_i} (r + wt_i) + c_{\phi_2}^{\mathcal{H}_i} + c_{\chi_L}^{\mathcal{H}_i} t_i \right) \\
 d_{H_3} &\equiv \left( c_{\phi_1}^{H_3^0} \right)^2 + \left( c_{\phi_2}^{H_3^0} \right)^2 + \left( c_{\chi_L}^{H_3^0} \right)^2 \tag{C.7}
 \end{aligned}$$

where  $\mathcal{H}_i$  stands for any scalar boson  $H_i, A_i, H_3^0$  and  $h^0$  and the upper/lower signs correspond to the CP even/odd scalar.

Also the following relation holds

$$\sum_i \frac{(r + wt_i)}{1 + t_i^2 + (r + wt_i)^2} = \frac{r^2}{k^2} \tag{C.8}$$

## D More on renormalisation

- **Renormalisation of the charge.**

The bare charge reads

$$\frac{1}{e^2} = \frac{1}{g_X^2} + \frac{1}{g_L^2} + \frac{1}{g_R^2} \tag{D.1}$$

leading to

$$\frac{\delta e}{e} = \delta Z_1^\gamma - \frac{3}{2} \delta Z_2^\gamma \tag{D.2}$$

$\delta e/e$  can be re-expressed using three different relations. First the renormalisations of field and coupling constant for the  $U(1)$  part are related

$$\delta Z_1^B = \delta Z_2^B \tag{D.3}$$

due to the  $U(1)_{B-L}$  Ward identity.<sup>18</sup> Second, the expressions for  $\delta Z_i^{\gamma Z(Z')}$  are given by the coefficient of the  $\partial_\mu A^\nu \partial^\mu Z_\nu(Z'_\nu)$  term of the renormalised Lagrangian

$$\begin{aligned}
 \delta Z_i^{\gamma Z} &= \frac{c_W s_W}{c_W^2 - s_W^2} (\delta Z_i^Z - \delta Z_i^\gamma) + \epsilon^2 k z_h \frac{s_R^2 c_R^2 c_W}{s_W (c_W^2 - s_W^2)^2 (c_R^2 - s_R^2)} (s_W^2 \delta Z_i^Z - c_W^2 \delta Z_i^\gamma + \delta Z_i^{Z'}) \\
 \delta Z_i^{\gamma Z'} &= \frac{c_R s_R c_W}{(c_R^2 - s_R^2)(c_W^2 - s_W^2)} ((c_W^2 - s_W^2) \delta Z_i^{Z'} - c_W^2 \delta Z_i^\gamma) - \frac{s_R s_W^2}{c_W^2 - s_W^2} \delta Z_i^Z \\
 &\quad + \epsilon^2 k z_h \frac{c_R c_W s_R}{(c_R^2 - s_R^2)^2 (c_W^2 - s_W^2)^2} \left( (1 + 2c_W^2(-1 + c_R^2(-1 + 3c_W^2)s_R^2)) \delta Z_i^Z \right. \\
 &\quad \left. + (-1 + 2c_W^2 + 2c_R^2(2 - 6c_W^2 + 3c_W^4)s_R^2) \delta Z_i^\gamma \right. \\
 &\quad \left. - 2s_R^2 c_R^2 (-1 + 2c_W^2)(-2 + 3c_W^2) \delta Z_i^{Z'} \right) \tag{D.4}
 \end{aligned}$$

<sup>18</sup>This relation is formally identical to the QED one [47].



Finally, one can use the on-shell relations for  $\hat{\Sigma}^{\gamma Z}(0)$  and  $\hat{\Sigma}^{\gamma Z'}(0)$  in eq. (4.5). Combining these three relations yields:

$$\frac{\delta e}{e} = \frac{1}{2}\Pi^\gamma(0) + \frac{s_W}{c_W} \frac{\Sigma^{\gamma Z}(0)}{M_Z^2} \left(1 + \frac{s_R^2 k z_h}{s_W^2} \epsilon^2\right) + \frac{s_R}{c_R c_W} \frac{\Sigma^{\gamma Z'}(0)}{M_{Z'}^2} \left(1 - \frac{s_W^2 z_h}{k s_R^2} (c_R^2 k z_h + s_R^2 w^2) \epsilon^2\right) \quad (\text{D.5})$$

with  $\Pi^\gamma$  defined in eq. (4.6).<sup>19</sup>

• **Renormalisation of  $s_W$ .**

It is given by

$$\frac{\delta s_W^2}{s_W^2} = \delta Z_2^W + \Pi^\gamma(0) - 2 \frac{c_W}{s_W} \frac{\Sigma^{\gamma Z}(0)}{M_Z^2} + 2 \frac{c_W z_h}{k c_R s_R} (c_R^2 k z_h + s_R^2 w^2) \frac{\Sigma^{\gamma Z'}(0)}{M_{Z'}^2} \quad (\text{D.6})$$

• **Expressions at leading order in  $\epsilon$  for the heavy particles.**

These are:

$$\begin{aligned} \delta Z_2^{W'} &= -\Pi^\gamma(0) + \delta Z_{Z'W'\gamma} - 2 \frac{s_W^2}{c_W^2} \delta Z_{ZW\gamma} + \mathcal{O}(\epsilon^2) \\ \delta Z_1^{W'} &= -\Pi^\gamma(0) + \frac{3c_R^2 + s_R s_W}{c_W c_R s_R} \frac{\Sigma^{\gamma Z'}(0)}{M_{Z'}^2} + \frac{c_R^2}{s_R^2} \left( \frac{\delta M_{Z'}^2}{M_{Z'}^2} - \frac{\delta M_{W'}^2}{M_{W'}^2} \right) \\ &\quad - 3 \frac{s_W}{c_W} \frac{\Sigma^{\gamma Z}(0)}{M_Z^2} - \left( \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) + \mathcal{O}(\epsilon^2) \end{aligned} \quad (\text{D.7})$$

It is easy to see that at leading order in  $\epsilon$  in the limit  $s_W = 0$  where Left and Right sectors decouple one recovers SM-like expressions with  $s_W \rightarrow s_R$  and  $W \rightarrow W'$ ,  $Z \rightarrow Z'$ . Indeed due to the custodial symmetry the difference  $\delta M_{Z'}^2/M_{Z'}^2 - \delta M_{W'}^2/M_{W'}^2$  is proportional to  $s_W$  and thus cancels in this limit.

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<sup>19</sup>This equation agrees with the one obtained in ref. [58], which was derived in the TLRM in the special case  $w = 0$  and  $g_L = g_R$  when using the relation between  $\Sigma^{\gamma Z^{(\prime)}}$  and  $\delta Z_{\gamma Z^{(\prime)}}$ . The agreement is obtained modulo a factor four which comes from the masses of the gauge bosons which differ by such a factor in the triplet and doublet models, i.e.,  $M_{Z'}^2 = (g_R^2 + g_X^2)v_R^2/4$  in the doublet case ( $M_{Z'}^2 = (g_R^2 + g_X^2)v_R^2$  in the triplet case), where  $v_R/\sqrt{2}$  is the vacuum expectation value of the  $\chi_R$  doublet (respect., triplet).

	$A_\mu^0$	$Z_\mu^0$	$Z_\mu^{\prime 0}$
$\bar{u}_L^i u_L^i$	$i\frac{2}{3}e\gamma^\mu$	$i\frac{e}{c_W s_W}\gamma^\mu \left(\frac{1}{2} - \frac{2}{3}s_W^2 - \frac{1}{6}\epsilon^2[(1+r^2)s_R^2 - k^2 s_R^4]\right)$	$i\frac{e s_R}{c_R c_W} \left(-\frac{1}{6}\right)\gamma^\mu$
$\bar{u}_R^i u_R^i$	$i\frac{2}{3}e\gamma^\mu$	$i\frac{e}{c_W s_W}\gamma^\mu \left(-\frac{2}{3}s_W^2 + \frac{(4c_R^2-1)}{6}\epsilon^2[-w^2 + c_R^2 k^2]\right)$	$i\frac{e}{s_R c_R c_W} \left(\frac{1}{2} - \frac{2}{3}s_R^2\right)\gamma^\mu$
$\bar{d}_L^i d_L^i$	$-i\frac{1}{3}e\gamma^\mu$	$i\frac{e}{c_W s_W}\gamma^\mu \left(-\frac{1}{2} + \frac{1}{3}s_W^2 - \frac{1}{6}\epsilon^2[(1+r^2)s_R^2 - k^2 s_R^4]\right)$	$i\frac{e s_R}{c_R c_W} \left(-\frac{1}{6}\right)\gamma^\mu$
$\bar{d}_R^i d_R^i$	$-i\frac{1}{3}e\gamma^\mu$	$i\frac{e}{c_W s_W}\gamma^\mu \left(\frac{1}{3}s_W^2 - \frac{(2c_R^2+1)}{6}\epsilon^2[-w^2 + c_R^2 k^2]\right)$	$i\frac{e}{s_R c_R c_W} \left(-\frac{1}{2} + \frac{1}{3}s_R^2\right)\gamma^\mu$
$\bar{\nu}_L^i \nu_L^i$	0	$i\frac{e}{c_W s_W}\gamma^\mu \left(\frac{1}{2} + \frac{1}{2}\epsilon^2[(1+r^2)s_R^2 - k^2 s_R^4]\right)$	$i\frac{e s_R}{c_R c_W} \left(\frac{1}{2}\right)\gamma^\mu$
$\bar{\nu}_R^i \nu_R^i$	0	$i\frac{e}{c_W s_W}\gamma^\mu \left(\frac{1}{2}\epsilon^2[(1+r^2) - k^2 s_R^2]\right)$	$i\frac{e}{s_R c_R c_W} \left(\frac{1}{2}\right)\gamma^\mu$
$\bar{\ell}_L^i \ell_L^i$	$-ie\gamma^\mu$	$i\frac{e}{c_W s_W}\gamma^\mu \left(-\frac{1}{2} + s_W^2 + \frac{1}{2}\epsilon^2[(1+r^2)s_R^2 - k^2 s_R^4]\right)$	$i\frac{e s_R}{c_R c_W} \left(\frac{1}{2}\right)\gamma^\mu$
$\bar{\ell}_R^i \ell_R^i$	$-ie\gamma^\mu$	$i\frac{e}{c_W s_W}\gamma^\mu \left(s_W^2 - \frac{(2c_R^2-1)}{2}\epsilon^2[-w^2 + c_R^2 k^2]\right)$	$i\frac{e}{s_R c_R c_W} \left(-\frac{1}{2} + s_R^2\right)\gamma^\mu$

**Table 5.** Couplings of neutral gauge bosons to quarks and leptons.

	$W_\mu^+$	$W_\mu^{\prime +}$
$\bar{u}_L^i d_L^j$	$i\frac{e}{\sqrt{2}s_W}V_L^{ij}\gamma_\mu$	$\mathcal{O}(\epsilon^2)$
$\bar{u}_R^i d_R^j$	$i\frac{e\sqrt{2}}{s_W}r\epsilon^2 V_R^{ij}\gamma_\mu$	$i\frac{e}{\sqrt{2}s_R c_W}V_R^{ij}\gamma_\mu$

**Table 6.** Couplings of charged gauge bosons to quarks (the adaptation to leptons is straightforward).

	$H_i^0$	$A_i^0$
$\bar{u}_R^i u_L^j$	$-\frac{i}{\epsilon v_R} \left(\frac{1}{1-r^2} \left(\frac{2r}{u_i} + w S_i\right) m_u^i \delta^{ij} + G_i V_R^{ia} m_d^a V_L^{ja*}\right)$	$\frac{1}{\epsilon v_R} \left(\frac{1}{1-r^2} \left(\frac{2r}{u_i} + w S_i\right) m_u^i \delta^{ij} + G_i V_R^{ia} m_d^a V_L^{ja*}\right)$
$\bar{u}_L^i u_R^j$	$-\frac{i}{\epsilon v_R} \left(\frac{1}{1-r^2} \left(\frac{2r}{u_i} + w S_i\right) m_u^i \delta^{ij} + G_i V_L^{ia} m_d^a V_R^{ja*}\right)$	$-\frac{1}{\epsilon v_R} \left(\frac{1}{1-r^2} \left(\frac{2r}{u_i} + w S_i\right) m_u^i \delta^{ij} + G_i V_L^{ia} m_d^a V_R^{ja*}\right)$
$\bar{d}_R^i d_L^j$	$\frac{i}{\epsilon v_R} \left(G_i V_R^{*ai} m_u^a V_L^{aj} - \frac{1}{1-r^2} \left(\frac{2r}{u_i} + w S_i\right) m_d^i \delta^{ij}\right)$	$\frac{1}{\epsilon v_R} \left(G_i V_R^{*ai} m_u^a V_L^{aj} - \frac{1}{1-r^2} \left(\frac{2r}{u_i} + w S_i\right) m_d^i \delta^{ij}\right)$
$\bar{d}_L^i d_R^j$	$\frac{i}{\epsilon v_R} \left(G_i V_L^{*ai} m_u^a V_R^{aj} - \frac{1}{1-r^2} \left(\frac{2r}{u_i} + w S_i\right) m_d^i \delta^{ij}\right)$	$-\frac{1}{\epsilon v_R} \left(G_i V_L^{*ai} m_u^a V_R^{aj} - \frac{1}{1-r^2} \left(\frac{2r}{u_i} + w S_i\right) m_d^i \delta^{ij}\right)$

**Table 7.** Couplings of neutral scalar bosons to quarks (the adaptation to leptons is straightforward).  $H_3^0$  does not couple to fermions at this order.

	$H_i^+$
$\bar{u}_R^i d_L^j$	$\sqrt{2} \times \frac{i}{\epsilon v_R} \left( \frac{1}{1-r^2} \left( \frac{2r}{u_i} + w S_i \right) m_u^i V_L^{ij} + G_i V_R^{ij} m_d^j \right)$
$\bar{u}_L^i d_R^j$	$-\sqrt{2} \times \frac{i}{\epsilon v_R} \left( G_i m_u^i V_R^{ij} - \frac{1}{1-r^2} \left( \frac{2r}{u_i} + w S_i \right) V_L^{ij} m_d^j \right)$

**Table 8.** Couplings of charged scalar bosons to quarks (the adaptation to leptons is straightforward).

	$G_W^+$	$G_{W'}^+$
$\bar{u}_R^i d_L^j$	$\frac{-i\sqrt{2}}{\epsilon v_R k} \left[ m_u^i V_L^{ij} \left( 1 + \frac{2r^2}{k^2} \epsilon^2 \right) - 2r \epsilon^2 V_R^{ij} m_d^j \right]$	$\frac{i\sqrt{2}}{v_R k} V_R^{ij} m_d^j$
$\bar{u}_L^i d_R^j$	$\frac{i\sqrt{2}}{\epsilon v_R k} \left[ V_L^{ij} m_d^j \left( 1 + \frac{2r^2}{k^2} \epsilon^2 \right) - 2r \epsilon^2 m_u^i V_R^{ij} \right]$	$\frac{-i\sqrt{2}}{v_R k} m_u^i V_R^{ij}$

**Table 9.** Couplings of Goldstone bosons associated with charged gauge bosons to quarks (the adaptation to leptons is straightforward).

	$G_Z^0$	$G_{Z'}^0$	$h^0$
$\bar{u}_L^i u_R^i$	$\frac{m_u^i}{\epsilon v_R k} \left[ 1 + \frac{w^4 - c_R^4 k^2}{2k^4} \epsilon \right]$	$-c_R^2 \frac{m_u^i}{v_R k}$	$i \frac{m_u^i}{\epsilon v_R k} + \mathcal{O}(\epsilon^2)$
$\bar{d}_L^i d_R^i$	$-\frac{m_d^i}{\epsilon v_R k} \left[ 1 + \frac{w^4 - c_R^4 k^2}{2k^4} \epsilon \right]$	$c_R^2 \frac{m_d^i}{v_R k}$	$i \frac{m_d^i}{\epsilon v_R k} + \mathcal{O}(\epsilon^2)$

**Table 10.** Couplings of Goldstone bosons associated with neutral gauge bosons to quarks (the adaptation to leptons is straightforward). The  $\mathcal{O}(\epsilon^2)$  corrections to  $h^0$  have a rather complicated expression, involving various contributions from the scalar potential.

	$H_i^0$	$A_i^0$
$W^\pm W'^\mp$	$ig_R M_W F_i(r, w, t_i)$	$\mp g_R M_W F_i(r, w, t_i)$
$W^\pm G_{W'}^\mp$	$i \frac{g_R}{2} \frac{M_W}{M_{W'}} F_i(r, w, t_i) (p' - p)_\mu$	$\frac{g_R}{2} \frac{M_W}{M_{W'}} F_i(r, w, t_i) (p' - p)_\mu$
$G_W^\pm W'^\mp$	$i \frac{g_R}{2} F_i(r, w, t_i) (p' - p)_\mu$	$\frac{g_R}{2} F_i(r, w, t_i) (p' - p)_\mu$
$G_W^\pm G_{W'}^\mp$	$-i \frac{g_R}{2} \frac{M_{H_0}^2}{M_{W'}^i} F_i(r, w, t_i)$	$\frac{g_R}{2} \frac{M_{A_0}^2}{M_{W'}^i} F_i(r, w, t_i)$
$Gh Gh'$	$-i \xi \frac{g_R}{\sqrt{2}} M_W F_i(r, w, t_i)$	$\xi \frac{g_R}{\sqrt{2}} M_W F_i(r, w, t_i)$

**Table 11.** Couplings of neutral (pseudo)scalar bosons to gauge bosons, Goldstone bosons and ghosts.

	$h^0$	$H_i^0 (i = 1, 2)$	$A_i^0 (i = 1, 2)$	$H_3^0$
$W_\alpha W_\beta$	$i \frac{2}{v_R k \epsilon} (1 - 2(w_h^2 - \frac{c^{h^0}}{k}) \epsilon^2)$	$i \frac{2}{v_R k} \epsilon (4r F_i(r, w, t_i) + \frac{c^{H_i}}{k})$	0	$\frac{2i}{k^2 v_R} c^{H_3}$
$W_\alpha^\mp W_\beta^\pm$	$i \frac{2w_h}{v_R k}$	$i \frac{2F_i(r, w, t_i)}{v_R}$	$\pm \frac{2F_i(r, w, t_i)}{v_R}$	$\mathcal{O}(\epsilon)$
$W'_\alpha W'_\beta$	$i \frac{2(1+r^2+k c_{\chi_{R,r}}^{h^0}) \epsilon}{v_R k}$	$i \frac{2\epsilon}{v_R} (-w S_i + c_{\chi_{R,r}}^{H_i})$	0	$i \frac{2}{v_R}$
$Z_\alpha Z_\beta$	$i \frac{2}{v_R k \epsilon} (1 - 2(z_h^2 - \frac{c^{h^0}}{k}) \epsilon^2)$	$i \frac{2}{v_R k} (2w S_i z_h + \frac{c^{H_i}}{k}) \epsilon$	0	$\frac{2i}{k^2 v_R} c^{H_3}$
$Z_\alpha Z'_\beta$	$-i \frac{2z_h}{v_R k}$	$i \frac{2}{v_R k} w S_i$	0	$\mathcal{O}(\epsilon)$
$Z'_\alpha Z'_\beta$	$i \frac{2\epsilon}{v_R k} (c_R^2 k z_h + s_R^2 w^2 + k c_{\chi_{R,r}}^{h^0})$	$i \frac{2\epsilon}{v_R} (-c_R^2 - s_R^2) w S_i + c_{\chi_{R,r}}^{H_i}$	0	$i \frac{2}{v_R}$

**Table 12.** Feynman rules relevant for setting unitarity bounds in the left-right model. All the couplings have to be multiplied by  $g_{\alpha\beta}$ . Only the LO terms up to  $\mathcal{O}(\epsilon)$  are shown except for the light Higgs boson where the NLO are also given. The couplings to  $H_3^0$  which are not needed are not given explicitly. The results are given in units of the masses of the two interacting gauge boson given in the first column.

	$h^0$	$H_i^0$	$H_3^0$	$A_i^0$	$H_i^\pm$
$W$	$i \frac{g_L^2}{2} (1 - 2\tilde{w}_h^2 \epsilon^2)$	$i \frac{g_L^2}{2} (1 + w_{H_i} \epsilon^2)$	$i \frac{g_L^2}{2} d_{H_3} \epsilon^2$	0	$i \frac{g_L^2}{2} (1 - k^2 F_i^2 \epsilon^2)$
$Z$	$i \frac{g_L^2}{2c_W^2} (1 - 2\tilde{z}_h^2 \epsilon^2)$	$i \frac{g_L^2}{2c_W^2} (1 + \tilde{z}_{H_i} \epsilon^2)$	$i \frac{g_L^2}{2c_W^2} d_{H_3} \epsilon^2$	0	$i \frac{g_L^2 (c_W^2 - s_W^2)^2}{2c_W^2} (1 - c_{H_i^\pm} \epsilon^2)$

**Table 13.** Couplings of  $W_\alpha W_\beta$  and  $Z_\alpha Z_\beta$  with various scalars. The expressions of the various coefficients of the  $\epsilon^2$  terms are given in eq. (C.7). All these couplings are multiplied by  $g_{\alpha,\beta}$ .

$W_\alpha Z'_\beta$	$W'_\alpha Z_\beta$	$W'_\alpha Z'_\beta$
$-i \frac{2}{v_R k} w S_i$	$-i \frac{2}{v_R} F_i$	$-i \frac{2s_R^2}{v_R} k F_i$

**Table 14.** Same as in table 12 but for the two charged scalars  $H_i^\pm$ .

	$Z_\mu$	$Z'_\mu$	$\gamma$
$W_\alpha^- W_\beta^+$	$ie \frac{c_W}{s_W}$	$-ie \frac{c_R s_R c_W}{s_W^2} k z_h \epsilon^2$	$ie$
$W_\alpha^- W_\beta'^+$	$-ie 2r \frac{s_R}{s_W^2} \epsilon^2$	$ie 2r \frac{c_R}{s_W} \epsilon^2$	$0$
$W_\alpha'^- W_\beta'^+$	$-ie (\frac{s_W}{c_W} - \frac{c_R^2}{c_W s_W} k z_h \epsilon^2)$	$ie \frac{c_R}{c_W} (\frac{1}{s_R} + s_R k z_h \epsilon^2)$	$ie$

**Table 15.** Triple gauge couplings involving either one  $Z$  or one  $Z'$  boson. These couplings have to be multiplied by  $g_{\alpha\beta}(p_+ - p_-)_\mu + g_{\mu\beta}(q - p_+)_\alpha + g_{\mu\alpha}(p_- - q)_\beta$  where  $(p_+, p_-, q)$  are the incoming momenta of the positively, negatively charged and neutral particles respectively.

	$Z_\alpha$	$A_\alpha$
$G^\pm W'_\beta$	$-ig_L \frac{w_h}{c_W} M_{W'} \epsilon$	$0$
$G'^\pm W_\beta$	$0$	$0$
$G'^\pm W'_\beta$	$-ig_L \frac{s_W^2}{c_W} (1 - \frac{1}{s_W^2} (1 + r^2 - s_R^2 k z_h) \epsilon^2) M_{W'}$	$ig_L s_W M_{W'}$
$G^\pm G'^\mp$	$ig_L \frac{1}{2c_W} w_h \epsilon$	$0$
$G'^\pm G'^\mp$	$ig_L \frac{s_W^2}{c_W} \left(1 - \frac{1}{2s_W^2} (1 + r^2 + (c_R^2 - s_R^2) k z_h) \epsilon^2\right)$	$-ig_L s_W$
$g_{W'} g_{W'}$	$g_L \frac{s_W^2}{c_W} (1 - \frac{c_R^2}{s_W^2} k z_h \epsilon^2)$	$-g_L s_W$
$g_{W'} g_W$	$0$	$-2g_L r s_R c_W \epsilon^2$
$Z_\beta W_\gamma^\pm W_\delta'^\mp$	$ig_L^2 \frac{s_W^4}{c_W^2} \left(1 - 2 \frac{c_R^2}{s_W^2} k z_h \epsilon^2\right)$	$-$
$Z_\beta G'^\pm G'^\mp$	$i 2g_L^2 \frac{s_W^4}{c_W^2} \left(1 + \frac{1}{4s_W^4} (c_g - 4s_W^2 d_g) \epsilon^2\right)$	$-2ig_L^2 \frac{s_W^3}{c_W} \left(1 - \frac{1}{2s_W^2} (1 + r^2 + d_g) \epsilon^2\right)$
$Z_\beta G'_0 G'_0$	$i \frac{g_L^2}{2c_W^2} (c_R^2 (1 + r^2) - s_R^2 k z_h) \epsilon^2$	$0$
$A_\beta G'^\pm G'^\mp$	$-2ig_L^2 \frac{s_W^3}{c_W} \left(1 - \frac{1}{2s_W^2} (1 + r^2 + (c_R^2 - s_R^2) k z_h) \epsilon^2\right)$	$2ig_L^2 s_W^2$
$A_\beta W_\gamma^\pm W_\delta'^\mp$	$-ig_L^2 \frac{s_W^3}{c_W} \left(1 - \frac{c_R^2}{s_W^2} k z_h \epsilon^2\right)$	$ig_L^2 s_W^2$

**Table 16.** Triple and quadruple gauge couplings involving a  $Z$  boson and at least one unphysical Goldstone boson.  $c_g = (1 - 4s_W^2)(1 + r^2)$ ,  $d_g = (c_R^2 - s_R^2) k z_h$  and  $M_{W'}$  is the mass of the  $W'$  up to order  $\epsilon^2$ . All the couplings are multiplied by  $g_{\alpha\beta}$  except the quadruple couplings involving only physical gauge bosons, the triple gauge couplings with two unphysical gauge bosons or two ghosts. The former should be multiplied by  $g_{\gamma\alpha} g_{\delta\beta} + g_{\gamma\beta} g_{\delta\alpha} - 2g_{\gamma\delta} g_{\alpha\beta}$  while the latter with unphysical gauge bosons should be multiplied  $(p_- - p_+)_\alpha$  with  $p_\pm$  the incoming momenta of  $G^\pm$ . The triple couplings with ghosts are multiplied by  $p_\alpha$ , the outgoing momenta of the ghost.

	$Z'_\alpha$
$G^\pm W'_\beta$	$-ig_L w_h \frac{s_R s_W}{c_R c_W} M_{W'} \epsilon$
$G'^\pm W_\beta$	$-i \frac{2r}{c_R} \epsilon^2$
$G'^\pm W'_\beta$	$-ig_L \frac{s_R s_W}{c_R c_W} (1 - \frac{1}{2}(1 + r^2 + c_R^2 k z_h) \epsilon^2) M_{W'}$
$G^\pm G'^\mp$	$ig_L \frac{s_R s_W}{c_R c_W} \frac{w_h}{2} \epsilon$
$G'^\pm G'^\mp$	$-ig_L \frac{s_W (c_R^2 - s_R^2)}{2 c_R c_W s_R} \left(1 - \frac{s_R^2}{c_R^2 - s_R^2} (1 + r^2 + 2c_R^2 k z_h) \epsilon^2\right)$
$g_{W'} g_{W'}$	$-g_L \frac{c_R s_W}{c_W s_R} (1 + s_R^2 k z_h \epsilon^2)$
$Z_\beta W'_\gamma{}^\pm W'_\delta{}^\mp$	$-ig_L^2 \frac{s_W^3 c_R}{c_W^2 s_R} (1 - \frac{1}{s_W^2} (c_R^2 - s_R^2 s_W^2) k z_h \epsilon^2)$
$A_\beta W'_\gamma{}^\pm W'_\delta{}^\mp$	$ig_L^2 \frac{s_W^2 c_R}{c_W s_R} (1 + s_R^2 k z_h \epsilon^2)$
$A_\beta G'^\pm G'^\mp$	$ig_L^2 (c_R^2 - s_R^2) \frac{s_W^2}{c_R c_W s_R} (1 + \frac{s_R^2}{c_R^2 - s_R^2} (1 + r^2 + 2c_R^2 k z_h) \epsilon^2)$
$Z_\beta G'^0 G'^0$	$ig_L^2 \frac{c_R s_R s_W}{2c_W^2} (1 + r^2 - w^2 + k z_h) \epsilon^2$
$Z_\beta G'^\pm G'^\mp$	$-ig_L^2 (c_R^2 - s_R^2) \frac{s_W^3}{c_R c_W^2 s_R} (1 + \frac{1}{c_R^2 - s_R^2} (1 + r^2$ $+ \frac{s_R^2 w^2}{2s_W^2} (-3 + 2c_W^2) - \frac{-2 + c_W^2}{s_W^2} (-1 + 2c_R^2 s_R^2) k z_h) \epsilon^2)$

**Table 17.** Triple and quadruple gauge couplings involving a  $Z'$  boson and at least one unphysical Goldstone boson. For the factor multiplying these couplings see table 16.

$W_\alpha G_0 W'_\beta$	$ig_L w_h \epsilon M_{W'}$
$W_\alpha G^\pm Z'_\beta$	$i \frac{g_L}{c_R} z_h \epsilon M_{W'}$
$W_\alpha G^\pm G'_0$	$-g_L \frac{z_h}{2} \epsilon$
$W_\alpha G'^\pm G_0$	$-g_L \frac{w_h}{2} \epsilon$
$W_\alpha W_\beta G'^\pm G'^\mp$	$i \frac{g_L^2}{2} (1+r^2) \epsilon^2$
$W_\alpha W_\beta G'_0 G'_0$	$i \frac{g_L^2}{2} (c_R^2 (1+r^2) - s_R^2 k z_h) \epsilon^2$

**Table 18.** Triple and quadruple gauge couplings involving a  $W$  boson and at least one unphysical Goldstone boson. Only the couplings contributing at  $\epsilon^2$  to  $\Delta r$  are shown. All these couplings are multiplied by  $g_{\alpha\beta}$  except the triple couplings to two unphysical gauge bosons which are multiplied by  $(p_\pm - p_0)_\alpha$  with  $p_{\pm,0}$  the incoming momenta of  $G_{\pm,0}$ .

$W_\alpha H_i^\pm$		$Z_\alpha$	
$h^0$	$\mathcal{O}(\epsilon^2)$	$A_i^0 h^0$	$\mathcal{O}(\epsilon^2)$
$H_i^0$	$i \frac{g_L}{2} (1 + \frac{1}{2}(w_{H_i} - F_i^2 k^2) \epsilon^2)$	$A_i^0 H_i^0$	$\frac{g_L}{2c_W} (1 + \frac{1}{2}(z_H + 2(b_{A_i} + b_{H_i})) \epsilon^2)$
$A_i^0$	$\frac{g_L}{2} (1 - \frac{1}{2}(w_{A_i} + F_i^2 k^2) \epsilon^2)$	$H_i^\pm H_i^\mp$	$i \frac{g_L (c_W^2 - s_W^2)}{2c_W} \left(1 - \frac{1}{c_W^2 - s_W^2} (\frac{z_H}{2} + F_i^2 k^2) \epsilon^2\right)$
$H_3^0$	$i \frac{g_L}{2} b_{H_3}^i \epsilon$	$A_i^0 H_3^0$	$\frac{g_L}{2c_W} b_{H_3}^i \epsilon$

**Table 19.** Couplings of  $W_\alpha$  and  $Z_\alpha$  with two scalars  $\mathcal{H}_1 \mathcal{H}_2$ . The couplings with  $i \neq j$  are  $\mathcal{O}(\epsilon^2)$  and thus irrelevant here. These couplings are multiplied by the difference of the incoming momenta of the two scalars  $(p_{\mathcal{H}_2} - p_{\mathcal{H}_1})_\alpha$  where  $\mathcal{H}_1 = H^0$  and  $\mathcal{H}_2 = H^+$  in the former case.

Obs.	$c_0$	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	$c_8$	$c_9$	$c_{10}$	$c_{11}$
$\Gamma_Z$ [MeV]	2495.16	-2.54	20.12	63.41	-19.73	-3.49	-54.57	9217.30	-0.33	-14.14	-3.53	-93.70
$\sigma_{had}$ [pb]	41478.8	1.81	49.24	-630.62	27.07	2.02	86.56	-85804.67	-1.99	4.88	0.81	223.91
$R_b$ *	215.833	0.03	-3.07	-0.04	-0.67	-0.07	0.76	-20.44	0.02	0.06	0.20	0.47
$R_c$ *	172.23	-0.03	1.01	2.33	1.25	0.38	-1.20	36.86	-0.01	-0.09	-0.18	-0.57
$R_e$ *	20739.2	-8.90	-28.99	788.79	-52.28	-8.88	-364.33	11635.59	0.80	-25.15	-5.80	-413.52
$R_\mu$ *	20739.4	-8.90	-28.99	788.79	-52.27	-8.88	-364.34	11635.29	0.80	-25.15	-5.80	-413.54
$R_\tau$ *	20786.3	-8.91	-29.12	790.58	-52.33	-8.89	-364.59	11549.52	0.80	-25.16	-5.80	-414.50
$A_{FB}(b)$ *	102.81	-2.84	15.14	-2.33	-1.33	-1.62	-115.77	3571.84	-0.22	-1.12	-1.14	-22.73
$A_{FB}(c)$ *	73.45	-1.75	9.49	-1.49	-0.27	-1.56	-91.45	2800.34	-1.29	-0.13	1.23	-10.99
$A_{FB}(\ell)$ *	16.13	-0.11	2.04	-0.31	-1.46	-0.12	-34.68	980.77	0.36	-1.16	-0.21	40.07
$\mathcal{A}_b$ *	934.637	-0.32	0.55	-0.18	-1.21	-0.09	-13.39	423.59	$1.2 \times 10^{-3}$	-0.95	-0.35	-11.17
$\mathcal{A}_c$ *	667.717	-1.75	9.42	-1.45	-6.96	-1.02	-71.76	2264.66	-0.15	-5.53	-1.84	-48.56
$\mathcal{A}_\ell$ *	146.673	-4.00	21.44	-3.30	-17.73	-2.41	-162.98	5149.15	-0.36	-13.90	-4.36	-141.83
$M_W$ [GeV]	80.3644	-0.06	0.53	-0.08	-0.10	-0.05	-1.09	114.74	-0.01	-0.09	-0.07	-0.79
$\Gamma_W$ [MeV]	2090.86	-4.66	41.95	47.91	-19.03	-4.11	-84.25	8942.90	-0.51	-14.92	-8.32	-106.34
$Q_W(Cs)$	-72.9586	-0.10	0.10	-0.23	-0.92	-0.15	-5.22	166.44	-0.01	-0.70	-0.25	-4.88
$Q_W(Tl)$	-115.469	-0.15	0.10	-0.35	-1.38	-0.24	-7.74	246.99	-0.02	-1.04	-0.38	-7.28
$Q_W(p)$ *	73.0053	-2.10	11.34	-1.63	-12.37	-1.18	-85.55	2700.33	-0.17	-9.68	-2.87	-73.06

**Table 20.** Values of the first 12 coefficients in the parametrization of various EWPO, see eq. (5.8). Above, \* =  $10^3$ .



Obs.	$c_{12}$	$c_{13}$	$c_{14}$	$c_{15}$	$c_{16}$	$c_{17}$	$c_{18}$	$c_{19}$	$c_{20}$	$c_{21}$	$c_{22}$	$c_{23}$
$\Gamma_Z$ [MeV]	11510.0	133.8	16.6	3377.0	362.5	11700.0	-2377.0	-1102.0	3304.0	-168.8	-3672.0	166900.0
$\sigma_{had}$ [pb]	341600.0	548.4	1642.0	-1344.0	53970.0	214100.0	$-1.8 \times 10^6$	-184600.0	$-2.1 \times 10^6$	-13570.0	-982300.0	445000.0
$R_b^*$	-73.3	-1.4	-0.2	-14.1	-14.5	-81.5	712.3	99.5	987.5	5.6	516.9	-41.7
$R_c^*$	101.4	-3.2	0.9	19.6	20.5	120.7	-1054.0	-147.6	-1478.0	-8.0	-776.1	980.6
$R_e^*$	-151300.0	587.3	-1251.0	5756.0	-39890.0	-146400.0	720200.0	112200.0	931400.0	9315.0	506900.0	-79820.0
$R_\mu^*$	-151300.0	587.3	-1251.0	5756.0	-39890.0	-146400.0	720200.0	112200.0	931400.0	9315.0	506900.0	-79930.0
$R_\tau^*$	-151800.0	584.7	-1256.0	5760.0	-40050.0	-147000.0	724700.0	112800.0	937100.0	9355.0	509600.0	-80730.0
$A_{FB}(b)^*$	3596.6	3.3	-25.6	1800.0	371.3	5303.9	12443.0	1596.6	5560.0	435.5	2023.4	-354.0
$A_{FB}(c)^*$	6586.8	-18.2	127.5	1407.8	238.4	5979.6	-48140.7	-4440.0	-50758.5	-1595.7	-25437.1	113211.1
$A_{FB}(\ell)^*$	14385.0	-15.1	98.8	495.0	2504.5	12994.3	-16064.1	-33899.1	-140962.4	-1469.0	-135554.3	-59309.6
$\mathcal{A}_b^*$	-1993.0	1.5	-7.3	212.1	-557.3	-1781.0	8844.0	1418.0	11250.0	114.5	6110.0	1780.0
$\mathcal{A}_c^*$	-10690.0	5.9	-42.5	1141.0	-3018.0	-9620.0	40190.0	6491.0	50090.0	589.0	27720.0	106100.0
$\mathcal{A}_\ell^*$	-25700.0	4.6	-36.7	2594.0	-7211.0	-23250.0	-16150.0	5536.0	-16030.0	1150.0	6526.0	$1.6 \times 10^6$
$M_W$ [GeV]	-66.6	-0.049	0.2	17.7	-30.0	-65.9	239.9	58.6	328.3	4.2	200.0	322.5
$\Gamma_W$ [MeV]	7131.0	99.0	25.5	3465.0	-761.3	7405.0	9060.0	1443.0	18570.0	89.7	6349.0	185300.0
$Q_W(Cs)$	-392.3	-0.4	-1.3	83.4	-121.3	-306.9	4069.0	619.9	5892.0	23.5	3185.0	-64540.0
$Q_W(Tl)$	-579.7	-0.7	-1.8	123.7	-179.3	-452.8	6120.0	931.5	8875.0	34.9	4797.0	-97900.0
$Q_W(p)^*$	-6804.0	1.2	-27.7	1363.0	-2072.0	-5419.0	51620.0	8026.0	72530.0	386.0	39410.0	-676800.0

**Table 21.** Same as in table 20 but for the next 12 coefficients.

Obs.	$c_{24}$	$c_{25}$	$c_{26}$	$c_{27}$	$c_{28}$	$c_{29}$	$c_{30}$	$c_{31}$	$c_{32}$	$c_{33}$	$c_{34}$	max. dev. [%]
$\Gamma_Z$ [MeV]	661800.0	$1.9 \times 10^6$	-57.1	132.4	65.2	$2.0 \times 10^6$	-42.4	-186.2	592900.0	$2.8 \times 10^6$	$-1.0 \times 10^6$	0.005
$\sigma_{had}$ [pb]	$3.0 \times 10^6$	$8.4 \times 10^6$	-6709.0	44420.0	25210.0	$8.1 \times 10^6$	-2578.0	118100.0	-32260.0	$1.3 \times 10^7$	$-8.6 \times 10^6$	0.007
$R_b^*$	-2019.0	-5477.0	2.7	-44.6	-16.5	-4874.0	2.0	-62.8	-1549.0	-6782.0	8356.0	0.002
$R_c^*$	3014.0	8255.0	-4.0	66.2	25.0	6374.0	-2.9	93.5	1612.0	10250.0	-10310.0	0.002
$R_e^*$	$-1.1 \times 10^6$	$-2.4 \times 10^6$	4678.0	-40130.0	-15960.0	$-2.1 \times 10^6$	1964.0	-58390.0	687300.0	$-3.4 \times 10^6$	$1.6 \times 10^6$	0.01
$R_\mu^*$	$-1.1 \times 10^6$	$-2.4 \times 10^6$	4679.0	-40130.0	-15970.0	$-2.1 \times 10^6$	1965.0	-58390.0	687300.0	$-3.4 \times 10^6$	$1.6 \times 10^6$	0.01
$R_\tau^*$	$-1.1 \times 10^6$	$-2.4 \times 10^6$	4698.0	-40360.0	-16040.0	$-2.1 \times 10^6$	1975.0	-58750.0	688400.0	$-3.4 \times 10^6$	$1.7 \times 10^6$	0.01
$A_{FB}(b)^*$	3568.8	2181.3	219.6	-3.9	-164.8	28008.9	-79.8	12.0	7046.5	6590.4	-14429.2	0.066
$A_{FB}(c)^*$	320931.0	422936.3	-752.8	-1358.9	250.6	544022.1	-5193.3	3974.9	88399.4	705771.5	191032.3	11.0
$A_{FB}(\ell)^*$	-208918.8	371316.2	-645.1	1609.5	573.0	$2.1 \times 10^6$	2840.4	1203.5	20210.1	$2.2 \times 10^6$	-103658.3	11.5
$\mathcal{A}_b^*$	-19660.0	-44070.0	57.5	-491.8	-196.5	-33490.0	25.0	-713.0	24000.0	-65120.0	63380.0	0.002
$\mathcal{A}_c^*$	474200.0	$1.1 \times 10^6$	295.5	17.2	-40.6	593100.0	-99.3	-785.6	150800.0	$1.3 \times 10^6$	-576900.0	0.007
$\mathcal{A}_\ell^*$	$1.4 \times 10^6$	$-6.4 \times 10^6$	579.6	3559.0	336.0	$2.4 \times 10^7$	6449.0	-3344.0	$1.1 \times 10^6$	$5.8 \times 10^6$	$-1.3 \times 10^6$	0.02
$M_W$ [GeV]	-553.8	-900.2	2.7	-15.1	-6.4	-465.3	-0.7	-18.5	1841.0	-1650.0	-926.5	0.0008
$\Gamma_W$ [MeV]	666200.0	$1.6 \times 10^6$	87.2	205.6	-78.3	$1.7 \times 10^6$	54.6	-83.2	692100.0	$2.2 \times 10^6$	$-1.3 \times 10^6$	0.007
$Q_W(Cs)$	-558000.0	$-1.7 \times 10^6$	11.9	101.5	17.4	$-1.0 \times 10^6$	25.7	38.7	31300.0	$-2.1 \times 10^6$	-103300.0	0.005
$Q_W(Tl)$	-845800.0	$-2.5 \times 10^6$	17.7	156.2	27.2	$-1.5 \times 10^6$	39.0	61.9	47170.0	$-3.2 \times 10^6$	-156200.0	0.005
$Q_W(p)^*$	$-5.9 \times 10^6$	$-1.8 \times 10^7$	194.2	681.9	53.9	$-1.1 \times 10^7$	253.7	-145.2	382200.0	$-2.3 \times 10^7$	$-1.2 \times 10^6$	0.04

**Table 22.** Same as in table 20 but for the last 11 coefficients. The maximum deviation is given in the last column.

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