CONSUMER INFORMATION, PRODUCT QUALITY, AND SELLER REPUTATION

by

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ABSTRACT

This thesis is concerned with the performance of markets in which buyers are unable to observe the quality of the products they buy prior to purchase. In such environments there is a temptation for sellers to reduce the quality of their items. An important force in such markets which gives firms an incentive to maintain quality is the formation of firm-specific reputations.

The recurring theme of this thesis is that while reputation can significantly prevent quality deterioration, it cannot work perfectly. Since a seller can always cheat on his customers (cut quality) without detection, at least for a little while, the incentives to maintain quality are not as strong as they are under perfect information.

The first essay analyzes a monopoly seller in this imperfect information environment. Viewing reputation as consumers' expectations of quality, it is shown that any self-fulfilling quality level is less than the perfect information quality level. Likewise, in a model where reputation adjusts towards previous quality, any steady state quality level is lower than the perfect information quality level. Furthermore, the slower is reputation adjustment, the lower is the steady-state quality level.

Thesis Supervisors: Richard Schmalensee, Professor of Applied Economics
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An analysis is also made of the pricing decisions of a monopolist who faces initial misperceptions of the quality of his product. It is shown that in many cases the initial imperfect information has lasting effects. There is a sharp distinction between initial over- and under-estimates of product quality.

A welfare analysis is made of the effects of imperfect information. Since monopoly power puts the analysis in a second-best world, the addition of informational problems can in general provide welfare benefits. The welfare effects of imperfect information are investigated both on the path to steady-state (while consumers' expectations are not fulfilled) and in the steady-state.

The second essay studies a perfectly competitive environment in which reputations play an important role. In this case, high quality items sell at a premium above cost, despite perfect competition. The quasi-rents which firms with good reputations earn provide a competitive rate of return to the reputation which is viewed as an asset. Since there is a divergence between price and cost for high quality items, there are welfare losses as a result of the imperfect information. An analysis is made of the welfare effects of various consumer information remedies.

Finally, an analysis of minimum quality standards in a model where producers choose what quality to produce is possible in the second essay. It is shown that raising the minimum quality standard provides benefits for consumers who prefer to consume high quality items. In general, optimal minimum quality standards exclude from the market products which some consumers would like to buy.

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This Thesis is Dedicated to

My Parents

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Consumer Information, Product Quality and Seller Reputation

Carl Shapiro*

M.I.T.

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* I would like to thank Peter Diamond, Joe Farrell, Drew Fudenberg, Eric Maskin, Steve Salop, Richard Schmalensee, and Jean Tirole for helpful comments and suggestions.

Economists are becoming increasingly aware of how imperfect information can cause a wide range of market imperfections. A great deal of effort has gone into careful analysis of uncertainty about the various prices at which a homogeneous product is being offered for sale.[1] Yet uncertainty about price is relatively minor and inexpensive to eliminate in comparison with uncertainty about other product characteristics (*e.g.*, durability, safety or taste). This paper is concerned with performance in markets where the products sold cannot be evaluated fully and accurately prior to purchase. The analysis will center on how a profit-maximizing firm chooses product quality in an environment where consumers cannot observe quality prior to purchase. I am particularly interested in how the quality of products provided depends on the manner and speed with which consumers gather information about products and how they enter and leave the market.

Qualitative uncertainty is a widespread and important feature of markets for most firms' goods and services. Virtually all services are impossible to evaluate until they are used. This includes medical and legal services, automobile repair, plumbing and electrical work, etc. Another important class of products whose quality cannot easily be judged prior to purchase is consumer durables. In fact, when a new model automobile comes out, there simply is no way of knowing what its repair record will look like. As a final class of examples, there are many products which we buy quite frequently which have unobservable attributes: restaurant meals (taste) and clothing (will it fade or shrink?) are two examples. The rise in both the complexity of products (consider hi fi equipment or ethical drugs) and the fraction of income spent on services has increased the importance of these informational problems.

The performance of the market in such a setting is essential to the evaluation of a wide variety of regulatory initiatives. These include occupational licensing (or minimum quality standards in general), occupational health regulations (the worker taking the role of the consumer), automobile safety rules, and a wide range of regulations by the Food and Drug Administration, Federal Trade Commission, and Consumer Product Safety Commission. Recent actions such as the FTC's proposed used car rule are designed precisely to improve the information in the market and thereby enhance performance. It is paramount, therefore, to describe market performance in the absence of public policy interference to be able to evaluate the worthiness of either mandatory standards or public provision of information.

The analysis in this paper is restricted to the case where a monopolist controls the quantity as well as quality of the good in question. Since imperfect information generally leads to some market power, this analysis is a necessary prerequisite to studying many suppliers in the presence of qualitative uncertainty. One treatment of such markets with many sellers is Shapiro [1980].

The paper is organized as follows: first I discuss a variety of modelling problems which arise once the perfect information assumption is removed. This is meant both to familiarize the reader with the problems and introduce some proposed solutions. Then I present the analysis in the case where the seller sets quality once and for all (a new product, for example). Finally, I treat the case where sellers can vary the quality of their products over time.

Product Characteristics and Quality

Since the issues described above revolve around product choice by a seller rather than simply pricing or output decisions for a given product, it is very useful to adopt Lancaster's (1966) framework and view products as bundles of characteristics. When products are viewed in this way, one question which arises is whether or not the market provides the socially optimal mix of product characteristics.[2] Since I wish to focus on product quality as opposed to product variety, I restrict quality to a single dimension. This should be thought of as some product attribute which is difficult or impossible to observe prior to purchase, but which consumers all like to have more of. Examples include durability, safety (probability of no accident), or speed of service. The restriction of quality to one dimension implies, in particular, that any two consumers will agree which of two products is preferred although they may disagree as to how much the added quality is worth.[3]

Enodenous Quality Choice: A Dynamic Problem

To understand the problems which occur when product quality is unobservable prior to purchase, it is important to distinguish the case where sellers choose product quality from the case where there is an exogenous supply of products of different qualities. The latter case is the one introduced by Akerlof in his seminal article [1970]. Unfortunately, models with exogenous quality supply are of limited usefulness in product markets. If the price offered depends only on average quality in the market, and higher quality items are more costly to produce, then in a static model there is no incentive for a given producer to provide anything other than minimal quality. Consequently, the market will be overrun by minimal quality items.[4] The same result occurs in a dynamic model if consumers do not learn about the quality of individual firms over time. The incentive to producing high quality items is that higher quality today will cause the demand curve in the future to shift out. So product quality choices by sellers are fundamentally dynamic.

<u>Information Flows</u>

It is apparent from these considerations that the technology of information flow will be essential to any story of market equilibrium. It is convenient to distinguish three facets of this technology: (1) How information flows among consumers, (2) How much a consumer learns about the product's quality from using it, and (3) How consumers enter and leave the market. Both the nature of the product and the institutions surrounding the market for the good influence the way in which information flows regarding each firm's products.

Personal Learning

The first stage in the process by which a firm's quality today is transmitted to potential future buyers is through the observations of those who have actually used the product. So the ability of consumers to observe product characteristics is crucial.

Product attributes which can be observed prior to purchase have been called search attributes (by Nelson [1970]). They present the same informational problems as do prices. In my opinion, these problems are minor in comparison with those involving attributes which require use to be observed. These latter characteristics have been divided into two classes: experience attributes which are observable after use (e.g., taste) and credence attributes which may remain unobservable even after use. (e.g., the structural integrity of an automobile).[5] It is intuitively clear that consumer information problems are most severe for credence properties of products. In fact, most regulations regarding product attributes are directed at credence attributes. Licensing of doctors, most regulations by the Food and Drug Administration and the Consumer Product Safety Commission are examples.

For most products the distinction between experience and credence properties is probably overdrawn. There is some learning about product attributes with use, but at varying speeds for various products. It is very useful in this context to think of the consumer as observing some outcome which depends on both the quality of the product and some unobservable random variable.[6] In the car safety example, if a car holds up well in an accident it is not clear whether the minimal damage is due to the type of accident or the way the car was made. A similar lack of observability holds true for drugs, services, etc. If a lawyer loses a case it is hard to know if he was ineffective or if the case was weak.

One way to incorporate learning into consumer choice is to assume that quality is positively related to the probability of repeat purchase. This approach has been taken in Schmalensee [1978] and Smallwood-Conlisk [1979]. One problem with it is that it gives no insight into how consumers respond to price.[7]

I have taken a different approach in what follows. At any point in time a consumer has some <u>expectations</u> regarding product quality. This determines the position of his demand curve. The learning then corresponds to adjusting expected quality towards true quality. Products for which quality is hard to observe even after use will display slow or lagged adjustment of consumer expectations. It should be noted that the ability of consumers to draw inferences about the firm's quality from using the product is not beyond the control of the firm. To the extent that firms have imperfect quality control, a consumer who gets a bad item may have difficulty knowing whether its poor quality is representative of that firm or not. For the purposes of this discussion, I will avoid this problem by assuming that all the products a firm sells on a given date have uniform quality.

Interpersonal Learning and the Market for Information

There are quite a few potential sources of information about product quality in addition to personal experience. These are (1) Experience of friends, (2) Publications, either public or private, such as Consumer Reports,[8] (3) Advertising, and (4) Potential signals of quality such as price, warranties, or advertising.

If the market for information about product attributes worked well, the informational problems in the markets for final goods and services would be substantially reduced or eliminated. There are several reasons why the information market cannot be expected to work well, however.

The first two reasons relate to the public good nature of information. The first is that it is difficult to prevent resale of information. A private firm that sells information may not be able to survive even though there is a substantial value to the information, because there would be no restrictions on individuals who buy the information passing it along (freely) to non-buyers. The second reason is due to the positive externality created by informed buyers which benefits uninformed buyers through raising quality (similar to what happens in Salop-Stiglitz [1977]). This quality influence is not accounted for by consumers when considering whether to buy information or not, and hence too little is purchased.

A final reason that private provision of information may not be possible is due to credibility problems. Consumers may fear that the information supplier is not entirely candid; perhaps he is being bribed by some producers, or under the influence of his advertisers. A governmental provision of information may be able to avoid these problems. This of course depends on how such an agency is set up to provide incentives for truthful provision of information.

In order to evaluate the desirability of information activities by public authorities, it is necessary first to understand the advantages which derive from improved information. That is the goal of the analysis to follow. The comments above are intended to highlight why the laissez-faire level of information can be expected to be suboptimal.

The possibility that warranties may provide information about quality will be ruled out in this paper. Spence [1977b] has shown that when quality refers to probability of failure of an item, warranties may serve as signals of quality. For many goods this is possible only to a very limited extent. Take a washing machine, for example. A warranty may hold for 1 or 2 years, but it is not possible even for manufacturers of good machines to offer comprehensive warranties. This is because the manufacturer cannot monitor maintenance or intensity of use. Such moral hazard and adverse selection problems are inevitable barriers to the absorption of product quality risks by the producer. In the example of legal services, it is possible to write incentive contracts, but adverse selection limits their scope.[9]

Finally, in this paper I do not include advertising, although its virtues as well as faults must be analyzed in the context of imperfect information. Advertising can be viewed as altering consumer's expectations of quality. Since there is an obvious incentive for producers to overrate their product, the key question here is why consumers pay any attention to such claims. The ability of advertising to convey information about product quality is something I hope to treat in the future.

I have tried in this section to identify factors which influence the speed of learning by consumers. The analysis below focuses on the relationship between that speed and the quality of product chosen by the monopolist. In examples where the only learning is personal learning, it is possible to be quite explicit about how the firm's demand curve shifts in response to learning. Such examples place an upper bound on the informational problems in the market, since there are in fact additional information sources, as discussed above.

General Formulation of Quality Choice by a Monopolist Under Imperfect Information

Consider the problem faced by a firm in setting its quality, q. At any point in time the firm can reap extra profits by cutting quality; there will be no loss in revenues until consumers can respond to the quality change.

Since the problem is essentially dynamic, let t denote time and call p(t) the price at time t, q(t) the quality, and x(t) sales. There is also a cost function c(x,q) in quantity and quality.

There may also be costs to changing quality i.e., once and for all costs to introducing a new quality line. I ignore these for the most part, except to note that they justify the attention paid to once-and-for-all quality choices below. In fact we do not usually observe frequent quality changes by sellers; price changes occur much more frequently. This is further justification for the treatment in the following section.

Profits at time t are given by

$\pi(t) = p(t)x(t) - c(x(t), q(t))$.

The crucial question is how x(t) depends on p(t) and previous quality choices.[10] One approach is to treat x(t) as the state variable representing a loyal set of patrons. Then the inflow of new customers and the outflow of dissatisfied customers will in general depend on quality, price, and the stock of customers itself. Smallwood and Conlisk [1979] took this approach in discrete time with quality being the probability any given customer would return the next period. This required them to take price as fixed and exogenous, and they could not generally determine what quality a firm would choose. Furthermore, they did not allow quality changes over time.[11] Farrell [1979] has also taken this approach and is able to study the optimal q(t) path for a firm, again taking price as fixed and exogenous.

The approach in this paper is quite different since I am interested in allowing price changes over time, and view the firm's <u>reputation</u> as the state variable rather than its sales. In this view, previous quality and sales influence what various consumers think about the quality of a firm's product, and it is this reputation, R(t), which determines the location of the firm's demand curve. Consequently, the firm's pricing decisions over time can be studied, and consumers' individual demand curves can be derived from utility functions and expectations of quality.[12]

This approach does require specifying the process by which reputation adjusts over time. I argue strongly for <u>adaptive</u> expectations by consumers in response to quality changes by a seller. There is no evidence at all to support a more sophisticated approach by consumers in which they solve out the firm's optimal control problem to figure out what to expect. Instead, evidence on consumer behavior suggests that to a large extent consumers extrapolate from recent experience to predict future product performance. This is a consequence of the information processing behavior consumers employ.[13]

Viewing a firm's reputation as the state variable in the dynamic setting outlined above, the firm's problem is:

$$\max_{x(t),\, q(t)} \int_{t=0}^{\infty} e^{-rt}[x(t)p(x(t),\, R(t)) - c(x(t),\, q(t))]dt$$

$$\text{subject to} \quad \dot{R}(t) = f(x(t),\, q(t),\, R(t)).$$

$$R(0) \text{ given.}$$

Here the inverse demand curve the firm faces at time t is $p(x(t), R(t))$; its location depends on reputation, not quality at time t (since that will not be observed until later). Now consumer learning will be embodied in the $f(x, q, R)$ function. The discussion above about information sources influences firm behavior through f. Consequently, it will be very important to see how the optimal choice of $q(t)$ and $x(t)$ depend on the specification of f.

<u>Once and for All Quality Choice: General Results</u>

In this section I shall consider the problem faced by a monopolist when he is choosing once and for all what quality product to produce. It is best thought of as introduction of a new product, but applies at any point where a long-run quality choice is being made.

Suppose consumers all expect the quality produced to be R_o. That is, suppose the firm has an initial reputation of R_o (R and q are measured in the same units). Initially I take R_o as beyond the control of the firm. If advertising is permitted, it may well operate through altering R_o. One could study how much the firm would advertise to influence R_o favorably. Of course, such advertising need not be informative. R_o may also depend on the quality of products already in the market (see p. 56 below). In the full model where quality changes over time are permitted, R_t will depend on previous quality and sales choices by the firm itself.

Suppose, given initial reputation R_o the firm elects to produce a product of quality q. It then chooses an optimal pricing path (and corresponding sales path) over time to maximize the present value of its profit stream. What such a path looks like will depend on how learning occurs by consumers. To remain perfectly general, denote the <u>optimized</u> present value of profits from choosing q given R_o by $V(q, R_o)$. So long as there is discounting we expect this to be finite, and initial reputation to be of some value, i.e.,

(A1) $V_2(q, R_o) > 0$ for all q, $R_o > 0$.

(here subscripts denote partial derivatives).

(A2) $V(q, R_o)$ is finite for all (q, R_o) and is continuously differentiable.

Such a V function permits very general learning behavior on the part of consumers.

In order to discuss the firm's optimal quality choice when facing initial reputation R_o, it is very useful to identify the quality which would be chosen in a perfect information world. Define

$$W(q) = V(q, q).$$

This is the present discounted profits from choosing q under perfect information.[14] I will assume

(A3) $W'(0) > 0$, and $W'(q) < 0$ for q large.

Consequently, there will be an optimal quality choice under perfect information. If we denote that by q*, then

(1) $W'(q^*) = 0$

(There may be several roots to (1), but q* is the best one).

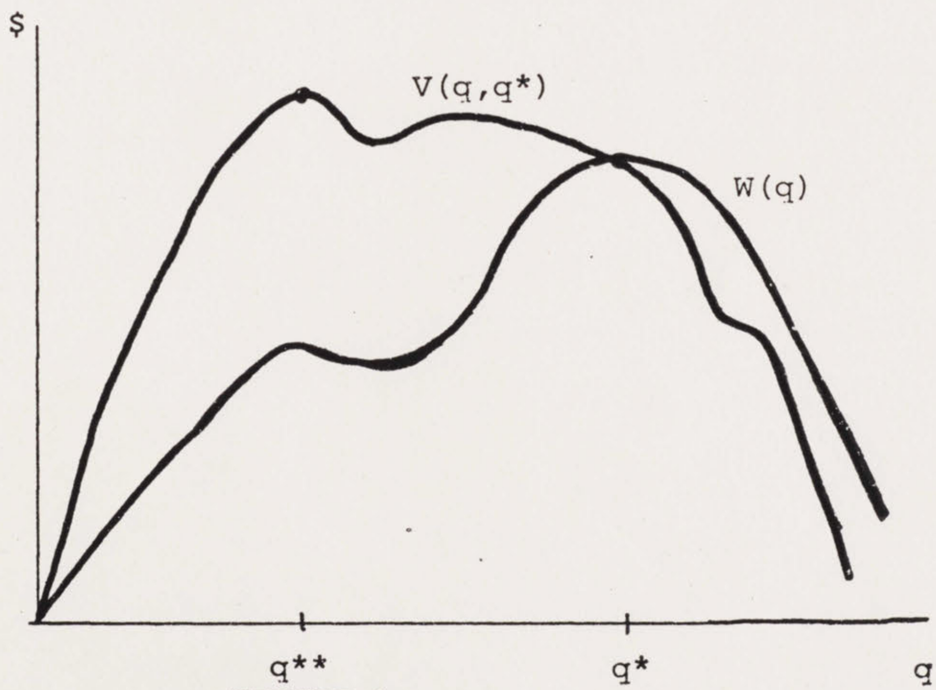
Finally, I will bound the quality choice from below by assuming that for $q \leqq 0$ (this is just a normalization) consumers can detect the poor quality and will not buy the product:

(A4) $V(q, R_o) = 0$ for $q \leqq 0$.[15]

It is now quite easy to identify one sense in which imperfect information will lead the firm to shade on quality:

<u>Theorem 1</u>. Under A1, A2, and A3 if consumers expect the firm to produce at the perfect information profit maximizing quality level q^*, it will be optimal for the firm to produce a lower quality.

<u>Proof</u>: We know $W'(q^*) = V_1(q^*, q^*) + V_2(q^*, q^*) = 0$. By A1 we know $V_2(q^*, q^*) > 0$. Consequently, $V_1(q^*, q^*) < 0$. Thus the curve $V(q, q^*)$, the relevant one for a firm facing initial reputation q^*, cuts $W(q)$ from above at q^*. Furthermore, again by A1, for $q>q^*$, $W(q) = V(q, q) > V(q, q^*)$ so $W(q)$ lies above $V(q, q^*)$. Similarly, for $q<q^*$, $V(q, q^*) > W(q)$. A consequence of these facts is that $V(q, q^*)$ must be as shown in Figure 1. Therefore the optimal choice of quality facing initial reputation q^*, q^{**} in the Figure, is less than q^*. ■



<u>FIGURE 1</u>

Note the great generality of Theorem 1. In particular it requires no concavity assumptions.

If we denote by $B(R_0)$ the firm's best choice of quality facing initial reputation R_0, the above argument shows that for $R_0 \geq q^*$, $B(R_0) < R_0$.[16] It is natural to ask whether there is some quality level $\hat{q}$ such that $B(\hat{q}) = \hat{q}$. That is, is there some quality level such that if consumers expect the firm to produce at that quality level it will be optimal for the firm to do so?

Without making further assumption about the V function it is not possible to conclude that such a quality level exists. If V is concave in its first argument, however, there will be such a self-fulfilling quality level:

<u>Theorem 2</u>. Under A1-A4 if V is concave in its first argument then there exists a quality level $\hat{q} < q^*$ such that if consumers expect the firm to produce $\hat{q}$, it will in fact be optimal for the firm to do so.

<u>Proof</u>: Consider the function $V_1(q, q)$. We know $V_1(0, 0) + V_2(0, 0) = W'(0) > 0$. Also $V_2(0, 0) = 0$ by A4. Therefore $V_1(0, 0) > 0$. Also, by the argument above, $V_1(q^*, q^*) < 0$. So, since $V_1(q, q)$ is continuous and is positive at $q=0$ and negative at $q=q^*$ there must be some $\hat{q} \varepsilon (0, q^*)$ such that $V_1(\hat{q}, \hat{q}) = 0$ by the intermediate value theorem. Now if V is concave in its first

argument and $V_1(\hat{q}, \hat{q}) = 0$, then $\hat{q}$ maximizes $V(q, \hat{q})$, so that facing initial reputation $\hat{q}$ the firm picks $q=\hat{q}$.

It is instructive to cast these results in terms of the $B(R_o)$ function mentioned above. The concavity assumption guarantees that $B(R_o)$ is a continuous function. Then Theorem 1 tells us that $B(R_o)$ lies below the 45° line for $R_o \geq q^*$. Assumption A4 insures that $B(0) > 0$. The simplest case is when there is a single solution to $B(q) = q$, as in Figure 2.

It is possible that $B(R_o)$ intersects the 45° line several times, however. There will always be an odd number of intersections, unless the $B(R_o)$ function happens to have a tangency with the 45° line. See Figure 3. The shape of the V function corresponding to Figure 2 is shown in Figure 4.

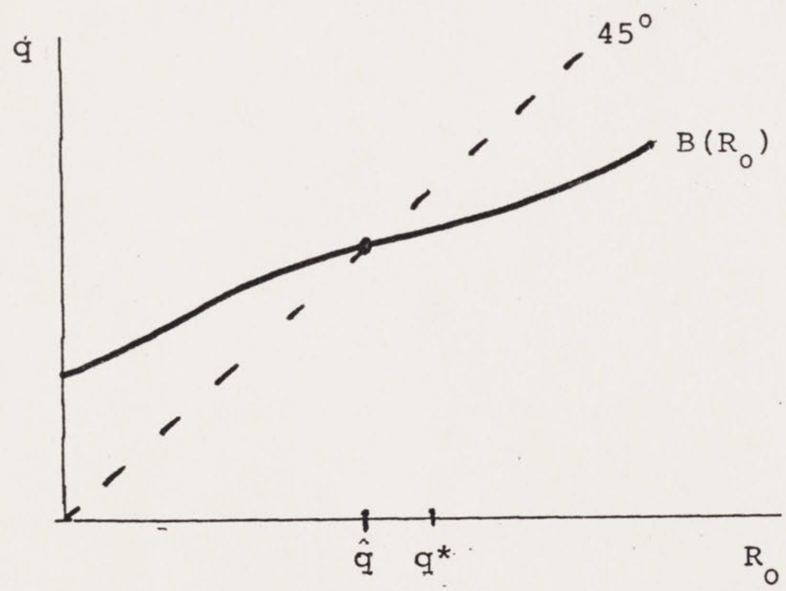


FIGURE 2

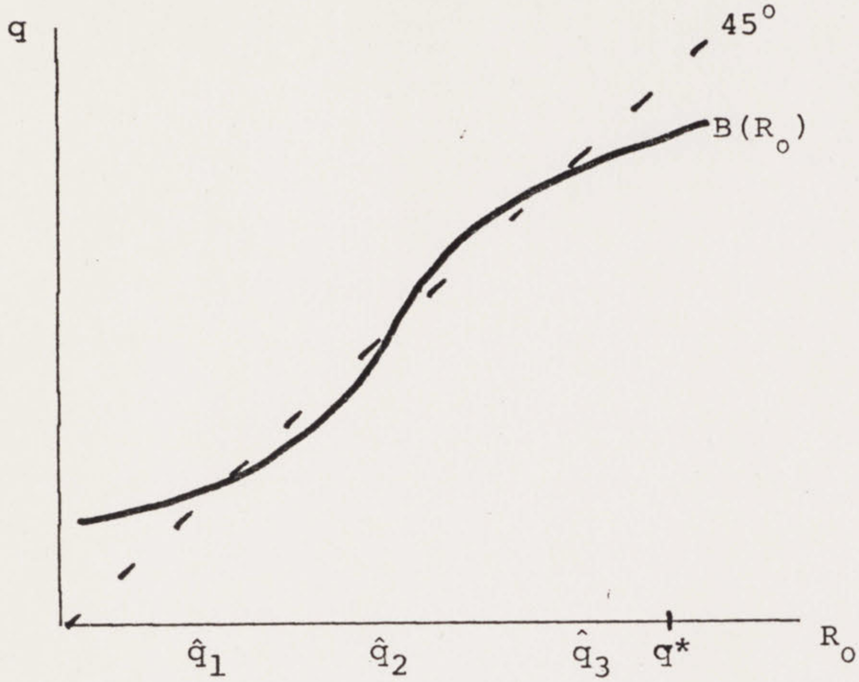


FIGURE 3

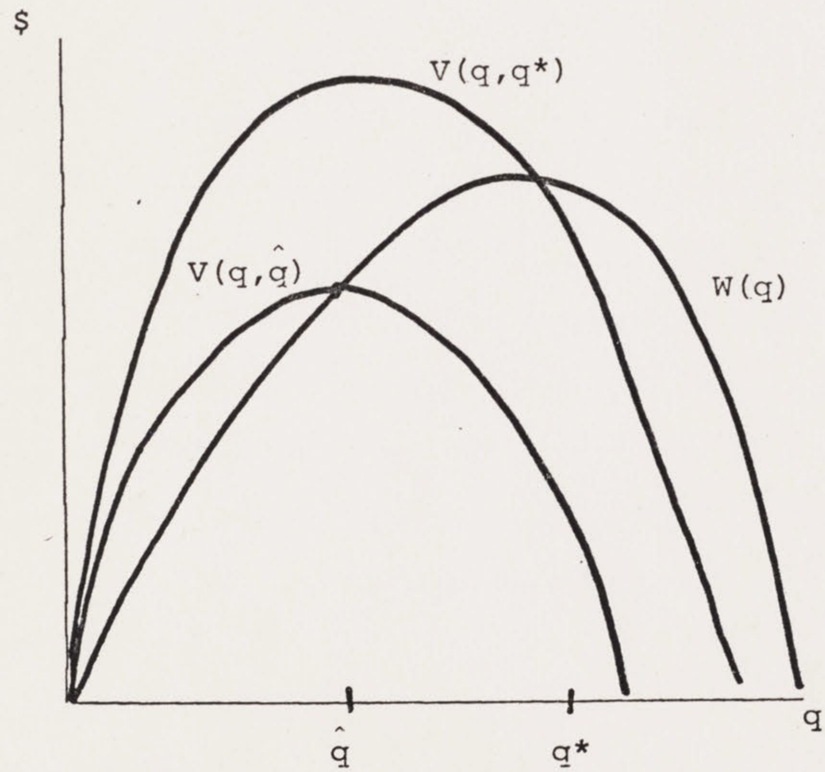


FIGURE 4

This self-fulfilling quality level (any of them if several exist) has some peculiar properties. First of all, the monopolist does worse at $\hat{q}$ than he would under perfect information, since $W(\hat{q}) < W(q^*)$. Therefore, if the monopolist could commit himself to producing q^* and convey this commitment to consumers, he could do better. This provides one justification for warranties; the monopolist could promise to pay customers if they get a product of quality less than q^*. For example he could give a money-back guarantee on items of quality less than q^*. He would then credibly commit himself to producing q^* and could achieve $W(q^*)$. Without the warranty, however, after inducing expectations q^* he would want to cut quality, by Theorem 1. It is instructive to note that the incentive to provide warranties does not arise from competitive pressures, but rather from a desire by the monopolist to commit himself.

If consumers are sophisticated and solve out the firm's optimal choice problem, we would expect to see $\hat{q}$ produced. It is a rational expectations equilibrium quality level in the following sense: if all consumers expect $\hat{q}$ the firm will fulfill their expectations.[17]

Such calculations by consumers are not consistent with observed consumer behavior, however, which is rather more adaptive and extrapolative.[18] Without the self-fulfilling expectations requirement it is not possible to pin down the quality that is chosen by the monopolist, since R_o is exogenous. One way to proceed is to study activities such as advertising which influence R_o. As we will see below, however, this analysis as it stands has important implications for the case where quality changes over time are permitted. In that case R_t is endogenous; it depends on quality choices in earlier periods.

It is possible to indicate in this framework how the speed of learning by consumers affects the firm's optimal quality choice. Suppose all consumers hold their initial expectations of q_o for κ periods, after which they will learn the true quality.[19] Denote by $\upsilon(q,q_o)$ the profits earned in one period when all consumers believe the quality is q_o but in fact it is q. Clearly $\upsilon_1<0$, $\upsilon_2>0$. If all learning occurs after κ periods and the discount factor is ρ, $0<\rho<1$, then the present value of profits from choosing q is

$$V(q,\ q_o,\ \kappa) = \frac{1-\rho^\kappa}{1-\rho}\upsilon(q,\ q_o) + \rho^\kappa W(q).$$

The first order condition for the choice of q is given by

$$V_1(q,\ q_o,\kappa) = \frac{1-\rho^\kappa}{1-\rho}\ \upsilon_1(q,\ q_o) + \rho^\kappa W'(q) = 0$$

Defining $s=\rho^{\kappa}$, the speed of learning (since when κ rises s falls because $\rho<1$) we can rewrite the first-order condition for q as

$$(1-s)\upsilon_1(q,\ q_o) + s(1-\rho)W'(q) = 0$$

Differentiating with respect to s yields

$$(1-s)\upsilon_{11}\frac{dq}{ds} - \upsilon_1 + (1-\rho)W' + s(1-\rho)W''\frac{dq}{ds} = 0$$

or

$$\frac{dq}{ds} = -\frac{(1-\rho)W'(q) - \upsilon_1(q,\ q_o)}{(1-s)\upsilon_{11}(q, q_o) + s(1-\rho)W''(q)}$$

From the first-order condition we know $W'(q)>0$ at the q chosen (because $\upsilon_1<0$ everywhere) so the numerator is positive. The denominator is negative by the second order condition defining the optimal q, so we can conclude[20] that $\frac{dq}{ds} > 0$ and have proven

Theorem 3 In the case where all learning occurs after κ periods, as κ increases the optimal quality level falls, for any given initial reputation.

Consequently, the quality chosen approaches q* monotonically from below as κ falls to 0.

This comparative static result can be shown in a more general setting. Denote the present value of profits from choosing q when the speed of learning is s by $V(q,\ q_o,\ s)$. Then the first-order condition for q is $V_q(q; q_o, s) = 0$ and the comparative statics computation gives

$\frac{dq}{ds} = -\frac{V_{qs}}{V_{qq}}$ so $\frac{dq}{ds} > 0$ if and only if $V_{qs} > 0$ i.e., if increased speed of learning increases the value of quality to the monopolist. In the case where $V(q, q_o, s) = (1-s)V(q, q_o) + sW(q)$ this will hold. In general one would need to look to see how s entered V to see if $V_{qs} > 0$.

It is only slightly more difficult to see how the self-fulfilling quality level is influenced by the learning speed of consumers. Let me consider the case where this quality is unique; uniqueness is guaranteed if we assume the function $V_1(q, q)$ is decreasing in q (so there can only be a single root to $V_1(q, q) = 0$). This assumption is equivalent to $V_{11}(q, q) + V_{12}(q, q) < 0$ for all q. The self-fulfilling quality level, $\hat{q}$, is defined as a function of s by $V_1(\hat{q}, \hat{q}, s) = 0$. Differentiating with respect to s gives

$$\frac{d\hat{q}}{ds} = \frac{-V_{1s}}{V_{11}+V_{12}}$$

So again we get $\frac{d\hat{q}}{ds} > 0$ exactly when $V_{1s} > 0$, under the additional assumption of $V_1(q, q)$ declining.

Welfare Implications

The analysis above indicates that imperfect information will tend to cause a reduction in the quality of products provided. This can lead to either a gain or loss in social welfare, using the criterion of consumers surplus plus profits to judge welfare. The welfare consequences of quality choice by a monopolist under perfect information are studied in Spence [1975] and Sheshinski [1976]. The conclusion of their analysis is that, depending on the shape of the inverse demand curve in quantity and quality, $p(x, q)$, a monopolist may either under- or over-supply quality. This is because the monopolist considers the effect on the marginal buyer of a change in quality, while social welfare requires looking at the effect on the average buyer. As a result, given the quantity produced, the monopolist quality is too low if and only if $p_{xq}<0$, i.e., if and only if the marginal consumer's evaluation of quality is less than the average consumer's. (See Spence [1975] p.419). Furthermore, the choice of quality may interact with the severity of output restriction if price elasticity is dependent on quality.

Consider the welfare consequences of the self-fulfilling equilibrium quality choice. If quality at the perfect information outcome, q* were less than is socially optimal, then the informational problems would exacerbate the welfare losses. It is perfectly possible, however, that q* exceeds the socially optimal quality level, in which case the lack of information could help matters. This is a typical second-best example of two imperfections counteracting each other.

The welfare analysis along the path to a steady state, i.e., while consumers are learning the true quality, is considerably more complex. For example, some consumers may purchase the product even though it in fact has little value to them, because they overestimated the quality. It is clear that the joint distribution of valuations of quality and expectations of quality, and how this distribution is updated, are central to a welfare analysis.

Consider a special case: everyone has common misperceptions about quality, expecting R_0 instead of q. Suppose initially that price elasticity of demand is unaffected by expected quality, so the firm's price is independent of reputation.[21] Then if $R_0>q$ more people will buy the product than would under perfect information. This seems like it would represent a welfare gain, since monopoly output is too low. Even though some of those who bought only because they expected $R_0>q$ regret having done so when they observe q, there is a social gain to their having purchased the product if their valuation exceeds the cost of production.

To sketch out this example more fully, assume constant return to scale with unit cost function c(q). Let consumers be described by their valuation of quality θ such that type θ has utility function $\theta q-p$ from buying a unit of quality q at price p.[22] The demand curve is generated by a distribution of θ's. Suppose the monopolist sets price p, quality q, and has initial reputation R_o. Then θ will buy if and only if $\theta R_o > p$. From a social welfare point of view, θ should buy if and only if $\theta q > c(q)$. Depending on the relative size of $\frac{c(q)}{q}$ and $\frac{p}{R_o}$ there may be too little or too much output. See Figure 5, which illustrates the case $R_o > q$.

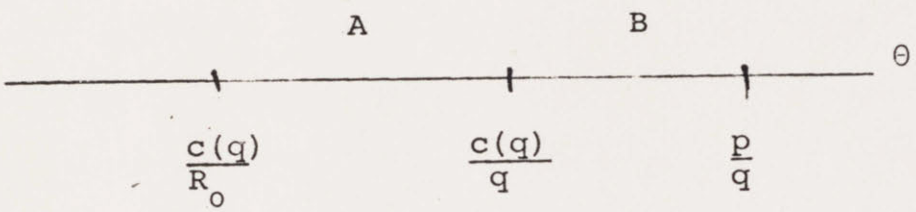


FIGURE 5

Over-Estimates of Quality:

$R_o > q$

If $\frac{p}{R_o}$ is in the interval A, then too many consumers purchase the product. This does not imply that the situation would be improved by eliminating the informational problem, however. We must compare purchases by $\theta \geq \frac{p}{R_o}$ with purchases by $\theta \geq \frac{p}{q}$. The latter suffers from the usual monopoly output restriction.

If $\frac{p}{R_o}$ is in the interval B, too few consumers purchase under imperfect information, but the imperfect information improves welfare. In this case $R_o > q$ causes more consumers, but not too many, to purchase the product. This is a welfare gain because too few consumers bought the product under perfect information, for the usual reason of monopoly power causing output reduction. So mild overestimates of quality improve welfare (under the assumption of price elasticity independent of reputation).

It is possible that the first best outcome is achieved under monopoly and imperfect information if $\frac{c(q)}{q} = \frac{p}{R_o}$. There will of course be distributional consequences of the imperfections, but those are not considered in our standard welfare measure.

The situation is quite different if $R_o < q$. Then the pessimistic misperceptions will reduce sales and exacerbate welfare problems. See Figure 6 below.

$$\underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} \;\; \theta$$

$$\frac{c(q)}{q} \qquad\qquad\qquad \frac{p}{q} \qquad\qquad\qquad \frac{p}{R_o}$$

FIGURE 6

Under-Estimates of Quality:

$R_o < q$

Now even fewer sales occur with expectations R_o than would under perfect information; there is an unambiguous welfare loss. The first best occurs when $\theta \geq \frac{c(q)}{q}$ purchase the product. Under monopoly and perfect information fewer θ's purchase since $p > c(q)$; only $\theta \geq p/q$ purchase. Adding under-estimates of quality further reduces the set of θ's who purchase to $\theta \geq p/R_0$. This causes a further reduction in welfare.

Allowing reputation to influence what price the firm charges permits almost anything to occur. For example, it could be the case that pessimistic expectations increase demand elasticity, and this causes price reductions which more than affect the losses mentioned above due to such expectations. That is, if

$\frac{c(q)}{q} \leq \frac{p(R_0)}{R_0} < \frac{p(q)}{q}$, even though $R_0 < q$, then there are welfare gains to having misperceptions of R_0 (here $p(q)$ is the price charged when consumers all believe the quality is q).[23]

Another welfare effect, explored in an example below, is caused when the firm's pricing decisions are made with recognition that they affect the learning process and hence future demand. Supposing that higher sales lead to more rapid learning by the market about true quality, there is an incentive to cut back on sales when reputation exceeds true quality. This exacerbates the usual monopoly welfare losses. Conversely, when reputation is below true quality, there is a benefit in addition to static marginal revenue from making another sale. Namely, there is a more rapid expansion of demand due to the increased speed of learning about the true quality. This effect leads the monopolist to expand output relative to its static profit maximizing level, causing a welfare gain along the path as consumers learn. It is worth noting that these effects are not small, because the monopolist is not at the social optimum in their absence.

Personal Learning: An Extended Example

In this section I analyze monopoly pricing and output decisions over time as consumers learn. The learning is restricted in two ways which limit the generality of the results: (1) All consumers begin with the same expected quality,[24] and (2) learning occurs only from personal experience: a given consumer learns nothing until he tries the product, at which time he learns its true quality.

Consumers differ in their tastes for quality. A consumer of type θ has utility function $\theta q-p$,[25] as above. Hence if all consumers expect quality R, then those θ's who buy will be $\{\theta = \theta \geq p/R\}$[26]. Denote by $F(\bar{\theta})$ the number of consumers of types $\theta \leq \bar{\theta}$. Let the range, of θ's lie in the closed interval $[\theta_L, \theta_H]$ where $0<\theta_L$; call $F(\theta_H)=N$. Suppose all consumers initially believe the quality to be R, when in fact it is q. The initial demand curve is thus $s(p) = N-F(p/R)$. Similarly, the fully informed demand curve is $z(p) = N - F(p/q)$. In the diagrams these are drawn as linear demand curves; that corresponds to taking F to be the uniform distribution, but is totally unnecessary.

Under the type of learning assumed, the position of the current demand curve depends only on which consumers have previously tried the product (and therefore learned). This in turn depends only on the lowest price previously charged.

Each period the firm chooses a price. Its objective is the present value of profits. In general the price charged will vary with time as more consumers learn the true quality of the product. The price charged will not generally be the one which maximizes static profits, because the firm must account for the effect of this period's price on the demand curve in future periods.

Since the situation is fundamentally different depending upon whether initial expectations are over- or under-estimates of true quality, I separate the analysis into two cases: $R > q$ and $R < q$.

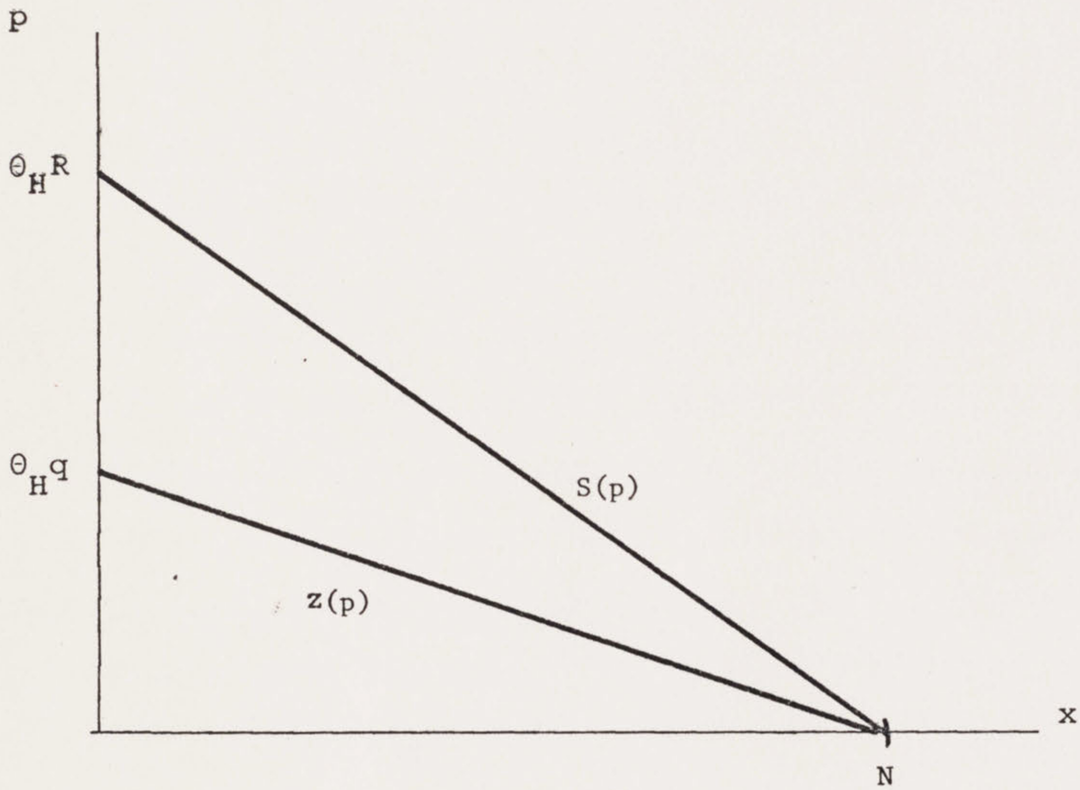


FIGURE 7

Initial Reputation Exceeds True Quality

(CASE I)

Case I: $R > q$ (Refer to Figure 7)

Call the price changed in period t, p_t.

Lemma 1: If for some T, $P_T \geq P_{T-1}$,

then $P_t = P_T$ for all $t \geq T$.

Proof: If $P_T \geq P_{T-1}$, then there is no shift in the demand curve due to the sales made during period T. This is because no new people tried the product during period T. Consequently, if it was optimal for the firm to charge P_T facing this demand curve during period T, it is optimal for it to do the same thing again in period T+1.

Since we know the pricing sequence is monotonically decreasing over time except possibly for a final jump in price, it is natural to ask how low p_t gets, and where it finally ends up.

It is at this point necessary to describe what the demand curve looks like as a function of the lowest price previously charged, $\hat{p}$. Denote this demand curve by $x(p,\hat{p})$. We must distinguish two subcases: (A) $\hat{p} > \theta_H q$ and (B) $\hat{p} < \theta_H q$.

Case I-A = $\hat{p} > \theta_H q$

The demand curve $x(p, \hat{p})$ in this case is shown in Figure 8. It is constructed as follows: There are $s(\hat{p})$ consumers who have previously tried the product. Since they updated from R to q, their demand is represented by the portion of the $z(p)$ curve from $x=0$ to $x=s(\hat{p})$. Those who have yet to try the product have demand represented by $s(p)$ from $x = s(\hat{p})$ to $x = N$. By adding these two demand curves together we get $x(p, \hat{p})$.

For $p > \hat{p}$ no one will buy, since all those who previously did now know better. For $\theta_H q < p < \hat{p}$ some uninformed consumers will buy, namely those who value the product enough to buy but not enough to have done so at $\hat{p}$: $\theta_H q < \theta R < p$. There are just $s(p)-s(\hat{p})$ of these, since $s(\hat{p})$ have already learned. For $z^{-1}(s(\hat{p}))<p<\theta_H q$ some informed consumers will buy and some will not. Exactly those θ s.t. $\theta R > \hat{p}$ (informed) and $\theta q>p$ will do so. There are $z(p)$ of these. Also some uninformed buy: those θ s.t. $\theta R<\hat{p}$ and $\theta R>p$. There are $s(p)-s(\hat{p})$ of these. Finally, for very low p $(p<z^{-1}(s(\hat{p})))$ all those who are informed buy, $s(\hat{p})$, and some uninformed do as well: $z(p)-s(\hat{p})$ of these. Summing up, we get the demand curve $x(p, \hat{p})$ shown in Figure 8. Algebraically, for $\hat{p} > \theta_H q$ we have

$$x(p,\hat{p}) = \begin{cases} 0 & p \geq \hat{p} \\ s(p)-s(\hat{p}) & \theta_H q<p<\hat{p} \\ s(p)-s(\hat{p})+z(p) & z^{-1}(s(\hat{p}))<p<\theta_H q \\ s(p) & p<z^{-1}(s(\hat{p})). \end{cases}$$

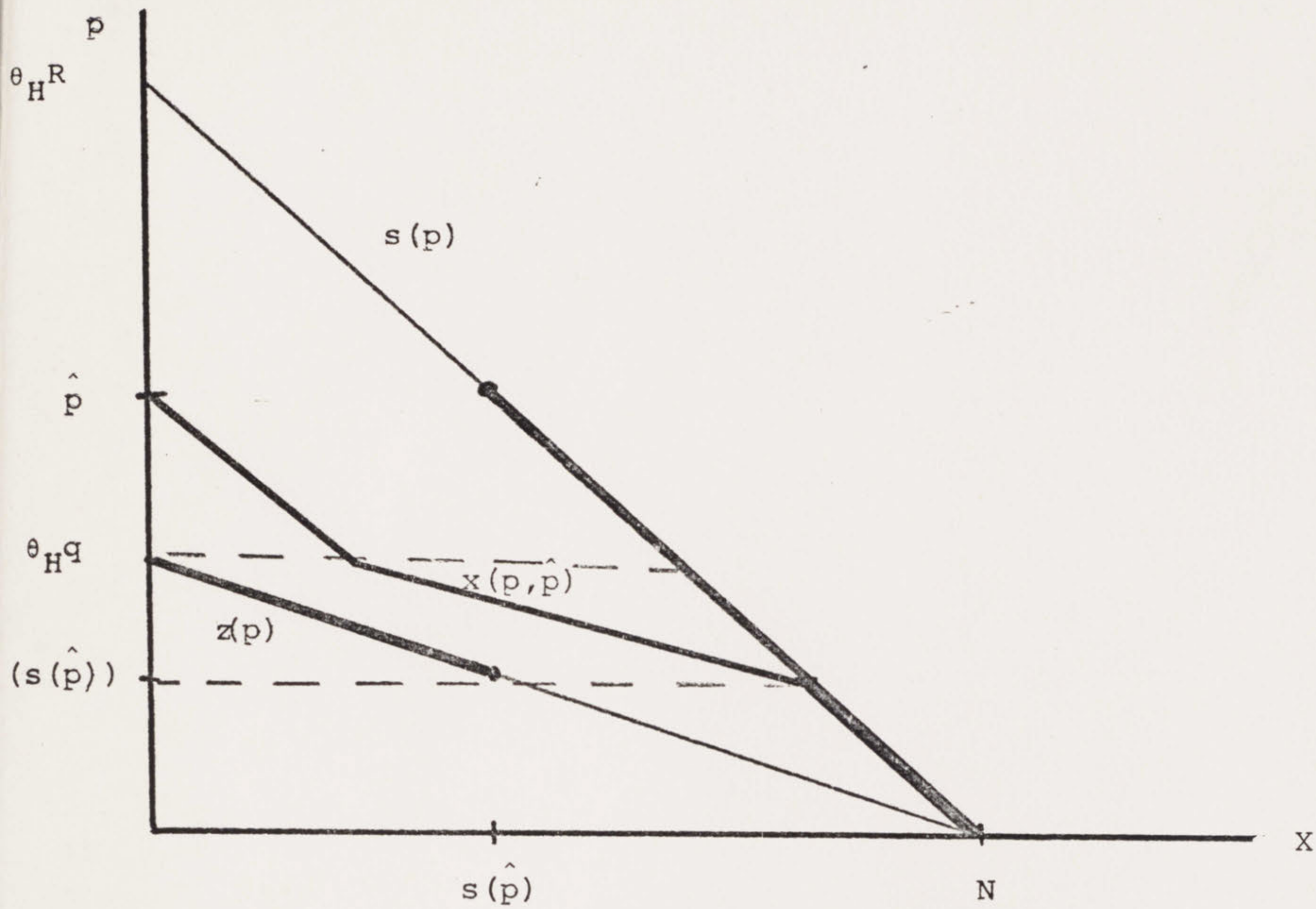


FIGURE 8

$R>q$ and $\hat{p} > \theta_H q$

Case I-B. $\hat{p} < \theta_H q$.

The description of demand is somewhat simpler in the case where $\hat{p} < \theta_H q$ because those willing to pay the most are now the informed who value quality the most, rather than the uninformed who are overly optimistic. See Figure 9 below for the shape of $x(p,\hat{p})$ in this case. Again this curve is derived by adding two other curves together horizontally: the $s(p)$ curve from $s(\hat{p})$ to N and the $z(p)$ curve from 0 to $s(\hat{p})$. Algebraically, when $\hat{p} < \theta_H q$ we get

$$x(p, \hat{p}) = \begin{cases} z(p) & p \geq \hat{p} \\ z(p)+s(p)-s(\hat{p}) & z^{-1}(s(\hat{p}))<p<\hat{p} \\ s(p) & p<z^{-1}(s(\hat{p})) \end{cases}$$

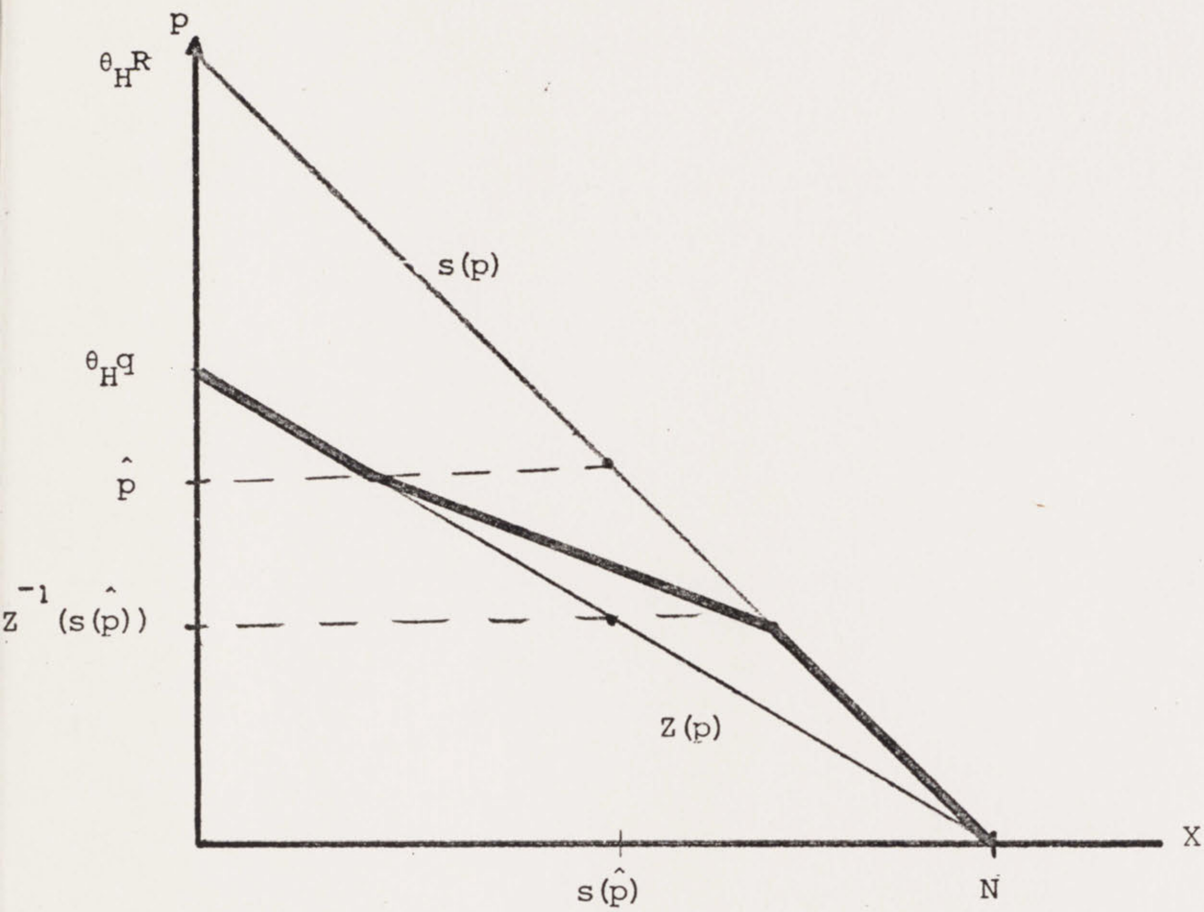


FIGURE 9

$R>q, \quad \hat{p} < \theta_H q$

Notice that even though both s(p) and z(p) exhibited declining marginal revenue, $x(p, \hat{p})$ need not.

Now we can proceed to analyze the optimal pricing sequence $\{p_t\}$ chosen by the monopolist. Denote by p^* the profit maximizing price facing the fully informed demand curve z(p). Then

Lemma 2 For some T, $p_t \leq p^*$ for all $t \geq T$.

Proof: By Lemma 1, there are two cases to consider. One where price falls monotonically, and another where price remains the same after some date. In the latter case, price could not remain at $p > p^*$ because the firm could gain profits both in the short- and long-run by charging p^* instead of p (charging p forever yields per period profits of $(p-c(q))z(p) < (p^*-c(q))z(p^*)$ and $x(p,\hat{p})$ always lies on or above $z(p)$ so it is feasible to earn $(p^*-c(q))z(p^*)$ every period.) In the case where $\{p_t\}$ declines monotonically, it must approach some $\bar{p}$. Then if $\bar{p}>p^*$ the per period profits approach $(\bar{p}-c(q))z(\bar{p}) < (p^*-c(q))z(p^*)$ so approaching $\bar{p}$ cannot be optimal.

Lemma 3: For some t, $p_t < p^*$.

Proof: Suppose not. Then $\{p_t\} \to p^*$ by Lemmas 1 and 2, and profits approach $(p^*-c(q))z(p^*)$. The demand curve approaches $x(p,p^*)$ which looks like Figure 10 below. I have drawn in the associated marginal revenue curve as well.

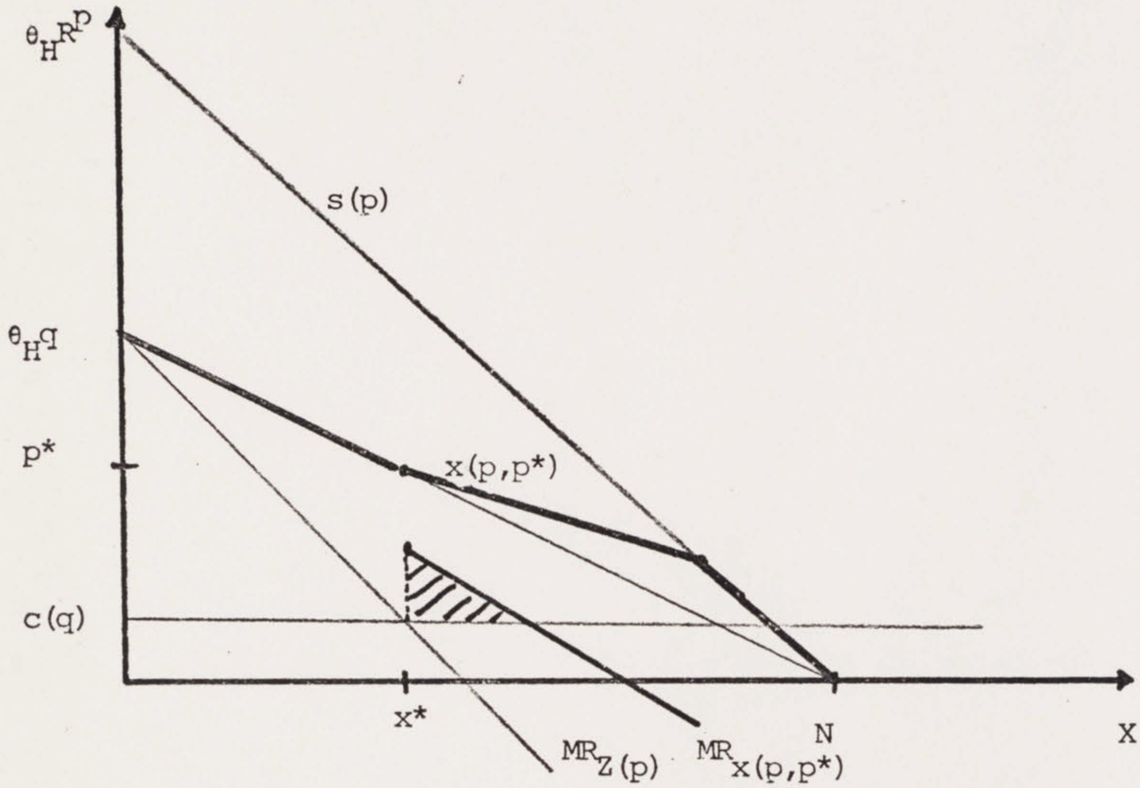


FIGURE 10

Since the curve $x(p,p^*)$ has a kink at p^*, the marginal revenue is not defined there. But the marginal revenue curve associated with $x(p,p^*)$ is shown as $MR_{x(p,p^*)}$ on Figure 10. It can be seen that the firm can make one period profits in excess of $(p^*-c(q))z(p^*)$ by setting $p<p^*$, since $MR_{x(p,p^*)}>c(q)$ for $p<p^*$. In fact, it could earn as much as the shaded triangle's area in a one-shot exploitation of consumer's initial misperceptions. Whether it will want to do this all at once depends on the exact shape of demand and the discount rate, but the basic point is established: at some point the price will fall below p^* to reap some gains from the misinformed. ∎

<u>Theorem 4</u> When consumers learn only from personal experience and begin with common overexpectations about product quality, the monopolist's prices over time fall monotonically,[27] eventually falling below the fully-informed monopoly price, and then jump back to that price forever after.

Proof: In view of the Lemmas, all that needs to be shown is that price does eventually return to p^* after it has fallen below it. If not, suppose $\{p_t\} \to \bar{p}<p^*$. Then profits approach $(\bar{p}-c(q))z(\bar{p})<(p^*-c(q))z(p^*)$ and it would be better for the firm to charge p^* forever rather than $\bar{p}$. ■

<u>Case II R<q</u>

The situation is entirely different when consumers are skeptical about a product's attributes. Now the firm will tend to sell more than the static monopoly profit maximizing level, because more sales today shifts out the demand curve tomorrow. While this might represent a welfare gain, it must be balanced against the fact that consumers are less likely to buy, even if the product is valuable to them, because they underestimate its value.

The initial demand curve $s(p)$ and the fully-informed demand curve $z(p)$ are shown below in Figure 11. I have also drawn in $x(p,\hat{p})$, the demand curve the firm faces when $\hat{p}$ is the lowest price previously charged.

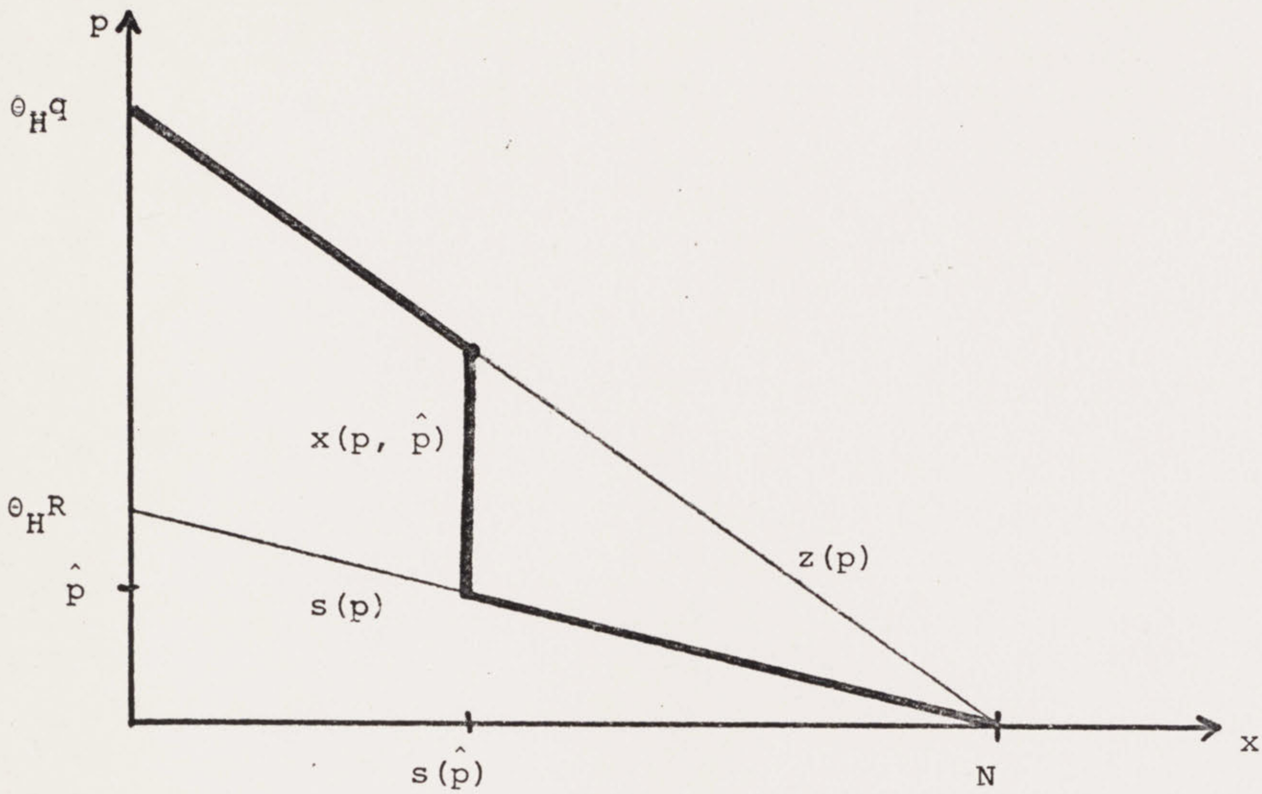


FIGURE 11

Quality Exceeds Initial Reputation

Let $p^*(R)$ be the static profit maximizing price facing common expectations R (i.e.,facing $s(p)$), and $p^*(q)$ be the profit maximizing price facing perfect information demand, $z(p)$. Let $x^*(R)$ and $x^*(q)$ be the corresponding levels of sales and $\pi^*(R)$, $\pi^*(q)$ the profit levels. Then there is a simple case which can be fully described:

Lemma 4 If $x^*(R) \geq x^*(q)$ then the monopolist first charges $p^*(R)$ and forever after charges $p^*(q)$.

Proof: Observe that the monopolist cannot possibly make more than $\pi^*(q)$ during any period, and cannot possibly make more than $\pi^*(R)$ the first period. So if making $\pi^*(R)$ followed by $\pi^*(q)$ is feasible, it is optimal. If

$x^*(R) \geq x^*(q)$ then after making $\pi^*(R)$ the first period he faces a demand curve such that the quantity-price combination $(p^*(q), x^*(q))$ is feasible. Hence the proposed regime must be optimal. ∎

While it is possible that $x^*(R) \geq x^*(q)$, it does not seem to be the usual case. This is because such a relationship implies that $p^*(R)$ is considerably below $p^*(q)$ <u>i.e.</u> the pessimistic expectations cause much more elastic demand than accurate expectations. If this inequality does not hold we can prove

<u>Theorem 5</u> If $x^*(R) < x^*(q)$, then in the long run the monopolist will not sell as many units when he faces initial expectations $R<q$ as he would under perfect information. His long-run per period profits will be below their perfect information level, and there will be an added welfare loss as a result.

Proof: The analysis is significantly aided by reference to Figure 12 below. There I have drawn the initial and fully informed profits as a function of sales. That is:

$$\pi(x;\ R) = [s^{-1}(x) - c(q)]x \qquad \text{and}$$

$$\pi(x;\ q) = [z^{-1}(x) - c(q)]x\ .$$

Since $R<q$, or equivalently $s(p)<z(p)$ so $s^{-1}(x)<z^{-1}(x)$, $\pi(x,R)<\pi(x,q)$. I have drawn the case to which this Theorem refers, namely $x^*(R)<x^*(q)$.

The profits obtainable as a function of sales when $\hat{x}$ is the maximum number of sales previously made (alternatively: $\hat{p}=s^{-1}(\hat{x})$ is the minimum price previously charged) is shown in Figure 13 as $\pi(x,\ \hat{x})$.

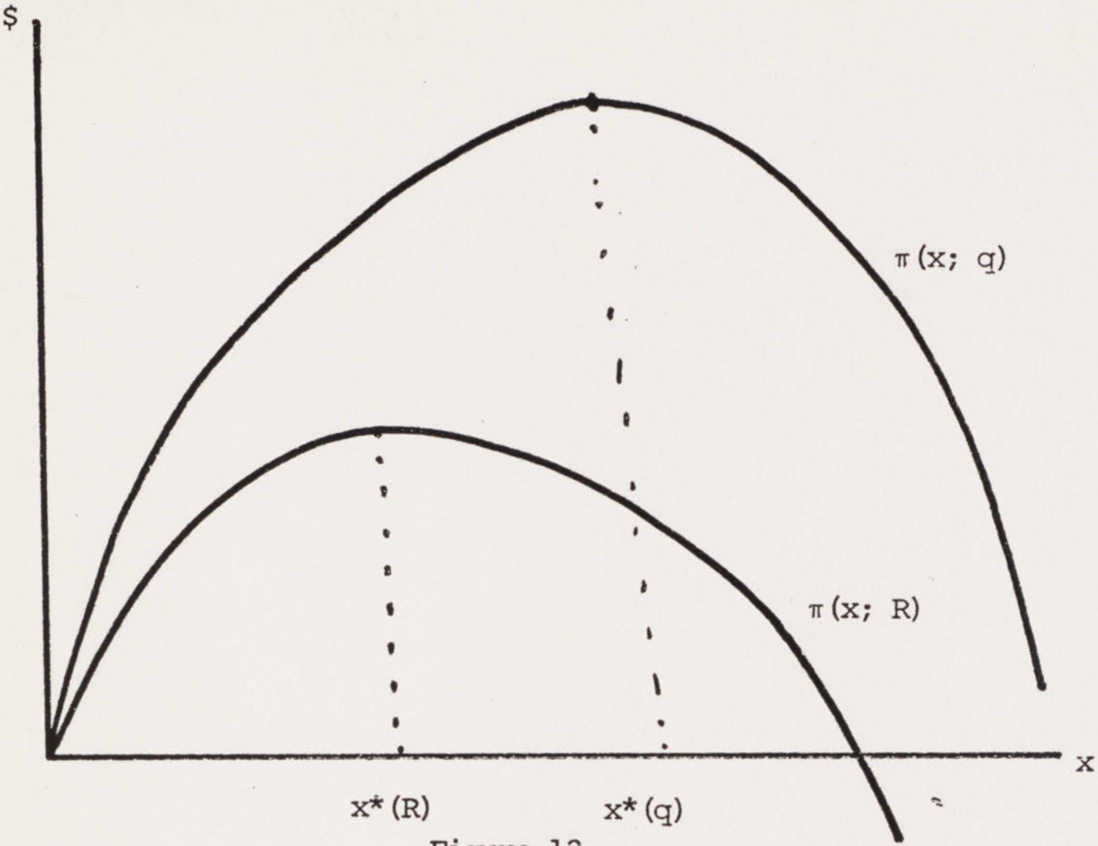


Figure 12

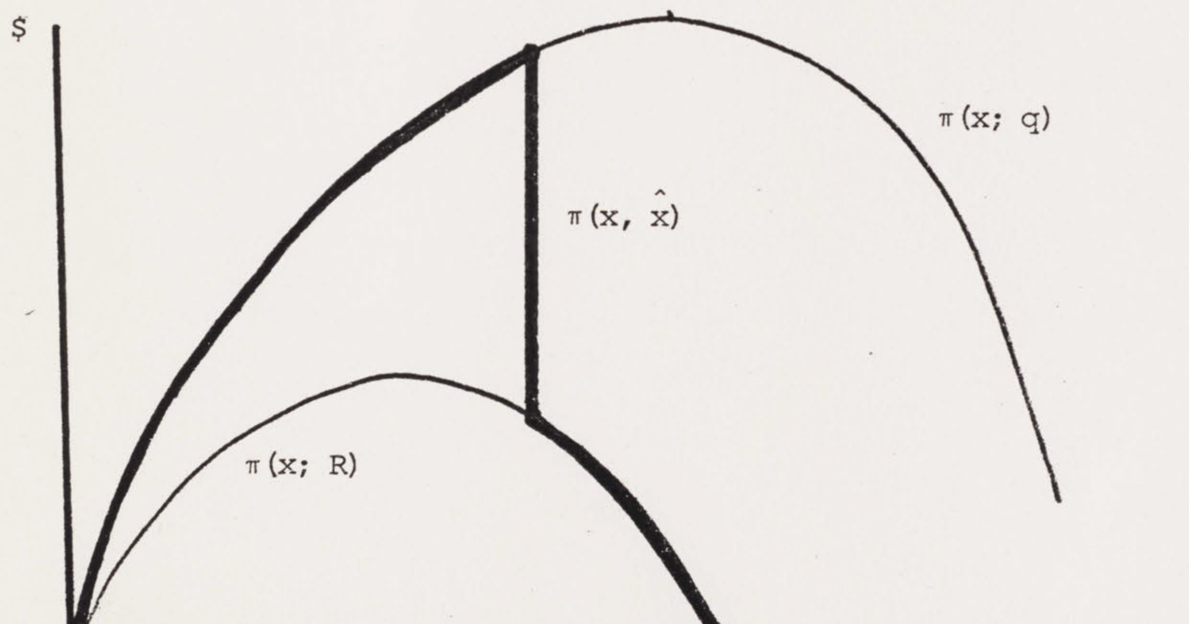


Figure 13

Several points can now be made clear. First of all, the monopolist will never sell more than $x^*(q)$ or less than $x^*(R)$. This is because either of these actions loses money in both the short- and long-run. Selling less than $x^*(R)$ sacrifices profits this period and fails to inform very many consumers that the product is better than they had thought. Selling more than $x^*(q)$ sacrifices short-run profits, whichever $\pi(x, \hat{x})$ the firm is facing, while <u>informing customers whom the firm will not want to sell to anyway</u>. That is the key: there is a cost to informing more customers that the product is better than they had believed. This is done through introductory offers in this example. I have implicitly assumed that it is impossible to cut price only to new customers; the firm sets one price each period.

Once $x^*(q)$ customers are informed, there is no point in informing more, because the firm does just as well facing $\pi(x, x^*(q))$ as facing $\pi(x; q)$.

What will the actual sales path look like? Again, the trade-offs to be made between short-run sacrifices and long-run gains depend on the discount rate. But it is easy to see that, so long as the discount rate lies between 0 and 1, (1) at some time the firm will sell more than $x^*(R)$, and (2) the firm will never sell as many as $x^*(q)$. The firm will not repeatedly sell $x^*(R)$ because, to the first-order, there are no losses from selling a bit more ($\pi_x(x^*(R); R) = 0$) but there are real long-run gains ($\pi_x(x^*(R);q)>0$).

Similarly, there is no point in pushing sales all the way up to $x^*(q)$ because the long-run gains are $\frac{1}{r}\pi_x(x^*(q); q) = 0$ and the short run gains are $\pi_x(x^*(q), \hat{x}) < 0$ (*i.e.*, there are short-run losses from expanding output). Consequently, the monopolist will utlimately sell $\hat{x}\varepsilon(x^*(R), x^*(q))$ forever after. This provides less profit than he would obtain under perfect information since $\pi(x^*(q); q) > \pi(\hat{x},\hat{x})$. And it entails a welfare loss since the monopolist already has restricted output below its socially optimal level. ∎

I should point out that this welfare loss is exacerbated by the inability of the monopolist to provide selective discounts to new customers. If he could do that, he would find it profitable to inform more consumers, through introductory offers, that the true quality is q.

The results in this example of personal learning carry over if learning is not immediate in response to use of the product, so long as it takes only finitely long. Slower learning will influence the optimal sales (price) path, but the qualitative conclusions of Theorems 4 and 5 carry over.

A Continuous Time Example of Once with For All Quality Choice

In this section I move away from studying how individual consumers learn to focus on one aspect of the monopolist's optimal sales path in the presence of aggregate consumer learning. That aspect, referred to above in the welfare section, is that when sales levels influence learning, if reputation exceeds quality the firm will cut back on sales relative to static profit maximization. As in the above section, this model takes quality choice as given and studies the sales path over time.

Denote quality by q, reputation by R, and sales by x. The inverse demand curve the firm faces depends on reputation, and is written $p(x,R)$. The firm faces a control problem of the type discussed in the introduction, with the restriction that quality is a once-and-for-all choice. This problem fits precisely into the framework analyzed in Theorems 1 and 2:

$$V(q,\ R(o)) = \max_{x(t)} \int_{o}^{\infty} e^{-rt}[p(x(t),\ R(t))x(t)\ -\ c(x(t),\ q)]dt$$

$$\text{s.t}\quad \dot{R} = sx(q-R)$$

$$R(o)\ \text{given.}$$

The analysis here will focus on what the path $x(t)$ looks like. To avoid other effects, I assume constant returns to scale: $c(q,\ x) = xc(q)$.

The specification of $\dot{R}$ is crucial to the optimal control problem. The idea behind the equatiòn $\dot{R} = sx(q - R)$ is that the speed with which reputation adjusts towards true quality depends positively on the level of sales. This occurs for two reasons: (1) A given customer updates his expectations more completely, the more experience he has with the product, and (2) the more customers who try the product, the more learning there will be regarding true quality (and the more subsequent interpersonal communication about the firm's quality). As we will see, this causes the monopolist to cut back on sales when $R>q$ in order to retard the deterioration of reputation. The reverse effect occurs when $R<q$. The parameter s measures the speed with which consumers learn from using the product.

The present value Hamiltonian for the firm's control problem is :

$$H(x, R, \lambda) = p(x, R)x - xc(q) + \lambda sx(q-R).$$

Assuming $p_R(x, R) > 0$, we know that reputation must have positive shadow value, i.e,, $\lambda>0$. The assumption in Theorem 1 that $V_2>0$ is exactly that $\lambda>0$.

Denote marginal revenue by $MR(x, R)$

(i.e $MR(x, R) = p(x, R) + xp_x(x, R)$).

Then, the necessary conditions for the optimal path include

$$MR(x, R) - c(q) = \lambda s(R-q)$$

$$xp_R(x, R) - \lambda sx = r\lambda-\dot{\lambda}$$

We can see from the first equation that $MR>c$ exactly when $R>q$. Assuming declining MR ($MR_x<0$), this means that sales are cut back relative to static maximization exactly when $R>q$.

Assuming that $MR_R > 0$, i.e, that increased reputation increases marginal revenue, we know the curve in x-R space along which $MR(x, R) = c$ is upward sloping, as in Figure 14. By the first equation above, since $\lambda > 0$ we know that $\text{sgn}(MR-c) = \text{sgn}(R-q)$, so the optimal regime never enters the shaded regions.

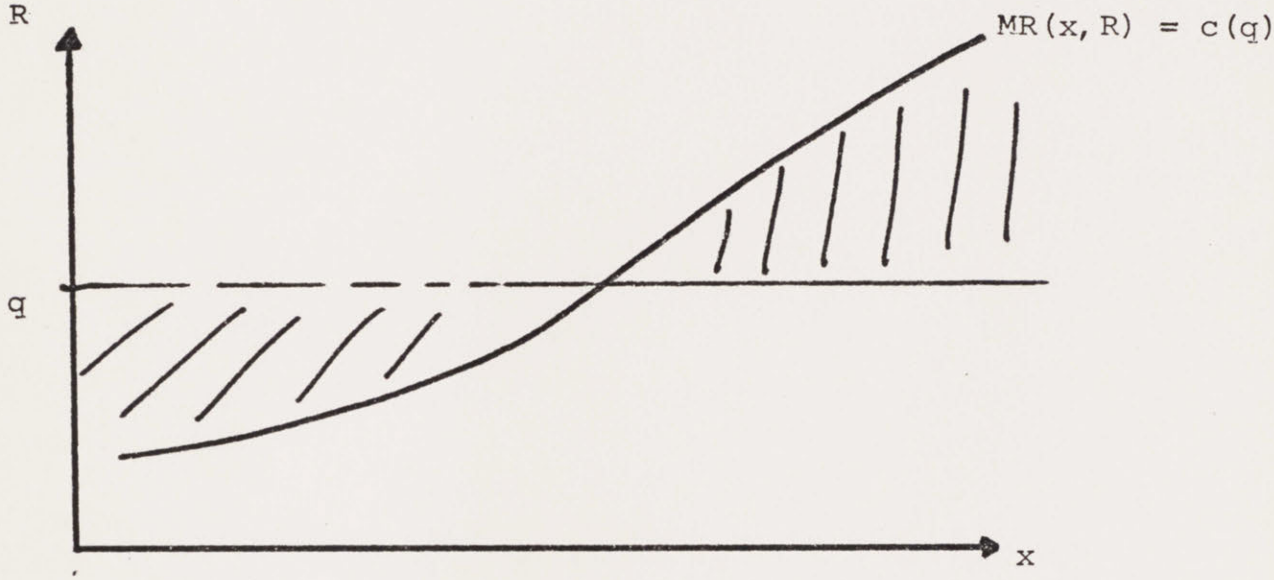


Figure 14

To investigate the dynamic system induced in x-R space by the necessary conditions above, eliminate λ and solve for $\dot{x}$ and $\dot{R}$ as functions of x and R. This is done in the Appendix. The resulting equations of motion are

$$\dot{x} = \frac{-sx}{MR_x} (q-R) \left[xp_{xR} + \frac{MR-c(q)}{R-q} \frac{r}{sx}\right]$$

$$\dot{R} = sx(q - R)$$

When $p_{xR}=0$, a central case, the $\dot{x}=0$ curve corresponds to MR=c. This case is drawn in Figure 15 below. Since MR-c

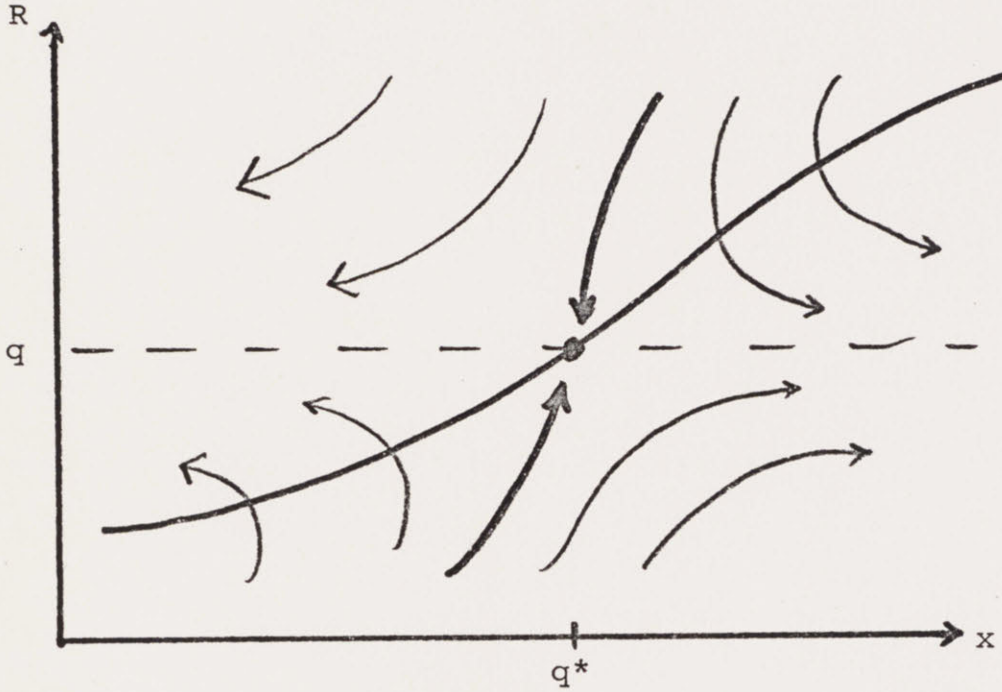


Figure 15

and R-q are of the same sign in the relevant regions, we then get sqn $(\dot{x})$ = sqn (q-R) outside the shaded regions. This says that sales levels will approach the perfect information level notonically. Unless p_{xr} is a large negative number, the expression in brackets in the $\dot{x}$ equation above will be positive and we will have sqn $\dot{x}$ = sqn (q-R).

To complete the solution to the optimal control problem, we must see which path is best, starting at a given R(0). Fortunately, it is easy to rule out all paths except the one leading to the steady state. Begin with the case R(0)>q. We know any path which enters the shaded regions from Figure 14 cannot be optimal. Consider those paths which lead to the x=0 boundary. These involve closing down with a high reputation.

This cannot be optimal under the assumption that positive profits can be earned in the steady-state, because a firm with $R>q$ can at least duplicate the actions of a firm with reputation q. This proves that the optimal regime when $R(0)>q$ involves following the separatrix into steady state from where it intersects $R=R(0)$.

The analysis is much the same when $R(0)<q$. Now we must rule out paths which lead off to $x=\infty$ as $q>R$. These lead to large losses as x grows and hence cannot be optimal. Again the optimal regime goes to the point (x^*, q) in the Figures.

The relationship between sales levels in the presence of learning, $x^*(R)$, and under static profit maximization, $x^S(R)$, is depicted below. The fact that $x^*(R) > x^S(R)$ if and only if $R<q$ does not depend on the assumption that $MR_R>0$. The welfare consequences of this behavior were discussed in a previous section.

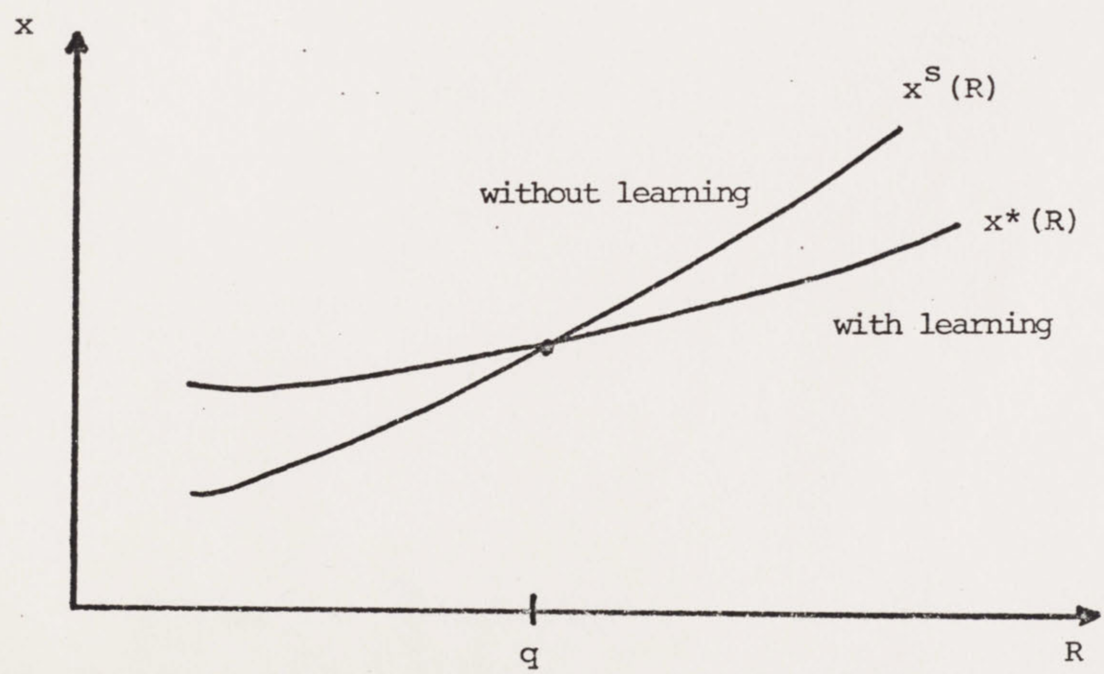


Figure 16

Product Quality Choice Over Time: Remarks and General Results

While in some cases producers choose product attributes once and for all, as studied above, in other cases it is possible for them to change their quality over time. In this section, I study the behavior of a monopolist who can alter his quality each period. This is one polar case - no costs to changing quality - while the earlier analysis is the other - large costs to changing quality so that a once and for all choice must be made.

A central issue in studying quality changes over time is the following: under what circumstances (cost functions, demand functions, and consumer learning) will a monopolist find it optimal to settle down to some steady-state quality level? The alternative is to oscillate, repeatedly running up a reputation and then milking it. If the oscillations were optimal, we might expect alternative mechanisms instead of reputation to arise to certify or control the monopolist's product quality.

First I discuss the necessary conditions for a quality level to constitute a steady state. Again we see that imperfect information causes the monopolist to reduce quality. The way in which consumers' expectation formation affects the steady-state quality level is analyzed. Finally I make some observations on when the sufficient conditions will be satisfied so that staying in or going to steady state is optimal.

The first main result regarding steady-state quality levels is the analogue of Theorem 1 in a world where quality can be changed over time:

Theorem 6. So long as reputation has positive value, any steady-state quality level must lie below the perfect information quality level.

Proof: One possible deviation from steady state is a once and for all change in quality. Since, by Theorem 1, for any quality level at least as great as q* (the perfect information quality level) it is preferable to cut quality (forever) rather than to maintain quality, such quality levels cannot be steady states. ∎

To analyze more carefully what the steady state quality level will be, consider the following set up: Each year t the firm can choose a quality level q_t. Consumer expectations at the beginning of the year are summarized in the reputation, R_t. The firm can vary prices (and sales) throughout the year if it desires. The resulting profits during the year are written $V(q_t\ R_t)$.[28] By the end of the year, reputation will adjust to R_{t+1} in a manner yet to be specified. The firm's objective function is

$$W = \sum_{t=0}^{\infty} \rho^t V(q_t, R_t)$$

where R_o is given.

Theorem 7. Let $W(q) = V(q, q)$ be concave in q. If $R_t = \gamma R_{t-1}+(1-\gamma)q_{t-1}$. where $0<\gamma<1$, then any steady-state quality level q^s must be less than the full information profit-maximizing quality level q^*, and greater than the lowest self-fulfilling quality level $\hat{q}$ (this q^* and $\hat{q}$ refers to the given V function).

Proof: Suppose the firm is in steady-state at quality q^s. Consider a small one time increase in q during period 0, followed by a return to q^s forever. The effect on the stream of profits is

$$\frac{dW}{dq_o} = V_1(q^s, q^s) + \sum_{t=1}^{\infty} \rho^t V_2(q^s, q^s) \frac{dR_t}{dq_o}$$

Now $R_1 = \gamma R_o + (1-\gamma) q_o$ so

$$\frac{dR_1}{dq_o} = 1-\gamma.$$

And $R_2 = \gamma R_1 + (1-\gamma) q_1$ so

$$\frac{dR_2}{dq_o} = \gamma \frac{dR_1}{dq_o} = \gamma(1-\gamma).$$

In general $\frac{dR_t}{dq_o} = \gamma^{t-1}(1-\gamma)$, $t \geq 1$.

So

$$\frac{dW}{dq_o} = V_1(q^s, q^s) + V_2(q^s, q^s) \left[\sum_{t=1}^{\infty} \rho^t \gamma^t \left(\frac{1-\gamma}{\gamma}\right) \right]$$

If q^s is to be a steady-state, it must be the case that $\frac{dW}{dq_o} = 0$ at $q = q^s$.

Therefore we get

$$V_1(q^s, q^s) + V_2(q^s, q^s) \frac{1-\gamma}{\gamma} \rho\gamma \frac{1}{1-\rho\gamma} = 0$$

or

$$V_1(q^s, q^s) + \frac{\rho(1-\gamma)}{1-\rho\gamma} V_2(q^s, q^s) = 0.$$

For $q \geq q^*$ we know $V_1(q, q) + V_2(q, q) < 0$ since $W(q)$ is concave. Since $V_2 > 0$ and $0 < \frac{\rho(1-\gamma)}{1-\rho\gamma} < 1$ we then know $V_1(q, q) + \frac{\rho(1-\gamma)}{1-\rho\gamma} V_2(q,q) < 0$ for $q \geq q^*$. Therefore no quality level at or above q^* can qualify as a steady state. (As we know from Theorem 6)

Next consider quality levels below the lowest self-filling quality level $\hat{q}$. They key thing about $\hat{q}$ is that for $q < \hat{q}$ we have $V_1(q,q) > 0$. Consequently we also have $V_1(q,q) + \frac{\rho(1-\gamma)}{1-\rho\gamma} V_2(q,q) > 0$, so these qualities cannot be steady-states.

■

Since the incentive to cut quality is greater when detection is delayed, there is lower quality in steady state under such circumstances. The self-fulfilling quality, $\hat{q}$, is the one which would be chosen if there were no future after this year. If there were no learning about quality until the year was over, $\hat{q}=0$ (Akerlof). In general, however, even ignoring future years it does not pay to produce the lowest possible quality: thinking of automobile model years as an example, word may get out within the year that the car performs poorly. Since future years <u>do</u> matter, quality chosen will exceed $\hat{q}$; higher future reputation is of positive value. At $\hat{q}$ there is no loss, up to the first-order, of an increase in quality, and there are real gains in the future.

Notice that as the interest rate approaches 0, ie., as $\rho \to 1$, the solution approaches q^*. Also, as $\rho \to 0$ so that future years matter very little at all, the steady-state quality level goes to $\hat{q}$. In fact, it is possible to see how the speed with which consumers update their expectations influences the steady-state quality level:

Theorem 8. When a steady state quality level exists in the context of Theorem 7, it is higher (1) the higher is the weight placed on recent quality by consumers in forming reputation and (2) the higher the discount factor (i.e., the lower the interest rate).[29]

Proof: The steady-state quality level, defined by (*) above, depends on the factor $s=\frac{\rho(1-\gamma)}{1-\rho\gamma}$; $0<s<1$. Observe that $\frac{ds}{d\gamma} < 0$ and $\frac{ds}{d\rho} > 0$. So higher discount factor and more weight on quality in reputation formation cause s to rise. All that needs to be shown, therefore, is that $\frac{dq^s}{ds} > 0$. Differentiate (*) with respect to s yielding

$$(V_{11}+V_{12}+sV_{12}+sV_{22})\frac{dq^s}{ds} + V_2 = 0 \text{ or}$$

$$\frac{dq^s}{ds} = \frac{-V_2}{V_{11}+V_{12}+S(V_{12}+V_{22})}$$

If q^s is a steady-state, the denominator is negative by the second order conditions. Therefore $\frac{dq^s}{ds} > 0$ since $V_2>0$ and the Theorem is proven. ∎

Example: Suppose the firm produces one project each period so that sales are not a control variable. This would apply to a lawyer, for example, who does one case each period and can choose how hard to work on the case. Let the price he can earn for one project be a function of his reputation p(R). Let the cost of producing a project of quality q be c(q). Assume $p'>0$, $p''<0$, $c'>0$, $c''>0$.

Under perfect information the seller would choose q to

$$\underset{q}{\text{maximize}} \quad p(q) - c(q)$$

So q^* satisfies $p'(q^*) = c'(q^*)$. Under imperfect information we have $V(q, R) = p(R) - c(q)$ so the first-order condition is $-c'(q) + \frac{\rho(1-\gamma)}{1-\rho\gamma} p'(q) = 0$ when $R_t = \gamma R_{t-1} + (1-\gamma)q_{t-1}$. Writing this as $c'(q) = sp'(q)$, $0<s<1$ we have $c''(q)\frac{dq}{ds} = p'(q) + sp''(q)\frac{dq}{ds}$ or $\frac{dq}{ds} = \frac{p'(q)}{c''(q) - sp''(q)} > 0$.

In the case $\gamma=0$ so $R_t=q_{t-1}$ we get $c'(q) = \rho p'(q)$.

See Figure 17 below.

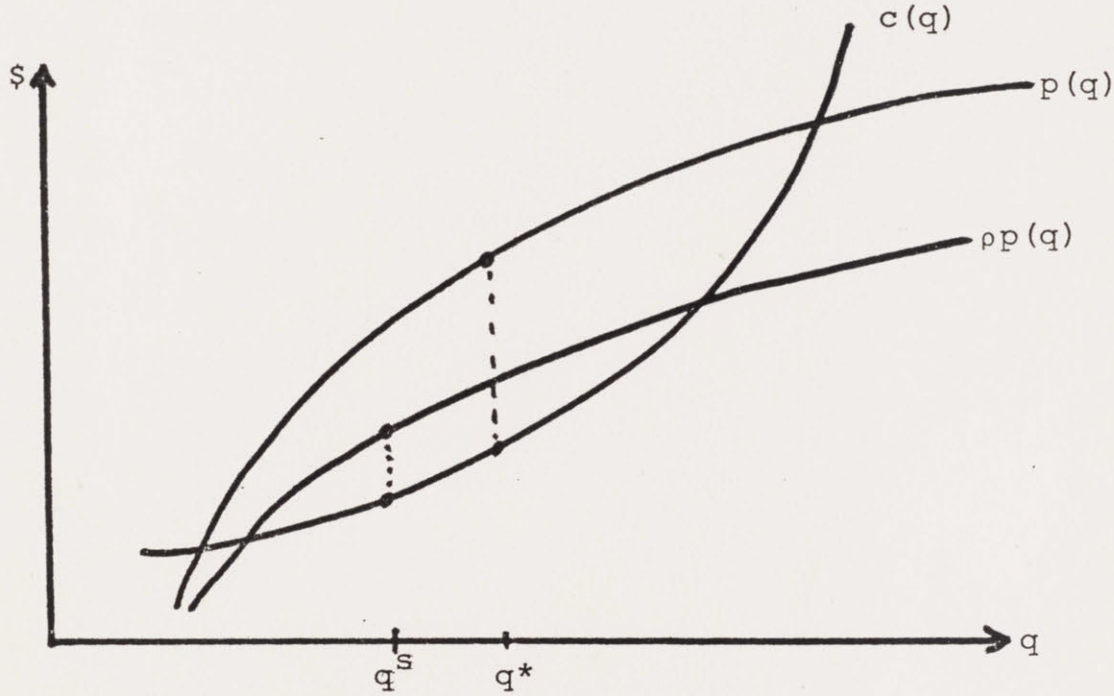


FIGURE 17

In general there is no guarantee that an optimal steady-state quality level exists. If V(q, R) is concave, then we can be sure that it does. Part of the problem, however, is that the specification of how reputation changes which was used in Theorems 7 and 8 does not incorporate a sales term in reputation adjustment. If the firm can build up a good reputation by selling one good item and yet sell many poor items before reputation diminishes, it will never be optimal to stay in steady state.

In general, oscillations will be desirable whenever a firm can earn more in the process of running down its reputation than it cost to build it up. This will depend on the precise mechanism by which reputation is formed. I consider one plausible specification of reputation adjustment in the model below. An open problem is how the activities of individual consumers - both information gathering and expectation adjustment- influence the way in which reputation moves. A model which addressed that problem would be able to trace through the impact of improved communication or information gathering on the firm's steady-state quality level.

<u>A Continuous Time Model with Variable Quality</u>

In this section I analyze another model of the form discussed in the introduction. It is very similar to the earlier continuous time model, with the addition of quality as a control variable. Formally, the monopolist faces the control problem

$$\max_{q(t),\, x(t)} \int_0^\infty e^{-rt} \left[B(x,R) - c(x,q)\right] dt$$

$$\text{s.t.} \quad R = sx(q-R). \quad R(o) \text{ given.}$$

The function $B(x,R)$ is the benefit or revenue function; it is $xp(x,R)$ where $p(x,R)$ is the inverse demand curve. The reputation adjustment equation was discussed above in the once and for all quality choice model.

The current value Hamiltonian is $H(x,q,R,\lambda) = B(x,R) - C(x,q) + \lambda sx(q-R)$. The necessary conditions for the optimal regime include

(1) $B_x(x,R) - c_x(x,q) + \lambda s(q-R) = 0$

(2) $-C_q(x,q) + \lambda sx = 0$

(3) $B_R\ (x,R) - \lambda sx = r\lambda - \dot{\lambda}$

(4) $\dot{R} = sx(q-R)$

I consider the case with constant returns to scale so $C(x,q) = xc(q)$. Then $C_q(x,q) = xc'(q)$ and (2) gives us

$$c'(q) = \lambda s.$$

Therefore $c''(q)\dot{q} = \dot{\lambda}s$. We can thus eliminate both $\dot{\lambda}$ and λ to get

(5) $B_x(x, r) - c(q) + c'(q)(q-R) = 0$

(6) $B_R(x, R) - xc'(q) = \frac{r}{s}c'(q) - \frac{1}{s} c''(q)\dot{q}$

Using (5) and (6), it is easy to write down the equations which must be satisfied by a steady state quantity - quality pair (x,q). In steady state q=R, $\dot{R}$=0 and we get

(7) $B_x(x, q) = c(q)$

(8) $B_q(x, q) = xc'(q) + \frac{r}{s} c'(q)$.

It is very instructive to compare these steady-state equations to the first-order conditions for quantity and quality choice under perfect information. With perfect information the monopolist solves

(**) $\max_{q,x} \; B(x, q) - xc(q)$

with first-order conditions

(9) $B_x(x, q) = c(q)$

(10) $B_q(x, q) = xc'(q)$

<u>Theorem 9</u>. So long as the perfect information problem (**) is concave, the steady-state quality level is strictly lower and is monotonically decreasing in $\frac{r}{s}$. That is, as the speed of learning falls or the interest rate rises the firm's steady state quality level declines.

Proof: Totally differentiate (7) and (8) to get

(11) $B_{xx}dx + B_{xq}dq = c'dq$

(12) $B_{xq}dx + B_{qq}dq = c'dx + xc''dq + (\frac{r}{s})c''dq + c'd(\frac{r}{s})$

Rewriting, we have

$$\begin{bmatrix} B_{xx} & B_{xq}-c' \\ B_{xq}-c' & B_{qq} - c''x-(\frac{r}{s})c'' \end{bmatrix} \begin{pmatrix} dx \\ dq \end{pmatrix} = \begin{pmatrix} 0 \\ c'd(\frac{r}{s}) \end{pmatrix}$$

The second-order condition, fulfilled by assumption, for the perfect information problem is that the matrix

$$\begin{bmatrix} B_{xx} & B_{xq}-c' \\ B_{xq}-c' & B_{qq}-c''x \end{bmatrix}$$

be negative definite. The addition of the term $(\frac{r}{s})c''$ will preserve negative definiteness since $c''>0$. Call the new matrix A. So $|A|>0$.

Now, using Cramer's Rule,

$$\frac{dq}{d(\frac{r}{s})} = \frac{B_{xx}c'}{|A|} < 0 \quad \text{since } B_{xx}<0.$$

We cannot generally sign $dx/d\frac{r}{(s)}$ because the relationship between x and R is ambiguous even in the perfect information case. But by (7) and (9) we can see that in steady-state the sales associated with a given quality are the same that would prevail under perfect information.

The result of Theorem 9 is a very intuitive one: for high speeds of learning the firm's incentives look much as they do under perfect information. Likewise, for low interest rates the short-run gains from cutting quality are relatively unimportant so quality in steady-state again nears its perfect information level.

This leaves us only with the question of whether it is in fact optimal for the firm to go to a steady state. One very special case where it is optimal to do so is when $c(q)=cq$. In this case we get a bang-bang solution to the control problem. In such a case one would want to put an upper bound on q as well as a lower bound of 0. Call q^s the quality level which satisfies the steady-state equations. Then if $R(0)>q^s$ the firm sets $q=0$ until $R=q^s$ at which point it sets $q=q^s$ forever. The opposite result occurs when $R(0)<q^s$. By the theorems above we know $q^s<q^*$.

The bang-bang example does not give any insight into the dynamics. Fortunately, since there is only one state variable, it is possible to eliminate oscillations and be guaranteed of convergence to some steady-state. The key is that the Hamiltonian is jointly concave in the control variables:

$$H_{xx} = B_{xx}$$

$$H_{qq} = -xc''(q) \qquad \text{and}$$

$$H_{xq} = -c'(q) + \lambda s = 0 \quad \text{by (2).}$$

So the second derivative matrix is

$$\begin{bmatrix} B_{xx} & 0 \\ 0 & -xc''(q) \end{bmatrix}$$ which is negative

definite. As a result, the optimal controls are continuous functions of the state variable: $x^*(R)$ and $q^*(R)$. (As in any control problem, we can "synthesize" the problem and write the optimal controls as a function of the state variables alone. Along the optimal path we also can write $\lambda = \lambda^*(R)$).

Now there are two fundamentally different possibilities. Case A is when $q^*(0)>0$; Case B is when $q^*(0)=0$.

Case A

Consider the function $q^*(R)$. We know that for R large enough it cannot pay to keep building up reputation. That is, for R large $q^*(R)<R$.[30] Since $q^*(R)$ is a continuous function, and it starts above $q=R$ at $R=0$ and eventually falls below $q=R$, at some point it must cross $q=R$. That is, for some q^s, $q^*(q^s)=q^s$. See Figure 18.

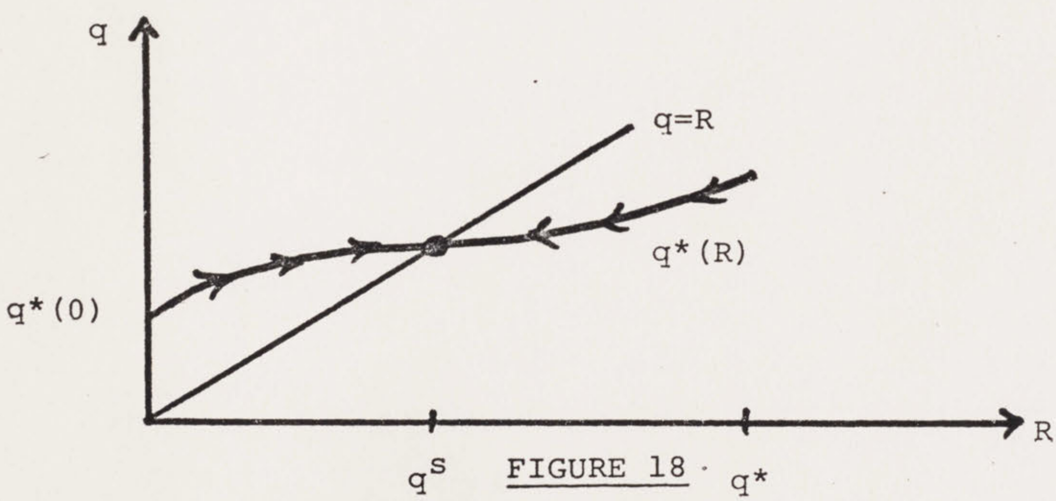


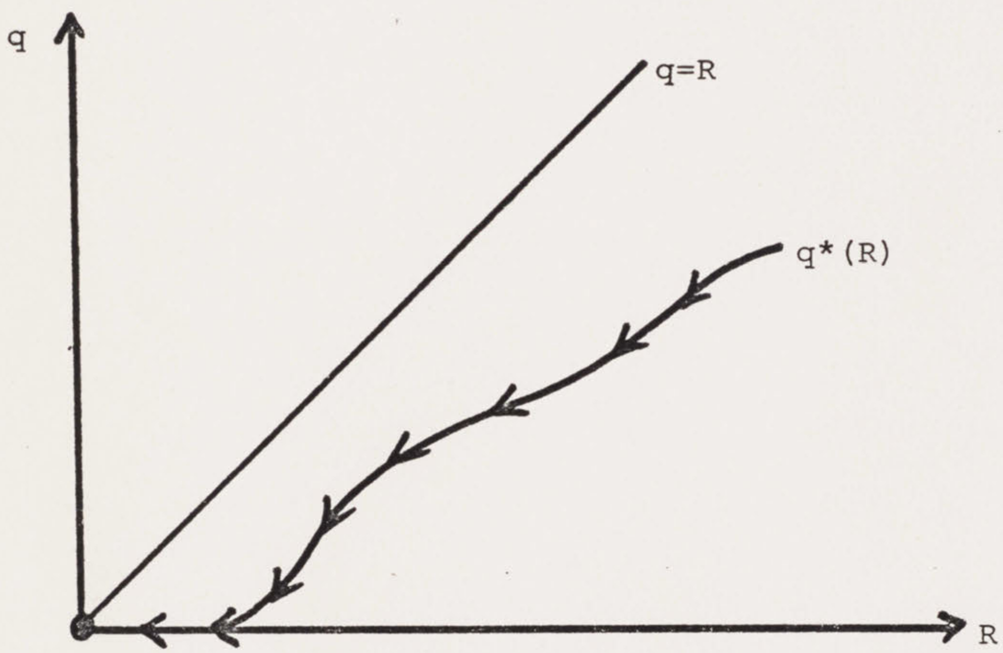
FIGURE 18

CASE A: Steady-State at q^s

The steady-state at q^s is stable. Since the perfect information problem has a unique solution, there will be only one such steady state. Theorem 9 has analyzed the position of the steady state.

<u>Case B</u> $q^*(0)=0$

It is possible that it is always optimal for the firm to run down its reputation. This corresponds to Figure 19 below.



<u>FIGURE 19</u>

Case B: $q^*(0) = 0$

This would occur if learning were very slow or the interest rate high. The firm simply runs down its reputation, eventually going out of business when $R=0$ (assuming $p(x,0) = 0$ for all x) It is interesting to note that just because a firm is intending to run down its reputation does not imply it will necessarily set $q=0$ while doing so. After all, since $c(q)$ is convex, there is little cost to improving quality slightly from 0, and it may significantly retard the deterioration of reputation.

Conclusions

This paper has presented a number of models which describe a monopolist's behavior when consumers cannot observe all the relevant attributes of his product prior to purchase. In a very general setting it has been shown that the quality the firm chooses to produce is lower under such circumstances than in a perfect information setting. The welfare consequences of this reduction in quality are generally ambiguous because we are in a second best world due to monopoly power.

The outstanding issues are many: How is product quality choice related to the information gathering activities of individual consumers or the information flows in the marketplace generally? Directly related to this is the question of under what circumstances a firm would find it profitable to alternately run up and then milk its reputation. Finally, firms themselves engage in a host of information-providing activities. There is no reason that they will provide information with socially desirable content or format. The relation between advertising and quality must be explored in an information environment of the type presented in this paper.

The desirability of public information provision can be evaluated using the results above.[31] Improved information increases the speed of learning by consumers; the effect this has on quality has been treated above. Equipped with this relationship and an estimate of consumers' preferences over quality as well as quantity, a welfare analysis of information provision can be made.

Notes

1. See, for example, Salop-Stiglitz, or Wilde-Schwartz.

2. See Lancaster [1975]. This question is only interesting when the variety of products is limited by the existence of some fixed costs.

3. With one dimensional quality q, the utility derived from consuming one unit can be written as θq where θ measures the intensity of preference for quality. So any two consumers θ_1 and θ_2 would agree that higher q was better.

   If $\theta_1 > \theta_2$, consumer 1 would be willing to pay more for increased quality than would consumer 2.

   With two (or more) attributes, consumers can disagree which of two products is preferred: one may prefer the restaurant with good food but slow service (sensitive taste but not in a rush) while another may prefer a fast food option.

4. Imagine that there is a minimal quality below which consumers can tell the product is shoddy.

5. See Darby and Karni and Nelson for a fuller discussion of these terms.

6. A treatment along these lines is given in Grossman, Kihlstrom, and Mirman.

7. And in fact the models mentioned take price as fixed and exogenous.

8. Interestingly enough, there does exist at least one regular publication providing information on the local level about services: Washington Checkbook.

   It is yet not possible to provide such a service without outside funding, however, due to public goods problems discussed below.

9. For very low qualities,malpractice suits may become relevant. In most cases, however, consumers respond to low quality by simply not buying the product again or by lowering their reservation price for it. I ignore product liability below.

10. In general x(t) may depend on the history of prices as well, since price changes may signal quality changes. Also, in a fully general formulation one would certainly want to include advertising.

11. Although they have a computation to show the firm's best choice of quality, I believe it is in error. This is because they ignore the effect quality has on the inflow of new consumers through its effect on market share.

12. This is in contrast to the approach with $x(t)$ as the state variable. Following the rule: "Switch brands if the product fails" is a useful rule of thumb but hardly optimal.

13. See, for example, Bettman or the consumer behavior or marketing literature.

14. I am assuming that under imperfect information consumers act as though their expectations were known with certainty. That is, R_o is a point expectation.

15. I have ruled out the possibility that the firm always want to produce minimal quality by assuming $V(0, R_o) = 0$. This requires some ability of consumers to observe quality if it is very low. Without this assumption it could well be the case that the choice for the firm is "minimal quality", whatever that may mean. This would hold for attributes which are completely unobserable (e.g., automobile safety features if no one took the trouble to run tests on new models). Theorem 2 still holds without assuming that $v(o,R)=o$ so long as $v_1(o,o)>o$. This condition would be guaranteed, for example, by assuming that there are no cost savings to be had from reducing quality below o.

16. I have shown above that $B(q^*)<q^*$. To see that $B(\bar{R}_o) < R_o$ for $R_o>q^*$, notice that for $q \geq R_o$, $V(q,R_o) \leq V(q,q) < V(q^*,q^*)$ $<V(q^*,R_o)$ which is a feasible present value of profits for a firm facing initial reputation R_o.

17. Individual consumers cannot "game" with the firm by changing their expectations because there are many consumers so that any one consumer cannot influence the firm's choice.

18. Such sophistication would not be logically inconsistent with the type of learning embodied in V, but is implausible.

19. This is of course a very special case of learning. It may well hold for attributes which no one can observe for some time (e.g., durability) but which then become public knowledge, perhaps through publication. Automobile repair records or other durability characteristics may fit this type of learning well.

20. This and the following comparative statics computation assume that the optimal q is unique so that $\frac{dq}{ds}$ exists and the argument goes through.

21. This is relaxed below.

22. I stick to {0,1} demands for simplicity.

23. Permitting diverse expectation among consumers further complicates the analysis and can lead to welfare gains or losses from the misinformation as well.

24. As discussed above, in general, the joint distribution of expectations and valuations of quality determine demand. The updating which occurs as consumers buy and learn in that more general context is considerably more complex. I make some remarks about it below.

25. Again, consumers have $\{0,1\}$ demands. This is not essential for the analysis, but provides a substantial simplification.

26. I treat consumers as having point estimates of quality, thereby assuming away the possibility of the consumer's buying a product he expects will not be worth the price in order to learn about its quality.

27. If the monopolist initially charges a price below p^*, he could then jump immediately to p^* forever. In this case the "monotonically delining" part of the price path is the single price charged during the first period.

28. So this $V(q,R)$ function is a truncated version of the one used in the once and for all quality choice problem.

29. In general, I can permit $R_t = \sum_{k=0}^{L} A_k q_{t-k}$. This has no effect on the qualitative results

The factor $\Sigma \rho^k A_k$ is the relevant speed of expectation formation.

30. It cannot pay to continually build up higher and higher reputation. Here is why: suppose $q^*(R) > R$ for all R. Then $R(t)$ would be strictly increasing. But when $R \geq q^*$ (the perfect information level of quality) we have
$B(x, R) - c(x, R) < B(x, q^*) - c(x, q^*)$.
Furthermore, if $q > R$
$B(x, R) - c(x, q) < B(x, R) - c(x, R)$.
Therefore, So long as $q > R > q^*$ we know the flow of profits is less than the perfect information profit flow. But when $R > q^*$ the firm could achieve at least $B(x, q^*) - c(x, q^*)$ forever. Hence continual building of reputation cannot be optimal.

31. The attractiveness of minimum quality standards, however, should not be treated in a monopoly model. This is because only one quality is provided, so the policymaker could just specify the socially optimal quality. Instead, quality standards must face the issue of diversity of preferences in the presence of many suppliers. See Shapiro [1980].

Appendix

We begin from

(A.1) $MR(x,R) - c(q) = \lambda s(R - q)$

(A.2) $xp_R(x,R) - \lambda sx = r\lambda - \dot{\lambda}$

(A.3) $\dot{R} = sx(q - R)$

From A.1, when $q \neq R$

(A.4) $$\lambda = \frac{MR(x,R) - c(q)}{s(R - q)}$$

So $$\dot{\lambda} = \frac{1}{s} \frac{(R - q)\ [\ MR_x\dot{x} + MR_R\dot{R}\ ] - [\ MR - c\ ]\dot{R}}{(R - q)^2}$$

Substituting for λ into (A.2) gives

(A.5) $$xp_R(x,R) - \frac{MR - c}{R - q}\, x = \frac{r}{s}\ \frac{MR - c}{R - q} - \dot{\lambda}$$

Multiply by $s(R - q)$ and substitute for $\dot{\lambda}$ to get

(A.6) $$sx\ [\ (R - q)\ p_R - (MR - c)\ ] = r[MR - c] - [MR_x\dot{x} + MR_R\dot{R}] + \frac{MR - c}{R - q}\,\dot{R}\ .$$

From (A.3) the last term is just $-sx(MR - c)$ and thus cancels the last term on the left - hand-side to give

(A.7) $sx(R - q)\ p_R = r(MR - c) - MR_x\dot{x} - MR_R\dot{R}$

Solving for $\dot{x}$, using (A.3) to substitute for $\dot{R}$, gives

(A.8) $$\dot{x} = \frac{sx(R - q)(MR_R - p_R) + r(MR - c)}{MR_x}$$

This can be rewritten as

$$\dot{x} = [-\frac{sx}{MR_x}](q - R) [MR_R - p_R) + \frac{r(MR - c)}{sx(R - q)}]$$

Finally, $MR = p + p_x$ so

$MR_R = p_R + p_{xR}$ or

$MR_R - p_R = p_{xR}$ so

(A.9) $$x = [\frac{-sx}{MR_x}](q - R) [xp_{xR} + \frac{r}{sx} (\frac{MR - c(q)}{R - q})].$$

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Premiums for High Quality Products as Rents to Reputation

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It has long been recognized that a firm which has a good reputation owns a valuable asset. This is often referred to as the "goodwill" value of the firm's brand name or loyal customer patronage. This paper develops a model which explores the implications of firm-specific reputations in a perfectly competitive environment.

The idea of reputation only makes sense in an imperfect information world. A firm has a good reputation if consumers believe their products to be of high quality. If product attributes were perfectly observable prior to purchase, then previous production of high quality items would not enter into consumers' evaluations of the firm's product quality. Instead, quality beliefs could be derived solely from inspection.

When product attributes are difficult to observe prior to purchase, consumers may plausibly use the quality of products produced by the firm in the <u>past</u> as an indicator of present or future quality. In such cases, the firm's decision to produce high quality items is a dynamic one: the benefits to doing so accrue in the future via the effect of building up a reputation.

Since reputation is a capital asset, it is natural that some rental income should accrue to it. These quasi-rents exactly compensate the firm for the costs incurred in building up its reputation so that there are zero profits ex-ante. In fact, were there no flow of profits to be earned from having a high reputation, it would not pay to maintain the reputation. Instead of continuing to produce high quality items, a firm with a good reputation could cut quality and take a short-run gain as a result. This "fly-by-night" strategy would always be attractive were "profits" not being earned by maintaining reputation. These "profits" are really just rents in return for building up the reputation initially. But the above discussion makes it clear that they are also necessary to prevent the firm from preferring to milk its reputation.

These ideas are utilized in the analysis below to derive an equilibrium price-quality schedule under imperfect information. Prices of high quality products must exceed their cost in order to provide the flow of quasi-rents to reputation. The welfare consequences of this price-cost gap are investigated. In particular, a welfare analysis of minimum quality standards is presented. The higher the minimum quality standard, the less a firm can earn while milking its reputation. Since the premiums for high quality items are exactly large enough to forestall this milking, they are lower, the higher is the minimum quality standard. Thus raising minimum quality standards benefits consumers who like to consume high quality items. Balanced against this is the direct effect of excluding products other consumers would like to use.

There are a number of effects which are ruled out in order to focus on reputation as a quality-assuring mechanism. For example, the assumption that costs of production are not time interdependent is a strong one. One main reason for the fact that different firms product different quality items is that some producers find it easier to produce quality products. This will typically be due to some capital good on the cost side. Examples include expensive machinery in an auto repair shop which makes it less costly to provide good service, or training on the part of a skilled worker which has the same effect (*i.e.*, reducing the marginal cost of quality). We might expect the capital goods to signal quality in such cases. I rule them out to focus only on a capital good on the demand side, namely reputation.

Similarly, a product line of high quality may already be designed and in production. This reduces the savings from a reduction in quality (which would involve some redesign efforts). This latter could be modelled as a cost to changing quality. However, so long as the firm can exit the market without taking losses, it will have to be the rents to reputation which forestall the temptation to do so. Firm specific capital other than reputation could serve some of this function as well.

Finally, I rule out guarantees as a quality-assuring mechanism. This is not because I believe them to be unimportant, but simply imperfect. A washing machine may have a one year guarantee, but the consumer is expecting a lifetime of 10 years from it. Therefore, there is room for potential quality cutting by the seller, the guarantee not withstanding. For a variety of moral hazard and adverse selection reasons, perfect guarantees are not feasible. Anytime the sellers could make some quick profits by reducing quality the analysis below will apply.

The paper is organized as follows: first I analyze in full the case where the seller chooses a quality of product each period but does not also have sales as a control variable. After defining and computing the equilibrium here, I analyze the welfare effects of improved information and minimum quality standards. Then I present a continuous time model where firms can choose sales as well as quality levels. This permits identification of some additional effects of imperfect information. In particular, firms do not operate at efficient scale. Finally, I provide a summary and conclusion.

General Set-Up and Definition of Equilibrium

The model to follow describes the situation discussed above. It is necessarily dynamic, since reputation formation occurs over time. It is set in discrete time, where the period is the length of production. For example, if the product is constructing a building, the period of time is however long it takes to put one building up.

Each period each seller can choose what quality product to provide. In this model the number of products a given firm produces per period is held fixed (at one). For a model where quantity is also a control variable of the firm, see the final section. The price a seller can charge is determined by his reputation, R, and is denoted by p(R). This emodies the perfect competition assumption. Since in equilibrium consumers know the price-quality schedule, no firm can exert control over its price; consumers will not purchase from any firm which offers a price-reputation pair above the schedule. The cost of production depends on quality and is called c(q). I assume $c'(q) > 0$ and $c''(q) > 0$. Sellers choose quality over time to maximize the present value of their profits.

Reputation formation will initially be assumed to be of a very simple form: a seller's reputation this period is exactly his quality last period:

(1) $R_t = q_{t-1}$.

I will discuss modifications of (1) below. It simply reflects the fact that quality cannot be observed prior to purchase, and hence sellers can, at least for one period, cheat on their customers by reducing quality.

Consumers differ in both their taste for quality, Θ, and in their underlying evaluation of the good v. Consumers purchase either 0 or 1 unit of the good. If a consumer of type (Θ, v) consumes a unit of quality q he achieves net utility $\Theta q + v - p$, where p is the price paid.

Finally, there is a minimum quality q_0. This may be given several interpretations, but the simplest is that it is illegal to sell items of quality below q_0. I will discuss q_0 at length below. The distribution of demands is such that q_0 is actually produced (we will see below that this will be the case for the optimal minimum quality standard).

Entry is permitted, but new firms must prove themselves in order to build up a reputation. Initially, they must sell their product at price $p(q_0)$. This assumption is necessary for any equilibrium to exist. In fact if new firms could sell for any price higher than the cost of producing minimum quality items, then entrants could make positive profits by producing items of quality q_0 and selling them for one period and then exiting.

Equilibrium

Equilibrium is a price-quality (or, equivalently, a price-reputation) schedule $p(q)$ such that

A. Each consumer, knowing $p(q)$, chooses his most preferred product on the schedule to consume (if he uses the product at all

B. Markets clear at every quality level (this determines the number of active firms in equilibrium).

C. A firm with reputation R finds it optimal to produce quality $q=R$ rather than to deviate (that is, consumers' expectations regarding quality are fulfilled).

D. No new entry is attractive.

Heuristic Derivation of Equilibrium Price-Quality Schedule

In this section I will derive the equilibrium price-quality schedule from elementary arguments. Just as the price of a good in perfect competition (in the long run, with perfectly elastic input supplies) is determined by the supply side alone - namely minimum average cost - so is the price which prevails at each quality level determined solely on the supply side in this model.

There are two conditions which are used to derive the schedule- conditions C. and D. from the definition of equilibrium above. First consider the condition that a firm with reputation q does not wish to milk its reputation. One way to milk reputation is to cut quality to the minimum, take short-run gains, and exit the market. This would yield profits of $p(q) - c(q_0)$. The alternative strategy of maintaining quality forever yields present discounted profits of $\frac{1+r}{r}(p(q) - c(q))$. In order that milking not be attractive we must have

$$\frac{1+r}{r}[p(q) - c(q)] \geq p(q) - c(q_0) \quad \text{i.e.}$$

(2) $$p(q) \geq c(q) + r(c(q) - c(q_0)).$$

As expected, the firm must be able to earn profits by maintaining quality in order not to wish to run down its reputation.

Turning to the free-entry condition, equilibrium requires that entry not be attractive. The profits to an entrant who produces quality q forever are

$$p(q_0) - c(q) + \frac{1}{r}[p(q) - c(q)].$$

Condition D then becomes

$$p(q_0) - c(q) + \frac{1}{r}[p(q) - c(q)] \leq 0 \quad \text{or}$$

(3) $p(q) \leq c(q) + r[c(q) - p(q_0)].$

Finally, it must be the case that

(4) $p(q_0) = c(q_0).$

Basically, there is no informational problem for products of quality q_0. If $p(q_0) < c(q_0)$ no firm would supply quality q_0. If $p(q_0) > c(q_0)$ any entrant could profitably undercut sellers of quality q_0 by simply offering a product of quality q_0 at a price between $p(q_0)$ and $c(q_0)$. Since consumers of quality q_0 know they will not face lower quality than q_0, they will be happy to buy from the entrant at the lower price.

Substituting $p(q_0) = c(q_0)$ into (3), we see that (3) is the reverse inequality of (2). Therefore, these two conditions together fully determine p(q), which is given by

(5) $p(q) = c(q) + r(c(q) - c(q_0)).$

Figure 1 shows the equilibrium p(q) schedule and its relationship to the perfect information schedule c(q).

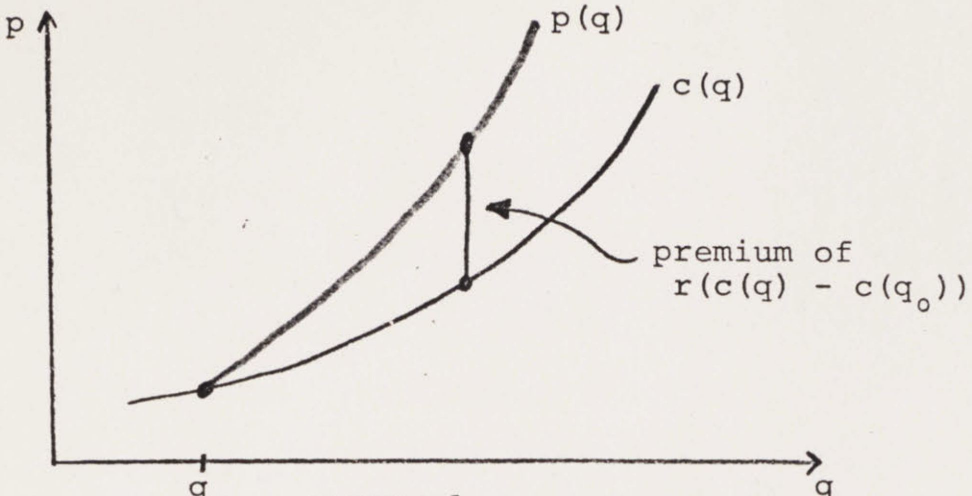

FIGURE 1

Equilibrium p(q) Schedule

Formal Derivation of p(q)

Consider a firm with initial reputation R_o. The firm can choose quality in each period to maximize present discounted profits:

$$\max_{q_o, q_1, \ldots} \sum_{t=0}^{\infty} \rho^t [p(R_t) - c(q_t)]$$

such that $R_t = q_{t-1}$

R_o given

Here $\rho = \frac{1}{1+r}$ is the discount factor. This problem can be re-written as

$$\max_{q_o, q_1, \ldots} \sum_{t=0}^{\infty} \rho^t [p(q_{t-1}) - c(q_t)]$$

with q_{-1} given.

Differentiating with respect to q_t gives

$$\rho^{t+1} p'(q_t) - \rho^t c'(q_t) = 0 \quad \text{or}$$

$$\rho \, p'(q_t) = c'(q_t) \quad \text{i.e.}$$

(6) $p'(q_t) = (1+r)c'(q_t)$.

For an arbitrary p(q) schedule, so long as the second-order conditions held everywhere, there would be a unique solution to (6) and all firms would choose to produce the quality which yielded the solution. Such a price schedule could not be an equilibrium because it would violate market clearing at other quality levels.

If (6) is used to define p(q), however, then all quality levels will by definition satisfy the steady-state condition. Therefore for any initial reputation, a firm would find it optimal to maintain its reputation.[1] This is exactly the condition needed for an equilibrium in which a variety of products is sold.

The differential equation for p(q) given by (6), along with the boundary condition (4) admits only (5) as its unique solution.

Since it is optimal for a firm with reputation R_0 to maintain quality, it is easy to figure out the asset value of reputation. This is just the present value of profits accruing to having the reputation, when the firm follows its optimal regime from that point on. We can compute this value as

$$V(R_o) = \frac{1+r}{r}\,[p(R_o) - c(R_o)]$$

$$= \frac{1+r}{r}\,[r(c(R_o) - c(q_o))]$$

$$V(R_o) = (1+r)\;(c(R_o) - c(q_o)).$$

Of course this is increasing in R_o. Also, $V(q_o) = 0$ as is necessitated by free entry with initial reputation q_o. Note that it is also increasing in r; we will see below that this implies that improved information decreases the asset value of reputation. Finally it is decreasing in q_0 so an increase in the minimum quality standard would cause a capital loss for firms with good reputations.

I should emphasize that these profits are only ex-post profits. The asset value of reputation R_0 exactly equals the cost of building up that reputation. Ex ante there are zero profits.

Information Provision and the Interpretation of r

At this point I would like to indicate how the equilibrium $p(q)$ schedule depends on information flows in the market. This will be important for studying the welfare consequences of improved information.

Information in this model is embodied in the reputation adjustment equation. As alternatives to (1), consider

(7) $R_t = q_{t-n}$

(8) $R_t = \gamma R_{t-1} + (1-\gamma)\, q_{t-1}$.

To see how these specifications alter the equilibrium schedule, I simply restate the optimal control problem discussed above with these alternative equations of motion of the state variable, R.

Looking first at (7) we get

$$\max_{q_0, q_1, \ldots} \sum_{t=0}^{\infty} \rho^t [p(R_t) - c(q_t)]$$

$$R_t = q_{t-n}$$

$$q_{-1}, \ldots q_{-n} \text{ given}$$

re-writing we have

$$\max_{q_0 q_1, \ldots} \sum_{t=0}^{\infty} \rho^t [p(q_{t-n}) - c(q_t)]$$

$$q_{-1}, \ldots q_{-n} \text{ given}$$

Differentiate with respect to q_t to get

$$\rho^{t+n} p'(q_t) - \rho^t c'(q_t) = 0 \quad \text{or}$$

(9) $p'(q_t) = (1+r)^n c'(q_t)$.

This replaces (6) when $R_t = q_{t-n}$.

For r small we get the approximation

(9)' $p'(q_t) \cong (1+rn)\, c'(q_t)$

Therefore increasing r in equation (5) can be thought of as increasing the length of time before quality is observed.

While (7) reflects a lag in observing quality, (8) captures two possible effects in reputation formation: The first is that consumers do not completely alter their judgment of the firm on the basis of one period's quality. Rather they may slowly adjust reputation towards observed quality. The second concerns the probability of observing true quality. Some product attributes are difficult to detect even after purchase - e.g., safety features. If γ is the probability that the true quality is not observed (in which case reputation is unaltered) then (8) will hold. The earlier case, (1), corresponds to $\gamma=0$ so that there is rapid or certain reputation adjustment.

To derive the steady-state necessary condition when $0<\gamma<1$, it is enough to compute the change in profits from a one-shot blip in quality at time 0 followed by a return to producing q every period. The effect of such a deviation is:

$$V = \sum_{t=0}^{\infty} \rho^t [p(R_t) - c(q_t)]$$

$$\left.\frac{dV}{dq_0}\right|_{\substack{R_0=q \\ q_t=q}} = -c'(q) + \rho p'(q)\frac{dR_1}{dq_0} + \rho^2 p'(q)\frac{dR_2}{dq_0} + \cdots$$

$$= -c'(q) + p'(q)\sum_{t=1}^{\infty} \rho^t \frac{dR_t}{dq_0}.$$

Now $R_t = \gamma R_{t-1} + (1-\gamma) q_{t-1}$

So $\dfrac{dR_1}{dq_0} = 1-\gamma$

$$\frac{dR_2}{dq_o} = \frac{dR_2}{dR_1}\frac{dR_1}{dq_o} = \gamma(1-\gamma)$$

And $\frac{dR_t}{dq_o} = \gamma^{t-1}(1-\gamma)$.

Substituting into $\frac{dV}{dq_o}$ we have

$$\frac{dV}{dq_o} = -c'(q) + p'(q) \sum_{t=1}^{\infty} \rho^t(1-\gamma)\gamma^{t-1}$$

$$= -c'(q) + (1-\gamma)\rho p'(q)\sum_{t=0}^{\infty} \rho^t\gamma^t$$

$$= -c'(q) + \frac{(1-\gamma)\rho p'(q)}{1-\rho\gamma}$$

If q is to be a steady-state quality level this expression must be zero. So

$$p'(q) = \frac{1-\rho\gamma}{\rho(1-\gamma)} c'(q)$$

$$= [1 + \frac{1-\rho}{\rho(1-\gamma)}]c'(q)$$

(10) $$p'(q) = (1 + \frac{r}{1-\gamma})c'(q).$$

When $\gamma=0$ this reduces to our original expression. Slow reputation adjustment (forgiving consumers) or difficult to detect attributes raise γ and, in the analysis below, can be treated by raising r.

Finally, perhaps the most important interpretation of r is as frequency of purchase. Taking as given the market discount rate per unit time, i, if the period is of length T, then $\rho=e^{-iT}$ or

$r = e^{iT} - 1$. Large T, namely infrequent production periods, is another interpretation of a large r. As one would expect, informational problems are more severe the larger is r.

Summarizing, large values of r can be interpreted as

(a) Infrequent production (i.e., lengthy production process)

(b) Long lags in detection of quality

(c) Slow updating of reputations or

(d) Difficult to detect quality attributes.

In the welfare analysis of r below, r can be thought of as a policy variable since information provision activities can influence r through several of the above channels.

<u>Description of Equilirbium p(q) Schedule</u>

From Figure 1 and equation (5) it is easy to see some of the qualitative characteristics of the equilibrium price - quality schedule. First notice that the premium paid for high quality products, $r(c(q) - c(q_0))$ is larger the higher the quality involved. So the imperfect operation of reputation as a quality - conveying mechanism is more severe for higher quality items.

Notice also

<u>Theorem 1</u>. As $r \to 0$, the equilibrium price-quality schedule approaches the perfect information schedule.

This reflects the fact that as $r \to 0$ the flow of profits necessary to forestall cheating on quality becomes smaller. For $r=0$ any positive flow of profits would be more than enough to cause a firm to prefer to maintain quality. Viewed differently, any positive flow of profits would be more than enough to compensate an entrant for the finite one-period loss involved in building up the reputation.

Keeping in mind the interpretations of r noted above, the larger is r the more of a gap between p(q) and c(q). The welfare consequences of this will be explored below. See Figure 2.

It is also easy to see how q_0 affects p(q). An increase in q_0 simply shifts the whole schedule down by a fixed amount without affecting the slope. Of course the schedule starts at $q=q_0$, so an increase in q_0 reduces the spectrum of products available in the market. See Figure 3.

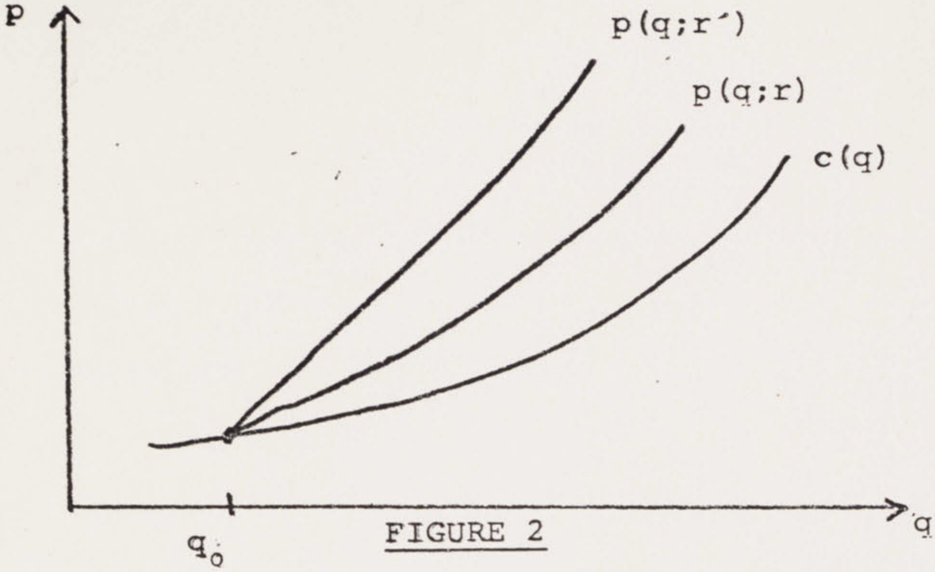

FIGURE 2

Effect of r on p(q): r'>r

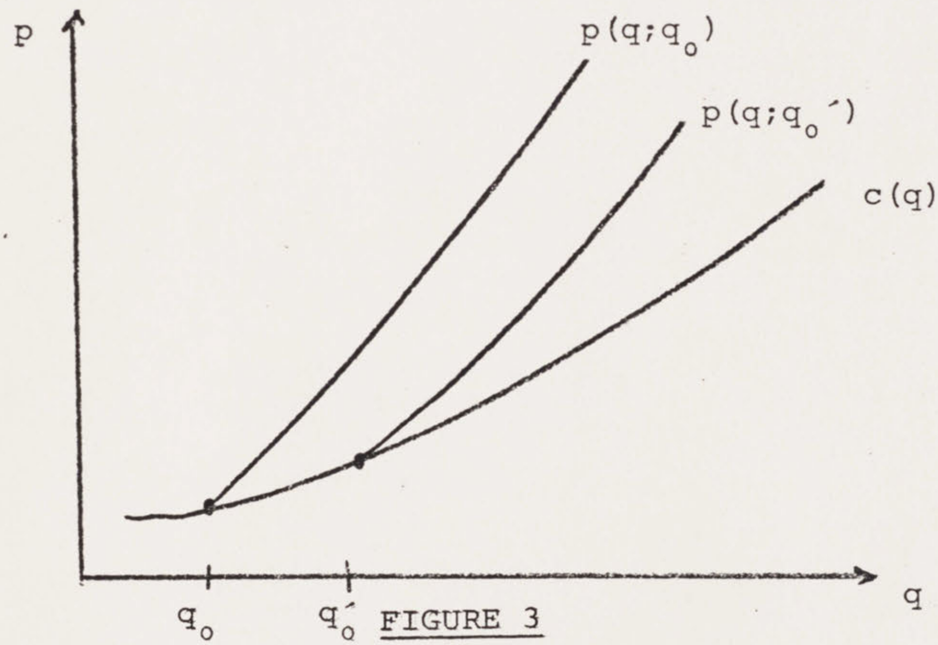

FIGURE 3

Effect of q_o on p(q): $q_o' > q_o$

Consumers

In this section I look more closely at how various consumers respond to a given $p(q)$ schedule. This is necessary in order to perform a welfare analysis.

As mentioned above, each consumer is described by two parameters, Θ and v. A consumer of type (Θ, v) achieves utility $\Theta q + v - p$ from purchasing one unit of quality q at price p.

Consumers buy either 0 or 1 unit of the good. There is a given distribution of types of consumers $f(\Theta, v)$. This distribution is confined to the box $[\underline{\Theta} \times \bar{\Theta}] \times [\underline{v} \times \bar{v}]$ where $\underline{\Theta} > 0$. (Multi-unit demands for the same quality can be treated via the f function).

Consumer (Θ, v), when facing the price-quality schedule $p(q)$, solves the following problem:

$$\max_{q>q_0} \Theta q + v - p(q).$$

Differentiating with respect to q we have

(11) $\Theta = p'(q)$

unless $\Theta < p'(q_0)$, in which case $q = q_0$. These describe the choice of q by Θ *if* the product is purchased.

So long as $p''(q) > 0$, which follows from the assumption that $c''(q) > 0$, we know that consumers with a greater taste for quality, higher θ's, consume higher quality items:

$$\frac{dq}{d\Theta} = \frac{1}{p''(q(\Theta))} > 0$$

Substituting our formula (5) for p(q) into (11) we have the quality choice by Θ given r, denoted $q(\Theta, r)$, defined by

$$\text{(12)} \quad \begin{cases} \Theta = (1 + r)\, c'(q(\Theta, r)) & \text{if } \Theta > (1 + r)\, c'(q_o). \\ q(\Theta, r) = q_o & \text{if } \Theta \leq (1 + r)\, c'(q_o). \end{cases}$$

It is important to note that q_o does not affect the slope of p(q) and thus does not affect $q(\Theta, r)$ except for those who choose to consume q_o. It will, however, affect the set of consumers who buy at all.

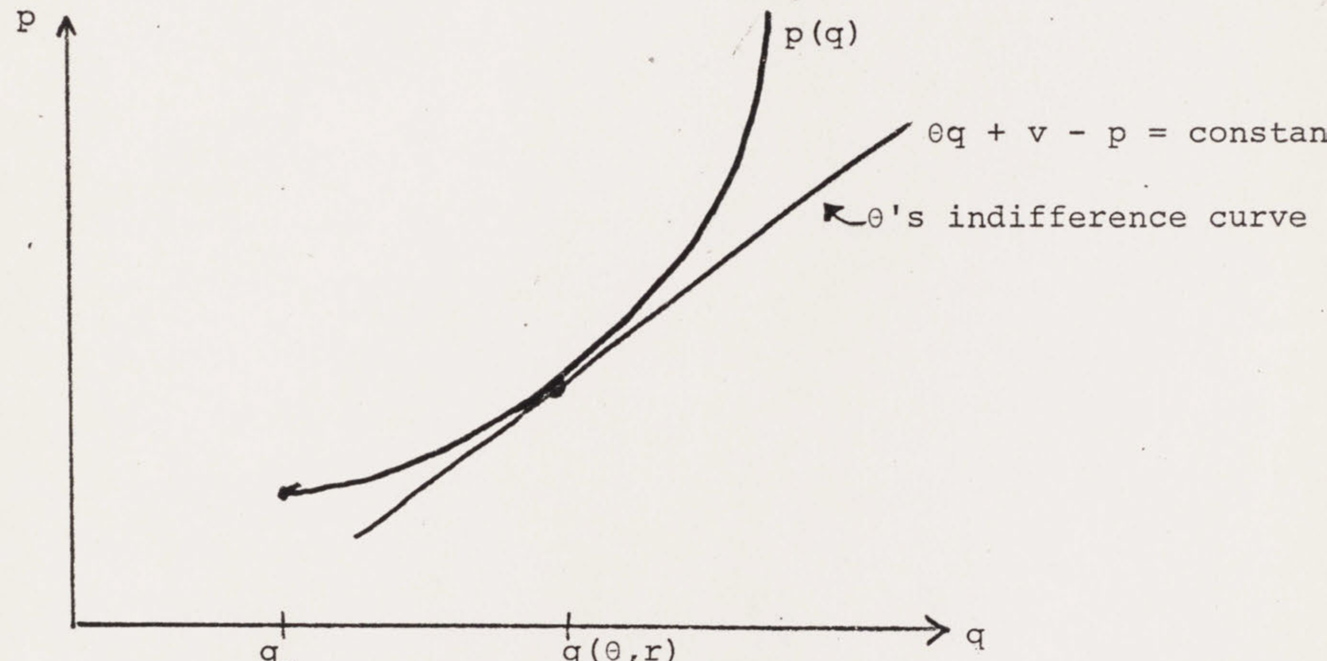

FIGURE 4

Choice of Quality by (Θ, v)

Type (Θ, v) will purchase the product if and only if

$\Theta q(\Theta, r) + v - p(q(\Theta, r)) > 0.$

Subtituting for $p(q)$ this becomes

$\Theta q(\Theta, r) + v - [(1 + r)\, c(q(\Theta, r)) - rc(q_o)] \geq 0$

Rewriting, we have:

(Θ, v) purchases the product if and only if

(13)

$v \geq (1 + r)\, c(q(\Theta, r)) - rc(q_o) - \Theta q(\Theta, r).$

Denote the right-hand side by $v(\Theta; q_o, r)$. Differentiating (13) with respect to q_o and using (12) we have

$$\text{(14)} \quad v_{q_o}(\Theta; q_o, r) = \begin{cases} -rc'(q_o) & \text{if } q(\Theta, r) > q_o \\ c'(q_o) - \Theta & \text{if } q(\Theta, r) = q_o \end{cases}$$

This is to be interpreted as follows: when q_o rises the $p(q)$ schedule shifts down (by $rc'(q_o)$). This causes consumers with high valuations of quality (Θ) to face a more attractive opportunity set and more of them buy. This is represented by region B in Figure 5 ($v(\Theta; q_o, r)$ falls for high Θ's). On the other hand low Θ's $(\Theta < c'(q_o)$ would like to consume products of quality less than q_o, so raising q_o makes them worse off, and only higher v's will purchase as a result. Those in region D leave the market when q_o is raised to q_o. For $\Theta \varepsilon (c'(q_o), (1+r)c'(q_o)$, (Θ, v) would like to purchase $q > q_o$ if he only had to pay the cost. He is unwilling to pay the premium as well, however, so purchases q_o. Since q_o sells at price $c(q_o)$, he prefers q_o to be raised. This is all summarized in Figure 5 below. (We know $v(\Theta, q_o, r)$ is declining in Θ since higher Θ's derive strictly greater utility from any given quality product, and thus would certainly buy a unit if lower Θ's did.)

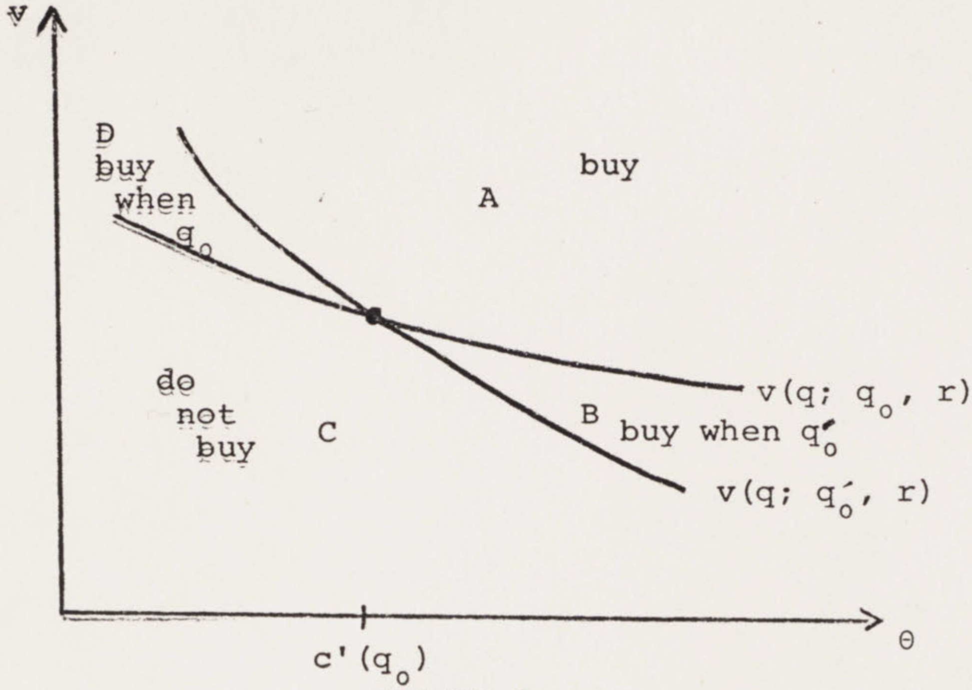

FIGURE 5

$v(\theta,\ q_o,\ r):\ \ q_o' \gtrsim q_o$

Having described how $(\theta,\ v)$'s response depends on q_o, let me look more closely at the effects of r. When r goes up any consumer who purchases $q>q_o$ finds he must pay more. Consequently, he is worse off; furthermore, since r affects the marginal cost of quality, $(1+r)\ c'(q)$, it will affect his quality choice, via (12). Differentiating (12) with respect to r we have (for

$\theta \geq (1+r)\ c'(q_o\ \ ),\ 0 = (1+r)\ c''(q(\theta,\ r))\ \ q_r(\theta,\ r) + c(q(\theta,\ r))$

or

(15) $$q_r(\theta,\ r) = \frac{-c'(q(\theta,\ r)}{(1+r)\ c''(q(\theta,\ r))} < 0.$$

As expected, increased r causes a given type of consumer to substitute towards lower quality products (unless θ was using q_o already, as would be the case for $\theta \leq (1+r)\ c'(q_o)$). See Figure 6.

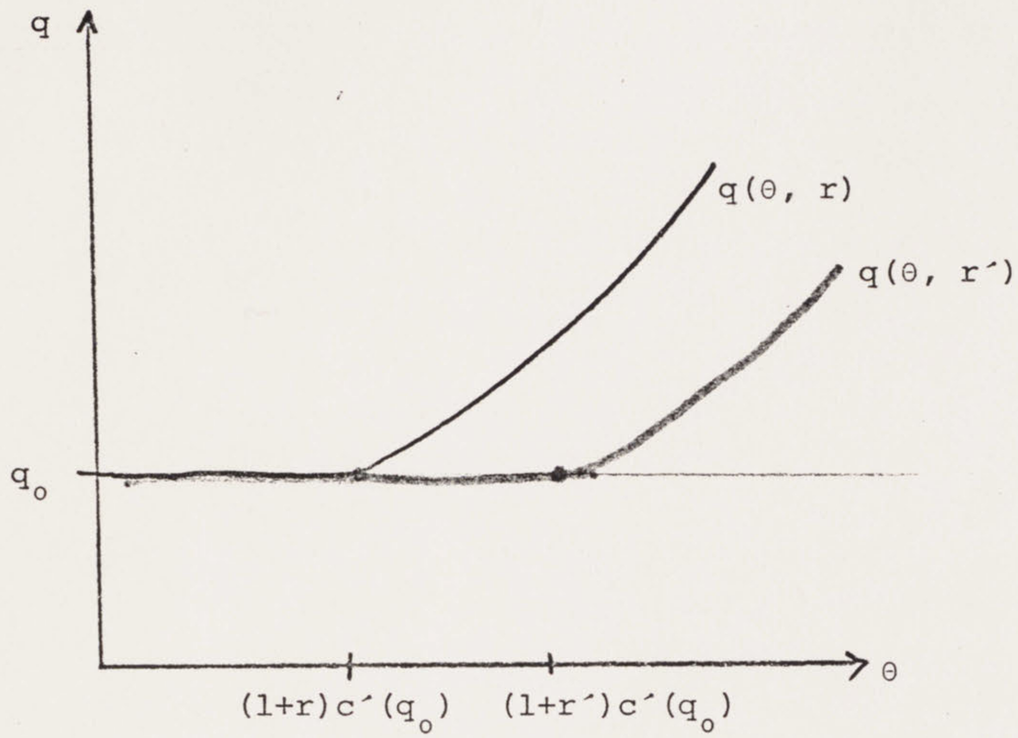

FIGURE 6

$q(\Theta, r)$: $r' > r$

Furthermore, the less favorable p(q) schedule which results from increased r causes fewer consumers of a given Θ-type to consume at all. Differentiate (13) with respect to r to get

$$v_r(\Theta\;\; q_o, r) = (1+r)\; c'(q(\Theta, r))q_r(\Theta, r) + c(q(\Theta, r))-c(q_o) - \Theta q_r(\Theta, r)$$

$$= [\; (1+r)\; c'(q(\Theta, r)) - \Theta]\; q_r(\Theta, r) + c(q(\Theta, r)) - c(q_o)$$

By (12) the first term in brackets is zero for $\Theta > (1+r)\; c'(q_o)$ and so

(16) $\quad v_r(\Theta; q_o, r) = c(q(\Theta, r)) - c(q_o) > 0$

for those Θ's. For $\Theta \leq (1+r)\; c'(q_o)$, $q(\Theta, r) = q_o$, $q_r(\Theta, r) = 0$ and

$v_r(\theta;\, q_o,\, r) = 0$. This is because low θ's continue to use q_o at price $c(q_o)$, whatever r is. See Figure 7. Those consumers in-between the two curves drop out of the market when r rises to r´.

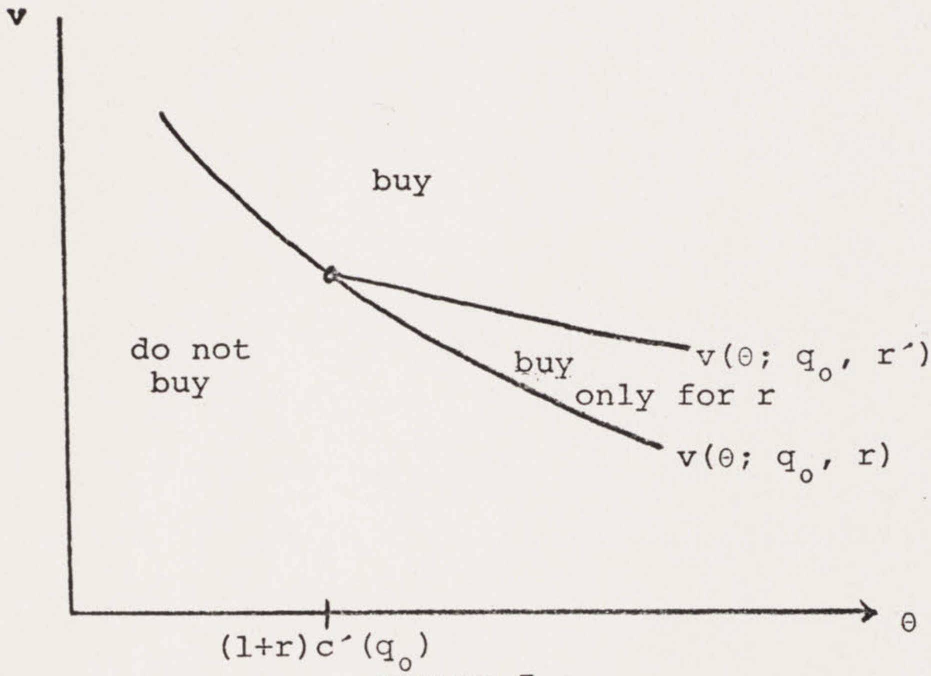

FIGURE 7

$v(\theta;\, q_o,\, r)$: $r' \gtrsim r$

Welfare Analysis of r

Utilizing the analysis of consumer behavior above, we can determine the welfare effects of changing r and q_0. This section studies r; the next will treat q_0. Keep in mind the section on the interpretation of r when considering changing r in this section. Since q_0 will be fixed in this section, it is suppressed in the notation when possible.

The idea behind the welfare theorem in this section is this: as r increases, the wedge between price and cost for high quality products rises. This is like a tax on high quality items. Increases in r lead to increases in the "tax", with associated distortions. Of course, information costs are as "real" as production costs, so this should not be viewed as a market failure so much as a cost due to imperfect information.

The welfare measure used is aggregate consumer surplus plus profits. Equivalently, I will write down expressions for gross utility minus the costs of production. Since producers earn zero profits *ex ante*, we can identify this aggregate welfare measure with consumer surplus. This requires inclusion of the transition period, during which firms take losses to build up reputations, in the welfare analysis. The easiest way to treat this period is to assume[2] that it differs from the steady-state only in the prices charged (all items sell at $c(q_0)$). With this convention there is no difference in social welfare between the transition period and the steady-state, because the same allocation prevails. Consequently, we can identify steady-state aggregate welfare with consumer surplus, by using the zero profit condition.

Look first at the set of all consumers of type Θ . For $\Theta < (1+r)\ c'(q_o)$ type Θ either uses $q=q_o$ or stays our of the market. The aggregate welfare of type Θ consumers is

$$W(\Theta,\ r) = \int_{v(\Theta,r)}^{\bar{v}} f(\Theta,\ v)\,[\Theta q_o + v - c(q_o)]\,dv.$$

For $\Theta < (1+r)\ c'(q_o),\ q(\Theta,\ r) = q_o$ so

$$v(\Theta,\ r) = c(q_o) - \Theta q_o \quad \text{and}$$

$$W(\Theta,\ r) = \int_{c(q_o)-\Theta q_o}^{\bar{v}} f(\Theta,\ v)\,[\Theta q_o + v - c(q_o)]\,dv. \tag{17}$$

Evidently, r has no influence on these consumers' utility since it neither affects the quality chosen (q_o) nor its price (and hence $v(\Theta,\ r)$).

The situation is very different for $\Theta > (1+r)\ c'(q_o)$. Now

$$W(\Theta,\ r) = \int_{v(\Theta,\ r)}^{\bar{v}} f(\Theta,\ r)\,[\Theta q(\Theta,\ r) + v - c(q(\Theta,\ r))]\,dv$$

Differentiating with respect to r we have

$$W_r(\Theta,\ r) = \int_{v(\Theta,\ r)}^{\bar{v}} f(\Theta,\ v)\,[\Theta q_r - c'q_r]\,dv$$
$$- v_r f(\Theta,\ v(\Theta,\ r))\,[\Theta q(\Theta,\ r) + v(\Theta,\ r) - c(q(\Theta,r))]$$

Using (12) to substitute for c', and (13) for $v(\Theta,\ r)$ we have

$$W_r(\Theta,\ r) = \int_{v(\Theta,\ r)}^{\bar{v}} f(\Theta,\ v)\ q_r\left(\Theta - \frac{\Theta}{1+r}\right)dv$$
$$- v_r f(\Theta,\ v(\Theta,\ r))\,r\,[c(q(\Theta,\ r)) - c(q_o)]$$

Finally, substituting for q_r and v_r from (15) and (16) we get

$$(18)\quad W_r(\Theta, r) = \frac{r\Theta}{(1+r)^2}\left[\frac{-c'(q(\Theta, r))}{c''(q(\Theta, r))}\right]\int_{v(\Theta,r)}^{\bar{v}} f(\Theta, v)dv$$

$$-f(\Theta, v(\Theta, r))r[c(q(\Theta, r)) - c(q_0)]^2$$

The first term here indicates the welfare loss due to the further distortion in quality choice by type Θ's as r increases. The second term reflects the fact that some type Θ's (namely type $(\Theta, v(\Theta, r))$) are forced out of the market by the increase in r. There is an unambiguous welfare loss as r increases. The gains from reducing r should be weighed against the costs of any information provision activities which could do so. This welfare analysis is summarized in

Theorem 2. There is a welfare loss as r increases for all consumers who consume qualities above q_0, given r. Increases in r also cause more consumers to leave the market altogether, with additional welfare losses resulting. Changes in r have no effect on consumers who purchase quality q_0. In general consumers substitute to lower quality items as r rises.

Notice that $W_r(\Theta, 0) = 0$; this reflects the fact that there is no loss, to the first order, from imperfect information when we first move away from perfect information (r=0). Notice also that the per-capita welfare losses as r increases tend to be greater for those who value quality the highest (high Θ). Finally, the curvative of the cost function, $\frac{c'}{c''}$, enters into the welfare loss.[3] This is because it determines how severe is the substitution towards lower quality items as a consequence of the premiums for higher quality products.

Welfare Analysis of q_0

In a perfect information world there is no justification for a minimum quality standard. After all, its only effect would be to artificially restrict the range of products offered for sale.

When product quality cannot be observed prior to purchase, however, there may well be justification for such standards. The usual story is that the minimum standard or licensing protects consumers from quacks, frauds, and rip-offs generally. This refers to a disequilibrium situation where consumers may be unpleasantly surprised by the quality of the product they buy.

While such a story is perfectly plausible, it is <u>not</u> the one I am telling in this paper. Rather, I am concerned with the desirability of a minimum quality standard, where the standard influences the equilibrium price-quality schedule. So, even granting that consumers are never surprised (in equilibrium) <u>i.e.</u>, that their expectations of quality are fulfilled, it is desirable to impose a minimum standard.

There are, as far as I know, no other formal analyses of minimum quality standards where the supply of products of various qualities is endogenous. The case with exogenous supplies has been treated by Leland [1979].

Since I have already shown how the minimum standard q_0, influences the equilibrium $p(q)$ schedule, (5), and how consumers respond to this, it is relatively easy to do the welfare analysis to determine the optimal minimum quality standard.

Looking at type Θ consumers, and taking $r>0$ as fixed, we can write down welfare of type Θ's (using the same convention as above to identify consumer surplus and aggregate welfare) as

$$W(\Theta, q_o) = \int_{v(\Theta,q_o)}^{\bar{v}} f(\Theta, v)[\Theta q(\Theta, r) + v - c(q(\Theta, r))]dv.$$

Let me again consider the two classes of Θ's separately:

first $\Theta \leq (1+r)\ c'(q_o)$ and

then $\Theta > (1+r)\ c'(q_o)$.

For the first group, $q(\Theta, r) = q_o$ so $v(\Theta,q_o) = c(q_o) - \Theta q_o$ and

$$W(\Theta,q_o) = \int_{c(q_o)-\Theta q_o}^{\bar{v}} f(\Theta, v)\ [\Theta q_o + v - c(q_o)]dv$$

And so

$$W_{q_o}(\Theta,q_o) = \int_{c(q_o)-\Theta q_o}^{\bar{v}} f(\Theta, v)[\Theta - c'(q_o)]dv$$

$$(19)\quad W_{q_o}(\Theta, q_o) = [\Theta - c'(q_o)] \int_{c(q_o)-\Theta q_o}^{\bar{v}} f(\Theta, v)dv.$$

For $\Theta < c'(q_o)$, consumer Θ would like to purchase $q<q_o$ and so is hurt by a rise in the standard. For $c'(q_o) < \Theta < (1+r)c'(q_o)$, Θ is happy to see q_o raised: so long as he only has to pay the cost of the item and not the premium payment he prefers $q > q_o$ Since minimum quality items sell at cost, these Θ's prefer to see the standard raised.

For the second group, $(\Theta > (1+r)\, c'(q_o))$, we have

$$W(\Theta, q_o) = \int_{v(\Theta, q_o)}^{\bar{v}} f(\Theta, v)[\Theta q(\Theta, r) + v - c(q(\Theta, r))]dv$$

Since q_o does not influence the quality choice $q(\Theta, r)$, it has an impact only through the number of consumers who purchase the good.

$$W_{q_o}(\Theta, q_o) = - v_{q_o}(\Theta, q_o) f(\Theta, v(\Theta, q_o))[\Theta q(\Theta, r) + v(\Theta, q_o) - c(q(\Theta. r))]$$

From (14) we have $v_{q_o}(\Theta, q_o) = -rc'(q_o)$ so

$$W_{q_o}(\Theta, q_o) = rc'(q_o) f(\Theta, v(\Theta, q_o)) \quad [\Theta q(\Theta, r) + v(\Theta, q_o) - c(q(\Theta, r))]$$

Now use (13) to substitute for $v(\Theta, q_o)$ to get

$$W_{q_o}(\Theta, q_o) = rc'(q_o) f(\Theta, v(\Theta, q_o)) r(c(q(\Theta, r) - c(q_o))$$

Rewriting this we have

$$W_{q_o}(\Theta, q_o) = r^2 c'(q_o)[c(q(\Theta, r)) - c(q_o)] f(\Theta, v(\Theta, q_o))$$

As expected, this is positive. Since $p(q) > c(q)$ for $q > q_o$, some consumers who would purchase the product under perfect information drop out of the market rather than pay the premium. As q_o increases the premium falls, and some of these consumers, whose valuation of the product exceeds its cost, re-enter the market. This constitutes a welfare gain.

The calculations above can be summarized in

<u>Theorem 3</u>. Given some minimum quality standard q_o, all consumers of type Θ such that $\Theta > c'(q_o)$ enjoy a welfare gain from raising q_o, while those Θ's for which $\Theta < c'(q_o)$ suffer as a result.

The calculation of the optimal minimum quality standard is not hard, now that we have computed the welfare achieved in supplying type Θ for every Θ.

$$W(q_o) = \int_{\Theta} W(\Theta, q_o)d\Theta$$

$$W'(q_o) = \int_{\Theta} W_{q_o}(\Theta, q_o)d\Theta$$

$$= \int_{\underline{\Theta}}^{c'(q_o)} W_{q_o}(\Theta, q_o)d\Theta + \int_{c'(q_o)}^{\bar{\Theta}} W_{q_o}(\Theta, q_o)d\Theta$$

We know the integrand is always negative in the first integral, and always positive in the second. The optimal q_o, q_o^*, satisfies $W'(q_o^*) = 0$.

For q_o such that $c'(q_o) \leq \underline{\Theta}$ we know $W'(q_o) > 0$. Likewise, for q_o such that $c'(q_o) \geq \bar{\Theta}$, $W'(q_o) < 0$. Consequently

<u>Theorem 4</u>. The optimal minimum quality standard is such that (1) there are some consumers who cannot get as low a quality item as they would prefer under perfect information, and (2) some consumers would prefer a higher standard i.e., would prefer a better product under perfect (or imperfect) information.[4]

In particular, setting a minimum quality standard $\underline{q}$ such that $\underline{\theta} = c'(\underline{q})$, so that no one would want to buy a lower quality than $\underline{q}$, is not optimal.

Continuous Time Model with Quality and Quantity as Controls

In this section I present a model in which firms can choose a sales level, x, as well as a quality, q, at each point in time. When sales are a control variable, the reputation adjustment process must be changed to reflect this fact.[5] I adopt a specification in which the speed of adjustment of reputation depends positively on the sales level.

The resulting model is naturally more complex than the one in which quality is the only control variable. The main reasons for presenting it are two: Firstly, it indicates that the qualitative characterization of the price-quality schedule derived above is not peculiar to a model in which quantity variables are absent. Since many producers of consumer goods can vary their sales over time as well as quality, this is important. Secondly, it allows us to identify an additional welfare loss which is a consequence of imperfect information: there is a production inefficiency induced by the fact that prices for high quality items sell above their minimum average cost. Specifically, active firms operate at above efficient scale. In equilibrium there are too few firms, each producing too much.

In particular, each firm faces an optimal control problem of the following form:

$$\max_{x(t),\, q(t)} \int_0^\infty e^{-rt}[p(R)x - c(x, q)]dt$$

$$\text{s.t.} \quad \dot{R} = sx(q - R)$$

$$R(0) \text{ given.}$$

Here $c(x, q)$ is the cost function in quantity and quality; I assume $c_x>0$, $c_{xx}>0$, $c_{qx}>0$, and that we have U-shaped average cost curves for any q. The parameter s represents the speed of learning by consumers, and p(R) is the price a firm can charge if its reputation is R. Again we have perfect competition, so the firm faces a perfectly elastic demand curve at price p(R).

The current value Hamiltonian for this control problem is

$$H(x, q, \lambda, R) = p(R)x - c(x, q) + \lambda sx(q-R).$$

The necessary conditions for an optimal regime include

(20) $H_x = p(R) - c_x(x, q) + \lambda s(q-R) = 0$

(21) $H_q = c_q(x, q) + \lambda sx = 0$

and

(22) $H_R = p'(R)x - \lambda sx = r\lambda - \dot{\lambda}.$

We can solve for the steady-state conditions by putting $\dot{\lambda}=0$, $q=R$ to get

(23) $p(q) = c_x(x, q)$

(24) $c_q(x, q) = \lambda sx$

(25) $p'(q)x = r\lambda + \lambda sx$

Solve for λ using (24) to get, finally,

(26) $p'(q) = \dfrac{c_q(x, q)}{x} [1 + \dfrac{r}{sx}]$

Notice the similarity between (26) and (6).

The reasoning now parallels the formal derviation of p(q) in the case where x was not a control variable: For an arbitrary p(q) schedule (23) and (26) would imply a unique steady state (x,q) pair, at which all firms would choose to produce (if they settle down at all). This would not satisfy the equilibrium conditions for the same reasons as in the earlier case. But if (23) and (26) are used to <u>define</u> p(q), with the auxilliary variable x(q) as well, then any firm would find it optimal to maintain q=R rather than to deviate.[6]

Before looking more closely at the solution to (23) and (26), it is helpful to define the perfect-information price-quality schedule. It is given by

$$\phi(q) = \min_{x} \frac{c(x,\ q)}{x}$$

i.e. quality q is supplied at its minimum average cost. The associated scale at which firms operate, z(q), satisfies

$$(27) \quad c_x'(z(q),\ q) = \frac{c(z(q),\ q)}{z(q)} .$$

since MC=AC at minimum AC.

Returning to the imperfect information case, we must add the natural boundary condition to solve the differential equation for p(q) given by (26), namely

$$(28) \quad p(q_0) = \phi(q_0).$$

This is analogous to $p(q_0) = c(q_0)$ in the earlier case.

<u>Theorem 5</u> As $r \to 0$ or $s \to \infty$ the equilibrium p(q) schedule approaches the perfect information schedule $\phi(q)$. For $(\frac{r}{s}) > 0$ $p(q) > \phi(q)$ for all $q > q_0$.

Proof: Let me first show that when $\frac{r}{s} = 0$, $(\phi(q), z(q))$ solves the system given by (23), (26) and (28). Note that for any set of parameters the system has a unique solution. Well, $\phi(q) = \frac{c(z(q), q)}{z(q)}$ by the definition of $\phi(q)$, and that equals $c_x(z(q), q)$ by the definition of $z(q)$ so (23) is satisfied. To verify (26) simply differentiate the equation defining $\phi(q)$, to get

$$\phi'(q) = \frac{z(q)[c_x z' + c_q] - cz'}{z^2}$$

$$= [\frac{c_x}{z} - c]z' + \frac{c_x}{z}$$

But $\frac{c_x(z, q)}{z} = c(z, q)$ so

$$\phi'(q) = \frac{c_q(z(q), q)}{z(q)}$$

which is exactly (26) when $x=z$, $(\frac{r}{s}) = 0$. Now, since the solution to the differential equation is continuous in $\frac{r}{s}$, we have proven the first part of the Theorem. The second part of the Theorem can be shown by a more basic argument. If $p(q) < \phi(q)$ firms selling quality q would be losing money and it could not be optimal to continue doing so. If $p(q) = \phi(q)$ they are breaking even, but they could make positive profits (at least for a little while if $s<\infty$) by running down reputation. Therefore maintaining quality can only be optimal if $p(q) > \phi(q)$.

It is interesting to note how r and s enter only through their ratio. This is very intuitive: for low interest rates or high learning speeds the informational problems are less important.

There is an interesting effect which comes up in this model which could not arise in the earlier model: since $p(q) > \phi(q)$ firms providing quality q operate at above efficient scale.[7]

Theorem 6 For $(\frac{r}{s}) > 0$, all firms providing non-minimal quality operate at a point above efficient scale. So, in addition to the welfare losses due to imperfect consumer quality matching, and some consumers dropping out of the market, there is a production inefficiency.

Proof: Since $p(q) > \phi(q)$ for $q>q_0$ by Theorem 5, we know that $x(q) > z(q)$ since $c_{xx}>0$ and $x(q)$ is defined by (23). See Figure 8 below. So the number of products of quality q which are sold in equilibrium is not produced in the cost-minimizing manner. There are too few firms, each of which produces too much.

Since average cost is $\frac{c(x(q),q)}{x(q)} > \phi(q)$, some of the premium to high quality items, $p(q) - \phi(q)$ is dissipated by the production cost inefficiency.

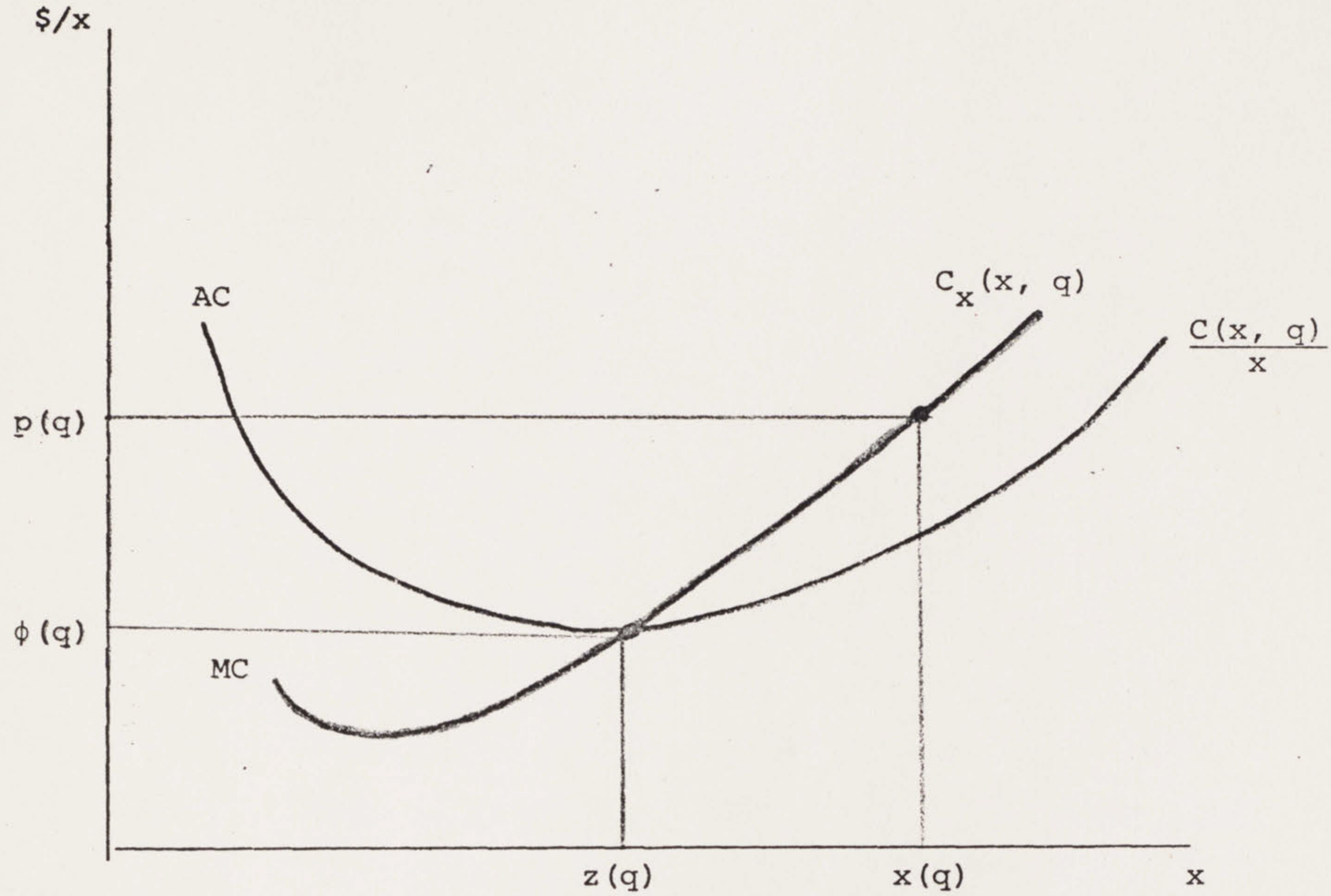

FIGURE 8

Production Inefficienty due to $p(q) > \phi(q)$

Conclusions

This paper has investigated the implications of reputation in a perfectly competitive environment. It has been shown that reputation can operate only imperfectly as a mechanism for assuring quality. High quality items sell for a premium above cost. This premium provides a flow of profits which compensate the seller for the resources expended in building up the reputation.

Several common but informal notions relating to reputations have been challenged by this analysis. First, a good reputation need not confer market power on its owner. Indeed, firms face perfectly elastic demand curves in the model presented above. Second, reputations need not imply a barrier to entry either. It is true that a firm must expand resources initially to build up a reputation, but it is not possible, at least in this model, to earn super-normal profits by virtue of having built up a reputation. In other models, which I hope to explore, it may be the case that there are first-mover advantages in reputation formation, and thus reputation could serve as a barrier. In this first simple model, however, it does not. Finally, care must be taken in evaluating profit data for consumer goods industries. If reputation is not included in the set of assets a firm owns, the calculations of its rate-of-return will exceed the market rate of return. This is misleading, as would be the conclusion based upon it that the firm enjoyed some degree of market power.

Finally, a welfare analysis of information remedies and minimum quality standards is made. There are welfare gains from improving information transmission; these must be balanced against the costs of such a program, of course. Optimal minimum quality standards are also studied. In general it is optimal to exclude from the market items which some consumers would like to purchase, i.e., the standard should be binding. This is because there are welfare gains to consumers who like high quality items which arise from raising the standard. These gains arise because a higher minimum quality standard reduces the premiums for high quality goods.

Notes

1. In fact, a firm would be indifferent to maintaining or deviating, but stability would be provided by any positive adjustment costs to changing quality. Such indifference is inevitable in a model in which identical firms choose a variety of actions in equilibrium.
2. The welfare theorems do not depend on this assumption. They only require that a consistent description of what happens during the transition period be maintained throughout the analysis.
3. Recall that is is really the curvative of c(q) relative to utility in q, but q has been scaled such that utility is linear in quality.
4. This Theorem holds no matter what weights are placed on the utilities of different consumers in the welfare measure so long as the weights are positive and finite.
5. It is not plausible that reputation adjustment is independent of sales. Furthermore, if it were, there would be no equilibrium. This is because a firm could build up reputation by selling, say, one good item and then sell a great many bad items when reputation is high. Since this strategy gives more profits from running down reputation than the costs of building it up, firms would never maintain quality. See Shapiro (1979).
6. I have been unable to verify the sufficiency conditions for the optimal control problem when sales levels are a control variable. The maximized Hamiltonian is not concave, but that does not mean the solution is not optimal.
7. This is in contrast to the traditional Chamberlinian result that firms operate below efficient scale in monopolistic competition.

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