Constructing Minimum-Energy Broadcast Trees In Wireless Ad Hoc Networks

Weifa Liang
Department of Computer Science
The Australian National University
Canberra, ACT 0200, Australia
wliang@cs.anu.edu.au

ABSTRACT

In this paper we assume that a multihop wireless network (also called a wireless ad hoc network) consists of nodes whose transmitting powers are finitely adjustable. We consider two fundamental problems related to power consumption in this kind of network. One is the minimum-energy broadcast tree problem, which broadcasts a message from a source node to all the other nodes in the network such that the summation of transmission powers at all nodes is minimized; and another is the minimum-energy multicast tree problem, which multicasts a message from a source node to the nodes in a given subset of nodes such that the summation of the transmission powers at all involved nodes is minimized.

We first show the minimum-energy broadcast tree problem is NP-complete. We then present an approximate algorithm for the problem in a general setting, which delivers an approximate solution with a bounded performance guarantee. The algorithm takes $O((k+1)^{1/\epsilon} n^{3/\epsilon})$ time, where n is the number of nodes in the wireless network, k is the number of power levels at each node, and ϵ is constant with $0 < \epsilon \leq 1$. For a special case of the problem where every node is equipped with the same type of battery, we propose an approximate algorithm which has a better performance ratio than that in the general case setting, and the algorithm takes $O(kn^2 \log n)$ time. We finally extend the technique for the minimum-energy broadcast tree problem to solve the minimum-energy multicast tree problem, which leads to a similar result. The technique adopted in this paper is to reduce the minimum-energy broadcast (multicast) tree problem on a wireless ad hoc network to an optimization problem on an auxiliary weighted graph, and solve the optimization problem on the auxiliary graph which in turn gives an approximate solution for the original problem.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design; C.2.2 [**Computer-Communication Networks**]: Network Protocols; F.2.2 [**Analysis of Algorithms and Problem Complexity**]: Nonnumerical Algorithms and Problems

General Terms

Algorithms, design, performance

Keywords

Wireless network, approximate algorithm, power awareness, ad hoc networks, energy consumption optimization, broadcast and multicast algorithm.

MOBIHOC'02, June 9-11, 2002, EPFL Lausanne, Switzerland.

1. INTRODUCTION

Mobile multihop radio networks, also termed peer-to-peer, all wireless, or wireless ad hoc networks, are expected to fulfill a critical role in applications in which wired backbone networks are not available or not economical to build [3]. These span the commercial, public as well as tactical communications sectors. The networks provide the only solution in situations where instant infrastructure is needed and no central system backbone and administration (like base stations and wired backbone in a cellular system) exist. Some of the applications of the networks include mobile computing in areas where other infrastructure is unavailable, law enforcement operations, disaster recovery situations, as well as ad-hoc networks for large events such as sporting events or congresses when it is not economical to build a fixed infrastructure for a short temporary usage. In tactical battlefield communications the hostility of the environment prevents the application of a fixed backbone network. Overall, we here consider a peer-to-peer mobile network consisting of a large number mobile nodes that create a network on demand and may communicate with each other via intermediate nodes in a multihop mode, i.e., every node can be a router.

1.1 Related Work

In this paper we study the problems of broadcasting and multicasting in wireless ad hoc networks. Most previous research and development work on multicasting has centered on tethered, point-to-point (typically high speed) networks. By contrast we address infrastratureless (peer-to-peer) applications. We incorporate the broadcast properties of wireless communication media into our algorithms and performance measure. Among the most crucial issues related to mobile wireless applications is that of operation in limited energy environments. Our focus therefore is on the development of algorithms for the formation of energy-efficient trees for broadcast and multicast communications.

A crucial issue in wireless networks is to trade-off between the "reach" of the wireless transmission and the resulting interference by the transmission. We assume that the power level of a transmission can be chosen within a given range of values. Therefore, there is a trade-off between reaching more nodes in a single hop by using higher power versus reaching fewer nodes in that single hop by using lower power. Another crucial issue is that of energy consumption, because of the nonlinear attenuation properties of radio signals.

The problem of multicast scheduling in cellular mobile networks was studied in [12], and a forwarding multicast protocol for noncellular networks was proposed in [6]. Although a number of studies have addressed multicasting specifically for wireless networks in the past [13, 9, 11, 14], most of these researches focused on the robustness of routing protocols to respond to dynamic change of network topology, without taking the power consumption issue into account. The one dealing with this issue can be found in [19]. Most of the previous works on routing in wireless ad hoc networks deal with the problem of finding and maintaining correct routes to the destination during the mobility and changing topology [1, 7, 9, 11]. In [13] the multicasting problem with a goal toward reaching efficient and near-minimum-cost algorithms for wireless networks was addressed, their approach was link-based, and hence does not take into account the node-based nature of wireless communications. In [1, 7] the authors presented a simple algorithm which guarantees strong connectivity and assumes limited node range. The problem of minimum energy routing has also been addressed in [1, 7, 17, 2, 18]. The approach of these works was to minimize the consumed energy to reach the destinations. Recently, theoretical modeling and minimizing energy consumption of wireless ad hoc networks have further been expanded. For example, Chlamtac and Farago [8, 3] introduced a new model and methodological approach for dealing with the probabilistic nature of mobile networks based on the theory of random graphs. Khanna and Kumaran [10] applied the node coloring theory to the radio channel assignment of wireless networks. Ramanathan and Rosales-Hain [15] studied the assignment of different transmit powers to different nodes to meet a global topological property (e.g. k-connectivity), and proposed algorithms for minimizing the power consumption in order to keep the network k-connectivity with $k \geq 1$. Singh et al [18] proposed several power-aware metrics based on battery consumption and demonstrated how to use these measures to determine routes for wireless ad hoc networks.

1.2 Contributions

In this paper we consider two fundamental problems related to power energy saving in a wireless ad hoc network. One is the minimum-energy broadcast tree problem; and another is the minimum-energy multicast tree problem. It is easy to show that the minimum-energy multicast tree problem is NP-complete by reducing the Steiner tree problem to it. Our major contributions in this paper are as follows. We first show the minimum-energy broadcast tree problem is NP-complete. We then present an approximate algorithm for the problem in the general setting, which delivers an approximate solution with a bounded performance guarantee. For a special case of the problem where each node is equipped with the same type of battery, we give an approximate algorithm with a better performance ratio, compared with its general counterpart. We finally extend the technique for the minimum-energy broadcast tree problem to solve the minimum-energy multicast tree problem, which delivers a similar result. In addition, we also show that the performance ratio of a previously known algorithm BIP for the minimum-energy broadcast tree problem [19] is $\Omega(n)$, which means the performance of BIP is worse in some cases, where n is the number of nodes in the network. The technique adopted in this paper is to reduce the minimum-energy broadcast (multicast) tree problem on a wireless ad hoc network to an optimization problem on an auxiliary weighted graph, and solve the optimization problem on the auxiliary graph which in turn gives an approximate solution for the original problem.

The rest of the paper is organized as follows. In Section 2 we introduce the wireless communication model and the problem definition. In Section 3 we show that the minimum-energy broadcast tree problem is NP-complete, which means that there is unlikely to be an exact solution for it in polynomial time unless $P = NP$. In Section 4 we first propose an approximate algorithm for the problem which delivers an approximate solution with a bounded performance guarantee, and then present an approximate algorithm for a special case where each node is equipped with the same type of battery, which has a better performance ratio than that in the general setting. In Section 5 we apply the technique for the minimum-energy broadcast tree problem to solve the minimum-energy multicast tree problem, which leads to a similar result. In Section 6 we analyze the performance ratio of a previously known algorithm BIP for the minimum-energy broadcast tree problem. In Section 7 we conclude the paper.

2. PRELIMINARIES

The wireless ad hoc networks studied here are quite different from the cellular systems and wireless LANs that have been developed in commercial markets. Cellular systems have fixed base stations, which communicate among themselves using dedicated non-wireless lines, thus, the only multicast problems that are new in those systems involve tracking the mobile users. Otherwise, wireless communication is limited to that between mobile users and base stations.

Unlike the wired networks in which the links and the capacities of the links are determined a priori, the wireless network is built dynamically, depending on factors such as the distances between nodes, transmitted powers of nodes, error-control schemes, other user interference, and background noise. Thus, even the physical locations of the nodes are fixed, many of the factors that affect network topology are influenced by the actions of the network nodes. Furthermore, in such networks no distinction can be made between uplink and downlink traffic, thus, greatly complicating the interference environment. Therefore, the wireless networking environment poses many challenges not encountered in non-wireless or cellular networks, even the mobility is not addressed. In this paper we assume that the node locations in a wireless network are fixed, and the channel conditions are unchanging. The wireless channel is distinguished by its broadcast nature; when omnidirectional antennas are used, every transmission by a node can be received by all nodes that lie within its communication range. Consequently, if the multicast group membership including multiple nodes in the immediate communication vicinity of the transmitting node, a single transmission suffices for reaching all these receivers. In addition to interference, another undesirable impact of the use of high transmitter power is that it results in increased energy usage, since the propagation loss varies nonlinearly with distance. Therefore, the energy saving for this type of network is of paramount importance.

2.1 Wireless communication model

In this subsection we model the wireless ad hoc network. Conventionally, a wireless ad hoc network is represented by a graph where two nodes have an edge if and only if the two corresponding nodes can communicate with each other. However, in this paper we consider source-initiated, circuit-switched multicast sessions. We model the wireless network as follows. A wireless ad hoc network is represented as $M = (N, L)$, where N is the set of nodes with $|N| = n$ and $L \mapsto (Z^+, Z^+)$ is a set of coordinates denoting the locations of nodes. The nodes are assumed to have the capability of packet forwarding, i.e., relaying an incoming packet to its neighboring nodes and the transmitting power level can be adjusted to a level appropriate for successful reception if the receiver is within the transmission range. Associated with each node, there are several transceivers and can thus support several multicast sessions simultaneously. Multicast requests and session durations are generated randomly at the network nodes. Each multicast group consists of a source node and multiple destination nodes. The set of nodes supporting a multicast session is referred to as a *multicast tree*.

The reachability of a source node in a wireless network is determined by the transmission powers at each node in the network. In this paper we assume that the transmission powers at each node is adjustable finitely. Without loss of generality, we assume that there are k adjustable power levels at each node. Among the k power levels, one is the minimum operational power $p_{\min}$; and another is the maximum operational power $p_{\max}$. Given a node $v_i \in N$, let $w_{i,1}, w_{i,2} \ldots, w_{i,k}$ be its k power levels. Assume that $w_{i,l_1} \leq w_{i,l_2}$ if $l_1 < l_2$, $1 \leq l_1, l_2 \leq k$.

The connectivity of the network depends on the transmission powers at the nodes in the network. We assume that each node can choose its power level from level 1 (the minimum operational power) to level k (the maximum operational power). The nodes in any particular multicast tree do not necessarily have to use the same power levels; moreover, a node may use different power levels for various multicast trees in which it participates.

The *propagation function* is represented as $\gamma : L \times L \mapsto Z^+$, where L is a set of location coordinates of nodes. $\gamma(l_i, l_j)$ gives a loss in dB due to propagation at location $l_j \in L$ when a message is originated from location $l_i \in L$. The propagation function captures the environmental characteristics determining the formation of a link. It could be measured as described as in [16] or approximately modeled with a function. The successful reception of a transmitted signal depends, along with the propagation function on the transmit power p, and the receiver sensitivity ϑ. The receiver sensitivity is the threshold signal strength needed for reception and is assumed to be an apriori known constant, same for all nodes. In particular, for successful reception,

$$p - \gamma(l_i, l_j) \quad \geq \quad \vartheta. \tag{1}$$

We assume that γ is a monotonically increasing function of the geographical distance $d(l_i, l_j)$ between l_i and l_j. This is generally true for free space propagation or when environmental clutter cause the same amount of signal degradation in all directions [16]. In [19] it assume that the received signal power varies as r^α, where r is the range and α is a parameter that typically takes on a value between 2 and 4, depending on the characteristics of the communication medium. Based on this model the transmitted power required to support a link between two nodes separated by range r is proportional to r^α. For simplicity, in this paper we say that a node S_i is within the transmission range of a node S_j while S_j uses the transmitting power p to broadcast (multicast) a message, which means that S_i is able to receive the signal from S_j when S_j sends its message using power p.

2.2 The minimum-energy multicast tree problem

The minimum-energy multicast tree problem is defined as follows. Given a wireless ad hoc network $M = (N, L)$, a source node, and a terminal set D ($\subset N$), to broadcast a message from the source node to the nodes in D such that the sum of transmission powers consumed at all involved nodes (some nodes are relay nodes) is minimized. Formally speaking, the task is to construct a multicast tree rooted at the source node including all the nodes in D such that the sum of transmission powers at non-leaf nodes in the tree is minimized, for every new multicast session. It involves the choice of transmitting nodes as well the transmitter-power level at every chosen transmitting node. Note that the leaf nodes which do not transmit do not contribute to the energy consumption.

2.3 The minimum-energy broadcast tree problem

Assume that s is a source node, the minimum-energy broadcast tree problem is a special case of the minimum-energy multicast tree problem where $D = N - \{s\}$, which is defined as follows. Given a wireless ad hoc network $M = (N, L)$ and a source node s, to broadcast a message from s to all the other nodes such that the sum of transmission powers at all nodes is minimized. Formally speaking, the task is to construct a broadcast spanning tree rooted at the source node and spans all the other network nodes such that the sum of transmission powers at non-leaf nodes is minimized, for every new broadcast session.

3. THE MINIMUM-ENERGY BROADCAST TREE PROBLEM IS NP-COMPLETE

In this section we show that the minimum-energy broadcast tree problem is NP-complete by reducing the 3-conjunctive normal form satisfiability (3-CNF SAT) problem to it.

THEOREM 1. *Given a wireless network $M(N, L)$ with k adjustable power levels at each node, a source node, and a positive value W, to determine whether there is a broadcast spanning tree rooted at the source node such that the sum of transmission powers at the non-leaf nodes in the tree is no greater than W is NP-Complete. This problem is also referred to the minimum-energy broadcast tree problem.*

PROOF. First, it is easy to verify the minimum-energy broadcast tree problem is NP-hard. Given a directed tree rooted at the source and each non-leaf node has been assigned a power level, the sum of transmission powers at the non-leaf nodes can be computed and checked to see whether it is no greater than W, which can be done in polynomial time, where W is a given value in advance. Therefore, the problem is NP-hard. For convenience, here we consider a special case of the problem with $k = 2$, i.e., there are at most two adjustable power levels at each node. In the following we show that the problem is NP-complete, even for this special case. The problem for the general case, therefore, is NP-complete too.

Given an instance of a 3-CNF SAT problem, the objective is to construct an instance of the minimum-energy broadcast tree problem such that the 3-CNF SAT instance is satisfiable if and only if there is a solution for the minimum-energy broadcast tree instance.

Suppose that there is a 3-CNF SAT instance consisting of n boolean variables x_1,x_2,..., x_n, and m conjunctive normal forms (CNFs) $C_1, C_2, \ldots, C_m$, where $C_j = y_{j,1} \vee y_{j,2} \vee y_{j,3}$ and the three literals $y_{j,1}, y_{j,2}, y_{j,3} \in \{x_1, \bar{x}_1, x_2, \bar{x}_2, \ldots, x_n, \bar{x}_n\}$, $1 \leq j \leq m$. The corresponding minimum-energy broadcast tree instance $M = (N, L)$ for this 3-CNF SAT instance is constructed as follows:

(1) There is a source node $S_{0,0}$ which has a power level with value 1. (2) For each boolean variable x_i, there are two corresponding nodes $S_{i,1}$ and $S_{i,2}$, and each of the nodes has two power levels with values 1 and 2. If x_i is true, the power value of $S_{i,1}$ is 1; otherwise, the power value of $S_{i,1}$ is 2. If $\bar{x}_i$ is true, the power value of $S_{i,2}$ is 1; otherwise, the power value of $S_{i,2}$ is 2. In addition, there is a node X_i which will be used to determine whether either x_i is true or $\bar{x}_i$ is true, and the transmission power of X_i is zero, $1 \leq i \leq n$. (3) For each 3-CNF $C_j = y_{j,1} \vee y_{j,2} \vee y_{j,3}$, there is a corresponding node SC_j with a transmission power 0, $1 \leq j \leq m$. Therefore, the node set $N = \{S_{0,0}\} \cup \{S_{i,1}, S_{i,2}, X_i \ : \ 1 \leq i \leq n\} \cup \{SC_i \ : \ 1 \leq i \leq m\}$.

Now, given a node in N, we define which nodes are within its transmission range of the node when it uses either one of its two transmission power levels to broadcast messages, as follows.

(i) Assume that node $S_{0,0}$ is the source. For all i and j, node $S_{i,j}$ is within the transmission range of $S_{0,0}$ when $S_{0,0}$ uses power value 1 to broadcast messages, $1 \leq j \leq 2$, $1 \leq i \leq n$. (ii) Node X_i is always within the transmission range of both $S_{i,1}$ and $S_{i,2}$, no matter which power level with value 1 or 2 is used to broadcast messages by $S_{i,1}$ or $S_{i,2}$. (iii) Let $C_j = y_{j,1} \vee y_{j,2} \vee y_{j,3}$. If literal $y_{j,l} = x_i$, then node SC_j is within the transmission range of $S_{i,1}$ when $S_{i,1}$ uses either one of its two power levels to broadcast messages; if literal $y_{j,l} = \bar{x}_i$, then node SC_j is within the transmission range of $S_{i,2}$ when $S_{i,2}$ uses either one of its two power levels to broadcast messages, $1 \leq j \leq m$, and $1 \leq i \leq n$. Accordingly, the wireless network $M = (N, L)$ has been built. Let $W = n + 1$. We claim that if there is a directed broadcast spanning tree T in $M(N, L)$ rooted at $S_{0,0}$ such that the sum of the transmission powers at non-leaf nodes in T is no greater than W, then the 3-CNF SAT instance is satisfiable. Clearly, T has the following properties.

(I) The edge $\langle S_{0,0}, S_{i,j} \rangle$ must be in T, because this is the only incoming edge into the node $S_{i,j}$, and every node in M must be included in T, $1 \leq i \leq n$ and $1 \leq j \leq 2$.

(II) The edge either $\langle S_{i,1}, X_i \rangle$ or $\langle S_{i,2}, X_i \rangle$ must be in T but not both of them are in T, because there are the only two incoming edges into node X_i, and one of the edges must be in T in order to include X_i in T.

(III) Either node $S_{i,1}$ or node $S_{i,2}$ is a leaf node in T for each i, $1 \leq i \leq n$. From the above discussion, we know that one of the two nodes must be a non-leaf node of T. Assume that both of the nodes are non-leaf nodes in T, then there are at least two outgoing edges in T from the two nodes, which can be one of the following forms: (a) $\langle S_{i,1}, X_i \rangle$ and $\langle S_{i,2}, C_j \rangle$, where the corresponding boolean variable $\bar{x}_i$ of $S_{i,2}$ is a literal in C_j; (b) $\langle S_{i,2}, X_i \rangle$ and $\langle S_{i,1}, C_j \rangle$, where the corresponding boolean variable x_i of $S_{i,1}$ is a literal in C_j. We now show case (a) is impossible in T. Due to that both $S_{i,1}$ and $S_{i,2}$ are the non-leaf nodes in T, then the sum of the transmission powers at non-leaf nodes in T is at least $n + 2$ ($> W$), which is explained as follows. For every j with $j \neq i$, X_j is a leaf node and either $S_{j,1}$ or $S_{j,2}$ is the parent of X_j in T, the sum of transmission powers of all $S_{j,1}$ and $S_{j,2}$ is at least $n - 1$, $1 \leq j \leq n$, while the sum of transmission powers of both $S_{i,1}$ and $S_{i,2}$ is at least 2, and the transmission power of source node is 1. As results, the sum of transmission powers of non-leaf nodes in T is at least $n + 2$, which is greater than W. This contradicts our initial assumption, so, either $S_{i,1}$ or $S_{i,2}$ must be a leaf node in T for each i, $1 \leq i \leq n$. Case (b) can be dealt similarly and omitted.

(IV) For each SC_j, there must be an incoming edge into node SC_j in T such that the transmission power of another endpoint of the edge is 1 and the edge is either (a) $\langle S_{i,1}, C_j \rangle$, where the corresponding boolean variable x_i of $S_{i,1}$ is a literal in C_j; or(b) $\langle S_{i,2}, C_j \rangle$, where the corresponding boolean variable $\bar{x}_i$ of $S_{i,2}$ is a literal in C_j. It is obvious that there is such an edge in M incoming to SC_j. We show that the transmission power of another endpoint S of the edge in T must be 1. Assume that this is not true. Then, the transmission power of S in T is 2 by the definition of the wireless network. The sum of transmission powers non-leaf nodes is at least $n + 2$, because the transmission power at source node is 1, the sum of transmission powers of all nodes $S_{i,1}$ and $S_{i,2}$ except node S (either $S = S_{i',1}$ or $S = S_{i',2}$) is at least $n - 1$ due to the fact that X_i is a leaf node in T, while the transmission power of S is 2 by the assumption. As results, the sum of transmission powers of non-leaf nodes in T is at least $1 + (n - 1) + 2 = n + 2 > W$, which contradicts the initial assumption that the transmission powers in T is no greater than W. Therefore, the transmission power of another endpoint S of the edge in T must be 1.

Having the tree T, we can assign the boolean variables with values such that the 3-CNF SAT instance is satisfiable. For each X_i, if $S_{i,1}$ is not a leaf node in T, then x_i is true; otherwise, $S_{i,2}$ is not a leaf node in T and $\bar{x}_i$ is true. Thus, every boolean variable of the n variables in the 3-CNF SAT instance is assigned either "true" or "false". For such an assignment, we claim that every 3-CNF clause C_j must be true, $1 \leq j \leq m$. We prove this claim by contradiction. Assume that there is a clause C_{j_0} which is false under the assignment and $\langle S_{i,1}, C_{j_0} \rangle$ is an edge in T (if the edge $\langle S_{i,2}, C_{j_0} \rangle$ is in T, it can be dealt similarly and omitted), where $S_{i,1}$ is one of the corresponding nodes of the three literals in C_{j_0} and must be a non-leaf node in T. Otherwise, SC_{j_0} cannot receive any message from any other nodes by the construction of the wireless network. Then, the transmission power of $S_{i,1}$ in T must be 2, otherwise, $S_{i,1}$ would not be chosen as the parent of C_{j_0} if any of the transmission power of the other corresponding nodes of the other two literals in C_{j_0} is 1. Thus, the sum $p(T)$ of the transmission powers in T, therefore, is

$$p(T) \quad \geq \quad 1 + (n-1) + 2 = n + 2 > W, \qquad (2)$$

where in the right hand side of Inequality (2), the first term is the source's transmission power; the second term is the minimum sum of the transmission powers of either $S_{j,1}$ or $S_{j,2}$ for all j with $j \neq i$, $1 \leq j \leq n$ because X_j is a leaf node in T and can be received messages from $S_{j,1}$ or $S_{j,2}$ only; the third term is the transmission power of $S_{i,1}$. This contradicts the initial assumption that $p(T) \leq W$. Thus, all clauses C_j are true under the assignment, $1 \leq j \leq m$.

Suppose that a given 3-CNF SAT instance is satisfiable, we show there is a directed broadcast spanning tree T rooted at the source such that the sum of transmission powers at the non-leaf nodes in T is no greater than W, which is constructed as follows.

First, the source node $S_{0,0}$ is chosen as the root and its power is 1. There are directed edges $\langle S_{0,0}, S_{i,1}\rangle$ and $\langle S_{0,0}, S_{i,2}\rangle$ in T for all i, which means that $S_{i,j}$ is able to receive messages sent by $S_{0,0}$ when $S_{0,0}$ uses power 1 to broadcast messages, $1 \leq j \leq 2$, $1 \leq i \leq n$.

Then, for each boolean variable x_i, if its value in the 3-CNF SAT instance is true, then the power level at $S_{i,1}$ is 1, and $\langle S_{i,1}, X_i\rangle$ is in T. Otherwise, the power level at $S_{i,2}$ is 1, and $\langle S_{i,2}, X_i\rangle$ is in T.

Finally, for a given 3-CNF $C_j = y_{j,1} \vee y_{j,2} \vee y_{j,3}$, at least one literal $y_{j,l}$ is true, because C_j is true under the assignment, $1 \leq l \leq 3$. If more than one literals in C_j are true, one of them is chosen as the parent of SC_j in T. Let $y_{j,l}$ be true. If $y_{j,l} = x_i$, then the directed edge $\langle S_{i,1}, SC_j\rangle$ is in T; otherwise $(y_{j,l} = \bar{x_i})$, the directed edge $\langle S_{i,2}, SC_j\rangle$ is in T, $1 \leq j \leq m$ and $1 \leq i \leq n$.

As a result, T is a directed, spanning tree rooted at $S_{0,0}$, and the sum $p(T)$ of the transmission powers at the non-leaf nodes of T is

$$P(T) \quad \leq \quad 1 + n \leq W, \qquad (3)$$

where in the right hand side of Inequality (3), the first term is the source's transmission power; the second term is the sum of the transmission powers of either $S_{i,1}$ or $S_{i,2}$ for all i, $1 \leq i \leq n$. The construction of the minimum-energy broadcast tree instance from the 3-CNF SAT instance can be done in polynomial time in terms of n and m. While it is well known that the 3-CNF SAT problem is NP-complete, the minimum-energy broadcast tree problem is NP-Complete too. □

4. APPROXIMATE ALGORITHMS FOR MINIMUM-ENERGY BROADCAST TREE PROBLEM

As shown the minimum-energy broadcast tree problem is NP-complete, it is unlikely to solve it in polynomial time unless P=NP. Instead, we focus on devising an approximate algorithm for it. The idea is first to reduce the problem to an optimization problem on an auxiliary graph. Then, solve the optimization problem on the auxiliary graph. Finally, the approximate solution for the auxiliary graph gives an approximate solution for the original problem.

4.1 Broadcast tree with k adjustable power levels at each node

Given the wireless network $M(N, L)$ with k adjustable power levels at each node, an auxiliary, weighted, directed graph $G = (V, E, \omega_1)$ is constructed as follows.

For each node S_i, let $w_{i,1}, w_{i,2} \ldots, w_{i,k}$ be the k adjustable power levels at S_i with $w_{i,l_1} < w_{i,l_2}$, $1 \leq l_1 < l_2 \leq k$. A widget $G_i = (V_i, E_i)$ for S_i is built and shown in Fig 1, $V_i = \{s_i, v_{i,1}, v_{i,2}, \ldots, v_{i,k}\}$ and $E_i = \{\langle s_i, v_{i,l}\rangle \ : \ 1 \leq l \leq k\}$, where s_i represents the node S_i, $v_{i,l}$ represents node i working at transmission power level l, the directed edge from s_i to $v_{i,l}$ represents the lth power level and the weight assigned to it is the power $w_{i,l}$, $1 \leq l \leq k$. Having been

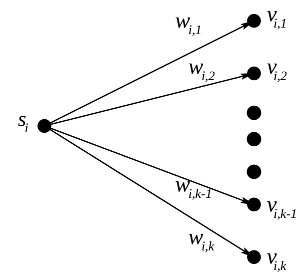

Figure 1: The widget $G_i = (V_i, E_i)$ for node S_i

built the widgets, it is ready to construct G. $V = \cup_{i=1}^{n} V_i$, $E = \cup_{i=1}^{n} E_i \cup E_{dist}$, and $\omega_1 : E \mapsto \mathbb{R}$, where E_{dist} is the set of edges between the nodes of different widgets, which is defined as follows. For two nodes S_i and S_j with $i \neq j$, let $d(S_i, S_j)$ be the distance from S_i to S_j. If S_i uses power $w_{i,l}$ at level l to broadcast a message and S_j is able to receive the message, then there is a directed edge $\langle v_{i,l}, s_j\rangle$ in E_{dist} with weight zero. In other words, S_j is within the transmission range of S_i when S_i uses power $w_{i,l}$ to broadcast messages, $1 \leq i \leq n$ and $1 \leq l \leq k$. Now consider the weight assignment of edges in G. For each $\langle s_i, v_{i,l}\rangle \in E_i \subset E$, $\omega_1(s_i, v_{i,l}) = w_{i,l}$; and for each $\langle v_{i,l}, s_j\rangle \in E_{dist}$, $\omega_1(v_{i,l}, s_j) = 0$. As results, the directed weighted graph G has been built and has the following properties.

LEMMA 1. *Given the directed graph $G(V, E, \omega_1)$ defined above, then (i) G contains $(k+1)n$ nodes and at most kn^2 directed edges, i.e., $|V| = (k+1)n$ and $|E| \leq kn^2$. (ii) Let $N(v_{i,l})$ be the set of neighboring nodes of $v_{i,l}$ in G, then, $N(v_{i,l_1}) \subseteq N(v_{i,l_2})$ if $1 \leq l_1 < l_2 \leq k$. (iii) G contains two types of nodes; type-1 node $s_i \in V$, whose outgoing degree is k and incoming degree is $deg_{in}(s_i) = |\{v_{j,l} \ : \ S_i$ is within the transmission range of S_j if S_j uses power $w_{j,l}$ to broadcast messages, $1 \leq l \leq k, 1 \leq j \leq n$, and $i \neq j\}|$; and type-2 node $v_{i,l} \in V$, whose incoming degree is 1 and outgoing degree is $deg_{out}(v_{i,l}) = |\{s_j \ : \ S_j$ is within the transmission range of S_i if S_i uses power $w_{i,l}$ to broadcast messages, $1 \leq l \leq k, 1 \leq j \leq n$, and $i \neq j\}$, $1 \leq i \leq n\}|$.*

PROOF. We first show property (i). For each node s_i in G, there are at most k incoming edges derived from another node s_j which corresponds to the k transmission power levels of S_j with $i \neq j$, $1 \leq j \leq n$. Thus, the total number of incoming edges of s_i is at most $k(n-1)$, $1 \leq i \leq n$. There are n such nodes in G. Therefore,

$$|E_{dist}| \quad \leq \quad k(n-1)n = kn^2 - kn. \qquad (4)$$

Since $E_i \cap E_j = \emptyset$, $E_i \cap E_{dist} = \emptyset$ and $|E_i| = k$, $i \neq j$, $1 \leq i, j \leq n$,

$$|E| = |\cup_{i=1}^{n} E_i \cup E_{dist}| = \sum_{i=1}^{n} |E_i| + |E_{dist}| \leq kn + kn^2 - kn = kn^2. \quad (5)$$

We then show property (ii). Following the construction of G, let $s_j \in N(v_{i,l_1})$, which means that the node S_j is within the transmission range of S_i if S_i uses power w_{i,l_1} to broadcast a message and S_j is able to receive the message. Certainly, S_j is still within the transmission range of S_i if S_i uses power w_{i,l_2} to broadcast messages due to that $w_{i,l_1} < w_{i,l_2}$. Thus, for any $s_j \in N(v_{i,l_1})$, it must be in $N(v_{i,l_2})$ too.

We finally show property (iii). For type-1 node s_i which is the representative of node S_i, its incoming degree $deg_{in}(s_i)$ follows the definition of G, and its outgoing degree is k which corresponds to its k transmission power levels. While for every power level node $v_{i,l}$, its incoming degree is 1 and outgoing degree is determined by its transmission power $w_{i,l}$ and the number of the other nodes that are within its transmission range of S_i when S_i uses power $w_{i,l}$ to broadcast messages, $1 \leq i \leq n$, $1 \leq l \leq k$. □

Having the auxiliary graph G, without loss of generality we assume that the node S_1 is the source and every other node is reachable from s_1. Otherwise, no such a minimum-energy broadcast tree in the wireless network exists. The rest is to find a directed Steiner tree in G rooted at s_1 including each terminal node in $D = \{s_i \mid 2 \leq i \leq n\}$. In the following we show the exact solution of the minimum-energy broadcast tree problem on the wireless network can be found through finding a Steiner tree in G.

LEMMA 2. *The Steiner tree in $G(V, E, \omega_1)$ defined gives an exact solution for the minimum-energy broadcast tree problem.*

PROOF. Assume that T_{opt} is a minimum-energy broadcast tree with source S_1 for the wireless network. Following the construction of G, there is a corresponding tree T in G rooted at s_1 including the nodes in S for T_{opt}. Let T_{st} be a Steiner tree in G rooted at s_1 including the nodes in D. Then, we have $W(T) \leq W(T_{st})$ because the sum of transmission powers at all nodes in T_{opt} is the minimum one and equal to $W(T)$, where $W(T')$ is the weighted sum of the edges in a tree T' of G. While T_{st} is such a minimum weighted tree rooted at s_1 that includes the nodes in D by the Steiner tree definition, $W(T_{st}) \leq W(T)$. Therefore, $W(T_{st}) = W(T)$. □

The Steiner tree T_{st} defined in $G(V, E, \omega_1)$ has the following unique properties.

LEMMA 3. *Given the Steiner tree T_{st} in $G(V, E, \omega_1)$ defined, (i) T_{st} is a tree in which no two directed edges derived from a single node S_i are included. In other words, for a given node S_i, not both $\langle s_i, v_{i,x}\rangle$ and $\langle s_i, v_{i,y}\rangle$ are included in T_{st}, $1 \leq x, y \leq k$, $1 \leq i \leq n$, and $i \neq j$. (ii) For every other node s_j, the number of edges in the path from the root s_1 to s_j is even, and for every $s_{j'}$ on the path, both its incoming degree and outgoing degree are one except the incoming degree of s_1 is zero, $1 \leq j' \leq n$, and $j' \neq 1$.*

PROOF. We show property (i) by contradiction. Assume that both of the directed edges are included in T_{st} with $x < y$. By the construction of T_{st}, neither $v_{i,x}$ nor $v_{i,y}$ is the leaf node of T_{st}; otherwise, both $v_{i,x}$ and $v_{i,y}$ are not included in T_{st}. Now, assume $\langle v_{i,x}, s_j\rangle$ is a directed edge from $v_{i,x}$ in T_{st} with weight zero, by the definition of G. Then, there must be a directed edge $\langle v_{i,y}, s_j\rangle$ with weight zero in G by case (ii) of Lemma 1 due to that $w_{i,x} \leq w_{i,y}$. Thus, another tree $T_{st} - \{\langle s_i, v_{i,x}\rangle, \langle v_{i,x}, s_j\rangle\} \cup \{\langle v_{i,y}, s_j\rangle\}$ in G can be found, in which s_1 is still the root and all nodes in D are included. The weight of this resulting tree is $W(T_{st}) - w_{i,x}$ ($< W(T_{st})$), which contradicts that T_{st} is the Steiner tree rooted at s_1 including the nodes in D. Therefore, no two directed edges derived from a single node are included in T_{st}, which means that there is only a power level chosen at each node if the Steiner tree can be found.

We now show property (ii). Let P be a directed path from s_1 to s_j in T_{st} consisting of edges $e_1, e_2, \ldots, e_m$ where the tail of e_1 is $tail(e_1) = s_1$ and the head of e_m is $head(e_m) = s_j$ and $tail(e_i) = head(e_{i-1})$, $2 \leq i \leq m$. Observe that for every edge in P, $e_{2i-1} \in E_{j'}$ with a positive weight, which is an edge in the widget of a node $S_{j'}$, and $e_{2i} \in E_{dist}$ with weight zero, $1 \leq i \leq \lfloor m/2 \rfloor$. While $e_1 \in E_1$ and $e_m \in E_{dist}$, m must be even, i.e., the number of edges in P is even. Since T_{st} is a directed tree, the incoming degree of every node including $s_{j'}$ is one. The outgoing degree of $s_{j'}$ is one too, following the argument for property (i). □

Given the Steiner tree T_{st}, to set power at each node is easy, which is presented as follows. For a node S_i, if its corresponding edge $\langle s_i, v_{i,l}\rangle$ in T_{st}, then the power at S_i is adjusted to $w_{i,l}$, $1 \leq i \leq n$ and $1 \leq l \leq k$. Therefore, the rest is how to find T_{st} in G efficiently. However, it is well known that there is unlikely to have a polynomial algorithm for finding a directed Steiner tree in G unless $P = NP$. Instead, we will focus on finding an approximate Steiner tree in G.

Let T_{app} be an approximate Steiner tree in G rooted at s_1 including the nodes in D, which has been obtained by applying the algorithm in [5], and let $W(T_{app})$ be the weight of T_{app}. Note that T_{app} may not be the broadcast tree that we wanted, because it may contain two directed edges derived from a single node. One such an example is shown in Fig 2, where an approximate Steiner tree in G is found.

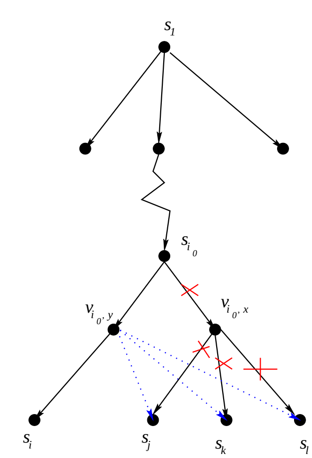

Figure 2: An approximate Steiner tree

From which we can see that both $\langle s_{i_0}, v_{i,x}\rangle$ and $\langle s_{i_0}, v_{i,y}\rangle$ are included in T_{app}. This means that two different power levels of node S_{i_0} are needed for this broadcast session, which obviously violates the restriction imposed in case (i) of Lemma 3. To remove such violation, we modify T_{app} to make it become another tree T'_{app} which obeys the restriction in Lemma 3 as follows. Assume that $x < y$, then we have $w_{i_0,x} < w_{i_0,y}$. Delete the edges $\langle s_{i_0}, v_{i,x}\rangle$ of weight $w_{i_0,x}$, $\langle v_{i_0,x}, s_j\rangle$, $\langle v_{i_0,x}, s_k\rangle$, and $\langle v_{i_0,x}, s_l\rangle$ of weight zeros from T_{app}. Add edges $\langle v_{i_0,y}, s_j\rangle$, $\langle v_{i_0,y}, s_k\rangle$, and $\langle v_{i_0,y}, s_l\rangle$ of weight zeros which are all in G into T_{app}. As results, the modified tree is a tree rooted at s_1 including all the nodes in D and its weight $W(T'_{app})$ $(= W(T_{app}) - w_{i_0,x} < W(T_{app}))$ is strictly less than $W(T_{app})$. In case more than two edges derived from a single node are in T_{app}, to deal with the case, among the edges, only the edge with the maximum weight will be kept, and all the other edges will be deleted using the similar construction as the above. Consequentially, T'_{app} is the tree needed, from which an assignment of power level for every node is straightforward and omitted. In summary, the proposed algorithm is given below.

Algorithm Mini_Broad_Tree(N, L, k)
begin
1. Construct the auxiliary graph $G(V, E, \omega_1)$;
2. Find an approximate Steiner tree T_{app} on G by applying the algorithm in [5];
3. Construct T'_{app} by modifying T_{app} if needed;
4. Set power level at each node, according to the information given by T'_{app}.

end

Thus, we have the following theorem.

THEOREM 2. *Given a wireless network $M(N, L)$ with k adjustable power levels at each node, there is an approximate algorithm to find a minimum-energy broadcast tree rooted at a source. The sum of transmission powers is $O(n^\epsilon)$ times of the optimum. The time complexity of the proposed algorithm is $O((k+1)^{\frac{1}{\epsilon}} n^{\frac{3}{\epsilon}})$, where k is the number of power levels at each node on the wireless network and ϵ is constant with $0 < \epsilon \leq 1$.*

PROOF. Following algorithm `Mini_Broad_Tree`, Step 1 takes $O(kn^2)$ time because G contains $(k+1)n$ nodes and at most kn^2 directed edges. Step 2 is the dominant which takes $O((k+1)^{\frac{1}{\epsilon}} n^{\frac{3}{\epsilon}})$ time by Theorem 4 in [5], because G contains $(k+1)n$ nodes and n terminal nodes. Step 3 takes $O(n)$ time by examining the outgoing edges of each non-leaf node in T_{app}. Step 4 takes $O(n)$ time by checking each non-leaf node with outgoing degree 1. Thus, the entire algorithm takes $O((k+1)^{\frac{1}{\epsilon}} n^{\frac{3}{\epsilon}})$ time.

Now we analyze the performance ratio of the proposed algorithm as follows. It has been already known that $W(T'_{app}) < W(T_{app})$, following the proof of Lemma 3, while $W(T_{app}) \leq cn^\epsilon W(T_{st}))$ by the approximate algorithm in [5]. Thus, $W(T'_{app}) < cn^\epsilon W(T_{st}) = cn^\epsilon W(T) = O(n^\epsilon W(T))$, where c is constant. □

The proposed algorithm above is a centralized algorithm, we here propose a distributed implementation for it in a distributed environment, in which each node A knows its neighboring nodes B, where B is within the transmission range of A when A uses its maximum power level to transmit messages. The key is to map the auxiliary graph G into the physical wireless network, which is described as follows. Each node (mobile) S_i contains the widget G_i, and there are at most k virtual links $\langle v_{i,l'}, s_j\rangle$,$\langle v_{i,l'+1}, s_j\rangle$, ..., $\langle v_{i,k}, s_j\rangle$ between physical node S_i and node S_j if S_j is within the transmission range of S_i, $1 \leq l' \leq k$, $1 \leq i, j \leq n$. Thus, the auxiliary graph G has been mapped to the physical wireless network. Having the auxiliary graph G, run a distributed algorithm for finding a directed Steiner tree including all nodes s_i, $1 \leq i \leq n$, on G. We here use the physical wireless network to simulate the steps in G, i.e., each mobile node (S_i) will act as $(k+1)$ nodes s_i, $v_{i,1}$, ..., $v_{i,k}$ in G. Therefore, the performance ratio between the approximation solution and the exact solution of the problem is determined by the distributed implementation of the directed Steiner tree algorithm. The simplest distributed implementation of the Steiner tree algorithm is through running a distributed single-source shortest path algorithm, which delivers an approximate solution within $O(n)$ times of the optimum. Having constructed the approximate Steiner tree, setting the power level for each mobile node is easy, following the information given by the tree.

4.2 Broadcast tree with the same type of battery at each node

In this section we consider a special case where each node is equipped with the same type of battery, i.e., each node has identical k-level powers, i.e., $w_{i,l} = w_{j,l}$ for $i \neq j$, $1 \leq l \leq k$, $1 \leq i, j \leq n$. We show that there is a better approximate algorithm for the minimum-energy broadcast tree problem, which delivers a solution within $O(\log^3 n)$ times of the optimum. Following the same spirit of the algorithm in the preceding section, the approximate solution for the problem is found through finding an approximate solution for an optimization problem on an auxiliary graph. To this end, in the following we focus on the construction of the auxiliary graph.

Let $\mathcal{V}_1$,$\mathcal{V}_2$,..., $\mathcal{V}_p$ be a partition of the nodes in the wireless network. Then, $\mathcal{V}_i \cap \mathcal{V}_j = \emptyset$, $\mathcal{V}_i \neq \emptyset$, and $\cup_{i=1}^{p} \mathcal{V}_i = \{s_1, s_2, \ldots, s_n\}$, where s_l represents node S_l, $1 \leq i, j \leq p$ with $i \neq j$, $1 \leq l \leq n$. Based on the node partition defined, a weighted, bipartite graph $G = (X, Y, E, \omega)$ is constructed as follows. $X = \cup_{i=1}^{n}\{v_{i,1}, v_{i,2}, \ldots, v_{i,k} \mid s_i \text{ is a node}\}$ where node $v_{i,l}$ represents the lth level power of S_i and its weight is $\omega(v_{i,l}) = w_{i,l}$, $1 \leq l \leq k$, $1 \leq i \leq n$. $Y = \{\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_p\}$ and $\omega : X \mapsto \mathbb{R}$. An edge $(v_{i,l}, \mathcal{V}_j) \in E$ if and only if at least a node $s_{j'} \in \mathcal{V}_j$ is within the transmission range of S_i when S_i uses power $w_{i,l}$ to broadcast messages. Obviously, $(v_{j',l}, \mathcal{V}_j) \in E$ if $s_{j'} \in \mathcal{V}_j$ for all l, $1 \leq l \leq k$. If multiple nodes in $\mathcal{V}_j$ are within the transmission range of S_i when S_i uses power $w_{i,l}$ to broadcast messages, there is only an edge $(v_{i,l}, \mathcal{V}_j) \in E$. Clearly, G is a simple, bipartite graph, and has the following properties.

LEMMA 4. *Let X_1 be a subset of X in G covering all nodes in Y such that the weighted sum $W(X_1)$ $(= \sum_{v \in X_1} \omega(v))$ of nodes in it is minimized. Then, (i) no two nodes v_{i,l_1} and v_{i,l_2} derived from a node S_i are included in X_1, $1 \leq l_1 < l_2 \leq k$. (ii) Denote by $W_{\min}$ the weighted sum of nodes in X_1, i.e., $W_{\min} = \sum_{v \in X_1} \omega(v)$. Let P_{opt} be the minimum power needed for broadcasting messages source at a node to* *all the other nodes in the network. Then,*

$$W_{\min} \leq P_{opt}. \quad (6)$$

PROOF. Let $N(v_{i,l})$ be the set of neighboring nodes of $v_{i,l}$ in G. We show case (i) by contradiction. Assume that both $v_{i,l_1} \in X_1$ and $v_{i,l_2} \in X_1$ with $l_1 < l_2$, then $w_{i,l_1} < w_{i,l_2}$ by the definition. Thus, $N(v_{i,l_1}) \subseteq N(v_{i,l_2})$. We now have $X_1' = X_1 - \{v_{i,l_1}\}$, which also covers all nodes in Y and the weighted sum of the nodes in X_1' is $W(X_1') = W(X_1) - w_{i,l_1} < W(X_1) = W_{\min}$. This contradicts the fact that X_1 is the set of the minimum weighted sum of nodes satisfying the constraint.

We now show case (ii). Let X_2 be a subset of X which corresponds to the nodes in the optimal broadcast tree. We show that the nodes in X_2 must cover all nodes in Y. Otherwise, assume that there is a node $\mathcal{V}_j \in Y$ which is not covered by any node in X_2. Since $\mathcal{V}_j \neq \emptyset$, let $s_{j'}$ be any node in $\mathcal{V}_j$, then $(v_{j',l}, \mathcal{V}_j) \in E$ by the definition of G and $v_{j',l} \notin X_2$ by the assumption, for all l, $1 \leq l \leq k$. Therefore, $s_{j'}$ is a leaf node in the broadcasting tree and is within the transmission range of another node S_i where S_i in the optimal broadcast tree uses a power level $w_{i,l}$ to broadcast messages. Thus, $v_{i,l} \in X_2$, and there is an edge $(v_{i,l}, \mathcal{V}_j) \in E$, i.e., $\mathcal{V}_j$ is covered by node $v_{i,l}$. This contradicts the initial assumption that $\mathcal{V}_j$ is not covered by any node in X_2. Therefore, X_2 is a covering set which covers all nodes in Y. While X_1 is a such minimum covering set, so, $W(X_1) \leq W(X_2)$. Furthermore, $W(X_1) = W_{\min}$ and $W(X_2) = P_{opt}$, therefore, we have $W_{\min} \leq P_{opt}$. □

Now, given a node partition $P = \{\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_p\}$ and a covering set X' $(\subseteq X)$ of $G(X, Y, E, \omega)$ that is not necessary to be the minimum one, we show how to construct a new node partition P'. For convenience, assume that the degree of every node $v_{i,l} \in X$ of G is at least 2. If there is a $v_{i,l} \in X$ with degree 1, let $(v_{i,l}, \mathcal{V}_j)$ be the unique edge, then $s_i \in \mathcal{V}_j$ by the definition of G. This means that no other nodes are able to receive the message of S_i if S_i uses power $w_{i,l}$ to broadcast a message. So, such node is useless for message broadcasting purpose and will be deleted from G. Without loss of generality, from now on we assume that every node $v_{i,l} \in X$ of G has degree of at least 2. The new node partition P' based on the information supplied by X' and the current node partition P, is constructed as follows.

Algorithm `Node_Partition`(G, X', P')
begin
1. Let $P = \{\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_p\}$;
2. A subgraph $G[X'] = (X', Y, (X' \times Y) \cap E, \omega)$ of G is constructed, which is an induced subgraph by the nodes in $X' \cup Y$;
3. Find connected components in $G[X']$. Let $CC_1, CC_2, \ldots, CC_{p'}$ be the node sets of connected components in $G[X']$.
4. A new partition $P' \leftarrow \{CC_1 \cap Y, CC_2 \cap Y, \ldots, CC_{p'} \cap Y\}$ is constructed.

end

It is easy to show that $p' \leq \lfloor p/2 \rfloor$ due to that every node in X of G has a degree at least 2. Having constructed the node partition, we are ready to present an algorithm for finding an approximate minimum-energy broadcast tree, which is presented as follows.

Algorithm `Mini_Broad_Tree_Equal`(N, L, k)
begin
Let $P = \{\{s_1\}, \{s_2\}, \ldots, \{s_n\}\}$;
for $i \leftarrow 1$ **to** n **do**
$p(s_i) \leftarrow 0$; /* the initial power assigned to node S_i */
endfor;
$i \leftarrow 0$; /* the number of iterations */
repeat
1. Construct an auxiliary weighted bipartite graph $G(X, Y, E, \omega)$ using P;
2. Find a covering set $X_1 \subseteq X$ in $G(X, Y, E, \omega)$ such that the weighted sum of nodes in it is minimized;
3. Check whether more than one node in X_1 is derived from a node. If so, delete all the other nodes except the one with the maximum weight. Let X_1' be the resulting subset of X_1;
4. **for** each node $v_{i,l} \in X_1'$ **do** $p(s_i) \leftarrow \max\{p(s_i), w_{i,l}\}$; **endfor**;
5. Construct a new partition P' by calling `Node_Partition`(G, X_1', P');
6. $P \leftarrow P'$; $i \leftarrow i + 1$;

until $(i = \lfloor \log n \rfloor)$;
7. Assign the edges in the tree directions. Set the power level for each node, following the given information.

end;

Before we continue, we notice the wireless network has the following transmission power symmetric property.

LEMMA 5. *Given a wireless network where each node has identical, adjustable k power-levels, then for any two nodes S_i and S_j with $i \neq j$, if S_j is within the transmission range of S_i when S_i uses power $w_{i,l}$ to broadcast a message, then S_i is also within the transmission range of S_j when S_j uses power $w_{j,l}$ to broadcast a message due to $w_{i,l} = w_{j,l}$, $1 \leq i, j \leq n$ and $1 \leq l \leq k$.*

We now have the following theorem.

THEOREM 3. *Given a wireless network $M(N, L)$ with k adjustable, identical power levels at each node, there is an approximate algorithm to find a minimum-energy broadcast tree rooted at a source. The total transmission power needed for maintaining the broadcast tree is $O(\log^3 n)$ times of the optimum. The time complexity of the proposed algorithm is $O(kn^2 \log n)$, where k is the number of power levels at each node in the wireless network.*

PROOF. The computational complexity of the proposed algorithm is analyzed as follows. The construction of the bipartite graph $G(X, Y, E, \omega)$ takes $O(kn^2)$ time because G contains at most $|X| + |Y| \leq kn + n$ nodes and kn^2 edges. The algorithm proceeds at rounds, and there are at most $\lceil \log n \rceil$ rounds. At each round, the graph G is constructed, and a covering set X_1 $(\subseteq X)$ with the minimum weighted sum needs to be found. However, it is well known that there is unlikely to have a polynomial algorithm for finding such a set unless P=NP. Instead, we use Chvatal's algorithm [4] to find a covering set which delivers an approximate solution within $O(\log n)$ times of the optimum. While finding such an approximate solution takes $O(kn^2)$ time due to that G contains at most $O(kn^2)$ edges, the running time of the entire algorithm is therefore $O(kn^2 \log n)$.

The rest is to analyze the performance ratio of the proposed algorithm. As mentioned, the algorithm proceeds in rounds. At each round, it actually performs merging directed broadcast trees. Initially, there are n broadcast trees. At each round, the nodes in a single partition component $\mathcal{V}_j$ form a broadcast tree. Once a covering set X_1' in G is found, the new node partition can then be found, and several broadcast trees are merged into a single broadcast tree if their corresponding partition components are in the same connected component of the induced subgraph $G[X_1']$. To perform the merge of the two trees, we need changing the directions of edges in the path from the tree root to a node in it. In the following we give more details of tree merge.

Assume that $\mathcal{V}_i$ and $\mathcal{V}_j$ are two partition components in P and $s_x \in \mathcal{V}_i$. Let T_1 be the broadcast tree consisting of the nodes in $\mathcal{V}_i$ and T_2 be the broadcast tree rooted at s_{j_1}, consisting of the nodes in $\mathcal{V}_j$. Assume that $v_{x,l} \in X_1'$ and there is an edge $(v_{x,l}, \mathcal{V}_j) \in E$. To merge T_1 and T_2 into a directed broadcast tree T, without loss of generality, assume that the root of T_1 will be the root of T (if the root of T_2 will be the root of T, the discussion is similar and omitted). Assume that the edge $(v_{x,l}, \mathcal{V}_j) \in E$ is derived due to that $s_{j_y} \in \mathcal{V}_j$ is within the transmission range of S_x when S_x uses power $w_{x,l}$ to broadcast messages. T can be constructed through adding a directed edge $\langle s_x, s_{j_y}\rangle$ and changing the directions of the edges in the path in T_2 from the root s_{j_1} to node s_{j_y}, which is shown in Fig. 3.

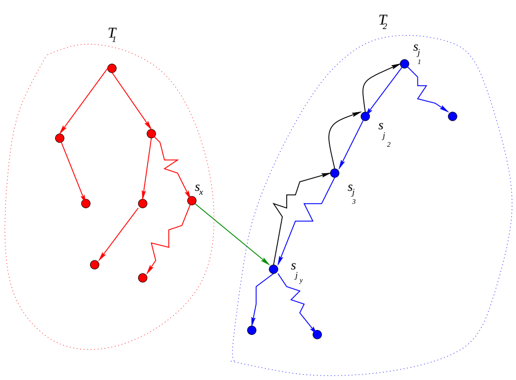

Figure 3: An illustration of tree merges

Suppose that the algorithm has executed the first $(i-1)$th rounds, now consider it is going to run the ith round. We distinguish extra power needed for tree merge at this round into the following four cases.

Case (i). The directed edge $\langle s_x, s_{j_y}\rangle$ is an edge from a leaf node s_x of T_1 to a non-leaf node s_{j_y} of T_2. To change the directions of edges in path from s_{j_1} to s_{j_y} in T_2, the transmission power of S_{j_i} must be adjusted to $p(s_{j_i-1})$, $2 \leq i \leq y$ by Lemma 5. Thus, if the power $p(s_{j_y})$ of S_{j_y} is less than the power $p(s_{j_y-1})$ of its parent node S_{j_y-1} in T_2, then, the extra power (except the powers used in X_1') for this tree merge is $p(s_{j_y-1}) - p(s_{j_y}) \leq p(s_{j_y-1})$.

Case (ii). The directed edge $\langle s_x, s_{j_y}\rangle$ is an edge from a leaf node s_x of T_1 to a leaf node s_{j_y} of T_2. For this case, the extra power for this tree merge is $p(s_{j_y-1})$ due to that in T_2 $p(s_{j_y})$ is a leaf node and its power is zero. Now its power in T must be at least $p(s_{j_y-1})$ to be able to broadcast messages to its parent S_{j_y-1} in T_2.

Case (iii). The directed edge $\langle s_x, s_{j_y}\rangle$ is an edge from a non-leaf node s_x of T_1 to a leaf node s_{j_y} of T_2. If $p(s_x) > w_{x,l}$, then node S_x still uses power $p(s_x)$ to broadcast messages, where $w_{x,l}$ is the power used by S_x at the current round. For this case, the extra power for this tree merge is $p(s_{j_y-1})$ due to that s_{j_y} is a leaf node in T_2 and its power is zero in T_2.

Case (iv). The directed edge $\langle s_x, s_{j_y}\rangle$ is an edge from a non-leaf node s_x of T_1 to a non-leaf node s_{j_y} of T_2. For this case, if the current power $p(s_{j_y})$ of S_{j_y} is greater than the power $p(s_{j_y-1})$ of its parent node S_{j_y-1} in T_2, no extra power is needed for such merge. Otherwise, the extra power for this merge is $p(s_{j_y-1}) - p(s_{j_y}) \leq p(s_{j_y-1})$. In the end, the total extra power needed for the implementation of tree merges at the ith round is no more than the transmission power sum of the broadcast trees at the $(i-1)$th round. Let Δ_i be the transmission power sum of the broadcast trees at the ith round. Then,

$$\Delta_i \quad \leq \quad \log(\frac{n}{2^i})W_{\min} + \Delta_{i-1}, \tag{7}$$

where the first term in the right hand side of Inequality 7 is the weighted sum of nodes in X_1', which is no greater than $\log |Y| \times W_{\min}$, while $|Y| \leq \frac{n}{2^i}$ at the ith round. $\Delta_0 \leq \log nP_{opt}$. As results, we have

$$\Delta_i \quad \leq \quad (i+1)\log nP_{opt}, \tag{8}$$

$1 \leq i \leq \lfloor \log n \rfloor$. Thus, the total transmission power of an approximate minimum-energy broadcast tree is

$$\sum_{i=0}^{\lfloor \log n \rfloor} \Delta_i \quad \leq \quad \sum_{i=0}^{\lfloor \log n \rfloor} (i+1)\log nP_{opt} = O(\log^3 nP_{opt}).$$

□

5. APPROXIMATE ALGORITHMS FOR MINIMUM-ENERGY MULTICAST TREE PROBLEM

It is well known that finding a minimum-cost multicast tree in a wired network is NP-complete. The problem in a wireless ad hoc network is at least as hard as its counterpart in the wired network. Therefore, finding a minimum-energy multicast tree in a wireless network is a difficult problem. In the following we apply the design methodology for the minimum-energy broadcast tree problem to solve the minimum-energy multicast tree problem.

5.1 Multicast tree with k adjustable power levels at each node

Following the same spirit as we construct a minimum energy broadcast tree in the wireless network, here we first construct an auxiliary, weighted, directed graph $G(V, E, \omega_1)$. Let $D = \{s_i$: if node S_i is a terminal node$\}$, then find an approximate directed Steiner tree in G rooted at the source including all nodes in D. Let T_D be the approximate Steiner tree. Check the edges in T_D to see whether they satisfy the restriction imposed by case (i) of Lemma 3. Let T_D' be the resulting tree after modifying T_D such that it meets the restriction. Accordingly, an approximate minimum-energy multicast tree is built and the power levels of the nodes involved in the network are determined. Thus, we have the following theorem.

THEOREM 4. *Given a wireless network $M(N, L)$ with k adjustable power levels at each node, a source node S_1, and a set D of terminal nodes, there is an approximate algorithm to find a minimum-energy multicast tree rooted at S_1. The total transmission power needed is $|D|^\epsilon$ times of the optimum. The time complexity of the proposed algorithm is $O(((k+1)n)^{\frac{1}{\epsilon}}|D|^{\frac{2}{\epsilon}} + kn^2)$, where k is the number of power levels at each node in the wireless network and ϵ is constant with $0 < \epsilon \le 1$.*

PROOF. The proof is exactly the same as that for Theorem 2, omitted. □

5.2 Multicast Tree with equal power levels at each node

We here consider a special case of the problem where each node has identical k-level powers, i.e., $w_{i,l} = w_{j,l}$ for $i \ne j$, $1 \le l \le k$, $1 \le i, j \le n$. We show that there is a better approximate algorithm for the minimum-energy multicast tree problem, which delivers a solution within $O(\log^3 |D|)$ times of the optimum, where D is the set of terminal nodes. Following the same idea of the algorithm `Mini_Broad_Tree_Equal`, the approximate solution for the problem is found through finding an approximate solution for an optimization problem on an auxiliary graph. The only difference lies on the construction of the auxiliary bipartite graph $G = (X, Y, E, \omega)$. In this case, Y is a partition of the terminal nodes. Initially $|Y| = |D|$. In each subsequent round, any partition of Y is a partition of the nodes corresponding to the terminal nodes in D. Thus, we have the following theorem.

THEOREM 5. *Given a wireless network $M(N, L)$ with k adjustable, identical power levels at each node, a source node and a set D of terminal nodes, there is an approximate algorithm to find a minimum-energy multicast tree rooted at a source. The total transmission power consumed of the multicast tree is $O(\log^3 |D|)$ times of the optimum. The time complexity of the proposed algorithm is $O(kn|D|\log|D|)$, where k is the number of power levels at each node in the wireless network.*

PROOF. The proof is similar to that for Theorem 3, omitted. □

6. THE ANALYSIS OF THE BIP ALGORITHM

In [19] they introduced an approximate algorithm called the Broadcast Incremental Power (BIP) algorithm for the minimum-energy broadcast tree problem. Unfortunately, they didn't give the performance of the BIP algorithm. In this section we analyze the performance of the BIP algorithm by showing that it is $\Omega(n)$ times of the optimum in the worst case. In contrast to this known algorithm for the minimum-energy broadcast tree problem, the performance ratio of our approximation algorithm is $O(n^\epsilon)$, $0 < \epsilon \le 1$, which trade-offs the time spent for finding an approximate solution and the accuracy of the approximate solution obtained.

6.1 The BIP algorithm

BIP is similar in principle to Prim's algorithm for the minimum spanning tree problem, in the sense that new nodes are added to the broadcast tree one at a time (on a minimum cost basis) until all nodes are included in the tree. Initially, the source node is added into the tree. The major difference between Prim's algorithm and the BIP algorithm is that BIP must dynamically update the costs at each step to reflect that the cost of adding new nodes to a transmitting node's list of neighbors is the incremental cost, whereas the inputs to Prim's algorithm are the link costs $D_{i,j}$ which remain unchanged throughout the execution of the algorithm. Consider an example in which node S_i is already in the tree and it may be either a transmission node or a leaf node, and a node S_j is not yet in the tree. For all such nodes S_i and S_j, the BIP algorithm evaluates

$$D'_{i,j} = D_{i,j} - p(S_i) \qquad (9)$$

where $D_{i,j}$ is the link-based cost of transmission between node S_i and node S_j, and $p(S_i)$ is the power level at which node S_i is already transmitting (prior to the addition of node S_j; if node i is currently a leaf node, $p(S_i) = 0$). The quantity $D'_{i,j}$ represents the incremental cost associated with adding node S_j to the set of nodes to which node S_i already transmits. The pair $\{i, j\}$ that results in the minimum value of $D'_{i,j}$ is chosen. Thus, a new node is added to the broadcast tree at every step of the algorithm.

6.2 The performance ratio of the BIP algorithm

THEOREM 6. *Given a wireless network $M(N, L)$, the performance ratio between the solution delivered by the BIP algorithm and the exact solution is no less than $\frac{cp_{\min}}{2p_{\max}} \times n$, where $p_{\min} = \min\{w_{i,l} \mid 1 \le i \le n, 1 \le l \le k\}$, $p_{\max} = \max\{w_{i,l} \mid 1 \le i \le n, 1 \le l \le k\}$, and c is constant.*

PROOF. We show that there is a wireless network such that the solution delivered by the BIP algorithm is no less than $\frac{cp_{\min}}{2p_{\max}} \times n \times OPT$, where OPT is the optimal solution of the problem.

Consider a wireless network consists of nodes layered within 3 levels, where there is a source node at layer 1, there are g nodes at layer 2, there are βg nodes at layer 3, both β and g are positive integers, and β is constant.

Assume that the source node has two power levels with values w_1 and $(1+\alpha)(w_1+w_2)$ respectively, where α is a constant with $0 < \alpha \le 1/2$. When the source node uses power w_1 to broadcast messages, all the nodes at layer 2 are within its transmission range; while the source node uses power $(1+\alpha)(w_1+w_2)$ to broadcast messages, all the nodes at layers 2 and 3 are within its transmission range.

The nodes at layer 3 are partitioned into g groups and each group contains exactly β nodes. All of the nodes at layer 3 are no transmission powers. For each node v at layer 2 using power w_2 to broadcast messages, there is a corresponding group at layer 3 in which the nodes are within the transmission range of v. Thus, the wireless network consists of $n = 1 + g + \beta g$ nodes. Thus,

$$g = \frac{n-1}{1+\beta}. \qquad (10)$$

Having constructed the wireless network, the BIP algorithm is applied on it. Then, the sum of transmission powers in the broadcast spanning tree obtained by the BIP algorithm is $w_1 + gw_2$. It is obvious that the optimal solution for the problem is a star centered at the source node using the power $(1+\alpha)(w_1+w_2)$ to broadcast messages. Let $\mathcal{A}_{BIP}$ be the approximate solution delivered by the BIP algorithm and $\mathcal{A}_{OPT}$ be the optimal solution of the problem. Then, the performance ratio between the approximate solution and the optimal solution is as follows.

$$\frac{\mathcal{A}_{BIP}}{\mathcal{A}_{OPT}} = \frac{w_1+gw_2}{(1+\alpha)(w_1+w_2)} \geq \frac{p_{\min}(1+g)}{2p_{\max}(1+\alpha)} \quad (11)$$

$$= \frac{p_{\min}(\frac{\beta+n}{1+\beta})}{2p_{\max}(1+\alpha)} \quad (12)$$

$$> \frac{p_{\min}\times n}{2p_{\max}(1+\alpha)(1+\beta)} \quad (13)$$

$$= \frac{p_{\min}}{4p_{\max}(1+\alpha)}n, \quad when \;\; \beta=1 \quad (14)$$

$$= \Omega(n) \quad (15)$$

Throughout the above analysis, it can be seen that the performance of the BIP algorithm is not good in some cases if the power values of the network are independent of the problem size n. □

7. CONCLUSIONS

In this paper we first have shown that the minimum-energy broadcast tree problem is NP-complete. We then have proposed an approximate algorithm for the problem with a bounded performance guarantee. We have also given an approximate algorithm for a special case of the problem where each node is equipped with the same type of battery, which delivers a better approximate solution. The technique for the minimum-energy broadcast tree problem have been extended to solve the minimum-energy multicast tree problem. Despite that a better approximate solution for the minimum-energy broadcast tree problem has been proposed in this paper, it is interesting and challenging to answer the following questions in a positive or negative way. (1) Whether there is an approximate algorithm for the problem with a constant performance ratio. (2) Whether the problem is still NP-hard if the power level at each node is adjustable infinitely.

8. ACKNOWLEDGMENTS

We appreciate Yingyu Wan for fruitful discussion and comments on the NP-completeness of the minimum-energy broadcast tree problem and analysis of the BIP algorithm. We also would like to thank an anonymous referee for bringing the references [9, 14, 11] to our attention.

9. REFERENCES

[1] D. J. Baker and A. Ephremides. The architectural organization of a mobile radio network via distributed algorithm. *IEEE Trans. Commun*, Vol. COM-29, pp. 56–73, 1981.

[2] J-H Chang and L. Tassiulas. Fast approximate algorithms for maximum lifetime routing in wireless ad-hoc networks. *IFIP-TC6/European Commission Int'l Conf.*, Lecture Notes in Computer Science, Vol. 1815, pp.702-713, Springer, 2000.

[3] I. Chlamtac and A. Farago. A new approach to the design and analysis of peer-to-peer mobile networks. *Wireless Networks*, Vol. 5, pp. 149–156, 1999.

[4] V. Chvátal. A greedy heuristic for the set-covering problem. *Mathematics of Operations Research*, Vol. 4(3), pp. 233–235, 1979.

[5] M. Charikar, C. Chekuri, T-Y Cheung, Z. Dai, A. Goel, S. Guha and M. Li. Approximation algorithms for directed Steiner problems. *J. Algorithms*, Vol. 33(1): pp. 73–91, 1999.

[6] C.-C Chiang, M. Gerla, and L Zhang. Forwarding group multicast protocol(FGMP) for multihop, mobile wireless networks. *Cluster Computing*, Vol. 1, pp. 187–196, 1998.

[7] A. Ephremides, J. E. Wieselthier, and D. J. Baker. A design concept for reliable mobile radio networks with frequency hopping signaling. *Proceedings of the IEEE*, Vol. 75, pp. 56–58, 1987.

[8] A. Farago, I. Chlamtac, and S. Bassagni. Virtual path network topology optimization using random graphs. *Proc. of INFOCOM'99*, pp. 491–496, 1999.

[9] J.J. Garcia-Luna-Aceves and E. L. Madruga. A Multicast routing protocol for ad-hoc networks. *Proc. of INFOCOM'99*, New York, 1999.

[10] S. Khanna and K. Kumaran. On wireless spectrum estimation and generalized graph coloring. *Proc. of INFOCOM'98*, pp. 1273–1283, 1998.

[11] S.-J. Lee. *Routing and Multicast Strategies in Wireless Mobile Networks*. Dept. of Computer Sci., UCLA, Los Angeles, 2000.

[12] M. Nagy and S. Singh. Multicast scheduling algorithms in mobile networks. *Cluster Computing*, Vol. 1, pp. 177–185, 1998.

[13] K. Makki, N. Pissinou, and O. Frieder. Efficient solutions to multicast routing in communication networks. *Mobile Networks and Applications*, Vol. 1, pp. 221–232, 1996.

[14] C. Perkins and E. M. Royer. Ad Hoc on Demand Distance Vector (AODV) routing. *Internet Draft*, $http://www.ietf.org/internet-drafts/draft-ietf-manet-aodv-01.txt$, Aug., 1998.

[15] R. Ramanathan and R. Rosales-Hain. Topology control of multihop wireless networks using transmit power adjustment. *Proc. of INFOCOM'00*, 2000.

[16] T. S. Rappaport. *Wireless Communications, Principles and Practice.* Prentice-Hall, 1996.

[17] V. Rodoplu and T. H. Meng. Minimum energy mobile wireless networks. *Proc. of ICC'98*, Vol. 3, pp. 1633-1639, 1998.

[18] S. Singh, M. Woo, and C.S. Raghavendra. Power-aware routing in mobile ad hoc networks. *Proc. of MOBICOM'98*, pp. 181–190, 1998.

[19] J.E. Wieselthier, G. D Nguyen and A. Ephremides. On construction of energy-efficient broadcast and multicast trees in wireless networks. *Proc. of INFOCOM'00*, 2000.