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Constructing Summary Indices of Social Well-Being:
A Model for the Effect of Heterogeneous Importance Weights[1]

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Abstract

We consider problems associated with the construction of summary indices for a social unit (e.g., cities, states, nations). These problems are motivated by the question of how to construct a social well-being or Quality-of-Life (QOL) index that summarizes many social indicators, and that a majority of individuals can agree with. We specify a model for measuring the extent to which individuals with differing importance weights for the component indicators agree on a summary QOL index, and derive conditions under which an index will be endorsed by a majority of a social group. We show that, in every case, intuition greatly underestimates the extent of agreement among individuals who have different importance weights for the components. Two types of QOL indices are distinguished: (1) those rating multiple social units (e.g., cities, states, countries) in the same time period (cross-sectional data), and (2) those rating a single social unit on multiple time periods (time-series data). In the first case, we show that it is easy to create a QOL index on which most people in society agree. In the second case, we show that it is more difficult, but define conditions under which it is possible. In particular, we show that the equal-weighting strategy is privileged in that it minimizes disagreement among all possible individuals' weights. When the actual distribution of individuals' weights is known, one can improve agreement further by using the mean weights applied by individuals. Finally, we examine nationally representative surveys of importance weights and show that they meet the conditions for successful construction of a QOL index that will be endorsed by a majority of individuals in a country. We conclude with recommendations for measuring weights and creating QOL indices that have high levels of support among individual members of social units.

KeyWords: *Summary index construction; quality-of-life indices; heterogeneous importance weights*

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INTRODUCTION

Sociologists have constructed summary indices for the comparison of social units (e.g., cities, states, nations) with respect to multiple dimensions of social life at least since the mid-twentieth century work of Angell (1942, 1947, 1949, 1951, 1972) on the social and moral integration of American cities. The last decade has seen increased interest among sociologists and other social scientists in the construction of summary indices of social well-being, or as they have come to be termed, "Quality-of-Life" (QOL) indices.[2] This work coincides with a general interest in the subject among individuals, policy makers, and political leaders. For instance, the term "Quality of Life" has been invoked on the floor of the US Congress more than 20 times per week in recent years (Government Printing Office 1999), and the National Academies are working with Congress to develop key national indicators (Government Accounting Office 2003).

There is, however, little agreement among sociologists and other social scientists on methods for aggregating social indicators to create a QOL index that is useful for public discourse on social well-being and policy issues relevant thereto. Some researchers even argue that no summary index should ever be computed (Johansson 2002; Erickson 1993). They cite two important barriers to QOL indices. The first is that the concept of QOL is too general to be useful. Critics point to the problem that QOL is a composite indicator whose components (e.g., crime rate, GDP/capita, environmental damage) are not highly correlated, nor are their causes identical. Hence traditional factor analysis would recommend that these components be treated as separate factors.

While these diverse components probably should not be combined into a single first-order factor, it is possible that QOL could be considered a higher-order factor (a factor analysis of first-order factors). Such a higher-order QOL factor can be useful in considering how people make emigration decisions ("is the QOL of one state higher than my current residence?"), and in how people make political decisions ("Am I better off today than 4 years ago?"). Therefore, this paper addresses issues raised by the stream of research (Inkeles 1993; Land 2000; Hagerty, Cummins, Ferriss, Land, Michalos, Peterson, Sharpe, Sirgy, and Vogel 2001) that considers QOL to be a measurable and useful concept.

A second fundamental reason for questioning the usefulness of QOL is that individuals, policy makers, and researchers themselves disagree on the relative importance weights that each social indicator should have in a summary QOL index. Without agreement on the importance of each social indicator, chances for agreement on the overall QOL index would seem slim based on an intuitive analysis. However, formal analysis has not previously been done on this problem. The present paper fills this gap and proves results that are not predicted by intuition. In particular, the present paper (1) specifies a model for how individuals disagree with each other on QOL judgments, (2) predicts how much disagreement results from various types of QOL indices and various distributions of weights, and (3) recommends QOL weights that maximize agreement among individuals.

We hasten to note that much research on social well-being can be conducted without any overall index of QOL – by examining individual components of quality of life (e.g., public health, education, income, etc.). It is more parsimonious to avoid assuming any higher-level construct when interest is restricted to one component of QOL, or when all lower-level components agree.

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[2] For a review of recent developments in the field of social indicators, see Land (2000); for a review and evaluation of 22 QOL indices, see Hagerty, Cummins, Ferriss, Land, Michalos, Peterson, Sharpe, Sirgy, and Vogel (2001). For a statement on the uses of social indicators, see Ferriss (1988).

For excellent examples of such research, see Inkeles (1993) for the effect of modernization on QOL, Weede (1993) for the effect of democracy on QOL, and Stokes and Anderson (1990) for the effect of disarticulation on QOL. But when lower-level components of QOL *disagree* in sign, the inevitable question arises, "What is the *net effect* of these conflicting social indicators on individuals' QOL?" This query sometimes is posed more brutally by individuals and politicians as in the question, "Are we better off than x years ago?" QOL indices could provide powerful shorthand descriptions for overall trends in QOL, much as the Dow-Jones Industrial Average is a powerful summary of the performance of more than 5,000 stocks in the U.S.

Previous Research on QOL Indices

Inkeles' (1993) review of the QOL concept proposed a total of nine categories of goods and services (food; housing and amenities such as piped water and sewage; medicine and health; education; communications and information; time available, as for leisure; physical security of the person; social security of the person, and environmental concerns), and six categories of freedom (freedom of movement; of belief, as in religion or politics; of association; of political determination to chose political leaders; economic freedom to work and spend, and freedom from discrimination). From these categories, a myriad of QOL indices have been created.

Hagerty et al. (2001) reviewed 22 QOL indices that have been developed in recent years, many of which have received attention from media and government. *A surprising observation is that none of these indices explicitly considers how individuals themselves weight each component of QOL, nor do they deal with the likely variation in weights over people.* Instead, each index solves the problem of weighting without reference to peoples' actual importance weights. Since individuals are the final consumer and ultimate arbiter of their own sense of QOL, it seems crucial

to investigate the problem of how individuals themselves weight various social indicators to judge their QOL.

Two types of QOL indices can distinguished: (1) those rating multiple social units (e.g., cities, states, countries) in the same time period (cross-sectional data), and (2) those rating a single social unit on multiple time periods (time-series data). Examples of QOL indices that illustrate these two types are the Human Development Index (HDI) of the United Nations Development Program (2001) and the Miringoff and Miringoff (1999) Index of Social Health (ISH). Each will be described briefly here and then further analyzed later herein.

The Human Development Index. The Human Development Index (HDI) is an example of a QOL index that can be used to make cross-sectional comparisons among social units – in this case, nations. The HDI is published annually by the United Nations Human Development Program.[3] It is calculated from three social indicators: log (Gross Domestic Product) in Purchasing Power Parity, life expectancy in years, and education (a weighted average of literacy rate and school enrollment rate). These three indicators are first transformed so that their ranges are equal and then are averaged (with equal weights) to derive the HDI index. A HDI score is calculated for each nation for which data on these three indicators are available. Nations then are arrayed from the most to the least developed with respect to these indicators of human development. The annual United Nations Human Development Program reports do not justify why the indicators are weighted equally. How much would the HDI change if the weights change? Do individuals (or members of any social group) hold equal weights for those indicators? Do individuals hold such diverse weights that no index can capture the views of the group? Unless we know the answer to this, computing a summary index seems premature.

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[3] See also Hagerty et al. (2001) for a review and evaluation of the HDI.

The Index of Social Health. The Index of Social Health (ISH) was developed by Miringoff and Miringoff (1999). Using the United States as the social unit to be indexed, the ISH is based on 16 social indicators tracked annually from 1970 to the most recent year available: average weekly earnings, life expectancy at age 65, gap between rich and poor, violent crime rate, infant mortality, child abuse, children in poverty, teenage suicide, drug abuse, high-school dropout rate, teenage births, unemployment, health insurance coverage, poverty among those over 65, alcohol-related traffic fatalities, and housing affordability. Rates for these indicators for any specific year are indexed as percentages of their values for the year in which they had their "best practice" or best performance value. They are then averaged with equal weights to obtain the value of the ISH for a specific year.

Not only the HDI and the ISH, but also Diener's (1995) Value Based Index of National Quality of Life, Estes' (1984, 1988, 1997) Index of Social Progress, Johnston's (1988) Comprehensive Quality of Life Index, Land, Lamb, and Mustillo's Index of Child and Youth Well-Being (2001), Morris' (1979) Physical Quality of Life Index (PQOL), Veenhoven's (1996) Happy Life-Expectancy Index, and numerous others use equal-weighting schemes. Many of these indices apply equal weights without stating why, and none consider whether individuals themselves would weight these components equally.[4]

In contrast, some QOL indices use factor analysis to weight components. *Money* magazine's "Best Places to Live" uses a combination of factor analysis and surveys of readers' importance weights for 40 components of QOL. Unfortunately, Guterbock (1997) shows that the economic factors are greatly overvalued in their index because their survey includes *more items* related to economics, despite the fact that readers rated crime, environment, and health as more

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[4] To assure that no indicator will dominate, many of these indices adjust the range of each indicator by dividing by the standard deviation or the range of each.

important than economics. Another index that uses factor analysis is Estes' Index of Social

Progress (1988), which uses a two-stage varimax factor analysis to assign weights to 40 indicators.

The basic difficulty with using factor analysis is that the weights are derived to maximize the

variance explained in the social indicators, *without* any reference to individuals' weights. If items

are carefully sampled from individuals' and decision-makers' concerns, then this practice can be a

proxy for weights. But no QOL index specifically adopts this practice. Guterbock (1997, p.355)

concludes, "The relative weights given to economics and the other eight factors *should be made*

part of the research problem. They should not be decided in advance by editorial fiat or as an

inadvertent by-product of initial questionnaire design." In this paper we provide a framework to

jointly consider weights and social indicators as part of the research problem of constructing a QOL

index that will be approved both by individuals and by researchers.

The problem of weighting becomes even more pressing when comparing alternative indices,

because alternative indices often use quite different social indicators, and show different trends for

countries.[5] For example, the Miringoffs' ISH uses 16 indicators, none of which overlaps with the

HDI indicators. (The ISH does not include any indicator of GDP/capita, and uses different

indicators for education and life expectancy.) Though the HDI index concludes that QOL in the US

is increasing, the indicators from ISH show that QOL in the US is decreasing. Clearly, the choice

of weights for social indicators is crucial to its acceptance by individuals and policy makers.

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[5] The problem of selecting the component indicators that comprise a QOL index is a perennial one where ideal
procedures often crash against the reality of the data available for comparisons among social units in the cross-section
and/or over time. Indeed, the selection of component indicators all too often has been arbitrary and not justified on the
basis of a theoretical conceptualization and/or prior research evidence (see, e.g., Booysen 2002 for a discussion of this
problem in the context of composite indices of development such as the HDI). Since the pioneering work of Andrews
and Withey (1976) and Campbell, Converse, and Rodgers (1976) over 25 years ago, however, there has accumulated
numerous social psychological studies of the components and determinants of subjective well-being, life satisfaction,
and happiness. The results of reviews and syntheses of these various studies (e.g., Cummins 1996, 1997) now can be
used to inform the selection of the components of summary well-being indices (as, e.g., in the work of Land et al. 2001
on a summary index of child and youth well-being). In this way, while the constraints of available data always will
force compromises, the evidence from studies of what leads to individual subjective well-being, life satisfaction, and
happiness can be used as an empirical basis to guide the selection of component indicators.

We should point out that agreement with individuals' judgment is not the only criterion for a good QOL index. There are good arguments for considering some indicators that individuals do NOT consider, if sociologists identify them as leading indicators of QOL (social capital, deterioration of child-rearing practices). Other criteria for QOL indices are listed in Hagerty et al. (2001), and include reliability, concurrent validity, etc. But QOL ultimately must be assessed by individuals, to whom the United Nations Charter guarantees self-determination.

Arrow's Impossibility Theorem is Not Binding

Previous research by economists and philosophers has long considered the problem of creating a summary "social welfare index". Their research received a setback in the 1950's with Arrow's famous Impossibility Theorem (Arrow 1951), which states that no social welfare index can exist under certain minimal information conditions. When the utility of individuals can be measured only on an ordinal scale, and interpersonal utility comparisons (i.e., how much happier one person is than another) are not allowed, then a majority of individuals can *never* agree on a social welfare index. But the field has been reinvigorated by Amartya Sen's (1982, 1993) Nobel Prize-winning work. He showed that *individuals must make some interpersonal utility comparisons* to guarantee the existence of a social welfare function, and he proposes several minimal-information measures that individuals can agree on. In practical terms, all of the commonly used social indicators already fulfill these conditions for existence. For example, Gross Domestic Product per capita (GDP/capita) implicitly assumes that utility is measured in dollars of income, and that utility of each person can be summed to get social welfare. Many sociologists would disagree with this and want to supplement GDP/capita with, say, a Gini coefficient that assigns measures levels of income inequality. But both measures compare interpersonal utility sufficient to define a

social welfare index. Hence Arrow's impossibility theorem is irrelevant to practical social indicators.

In contrast to the work in economics, our paper is not concerned with minimal conditions for existence of QOL index. In fact, every social indicator that computes mean or variance over the population meets Sen's minimal criterion of interpersonal comparability. Instead, we focus on the problem that *individuals weight the information differently* to evaluate QOL. This is an area that has not been considered by social welfare economists, but is crucial when people's perceptions and importances vary.

In this paper, we examine how much indices vary in the presence of a distribution of different people's values. In the next section, we specify a model for how individuals disagree with each other on QOL judgments and develop a formula for computing the correlation between any two weighting systems. These results then are applied to existing QOL indices for the first time to assess agreement among them. We also examine empirical distributions of individuals' values in 48 different nations. A final section concludes with recommendations for constructing QOL indices.

A MODEL OF AGREEMENT BETWEEN TWO QOL INDICES WHEN WEIGHTS DIFFER

AMONG INDIVIDUALS

Define **X** as a matrix with K columns and N rows. The columns record the scores from K social indicators (e.g., GDP/capita, Gini coefficient of income inequality, divorce rate, etc.) on each of the N social units (e.g., cities, states, nations). Define $\mathbf{W_i}$ as the weighting (column) vector of individual i, measuring how important each social indicator is to that person. Then i's QOL judgment of social unit n is the sum of the K social indicators, weighted by person i's *importance weights for each indicator k* or, for short, *importances*:

$$Q_{in} = \sum_{K} w_{ik}\, x_{kn}, \qquad w_{ik} > 0, \text{ for } n = 1,\dots, N \text{ social units} \tag{1}$$

Though this model may appear restricted to linearity, it can also incorporate non-linear effects by

adding a new variable that is some function of the old indicator (e.g., log (GDP/capita)), as in the

HDI). The general additive model has been successful at approximating many more complex

functions, and Sastre (1999) reports that people use an additive model in direct tests of how people

judge others' quality of life.

We constrain each weight to be a non-negative number (that is, only positive or zero weights

are allowed). Hence we assume that any indicators that are negatively related to QOL (e.g., infant

mortality) are reversed in sign to allow positive importance weights. This assumption that everyone

has positive weights is probably not controversial for social indicators such as GDP/capita and

infant mortality, where everyone prefers more money and better health, given that all else is held

constant. But it may be controversial for indicators such as divorce rate, where some people may

view higher divorce rate as reflecting more freedom for women, but others view it as a decline in

support for children. In such a situation, one could add indicators for the omitted variables

(women's freedom, support for children) to assure that weights are positive for all individuals.[6]

Note that multiplying all weights by a constant c simply expands the QOL index by the

factor c and does not change the ordering of the social units being rated. Therefore, without loss of

generality, we divide each person i's weights by Σw_{ik} so that all k weights sum to one for each i.

Finally, the linear model in Eq. 1 should not be confused with the simple utilitarian model of

Bentham, where utilities of *individuals* are summed to get social welfare, ignoring inequality among

individuals. In contrast, Eq. 1 allows some of the indicators to be measures of the overall

stratification of income or wealth (positional information), as well as measures of individuals'

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[6] Additional observations on the implications of the positive weights assumption are given below in the Discussion and Conclusions section.

freedom (called non-utility information by Sen 1993). Hence the model in Eq. 1 offers great

flexibility in modeling individuals' evaluations of QOL.

As noted previously, QOL indices may be used either in cross-sectional or over-time

comparisons. The goal in cross-sectional comparisons is to evaluate a social unit relative to other

social units. This may, for example, allow people to decide in *which* nation they should live (e.g.,

the International Living Index) or which nation is in more need of development assistance (e.g.,

Estes' Index of Social Progress). In the case of over-time comparisons, QOL indices rate multiple

time periods in the *same* social unit (time-series), to decide whether QOL has increased or

decreased over time in that entity. The goals in the time-series case are to provide information for

informing individuals about QOL changes over time, to fuel a public policy debate, and to decide

whether policies are successfully improving QOL within a given country. It is desirable to find a

measure of agreement that will be useful in both of these cases.

For this purpose, we propose to use the familiar Pearson correlation coefficient to measure

levels of agreement between the QOL judgments of individuals i and j, denoted by A_{ij}. The

correlation coefficient has a number of desirable properties for measuring agreement. It has finite

limits between –1 (complete disagreement) and 1 (complete agreement), and its statistical properties

are well researched. It already has been widely used as a measure of inter-rater agreement, and as a

measure of similarity between persons in cluster analyses.[7] Another attractive property of the

Pearson correlation coefficient is that certain values represent important levels of agreement among

people.[8] The first is naturally $A_{ij} = 1$, where perfect agreement occurs between the QOL indices of i

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[7] Alternative measures of association are Spearman's rank correlation coefficient and Kendall's coefficient of concordance (Hollander and Wolfe 1973). Both of these are restricted to rank order properties and cannot easily incorporate importance weights as we do. In addition, while the qualitative properties of the model we specify below generally would apply to these alternative correlation coefficients, the algebra would become hopelessly complex and make difficult, if not impossible, the derivation of the results we obtain.

[8] The use of the Pearson coefficient to measure agreement between individuals' importance weights is appropriate even in the case that the weights are measured by the conventional 1 to 3 (or 5) rating scales of sample surveys – as long as

and j. The second is $A_{ij} = .7$, which is the common cutoff among researchers for assessing

agreement between raters. Agreement between raters is not expected to be perfect, but the .7 cutoff

implies that about 50% of the variance in one rater should be predictable from the other ($r^2 > .5$).

The third noteworthy level of A_{ij} is zero, because this is the point above which the QOL index of

individual i agrees *in direction* with that of j. To see this, take the limiting example of QOL

evaluations of year t and year t+1. Then $A_{ij} > 0$ implies that the raters agree on whether the nation's

QOL has *increased or decreased during that time*. This is a fundamental question that often helps

define similar political parties, social classes, and interest groups. In the next section, we use the

Pearson correlation coefficient to calculate the agreement between any two individuals whose

importance weights differ among social indicators.

Agreement Between QOL Indices When Importance Weights Differ

The weighted sum in Eq. 1 is more compactly designated in matrix notation as:

$$\mathbf{Q_i} = \mathbf{XW_i}, \tag{2}$$

where $\mathbf{Q_i}$ is a $N \times 1$ column vector of summary (or composite) index values (or scores) of individual

i for each of the n = 1, …, N social units, $\mathbf{X}$ is a $N \times K$ matrix of values of the K social indicators for

each of the N social units, and $\mathbf{W_i}$ is a $K \times 1$ column vector of weights of person i for the K social

indicators in $\mathbf{X}$. We assume, without loss of generality, that each social indicator in $\mathbf{X}$ has already

one seeks only to draw conclusions about the measurements (i.e., the 1-to-3 (or5) ratings) themselves (Sarle 1995). For example, if we want to test the hypothesis that the mean importance weights of two component indicators are equal, then we need not be further concerned about measurement models. If, however, we want to draw conclusions about the underlying latent dimension of importance of the component indicators to the individuals surveyed, then we either must use a measurement procedure for the importance weights (such as conjoint measurement; see Krantz, Luce, Suppes, and Tversky 1971) that gives interval-scale properties to the measured importance weights or use a measurement model such as a Rasch model (Arminger, Clogg, and Cheng 2000) that relates the measured weight scores to the latent dimension in a possibly nonlinear way and thus produces non-equal-intervals among the measured weights. To date, there are no studies of the relative importance of component indicators of QOL composite scores that use anything other than the standard rating scales of sample surveys. Accordingly, the model and analyses we present can be regarded as pertaining to the properties of these weights viewed as measurements themselves and as approximations to the

been standardized, so that the mean of each column of $\mathbf{X}$ is zero and standard deviation is one. The resulting composite scores $\mathbf{Q_i}$ will also have a zero mean, since the original indicators had zero means. But in general the composite scores will *not* have standard deviation of one. Our goal then is to find the *correlation A_{ij} between the QOL indices of individuals i and j, with different weight vectors*, $\mathbf{W_i}$ and $\mathbf{W_j}$.

By definition of the correlation coefficient,

$$A_{ij} = 1/(N-1) \ S_i \ \mathbf{Q_i^T} \ \mathbf{Q_j} \ S_j , \tag{3}$$

where N is the number of social units rated, S_i is the inverse of the standard deviation of the QOL index for person i (used to standardize the scores $\mathbf{Q_i}$), and $\mathbf{Q_i^T}$ denotes the matrix transpose of $\mathbf{Q_i}$. The term $1/(N-1) \ \mathbf{Q_i^T} \ \mathbf{Q_j}$ is the covariance of the QOL indices, which after standardization is the correlation coefficient between $\mathbf{Q_i}$ and $\mathbf{Q_j}$.

ANALYSIS OF THE MODEL

We now state several propositions that summarize properties of this model for measuring agreement between QOL indices for different individuals. When the propositions can be derived discursively and with no advanced mathematics, they are done so as part of this text. For Propositions 3 and 5, the mathematical arguments necessary to prove the propositions are more demanding and thus appear in the Appendix.

Equation 3 can be simplified by substituting definitions of $\mathbf{Q_i}$ from Eq. 2:

$$A_{ij} = 1/(N-1) \ S_i \ (\mathbf{XW_i})^\mathbf{T} \ \mathbf{XW_j} \ S_j \ = \ 1/(N-1) \ S_i \ \mathbf{W_i^T}(\mathbf{X^TX})\mathbf{W_j} \ S_j$$

$$= \ S_i \ \mathbf{W_i^T} \ \mathbf{R_x} \ \mathbf{W_j} \ S_j , \tag{4}$$

individuals' underlying latent dimension of importance of the component indicators that may not take into account possible nonlinear relationships to the underlying dimension.

where the prime denotes matrix transposition and $\mathbf{R_x}$ is the $K \times K$ matrix of correlations among the K social indicators. Next, we can expand on the definition of the inverse of the standard deviation for the S_i in Eq. 4 by use of Eq. 2:

$$S_i = [(1/(N-1)\,\Sigma_n Q_{in}^2)]^{-1/2} = (1/(N-1)\,(\mathbf{XW_i})^{\mathbf{T}}\,\mathbf{XW_i})^{-1/2} = (\mathbf{W_i^T R_x W_i})^{-1/2}. \quad (5)$$

Hence Eq. 3 can be rewritten as:

$$A_{ij} = \mathbf{W_i^* R_x W_j^*}\,, \quad (6)$$

where

$$\mathbf{W_i^*} = \mathbf{W_i} / (\mathbf{W_i'\, R_x\, W_i})^{1/2} \quad (7)$$

denotes the standardized weights vector. This shows that the correlation among summary indices i and j can be written as a function of the matrix of correlations among the original social indicators $\mathbf{R_x}$, and some normalized function $\mathbf{W_i^*}$ of the weights. Eq. 6 shows that $\mathbf{W_i^*}$ is proportional to the original weights $\mathbf{W_i}$, but adjusted by the squared weights and covariances to yield a standard deviation of one on the new QOL indices.[9] This gives rise to:

Proposition 1: The correlation A_{ij} between any two individuals' QOL indices is a function not only of the two individuals' weights, but also is moderated by the correlations among the social indicators R_x.

In fact, we will show that, even when two persons' weights are diametrically opposed, A_{ij} can be surprisingly high because $\mathbf{R_x}$ acts as a lower limit on agreement. We begin by examining the common situation where all correlations in $\mathbf{R_x}$ are positive. For the simplest two-indicator case, the matrix notation in Eq. 6 can be expanded to:

$$A_{ij} = w^*_{i1}(w^*_{j1} + w^*_{j2}\, r) + w^*_{i2}\,(w^*_{j1}\, r + w^*_{j2}) \quad (8)$$

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[9] The calculation of A_{ij} is analogous to a rotation of axes, where the original axes are not orthogonal. In the usual case where the original axes are orthogonal, the normalization is simply the sum of squares of the weights $\mathbf{W_i}$. But here because the original axes are not orthogonal, the cross products are not zero and must be included in the computation of $\mathbf{W_i^*}$.

All standardized weights w* are nonnegative, because the raw weights themselves are always nonnegative.[10] If the correlation r in Eq. 8 is also nonnegative, then all variables in Eq. 8 are greater than (or equal to) zero, requiring that Eq. 8 be greater than (or equal to) zero. More generally, for any number of social indicators, the matrix multiplication in Eq. 6 can always be expanded as sums and products of w^*_{ik} (always nonnegative) and r_{ij}. This leads to the important result:

Proposition 2: When all correlations among social indicators R_x are positive, then all individuals will agree on the direction of the QOL index ($A_{ij} > 0$), regardless of the distribution of weights.

This result will be useful in later examples, because many QOL indices have social indicators that are all positively correlated.

To generalize further, some elements in R_x may be negative, so that A_{ij} may be less than zero. How low (and how high) can agreement go, and under what conditions is agreement lowest (highest)? To answer these questions, we calculate both the minimum and the maximum A_{ij} :

Proposition 3: The maximum agreement A_{ij} is 1 and occurs when $W_i = W_j$, (when the individuals' weights agree). The minimum value of A_{ij} for two social indicators (K = 2) is r, and occurs when W_i is orthogonal to W_j, (i.e., $W_i = [1,0]^T$ and $W_j = [0,1]^T$ so that each individual places all their weight on different indicators). When K > 2, the upper bound on the minimum is r_{min} , the minimum correlation between the social indicators. (Proofs are shown in the Appendix.)

Proposition 3 confirms the common intuition that agreement is maximized when people have the *same* weights on social indicators, and agreement is minimized when people have *opposing* (orthogonal) weights on social indicators. But intuition does not reveal the magnitude of the minimum $A_{ij} = r_{min}$. Note that the minimum correlation is *not zero*, but may be higher or lower,

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[10] More formally, $W_i^{'}R_xW_i$ in Eq. (7) is always nonnegative because R_x is positive definite in a quadratic form.

depending on r_{min} . We show later that this is important in estimating agreement on many actual

QOL indices.

The next proposition considers not just the minimum and maximum for A_{ij} , but the entire

area where agreement is positive ($A_{ij} > 0$).

Proposition 4: When some correlations among social indicators R_x are negative, then some

persons in the group may disagree on the direction of the QOL index ($A_{ij} < 0$). But the area where

people agree appears to rise quickly toward 100% as r_{min} rises toward zero. Specifically for the

case of two social indicators (K = 2), even when r is extremely negative (r = -.9), over one half of

the area (59.8%) results in agreement on the direction of the QOL index.

To prove this proposition, one must first find the points where $A_{ij} = 0$. From Eq. 6, this is:

$$A_{ij} = W^*_i R_x W^*_j = 0 \quad \Rightarrow \quad W_i R_x W_j = 0 \tag{9}$$

The last equality is true because, by Eq. 7 of the main text, W^*_i is equal to W_i divided by a

constant. Hence multiplying both sides by the constant retains the equality. Solving Eq. 9 in the

general case is difficult, but the special case for K=2 will be informative. When K=2 component

indicators, then the fact that weights must sum to one implies that $w_{i2} = 1-w_{i1}$ and $w_{j2} = 1-w_{j1}$.

Making these substitutions in Eq. 9 yields:

$$w_{i1}(w_{j1} + (1-w_{j1}) r) + (1-w_{i1}) (w_{j1} r +(1-w_{j1})) = 0 \tag{10}$$

Then we can solve for w_{i1} in terms of w_{j1} and any r as:

$$w_{i1} = (w_{j1}(1-r) -1) / (2w_{j1}(1-r) + r -1) \tag{11}$$

This is a hyperbolic function in w_{j1}, and can be graphed for any choice of [W_{i1}, W_{j1}] and for any

value of r. Figure 1 graphs this function for all possible weights of person i (w_{i1}) and person j (w_{j1})

on the unit square, using the value of r = -.7 for demonstration. The center region of the square

(between the two hyperbolas) is the region where the two people agree on direction of the QOL

index ($A_{ij} > 0$), and the areas in the upper left and lower right are the regions where the two people

disagree. Note that the diagonal line from [0,0] to [1,1] always results in perfect agreement between people, because this represents the line where the two people agree on their weights. The region of agreement always spreads from this maximum on the diagonal line toward the minimum on the corners at [0,1] and [1,0]. Note also that the area in this graph where people agree is much larger than the area where people disagree. In this graph, the area where people agree corresponds to 74.6% of the total area (of all possible weights). The percent of area where people agree is a useful index, because when people are distributed uniformly on the unit square, it predicts the actual percent of people whose QOL indices will agree.

Insert Figure 1 About Here

Table 1 calculates the area where QOL indices agree in the case of two social indicators, as a function of r. The top row shows that when the correlation between social indicators is -.90 (very extreme), the proportion of the unit square where $A_{ij} > 0$ (minimal agreement exists on the direction of the index) is 59.8%. When the correlation between social indicators is -.80 the percentage agreeing is higher at 66.3%. One can see that the percentage area increases rather quickly, so that when the correlation is -.5, fully 84.6% of all possible weights result in agreement ($A_{ij} > 0$). If the distribution of weights in the population is *uniform*, then Table 1 also gives the proportion of people in the population who agree. *Note that even for extremely low r (-.9), a majority of people still agrees on the direction of the QOL index.*

Insert Table 1 About Here

We now consider whether researchers can construct a QOL index that will maximize agreement among individuals. Let $\mathbf{Z}$ be any vector of weights that is a linear function of individuals' weights $\mathbf{W_1}, \mathbf{W_2}, \mathbf{W_3}, \dots$. Then we prove (in the Appendix) that the choice of $\mathbf{Z}$ that maximizes agreement over all individuals is simply the mean weight vector across individuals, $\underline{\mathbf{W}}$.

Proposition 5: There is a unique weighting for any QOL index that maximizes the agreement A with the index over all individuals i in the population. This unique weighting for the QOL index is $\underline{\mathbf{W}}$, or the average weights (calculated over all individuals in the population).

Proposition 5 is helpful only if the distribution of weights is already known, as from a survey. If individuals' importances are not known, then what weights should be used to create the QOL index in order to reduce the risk of disagreement most? This question defines a mini-max estimator, which minimizes maximum possible disagreement. We can show that the *equal weighting* policy $\mathbf{W_E} = [1/K, 1/K, 1/K, \dots]^T$ is the mini-max estimator, and therefore reduces the risk of disagreement most when weights are not known.

Proposition 6: When individuals' weights are not known, then the unique weights $\mathbf{Z}$ that minimizes maximum possible disagreement over all possible distributions is equal weighting: $\mathbf{W_E} = [1/K, 1/K, 1/K, \dots]^T$.

The proof is shown in the Appendix. We note that equal-weighting is a privileged strategy because, besides being the minimax estimator, it also minimizes disagreement under a Bayesian prior distribution of uniform weights. (This can easily be seen from Proposition 5 under the prior assumption that weights are uniformly distributed.) Hence if no surveys have been done to estimate the importance that the population places on each attribute (a common occurrence), then equal-weighting is optimal under both minimax and uniform prior assumptions. If surveys have been done to develop a better posterior distribution of the true weights, then in general the mean weight vector $\underline{\mathbf{W}}$ in Proposition 5 will achieve better agreement for the QOL index than equal weighting.

The above propositions state generally how agreement A_{ij} varies when individuals apply different weights to social indicators. We now analyze three specific examples to show how these results apply to actual QOL indices. The examples will show that, for many indicators, a very large majority will agree on QOL judgments. However, in other situations a substantial possibility for disagreement on QOL indices exists.

APPLICATIONS TO QOL INDICES

Example 1: Human Development Indicators 2001. Earlier we described the HDI and its three component indicators. We also noted that the HDI weights these three indicators equally to derive the HDI index. How important is the equal-weighting assumption? How much disagreement would result if individuals apply different weights to the social indicators?

First the correlations among the individual social indicators R_{xx} must be computed. We computed these from the published data for the HDI 2001 for 162 nations, as shown in Table 2(a). The correlations are all significantly different from zero and are quite high. These high correlations are consistent with previous findings on cross-sectional social indicators at the nation-level (Morris 1979). What Morris did not comment on was that any resulting QOL index formed from these social indicators also would have high agreement among individuals.

To see this, we can use Proposition 3, which states that the minimum agreement will be r_{min} in Table 2a, or +.77. Thus, we have the surprising conclusion that even people with diametrically opposed weights would have QOL indices that have correlation $r_{ij} = +.77$. The intuitive reason for this is that the underlying social indicators are near substitutes for each other. *Hence even people who disagree on the ethically appropriate weights can still agree on their QOL indices for the specific countries and time periods in question.* The correlation of +.77 is within the common

findings for test-retest reliability of a single measure. Hence, for the HDI, even worst-case weights

will yield indices that are equivalent for most purposes.

Proposition 3 gives the minimum of A_{ij} for the HDI 2001. But it is important to find the

entire distribution of agreement among all pairs of individuals, to gauge overall agreement in the

population. To estimate the distribution, Proposition 1 states that we must know not only the

correlations among social indicators R_x, but also the distribution of individuals' weights. We

therefore examine several benchmark distributions of weights. The first is a uniform distribution,

which will be presented next. The second is an actual distribution of importance weights that are

drawn from a survey, which will be presented in a later section.

Insert Table 2 About Here

To specify a first reasonable benchmark for distribution of weights in the population, we

assume a uniform distribution, simulating 100 draws from a population whose importance weights

are uniformly distributed along the unit interval [0,1]. We used the method of Becker, Denby,

McGill, and Wilks (1987) to create random draws from this multivariate distribution known as the

Dirichlet distribution. The resulting distribution of A_{ij} over all 4950 possible pairs of the 100

individuals is shown in Figure 2. As predicted by Proposition 3, all correlations are positive,

despite the fact that some individuals had diametrically opposed weights. As predicted by

Proposition 3, the minimum A_{ij} in the simulation is .82, above the theoretical minimum of .77. In

fact, the distribution itself is more positive than predicted by the propositions, because it is skewed

toward the maximum of one. Despite the intuition that the distribution of correlations among pairs

of individuals on a QOL index composed from uniformly distributed weights might itself be

uniform, the actual distribution is heavily skewed toward the maximum of one. This is good news

for agreement among individuals. The average correlation A_{ij} in Fig. 2 among people was +.97,

with standard deviation of .028. Over 93% of all possible pairs had correlations above +.90. This is

far higher than many would expect, when weights are distributed uniformly.

Insert Figure 2 About Here

In summary, our analysis of the effects of various weighting schemes for the HDI 2001

shows that the vast majority of possible weights (98% of the total volume) result in correlations

between indices (A_{ij}) that are very high (greater than +.90). For the HDI 2001, different weights are

simply not an impediment to agreement on a QOL index.

Example 2: GDP per capita and Income Equality. One of the reasons that weights don't

matter in the HDI 2001 is that the underlying social indicators are highly correlated (e.g., the

correlation between GDP per capita and health was .82). This is reasonable because all of the social

indicators collected are meant to be measures of human development. When multiple measures of

the same underlying construct are used, then we would expect them to have high correlations with

each other. A more challenging example is the relationship between income equality and GDP per

capita. These concepts are clearly different, and theorists have argued whether the direction of the

relationship is positive or negative (see Firebaugh 1999 for a review).   The United Nations Human

Development Report (2001) reports, in a supplementary table, income equality measures for 111

nations – the largest number ever reported in a single source. We extracted the most common

measure of inequality, the Gini coefficient of income distribution. Since the Gini coefficient varies

from 0 (no inequality) to 1 (maximum inequality), we reversed its direction by using the

transformation (1-Gini). Hence all importance weights remain in the positive quadrant. The

correlations between Equality (1-Gini) and the three HDI indicators over the 111 nations are shown in Table 2(b).

Note first that the intercorrelations among the three HDI indicators for the 111 nations (Table 2(b)) are quite similar to the ones computed over all 162 nations (Table 2(a)), and all correlations in the table are significantly different from zero. Note also that the simple correlation between GDP/capita and income equality is +.4. This figure is lower than those in the HDI, but is significantly greater than zero, and is consistent with multivariate results. How do different weightings affect a QOL index that includes not only HDI but also Equality?

As in Example 1, a benchmark distribution of 100 random individuals with uniformly distributed weights was generated. The resulting distribution of A_{ij} of all 4950 possible pairs of the 100 individuals is shown in Figure 3. Again, as predicted by Proposition 2, all correlations are positive, despite the fact that some individuals had diametrically opposed weights. As predicted by Proposition 3, the minimum A_{ij} in the simulation is .40, equal to the theoretical minimum of .40. Again, the distribution itself is more positive than predicted by the propositions, because it is skewed toward the maximum of one. The average correlation A_{ij} in Fig. 3 among people is +.91, with standard deviation of .01. Over 94% of all possible pairs had correlations above +.70 (the usual criterion for assessing good inter-rater reliability).

Insert Figure 3 About Here

Proposition 6 predicts that we can generate even more agreement among individuals by constructing the equal-weights QOL index of [.25,.25,.25,.25]. The distribution of agreement between the equal-weight QOL index and the 100 simulated individuals is shown in Figure 4. As

predicted by Proposition 6, average agreement increases. What was not predicted was the skew

toward one, resulting in 93% of individuals with $A_{E,i} > .9$.

Example 3: The Index of Social Health. We previously described the ISH and its 16

component social indicators and we noted that the indicators pertain to data for the United States for

multiple years since 1970. To our knowledge, the correlations among these indicators have not

been published. Using the raw data from Miringoff and Miringoff (1999), correlations were

computed and are shown in Table 3 for the 16 indicators. Note that, contrary to the previous cross-

sectional examples, Table 3 displays many large negative correlations. For example, average

weekly earnings is correlated at -.921 with life expectancy at age 65.(while life expectancy has been

increasing over time, weekly earnings of hourly workers has been declining). These large negative

correlations provide the conditions for conflicting policy recommendations and for very low

agreement among individuals on the resulting QOL index.Proposition 3 predicts that the lowest

agreement among pairs of individuals will be r_{miin} from Table 3, or -.94 . How much agreement

would actually result from this QOL index with a population whose weights were uniformly

distributed? Using the technique in Example 1 to generate individuals with uniformly distributed

weights, we find levels of agreement that are surprisingly high. Average A_{ij} is +.40, with a standard

deviation of .45, but again the distribution is strongly skewed toward one, with fully 80% of the

4095 paired comparisons resulting in $A_{ij} > 0$, and 34% of paired comparisons with $A_{ij} > .7$. The

actual percentage of people who agree on the trend over time in the ISH ($A_{ij} > 0$) will depend on the

distribution of persons' weights in the unit square. In particular, if weights themselves are

negatively correlated, such that a person with a higher than average weight on W_1 has a lower than

average weight on W_2, then the distribution would tend to the upper left and lower right sections of

Figure 1, causing a decrease in the percentage of people agreeing on the index.

Insert Table 3 About Here

Proposition 6 predicts that the equal-weighting QOL should generate maximum agreement

among uniformly distributed individuals. The distribution of agreement between the equal-weight

QOL index and the 100 simulated individuals is shown in Figure 5. The mean $A_{E,i}$ is +.67 with

standard deviation of .39. What was not predicted was the skew toward one, resulting in 67% (a

super-majority) of individuals with $A_{E,i} > .7$, and 89% with $A_{E,i} > 0$.

Insert Figure 5 About Here

Summarizing the analyses of these three examples of QOL indices, two would experience

very high levels of agreement ($A_{ij} > .7$ for a large majority of pairs) when weights are distributed

randomly in the population. This is due to the consistently positive correlations among most social

indicators when *cross-sectional* data on countries is used to evaluate agreement. Morris (1979) first

pointed out that many social indicators are highly (and positively) correlated, and his finding is

reinforced here twenty years later with more countries. This result tends to argue for a "single

factor" explanation of modernization, a conclusion also shared by the consortium of sociologists

from the "Comparative Charting of Social Change" program (Langlois, Caplow, Mendras, and

Glatzer 1994) who consider many more social indicators than we do.

But Morris' conclusions referred only to social indicators in *cross-sectional* analyses, as in

the first two examples. In contrast, the third example is a *time-series* of social indicators on a single

nation. This type of data results in many negative correlations, which can result in much lower levels of citizen agreement (e.g., life expectancy above age 65 is negatively correlated with average weekly earnings in the U.S. since 1970). The reason for the conflicting findings from time-series versus cross-sectional studies may be due in part to "restriction of range" problems (e.g., life expectancy varied far less in the U.S. since 1970 than it does in a cross-sectional sample of nations, where Somalia has a life expectancy of only 40 years.) It may also be due to preferences of individual nations. For example, the U.S. seems to prefer higher GDP/capita at the expense of some loss in equality, compared to European nations. Such a policy could result in negative correlation between these indicators as inequality is pushed up in order to gain GDP/capita. Whatever their cause, negative correlations tend to work against citizen agreement on QOL indices. This fact is unfortunate, because national debates more often focus on time-series analyses ("Are you better off than 4 years ago?") than on cross-sectional analyses (Are we better off than Somalia?") Yet *even with large negative correlations from the third example*, a QOL index can be constructed for individuals with uniformly distributed weights that allows a super-majority of 67% to endorse it with $A_{E,i} > .7$, with only 11% of individuals with $A_{E,i} < 0$.

The analysis so far has imposed few restrictions on people's actual importance weights (simply that they are positive, and in some cases, that they are uniformly distributed. In the next section, we examine *actual* distributions of weights from individuals in the U.S., to test whether they conform to the conditions required for agreement on a QOL index.

APPLICATION TO SAMPLE SURVEY DATA ON INDIVIDUALS' IMPORTANCE WEIGHTS FOR

QOL COMPONENTS

The World Values Survey (WVS) (Inglehart 2000) asks respondents in 50 countries to rate the importance of: family, friends, leisure time, politics, work, and religion. The exact wording to

the questions in 1995 was "Please say, for each of the following, how important it is in your life. Would you say xxx is very important (3), rather important (2), not very important (1), or not at all important (0)?" The scale is usually assumed to be equal-interval, (hence the codes are equal-interval) and the anchoring at 'not at all important' may be assumed to represent a weight of near zero. Consistent with our model, no negative weights are allowed. Table 4 shows the distribution of the 6 importance scales and their intercorrelations for the U.S. sample in 1995. They represent 1502 US residents randomly selected and interviewed by telephone (39 of the original sample did not complete one or more of the ratings and were excluded). Note that the mean importance for family is highest, followed by friends, religion, work, leisure time, and politics. Inspection showed that all six distributions were single-peaked and not bi-modal. Further, correlations are all significantly positive (with the exception of leisure with religion) but were all less than .25.

Insert Table 4 About Here

The four social indicators from Example 2 were reevaluated using the surveyed importances from a random sample of 100 respondents from the WVS.[11] Agreement was much *higher* using actual weights from the WVS than from using a uniform distribution. The mean agreement A_{ij} among the 4950 pairs was +.99, with standard deviation of .001. Over 99% of all possible pairs had correlations above +.90. Similarly, the 16 social indicators from Example 3 were reevaluated using the surveyed importances of 100 randomly sampled WVS respondents.[12] Again, agreement was

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[11] The exact pairing for the analysis reported was: GDP/capita with the importance of work, life expectancy with importance of family, education with importance of leisure time, and Gini with importance of politics. This pairing is far from ideal, but surveys assessing importance of these 4 indicators do not exist. Sensitivity analysis showed that alternate pairing yielded very similar overall agreement.

[12] The 16 indicators were paired with the 6 importance ratings as follows: The indicators (1) wages, (5) housing affordability, (8) employment were assigned to work importance. The indicators (2) life expectancy, (6) Infant mortality, (14) birth rate to teenage mothers, (12) children below the poverty line, (15) child abuse and (16) % covered by health insurance were assigned to Family importance. Indicators (4) Gini, (7) poverty rate over 65 were assigned to Political importance. Ideally a survey should directly assess importance of each of the 16 indicators for each

much higher using actual weights. The mean agreement A_{ij} among the 4950 pairs was +.94, with standard deviation of .06. Over 82% of all possible pairs had correlations above +.90.

Why is agreement so high when using weights from the WVS? One important reason is that the distributions of weights were neither uniform nor bimodal, but all were strongly unimodal. Hence instead of a uniform spread across the entire response scale, most people clustered near a single point on the response scale. To take the most extreme example, 95.1% of respondents said that family is "very important". Even for the variable with highest standard deviation, 53.7% of respondents said that work is "very important", and only 16% responded in the lowest 2 categories. Inspection of the weights for the other 40 countries in the WVS showed similar distributions, and resulted in similar levels of agreement. Even when all countries were pooled, a random sample of 100 respondents yielded mean agreement A_{ij} among the 4950 pairs was +.88, with larger standard deviation of .11. Over 92% of all possible pairs had correlations above +.70.

Is this distribution of weights typical of weights collected from survey respondents? One defect in the WVS questions was that the sum of each person's weights was not forced to equal one. We therefore recalculated the distribution after dividing each person's weights by their sum, to force the weights to sum to one. The resulting correlation matrix then yielded many negative coefficients as expected, though they were not large (the most negative coefficient was -.33, between leisure and religion). However, the resulting mean agreement A_{ij} barely differed from the previous, at $A_{ij} = .99$, and over 99% of pairs showed agreement above +.90. We caution that these survey questions were not designed to measure the importance of the actual social indicators

respondent. But that survey does not exist.   Sensitivity analysis showed that overall agreement changed very little when alternate importances were assigned to the indicators.

GDP/capita, education, etc., and that a dedicated question would surely get better data. We suggest

such a survey in the conclusions.[13]

DISCUSSION AND CONCLUSIONS

Of the many QOL indices that have been proposed to date, none have explicitly considered

whether individuals would agree with their choice of indicators and weights. The present paper

proposes a simple model for predicting the extent of individuals' agreement on QOL judgments

with other individuals, and investigates whether it is possible to create a QOL index from real social

indicators that will be endorsed by a majority of individuals. In every case we examined, using

both real surveys of individuals' importance weights as well as a more general uniform distribution,

it was possible to create a QOL index that a majority of individuals endorse (i.e., they agree at least

with the direction of the QOL index). Specifically:

1. When correlations among social indicators are all positive (as in all cross-sectional data sets

   considered here), then agreement will be high regardless of the variation in weights. This

   highlights the paradoxical result that people may argue *in theory* about whose weights are

   more ethically appropriate, but in *practice* their conflicting weights will yield substantial

   agreement on the overall QOL index. This result is well known in regression analysis, but

   has not been observed in the context of social indicators.

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[13] Johansen (2002) argues that weights collected from many surveys are suspect because individuals have not devoted much thought to the tradeoffs, and require further education in the form of "town meetings" and education by experts. We therefore attempted to survey a population that has devoted their lives to education and to research on the tradeoffs and interactions among social indicators – sociologists themselves. A convenience sample of 26 professional sociologists at an international conference in Europe completed the same questions as in the WVS. The resulting distributions appeared similar to those of WVS respondents in that all distributions were unimodal, and correlations among weights were mild and close to zero. Actual weights from all 325 possible pairs of 26 sociologists yielded a mean r^2_{ij} of +.98 with a standard deviation of .02. Ninety-five percent of the correlations were above .94, and the minimum was .88.

2. When some correlations among social indicators are negative (as in time-series data sets where trends diverge for some indicators), intuition suggests and Johansen (2002) predicts that chances for agreement are slim. However, our results are the first to show that disagreement is much rarer than expected, and occurs only when the distribution of individuals' weights are (1) bimodal, and (2) negatively correlated (that is, when individuals' weights are diametrically opposed). These conditions did not occur in the surveys of real importance weights, nor in the more general uniform distributions, with the result that agreement on the QOL indices was much higher than expected from simple intuition or from previous work. The reason that the uniform distribution generates such high agreement is because it is *not* bimodal. It contains a broad "middle segment" in the center whose weights are near enough to each other to generate agreement at the average $\underline{W}$. The surveys of real weights are very strongly unimodal, and so generate even higher agreement. However, polarized issues such as abortion are more likely to show bimodal weight distributions, generating insufficient agreement for a majority to endorse.

3. We also have shown that researchers can increase the level of agreement for a QOL index by weighting the components appropriately. Agreement is maximized by using the average weights from a survey of individuals' importances. Alternatively, if no surveys exist, equal-weighting of indicators is the minimax estimator that minimizes disagreement even among diametrically opposed individuals. Note that in current practice, many QOL indices already use equal-weighting of indicators, though their authors admit that they do not know whether this weighting is correct. The current results can now place current practice on a sound theoretical footing, and show how it is possible to further increase agreement through surveys.

Implications for QOL Indices. Our results predict high agreement among QOL indices that are constructed according to the assumptions in Eq. 2. These assumptions are: (1) all individuals place positive weights on each attribute, and (2) all individuals use general additive models to judge QOL. With respect to the first assumption, many existing QOL indices already conform. For example, everyone prefers more longevity, higher income, and more education (all other things being equal) in the Human Development Index and hence the positivity requirement is met. Another conforming survey is Inglehart's (2000) longitudinal study of values because the *World Values Survey (WVS)* allows only non-negative weights.

However, there are indices that fail the positive weights assumption. For example, *Money* magazine's index of Best Places to Live includes an indicator "average price of a 3-bedroom home". Some people (homeowners) would place a high positive weight on this, but others (homebuyers) would place a high negative weight, violating Eq. 2. In fact, this is an example of a zero-sum negotiation game where every gain for a buyer is a loss for the seller, and joint gains are always zero regardless of the price. *Money* magazine probably included this indicator because their readers are primarily home buyers, but this indicator is not suitable for a QOL index because (1) QOL does not change with this indicator since the joint sum is always zero, and (2) sharp disagreement would result because Eq. 2 is violated. Negotiation researchers (Pruitt and Kim 2004; Carnevale and Pruitt 1992) recommend instead including indicators that allow positive joint gains to enhance the framing of shared interests. Much research has shown that this increases the likelihood of agreement and increases joint gains in negotiations. Applying these principles to the *Money* magazine example, a simple "laddering" procedure ("what deeper goals are you trying to achieve with lower housing prices/ higher housing prices?") could replace the single zero-sum attribute (price) with *two* shared goals: lower cost per square foot of new construction, and higher personal

income. Both of these new indicators would conform to our assumptions and would result in higher likelihood of agreement.

This example points out that not all social indicators are appropriate in QOL indices, and inclusion should be contingent on each indicator's (1) reliability, (2) perceived importance by citizens, and (3) likelihood of agreement on the resulting QOL index, as derived here. Another important example of indicators to exclude from QOL indices are tax policy, because conservatives place a negative weight on average tax burden, and liberals tend to place a positive weight. Tax policy is better viewed as a means to an end, and a successful QOL index would again apply laddering to include the end-state variables (e.g., better health care, education, pollution control, and economic growth. These examples show that a QOL index would *not* remove the need for policy analysis and political discussion, but it would *focus* policy analysis and politics by forcing proponents to estimate each policy's results on the QOL index.

The second assumption from Eq. 2 is that individuals use a simple additive model to form judgments about QOL. While this model is confirmed by Sastre's (1999) study of how individuals evaluate well-being and by Srinivasan and Park's (1997) results predicting product preferences, it needs more empirical research. In particular, substitutability or complementarity may exist between social indicators that would require modeling *interactions* among indicators. For example, an individual with higher average income may consider life expectancy more important than an individual with very low income (as life becomes more "worth living," longer life may be more valuable). Such complementarity could be added to Eq. 2 by constructing an interaction term, though its importance weight would be more difficult to measure in surveys. Empirical tests for these interactions could be done by surveying individuals and determining their preferences for hypothetical "bundles" of social indicators for their social unit. To our knowledge, no such studies have been done for representative samples of any social units. Such work would be invaluable for

constructing a QOL index that correctly mirrors the preferences of the social unit.[14] The methods

we outline here also allow deeper analysis of the more than 20 QOL indices that have been

proposed. None of them agrees perfectly with each other, and some disagree even in direction with

others. Our analysis in Eq. 6 now allows researchers to "decompose" the sources of disagreement

into those due to selection of different (though correlated) indicators (R_x), those due to use of

different weights to construct the indicator W_z, and those due to different importance weights in the

target population W_i.

Our conclusions must be viewed with caution for several reasons. First, we made use of

existing surveys of individuals' weights that were not specifically designed to measure weights for

the QOL indexes reviewed here. Most importantly, the weights in Eq. 2 must be correct to a ratio

scale (because the zero point is meaningful) whereas the Likert scales in the WVS are often

considered correct only to an interval scale. However, the particular anchoring in WVS ("not at all

important" = 0) appears to assign the appropriate response to the zero point, and validation studies

of Eq. 2 in product-choice surveys (Srinivasan and Park 1997) show that this type of scale predicts

preferences quite successfully. Another limitation of the WVS survey is that it contained only 6

general importance weights (family, work, etc.) measured on a scale with only four points.

However, the finer gradations available with a 10-point scale are unlikely to change our results. We

show that agreement is most likely when: (1) weight distributions are all unimodal rather than

uniform or bimodal, (2) correlations are mild and positive, and (3) few people use the zero point of

the scale. All three of these conditions are true in the surveys we examined, and it seems unlikely

that an expanded rating scale or a different zero point would change these properties.

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[14] *Money* magazine now surveys a representative sample in the U.S. for ratings of importance for various indicators in their "Best Places to Live" index. However, they perform no tests for possible interactions and omit many indicators altogether.

Throughout this paper we have assumed that individuals are members of a political state, but our results can be directly generalized to expert committees, such as a task force of analysts attempting to agree on the effectiveness of a mix of government policies. Then the prospects for finding agreement on the correct mix of policies are given by the above propositions (assuming that the committee members reveal in good faith their beliefs about the effectiveness of each policy).

Finally, we note that future sociological research will help to reduce heterogeneity to some extent by clarifying the effects of each social indicator on QOL. For example, future research on the effects of education will estimate its effects on earnings and job satisfaction, and ultimately its higher-order effect on QOL. This could reduce heterogeneity due to differing *beliefs* on effectiveness. But research can never eliminate heterogeneity due to differing *endowments*. For example, even if research identifies the *average* effect of education on QOL, each individual will deviate from that average in a random-effects model (in this case, due to their childhood endowments, achieved education levels, etc.). Hence the common call for "doing more research" is not likely to eliminate all heterogeneity from individuals' judgments of QOL. Heterogeneity will not go away, and thus its effects must continue to be studied.

Researchers have debated the appropriateness of forming summary indices of social well-being for years. But they have investigated only extreme cases that predict high levels of disagreement among individuals with differing weights. In contrast, we examine the entire range of possible conditions and then study the resulting agreement among individuals for several real social indices. In every case, substantial *agreement* exists and is much higher than expected by intuition. In every case, a QOL index could be constructed that a large majority of individuals would endorse, because they would agree when the QOL index is rising and when it is declining – of prime importance for policy makers.

The following are proofs of propositions not given in the main text.

Proof of Proposition 3

      To prove the location of the maximum and minimum, one can compute the derivatives of A_{ij} with respect to weights $\mathbf{W_i}$ and $\mathbf{W_j}$. Taking the derivative of Eq. 6 in the main text subject to the constraint that the weights are standardized to one $(\mathbf{W_i}^T \mathbf{R_x} \mathbf{W_i})^{1/2} = 1$) gives the set of Lagrangian equations:

$$dA/d\mathbf{W_i} = \mathbf{RW_j} - 2\lambda_1 \mathbf{RW_i} = 0$$

$$dA/d\mathbf{W_j} = (\mathbf{W_i}^T\mathbf{R})^T - 2\lambda_2 \mathbf{RW_j} = 0$$

$$dA/d\lambda_1 = \mathbf{W_i}^T\mathbf{RW_i} - 1 = 0$$

$$dA/d\lambda_2 = \mathbf{W_j}^T\mathbf{RW_j} - 1 = 0 \qquad\qquad\qquad (A1)$$

Combining the first two equations in Eq. A1 gives the condition for the optimum as: $\mathbf{W_i} = 2\lambda_2\mathbf{W_j}$, or that $\mathbf{W_i}$ must be proportional to $\mathbf{W_j}$. The third and fourth equations require that both $\mathbf{W_i}$ and $\mathbf{W_j}$ be standardized to unit length. Hence $\mathbf{W_i}$ must not only be proportional to $\mathbf{W_j}$, but must be *equal* so that $\lambda_2 = 1/2$. Substituting $\mathbf{W_i} = \mathbf{W_j}$ in Eq. 6 of the main text shows that this point is a maximum and that $A_{ij} = 1$ there. This completes the proof for the maximum.

      Since the only interior optimum in Eq. 6 is a maximum, then the minimum A_{ij} must be found at the extreme points of the constrained function. In general the extreme points lie along all edges of the unit hypercube $[w_1,w_2,w_3,\ldots]$ such that all $w_i > 0$ and $\Sigma_i w_i = 1$. To search all of these points would require an extensive program with non-linear constraints. However, the minimum for the special case when $K = 2$ is easy to calculate, and it provides a good approximation to the minimum agreement for $K > 2$ in later examples.

The extreme points for the case when K = 2 are just: $\mathbf{W_i} = \{0,1\}$, $\mathbf{W_j} = \{0,1\}$. Evaluating A_{ij} at each of these points reveals a minimum at $\mathbf{W_i} = [1,0]^T$ and $\mathbf{W_j} = [0,1]^T$, with $A_{ij} = r$ at that point. Generalizing to K > 2, boundary conditions will always occur at the vertices of the unit hypercube where $\mathbf{W_i}$ places all weight on one social indicator (which we label indicator m), and $\mathbf{W_j}$ places all weight on a different social indicator (which we label n), such that $\mathbf{W_i}$ is orthogonal to $\mathbf{W_j}$. Evaluating A_{ij} at the vertex where $\mathbf{W_i}$ places all weight on the indicator m and $\mathbf{W_j}$ places all weight on indicator n yields $A_{ij} = r_{mn}$. The minimum of these vertices is simply r_{min} , the minimum correlation among social indicators. We caution that r_{min} is only an upper bound on the global minimum, because even smaller values of A_{ij} might be found by evaluating the edges rather than just the vertices of the unit hypercube, where individuals may place non-zero weights on several social indicators. But analysis of the examples later shows that r_{min} is a good approximation to the overall minimum A_{ij}.

Proof of Proposition 5

Compute the sum of squared errors between $\mathbf{Q_z}$ (the QOL scores of an arbitrary weighting vector $\mathbf{Z}$) and $\mathbf{Q_i}$ over all I individuals:

$$\text{SSE} = \Sigma_i \ (\mathbf{Q_z} - \mathbf{Q_i})^T(\mathbf{Q_z} - \mathbf{Q_i}) \tag{A3}$$

From Eq. 2 of the main text, this expands to:

$$\text{SSE} = \Sigma_i (\mathbf{XZ} - \mathbf{XW_i})^T (\mathbf{XZ} - \mathbf{XW_i}) \tag{A4}$$

which by transposing simplifies to:

$$\text{SSE} = \Sigma_i ((\mathbf{Z}\text{-}\mathbf{W_i})^T\mathbf{X}^T) \ \mathbf{X}(\mathbf{Z}\text{-}\mathbf{W_i}) \ = \ \Sigma_i (\mathbf{Z}\text{-}\mathbf{W_i})^T\mathbf{R_x} (\mathbf{Z}\text{-}\mathbf{W_i}) \tag{A5}$$

Note that Eq.A5 is a quadratic form with $\mathbf{R_x}$ symmetric. The minimum SSE can then be found as the point at which the derivative of SSE with respect to Z is zero:

$$\text{dSSE} / \text{d}\mathbf{Z} \ = \ 0 \ = \Sigma_i 2\mathbf{R_x} (\mathbf{Z}\text{-}\mathbf{W_i}) \tag{A6}$$

Since both sides can be multiplied by $(2\mathbf{R_x})^{-1}$ with no change, this simplifies to:

$$0 = \Sigma_i \, (\mathbf{Z}\text{-}\mathbf{W_i}) \quad \Rightarrow \quad \mathbf{Z} = \Sigma_i \, \mathbf{W_i} \, /\mathrm{I} \; = \; \underline{\mathbf{W}} \; . \tag{A7}$$

This proves that $\underline{\mathbf{W}}$ is the unique weighting vector for a QOL index that minimizes the sum of squared errors $\mathrm{SSE_{Zi}}$ between $\mathbf{Q_Z}$ and $\mathbf{Q_i}$, where i ranges from 1 to I across all individuals in the population.   Following the usual results from regression that minimizing $\mathrm{SSE_{Zi}}$ is equivalent to maximizing the correlation coefficient $\mathrm{A_{Zi}}$, this completes the proof.

Proof of Proposition 6

To obtain the minimax estimator, find the estimator Z that minimizes the maximum disagreement among all possible W's:

$$\begin{array}{lll} \text{Min} & \text{Max} & \Sigma \, (\mathrm{Z}\text{-}\mathrm{W})^{\mathrm{T}} \, \mathrm{R_x} \, (\mathrm{Z}\text{-}\mathrm{W}) \\ \mathrm{Z} & 0 \leq \mathrm{W} \leq 1 & i \end{array} \tag{A8}$$

From Proposition 3, the inner maximization (maximum disagreement) occurs when individuals' weights are diametrically opposed. That is, when the weights lie at the vertices of the unit hypercube where $\mathbf{W_i}$ places all weight on one social indicator (which we label indicator m), and $\mathbf{W_j}$ places all weight on a different social indicator (which we label n), such that $\mathbf{W_i}$ is orthogonal to $\mathbf{W_j}$. For an arbitrary number of individuals I, maximum disagreement occurs when each group gives all their weight to one of the K indicators and ignores all others:

$$\mathbf{W_i} \; = [1,0,0,\ldots 0]^{\mathrm{T}} \text{ for } [1 \leq i \leq \mathrm{I/K}] \tag{A9}$$

$$= [0,1,0,0,\ldots 0]^{\mathrm{T}} \text{ for } [\mathrm{I/K}+1 \leq i \leq 2\mathrm{I/K}] \; \ldots$$

$$=[0,0,\ldots 0,1]^{\mathrm{T}} \text{ for } [\mathrm{I(K\text{-}1)/K}+1 \leq i \leq \mathrm{I}]$$

Now examining the outer minimization in (A7), Proposition 5 shows that this minimum exists for

any $\mathbf{W_i}$ and the weights $\mathbf{Z}$ that achieve this minimization are just: $\mathbf{Z} = \Sigma_i \mathbf{W_i}/I$. Substituting (A9)

into this yields the minimax estimator:

$\mathbf{Z} = \Sigma_i \mathbf{W_i}/I = [I/K/I, I/K/I, \ldots I/K/I] = [1/K, 1/K, 1/K, 1/K\ldots]$. . This is simply the equal-

weighting policy.

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Table 1. Percent of all possible weights for two social indicators where two individuals agree

on the index ($A_{ij} > 0$), as a function of correlation R_x between the social indicators.

R_x	% where $A_{ij}>0$
-.9	59.8
-.8	66.3
-.7	74.6
-.6	78.5
-.5	84.6
-.4	88.7
-.3	94.8
-.2	96.0
-.1	99.5
>0.	100.

Table 2. Correlations among social indicators from (a) the HDI 2001 and (b) the HDI and Gini Coefficient.

(a)

	(1)	(2)	(3)
(1)log(GDP/capita)	1		
(2)Life Expectancy	.82	1	
(3)Education	.77	.79	1

(b)

	(1)	(2)	(3)	(4)
(1)log(GDP/capita)	1			
(2)Life Expectancy	.85	1		
(3)Education	.80	.82	1	
(4)1-Gini Index	.40	.40	.30	1

Table 3. Correlations among 16 social indicators of the ISH.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(1) Wages (average weekly earnings)	1.00	-0.85	-0.79	0.83	-0.27	-0.94	-0.59	-0.04	-0.81	-0.11	0.88	0.85	0.46	0.66	0.89	0.85
(2) Life Expectation at age 65	-0.85	1.00	0.83	-0.89	0.61	0.93	0.72	0.27	0.92	0.43	-0.91	-0.66	-0.67	-0.83	-0.94	-0.89
(3) (1-% Dropouts from high school	-0.79	0.83	1.00	-0.87	0.64	0.89	0.75	0.24	0.88	0.31	-0.82	-0.67	-0.72	-0.65	-0.93	-0.91
(4) (1-Gini)	0.83	-0.89	-0.87	1.00	-0.68	-0.94	-0.82	-0.28	-0.82	-0.48	0.80	0.69	0.74	0.64	0.94	0.94
(5) Housing Affordability Index	-0.27	0.61	0.64	-0.68	1.00	0.53	0.69	0.59	0.65	0.72	-0.53	-0.06	-0.84	-0.57	-0.65	-0.57
(6) (1-Infant Mortality rate)	-0.94	0.93	0.89	-0.94	0.53	1.00	0.80	0.23	0.87	0.30	-0.87	-0.77	-0.62	-0.67	-0.98	-0.95
(7) (1-% in poverty over 65 yrs.)	-0.59	0.72	0.75	-0.82	0.69	0.80	1.00	0.47	0.73	0.34	-0.56	-0.46	-0.68	-0.43	-0.84	-0.83
(8) (1-Unemployment %)	-0.04	0.27	0.24	-0.28	0.59	0.23	0.47	1.00	0.29	0.36	-0.31	0.40	-0.55	-0.29	-0.34	-0.12
(9) (1-Drug use rate of 12th Graders)	-0.81	0.92	0.88	-0.82	0.65	0.87	0.73	0.29	1.00	0.31	-0.91	-0.63	-0.69	-0.84	-0.93	-0.84
(10) (1-Traffic Fatalities from Alcohol)	-0.11	0.43	0.31	-0.48	0.72	0.30	0.34	0.36	0.31	1.00	-0.31	0.06	-0.61	-0.54	-0.35	-0.37
(11) (1-Violent Crime Rate)	0.88	-0.91	-0.82	0.80	-0.53	-0.87	-0.56	-0.31	-0.91	-0.31	1.00	0.59	0.64	0.88	0.89	0.76
(12)(1-% Children below Poverty Line)	0.85	-0.66	-0.67	0.69	-0.06	-0.77	-0.46	0.40	-0.63	0.06	0.59	1.00	0.20	0.40	0.69	0.80
(13) (1-Suicide Rate among 15-24 yrs.)	0.46	-0.67	-0.72	0.74	-0.84	-0.62	-0.68	-0.55	-0.69	-0.61	0.64	0.20	1.00	0.66	0.73	0.63
(14) (1-Birth rate to teenage mothers)	0.66	-0.83	-0.65	0.64	-0.57	-0.67	-0.43	-0.29	-0.84	-0.54	0.88	0.40	0.66	1.00	0.72	0.60
(15) (1-Child Abuse report rate)	0.89	-0.94	-0.93	0.94	-0.65	-0.98	-0.84	-0.34	-0.93	-0.35	0.89	0.69	0.73	0.72	1.00	0.94
(16) (1-% covered by health insurance)	0.85	-0.89	-0.91	0.94	-0.57	-0.95	-0.83	-0.12	-0.84	-0.37	0.76	0.80	0.63	0.60	0.94	1.00

Table 4. Means, standard deviations, and correlations among the 6 importance scales from World Values Survey, n=1502 US residents 1995 only.

	Family	Friends	Leisure time	Politics	Work	Religion
Family	1					
Friends	.15	1				
Leisure time	.09	.20	1			
Politics	.12	.12	.12	1		
Work	.10	.10	.09	.12	1	
Religion	.22	.16	.04	.12	.06	1
Mean	2.94	2.65	2.29	1.68	2.31	2.37
Standard Deviation	.27	.57	.70	.88	.90	.87

Figure 1. Regions of W_i and W_j where $A_{ij} > 0$ for two indicators that are negatively correlated ($r_x = -.7$).
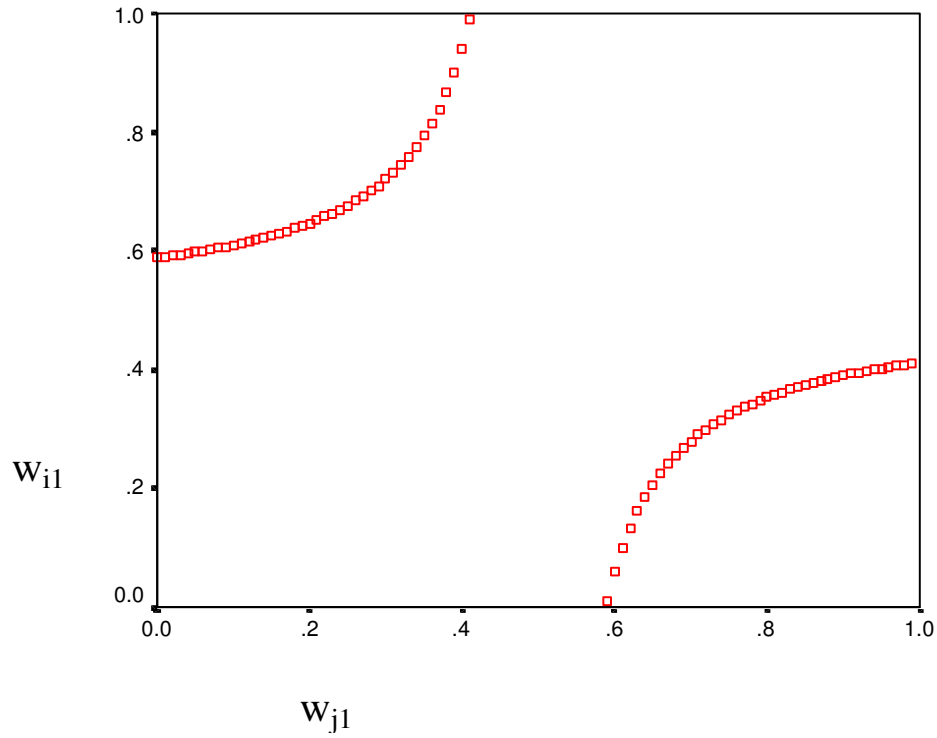
Figure 2. Histogram of A_{ij} over all 4950 possible pairs of 100 simulated individuals with weights generated from a uniform distribution for social indicators: log(GDP/capita), life expectancy, and education in Example 1.
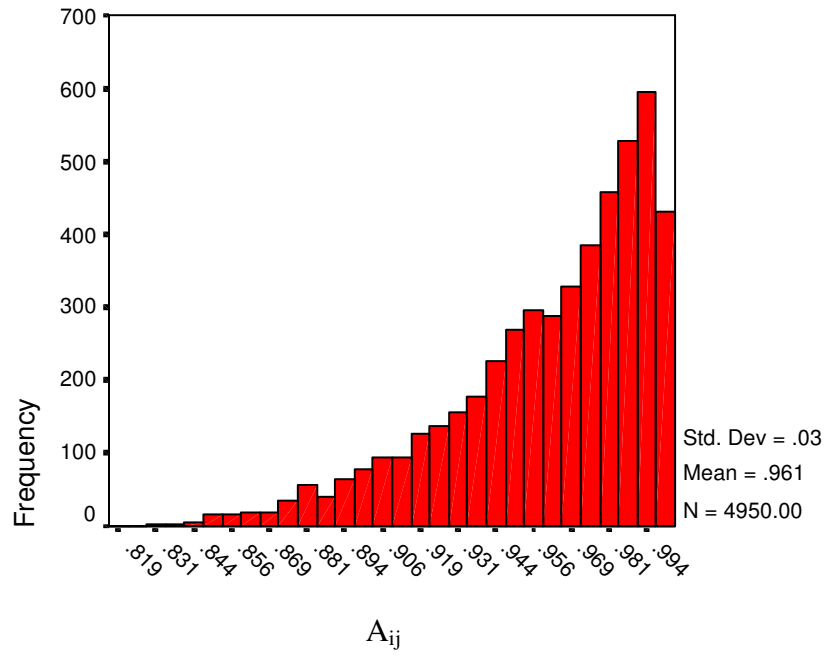
Figure 3. Histogram of A_{ij} over all 4950 possible pairs of 100 simulated individuals with weights generated from a uniform distribution for social indicators: log(GDP/capita),life expectancy, education, and (1-Gini).
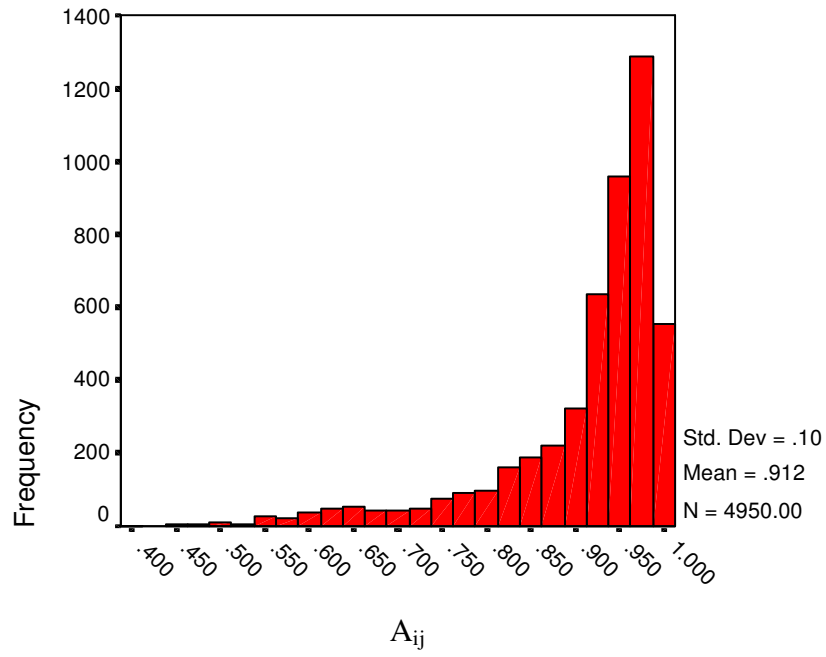
Figure 4. Distribution of agreement $A_{E,i}$ between the equal-weights QOL index and the 100 simulated individuals in Example 2.
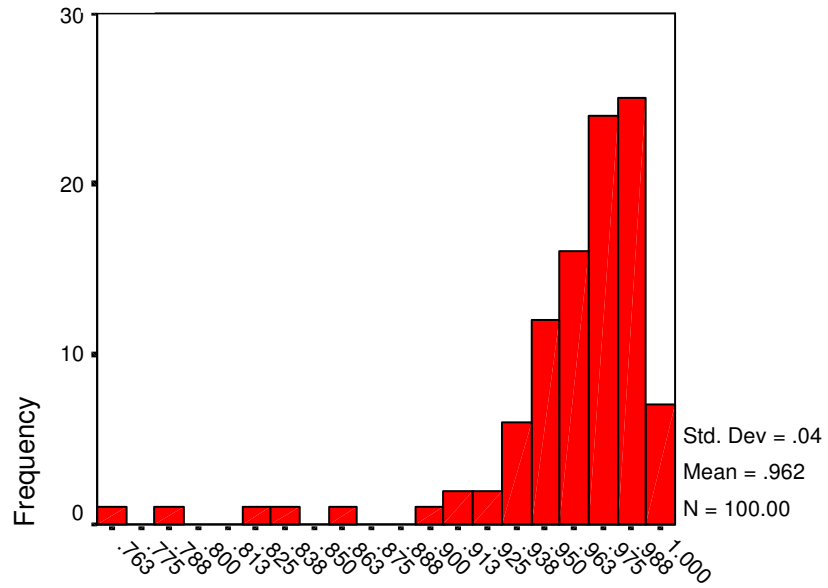

Std. Dev = .04

Mean = .962

N = 100.00

Figure 5. Distribution of $A_{E,i}$ agreement between the equal-weight QOL index and the 100 simulated Individuals for the 16 social indicators of the Index of Social Health.