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ABSTRACT

The overall intent is to clarify relationships between psychological constructivist, sociocultural, and emergent perspectives by grounding them in attempts to understand what might be happening in a variety of teaching and learning situations. The first part of the paper outlines an interpretive framework developed in the course of a classroom-based research project. At the level of classroom processes, the framework involves an emergent approach in which psychological constructivist analyses of individual activity are coordinated with interactionist analyses of classroom interactions and discourse. At the level of school and societal processes, the perspective taken is broadly sociocultural and focuses on the influence of individuals' participation in culturally-organized practices. In the second part of the paper, the framework is taken as background against which to compare and contrast the three theoretical perspectives. The emergent approach augments the psychological constructivist perspective by making it possible to locate analyses of individual students' constructive activities in social context. In addition, the purposes for which emergent and sociocultural perspectives might be appropriate are considered and observed to span the individual students' activities, the classroom community, and broader communities of practice. Contains 75 references. (Author/MKR)

Constructivist, Emergent, and Sociocultural Perspectives in the Context of Developmental Research

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CONSTRUCTIVIST, EMERGENT, AND SOCIOCULTURAL PERSPECTIVES IN THE CONTEXT OF DEVELOPMENTAL RESEARCH[1]

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Our overall intent is to clarify relationships between the psychological constructivist, sociocultural, and emergent perspectives by grounding them in our attempts to understand what might be happening in a variety of teaching and learning situations. In the first part of the paper, we therefore outline an interpretive framework that we have developed in the course of a classroom-based research project. At the level of classroom processes, the framework involves an emergent approach in which psychological constructivist analyses of individual activity are coordinated with interactionist analyses of classroom interactions and discourse. At the level of school and societal processes, the perspective taken is broadly sociocultural and focuses on the influence of individuals' participation in culturally-organized practices. In the second part of the paper, the framework is taken as background against which to compare and contrast the three theoretical perspectives. We discuss how the emergent approach augments the psychological constructivist perspective by making it possible to locate analyses of individual students' constructive activities in social context. In addition, we consider the purposes for which the emergent and sociocultural perspectives might be appropriate and observe that they together span the individual students' activities, the classroom community, and broader communities of practice.

It can be argued that one of the most significant developments in educational research during the past decade has been the increasingly prominent role played by constructivist and sociocultural approaches. In contrast to the initial claims made by adherents to each perspective for the hegemony of their own views, there appears to be a growing consensus that the perspectives are at least partially complementary (Cobb, 1994; Confrey, 1995; Hatano, 1993; Smith, in press; Steffe, 1995). Our interest in the relationship between sociocultural theory and various forms of constructivism is pragmatically based and stems from our involvement in a classroom-based research and development project. In the course of our work with teachers and their students, we addressed a variety of issues by drawing on several different interpretive perspectives. The views we will advance in this paper about the relationships between interpretive perspectives are therefore rooted in our activity of attempting to understand what might be going on in a range of specific teaching and learning situations.

In the first part of this paper, we describe the interpretive framework that we currently use when analyzing teachers' and students' activity. Our intention in doing so is to ground the proposed relationships between perspectives in the settings from which they first emerged. At the level of classroom processes, this frame-

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work represents an emergent or social constructivist approach that evolved from an initial psychological constructivist position. The framework was subsequently extended beyond the classroom level to the school and societal levels by drawing on sociocultural theory. In the course of the discussion, we justify the framework by indicating the unanticipated problems that we found ourselves addressing and the interpretive stances that we eventually took. In the second part of the paper, this presentation of the framework is then used as a backdrop against which to compare and contrast psychological constructivist, emergent, and sociocultural perspectives.

The approach we will take of attempting to ground theory in practice reflects the view that the relationship between theory and practice is reflexive (Cobb, in press; Simon, 1995). Theory is seen to grow out of practice and to feed back to inform and guide practice. This approach can be contrasted with more traditional styles of presentation in which the basic principles or tenets of theoretical positions are stated, and then implications are deduced for practice. As Schön (1983) observes, this rhetorical style elevates theory over practice and enacts a positivist epistemology of practice, thereby devaluing the relation between theory and practice as it is lived by reflective practitioners (Ball, 1993; Lampert, 1990; Simon & Blume, 1994). Further, it positions researchers and practitioners in superior and subordinate roles as producers and consumers of theory. In contrast, alternative approaches that attempt to ground theory in practice tend to position researchers and practitioners in more collaborative roles and to treat their areas of expertise as complementary (Nicholls & Hazzard, 1993). Approaches of this type also acknowledge the importance of developing a basis for communication between researchers and practitioners. As a consequence, they seem to have greater potential to contribute to current reform efforts.

The Interpretive Framework

The interpretive framework we will outline was developed in the course of an ongoing program of developmental research (Gravemeijer, 1994). The basic developmental research cycle consists of two closely related phases (see Figure 1). At the most global level, our goal has been to investigate ways of supporting elementary school students' conceptual development in mathematics. As part of this process, we and our colleagues have developed both sequences of instructional activities for students and an approach to professional development for teachers. The general methodology employed is that of the classroom teaching experiment conducted in collaboration with a practicing teacher who is a member of the research and development team. In the past nine years,we have completed a series of these experiments at the first-, second-, and third-grade levels. It became apparent in the first of these experiments that the individualistic psychological constructs that we had intended to use to account for mathematical learning were inadequate for our purposes. As a consequence, one of our primary theoretical objectives became that of exploring ways to account for students' mathematical development *as it occurs in the social context of the classroom.* Analyses of this type are central to the second of the two phases of the development research cycle shown in Figure

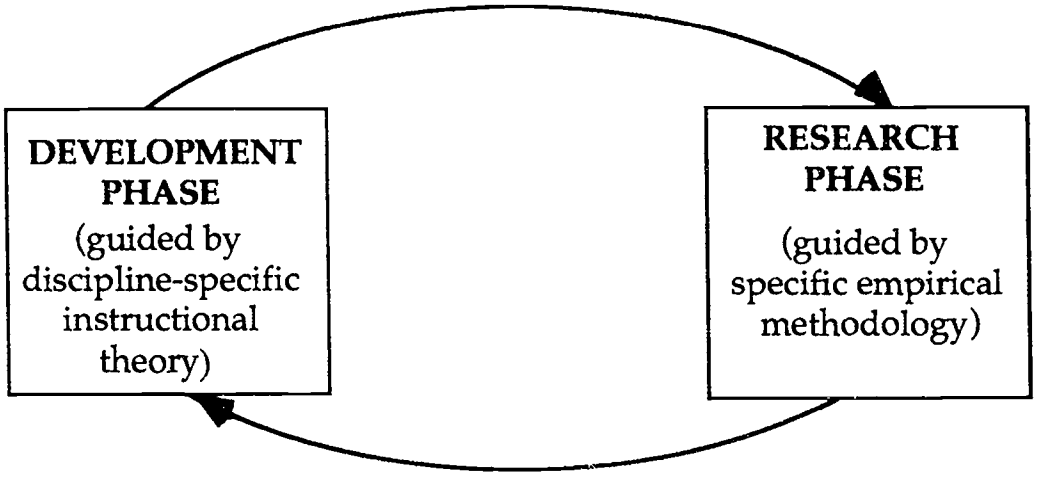

Figure 1. Phases of the Developmental Research Cycle[2]

1 and feed back to inform ongoing development efforts. The interpretive framework we will outline should be viewed as a response to this issue of accounting for learning in social context. Although our focus has been on students' development, there is some indication that the framework might be appropriate for analyses of teachers' socially-situated activity (Simon, 1995). However, we want to avoid the essentialist implication that the framework might somehow capture the structure of individual and collective activity independent of history, situation, and purpose. Our strongest claim is that we have found the framework useful when attempting to support change at the classroom and school levels.

The Classroom Level

It is important to clarify that when we speak of the classroom level, we do not mean this as a physical location. Instead, our intent is to indicate that explanations are formulated in terms of processes that occur in the classrooms—individual interpretations and actions, and face-to-face interactions and discourse. Thus, these explanations of students' activity in the classroom do not make reference to their participation in practices outside of the classroom. The interpretive framework at this level is shown in Figure 2.

The column heading "Social Perspective" refers to an interactionist perspective on communal or collective classroom processes (Bauersfeld, Krummheuer, & Voigt, 1988). The column heading "Psychological Perspective" signifies a psychological constructivist perspective on individual students' (or the teacher's) activity as they participate in and contribute to the development of these communal processes (von Glasersfeld, 1984, 1992). The version of social constructivism to which we sub-

[2] The domain-specific instructional theory referred to is that of Realistic Mathematics Education developed at the Freudenthal Institute (Streefland, 1991; Treffers, 1987). This developmental research cycle is, in many ways, analogous to the mathematical teaching cycle described by Simon (1995).

Social Perspective	Psychological Perspective
Classroom social norms	Beliefs about own role, others' roles, and the general nature of mathematical activity in school
Sociomathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical conceptions

Figure 2. An Interpretive Framework for Analyzing Individual and Collective Activity at the Classroom Level.

scribe is called the *emergent approach* or the *emergent perspective* and involves the explicit coordination of interactionism and psychological constructivism (Cobb & Bauersfeld, 1995). In the following paragraphs, we outline the framework by discussing social norms, then sociomathematical norms, and finally classroom mathematical practices.

Social norms. When we conducted our first classroom teaching experiment during the 1986-87 school year, we initially viewed learning in almost exclusively psychological constructivist terms. This methodology was in fact devised as an extension of the constructivist teaching experiment in which the researcher interacts one-on-one with a single child and attempts to influence the child's constructive activities (Cobb & Steffe, 1983; Steffe, 1983). In the case of the constructivist teaching experiment, the goal is to account for the child's development of increasingly powerful mathematical ways of knowing by analyzing the cognitive restructurings he or she makes while interacting with the researcher. In a similar manner, we intended to account for individual children's learning in the classroom by analyzing the conceptual reorganizations they made while interacting with the teacher and their peers. With hindsight, it is apparent that the relation between social interaction and children's mathematical development implicit in this approach was neo-Piagetian. We assumed that conflicts in individual students' mathematical interpretations might give rise to internal cognitive conflicts, and that these would precipitate mathematical learning (cf. Doise & Mugny, 1984; Perret-Clermont, 1980). In this account, social interaction was viewed as a catalyst for otherwise autonomous cognitive development in that it influenced the process of mathematical development but not its products, increasingly sophisticated mathematical ways of knowing.

The first unanticipated issue that we addressed in the classroom teaching experiment arose within the first few days of the school year. The second-grade teacher with whom we collaborated engaged her students in both collaborative small group work and whole class discussions of their mathematical interpretations and solutions. However, it soon became apparent that the teacher's expectation that the children would publicly explain how they had actually interpreted and solved tasks

ran counter to their prior experiences of class discussions in school. The students had been in traditional classrooms during their first-grade year and seemed to take it for granted that they were to infer the response the teacher had in mind rather than to articulate their own understandings. The teacher coped with this conflict between her own and the students' expectations by initiating a process that we subsequently came to term the renegotiation of classroom social norms. Examples of social norms for whole class discussions that became explicit topics of conversation included explaining and justifying solutions, attempting to make sense of explanations given by others, indicating agreement and disagreement, and questioning alternatives in situations where a conflict in interpretations or solutions has become apparent. In general, social norms can be seen to delineate the classroom participation structure (Erickson, 1986; Lampert, 1990).

A detailed account of the renegotiation process has been given elsewhere (Cobb, Yackel, & Wood, 1989). For our purposes, it suffices to note that a social norm is not a psychological construct that can be attributed to any particular individual, but is instead a joint social construction. As a consequence, we would object to accounts framed in individualistic terms in which the teacher is said to establish or specify social norms for students. To be sure, the teacher is necessarily an institutionalized authority in the classroom (Bishop, 1985). However, in our view, the most the teacher can do is express that authority in action by initiating and guiding the renegotiation process. The students also have to play their part in contributing to the establishment of social norms.[3] One of our primary contentions is in fact that in making these contributions, students reorganize their individual beliefs about their own role, others' roles, and the general nature of mathematical activity (Cobb et al., 1989). As a consequence, we take these beliefs to be the psychological correlates of the classroom social norms.

It is important to clarify that, in the view we are advancing, neither the social norms nor individual students' beliefs are given primacy over the other. Thus, it is neither a case of a change in social norms causing a change in students' beliefs, nor a case of students first reorganizing their beliefs and then contributing to the evolution of social norms. Instead, social norms and beliefs are seen to be reflexively related such that neither exists independently of the other. We can further clarify our position by building on Whitson's (in press) observation that what are seen are human processes that can be analyzed in either psychological or social terms depending on the issues at hand. A social analysis conducted from the interactionist perspective documents the evolution of social norms, and an analysis conducted from the psychological constructivist perspective documents students' reorganization of their beliefs. The social constructivist or emergent approach to which we subscribe draws on both these analyses and treats them as

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[3] Cooney's (1985) analysis of Fred, a beginning mathematics teacher, provides an excellent illustration of this point. In our terms, Cooney documents the difficulties that Fred encountered when he attempted to initiate the renegotiation of social norms and institute a problem solving approach.

complementary. In this joint perspective, the social norms are seen to evolve as students reorganize their beliefs and, conversely, the reorganization of these beliefs is seen to be enabled and constrained by the evolving social norms.

Sociomathematical norms. Thus far, in describing our initial interest in classroom social norms, we have explained why we found it necessary to go beyond an exclusively individualistic psychological perspective. We should again stress that we did not analyze social norms as an end in itself. Instead, our overriding motivation was to account for students' mathematical development as it occurred in the social context of the classroom. In this regard, one aspect of our analysis of social norms that proved disquieting was that it was not specific to mathematics, but applied to almost any subject matter area. For example, one would hope that students would challenge each other's thinking and justify their own interpretations in science and literature lessons as well as in mathematics lessons. As a consequence, our focus shifted in subsequent analyses to the normative aspects of whole class discussions that are specific to students' mathematical activity (Lampert, 1990; Voigt, 1995; Yackel & Cobb, in press). Examples of such *sociomathematical norms* include what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation.

As part of the process of guiding the development of an inquiry approach to mathematics in their classrooms, the teachers with whom we have worked regularly asked the students if anyone had solved a task a different way and then questioned contributions that they did not consider were mathematically different. It was while analyzing classroom interactions in these situations that sociomathematical norms first emerged as an explicit focus of interest. The analysis indicated that, on the one hand, the students did not know what would constitute a mathematical difference until the teacher and other students accepted some of their contributions but not others. Consequently, in responding to the teacher's requests for a different solution, the students were both learning what counts as a mathematical difference and helping to interactively constitute what counts as a mathematical difference in their classroom. On the other hand, the teachers in these classrooms were themselves attempting to develop an inquiry form of practice and had not, in their prior years of teaching, asked students to explain their thinking. Consequently, the experiential basis from which they attempted to anticipate students' contributions was extremely limited. Further, they had not necessarily decided in advance what would constitute a mathematical difference. Instead, the teachers seemed to clarify their own understanding of mathematical difference as they interacted with their students. Viewed in this way, the sociomathematical norm of mathematical difference appeared to emerge in the course of joint activity via a process of often implicit negotiation. A similar conclusion also holds for the other sociomathematical norms we have analyzed (Yackel & Cobb, in press).

The analysis of sociomathematical norms has proved to be pragmatically significant in that it has helped us understand the process by which the teachers with whom we have collaborated fostered the development of intellectual autonomy in

their classrooms. This issue is particularly significant to us in that the development of student autonomy was a goal of our work in classrooms that was explicitly stated from the outset. We originally characterized intellectual autonomy in terms of students' awareness of and willingness to draw on their own intellectual capabilities when making mathematical decisions and judgments. This view of intellectual autonomy was contrasted with intellectual heteronomy wherein students rely on the pronouncements of an authority to know how to act appropriately (Kamii, 1985; Piaget, 1973). As part of the process of supporting the growth of autonomy, the teachers with whom we have worked guided the development of a community of validators in their classrooms. In doing so, they necessarily had to encourage the devolution of responsibility (cf. Brousseau, 1984). However, their students could assume these responsibilities only to the extent that they had developed personal ways of judging that enabled them to know-in-action both when it was appropriate to make a mathematical contribution and what constituted an acceptable contribution. This required, among other things, that the students could judge what counted as a different mathematical solution, an insightful mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation. However, these were precisely the types of judgments that they and the teacher negotiated when establishing sociomathematical norms. We therefore inferred that students constructed specifically-mathematical beliefs and values that enabled them to act as increasingly autonomous members of classroom mathematical communities as they participated in the renegotiation of sociomathematical norms (Yackel & Cobb, 1993). These beliefs and values, it should be noted, are psychological constructs and constitute what the National Council of Teachers of Mathematics (1991) calls a mathematical disposition. We view them as the psychological correlates of the sociomathematical norms and consider the two to be reflexively related (see Figure 2).

It is apparent from the account we have given that we revised our conception of intellectual autonomy in the course of the analysis. At the outset, we defined autonomy in psychological terms as a characteristic of individual activity. However, by the time we had completed the analysis, we came to view autonomy as a characteristic of an individual's participation in a community. Thus, although the development of autonomy continues to be a central pragmatic goal for us, we have redefined our view of what it means to be autonomous by going beyond our original psychological constructivist position. This shift in perspective has enabled us to be more effective in helping teachers support the development of autonomy in their classrooms (McClain, 1995).

Classroom mathematical practices. The third aspect of the interpretive framework, that concerning classroom mathematical practices, was motivated by the realization that one can talk of the mathematical development of a classroom community as well as of individual children. For example, in the second-grade classrooms in which we have worked, various solution methods that involve counting by ones are established mathematical practices at the beginning of the school year. Some of the students are also able to develop solutions that involve the conceptual creation of units of ten and one. However, when they do so, they are obliged to

explain and justify their interpretations of number words and numerals. Later in the school year, solutions based on such interpretations are taken as self-evident by the classroom community. The activity of interpreting number words and numerals in this way has become an established mathematical practice that no longer stands in need of justification. From the students' point of view, numbers simply are composed of tens and ones—it is a mathematical truth.

This illustration from the second-grade classrooms describes a global shift in classroom mathematical practices that occurred over a period of several weeks. An example of a detailed analysis of evolving classroom practices can be found in Cobb et al. (in press). We contend that analyses of this type are appropriate for the purposes and interests of developmental research in that they document instructional sequences as they are realized in interaction in the classroom. They therefore draw together the two general phases of developmental research, instructional development and classroom-based research, and feed back to inform ongoing development efforts (see Figure 1).

Analyses of this type are also of theoretical significance in that they bear directly on the issue of accounting for mathematical learning as it occurs in the social context of the classroom. Viewed against the background of classroom social and sociomathematical norms, the mathematical practices established by the classroom community can be seen to constitute the immediate, local situations of the students' development. Consequently, in identifying sequences of such practices, the analysis documents the evolving social situations in which students participate and learn. Individual students' mathematical conceptions and activities are taken as the psychological correlates of these practices, and the relation between them is considered to be reflexive. In particular, students actively contribute to the evolution of classroom mathematical practices as they reorganize their individual mathematical activities and, conversely, these reorganizations are enabled and constrained by the students' participation in the mathematical practices.

As a point of clarification, we should stress that psychological analyses typically reveal qualitative differences in individual children's mathematical interpretations even as they participate in the same mathematical practices. In general, analyses conducted from the psychological constructivist perspective bring out the heterogeneity in the activities of members of a classroom community. In contrast, social analyses of classroom mathematical practices conducted from the interactionist perspective bring out what is jointly established as the teacher and students coordinate their individual activities. In drawing on these two analytic perspectives, the emergent approach focuses on both the individual and the community. This approach seeks to analyze both the development of individual minds and the evolution of the local social worlds in which those minds participate (Balacheff, 1990).

Summary. We pause to make two points about the interpretive framework as we have outlined it thus far. The first point concerns a possible misinterpretation. In the past, we have sometimes been interpreted as saying that students' mathematical activity is essentially psychological and individualistic, but is constrained by social and cultural processes such as social norms. We therefore emphasize that

we consider students' mathematical activity to be social through and through in that it develops as they participate in classroom mathematical practices. More generally, our intent is not to classify the teacher's and students' activities into those that are intrinsically individual and those that are intrinsically communal. Instead, our proposal is to *coordinate* analyses of classroom processes that are conducted in psychological and in social terms.

The second point we want to make is methodological and concerns the notion of replicability in the context of developmental research. The results of developmental research consist of a variety of products and analyses. These include sequences of instructional activities and analyses of students' learning in social context as the sequences are realized in interaction in classrooms. On the one hand, the assumption that productive patterns of learning can occur when an instructional sequence is enacted in other classrooms is central to the developmental research enterprise. On the other hand, the conception of the teacher as one who continually adjusts his or her plans on the basis of ongoing assessments of students' understandings implies that complete replicability is neither desirable nor, perhaps, possible (cf. Ball, 1993; Simon, 1995). The enactment of an instructional sequence is therefore assumed to involve experimentation on the part of the teacher in the course of which the sequence as intended by its developers is deliberately revised and modified.

Taking account of this formulation of the issue, we observe that educational research is replete with more than its share of wildly disparate and irreconcilable findings. The primary source of difficulty in our view is that the independent variables of traditional experimental research are relatively superficial and have little to do with either context or meaning. Such approaches are difficult to justify if one follows Lemke (in press) and considers that the ecology of the classroom is semiotic and involves meaning-making in which one thing is taken as a sign for another. Lemke calls systems with semiotic ecologies ecosocial systems. From this point of view, students are seen to always perceive, act, and learn by participating in the self organization of a system which is larger than themselves—the community of practice established in the classroom. Learning can therefore be characterized as "an aspect of self organization, not just of the human organism as a biological system, but of ecosocial systems in which the organism functions as a human being" (Lemke, in press). It is precisely this sense of participation in an evolving community of practice that is typically ignored in traditional educational research.

These considerations lead us to suggest that the relevant concept is commensurability rather than replicability. The difficulty is not so much that past findings have been disparate, but that they have been irreconcilable—it has not been possible to account for differences in findings when different groups of students received supposedly the same instructional treatment. The challenge as we see it is not that of replicating instructional sequences by ensuring that they are enacted in the same way in different classrooms. Instead, it is to develop ways of analyzing both instructional sequences and students' participation in them as they are realized in interaction in different classrooms. In this regard, we note that the framework as we have outlined it thus far illustrates one possible way to organize analy-

ses of both the classroom ecosocial system and the activity of the students (and teacher) who contribute to its development. For example, we have suggested that the constructs of social norms, sociomathematical norms, and classroom mathematical practices address aspects of the classroom microculture that are relevant to the purposes of developmental research. An analysis of classroom events organized in this way therefore might relate the emerging patterns of students' learning to their participation in sequences of instructional activities as they are realized in interaction. In addition, the teacher's role in guiding the development of both the classroom ecosocial system and the activity of the children who participate in it could become an explicit object of analysis, as could the broader institutional contexts in which such systems are embedded.

We should clarify that the intent of these comments is neither to recommend that others should necessarily use the specific framework we have outlined nor to claim that this framework resolves the commensurability issue once and for all. Instead, it is to illustrate the potential contribution of a framework of this type that is concerned with context and meaning. In particular, such a framework might support greater precision in developmental research by making it possible to compare, contrast, and relate different enactments of instructional sequences. This in turn would facilitate disciplined, systematic inquiry that embraces the messiness and complexity of the classroom.

The School and Societal Levels

In the course of our ongoing research and development activities, we have often been able to develop explanations that proved adequate for our purposes by referring solely to classroom processes. These analyses focus on the classroom ecosocial system as it is portrayed in the framework shown in Figure 2. There have, however, been occasions when we have found it essential to take account of the broader institutional contexts in which such systems are embedded. The elaborated version of the interpretive framework shown in Figure 3 synthesizes our reflections on these experiences. The central box replicates Figure 2 and corresponds to practices at the classroom level. The next box corresponds to practices at the school level, and the outermost box to practices at the societal level. In the following paragraphs, we provide a grounding for the elaborated framework. Later in the paper, we will reflect back on the discussion and use it as a setting in which to clarify distinctions between the emergent and sociocultural perspectives.

School level. The need to take account of broader institutional contexts first became apparent to us when we attempted to account for our experiences of working with approximately 50 first-, second-, and third-grade teachers at two action research sites. One of these sites was rural/suburban whereas the other served an almost exclusively inner-city student population. Our overall goal was to help these teachers revise the way in which they taught mathematics. To this end, we formulated an initial approach to teacher development at the rural/suburban site, where it proved to be reasonably successful. Our first priority when working with the teachers at this site was to help them make aspects of their textbook-based instruction prob-

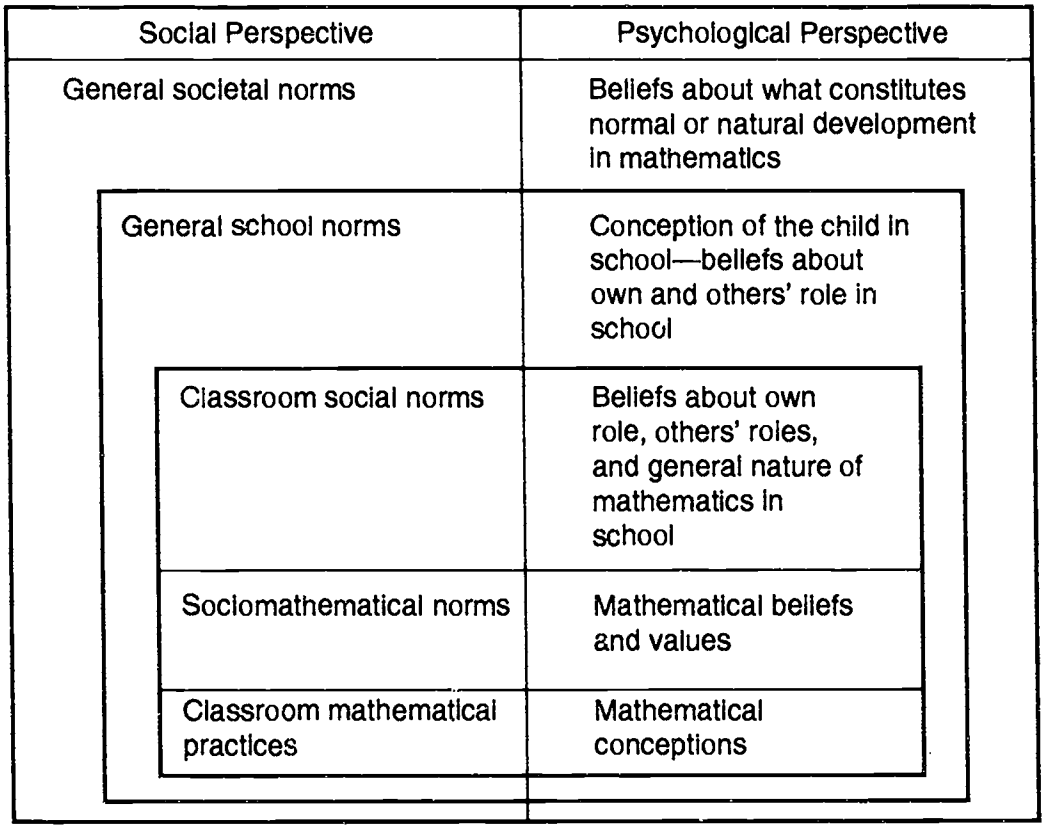

Figure 3. An Elaboration of the Interpretive Framework to the School and Societal Levels

lematic. We reasoned that only then would they have reason and motivation to want to reform their instructional practices while working with us. To this end, we used videorecordings of both individual interviews and classroom episodes to explore the consequences of traditional instruction. We have previously documented the success of this approach at the rural/suburban site. We observed, for example, that the teachers:

> began to differentiate between correct adherence to accepted procedures and [children's] mathematical activity that expressed conceptual understanding.
>
> As the teachers began to question the adequacy of textbook instructional activities and their current ways of teaching, they were then willing to consider alternative instructional activities designed to encourage meaningful mathematical activity. In doing so, they demonstrated the value they placed on children's mathematical sense-making. We did not have to convince them that children should learn with understanding. Rather, they had assumed that this kind of learning was occurring in their classrooms. *A shared desire to facilitate meaningful learning and a general concern for children's intellectual and social welfare*

> *constituted the foundation upon which we and the teachers began to mutually construct a consensual domain.* (Cobb, Wood, & Yackel, 1990, p. 140, emphasis added)

With our support during the school year, the 20 teachers referred to in the above passage radically revised the way they taught mathematics.

Shortly after this passage was written, we began working at the inner-city site. It soon became apparent that our initial approach to teacher development was not viable at that site. For the most part, an exploration of the consequences of traditional instruction did not lead these teachers to question their primarily drill-based approaches. It therefore appeared that whereas the teachers at the rural/suburban site assumed without question that students should learn mathematics with understanding, the beliefs and values of the teachers at the inner city site did not appear to be in conflict with traditional instructional practices. Subsequent efforts to support these teachers were more successful than our initial attempts in that several of them did develop forms of practice that were compatible with current reform recommendations in mathematics education. However, as we have documented elsewhere, the process by which these teachers reorganized their practices differed significantly from that of the teachers at the rural/suburban site (Feikes, 1992; Yackel & Cobb, 1993). This again indicates that there were differences in the two groups of teachers' underlying beliefs and values.

In reflecting on these experiences, we have come to realize that assumptions that we initially considered to be self-evident in fact reflect our culturally-specific beliefs and values. After working with teachers at the rural/suburban site, we had written that "a shared desire to facilitate meaningful learning and a general concern for children's intellectual and social welfare" constituted the foundation on which we and the teachers developed a basis for communication. At the time we wrote this statement, we assumed unquestioningly that engaging children in what for us counts as meaningful learning would necessarily be viewed as contributing to their welfare. However, our experiences at the inner-city site have led us to reconsider this assumption.

An analysis of observations made at the inner-city site during both classroom mathematics lessons and teacher induction sessions indicates that these teachers were deeply concerned about their students' intellectual and social welfare. However, there were crucial differences in what counted as intellectual and social welfare at the two sites (Yackel & Cobb, 1993). In particular, strictly enforced discipline seemed to be highly valued by teachers and administrators at the inner-city site. In addition, we did not observe instances where rules were discussed with students. Thus, although there were discussions of whether a rule had been violated, neither the appropriateness of the rules nor reasons for complying with them seemed to be topics of conversation.

In accounting for these differences between the two sites, we have come to the view that *what it means to be a child in school* is constituted by pedagogical communities (Banks, 1995; Walkerdine, 1988). Therefore this notion does not therefore appear to be fixed and universal, but is instead continually regenerated by the

members of a pedagogical community as they participate in the practices of schooling. At the inner city site, for example, to be a child in school was to follow specific rules and instructions. Further, to understand was to be able to verbalize relevant rules. Consequently, adults showed their concern for children's welfare by helping them learn to follow and verbalize rules. There was therefore no conflict at this site between the consequences of traditional mathematics instruction and the institutionalized views about what it means to be a child in school. This in turn implies that the teachers had no reason to revise their instructional practices.[4]

It is apparent that in the course of this discussion of our experiences at the two sites, we have viewed the teachers as representatives of particular communities of practice. As we will see, this approach of characterizing individuals in terms of community membership is typical of the sociocultural perspective. With regard to the implications of the analysis, we observe that core beliefs and values implicit in current reform recommendations are compatible with those of the teachers at the rural/suburban site but conflict with those of teachers at the inner-city site. This, for us, raises the very real possibility that reform efforts in which mathematics educators assume that their culturally-situated commitments are universal might well result in even greater disparities in the types of mathematics education that children experience than is currently the case. We therefore join Apple (1992) in calling for mathematics educators to explicate the ideological assumptions underpinning their reform recommendations. Only then might we be able to guard against the possibility that we will unknowingly foster even greater inequities.

Societal level. The grounding for the discussion of practices at this level is provided by an analysis reported by Yang and Cobb (1995). At the outset, our goal was simply to build on previous investigations of the mathematics achievement of Asian and American students by comparing the arithmetical learning of children in Taiwan and the U.S. However, in the course of the analysis, we came to the view that children in the two countries were participating in very different types of learning activities, and that these activities were culturally organized at the societal level.

With regard to the specifics of the investigation, the analysis covered preschool through second grade and dealt with arithmetical developments up to and including the construction of place value conceptions. Consistent with previous investigations, an analysis of videorecorded individual interviews indicated that there were significant differences in the quality of the two groups of children's arithmetical conceptions that favored the Chinese children in Taiwan (cf. Stevenson & Lee, 1990). In addition, an interactional analysis of classroom videorecordings made in the two countries indicated that there were important differences in the

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[4] We have been asked on several occasions whether the differences between the school communities reflect differences in the wider communities in which they were embedded. It would be inappropriate for us to address this issue for ethical reasons that pertain to the nature of the relationships we established with teachers and administrators at the two sites. As a consequence, a level corresponding to the wider community beyond the school is not included in Figure 3.

obligations that the children had to fulfill to appear competent (cf. Stigler, Fernandez, & Yoshida, 1992). However, the most relevant differences for our current purposes were those between the sequences of learning activities in which the children in the two countries participated. These sequences were identified by analyzing textbooks and by interviewing parents and teachers of the kindergarten, first-, and second-grade students. The issues addressed in these interviews included the types of learning activities that the teachers and parents considered most important for children's arithmetical development, the specific concepts and methods that children were expected to develop, the extent to which children needed either assistance or directed instruction, and the parents' and teachers' expectations for childrens' competencies at various age and grade levels.

The analysis indicated that there were important differences in the teachers' and parents' expectations for both the learning routes that the children would follow and the competencies they would develop, and in the extent to which the adults believed that it was necessary to provide direct support. In addition, there appeared to be differences in the internal consistency and coherence of the sequences of learning activities in the two countries. The American learning activities appeared to involve a major discontinuity in that the children's initial experiences in situations involving single-digit numbers did not appear to constitute a basis for their subsequent construction of place value conceptions. Significantly, the American teachers and parents considered that place value was a challenging concept and that it should be delayed until the second grade. In addition, the American teachers unequivocally stated that direct instruction was required. By way of contrast, the culturally-organized learning activities in Taiwan did not appear to have such contradictions. Further, the Chinese parents and teachers treated place value conceptions as relatively unproblematic developments that should begin in kindergarten. The tasks they posed and the questions they asked both seemed to reflect the view that it is natural for children to conceptualize numbers as composed of tens and ones at a relatively early point in their arithmetical development. In addition, they did not consider that this relatively easy developmental stage required direct instruction.

It is apparent from the analysis that the culturally-organized learning activities in which the Chinese students participated tended to enable the development of conceptual understanding in arithmetic to a far greater extent than did the learning activities in which the American students participated. Further, these differences in learning activities appeared to both corroborate and be supported by differences in the American and Chinese parents' and teachers' beliefs about what constitutes normal or natural development when children learn arithmetic. For example, the American parents and teachers had good reasons for believing that place value was a relatively late development. This belief in turn sustained pedagogical practices in which place value was experienced as a relatively challenging concept. Similarly, the Chinese teachers' and parents' beliefs were both expressed in and corroborated by the culturally-organized learning activities in which they and the children participated. It therefore seems reasonable to characterize these

two contrasting sets of beliefs about normal development as culturally-situated social constructions that are reflexively verified in practice.

The general position that we have arrived at is consistent with sociocultural approaches in that the American and Chinese children's contrasting arithmetical competencies are accounted for in terms of their participation in differing sequences of culturally-organized learning activities (cf. Cole, 1990; Lave & Wenger, 1991; Rogoff, 1994). Explanations of this type can be contrasted with an alternative orientation consistent with mainstream American psychology in which culture is treated as a cluster of variables that influences the course of essentially individualistic psychological processes. It should also be noted that the characterization of beliefs about psychological development as social constructions applies as much to widely accepted academic theories as it does to so-called folk theories (Lave, 1988; Newman, Griffin, & Cole, 1989). This, of course, does not imply that academic theories are mere myths or fictions, or that they are nothing more than arbitrary social conventions. Our point is instead that these theories are culturally-situated and that their development is guided by particular concerns and interests (Barnes, 1977). In our own case, for example, we have come to see the emergent approach we have outlined as grounded locally in the practices of developmental research, and as located more globally in an encompassing activity system that constitutes schooling in the U.S.

Summary. The interpretive framework shown in Figure 3 emerged relative to our purposes and offers a way to organize analyses conducted from different perspectives. It was in fact with these analyses in mind that we have previously discussed possible relationships between theoretical perspectives (e.g., Cobb, 1994). The order in which we have described the various perspectives, starting with a psychological constructivist perspective and working our way out, retraces the developments in our thinking over the last several years. The discussion of the framework at the classroom level focused on the relation between psychological constructivism and the emergent perspective. The sociocultural perspective came to the fore when we considered practices at the school and societal levels. In the remainder of this paper, we step back to compare and contrast these theoretical orientations more directly.

Coordinating Perspectives

Psychological Constructivism and the Emergent Perspective

We have seen that the emergent perspective augments psychological constructivism by coordinating it with interactionism. The relationship between psychological constructivism and interactionism can be clarified by considering, as an illustration, a situation in which a researcher is interacting with one student. To the extent that a psychological constructivist analysis takes account of the interaction, the focus is on the student's interpretations of the teacher's actions. An analysis of this type is made from the perspective of the researcher, who is inside the interaction and is concerned with the ways in which the student modifies his or

her activity. In contrast, an interactional analysis is made from the outside, from the point of view of an observer rather than of a participant in the interaction. From this perspective, the focus is on regularities in the student's and researcher's interactions, and on the consensual meanings that emerge between them, rather than on the student's personal interpretations. As Voigt (1994) makes clear, these consensual meanings are not psychological elements that capture the partial match of individual interpretations, but are instead located at the level of interaction. Examples of such constructs illustrated during the discussion of the interpretive framework include social norms, sociomathematical norms, and classroom mathematical practices.

Despite claims made to the contrary, we contend that researchers who typically take an individualistic focus are not conducting an interactional analysis merely because the students whose activity they are analyzing happen to be interacting with others. The researcher is conducting a psychological analysis as long as he or she focuses on the activity of each of the interacting individuals and fails to take their joint activity as an explicit object of analysis (Blumer, 1969). By the same token, it should be clear that the emergent approach does not merely involve bolting a social component onto an otherwise unchanged psychological approach. Instead, the relation between the interactionist and psychological constructivist perspectives is considered to be reflexive. The characterization of learning as an individual constructive activity is therefore relativized in that these constructions are seen to occur as students participate in and contribute to the practices of the local community.

The comments that we have made thus far do not delegitimize psychological analyses of, say, interviews or one-on-one teaching sessions. However, we do question whether such analyses capture individual students' conceptual understandings independently of situation and purpose. From the emergent perspective, interviews are viewed as social events in which the researcher and child negotiate their roles, their interpretations of tasks, and their understanding of what counts as a legitimate solution and an adequate explanation (Mischler, 1986; Schoenfeld, 1987; Voigt, 1995). As a consequence, we believe that it is important to view the students' activity as being socially situated even in settings such as interviews that are typically associated with psychological paradigms. The psychological analysis would then be conducted against the background of a social analysis that documents the interactively constituted situation in which the student is acting.

We have argued that the emergent approach is consistent with the purposes of classroom-based developmental research. We have also clarified that analyses conducted in line with this approach can give greater weight to either the psychological or the interactionist perspective depending on the issues and purposes at hand. In each case, one perspective comes to the fore against the background of the other. This reciprocity between the psychological and the social in turn serves to differentiate the emergent approach from sociocultural approaches.

Emergent and Sociocultural Perspectives

The emergent and sociocultural perspectives have a number of points in common. For example, both reflect the view that learning and understanding are inherently social and cultural activities. The two positions therefore reject the view that social interactions serve as a catalyst for otherwise autonomous intellectual development. Further, both attend to the role of symbols and artifacts in conceptual development. However, whereas the emergent perspective subsumes psychological constructivism, the sociocultural perspective constitutes an alternative to approaches that attribute a primary role to individual students' constructive activities.

We have seen that from an emergent perspective, learning is a constructive process that occurs while participating in and contributing to the practices of the local community. In the case of the interpretive framework, for example, students were seen to actively construct their mathematical ways of knowing as they participated in the mathematical practices of the classroom community. The link between collective and individual processes in this approach is therefore indirect in that participation enables and constrains learning, but does not determine it. Participation is therefore seen to constitute the conditions for the possibility of learning (Krummheuer, 1992). In contrast, a Vygotskian perspective such as that advanced by van Oers (in press) treats the link between collective processes and individual processes as a direct one: The qualities of students' thinking are *generated by* or *derived from* the organizational features of the social activities in which they participate. This conjectured direct linkage allows sociocultural theorists to be more directive when making instructional recommendations. For example, van Oers (in press) suggests that students should imitate culturally-established mathematical practices when they interact with the teacher or more capable peer. He goes on to argue that help should be gradually withdrawn so that students take over functions they could not initially perform alone, thereby internalizing the cultural activity. This recommendation instantiates Vygotsky's frequently cited general genetic law of cultural development.

> Any higher mental function was external and social before it was internal. It was once a social relationship between two people.... We can formulate the general genetic law of cultural development in the following way. Any function appears twice or on two planes....It appears first between people as an intermental category, and then within the child as an intramental category. (1960, pp. 197-8)

The contrasting emphases of the sociocultural and emergent perspectives are reflected in differing characterizations of the teacher's role. In sociocultural accounts, the teacher is typically portrayed as a representative of society who supports students' discursive reconstruction of culturally-approved meanings (cf. Forman, in press). This view leads to a treatment of negotiation that is partially at odds with emergent accounts of communication. From the emergent perspective,

negotiation is a process of mutual *adaptation* that gives rise to shifts and slides of meaning as the teacher and students coordinate their individual activities, in the process of constituting the practices of the classroom community. However, from the sociocultural perspective, negotiation is a process of mutual *appropriation* in which the teacher and students continually co-opt or use each others' contributions (Newman, Griffin, & Cole, 1989). The teacher is therefore typically expected to insert culturally-approved insights that students can co-opt, and to appropriate students' actions into the wider system of mathematical practices that they are to reconstruct. In this account, the teacher negotiates with students in order to mediate between their personal meanings and established cultural meanings. However, in the emergent approach, it is the local classroom community rather than the mathematical practices institutionalized by wider society that are taken as the immediate point of reference. From this point of view, the teacher negotiates with students in order to initiate and guide both students' individual constructions and the evolution of consensual mathematical meanings so that they become increasingly compatible with culturally-approved meanings. In general, whereas sociocultural approaches frame instructional issues in terms of the *transmission* of culture from one generation to the next, the emergent perspective is concerned with the *emergence* of individual and collective meanings in the classroom.

A further contrast between the two perspectives concerns the treatment of semiotic mediation. It is important to clarify that the emergent approach fully accepts Vygotsky's (1987) fundamental insight that semiotic mediation is crucially involved in students' conceptual development. The issue under consideration is that of explaining the nature of this involvement. In line with its central tenets, sociocultural accounts of semiotic mediation give precedence to social and cultural processes over individual psychological processes. For example, in one line of explanation most directly associated with Vygotsky, cultural tools such as conventional mathematical symbols are said to be internalized and to become cultural tools for thinking (Davydov & Radzikhovskii, 1985; Rogoff, 1990). In a second line of explanation associated with Leont'ev (1978), individuals are said to appropriate cultural tools to their own activity. Both formulations distinguish between students' personal meanings and sociohistorically developed cultural meanings inherent in the appropriate use of cultural tools. Further, both approaches contend that students will develop particular culturally-approved meanings as they learn to use language and other semiotic means appropriately (cf. Forman, in press). These approaches therefore characterize symbols as primary vehicles of the enculturation process in that they serve as carriers of meaning from one generation to the next when students use them while engaging in culturally-organized activities (van Oers, in press). It was in this sense that Vygotsky referred to symbols as "objective tools" (Bauersfeld, 1995). The underlying metaphor is again that of transfer or transmission in that learning is characterized as a process in which students inherit the cultural meanings that constitute their intellectual bequest from prior generations.

In the alternative emergent perspective, learning is viewed as a process of both active individual construction and enculturation. Further, processes of signi-

fication are considered to be integral to both classroom mathematical practices and the activities of students who participate in them. For example, the mathematical practices established by a classroom community might involve reasoning with physical materials, pictures, diagrams, computer graphics, or notations. An analysis of classroom mathematical practices can in fact delineate emerging chains of signification (Walkerdine, 1988) that constitute what Lemke (in press) calls the semiotic ecology of the classroom (Cobb et al., in press). When attention shifts from the interactionist to the psychological constructivist perspective, the physical materials, symbols, and notations that students use are viewed as constituent parts of their individual activities rather than as external tools (Bateson, 1973; Dewey, 1977; Prawat, in press). As a consequence, the use of particular materials and symbols is considered to profoundly influence both the nature of the mathematical capabilities that students develop and the processes by which they develop them.

We contend that the account of signification offered by the emergent approach is better suited to the purposes of developmental research in that it provides greater precision than sociocultural approaches. For example, a sociocultural analysis of a classroom teaching experiment might account for students' learning in terms of their appropriation or internalization of particular semiotic means. The difficulty from our point of view is that such an analysis does not specify in any detail the evolving social situation of the students' development by analyzing instructional sequences as they are realized in interaction in a particular classroom. In addition, this approach has difficulty in accounting for qualitative differences in individual children's mathematical interpretations except to the extent that they can be tied to the students' participation in different out-of-school communities of practice (Confrey, 1995; Hanks, 1991). In contrast, we illustrated when discussing the interpretive framework that an emergent approach addresses both of these issues. Analyses developed from this perspective therefore have implications for both the revision of instructional sequences and the development of follow-up teaching experiments (Cobb et al., in press).

In this discussion, we have questioned the relatively common view that a sociocultural stance must be adopted if the central role of language and other semiotic means are to be addressed. As an alternative, we have suggested that an emergent approach is appropriate for some purposes in that it admits a psychological constructivist view of learning but sees it as inextricably bound up with processes of signification (cf. Confrey, 1995; Kaput, 1991; Pirie & Kieren, 1994; Sfard, 1991; Thompson, 1992). An emergent analysis might in fact be said to "unpack" appropriation processes posited by sociocultural theorists by specifying how they are realized in interaction by members of specific classroom communities. What, at the global level of the reproduction of culture, is viewed as a process of transmission becomes, at the local level of the classroom community, a process of emergence in which students' constructive activities and the practices in which they participate are considered to be reflexively related.

Thus far, we have focused on situations in which an emergent approach might be particularly appropriate. We turn now to consider the sociocultural perspective and do so by first discussing an analysis reported by Crawford (in press). In pro-

posing to view "conscious behavior as a reflection of the socio-cultural environment in which an individual functions," Crawford makes it clear that she is taking a strong sociocultural perspective. One of her primary interests is to understand situations in which "there are conflicts and inconsistencies between the values and priorities of cultural experience at home and at school." As an illustration, she discusses the conflicts that arise when children growing up in traditional Aboriginal communities in Australia participate in school mathematics activities.

> The resistance of many Aboriginal students to learning mathematics in schools has been interpreted as lack of ability by many educators. In fact, for many Aboriginal people, the value conflicts that arise as a result of the world view that is implicit in the elementary mathematics curriculum are substantial barriers to learning....[For example,] the very high priority given in Western culture to quantity and to quantifiable variables was not supported by everyday activities and modes of categorical thinking in traditional Aboriginal communities. (Crawford, in press)

Crawford goes on to observe that

> Aboriginal communities find the educational practice, used frequently by teachers of mathematics, of asking students questions when the answer is already known to the teacher, extremely puzzling and distasteful.

In addition, there are "substantial differences between Aboriginal and non-Aboriginal categorical thinking even about such perceptually grounded concepts as color." As a consequence, for Aboriginal children, "the primary colors were not immediately evident as a means of classification [of counters and other manipulative materials]."

We find Crawford's analysis compelling and suggest that, for her purposes, it would be counter-productive to "unpack" the process by which Aboriginal children appropriate the values and priorities of their communities. In the analysis, these children are portrayed as "carriers" of the culturally-based understandings of their communities. The vantage point that Crawford seems to adopt is therefore that of an observer located outside the cultural group. From this perspective, thought and activity within a cultural group appear to be relatively homogeneous when compared with differences between groups. This, it will be recalled, was also the perspective that we took when conducting the school-level and societal-level analyses. In the case of the teachers at the two action research sites, for example, we viewed them as representatives of different pedagogical communities whose activity reflected the priorities and values of those communities. Similarly, in the comparison of the arithmetical learning in Taiwan and the U.S., the children, teachers, and parents in the two countries were viewed as "carriers" of distinct systems of cultural beliefs and values. In the course of the analysis, we did in fact point out the qualitative differences in the mathematical activity of children within each of

the two national groups (Yang & Cobb, 1995). However, these observations were tangential to the major emphasis of the analysis and merely served to illustrate the possibility of unpacking sociocultural processes, thereby focusing on the constructive activities of individual children.

Crawford subsequently clarifies that situations involving tensions in individuals' needs, expectations, and goals are not limited to conflicts between home and school experience, but also include attempts to reform instruction. In such cases, the tension is between the needs, experiences, and goals of the innovators and the teachers, or between those of the teachers and the students. For example, in the school level analysis, our interactions with the teachers at the inner-city site can be characterized in terms of a conflict between our own and the teachers' culturally-situated beliefs about what it means to be a child in school. Further, our experiences of working with the teachers at both action research sites can be seen to involve a tension between our own and the teachers' views about the general nature of mathematical activity in school. In this regard, Crawford observes that teachers tend to teach in the ways in which they were taught. She accounts for this phenomenon in sociocultural terms by contending that future teachers internalize attitudes and beliefs about how mathematics is learned and about the role of the teacher from their own participation as students in the culturally-organized activities of schooling. In conducting an emergent analysis, we, for our part, would attempt to "unpack" this internalization process. It can be noted, for example, that the beliefs and attitudes to which Crawford refers are the psychological correlates of classroom social and sociomathematical norms. Consequently, from an emergent perspective, future teachers are seen to actively construct the beliefs, suppositions, and assumptions that subsequently find expression in their pedagogical activity when, as students, they participate in the negotiation of classroom social and sociomathematical norms. In this account, a global process of internalization from the sociocultural environment is recast as one of negotiation and individual construction at the classroom level. The issue for us is not which of these two accounts gets things right. Instead, it is to consider the situations in which one type of analysis or the other might be more helpful. In our view, the precision of the emergent account is appropriate for certain purposes. However, in other situations, the globalness of sociocultural accounts has it own advantage. In this respect, the two theoretical perspectives can be seen to complement each other. The sociocultural approach that Crawford illustrates focuses on the social and cultural bases of personal experience whereas analyses developed from the emergent perspective account for the constitution of social and cultural processes by actively cognizing individuals.

Conclusion

We have used Crawford's work as a paradigm case to illustrate the relevance of sociocultural approaches to issues of cultural diversity and of reform at a more global level. It should be clear from the discussion that we consider both sociocultural and emergent perspectives to be viable positions. We would also note that a

central notion common both to these two perspectives and to psychological constructivism is that of activity. Differences between the perspectives concern the positioning of the researcher and thus the way in which activity is framed.

In psychological constructivist approaches, the analytical position taken by the researcher is inside an ongoing interaction, and the focus is on the ways in which individual students reorganize their activity. The emergent approach coordinates analyses of this type with those conducted from the interactionist perspective. We have suggested that the analytical position taken in this latter perspective is that of an observer of ongoing interactions located outside the local community but inside the broader cultural community. From this vantage point, individual activity is seen to be situated within the practices of a local community such as that constituted by the teacher and students in the classroom. In contrast, the positioning of the sociocultural theorist is outside the cultural group. From this perspective, individual activity is situated in broad sociocultural practices, and learning is characterized as a process of internalization or appropriation while participating in these practices.

In the course of the discussion, we have clarified that the emergent approach coordinates the psychological constructivist and interactionist perspectives. This led us to suggest that analyses whose primary purpose is psychological should be conducted against the background of an interactionist analysis of the social situation in which the student is acting. The contrasts we drew between the emergent and sociocultural perspectives paid particular attention to the kinds of issues that analyses conducted from each perspective might reasonably address. In addition, we considered how the two perspectives might complement each other. These possibilities are worth pursuing in our view given that the perspectives together span the individual student's activity, the classroom community, and broader communities of practice. The interpretive framework we have outlined represents one attempt to achieve such a coordination.

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