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Construction of Cryptographic S-Boxes Based on Mobius Transformation and Chaotic Tent-Sine System

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ABSTRACT Over the last few decades, different mediums of secure communication use chaos which is demonstrated by some nonlinear dynamical systems. Chaos shows unpredictable behavior and this characteristic is quite helpful in different encryption techniques and for multimedia security. In this work, the chaotic behavior of the improved Tent-Sine map is conferred and ultimately a new method to construct substitution-boxes is proposed. This new method explores the features of chaos through TSS map and algebraic Mobius transformation to generate strong S-boxes. The S-boxes are assessed using standard tests suit which includes nonlinearity, strict avalanche criterion, bit independence criterion, linear approximation probability and differential uniformity. Moreover, the proposed S-boxes show excellent statistical properties under majority logic criterions such as correlation, homogeneity, energy, entropy, contrast. The statistical encryption results are demonstrate the better performance of the proposed S-boxes when compared with some of state of the art S-boxes including AES, Gray, APA S8 AES, Skipjack and validate the suitability of anticipated method.

INDEX TERMS Substitution-box, block cipher, improved chaotic map, nonlinearity.

I. INTRODUCTION

The rapid increase in international networking provides many new options for the design and presentation in the form of digital data. This easy access and disposal to digital data which includes audio, video, electronic libraries, electronic advertising, web designing, and digital repositories develop the concern of security. The protection while transferring or saving data is indispensably important. For ensuring the security of such digital data and information, a specific field which is named as secure communication plays its vital role to counter this major problem. Secure communication can further be categorized into three main categories which include cryptography, watermarking and steganography. The basic purpose of both steganography and cryptography is similar but the difference lies in applying different methods. In these two methods, the basic purpose is to obscure the original information. Interestingly, the method adopted in both multimedia security techniques, steganography and watermarking is the same but they vary in purposes. Hiding of digital content in images is the goal of steganography whereas watermarking helps in declaring right ownership [1]–[4]. In symmetric-key cryptography, block cipher has a very important role to play in encrypting the information. By keeping the same dimensions, block cipher converts plaintext data into ciphertext data with the assistance of a user-provided an undisclosed key [5], [6], [29]. By following the reverse pattern on ciphertext data, decryption of the whole process is performed provided that secret keys of the process remain unchanged. The above-mentioned procedure is adopted in Advanced Encryption Standards (AES) [7]. It is considered as one of the strong cryptosystems which encrypt the plaintext and ensure

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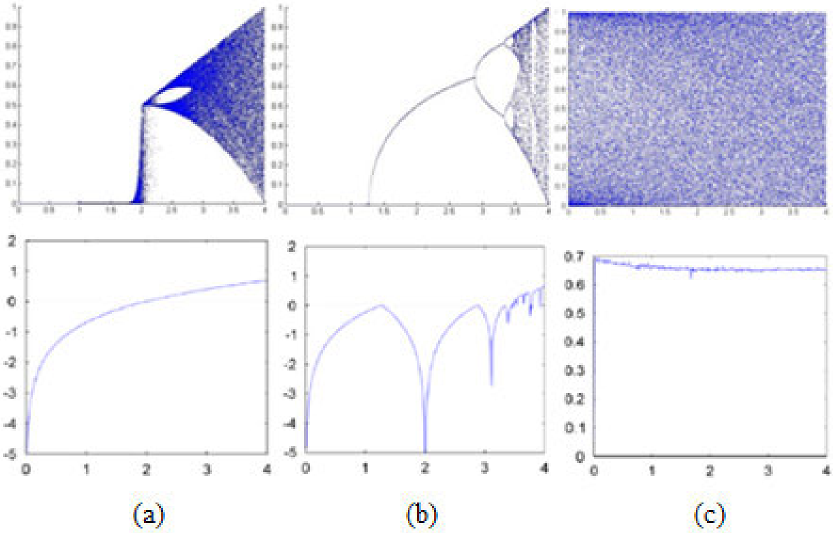


FIGURE 1. Bifurcation diagram (first row) and Lyapunov spectrum (second row) of chaotic (a) Tent map, (b) Sine map, and (c) TSS map versus parameter τ, α, and σ for (0, 4], respectively.

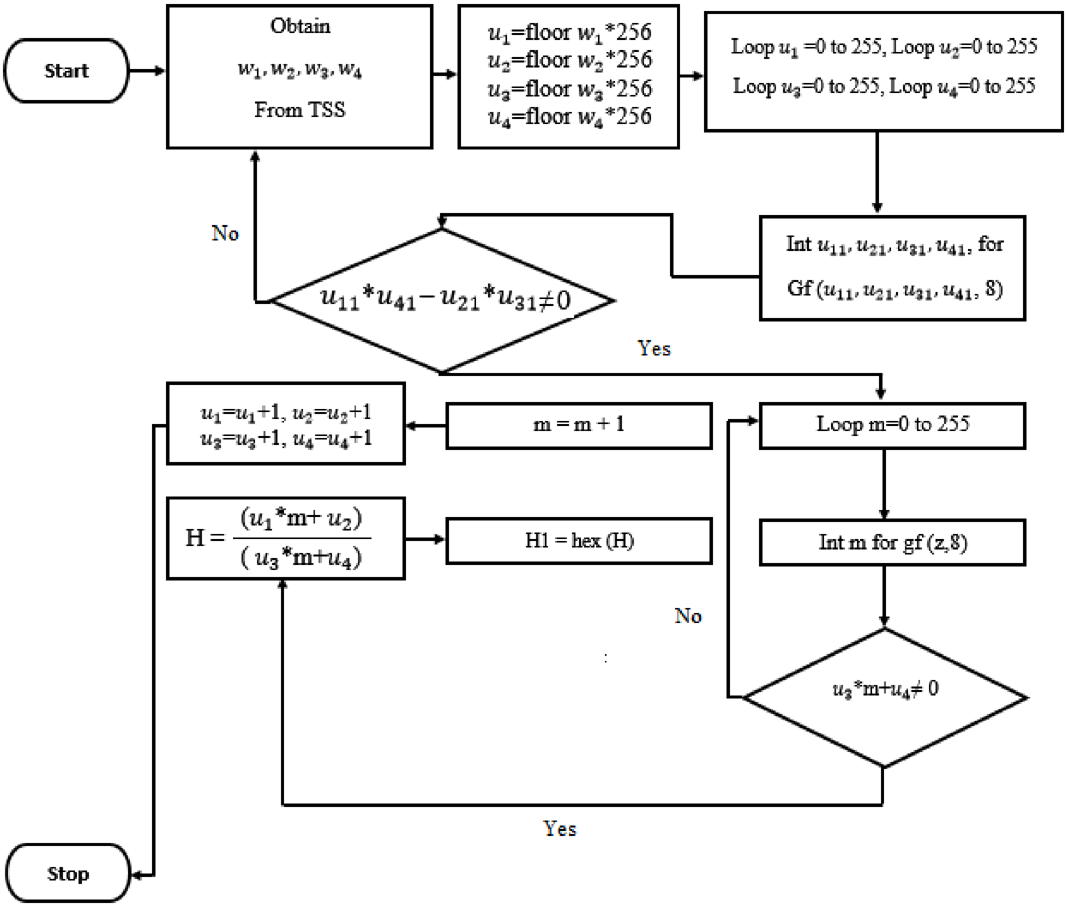
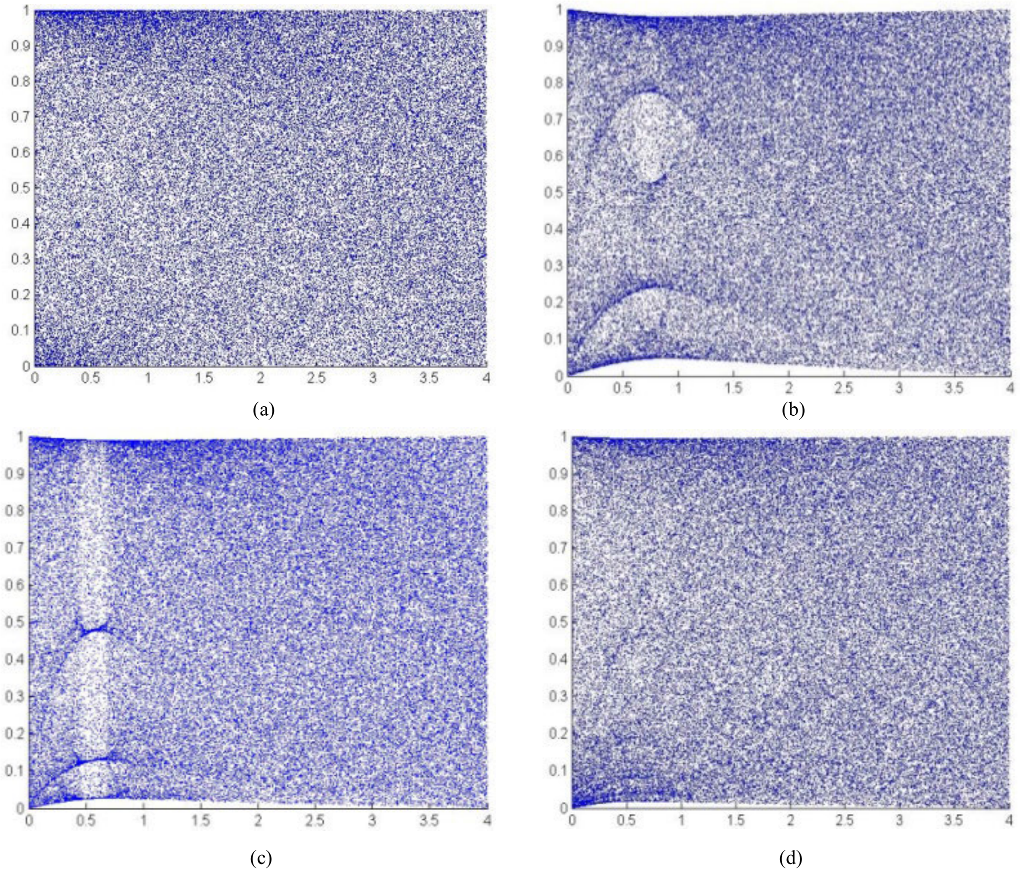


FIGURE 2. Flowchart of proposed S-box design method.

secure communication. The whole procedure consists of four steps. In the very first step byte substitution which is also named as substitution step, is done with the help of the substitution-box (S-box). This is the step that actually highlights the importance of S-box in the encryption procedure. It is the only nonlinear component and this byte substitution step creates confusion in the plaintext data that can be seen in encrypted data. Different applications of substitution



(a)

(b)

(c)

(d)

FIGURE 3. Bifurcation diagrams of chaotic TSS maps in (a) equation (6), (b) equation (7), (c) equation (8), and (d) equation (9), respectively.

and permutation process on plain text data provide encrypted ciphertext data [8]–[10]. The value and capability of producing confusion are measured by variations in the output bit pattern. The selected S-box must be robust and shows opposition against any attempt of cryptanalysis. Nonlinearity is considered as the foremost performing criterion of the S-box in any encryption method. Over the years, researchers are keen to get algebraically strong and cryptographically robust S-boxes. In addition to this, chaos-based S-boxes also have their importance for secure communication of data. These S-boxes exhibit different striking properties and offer interesting results to various ciphers. But the main focus is to improve the nonlinearity of these S-boxes [11], [12]. Mathematically, an S-box can be represented as:

$$S : GF(2^a) \rightarrow GF(2^b) \tag{1}$$

S-box holds one to one and onto relations which makes it a bijection mapping and hence its inverse is possible. A message symbol is replaced with one element of S-box. By equation (1), it is evident that an $a \times b$ S-box takes a bits as the input information and gives b bits as the output [13].

Due to the success of AES S-box, a number of proposals have been put forward for the design of strong S-boxes using Galois field based algebraic techniques. In [14], Cui *et al.* proposed an affine-power-affine (APA) structure in which same AES algebraic operations are performed but with modified affine transformation cycle. The obtained S-box found to have good cryptographic and algebraic features. Subsequently, M-T Tran *et al.* utilized the gray codes to improvise the S-box over algebraic coefficients in [15]. They added a preprocessing step to the structure of AES S-box by performing a gray-code transformation. In [16], Hussain *et al.* applied the algebraic permutation group S_8 on the elements of AES S-box which enables them to construct sequentially as much as 40320 S-boxes with similar performance strength as that of AES S-box. The same researcher proposed another algebraic method for S-box by applying the action of projective linear group on Galois field with a particular type of linear fractional transformation in [17]. In [18], a powerful algebraic method is suggested to construct 16 strong S-boxes built on the concept of Galois field extensions of order 256. The technique is purely algebraic and has ability to construct 8×8 S-boxes of high cryptographic strengths. Farwa *et al.* in [19] used multiplicative cyclic group of associated Galois field for the construction of algebraic S-box. They carefully formulated a bijective nonlinear iterative algebraic map defined on $GF(2^8)$ and the algebraic S-box provided acceptable properties for application in image encryption. A new S-box generation method based on both the algebraic and chaotic structures is proposed in [20]. Wherein, chaotic Chebyshev map and a

TABLE 1. **Chaotic S-box corresponds to the TSS by equation (4) for $\beta = 1$.**

83	27	78	85	30	124	26	239	153	109	48	57	154	38	191	46
118	7	73	230	201	213	94	144	41	250	216	9	242	121	101	127
50	63	234	252	126	199	174	225	217	52	21	233	86	88	135	3
106	180	238	223	18	214	28	95	205	227	240	162	105	37	49	131
31	29	237	114	155	65	96	139	246	173	198	147	67	54	138	120
12	80	68	241	167	145	132	210	99	158	89	22	11	192	134	149
218	79	181	71	219	8	69	60	87	248	91	133	34	90	39	32
130	251	16	245	76	122	156	108	171	159	23	228	254	110	44	142
19	169	148	189	58	6	35	123	72	200	194	36	222	116	64	186
232	10	17	188	236	202	14	168	229	176	2	212	4	129	74	42
215	195	104	207	221	92	5	235	77	208	47	187	20	119	193	197
226	220	166	163	255	141	182	128	93	211	102	66	125	62	61	33
97	253	179	175	40	164	70	185	151	25	112	137	157	177	13	165
247	55	56	84	24	161	15	51	117	231	190	244	146	152	206	209
150	1	53	0	45	172	178	81	59	111	82	249	98	203	224	183
113	243	43	75	143	196	115	160	136	170	103	204	140	107	100	184

TABLE 2. **Chaotic S-box corresponds to the TSS by equation (6) for $\beta = 8/9$.**

112	80	53	200	68	242	11	3	72	46	89	136	114	224	78	166
174	120	176	65	73	163	204	95	30	23	107	197	32	217	128	215
1	214	33	6	154	180	75	158	143	173	169	161	185	116	92	0
164	12	115	137	221	245	19	51	44	8	195	59	181	142	160	41
237	190	110	189	29	35	213	129	148	127	24	18	208	171	56	119
252	134	186	126	232	183	109	246	162	25	203	222	34	211	27	111
170	60	2	202	98	206	64	133	177	130	225	22	250	233	251	228
49	104	71	238	201	149	90	152	105	150	20	97	184	47	94	255
223	52	199	118	66	48	147	17	145	124	74	153	231	240	132	117
40	39	139	36	7	81	212	103	155	101	219	187	102	62	21	113
125	144	249	106	196	121	167	83	168	138	209	227	151	10	191	37
188	63	100	69	77	254	179	84	178	57	247	239	15	28	135	220
205	216	198	42	175	13	5	9	244	50	79	182	141	165	58	243
207	96	193	76	31	99	146	226	87	70	93	234	194	236	253	230
218	159	55	235	122	14	4	16	229	248	241	88	38	192	157	61

special class of permutation subgroups of symmetric group S_{16} are explored for S-box construction.

The available literature makes it evident that the S-box generation using any random approach, using chaos or some other pseudo-random source, doesn't found to have good cryptographic strengths compared to S-boxes generated via algebraic methods. The only merit with random S-boxes is that it is quite easy to get a large quantity of S-boxes. However, cryptographically strong S-boxes are found to be easily obtainable through algebraic techniques [21]. But, many of them are keyless techniques and yields static S-boxes. In literature, some of the well-known S-boxes are AES [7], APA [14], Gray [15], S8 AES [16], S8-APA [22], [23]. The performance of optimization based S-box methods lies in between the random-based and algebraic methods. With the applied heuristics, many of the researchers have obtained S-boxes better than random or chaotic S-boxes. But, the associated demerit is that they are time consuming as the optimization process takes significant time to get notable configuration of optimized S-box [24].

In this work, combination of two 1D chaotic maps to improve their chaotic range is used to construct different S-boxes. In addition, the group action of a projective general linear group is also performed on the elements of $GF(2^8)$. Hence, an improved chaos-assisted search for strong S-boxes construction using the algebraic Mobius transformation is put forward. The proposed method is key-dependent, means it also has its own set of key space. Our method is a blend

TABLE 3. Chaotic S-box corresponds to the TSS by equation (7) for $\beta = 4/5$.

51	188	63	187	122	179	183	101	24	1	55	147	254	111	211	62
226	66	225	189	250	169	39	120	213	108	82	215	204	84	58	96
95	253	75	22	30	159	127	228	28	85	117	121	57	232	25	13
4	103	19	99	136	78	5	202	7	10	64	114	243	23	90	61
129	67	148	138	139	182	170	80	42	155	110	91	145	115	151	68
43	105	0	245	16	252	89	171	178	227	222	153	162	164	168	231
104	247	210	251	35	165	27	69	37	249	2	191	40	156	207	208
41	83	32	199	74	205	152	125	220	94	234	17	8	255	123	106
229	132	11	244	175	79	36	15	100	87	190	157	52	173	137	59
48	65	167	236	18	146	72	192	38	246	216	12	238	174	131	235
112	26	154	130	161	6	20	172	98	107	212	50	73	77	116	185
119	224	31	181	197	109	166	209	180	46	124	242	53	186	214	128
218	184	21	150	140	86	92	102	240	237	60	143	163	221	195	194
56	158	93	54	160	3	126	134	49	29	47	217	70	97	141	206
113	248	200	9	177	233	198	76	203	142	71	241	230	45	144	239
196	81	193	133	223	118	34	176	33	201	149	135	88	44	219	14

TABLE 4. Chaotic S-box corresponds to the TSS by equation (8) for $\beta = 6/7$.

232	166	125	1	10	184	148	152	143	192	196	237	118	208	204	7
211	36	223	173	44	156	239	108	3	12	112	134	210	115	214	76
249	142	60	104	67	0	102	128	56	171	114	121	73	93	22	14
250	219	97	172	50	207	254	47	199	151	34	203	99	225	24	11
244	246	113	18	58	64	168	52	187	96	138	15	19	130	202	127
234	154	123	53	227	215	139	31	61	98	33	157	101	40	158	229
235	169	253	9	178	92	120	82	129	20	65	163	85	245	39	160
141	212	135	54	68	186	13	221	57	6	147	78	165	35	21	194
80	79	81	41	176	226	137	87	133	161	164	195	198	200	62	136
8	149	77	48	122	174	222	117	109	231	188	177	159	89	51	30
233	100	205	4	181	119	32	182	243	86	71	213	209	185	45	75
106	111	220	228	124	90	251	72	91	17	224	230	59	94	206	43
247	23	216	238	74	162	144	16	193	183	28	201	170	37	167	84
189	69	150	105	236	131	83	255	155	179	29	242	95	140	153	145
110	240	88	116	107	146	63	252	26	42	197	103	46	27	66	126
241	132	25	217	5	175	70	180	248	190	218	2	38	55	191	49

of chaos-based random and algebraic techniques. Therefore, it holds the advantages of ease of constructing S-boxes with cryptographic strengths similar to AES and key-dependent. The exhaustive comparison of S-boxes performance with a number of chaos-based., algebraic-based, and optimization-based methods is also done to reflect the superiority of our method over many of the standing methods. Moreover, an image encryption application of proposed S-boxes is also carried out to show their suitability to multimedia security.

II. REVIEW OF VARIOUS CHAOTIC MAPS

In the literature, various chaotic maps have been applied for encryption, watermarking and steganography techniques. Here, two chaotic maps i.e., Tent map and Sine map will be discussed and analyzed. The combination of these maps form a Tent-Sine system (TSS) which is used for the proposed chaotic S-box method.

A. CHAOTIC TENT MAP

The Tent map is expressed as [25]:

$$y_{n+1} = \begin{cases} \tau \dfrac{y_n}{2} & y_i < \dfrac{1}{2} \\ \tau \dfrac{(1 - y_n)}{2} & y_i \geq \dfrac{1}{2} \end{cases} \tag{2}$$

where, the range of parameter τ lies in the interval $0 < \tau \leq 4$, and state variable $y_n \in [0, 1]$. It is obvious from bifurcation diagram of chaotic tent map; the map name is due to its tent map like shape. The interval of chaotic behavior of tent map is [2, 4]. The bifurcation diagram and lyapunov exponent are shown in Fig 1(a). The behavior of Tent map is chaotic for

TABLE 5. Chaotic S-box corresponds to the TSS by equation (9), $\beta = 10/9$.

51	244	119	243	122	179	183	45	80	1	55	147	254	111	155	118
170	10	169	245	250	225	39	120	157	108	26	159	204	28	114	40
95	253	75	22	86	215	127	172	84	29	61	121	113	232	81	69
4	47	19	43	192	78	5	202	7	66	8	58	187	23	90	117
129	11	148	194	195	182	226	24	98	211	110	91	145	59	151	12
99	105	0	189	16	252	89	227	178	171	222	209	162	164	224	175
104	191	154	251	35	165	83	13	37	249	2	247	96	212	207	152
97	27	32	143	74	205	208	125	220	94	234	17	64	255	123	106
173	132	67	188	231	79	36	71	44	31	246	213	52	229	193	115
48	9	167	236	18	146	72	136	38	190	216	68	238	230	131	235
56	82	210	130	161	6	20	228	42	107	156	50	73	77	60	241
63	168	87	181	141	109	166	153	180	102	124	186	53	242	158	128
218	240	21	150	196	30	92	46	184	237	116	199	163	221	139	138
112	214	93	54	160	3	126	134	49	85	103	217	14	41	197	206
57	248	200	65	177	233	142	76	203	198	15	185	174	101	144	239
140	25	137	133	223	62	34	176	33	201	149	135	88	100	219	70

TABLE 6. Nonlinearity of proposed S-boxes.

Proposed S-box	nl_1	nl_2	nl_3	nl_4	nl_5	nl_6	nl_7	nl_8	Min nl
S-box-1	108	106	108	110	110	108	104	100	100
S-box-2	108	106	108	104	104	104	108	104	104
S-box-3	112	112	112	112	112	112	112	112	112
S-box-4	112	112	112	112	112	112	112	112	112
S-box-5	112	112	112	112	112	112	112	112	112

the specific interval only as clear from Fig. 1(a) which shows the limitations of this chaotic map.

B. CHAOTIC SINE MAP

The chaotic behaviour of well-known Logistic map and Sine map is somewhat similar to each other. This can be seen in both bifurcation and lyapunov exponent diagrams of Sine map given in Fig 1(b). The Sine map has following governing equation [26].

$$y_{n+1} = \alpha sin\left(\pi y_n\right)/4, \quad 0 < \alpha \leq 4; \tag{3}$$

Like Tent map, the state variable $y_n \in [0, 1]$ and α is system parameter. Both the Tent map and Sine map have almost identical behavior and they have common problems as well. The range of chaos in Sine map is also limited as depicted in bifurcation diagram. Moreover, the non-uniformity of its trajectory-points combine with limited chaotic range makes the application of Sine map limited.

C. CHAOTIC TENT-SINE SYSTEM

The short chaotic spread of both Tent map and Sine map demands a chaotic map whose chaotic spread is bigger than two seed maps. A unique nonlinear combination of these two maps gives a chaotic Tent-Sine system (TSS). This arrangement of chaotic maps plays an extremely complicated chaotic role [27]. The mod1 operator is required to keep the output range lies in between 0 to 1. The assimilation of parameters of both the chaotic maps, the expression is given in equation (4).

$$\begin{aligned} y_{n+1} &= F\left(y_n, \sigma, \beta\right) \\ &= \begin{cases} (\sigma \dfrac{y_n}{2} + (4-\sigma)\sin(\pi y_n^{\beta})/4) mod 1 & y_i < 1/2 \\ \Big(\sigma\left(1-y_n\right)/2 + (4-\sigma)\sin(\pi y_n^{\beta})/4\Big) & y_i \geq 1/2 \end{cases} \end{aligned} \tag{4}$$

where $0 < \sigma \leq 4$, $\beta > 0$ are two parameters and $y_n \in [0, 1]$ is the state variable of the TSS chaotic system. The chaotic limit of the TSS system is increased remarkably well and the output sequences are distributed uniformly which can be seen from its bifurcation and lyapunov exponent diagrams shown in Figure 1(c) for $\beta = 1$.

III. PROPOSED METHOD

A. MOBIUS TRANSFORMATION BASED CONSTRUCTION OF CHAOTIC S-BOX

The important step in any cryptographic technique is the appropriate selection of S-box. This careful selection also restricts the linear and differential attacks. With a higher chaotic range and complex properties, Tent-Sine system is considered for the structuring proposed S-boxes. The flow chart in Fig. 2 shows that the primary input for the structure of S-box is taken from chaotic Tent-Sine map.

TABLE 7. Comparison of different S-boxes performance features.

S-box	Nonlinearity			SAC	BIC-NL	BIC-SAC	DU	LP
	min	max	avg					
Proposed S-box-1	100	110	106.75	0.5002	104	0.4988	30	0.125
Proposed S-box-2	104	108	105.75	0.4927	98	0.5052	10	0.1328
Proposed S-box-3	112	112	112	0.504	112	0.504	4	0.0625
Proposed S-box-4	112	112	112	0.504	112	0.504	4	0.0625
Proposed S-box-5	112	112	112	0.504	112	0.504	4	0.0625
CHAOS-BASED RANDOM METHODS								
Lambic [33]	108	112	109.25	0.5012	104	0.5056	8	0.0937
Lambic [34]	106	108	106.75	0.5034	100	0.4951	10	0.1328
Lambic [35]	106	108	106.5	0.4978	100	0.5029	10	0.1328
Khan [36]	100	108	106	0.4946	96	0.5018	10	0.1328
Attaullah [37]	106	108	107.25	0.5034	98	0.498	12	0.1328
Özkaynak [38]	106	108	106.75	0.4941	98	0.4957	10	0.125
Tian [39]	104	108	106.75	0.4076	98	0.5022	10	0.1328
Solami [40]	106	110	108.5	0.5017	100	0.5026	10	0.1328
Silva-Garcia [8]	105	107	106	0.5066	96	0.5065	12	0.1445
Yi [41]	106	110	107.75	0.4976	100	0.5023	10	0.125
ALGEBRAIC METHODS								
AES [7]	112	112	112	0.5058	112	0.504	4	0.0625
APA [14]	112	112	112	0.4987	112	0.4993	4	0.0625
Gray [15]	112	112	112	0.5058	112	0.502	4	0.0625
S8 AES [16]	112	112	112	0.504	112	0.502	4	0.0625
Skipjack [42]	104	108	105.25	0.5026	100	0.5002	12	0.1172
Ali [23]	112	112	112	0.5031	112	0.5028	4	0.0625
Hussain [43]	100	108	104.75	0.4931	102	0.5	10	0.125
Jamal [44]	104	110	106.75	0.4988	102	0.5010	30	0.125
Razaq [45]	104	108	106.75	0.5031	96	0.5074	12	0.1484
Shuai [46]	108	112	110	0.4861	108	0.5020	6	0.0859
CHAOS-ASSISTED OPTIMIZATION METHODS								
Wang [47]	108	110	109	0.5026	102	0.5026	10	0.1406
Ahmad [48]	108	112	109.25	0.4985	98	0.4992	8	0.125
Tian [49]	106	110	108	0.5073	100	0.502	10	0.1523
Ahmad [50]	106	110	107	0.5015	98	0.5016	10	0.1484
Alzaidi [51]	108	110	109.5	0.4985	98	0.5020	10	0.1328
Farah [52]	104	110	106.5	0.4995	98	0.4983	10	0.1172
Ahmed [53]	106	108	107.5	0.4943	98	0.4982	10	0.125
Zhang [54]	108	110	108.75	0.4946	94	0.5054	10	0.1328
Alzaidi [55]	110	112	110.25	0.5	104	0.5052	10	0.125

Moreover, the mathematical foundation of the proposed method is defined by the concept of group action of a projective general linear group over a Galois finite field $GF(2^8)$ with the help of Mobius transformation. The group action and Mobius transformation is expressed as follows:

$$g : PGL(2, GF\left(2^8\right)) \times GF\left(2^8\right) \rightarrow GF\left(2^8\right)$$

$$g\left(m\right) = \frac{u_1 \times m + u_2}{u_3 \times m + u_4}, \quad 0 \leq m \leq 255 \tag{5}$$

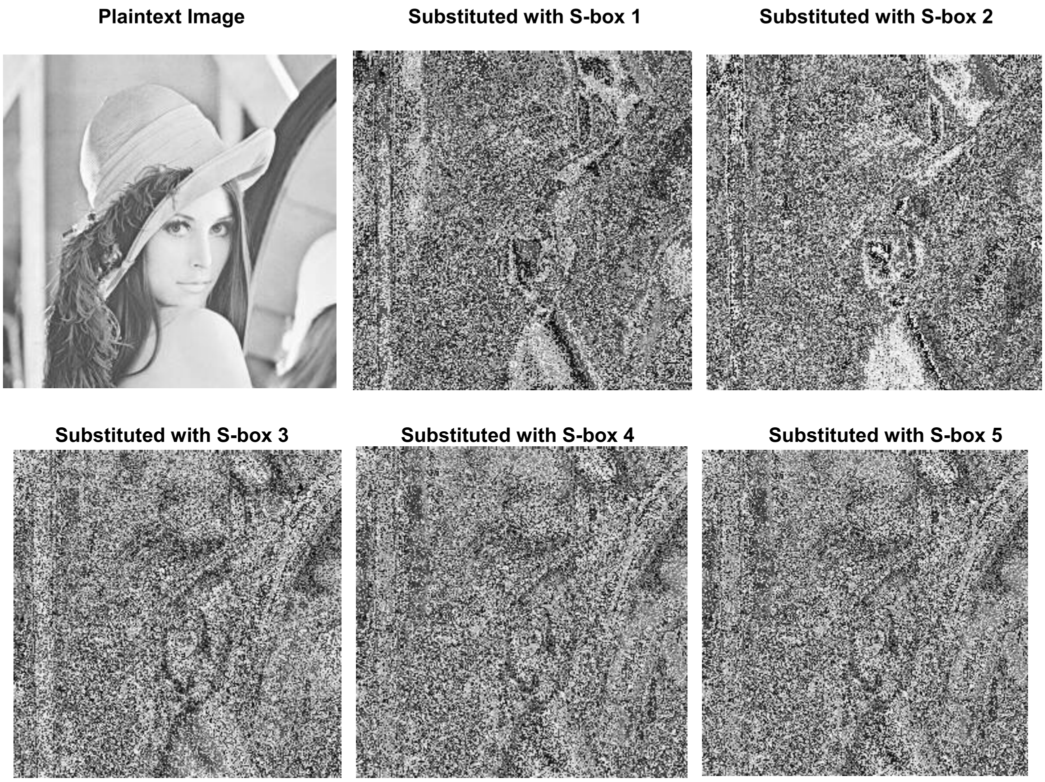


FIGURE 4. Plain image and substituted images with proposed S-box 2 to S-box 5.

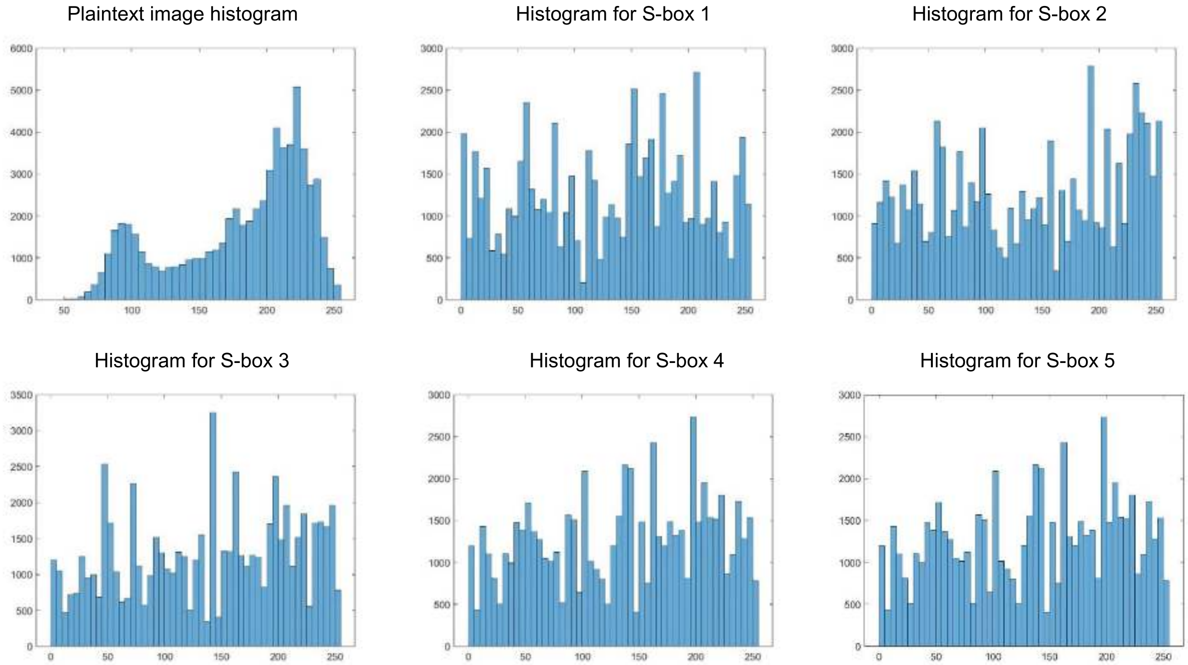


FIGURE 5. Histogram of Lena plain text image and substituted images with proposed S-box I to S-box V.

where, the four values u_1, u_2, u_3, and u_4 are from a finite field $GF(2^8)$. The Mobius transformation has a combined effect of inversion, dilation, rotation and translation in the Galois field domain. It also serves as one to one transformer provided that its inherent conditions are satisfied. The Mobius transformation $g(m)$ has point of discontinuity at $u_3 \times m + u_4 = 0$. This point has to be avoided while using it to generate the output value $g(m)$. The existence of discontinuity point of Mobius transformation is carefully checked when generating the S-box elements during operation of proposed scheme.

TABLE 8. Majority logic criterion results for plain and substituted images.

Images	Contrast	Correlation	Entropy	Energy	Homogeneity
Plain-image	0.445343	0.910666	7.279584	0.135318	0.857543
Proposed S-box-1	9.314369	0.089245	7.279584	0.017117	0.437896
Proposed S-box-1	10.76492	0.078912	7.279584	0.018535	0.434900
Proposed S-box-3	9.400597	0.075878	7.279584	0.017137	0.428206
Proposed S-box-4	9.56152	0.028254	7.279584	0.017690	0.434896
Proposed S-box-5	9.426823	0.056952	7.279584	0.017380	0.434357
AES	9.514212	0.065412	7.279584	0.017541	0.431805
APA	9.412510	0.058742	7.279584	0.017621	0.427469
S8 AES	9.741021	0.074290	7.279584	0.017200	0.427540
Gray	9.987453	0.073548	7.279584	0.017870	0.428410
Skipjack	10.2547	0.065419	7.279584	0.017412	0.435846

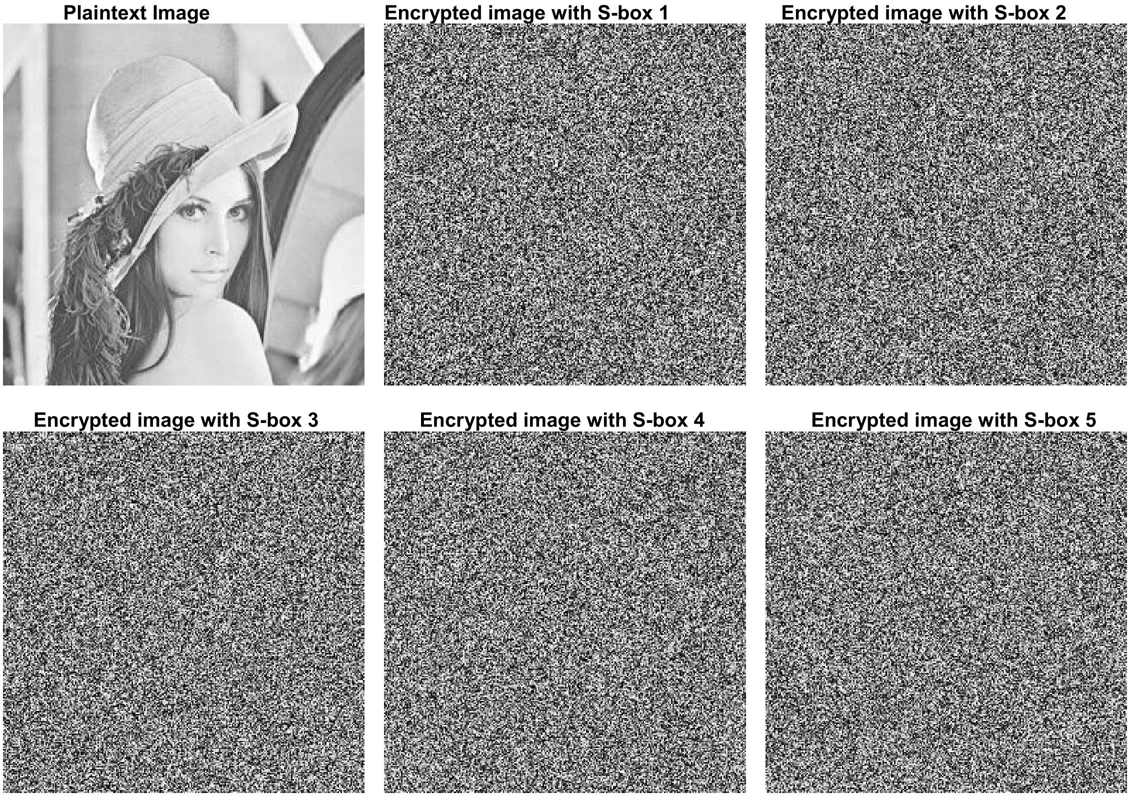


FIGURE 6. Lena plaintext image and its encrypted images with proposed S-boxes of 1, 2, 3, 4, and 5.

The scheme proceeds further only if $u_3 \times m + u_4 \neq 0$, else the scheme needs to loop back to avoid the possibility of this discontinuity.

The choice of four u_1, u_2, u_3, and u_4 values, allocated to Mobius transformation is selected from chaotic Tent-Sine system. The products of the proposed method are chaotic S-boxes. The algorithm depicts that loop applied in it takes values of u_1, u_2, u_3, u_4 and m from interval 0-255. The algorithm goes to the next step once it satisfies the condition of $u_1 \times u_4 - u_2 \times u_3$ is not equal to zero. This condition makes the transformation preclude the possibility that Mobius transformation $g(m)$ reduces to a constant. The operational steps of proposed method are the following.

Step I: Initially, random values w_1, w_2, w_3, w_4 are obtained by iterating the TSS map $F(y_0, \sigma, \beta)$ four times by using the control parameter σ having interval [0,4] and the initial value $y_0 \in [0, 1]$.

Step II: The four values u_i are calculated out of chaotic w_i for $i = 1,2,3,4$ obtained in Step I as:

$$u_i = mod(floor(w_i \times 10000), 256)$$

If the resulting value of $u_1 \times u_4 - u_2 \times u_3$ is any number other than zero, we will go to the next Step else we need to start from Step I again.

Step III: Here,we generate a sequence m of length 256 from different entries of the finite field $GF(2^8)$. This sequence of 256 entries will be in ascending order.

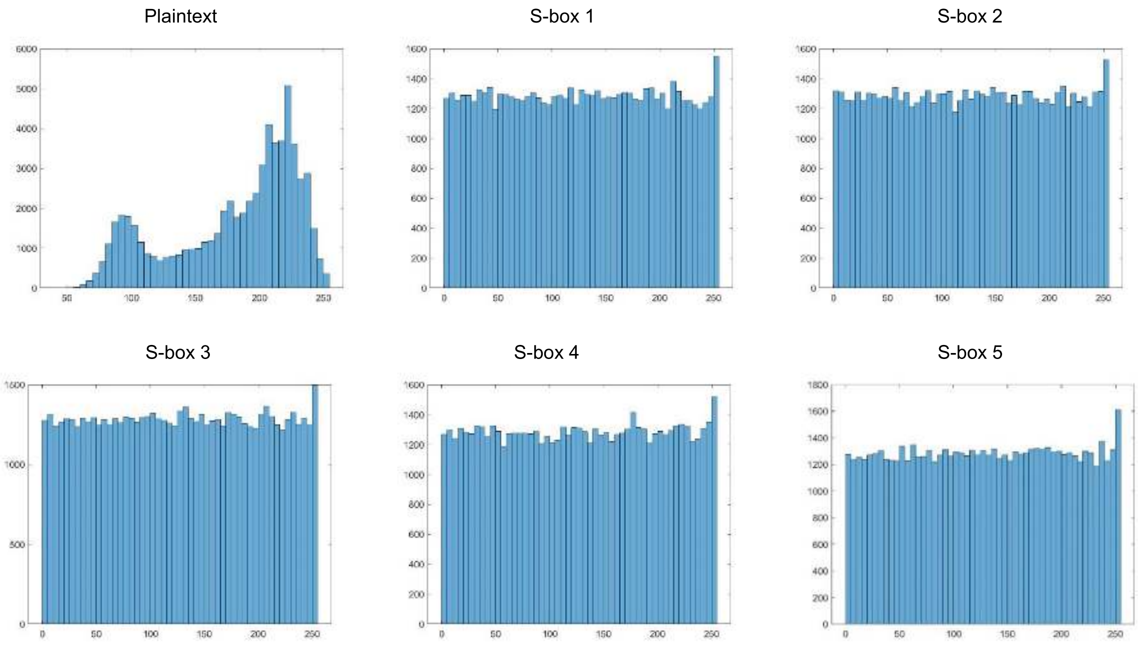


FIGURE 7. Histogram of Lena plain text image and encrypted images with proposed S-box 1 to S-box 5.

Step IV: We need to go back to step III provided that $u_3 \times m + u_4$ is equal to zero. If the result of the expression is non-zero then we would take group action of the projective general linear group $PGL(2, GF(2^8))$ on the elements of m. This group action is defined in equation (5).

Step V: Lastly, by iterating the above-mentioned four steps will provide us the sequence S having 256 different entries. Transform its elements into 16×16 matrix. This matrix S is the proposed S-box.

The flowchart of the proposed method using the TSS chaotic map and Mobius transformation is shown in Fig. 2.

B. PROPOSED S-BOXES

S-boxes are constructed by using various powers β of chaotic TSS map $F(y_0, \sigma, \beta)$. The initial values for $y_0 = 0.7$ and $\sigma = 3$ with specified β is taken for simulation. The detailed description of the map relates to the first S-box for $\beta = 1$ is specified in equation (4) and the maps of rest of the four S-boxes are given in equations (8) to (11). The bifurcation diagrams of different forms of TSS maps having different exponents are shown in Figure 3. The exponent β of TSS map is used as a parameter for constructing different S-boxes. The tabular form of proposed chaotic S-boxes for $\beta = 1, \beta = 8/9$, $\beta = 4/5$, $\beta = 6/7$, and $\beta = 10/9$ are provided in Tables (1) to (5), respectively. The exponents used for the construction of these S-boxes are mentioned with Tables.

$$y_{n+1} = \begin{cases} (\sigma \dfrac{y_n}{2} + (4-\sigma)\sin(\pi y_n^{8/9})/4) mod 1 \\ \quad y_i < 1/2 \\ \left(\sigma\,(1-y_n)/2 + (4-\sigma)\sin(\pi y_n^{8/9})/4\right) mod 1 \\ \quad y_i \geq 1/2 \end{cases} \tag{6}$$

$$y_{n+1} = \begin{cases} (\sigma \dfrac{y_n}{2} + (4-\sigma)\sin(\pi y_n^{4/5})/4) mod 1 \\ \quad y_i < 1/2 \\ \left(\sigma\,(1-y_n)/2 + (4-\sigma)\sin(\pi y_n^{4/5})/4\right) mod 1 \\ \quad y_i \geq 1/2 \end{cases} \tag{7}$$

$$y_{n+1} = \begin{cases} (\sigma \dfrac{y_n}{2} + (4-\sigma)\sin(\pi y_n^{6/7})/4) mod 1 \\ \quad y_i < 1/2 \\ \left(\sigma\,(1-y_n)/2 + (4-\sigma)\sin(\pi y_n^{6/7})/4\right) mod 1 \\ \quad y_i \geq 1/2 \end{cases} \tag{8}$$

$$y_{n+1} = \begin{cases} (\sigma \dfrac{y_n}{2} + (4-\sigma)\sin(\pi y_n^{10/9})/4) mod 1 \\ \quad y_i < 1/2 \\ \left(\sigma\,(1-y_n)/2 + (4-\sigma)\sin(\pi y_n^{10/9})/4\right) mod 1 \\ \quad y_i \geq 1/2 \end{cases} \tag{9}$$

IV. SUBSTITUTION BOXES ANALYSIS

The assessment of the S-box defines its further application in various cryptographic schemes and multimedia security [28]–[30]. For this purpose, different theoretic and statistical performance measures are being utilized to evaluate the strength of S-boxes [31]. A comprehensive demonstration of such measures, involving differential characteristics of the block cipher is discussed in [32]. These types of attacks are used in block cipher-based S-boxes like DES and AES. The cipher can be scrutinized by using information theory approach [31]. Different tests like nonlinearity score, strict avalanche criteria (SAC), bit independent criterion (BIC),

TABLE 9. Majority logic criterion analyses for plain and encrypted images (AES algorithm).

Images	Contrast	Correlation	Entropy	Energy	Homogeneity
Plain-image	0.445343	0.910667	7.279584	0.135318	0.857543
Proposed S-box-1	10.51509	-0.00114	7.996877	0.015647	0.389625
Proposed S-box-2	10.46624	0.001681	7.996711	0.015641	0.390036
Proposed S-box-3	10.43909	0.001395	7.997025	0.015638	0.390972
Proposed S-box-4	10.56595	-0.004463	7.997268	0.015638	0.388188
Proposed S-box-5	10.41189	0.006517	7.997024	0.015637	0.390352
AES	10.62102	-0.000250	7.998521	0.015640	0.392051
APA	10.41298	0.002151	7.996584	0.015632	0.386540
S8 AES	10.52987	-0.003251	7.997201	0.015625	0.390638
Gray	10.53687	0.004152	7.997259	0.015690	0.385021
Skipjack	10.48999	0.005421	7.996980	0.015605	0.386930

linear and differential approximation probabilities. The all eight nonlinearity scores of proposed five S-boxes are given in Table 6. The minimal score of nonlinearity is also shown to highlight that proposed S-boxes have high score of minimal nonlinearity and capable to mitigate the minimal nonlinearity based attack. Moreover, the cryptographic performance features of proposed five S-boxes are also compared with an exhaustive list of state of the art S-boxes in Table 7. We selected those S-boxes whose average nonlinearity score is about 106 for comparison in Table 7. From comparison Table, it is clear that the proposed S-boxes (preferably the S-box-3, Sbox-4 and S-box-5) have remarkably better performance compared to almost all of the S-boxes (including recent ones) listed in the comparison Table. They show exhibits similar strengths and features as that of AES, APA, Gray, S8-AES S-boxes.

V. STATISTICAL ANALYSIS

To analyze the quality of the S-box constructed with the help of chaotic tent-sine system, the *Lena* plaintext image is substituted with five different proposed S-boxes. Moreover, we used our proposed S-boxes in the encryption technique (encryption technique of AES is followed). Fig. 4 gives the pictorial representation of the *Lena* plain-image and substituted images using proposed S-boxes. While Fig. 5 shows the corresponding histograms of plain-image and substituted images. The plain-image *Lena* and its encrypted images using proposed S-boxes in the AES encryption scheme are shown in Fig. 6. The histograms of the original and encrypted images are given in Fig. 7. To show the strength of our technique, some statistical analyses under Majority Logic Criteria (MLC) [56] are described below.

A. CORRELATION

Correlation is considered as one of the basic methods to calculate the similarity between two images. The correlation is given by:

$$Corr = \sum \frac{(i-\mu i)\,(j-\mu j)\,p\,(i,j)}{\sigma_i \sigma_j} \tag{10}$$

where, $p(i, j)$ indicates the pixel value and i represent the position of row and j indicates its column value of digital images. The parameters μ *and* σ are the variance and standard deviation respectively.

B. ENTROPY

The magnitude of the improbability of a random variable to become the part of a random process is done in entropy. This analysis is used to depict the randomness of digital images. It can be defined as:

$$C = -\sum p(x_i) log_2 p(x_i) \tag{11}$$

where, probability of random variable is given by $p(xi\ )$.

C. CONTRAST

Contrast analysis facilitates the user to see objects vividly to identify the texture of an image. The general value of contrast is given by:

$$C = \sum |i-j|^2 p(i,j) \tag{12}$$

D. HOMOGENEITY

The nearness of the distribution in the gray level co-occurrence matrix (GLCM) to GLCM diagonal is measured in homogeneity analysis. This matrix shows the calculations of combinations of pixel brightness outcomes in tabular form. It can be given as:

$$Hom = \sum \frac{p(i,j)}{1+|i-j|} \tag{13}$$

E. ENERGY

In a digital image, squaring and taking the sum of gray pixels give the energy of the image. It is defined as:

$$E = \sum p(i,j)^2 \tag{14}$$

These different MLC analyses are performed to assess the best suited S-box for encryption techniques and multimedia security purposes. The comparison of the results of these analyses on the proposed technique with S-boxes such as AES, APA, S8 AES, Gray, and Skipjack are also performed. The MLC results after performing substitution operation are listed in Table 8 for proposed S-boxes and AES, APA, S8-AES, Gray and Skipjack S-boxes as well. The results indicate that the proposed S-box offers better statistical visual distortion effect than conventional S-boxes. The value of entropy is same for plain-image and all substituted images due to the same distribution of pixels. Whereas, Table 9 is maintained to provide the results of entropy, correlation, homogeneity, energy, contrast for plain-image and encrypted images. Here, the encryption is performed by AES algorithm using mentioned S-boxes. Again, the encryption results show the strength of our proposed S-boxes compared to others. The encryption outcomes of our proposed S-boxes are sufficiently satisfactory for secure communication applications.

VI. CONCLUSION

The most crucial components in the block encryption algorithms are substitution-boxes. They play vital role in the substitution-permutation network to offer sufficient nonlinearity and confusion. In this paper, a chaotic Tent-Sine system is applied for the construction of strong S-boxes. The Mobius transformation is applied to random values obtained through the chaotic map and provides 256 unique elements of the generated S-box. The randomness produced due to the inclusion of improved chaos increases the unpredictability of the cipher. The algebraic transformation fetches strength for the S-boxes. The results of the different statistical analyses indicate the extremely good cryptographic performance of our new S-boxes. The generated S-boxes show good results as compared to some well-known S-boxes, as apparent from the different statistical analyses.

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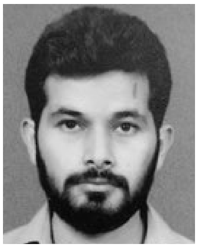

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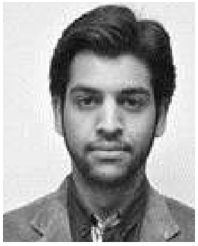

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