

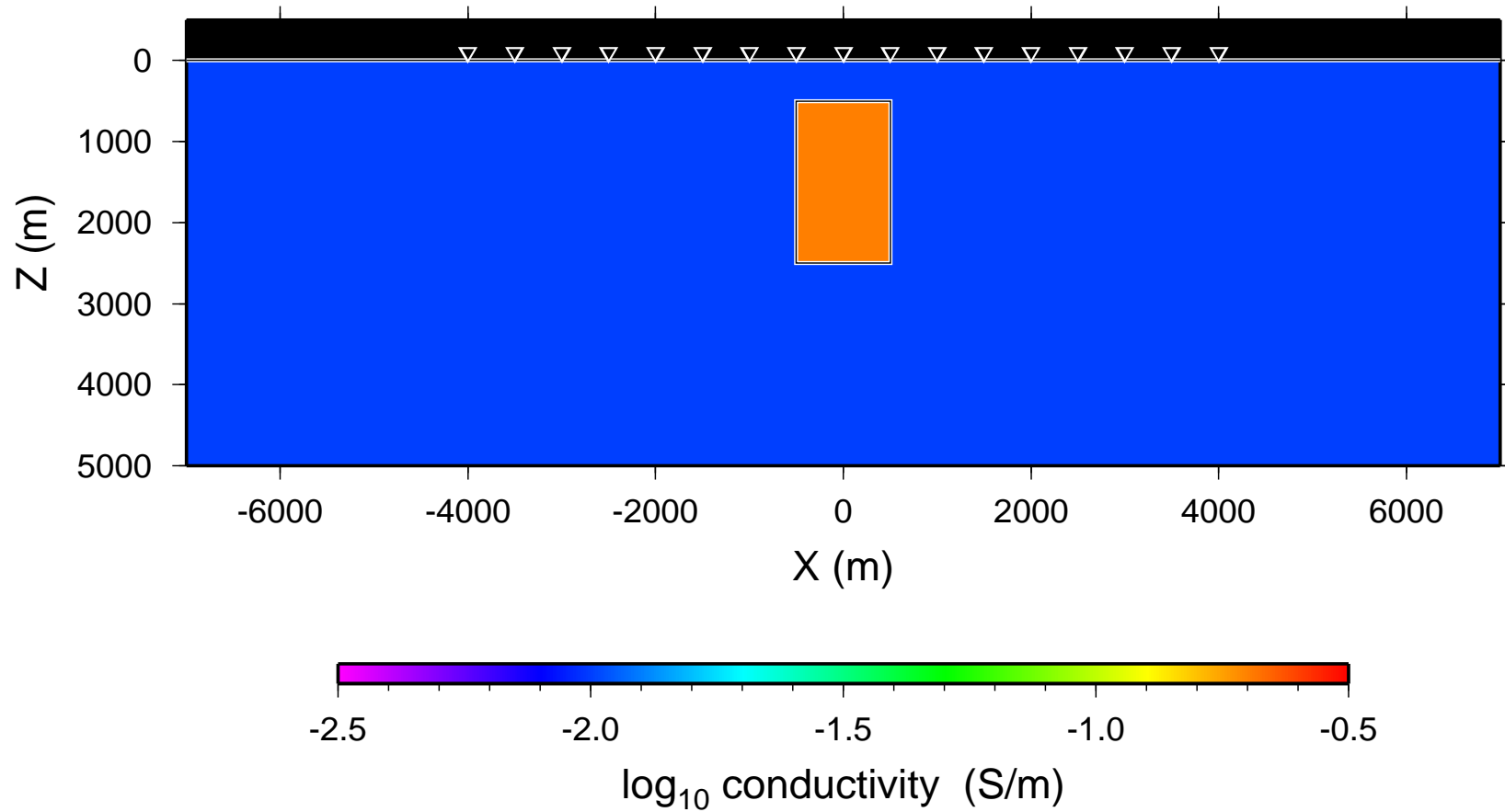
Constructing piecewise-constant models in multi-dimensional minimum-structure inversions

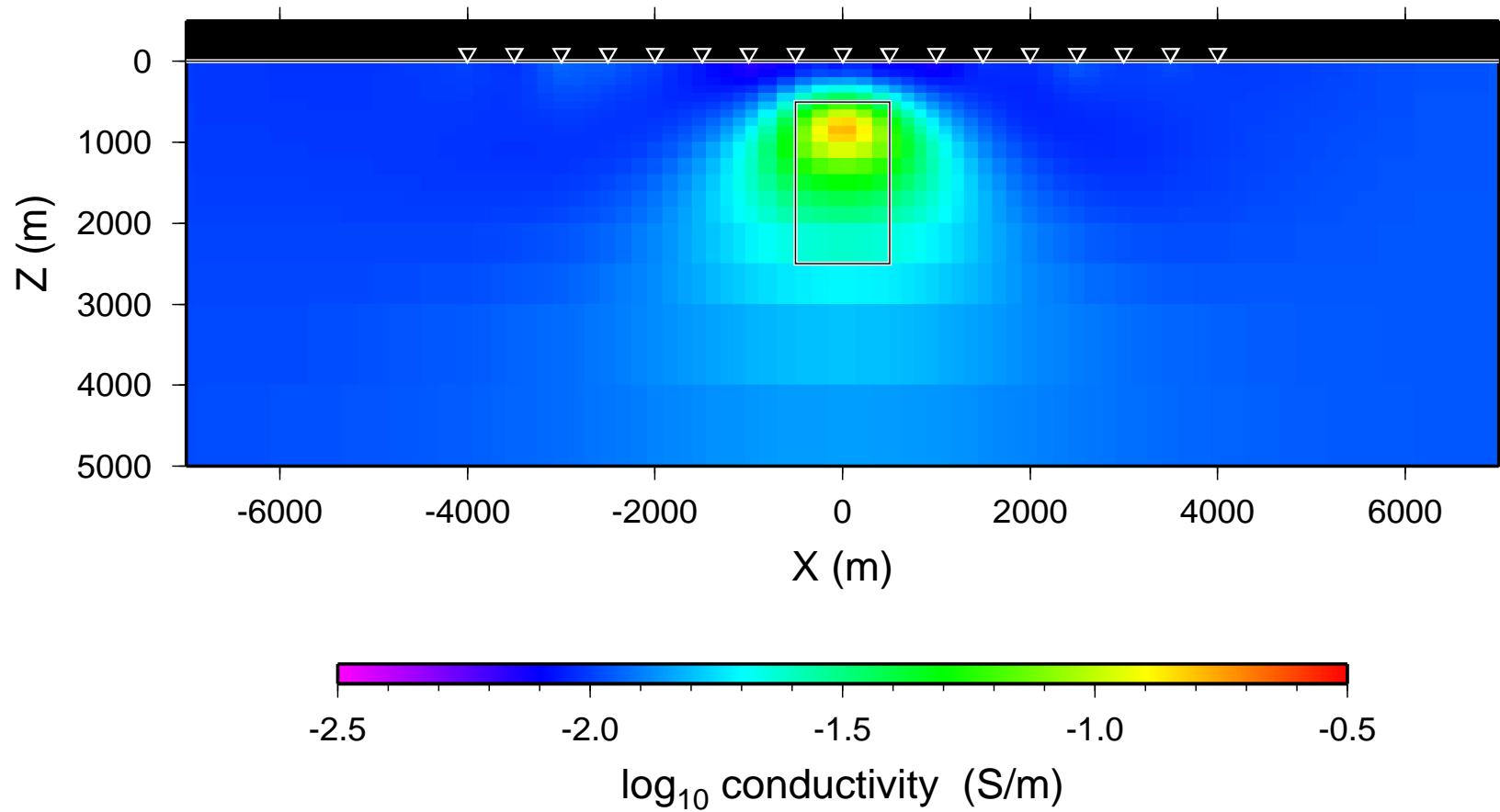
Colin G. Farquharson

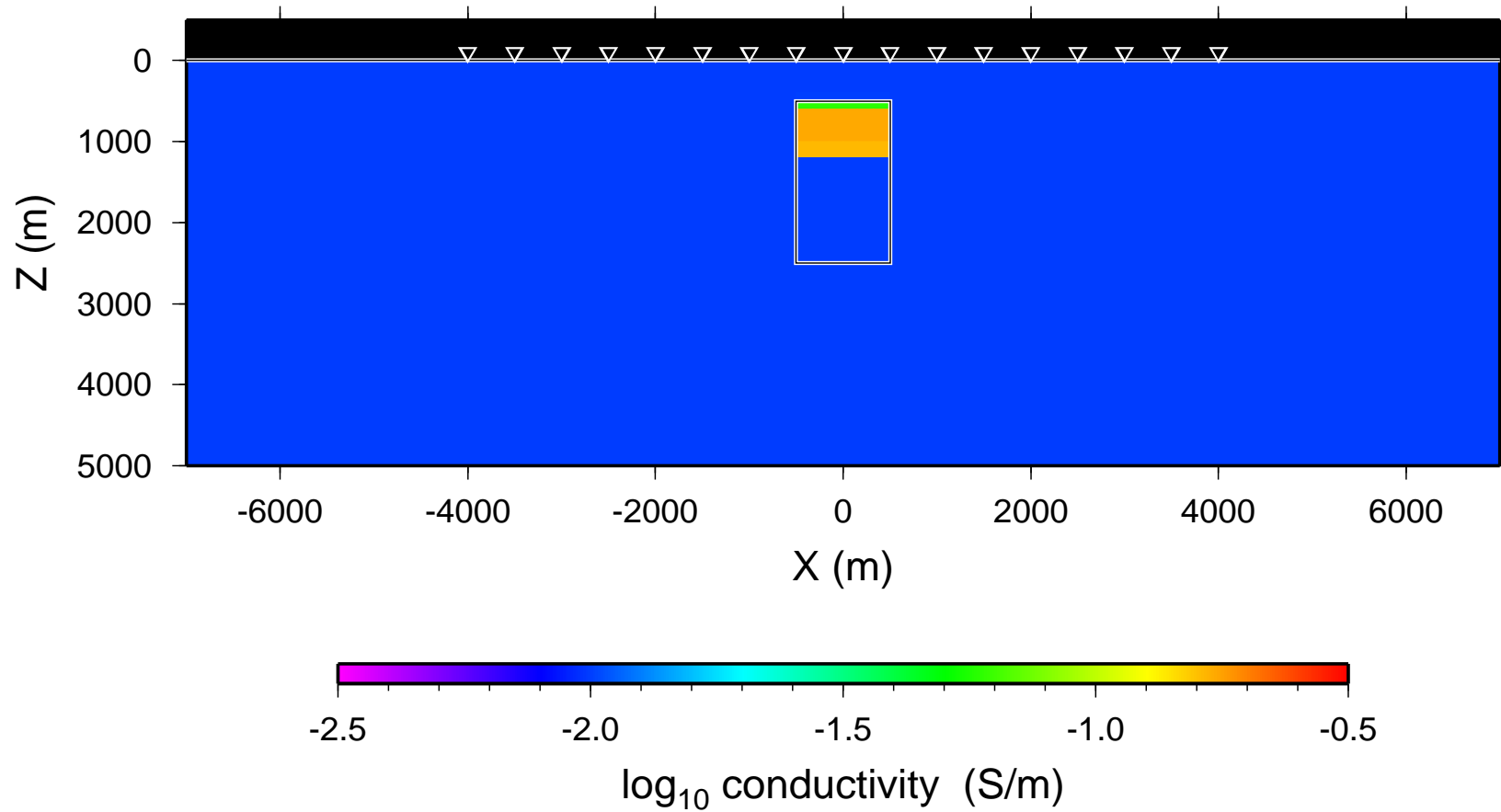
Inco Innovation Centre, and
Department of Earth Sciences,
Memorial University of Newfoundland.
St. John's, NL, Canada.



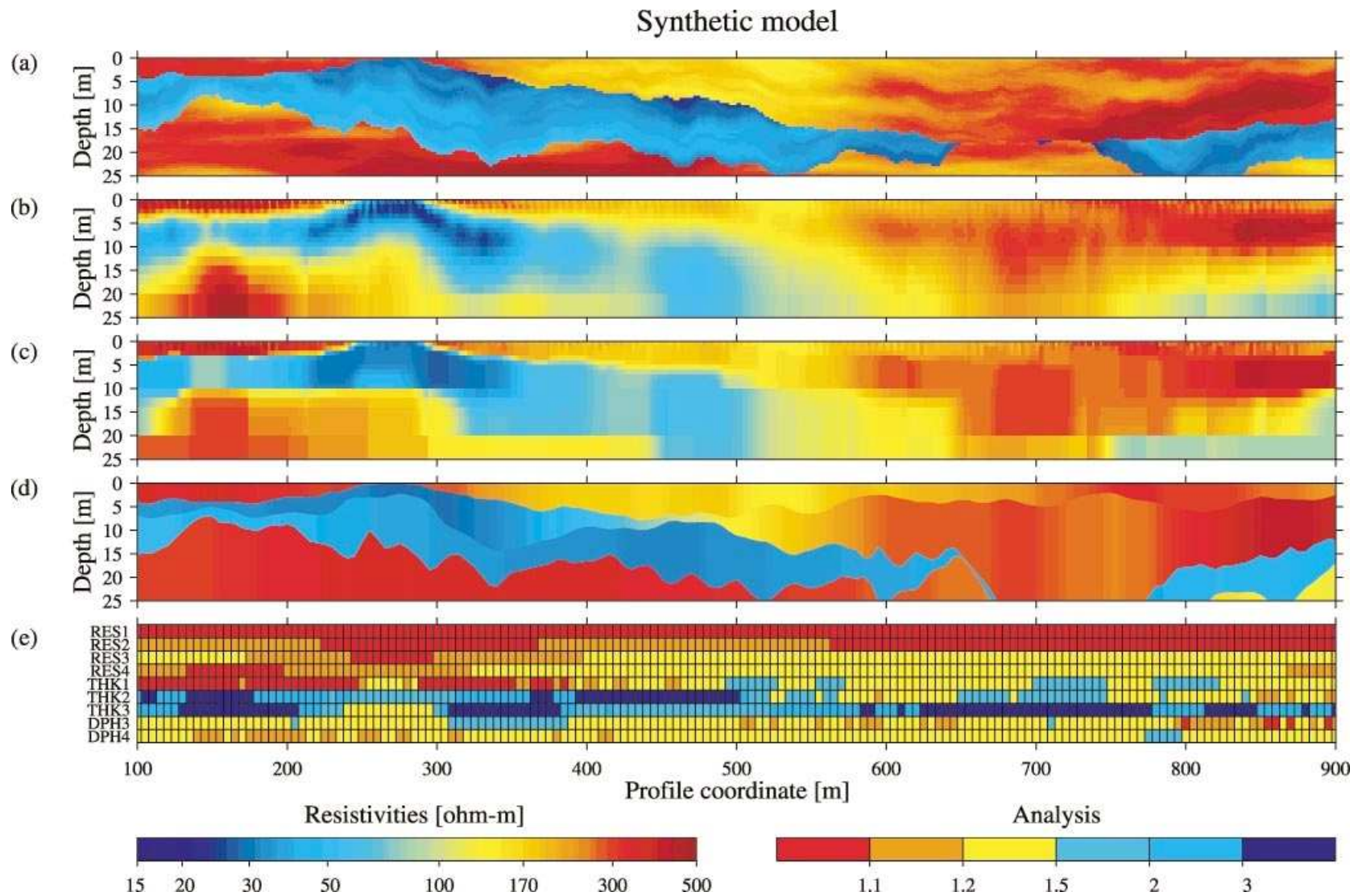
Motivation and Conclusions

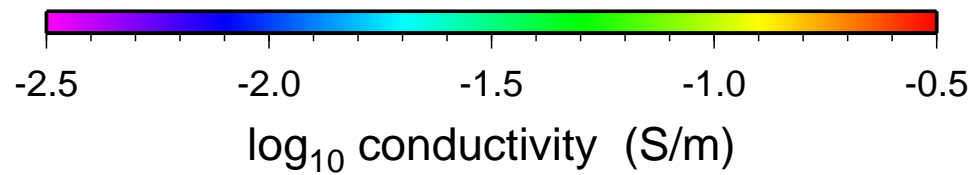
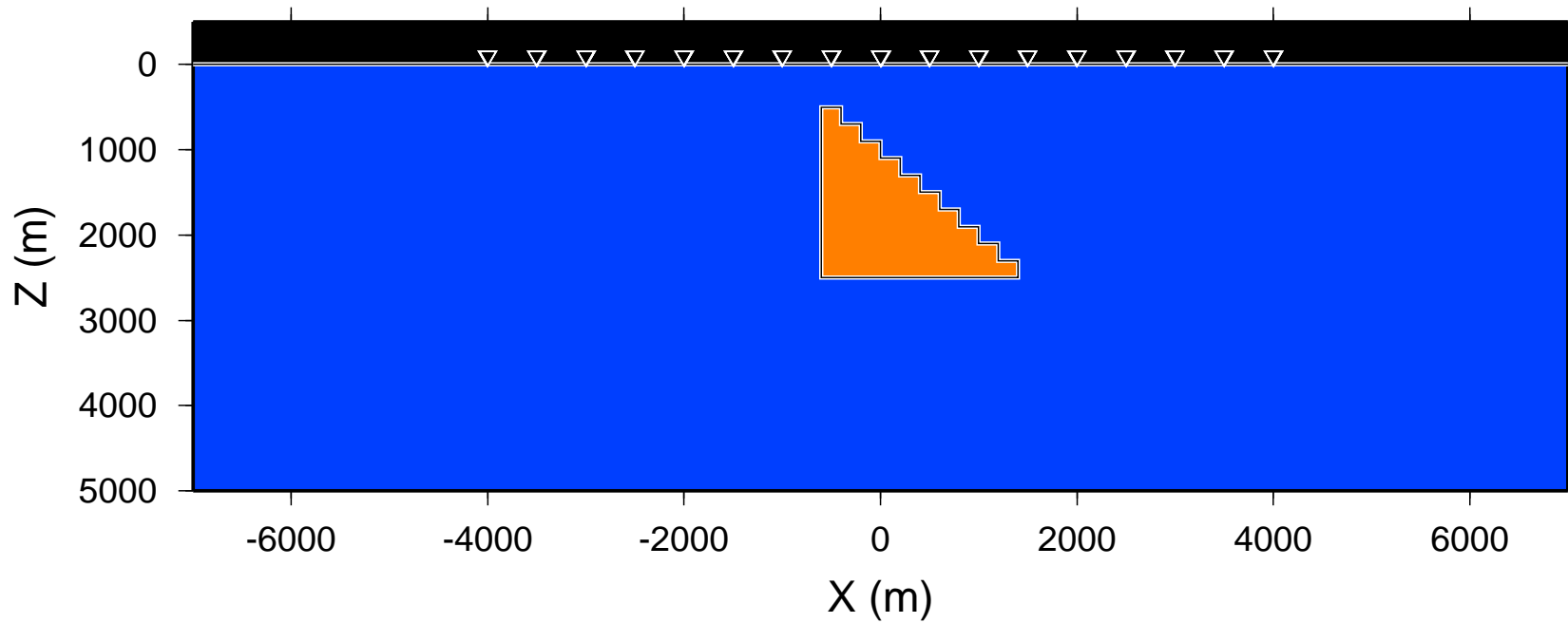


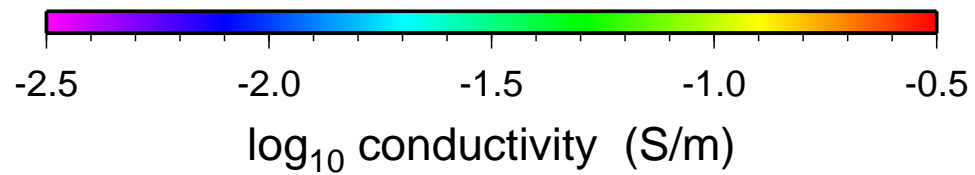
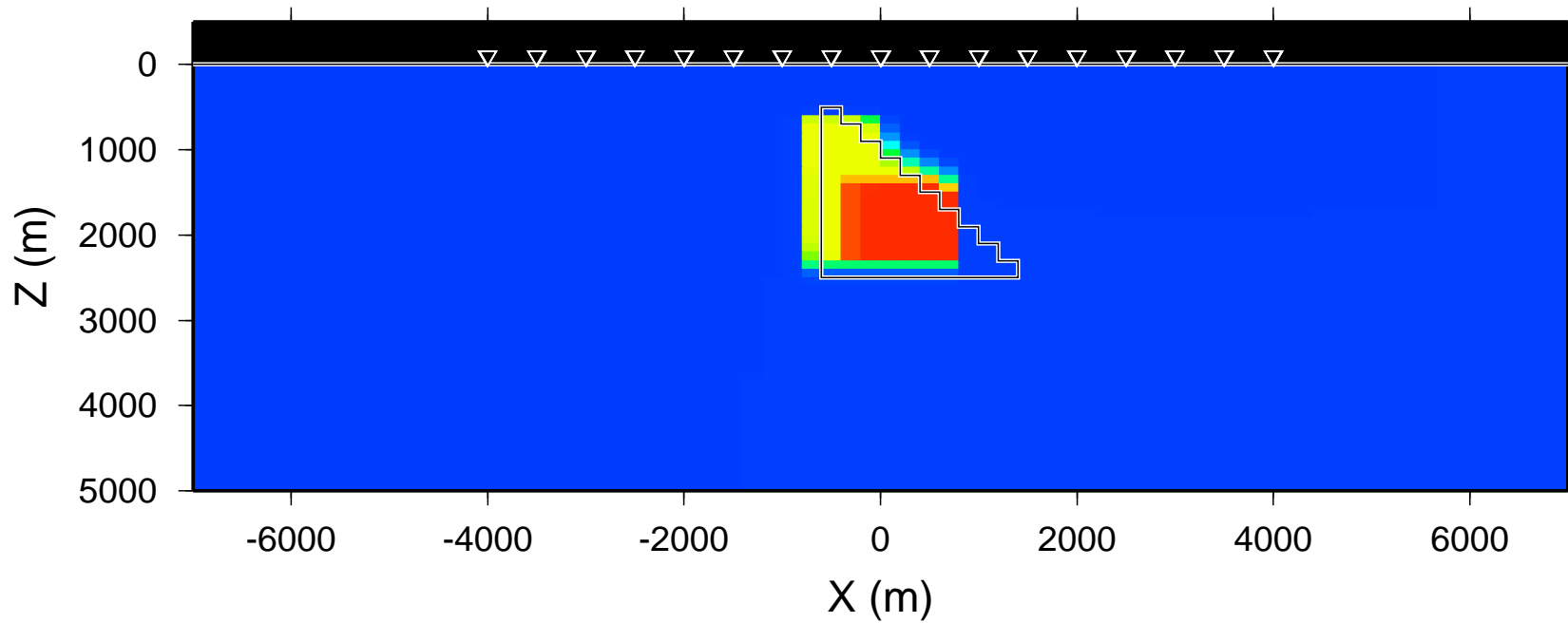


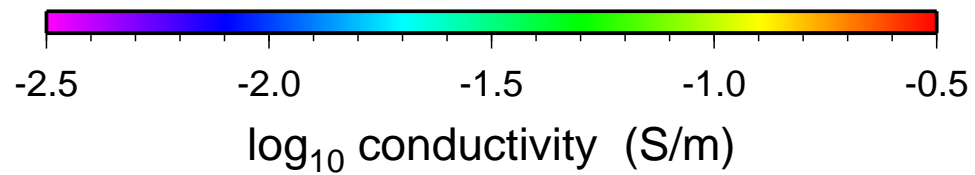
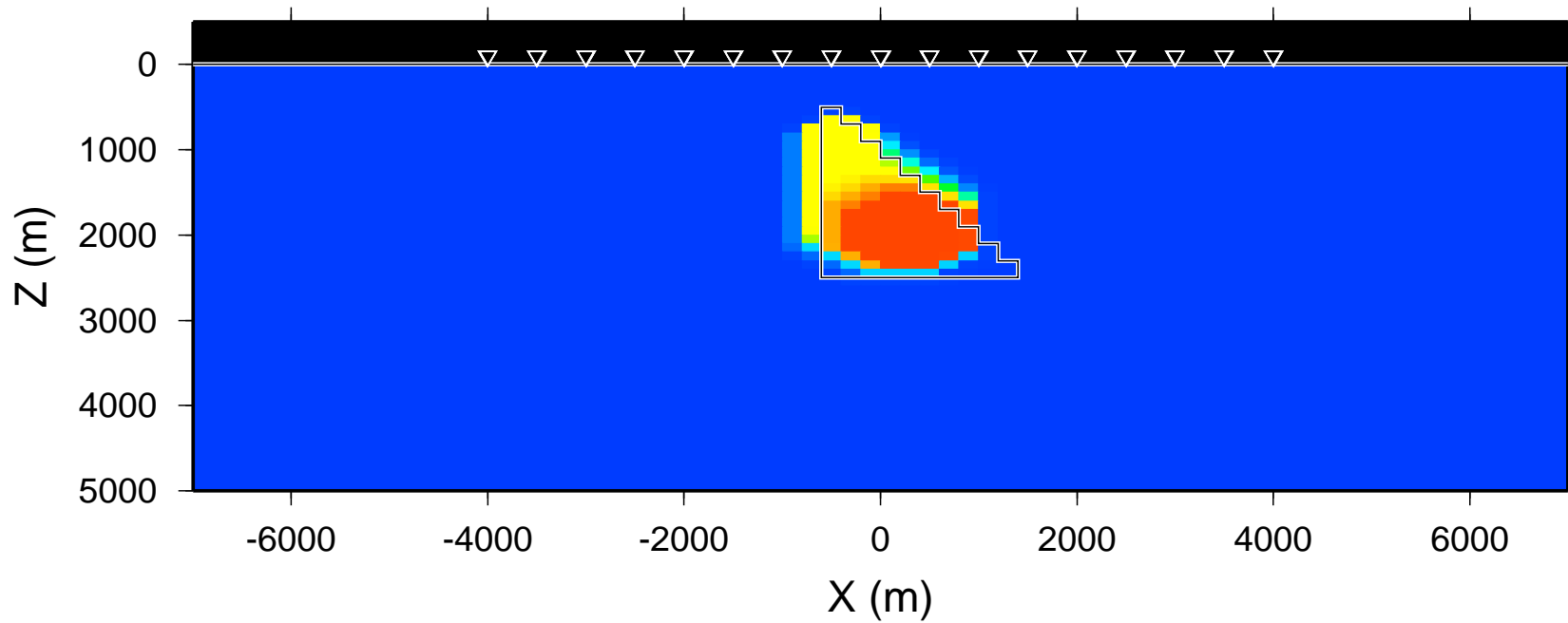


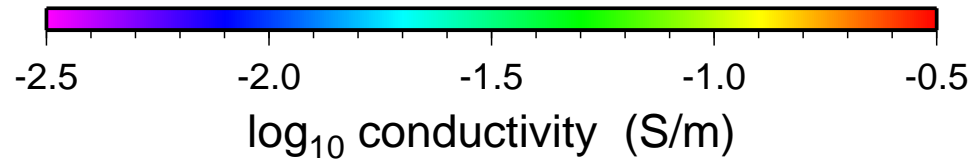
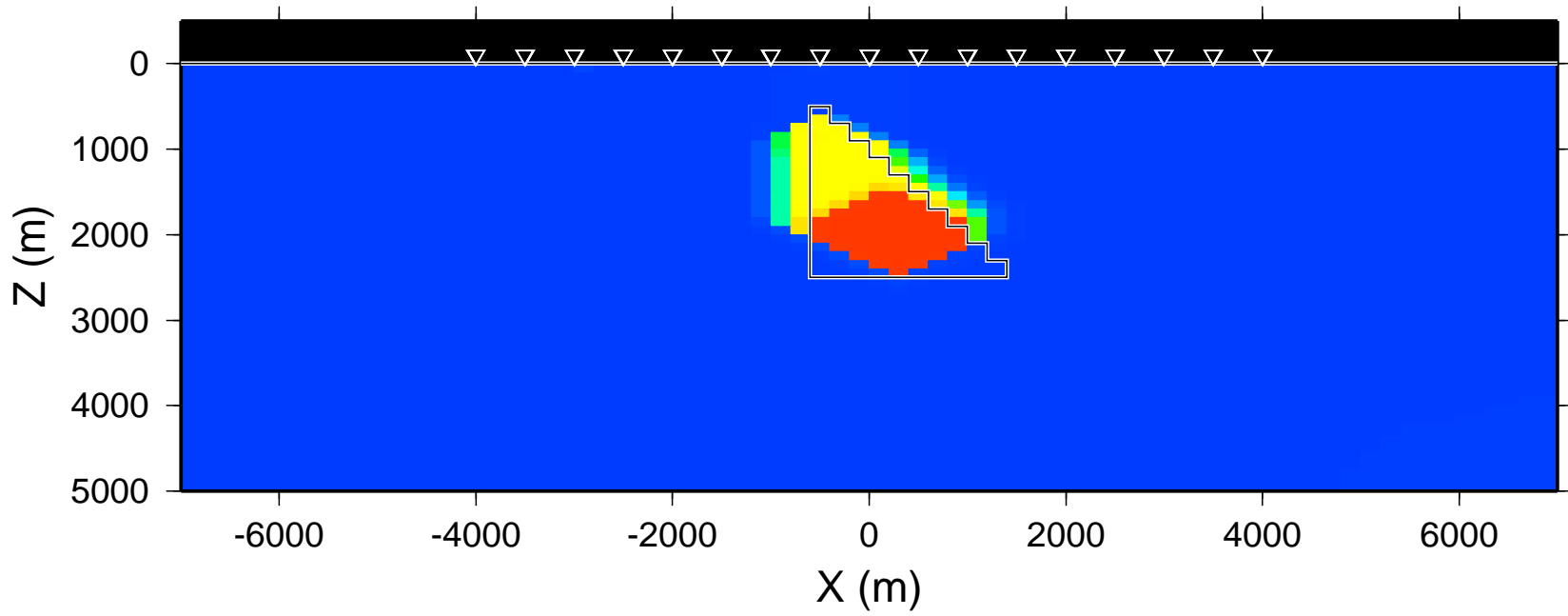
Auken & Christiansen (2004, Geophysics, 69, p752–761):











Outline

- Motivation and Conclusions.
- Previous work.
- General minimum-structure inversion strategy.
 - General measures.
 - Iterative solution procedure.
 - Measure of model structure.
- Example.
- Conclusions.

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Previous work

“True” minimum-structure algorithms:

Farquharson & Oldenburg (1998, GJI), 1-D EM;

Portniaguine & Zhdanov (1999, Geophysics), 3-D focusing;

Loke, Acworth & Dahlin (2003, Expl. Geop.), 2-D resistivity;

Farquharson & Oldenburg (2003, SEGJ), 2-D resistivity.

Laterally constrained layered inversions:

Smith et al. (1999, Geophysics), 2-D MT;

Auken & Christiansen (2004, Geophysics), 2-D resistivity;

de Groot-Hedlin & Constable (2004, Geophysics), 2-D MT.

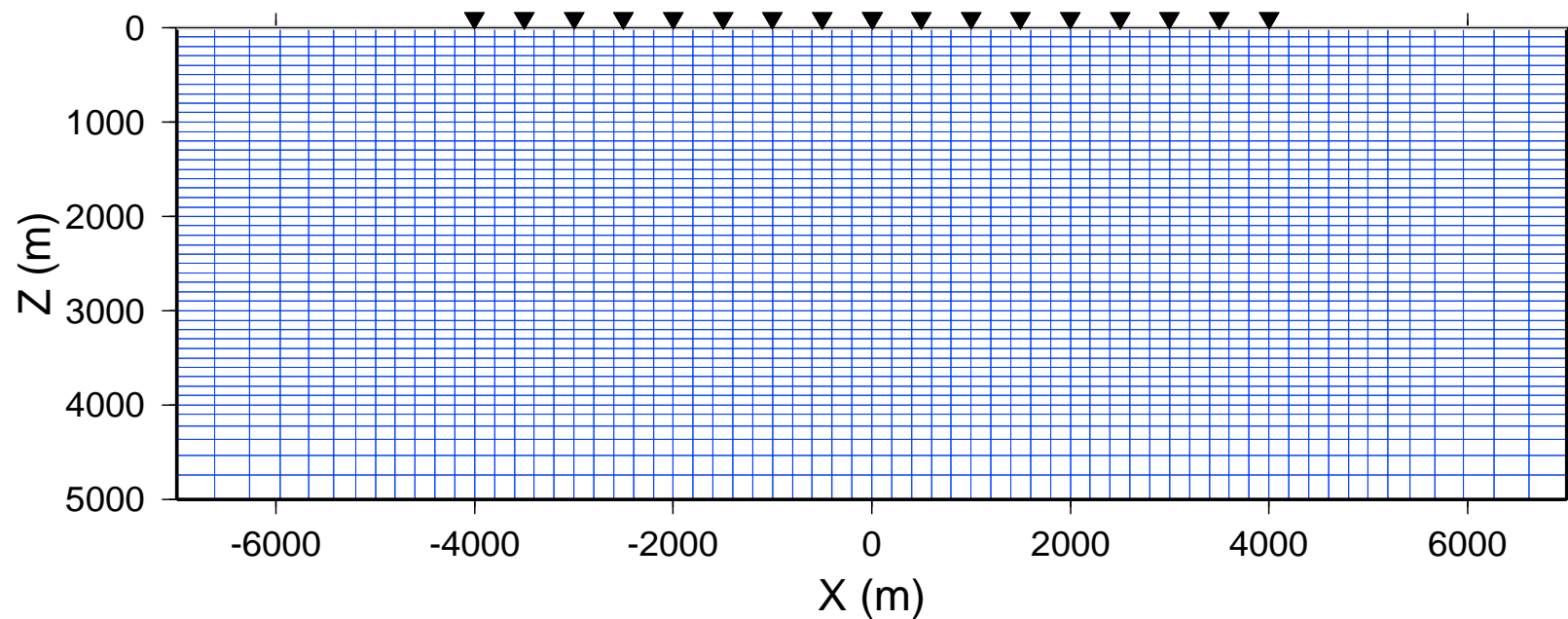


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General minimum-structure inversion strategy

- Mesh fixed during inversion; fine discretization.



General minimum-structure inversion strategy

- Minimize objective function:

$$\Phi = \phi_d + \beta \phi_m,$$

where ϕ_d is measure of data-misfit,

$$\phi_d = \phi_d(\mathbf{u}) \quad \mathbf{u} = \mathbf{W}_d(\mathbf{d}^{\text{obs}} - \mathbf{d}^{\text{prd}}),$$

and ϕ_m is measure of structure in model,

$$\phi_m = \sum_k \alpha_k \phi_k(\mathbf{v}_k) \quad \mathbf{v}_k = \mathbf{W}_k(\mathbf{m} - \mathbf{m}_k^{\text{ref}}).$$

General measures

- A general form for ϕ_d and ϕ_m is:

$$\phi(\mathbf{x}) = \sum_{j=1}^N \rho(x_j).$$

For example, the l_2 -norm: $\rho(x) = x^2$;

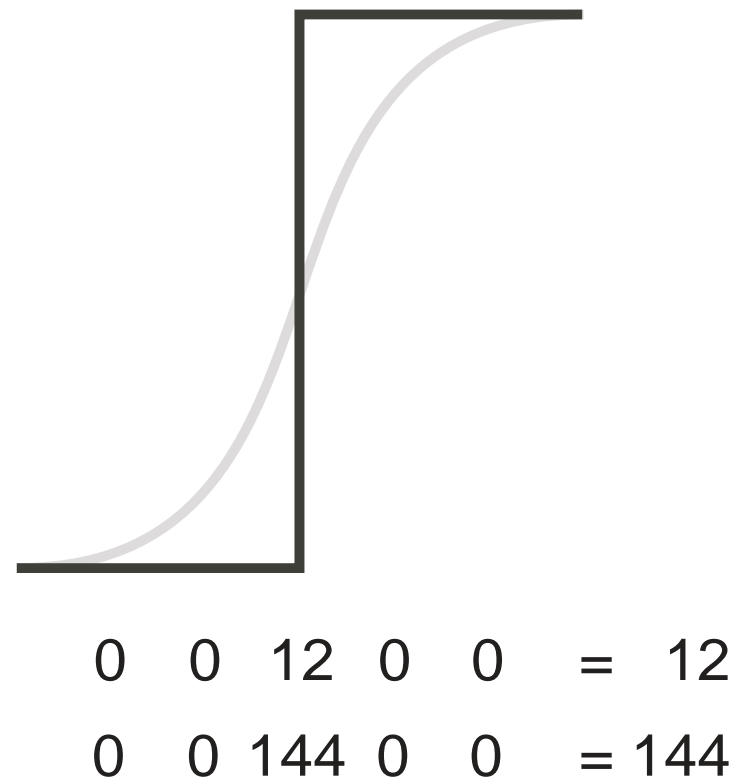
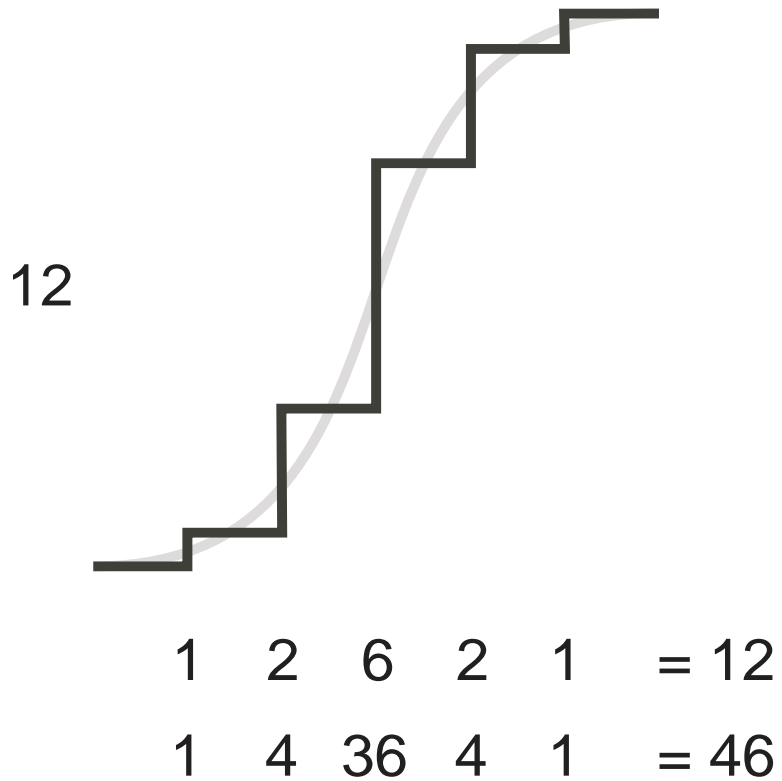
the l_p -norm: $\rho(x) = |x|^p$;

Ekblom's l_p -like measure: $\rho(x) = (x^2 + \epsilon^2)^{p/2}$;

Huber's M -measure: $\rho(x) = \begin{cases} x^2 & |x| \leq c, \\ 2c|x| - c^2 & |x| > c. \end{cases}$



General measures



Iterative solution procedure

- Differentiate Φ with respect to model parameters and equate to zero.

Get normal system of equations:

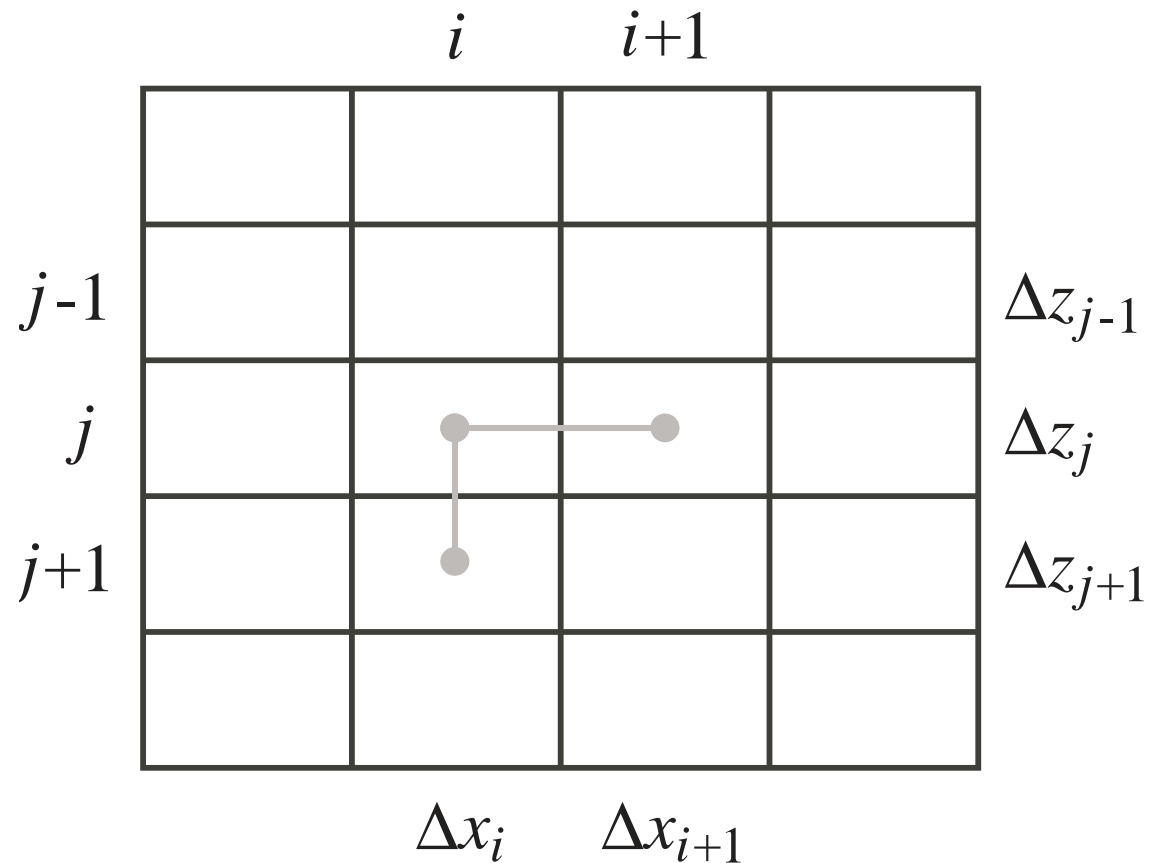
$$\begin{aligned} & \left[\mathbf{J}^T \mathbf{W}_d^T \mathbf{R}_d \mathbf{W}_d \mathbf{J} + \beta^n \sum_k \alpha_k \mathbf{W}_k^T \mathbf{R}_k \mathbf{W}_k \right] \delta \mathbf{m} \\ & = \mathbf{J}^T \mathbf{W}_d^T \mathbf{R}_d \mathbf{W}_d (\mathbf{d}^{\text{obs}} - \mathbf{d}^{n-1}) + \\ & \quad \beta^n \sum_k \alpha_k \mathbf{W}_k^T \mathbf{R}_k \mathbf{W}_k (\mathbf{m}_k^{\text{ref}} - \mathbf{m}^{n-1}). \end{aligned}$$

Update \mathbf{R}_d and \mathbf{R}_k as well as \mathbf{J} .

Measure of model structure

- Regularization via finite-difference matrices.

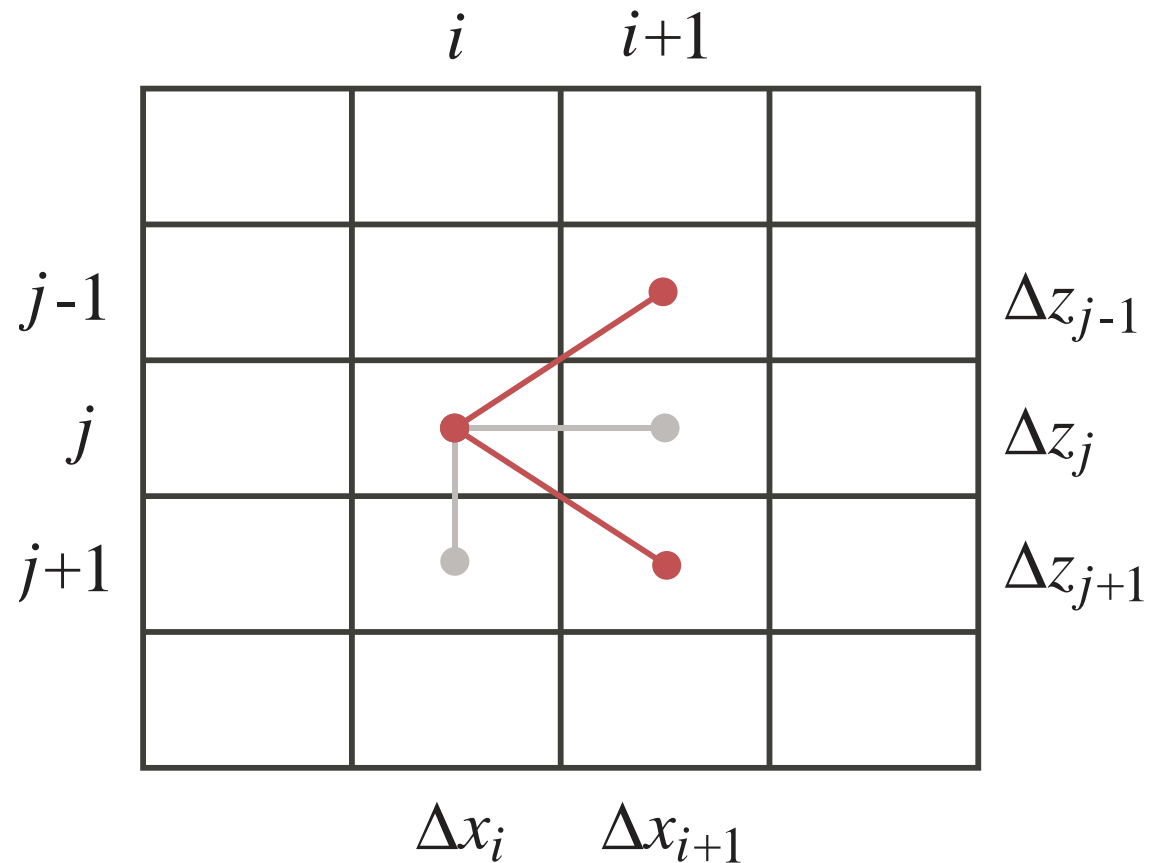
Old way:



Measure of model structure

- Regularization via finite-difference matrices.

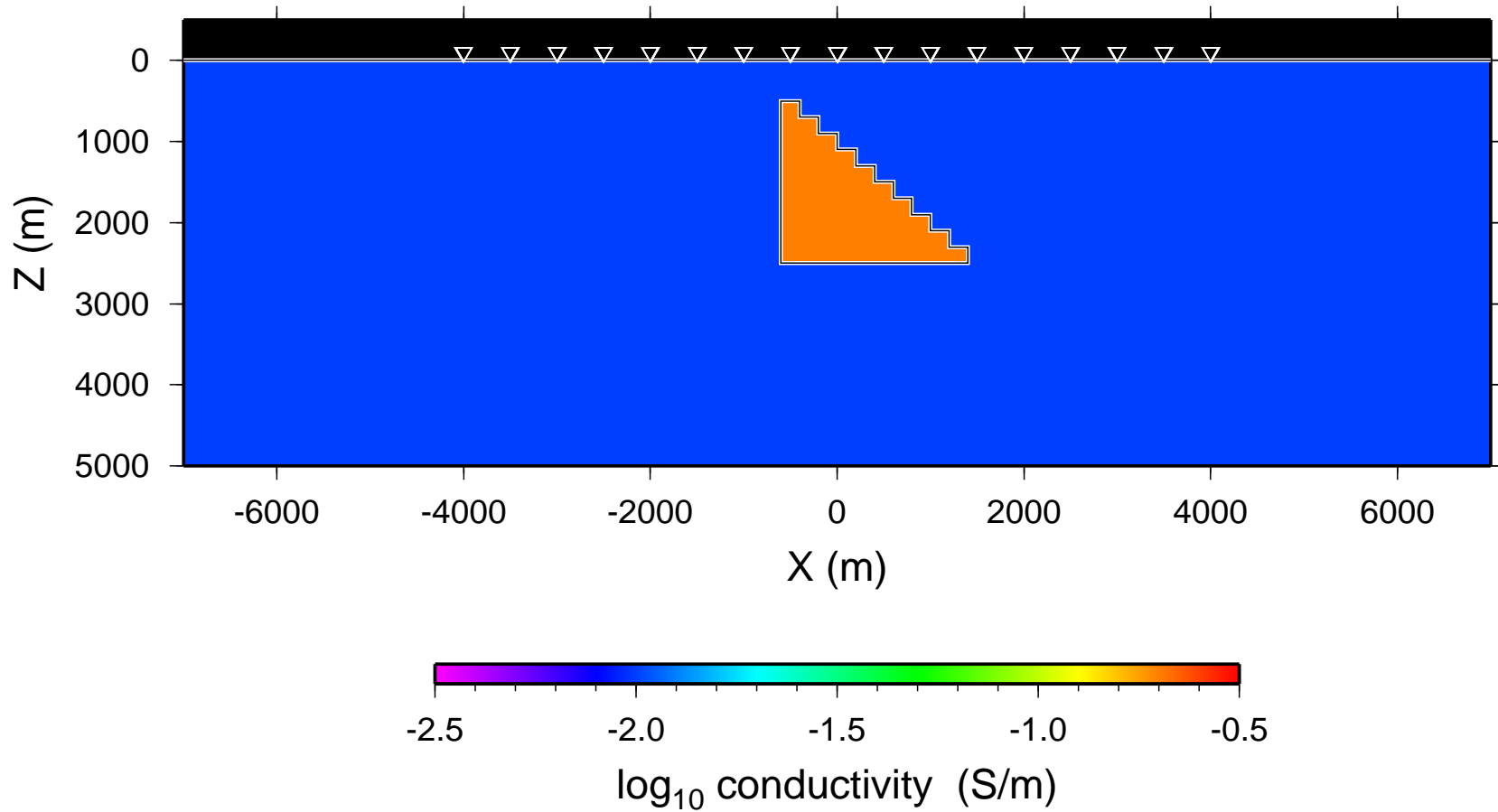
New way:

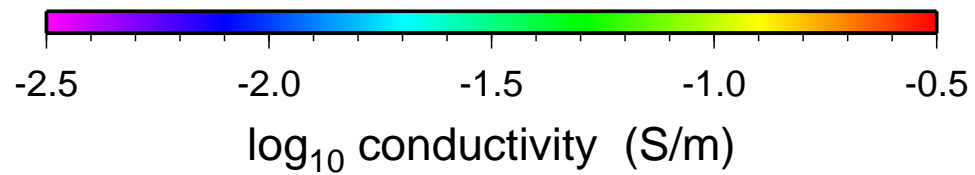
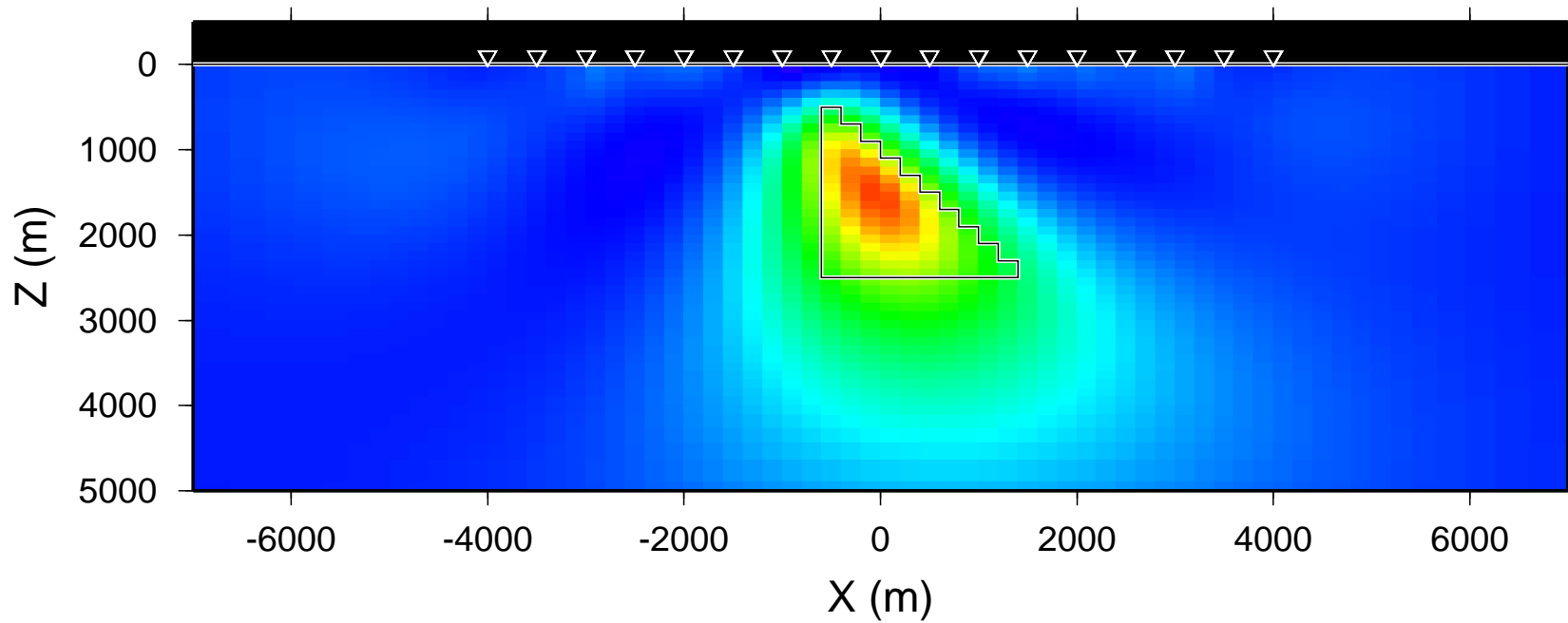


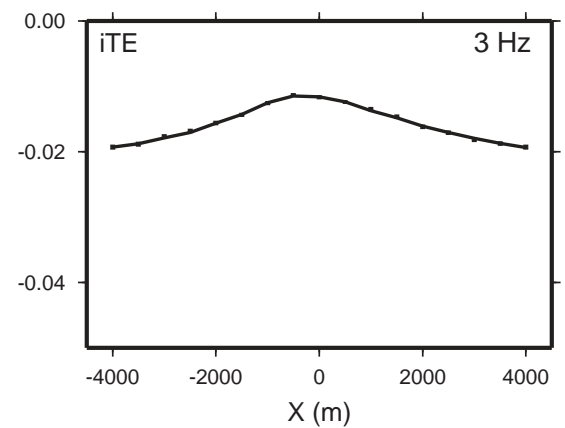
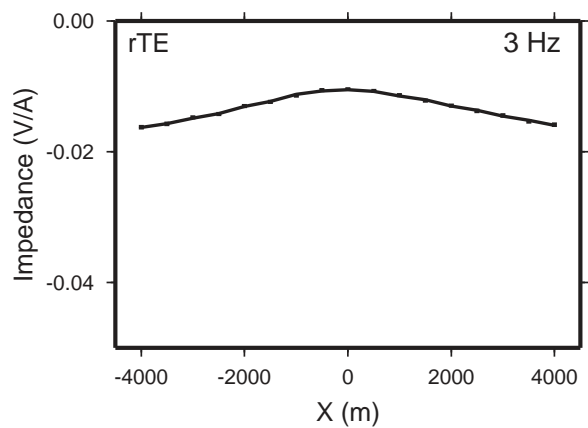
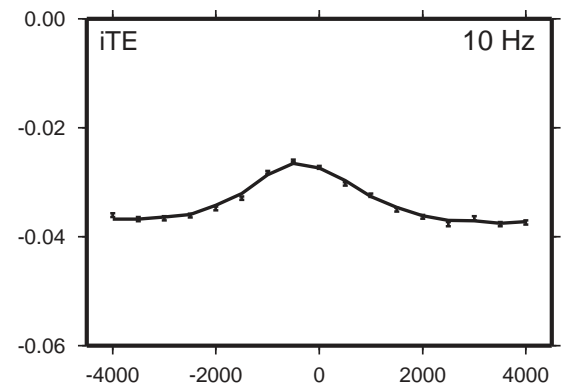
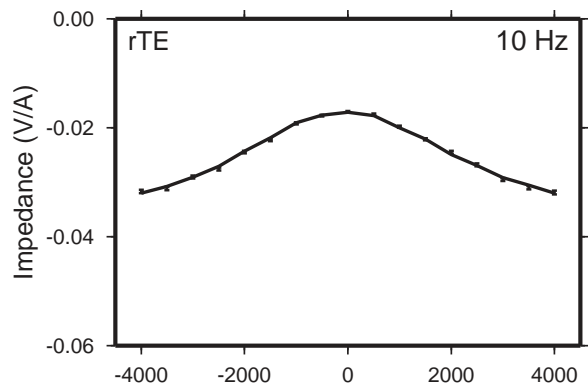
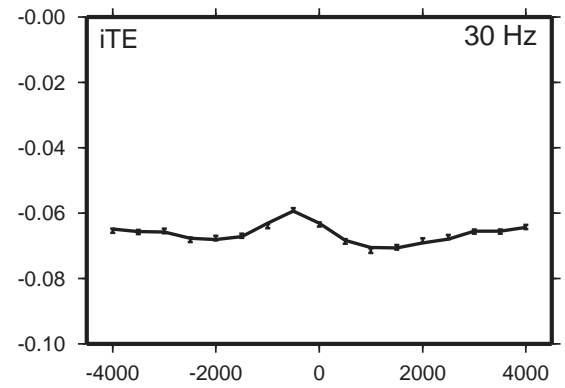
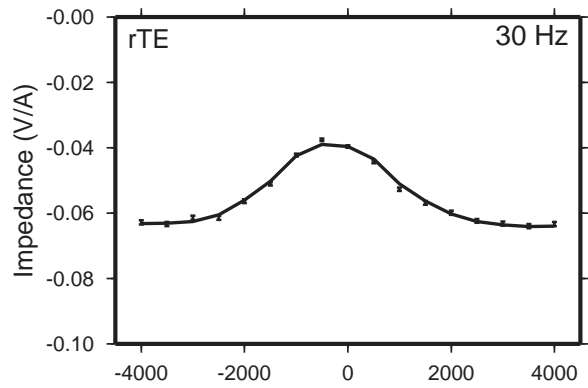
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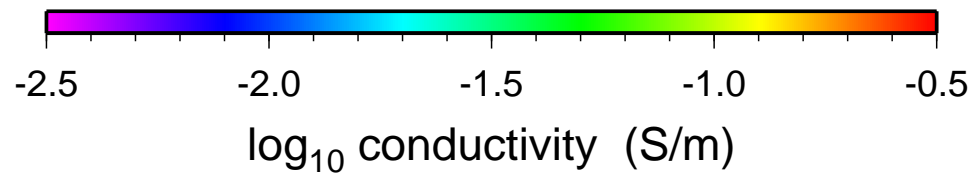
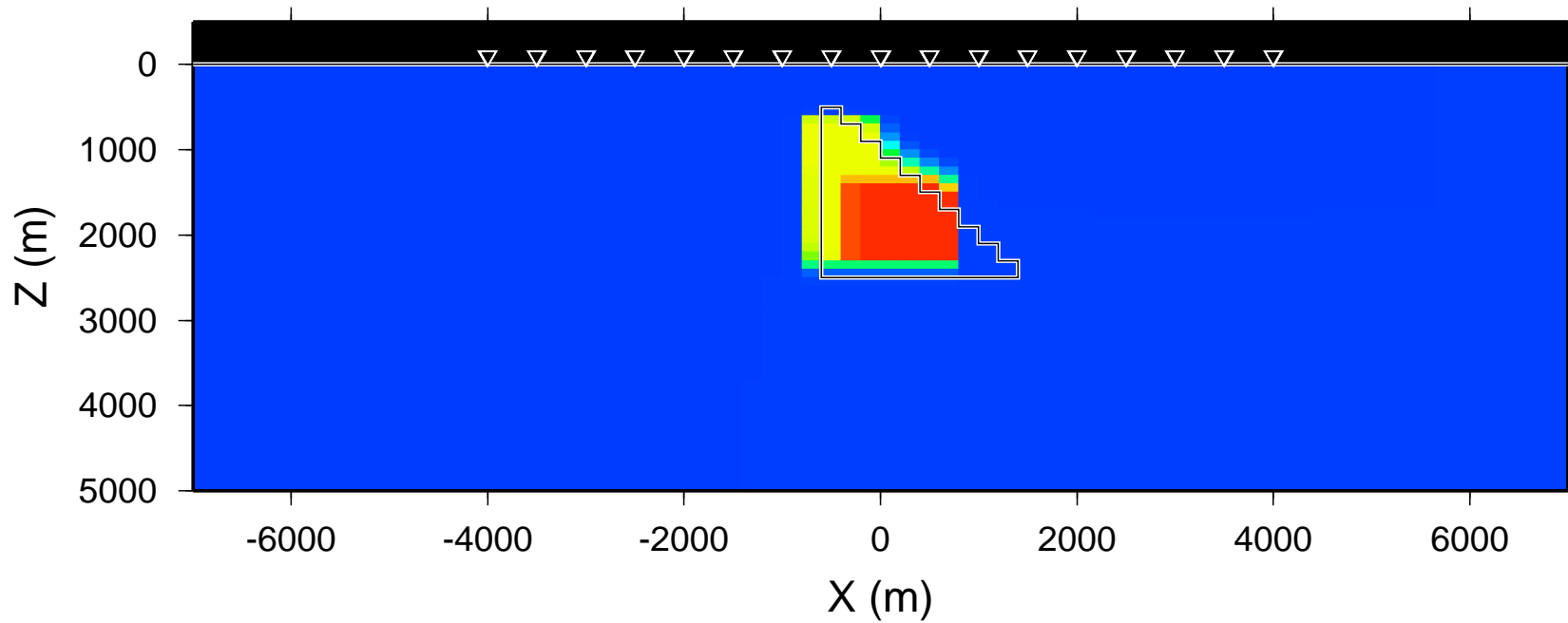
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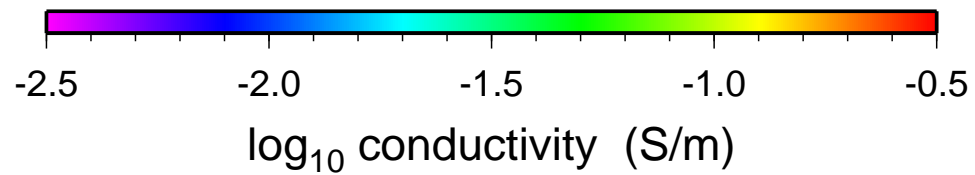
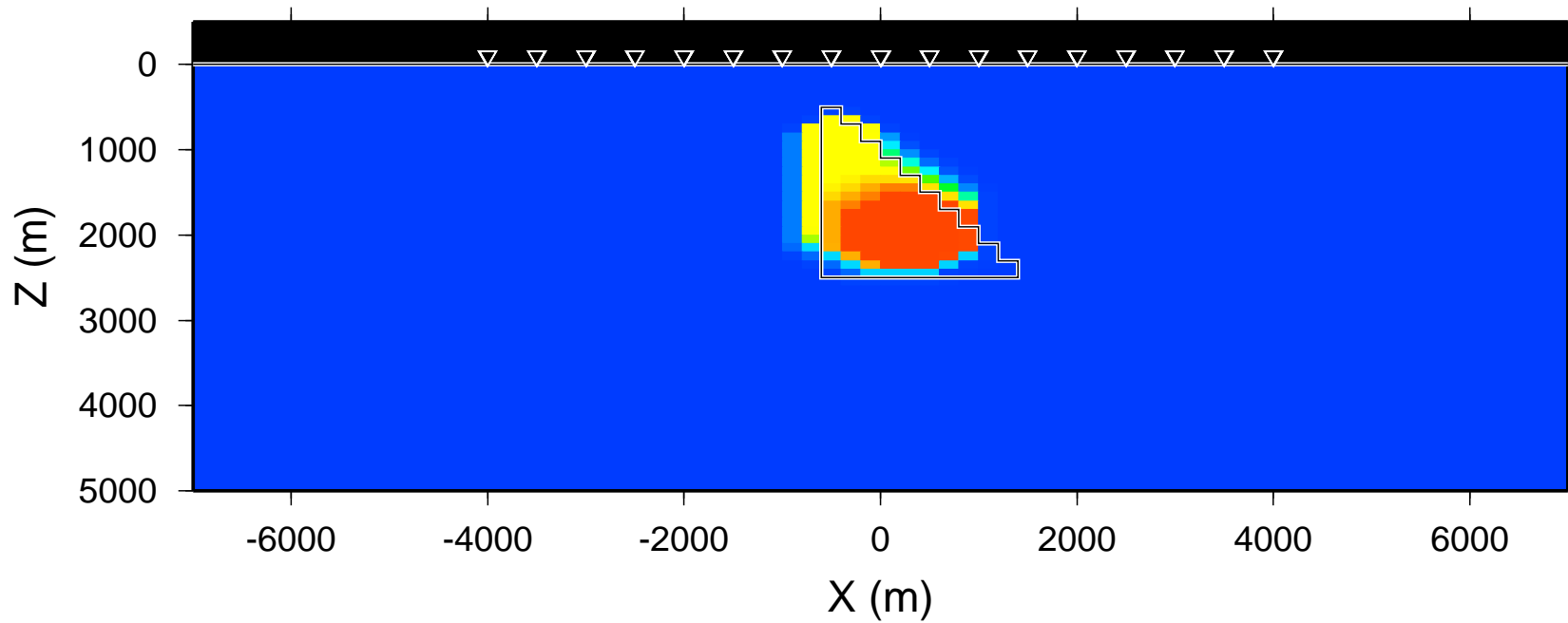
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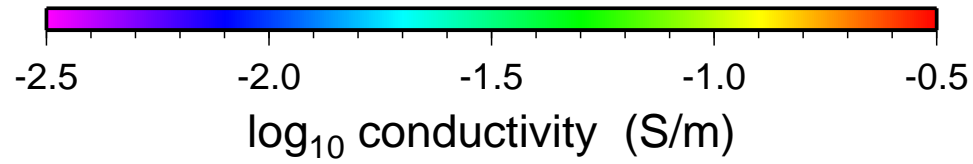
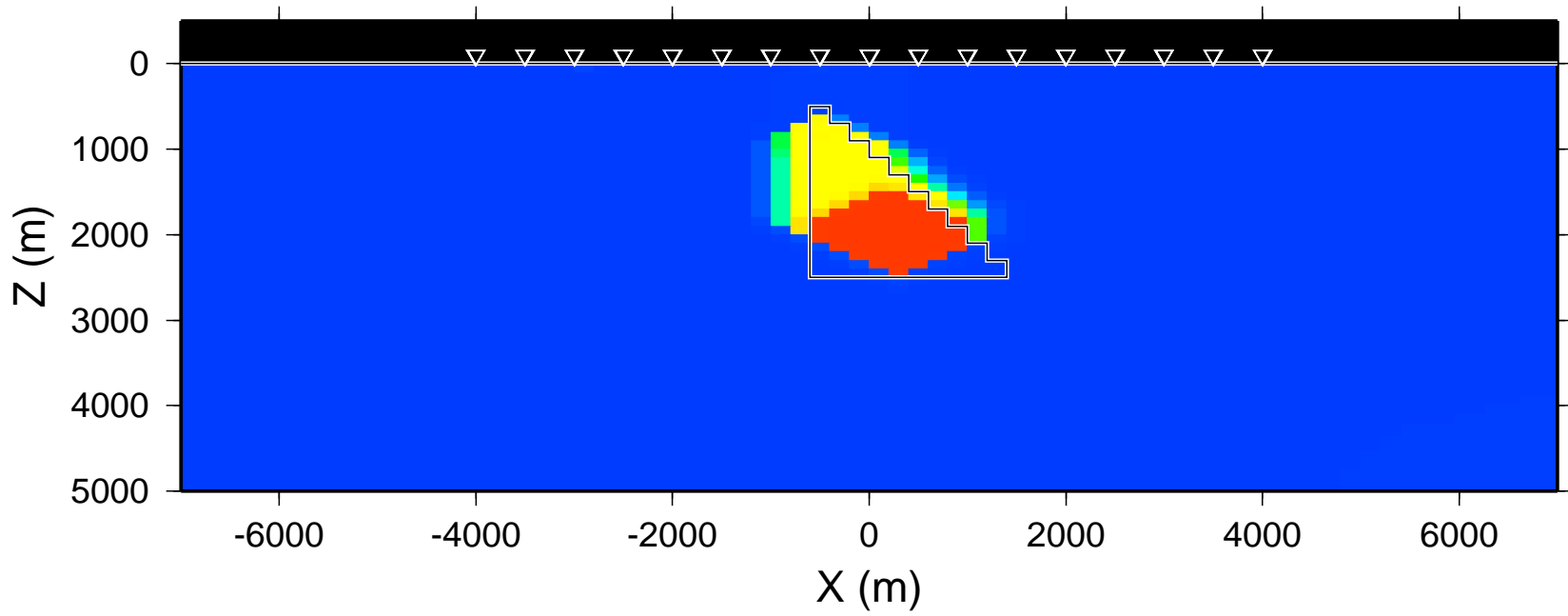












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Conclusions

- General measures and IRLS can be used to give blocky models in minimum-structure inversion algorithms.
- Diagonal differences in the regularization term give dipping interfaces.