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Constraint on the Mass Scale of a Left-Right-Symmetric Electroweak Theory from the K_L-K_S Mass Difference

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The K_L-K_S mass difference provides a stringent constraint on the mass (M_R) of the charged right-handed gauge field occurring in a "manifest" left-right-symmetric electroweak theory, yielding $M_R \gtrsim 1.6$ TeV. Taken in the context of a grand-unifying gauge theory, e.g., O(10), such a large bound on M_R, along with the measured value of $\sin^2\theta_W$, implies that $M_R \gtrsim 10^9$ GeV.

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The standard model of electroweak interactions is currently enjoying great success in being able to account for the existing experimental data.[1] However, the purely left-handed nature of that model is widely considered to be highly asymmetrical and unaesthetic leading to the suggestion that the standard model may be only an effective theory valid at current energies and that at higher energies an underlying left-right-symmetric (LRS) theory may make its presence felt.[2] It is therefore important to seek an understanding of the energy scales at which one may hope to see some evidence of such a symmetric theory.

A comprehensive study of the experimental constraints on a LRS theory from the low-energy charged-current sector was first conducted by Bég *et al.*,[3] leading them to conclude (among other things) that

$$\beta \equiv (M_L/M_R)^2 \lesssim 0.13 \Rightarrow M_R \gtrsim 220 \text{ GeV}, \tag{1}$$

where M_L and M_R are the masses of the charged left- and right-handed gauge bosons. In this work we present constraints imposed on the mass scale of a LRS theory deduced from the precisely determined mass difference, Δm_K, between K_L and K_S.[4] We find that, even allowing for the theoretical uncertainties in the calculation of Δm_K, the constraints on β are by far the most stringent to date, surpassing not only the existing constraints derived from β decay, muon decay, and neutral currents, but also projected improvements from forthcoming precision experiments.

We consider a six-quark model based on $G_W = \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)$ with the charged-current eigenstates placed in left- and right-handed doublets.[5] To calculate Δm_K, we first calculate the free-quark amplitude $A(d\bar{s} \to \bar{d}s)$ and then, taking that amplitude to be the effective Lagrangian, $\mathcal{L}_{\text{eff}}$, we evaluate the matrix element $\langle \bar{K}_0 | \mathcal{H}_{\text{eff}} | K_0 \rangle$, where $\mathcal{H}_{\text{eff}} = -\mathcal{L}_{\text{eff}}$.

The leading correction to the result obtained in the standard model[6] arises from the box graph in Fig. 1, A_{LR}, plus its equivalent $L \leftrightarrow R$ exchange. In the limit where the external quark momenta are assumed to be negligible compared to the loop momenta, we obtain, in the 't Hooft–Feynman gauge

$$A_{LR}(d\bar{s} \to \bar{d}s) = \left(\frac{g}{\sqrt{2}}\right)^4 \left(\frac{O_{LR}}{8\pi^2 M_R^{\,4}}\right) \sum_{i,j=u,c,t}\sum m_i U_{is}^{R*} U_{id}^{L} m_j U_{js}^{L*} U_{jd}^{R}$$
$$\times \left[\frac{\beta \ln\beta}{(1-\beta)(\epsilon_i - \beta)(\epsilon_j - \beta)} + \frac{\epsilon_i \ln\epsilon_i}{(1-\epsilon_i)(\beta - \epsilon_i)(\epsilon_j - \epsilon_i)} + \frac{\epsilon_j \ln\epsilon_j}{(1-\epsilon_j)(\beta - \epsilon_j)(\epsilon_i - \epsilon_j)}\right], \tag{2}$$

where $\epsilon_i = m_i^{\,2}/M_R^{\,2}$, U^L and U^R are 3×3 unitary matrices relating the left- and right-handed charged-current eigenstates to the mass eigenstates, and

$$O_{LR} \equiv [\bar{\psi}_s^{\,\alpha} \tfrac{1}{2}(1-\gamma_5)\psi_d^{\,\alpha}][\bar{\psi}_s^{\,\beta} \tfrac{1}{2}(1+\gamma_5)\psi_d^{\,\beta}],$$

where α and β are summed color indices. If we now assume $1 \gg \beta \gg \epsilon_{i,j}$, and $m_t^{\,2} \gg m_c^{\,2} \gg m_u^{\,2} \simeq 0$, we obtain

$$A_{LR}(d\bar{s}\to\bar{d}s) = -\beta(g^4 O_{LR}/32\pi^2 M_L^4) \times[\lambda_c\lambda_c' m_c^2(\ln\eta_c+1)+\lambda_t\lambda_t' m_t^2(\ln\eta_t+1)+(\lambda_c\lambda_t'+\lambda_t\lambda_c')m_c m_t \ln\eta_t], \quad (3)$$

where $\eta_i \equiv m_i^2/M_L^2$, $\lambda_i \equiv U_{is}^{R*}U_{id}^{L}$, and $\lambda_i' \equiv U_{is}^{L*}U_{id}^{R}$.

To proceed further we must make an assumption about the nature of the mixing angle in the right-hand sector. The simplest and most natural assumption is that of "manifest" LRS,[3] wherein the quark mass matrix is Hermitian and diagonalizable by a single unitary transformation, i.e., $U^R = U^L$. This implies $\lambda_i = \lambda_i'$, allowing us to sum the contributions coming from the exchange of two W_L's (i.e., the standard-model result[6]), two W_R's, and one W_L and one W_R. We thus obtain

$$A(d\bar{s}\to\bar{d}s) = -\frac{G_F^2}{4\pi^2}\Big\{\lambda_c^2 m_c^2[(1+\beta^2)O_{LL}+8\beta(\ln\eta_c+1)O_{LR}]+\lambda_t^2 m_t^2[(1+\beta^2)O_{LL}+8\beta(\ln\eta_t+1)O_{LR}] + 2\lambda_c\lambda_t m_c^2\Big[(1+\beta^2)\ln\Big(\frac{m_t^2}{m_c^2}\Big)O_{LL}+8\beta\frac{m_t}{m_c}(\ln\eta_t)O_{LR}\Big]\Big\}, \quad (4)$$

where

$$O_{LL} \equiv [\bar{\psi}_s{}^{\alpha}\gamma_\mu\tfrac{1}{2}(1-\gamma_5)\psi_d{}^{\alpha}][\bar{\psi}_s{}^{\beta}\gamma^\mu\tfrac{1}{2}(1-\gamma_5)\psi_d{}^{\beta}].$$

This sum omits two classes of box graphs whose contributions are small. First are the graphs, allowed under the assumption of mixing between W_L and W_R, in which the W's connect left- and right-handed currents. These are $\propto \tan^2\zeta < (0.06)^2$, where ζ is the left-right mixing angle.[7] Because of our choice of gauge, there is also the set of graphs wherein one or both of the W's are replaced by unphysical scalars. These graphs, however, are suppressed by the mass-dependent scalar couplings.[8]

We must now evaluate the matrix elements $\mathfrak{M}_{LL} \equiv \langle\bar{K}^0|O_{LL}|K^0\rangle$ and $\mathfrak{M}_{LR} \equiv \langle\bar{K}^0|O_{LR}|K^0\rangle$. A precise calculation is not feasible because of the traditional difficulties arising from hadron structure and strong interactions. The simplest approximation is that of vacuum saturation wherein one formally inserts a complete set of intermediate states in all possible ways, but then assumes that the contribution of the vacuum will dominate. For the purely left-handed (or right-handed) case one obtains the standard result[6]:

$$\mathfrak{M}_{LL} \simeq \mathfrak{M}_{LL}^{\text{vac}} = \tfrac{2}{3} f_K^2 m_K^2/2m_K. \quad (5)$$

To evaluate the LR contribution we also use the divergence equation $\bar{\psi}_1\gamma_5\psi_2 = -i\partial_\mu(\bar{\psi}_1\gamma^\mu\gamma_5\psi_2)/(m_1+m_2)$ to obtain

$$\mathfrak{M}_{LR} \simeq \mathfrak{M}_{LR}^{\text{vac}} = \tfrac{1}{2}[m_K^2/(m_s+m_d)^2+\tfrac{1}{6}]f_K^2 m_K^2/2m_K \simeq 7.7\mathfrak{M}_{LL}^{\text{vac}}, \quad (6)$$

where we have used the current-quark masses $m_s = 150$ MeV and $m_d = 7$ MeV. Combining this with Eq. (4) yields

$$A(K^0\to\bar{K}^0) = \langle\bar{K}^0| -\mathcal{L}_{\text{eff}}(d\bar{s}\to\bar{d}s)|K^0\rangle \simeq (3.4\times10^{-12}\ \text{GeV})\{0.01\lambda_c^2(1-430\beta)+(1.8,7.3)\lambda_t^2[1-(110,24)\beta] + 0.02\lambda_c\lambda_t[(5.2,6.6)-(2300,2300)\beta]\}, \quad (7)$$

where we have used $m_c = 1.5$ GeV, $\mathfrak{M}_L = 80$ GeV, and where in (a,b) a refers to $m_t = 20$ GeV and b refers to $m_t = 40$ GeV.

The lack of knowledge of λ_t and m_t makes it difficult to evaluate the contributions of the second and third terms above. However, for the case of a four-quark model ($\lambda_c^2 = \sin^2\theta_C\cos^2\theta_C$, $\lambda_t = 0$) we find

$$\Delta m_K = 2\,\text{Re}[A(K^0\to\bar{K}^0)] \simeq 0.32\times10^{-14}[1-430\beta]\ \text{GeV}, \quad (8)$$

to be compared to the experimental value: $\Delta m_K(\text{expt.}) = 0.35\times10^{-14}$ GeV. It therefore seems likely that unless there are some highly contrived cancellations[9] between the charm and the top quark con-

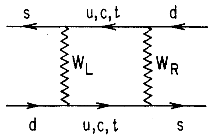

FIG. 1. The "scattering" graph for the transition $\bar{s}d \to s\bar{d}$ via W_L, W_R exchange. The "annihilation" term resulting from the crossed graph must also be included in the calculation.

tributions the charm quark contribution would dominate in Eq. (7). We thus obtain as a conservative constraint

$$\beta \lesssim \tfrac{1}{430} \quad \text{or} \quad M_R \gtrsim 1.6\ \text{TeV}, \tag{9}$$

corresponding to the extreme situation that the charm quark contribution to Δm_K is zero and the entire contribution to Δm_K arises from the terms in (7) containing the top quark.

QCD corrections reduce the free-quark amplitudes for the box graph given in Eq. (4). However, a calculation based on the work of Gilman and Wise[10] but including the additional renormalization between M_R and M_L indicates that QCD corrections for the LRS theory are very similar to those for the purely left-handed case. Since the constraints on β and M_R, given in (9), result from a ratio of the left-right contribution to the left-left contribution, they are expected to remain essentially intact once QCD corrections are taken into account.

As a test of the reliability of the vacuum-saturation approximation we have evaluated the matrix elements $\mathfrak{M}_{LL}{}^{1\pi}$ and $\mathfrak{M}_{LR}{}^{1\pi}$ for a single-pion intermediate state. The exact magnitude of these contributions cannot be determined because of uncertainties in the $K \to \pi$ form factors, though, as has been previously observed,[11] they may possibly be of the same order as $\mathfrak{M}_{LL}{}^{\text{vac}}$ and $\mathfrak{M}_{LR}{}^{\text{vac}}$, respectively. In the standard left-handed model this would lead to a cancellation. However, the combination of the $L \times R$ current structure and the opposite charge-conjugation property of vector and axial-vector currents ensures us that the matrix element $\mathfrak{M}_{LR}$, unlike $\mathfrak{M}_{LL}$,[11] has the same sign whether one saturates the intermediate states with vacuum or with one pion. Thus the single-pion contribution would strengthen the constraint given in (9), possibly quite substantially.

As another test we have also evaluated $\mathfrak{M}_{LR}$ using the Massachusetts Institute of Technology bag model.[12] McWilliams and Shanker[13] have calculated the $\bar{K}^0 \to K^0$ transition amplitude for a general S, P, V, A, and T current-current interaction. Using their work we find that $\mathfrak{M}_{LR}/\mathfrak{M}_{LL}$ is larger, by a factor of about $\frac{5}{3}$, in that model than in the vacuum estimate in (6). This, along with the single-pion insertion, supports the suggestion that despite the uncertainty arising from our inability to evaluate accurately the effect of the top quark, the constraint given in (9) is likely to be a conservative one unless, as stated previously, there are highly artificial cancellations in Eq. (7).

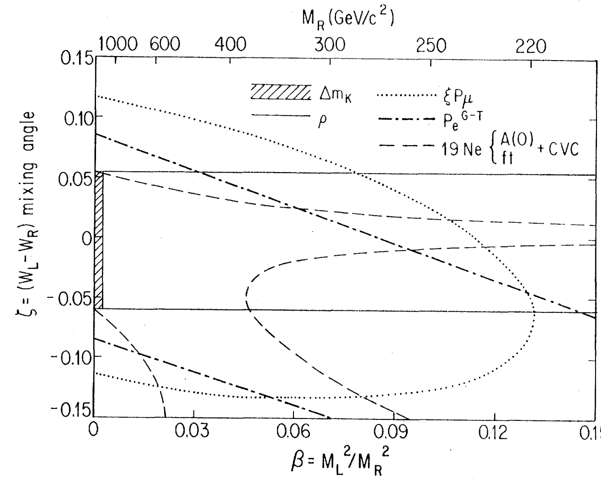

FIG. 2. Limits on the parameters ζ (W_L-W_R mixing angle) and β ($=M_L^2/M_R^2$) of a manifest left-right-symmetric theory resulting from the existing experiments (see Ref. 15) on β and muon decay are compared with that obtained from the K_L-K_S mass difference, Δm_K. The shaded region results from the constraint [Eq. (9)] on β from Δm_K and the limit on ζ set by the Michel parameter (ρ).

In Fig. 2 we present a comparison of our results with the previous[14] constraints on the parameters (ζ and β) of a manifest LRS electroweak theory. The Michel parameter ρ in muon decay limits the LR mixing angle to $|\zeta| < 0.06$. The constraints from experiments on the angular asymmetry ξP_μ in muon decay, polarization measurements from β decay, and measurements of the angular asymmetry of electrons for polarized ^{19}Ne nuclei are summarized in the figure.[15] From these existing experiments one gets a bound $M_R \gtrsim 250$ GeV. The allowed region ($\beta \lesssim 0.0023$ or $M_R \gtrsim 1.6$ TeV) resulting from the K_L-K_S mass difference is shown as a dashed line. To put the latter constraint on M_R in perspective we note that precision experiments, proposed or in progress, involving high-statistics measurements of ρ by Anderson *et al.*[16] and ξP_μ by Gidal *et al.*[17] promise to yield a limit on M_R around 500 GeV. A possible experiment using a polarized e^- beam to measure $e_R^- + p \to \nu_e + X$ in a new ep colliding machine with, say, 30-GeV e^- on 800-GeV protons is estimated to yield a comparable limit on M_R (i.e., around 500 GeV).

It should be noted that the constraints coming from low-energy decays (β or muon) become invalid if the mass of the right-handed neutrino is greater than the available energy. Our limit, on the other hand, is independent of the neutrino mass.[18]

Finally we consider the implication of such a constraint on M_R for the embedding of a LRS electroweak theory in a grand-unifying gauge group such as O(10). Rizzo and Senjanović[19] have recently reexamined the neutral-current data in the context of a LRS theory and have found it to be consistent with a low value of M_R of order 100 to 250 GeV if one also takes a relatively large value of $\sin^2\theta_W$ in the range of 0.31 to 0.25. They then observed that this particular range of values is compatible with the evolution equation arising from the embedding of the LRS electroweak gauge group in O(10). Values of $M_R \gtrsim 300$ GeV, however, leave the experimental value of $\sin^2\theta_W \simeq 0.22$ unaffected. The evolution equation then implies that if the unification mass is not to exceed the Planck mass we must have $M_R \gtrsim 10^9$ GeV. The bound $M_R \gtrsim 1.6$ TeV resulting now from the K_L-K_S mass difference would thus mean that, even for the nonminimal grand-unifying gauge group O(10) which has manifest LRS embedded into it, the intermediate mass scale (beyond M_L) is so large that, just as in SU(5), one has an effective "desert."[20]

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[5]We use a six-quark extension of the model proposed by Mohapatra and Pati, Ref. 2.

[6]Gaillard and Lee, Ref. 4; Ellis, Gaillard, and Nanopoulos, Ref. 4.

[7]In the left-right-symmetric model described in Ref. 3 the left- and right-coupling charged gauge bosons are related to the mass eigenstates by $W_L = W_1\cos\zeta + W_2\sin\zeta$, $W_R = W_2\cos\zeta - W_1\sin\zeta$.

[8]The dominant contribution from scalar exchanges arising from the inclusion of right-handed currents [for the contribution due to purely left-handed currents, see T. Inami and C. S. Lim, Prog. Theor. Phys. 65, 297 (1981)] comes from the analog to Fig. 1 with $W_L \to S_L$ (a left-coupling unphysical scalar). In a four-quark model this leads to an enhancement of $A_{LR}(d\bar{s} \to \bar{d}s)$ of $\simeq 8\%$ for $\beta = \frac{1}{10}$ and $\simeq 22\%$ for $\beta = \frac{1}{500}$.

[9]We cannot, of course, rule out the possibility of such a cancellation. A value of β n times that given in (9) would be obtained if the second and the third terms in (7) contributed $n \times \Delta m_K(\text{expt.})$ while the first term gave $-(n-1)\times\Delta m_K(\text{expt.})$. However, we consider such "fine-tuned" cancellations to be unlikely, especially for large n.

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[18]We thank H. Harari for discussion on this point.

[19]T. G. Rizzo and G. Senjanović, Phys. Rev. D 24, 704 (1981), and Phys. Rev. Lett. 46, 1315 (1981).

[20]Note that the value $\beta \gtrsim 0.1$ required by Ref. 19 to avoid this desert would necessitate a cancellation in (7) to approximately one part in forty (see Ref. 9). This possibility has to be considered extremely remote.