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1. INTRODUCTION AND THE MODEL

The connection between imperfect information and imperfect competition has received much attention in recent literature. A variety of equilibrium outcomes have been obtained, for example perfect competition in Fisher (1972), pure monopoly in Diamond (1971), and price dispersion in Salop and Stiglitz (1977). This reflects corresponding differences in assumptions regarding consumers' information costs and demand functions, and firms' production costs and oligopolistic interactions. The purpose of this paper is to build a model sufficiently general to encompass these earlier results as special cases, and so bring out their mutual relationships. In doing so we compare the methodology involved in generating monopolistic competition due to consumers' imperfect information, with the methodology involved in generating monopolistic competition due to product differentiation originated by Chamberlin (1948).

The present model considers a particular problem of limited price information concerning a *homogeneous* product. It is supposed that *identical* consumers know the distribution of prices charged in the market, but do not know which store charges which price. This information may be obtained at a cost which *differs* among consumers. The probability distribution of information costs over consumers is known to the stores. Given the stores' price distribution, each consumer decides whether to become informed. He enters the market only once.[1] Informed consumers go to the lowest-price store, and uninformed ones choose a store at random. A consumer becomes informed if the utility to be had from paying the information cost and buying at the lowest price is higher than the expected utility from remaining uninformed and purchasing randomly. If purchasing information generates the same utility as random selection, the consumer chooses the latter.[2] (Diamond's (1971) equilibrium can occur as a special case of this information structure.[3]) Each store sets its price to maximize its profit and assumes in the Bertrand–Nash manner that other stores will not change their prices in response.[4] However, it calculates the effect of its actions on the consumers' information-gathering and, hence, on its sales (i.e. the equilibrium is a Stackelberg equilibrium between producers and consumers). All stores have identical U-shaped cost curves (i.e. increasing marginal cost is assumed, but the implications of assuming constant marginal cost are discussed as well), and there is free entry.

These assumptions raise three natural questions: (i) How can consumers know the price distribution without knowing the specific price each firm charges? (ii) If one firm makes a small price change, how can the consumers know how this will affect the distribution without knowing which firm makes the change? (iii) How can firms, who are sophisticated enough to take into account the effects on consumer search behaviour of changes in the prices they charge, be simultaneously so naïve in their assumptions concerning other firms' pricing decisions? In Braverman (1976, pp. 29–30) different justifications for these assumptions are provided, but, clearly, they represent a weakness in this approach. The strength of the approach lies in its careful attention to the complete

general equilibrium of the industry, avoiding the serious shortcomings of some previous work involving perfect information, e.g. Stigler (1961), which postulated price dispersion without asking whether such a state would survive in a full equilibrium. (Rothschild (1973) demonstrated that Stigler's (1961) model results in a single price equilibrium when the producers' side of the market is analysed.)

Earlier work on these types of models made one or more of the following special assumptions: (i) The distribution of consumers' information costs is concentrated at one or two points; (ii) Each consumer's demand is perfectly inelastic up to a reservation price and zero beyond this price; (iii) Firms' marginal costs are constant; (iv) The number of firms is fixed. Our model generalizes all these features. The three possible equilibria are characterized:

(i) A perfectly competitive equilibrium (price equals minimum average cost) which is the *unique equilibrium*, if it exists.

(ii) A monopolistically competitive equilibrium with a single price which is *lower* than the monopoly price. (Only if (a) consumer demand is perfectly inelastic up to a reservation price, or (b) marginal cost is constant, is the monopolistically competitive price equal to the monopoly price.)

(iii) A price dispersion equilibrium which must entail only two prices, a low price equal to the perfectly competitive price, and a high price *lower* than the monopoly price.

Furthermore, we are able to show that monopolistically competitive equilibria (either with a single price or with price dispersion) entail *more* firms than the perfectly competitive equilibrium.

For expositional simplicity, some special assumptions are made. The first assumption is that there are no income effects on the industry in question; thus, consumer demand is unaffected by the information-acquisition decision. Formally, consider a partial equilibrium model of an industry with a homogeneous product. All other prices in the economy are assumed fixed; money income I is also exogenous. A consumer who pays price p for this product and spends a total of I' on goods gains utility $u = I' - f(p)$, where f is a strictly increasing, strictly concave function. By Roy's Identity, the downward sloping demand of a buyer at price p is $f'(p)$. I' equals the fixed amount I if the consumer does not buy price information, and $I - c$ if he buys such information at cost c. (Consumers differ in their information costs, c.) Thus we have the partial equilibrium assumption of zero income effects on the product in question. The qualitative results do not depend on this assumption, and the general case, modelled with a general indirect utility function, is discussed in Braverman (1976).

The second assumption is that the market is large relative to any one firm. This makes the Bertrand–Nash assumption about the firm's behaviour more plausible, and avoids the problem of instability of the kind discussed by Edgeworth. Furthermore, it permits the use of the approximation of treating the number of firms as a real number, and the resulting equilibrium condition of zero profit. Again, many results are valid as they stand or with minor modifications when we restrict the number of firms to be an integer; details are in Braverman (1976).

The third assumption is that for a given number of firms, a firm's maximization yields a unique optimal price. This assumption is discussed later.

The approach is as follows. Beginning with a postulated zero profit equilibrium, we determine a firm's perceived demand curve as it contemplates changing its price. We then check whether each firm's profit-maximization is consistent with the postulated equilibrium, both in general, and for particular distributions of search costs. This yields conditions for prevalence of particular kinds of equilibria. Issues of existence or uniqueness are considered only tangentially. They are discussed in detail in Braverman Guasch

(1977), and examples of non-existence are provided in Braverman (1976, pp. 98–100), and in Salop–Stiglitz (1977).

2. SINGLE PRICE EQUILIBRIA

Consider first a postulated equilibrium in which every firm charges an identical price, p^*. In such an equilibrium no consumers acquire information, and each identical firm has its equal share of customers.   Any one firm contemplating changing its price calculates its perceived demand curve, assuming that the other firms' prices remain at the equilibrium level, but recognizing that consumers acquire information as price dispersion appears. Since each firm maximizes profit, and entry reduces (excess) profit to zero, equilibrium requires that each firm's perceived demand curve must lie below its average cost curve, having only the equilibrium price and output point in common.   This more rigorous statement of the familiar tangency condition is required here because of the possible diversity in shapes of the perceived demand curves.   (Recall the assumption that for a given number of firms there corresponds *at most* one equilibrium price; i.e. equilibria, where a firm's perceived demand function touches the average cost curve more than once, are excluded.   However, this assumption does not exclude the possibility of having multiple single price equilibria corresponding to *different* industry sizes.)

Suppose one firm contemplates a small deviation from the assumed equilibrium price. The response of consumers to this slight price dispersion depends on the distribution of information costs in the neighbourhood of zero.   Three cases are possible which lead to three possible outcomes: A perfectly competitive equilibrium, non-existence of a single price equilibrium, and a monopolistically competitive equilibrium.   Before describing them, we shall derive technically the properties of a firm's perceived demand function.   A more intuitive discussion will follow later.

2.1. *Properties of firms' perceived demand functions*
When all n firms charge a common price p^*, each sells an amount $f'(p^*)/n$.   Now suppose one firm contemplates changing its price to $p^* \pm \varepsilon$, where ε is a small positive number. The two cases of a price rise and a price cut must be examined separately.

   (i) *Small price rise*.   A consumer with information cost c will obtain utility $I - c - f(p^*)$ if he becomes informed.   An uninformed consumer chooses a firm at random, and has expected utility $I - f(p^*)(n-1)/n - f(p^*+\varepsilon)/n$.   For small ε, this equals $I - f(p^*) - \varepsilon f'(p^*)/n$ to the first order.   Therefore the consumer will become informed if $c < \varepsilon f'(p^*)/n$.   If μ is the cumulative distribution function of search costs across consumers, then $\mu(\varepsilon f'(p^*)/n)$ consumers will be informed.   Normalize the total number of consumers L, to one.   Now our price-raising firm only gets its share of the uninformed consumers, and sells

$$[1 - \mu(\varepsilon f'(p^*)/n)] f'(p^*+\varepsilon)/n.$$

Comparing this with the amount when all firms charge p^*, $f'(p^*)/n$, we see that if the distribution of search costs has an *atom at* 0, even an infinitesimal price rise causes a *discontinuous* drop of amount $\mu(0+)f'(p^*)/n$ in sales.   If the search cost distribution has a density function around zero, then our firm's perceived demand has a one-sided derivative for a price rise, and differentiation with respect to ε easily shows this derivative to be

$$f''(p^*)/n - \mu'(0)[f'(p^*)/n]^2. \qquad \qquad ...(1)$$

   (ii) *Small price cut*.   An informed consumer has utility

$$I - c - f(p^* - \varepsilon), \quad \text{or} \quad I - c - f(p^*) + \varepsilon f'(p^*)$$

to the first order.   An uninformed consumer has expected utility

$$I - f(p^*)(n-1)/n - f(p^*-\varepsilon)/n, \quad \text{or} \quad I - f(p^*) + \varepsilon f'(p^*)/n$$

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to the first order. The criterion for acquiring information is $c < \varepsilon f'(p^*)(n-1)/n$, and there are $\mu(\varepsilon f'(p^*)(n-1)/n)$ informed consumers. The price-cutting firm gets all informed consumers and its share of uninformed ones, so its sales are

$$\mu(\varepsilon f'(p^*)(n-1)/n)f'(p^*-\varepsilon) + [1 - \mu(\varepsilon f'(p^*)(n-1)/n)]f'(p^*-\varepsilon)/n.$$

Again we compare this with $f'(p^*)/n$. If $\mu(0+) > 0$, there is a discontinuous gain of sales of $\mu(0+)f'(p^*)(n-1)/n$ following an infinitesimal price cut. If the distribution function is differentiable, the one-sided demand derivative for a price cut is

$$f''(p^*)/n - \mu'(0)[f'(p^*)(n-1)/n]^2. \qquad \dots(2)$$

We can compare (2) with (1), the one-sided derivative for a price rise found above. The effect of a price cut is numerically larger, save in the exceptional case of $n = 2$, or the case of $\mu'(0) = 0$, i.e. *zero density of consumers with zero search costs*. In this last case, the two-sided demand derivative is $f''(p^*)/n$. This is simply the derivative of a firm's share of the market demand when all firms change prices together.

Having characterized technically the properties of the firms' perceived demand functions for small price changes (larger price changes will, of course, result in different responses), we can describe the three possible cases.

2.2. *Perfectly competitive equilibrium*

Suppose there is an atom of consumers with zero information cost (i.e. $\mu(0+) > 0$). They find it worthwhile to acquire information in the presence of the slightest price dispersion. A firm which raises its price slightly loses its share of these customers; a firm which lowers its price slightly gains all such customers in the market rather than only its equal equilibrium share. Therefore, the perceived demand function exhibits a discontinuity at the
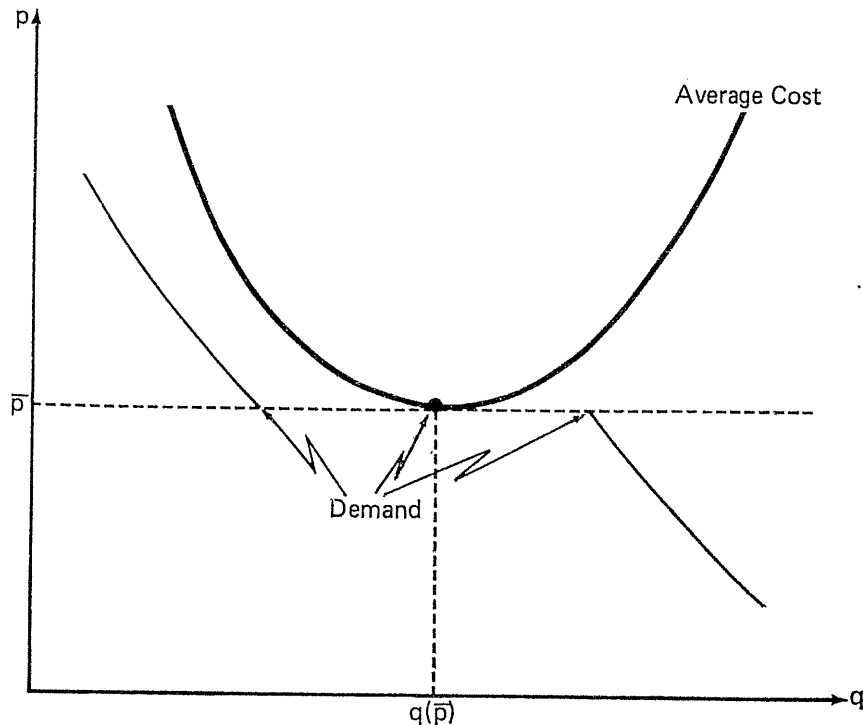

FIGURE 1

Perfectly competitive equilibrium.

postulated equilibrium price. If this discontinuity and the average cost curve have the kind of relative size and shape shown in Figure 1 (i.e. the demand facing a deviating firm lies below the average cost curve also for large price changes), we can have a single-price equilibrium at the perfectly competitive (minimum average cost) price, $\bar{p}$.[5]

With such a discontinuous perceived demand, no other price may be an equilibrium: at a lower price the zero-profit condition would clearly be incompatible; at a higher price a firm would cut price slightly to attract a lump of customers (rationing them if necessary), and earn a positive profit, thus contradicting profit maximization at the postulated equilibrium.

2.3. Non-existence of single price equilibria

Suppose the distribution of information costs has no atom, but has a positive density at zero (i.e. $\mu'(0) > 0$). Now price dispersion of the first order of smallness elicits information-gathering on the part of consumers, also forming a group of the same order of smallness. But there is an asymmetry between a price rise and a price cut((1) vs. (2)). First of all, in the case of one firm raising its price, the presence of $(n-1)$ now low-priced firms offers the consumer a high probability of meeting one low price firm at random, while in the case of price cut the one price-cutting firm becomes the sole low-priced one. Thus the incentive to become informed is $(n-1)$ times as large in response to a small price cut as it is in response to an equally small price rise. Second, observe the difference between a price rise and a price cut even for a given fraction μ of informed customers in the population. A price-raising firm gets its share of uninformed customers, i.e. $(1-\mu)/n$, representing a loss of μ/n as compared to its equilibrium share of $1/n$. A price-cutting firm gets all informed customers and its random share of uninformed ones, i.e. $\mu + (1-\mu)/n$, which is a gain of $\mu(n-1)/n$ over its equilibrium share, and this is $(n-1)$ times as large as the case of a rise. Combining these two effects, i.e. different relative numbers of informed customers and different responses out of a given number of these, we see that a small price cut gains $(n-1)^2$ times as many customers as an equally small price rise loses. Except for the case
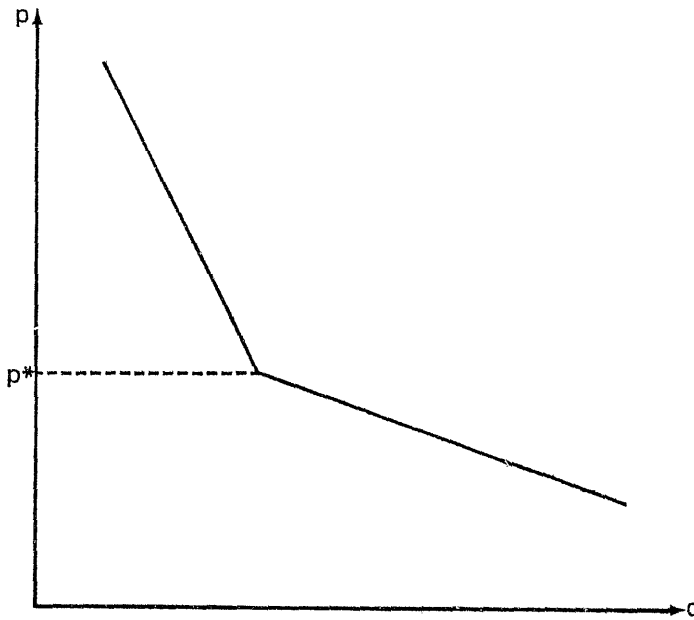

FIGURE 2
Kinked demand curve due to consumer imperfect information.

of duopoly (neglected by the assumption of a large market), the perceived demand curve has a kink, as pictured in Figure 2. This kink is in the *opposite* direction to the familiar Sweezy's (1939) kink, which arises from asymmetric conjectured responses of other firms.[6] Hence, this perceived dem ⸱⸱⸱ curve can never be tangential even in the generalized sense to a smooth average cost curve at any possible equilibrium price, P^*. This completely rules out the possibility of a single-price equilibrium in the present case, even at the perfectly competitive price.[7]

2.4. *Monopolistically competitive equilibrium*
Finally we have the case where the distribution of information costs has zero density at zero (i.e. $\mu'(0) = 0$). Now the fraction of consumers who become informed in response to a small price dispersion is of the *second order of smallness*. When one firm changes its price, therefore, the predominant influence on its sales is initially that due to changing industry demand. Hence, the firm's perceived demand function is differentiable at p^*,
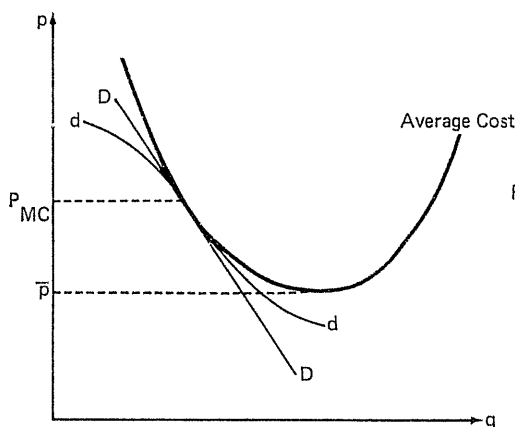

FIGURE 3a
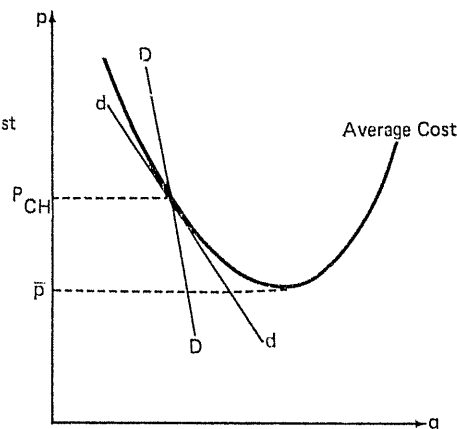Single price monopolistically competitive equilibrium due to imperfect information.

FIGURE 3b
Chamberlinian monopolistically competitive equilibrium due to product differentiation

i.e. both the R.H.S. derivative (1), and the L.H.S. derivative (2), are equal to $f''(p^*)/n$. The further effect of the change in a firm's share of the customers becomes important only for larger price changes. In other words, if we define the Chamberlinian DD curve as one firm's equal share of the industry demand curve, and the dd curve as the perceived demand curve for one firm when other firms maintain the equilibrium price but consumers respond by gathering information, then the two curves are tangential at the postulated equilibrium point. If the curvature of the demand and average cost curves are suitable, we can have a monopolistically competitive equilibrium at p_{MC} as shown in Figure 3a. It should be stressed that its nature is different from the conventional Chamberlinian notion, described in Figure 3b. In a Chamberlinian equilibrium the dd curve is constructed under the assumption that when an individual firm cuts (raises) its price (taking the prices of other firms as given), it gains (loses) customers. Therefore, at a Chamberlinian equilibrium the DD curve cuts the dd curve from above, while in monopolistic competition due to imperfect information, the *lack* of difference between the slopes of the DD and the dd curves at the equilibrium price is the necessary condition for the equilibrium.

An immediate consequence is that if an industry in such an equilibrium is monopolized, but is unable to change the number of firms and is required to maintain equal outputs from them all, then the monopoly will simply reproduce the monopolistically competitive equilibrium. This is unlike the standard Chamberlinian result. However, a monopoly

which is not so shackled will wish to alter the number of firms. This issue of more general comparisons between the different single price equilibria will be studied in a moment.

Corollaries. Note that the three cases: B, C and D, are mutually exclusive and exhaustive as regards their assumptions about the behaviour of the information cost distribution at zero, and that they provide necessary conditions for prevalence of the appropriate kind of single-price equilibrium. This gives the following clear conclusions: (a) *in the case of a positive density at zero, any equilibrium must show price dispersion*, (b) *a competitive equilibrium and a monopolistically competitive equilibrium with a single price can never occur together*. However, we can state even a stronger result:

Proposition 1. *If a perfectly competitive equilibrium exists, it is the unique (Bertrand-Nash) equilibrium. That is, following (b), if a structure supports a competitive equilibrium it cannot support a price dispersion equilibrium.*

The proof was first given in Braverman–Guasch (1977) and it is sketched in the appendix. The reader is advised to read the section on price dispersion before reading the proof. The proof is by contradiction. It demonstrates that a firm which deviates from the perfectly competitive equilibrium by increasing its price generates, at the high price of a postulated price dispersion equilibrium, more uninformed customers (its only customers) than does a high price firm of the postulated price dispersion equilibrium, i.e. the deviating perfectly competitive firm generates *higher demand* than the high price firm. However, if a perfectly competitive equilibrium exists, the Nash profit maximizing condition implies that any price increase generates losses, implying that the demand generated by the deviating firm lies below the average cost curve. Thus, the demand of a high price firm in price dispersion must be below the average cost curve, excluding the possibility of non-negative profits, which is a necessary condition for the existence of any equilibrium.

2.5. Comparison of single price equilibria

In this section a monopoly equilibrium, a monopolistically competitive equilibrium and a perfectly competitive equilibrium are compared on three grounds: (i) prices, (ii) number of stores, (iii) total surplus. Since we are assuming the size of the market to be " very large " compared to the efficient scale of a plant (store), $q(\bar{p})$, and treating the number of stores, n, as a continuous variable, we can equally well assume that the production function of a monopoly, which operates many stores, exhibits constant returns to scale at $\bar{p}$.

(i) *Prices.* The number of consumers in the market is L, normalized to 1. Thus consumers' inverse demand function is $p(X) \equiv f'^{-1}(p)$. We shall solve the optimal price problems in the quantity space and map the solution to the price space through $p(X)$. Thus, the monopoly problem is:[8]

$$\max_X Xp(X) - \bar{p}X \qquad \qquad ...(3)$$

implying the first order condition:

$$p'X + p - \bar{p} = MR(X) - \bar{p} = 0. \qquad \qquad ...(4)$$

The monopolistically competitive firm's problem is equivalent to (5):

$$\max_X Xp(X)/n - C(X/n) \qquad \qquad ...(5)$$

(since there is zero density of consumers with zero information cost) implying:

$$p'X + p - C'(X/n) = MR(X) - C'(X/n) = 0. \qquad \qquad ...(6)$$

Hence, the following proposition holds:

Proposition 2. *If revenue $R(X) \equiv Xp(X)$ is a concave function of X, then the monopoly price, p_M, is strictly higher than the monopolistically competitive price, p_{MC}.*

Proof. Let X_M and X_{MC} be the solutions to (4) and (6), respectively. The tangency of the DD curve to the downward sloping part of the AC curve in a monopolistically

competitive equilibrium, together with the assumption of increasing marginal cost, imply that $C'(X_{MC}/n) < \bar{p} = \min AC$. Hence, utilizing (4) and (6), $MR(X_M) > MR(X_{MC})$. Since marginal revenue is decreasing

$$X_M < X_{MC} \Rightarrow p_M > p_{MC}. \quad \|$$

Basically, the argument is that a monopoly's marginal cost must be greater than the marginal cost at the point where the monopolistically competitive firm is working, while a monopoly's store and a monopolistically competitive store consider the *same* marginal revenue schedule. In two special cases the monopoly price is equal to that in monopolistic competition: (i) if demand is inelastic up to a reservation price (Salop–Stiglitz (1977)), and (ii) if marginal cost is constant (Diamond (1971)). (If marginal cost is constant, (4) and (6) are exactly the same. Clearly in such a case, if there are fixed costs to set up a firm, the monopoly concentrates production in just one firm.)

(ii) *Number of firms.* We know that both a monopoly's store and a perfectly competitive firm produce the same output at minimum average cost, and monopoly produces smaller output than perfect competition. Hence, the number of stores operated by the monopoly, n_M, is lower than the number of stores in perfect competition, $\bar{n}$. Furthermore, the Chamberlinian "excess-capacity" theorem (that in monopolistic competition due to product differentiation a firm's output is smaller than the output corresponding to its minimum average cost) holds also for monopolistic competition due to imperfect information. Therefore, a monopolistically competitive firm produces less output than a perfectly competitive firm or a monopoly's store. However, since total output produced under monopolistic competition, $X(p_{MC})$, is smaller than the perfectly competitive output, $X(\bar{p})$, can we compare the industry size in both cases? The answer is yes.

Proposition 3. *There are more firms in monopolistic competition, n_{MC}, than in perfect competition, $\bar{n}$.*

Proof. Return to Figure 3a. By the Nash profit maximization condition, the *dd* curve must lie below the average cost curve for prices other than p_{MC}. For price cuts, the *DD* curve must lie below the *dd* curve.[9] Therefore at $\bar{p}$ the following relation holds:

$$DD(\bar{p}) = X(\bar{p})/n_{MC} < q(\bar{p}), \qquad\qquad ...(7)$$

where $q(\bar{p})$ denotes the quantity corresponding to minimum average cost. But an equilibrium condition for perfect competition is:

$$q(\bar{p}) \cdot \bar{n} = X(\bar{p}). \qquad\qquad ...(8)$$

Hence,

$$n_{MC} > \bar{n}. \quad \|$$

This proof is also valid for a Chamberlinian equilibrium as described by Chamberlin (1948), where he assumes all firms to be symmetric with one another.[10] To summarize, we have established the relation $n_{MC} > \bar{n} > n_M$.

(iii) *Total surplus.* We know from the standard theory that perfect competition generates more surplus than a monopoly or monopolistic competition due to imperfect information. In general, it is impossible to determine whether monopoly or monopolistic competition due to imperfect information generate more surplus. However, if the monopoly price equals the monopolistically competitive price, as occurs under the two assumptions mentioned above, a monopoly is welfare superior to monopolistic competition. It provides the same consumer surplus more efficiently than monopolistic competition which induces excessive fixed costs due to excessive entry.

3. EQUILIBRIUM WITH PRICE DISPERSION

3.1. *Characterization*

The information structure allows only two informational alternatives: complete information or random selection. Correspondingly, an equilibrium with price dispersion cannot involve more than two prices.

Proposition 4. *There is no price dispersion equilibrium with more than two different prices.*

Proof. By contradiction, assume there is an equilibrium with three different prices where all low, medium and high price firms generate zero profits. Medium and high price firms attract identical numbers of customers; all of whom are uninformed. A deviating high price firm which instead charges the medium price attracts at least as many customers as the medium price firm had before the deviation, since by cutting its price it decreases the price dispersion and motivates more consumers to remain uninformed. By the Nash profit maximizing condition, a deviating high price firm must incur losses from charging the medium price. Therefore, the medium price firms must earn negative profits since they sell no more than the high price deviant. Thus, we conclude that the only possible price dispersion equilibrium is a two-price equilibrium, (TPE). ‖

Now suppose an equilibrium with some firms, n_l, charging one price, p_l, and the remaining ones, n_h, the higher price, p_h. Write $n = n_l + n_h$, $\beta = n_h/n$, and $1 - \beta = n_l/n$. In this setting, an informed consumer has utility $I - c - f(p_l)$, and an uninformed consumer has expected utility $I - (1-\beta)f(p_l) - \beta f(p_h)$. Then the criterion for buying information is that $c < c^* \equiv \beta[f(p_h) - f(p_l)]$ the critical search cost. There are $\mu(c^*)$ informed consumers. The sales of a *high-price* firm and of a *low-price firm* are:

$$D_h(p_h) = [1 - \mu(c^*)]f'(p_h)/n \qquad\qquad ...(9)$$

and

$$D_l(p_l) = [1 - \mu(c^*)]f'(p_l)/n + \mu(c^*)f'(p_l)/n_l, \qquad ...(10)$$

respectively. (9) and (10) imply the following lemma.

Lemma 1. *There must be more than one low-price firm in a TPE; i.e. $n_l > 1$.*

Proof. By contradiction, assume $n_l = 1$. Then if this sole low-price firm changes its price to p_h, it collapses the postulated price dispersion equilibrium into a single price at p_h, and generates demand of $f'(p_h)/n > D_h(p_h)$. Since the postulated TPE entails zero profit, the move by the low price firm to p_h generates *strictly* positive profit. This is in contradiction to the assumption of Nash profit maximization. ‖

The next step in the characterization of a TPE is to derive technically the properties of the firms' perceived demand curves for small price deviations. A small change in one firm's price is going to change the number of informed customers. But the effect on demand is dependent on which firm changes its price.

(i) *Deviation by a high price firm.* Suppose one of the high price firms contemplates changing its price to $p_h \pm \varepsilon$. We suppose ε to be small enough that a price cut of ε will not make it the lowest-price firm. Using the method that should now be familiar, we see that to the first order, consumers with information costs below $c^* \pm \varepsilon f'(p_h)/n$ will become informed, and the sales of our firm will be $[1 - \mu(c^* \pm \varepsilon f'(p_h)/n)]f'(p_h \pm \varepsilon)/n$. The perceived demand will be continuous so long as μ does not have an atom at c^*. (However, having an atom at c^* is consistent with a TPE as well.[11]) Demand will be differentiable if μ has a density at c^*, and the demand derivative will be

$$[1 - \mu(c^*)]f''(p_h)/n - \mu'(c^*)[f'(p_h)/n]^2. \qquad ...(11)$$

i.e. if a high price firm makes a change, its sales change due to the negative slope of the consumer's demand and the change in the number of uninformed customers.

(ii) *Deviation by a low price firm.* Consider the case of a price rise and a price cut separately: (a) *Price rise.* When it raises its price to $p_l + \varepsilon$, consumers with information costs less than $c^* + \varepsilon f'(p_l)/n$ will become informed. Then the sales of this firm will be

$$[1 - \mu(c^* + \varepsilon f'(p_l)/n)]f'(p_l + \varepsilon)/n.$$

Letting ε go to zero, we have the limit $[1 - \mu(c^*)]f'(p_l)/n$. Compare this limit with (10). So long as $\mu(c^*)$ is positive (which simply states that a TPE exists, since a price dispersion equilibrium must entail informed customers), an infinitesimal price rise will lead to a discontinuous loss in sales of amount $\mu(c^*)f'(p_l)/n_l$, even when μ has no atom at c^*. Note that this argument needs $n_l > 1$, which is guaranteed by Lemma 1. (b) *Price cut.* Following the same method, it can be easily shown that an infinitesimal price cut, which makes the deviating firm the cheapest in the market, generates (in the limit) a demand of

$$[1 - \mu(c^*)]f'(p_l)/n + \mu(c^*)f'(p_l).$$

Comparing this limit to (10) (the demand of a low price firm at p_l), we see that an infinitesimal price cut leads to a *discontinuous* gain in sales of $\mu(c^*)f'(p_l)(n_l - 1)/n_l$.

Hence, if a low price firm raises its price, it loses all informed customers, while if it lowers its price it gains them all, no matter how small the change. Thus the lower-price firm's perceived demand curve has both R.H.S. and L.H.S. discontinuities at p_l, similar to the ones at a perfectly competitive equilibrium (recall Figure 1). With such discontinuities, the lower price, p_l, must equal minimum average cost, $\bar{p}$. The argument is the same one given for the perfectly competitive equilibrium. However, there it was necessary to assume $\mu(0+) > 0$ (i.e. atom of consumers with zero information cost) in order to generate the discontinuities. In a TPE there is no need to assume the existence of atom of consumers (at any c) at all.

(iii) *Deviation by a low price firm and a high price firm to the same price.* Finally, consider an intermediate price level, and compare the effect of a high price firm cutting its price to this level with that of a low price firm raising its price to the same level. Formally, n_l firms charge p_l, n_h firms charge p_h, and one firm charges p, where $p_l < p < p_h$, (i.e. $n = n_l + n_h + 1$). This enables us to compare the sales of a firm cutting price from p_h to p with those of a firm raising price from p_l to p. Now an informed consumer has utility

$$I - c - f(p_l),$$

and an uninformed consumer has expected utility

$$I - [n_l f(p_l) + n_h f(p_h) + f(p)]/(n_l + n_h + 1).$$

The critical information cost is

$$c^* = [n_h(f(p_h) - f(p_l)) + (f(p) - f(p_l))]/(n_l + n_h + 1).$$

With $\mu(c^*)$ informed customers, the mid-price firm has its random share of the rest, and makes sales of $[1 - \mu(c^*)]f'(p)/n$. Fixing values of p_l, p_h, p and $n = n_l + n_h + 1$, we see that c^* increases as n_h increases. Then $\mu(c^*)$ increases (or does not decrease, if there are no consumers with search costs between the initial c^* and the new c^*). Hence the sales of the mid-price firm fall (do not increase). Now with a given number of firms initially in a two-price situation, n, the case of a firm raising its price from p_l to p differs from that of a firm lowering its price from p_h to p merely in having a value of n_h higher by one, and correspondingly a value of n_l lower by one. Thus the price-raising firm has smaller (no larger) sales than the price-cutting firms. In other words, the price rise leaves slightly fewer low-priced firms, and so a slightly greater incentive to acquire information. Since in each case the firm is left only with its share of uninformed customers, the price-raising firms will have smaller (no larger) sales.

Given the above characterization of the firms' perceived demand functions, we describe graphically a two-price equilibrium (TPE) in Figure 4, which will arise if the shapes of the demand functions and cost curves are suitable.   Since we have an additional equilibrating variable, namely the proportions of firms charging the two prices, it is possible to construct such an equilibrium for a non-trivial range of distributions of information costs.[12]

Before concluding this section we derive an additional lemma which is essential to the analysis in the next section.   The lemma states that in a situation (not necessarily a TPE),
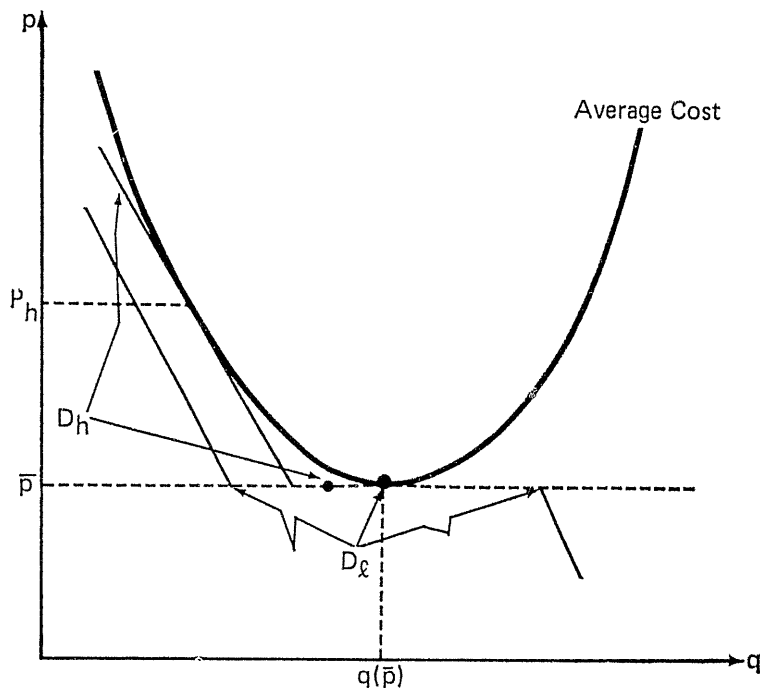
D_h: High-priced firm's perceived demand curve
D_ℓ: Low-priced firm's perceived demand curve

FIGURE 4

A two-price equilibrium.

where n_h firms earn zero profit at p_h and n_l firms earn zero profit at $\bar{p}$, a deviating high price firm which charges the low price, $\bar{p}$, generates *strictly* lower demand than the low price firm.   This is so since by cutting its price to $\bar{p}$, the deviating firm not only motivates less (at least no more) customers to become informed, but shares them with all the other low price firms, as well.

Lemma 2. $D_h(\bar{p}) < D_l(\bar{p})$.

Proof.

$$D_h(\bar{p}) = [1 - \mu(c_1^*)]f'(\bar{p})/n + \mu(c_1^*)f'(\bar{p})/(n_l + 1), \qquad \ldots(12)$$

and

$$D_l(\bar{p}) = [1 - \mu(c_0^*)]f'(\bar{p})/n + \mu(c_0^*)f'(\bar{p})/n_l, \qquad \ldots(13)$$

where c_0^* is the critical search cost in a TPE, and c_1^* is the critical search cost corresponding to price dispersion generated by $n_h - 1$ firms charging p_h and $n_l + 1$ firms charging $\bar{p}$.   Clearly,

$c_0^* > c_1^*$ which implies $\mu(c_0^*) \geqq \mu(c_1^*)$. Utilizing this relation, and engaging in simple arithmetic, we derive

$$[1/f'(\bar{p})][D_h(\bar{p}) - D_l(\bar{p})] = [\mu(c_0^*) - \mu(c_1^*)]/n - [\mu(c_0^*)(n_l+1)/n_l$$
$$- \mu(c_1^*)]/(n_l+1) < 0. \quad \| \quad ...(14)$$

The importance of this lemma is the following: Our definition of a Nash equilibrium does not allow a firm to maximize profit at more than one price, given industry size. For prices $p > \bar{p}$, this assumption seems reasonable, excludes unlikely cases and does not affect the industry size comparison of price dispersion to perfect competition (discussed in the next section). However, the case of a deviating high price firm charging $\bar{p}$, thus becoming a low price firm and gaining *informed* consumers, is significantly different. Also, whether $D_h(\bar{p}) < D_l(\bar{p})$ or $D_h(\bar{p}) \geqq D_l(\bar{p})$ is crucial to the analysis of industry size. Hence, rather than assuming it, Lemma 2 assures us that the zero profit condition by itself guarantees that a high price firm of a TPE is *never ending indifferent* to charging the low price, $\bar{p}$.

3.2. Comparison of price dispersion with single price equilibria

(i) *Prices.* The low price in a TPE equals the perfectly competitive price, $\bar{p}$. To discuss the high price, $\bar{p}_h$, consider the following maximization problem of a high price firm in a TPE.[13]

$$\max_X [1 - \mu\{c^*(p(X))\}]Xp(X)/n - C([1 - \mu\{c^*(p(X))\}]X/n) \qquad ...(15)$$

where $p(X) \equiv f'^{-1}(X)$. Assume for simplicity that the distribution of search costs is represented by a continuous density, μ'. Then (15) generates the first-order condition

$$\{Xp' + p - C'(\{1-\mu\}X/n)\}(1-\mu)/n - \mu'(c^*)\frac{\partial c^*}{\partial p}p'[p - C'(\{1-\mu\}X/n)]X/n = 0 \quad ...(16)$$

or

$$\{MR(X) - C'(\{1-\mu\}X/n)\}(1-\mu)/n - A\mu'(c^*) = 0 \qquad ...(17)$$

and $A < 0$ since $\partial c^*/\partial p > 0$, $p' < 0$ and $p > C'$.

Therefore the following proposition holds:

Proposition 5. *If $R(X) = Xp(X)$ is concave in X, then $p_h < p_M$.*

Proof. Define X_h as the optimal solution to (15), i.e. $p_h = p(X_h)$. From (17), $MR(X_h) \leqq C'(\{1-\mu\}X_h/n) < \bar{p} = \min AC$. (High price firm produces on the downward sloping part of its AC curve.) But from (4) $MR(X_M) = \bar{p}$. Hence,

$$MR(X_M) < MR(X_M) \Rightarrow X_h > X_M \Rightarrow p_M > p_h. \quad \|$$

Again, by transforming the problem to the quantity space in the above manner, we can compare all problems having the same marginal revenue schedule. By comparing (17) with (4), one can easily show that if $\mu'(c^*) > 0$, then $p_{MC} \neq p_h$. However, we cannot say more than that without too restrictive assumptions.

(ii) *Number of Stores.*

Proposition 6. *There are more firms in price dispersion equilibrium, n_d, than in perfect competition, $\bar{n}$.*

Proof. If a high price firm deviates to $\bar{p}$, then Lemma 2 and the zero profit condition imply:

$$D_h(\bar{p}) = X(\bar{p})(1-\mu)/n_d + \mu X(\bar{p})/(n_l+1) < D_l(\bar{p}) = q(\bar{p}). \qquad ...(18)$$

Furthermore, in a perfectly competitive equilibrium $q(\bar{p})\bar{n} = X(\bar{p})$. Hence, collecting terms, (18) is equivalent to:

$$X(\bar{p})[1/n_d + \mu(1/(n_l+1) - 1/n_d)] < X(\bar{p})/\bar{n}. \qquad ...(19)$$

a single price and a perfectly competitive equilibrium are mutually exclusive (see main text).

Thus, denote the critical search cost in a TPE as c_d^*. Denote the critical search cost generated by price dispersion due to one perfectly competitive firm raising its price from $\bar{p}$ to p_h (the high price of a TPE), as $\bar{c}^*$.

Lemma. $\bar{c}^* < c_d^*$.

Proof.

$$\bar{c}^* = \{f(p_h) - f(\bar{p})\}/\bar{n} \qquad \ldots(A.1)$$

and

$$c_d^* = (1 - n_l/n_d)\{f(p_h) - f(\bar{p})\}, \qquad \ldots(A.2)$$

where n_l denotes the number of low price firms in a TPE.

In order to prove the lemma, it is sufficient to prove that:

$$1 - n_l/n_d > 1/\bar{n}. \qquad \ldots(A.3)$$

From Proposition 6, $n_d > \bar{n}$. Furthermore $\bar{n} > n_l$, since there is a lower aggregate demand at $\bar{p}$ in a TPE than in perfect competition. Using these two relations (and only here utilizing the integer number property) (A.4) holds:

$$1 - n_l/n_d \geqq 1 - (\bar{n} - 1)/(\bar{n} + 1) = 2/(\bar{n} + 1) > 1/\bar{n}. \qquad \ldots(A.4)$$

since $\bar{n} > 1$. ‖

Using this lemma the following proposition holds.

Proposition. *If a model supports a perfectly competitive equilibrium, it does not support a TPE.*

Proof. By contradiction. Assume a model supports both equilibria. Consider the demand of a perfectly competitive firm, which raises its price from $\bar{p}$ to p_h. It generates demand, $\bar{D}(p_h)$, only from uninformed customers and the demand lies below the AC curve due to the Nash profit maximization condition, i.e.

$$\bar{D}(p_h) = (1 - \mu(\bar{c}^*))X(p_h)/\bar{n} < q(p_h). \qquad \ldots(A.5)$$

$$(q(p_h) \equiv AC^{-1}(p_h)). \qquad \ldots(A.6)$$

Now consider the demand facing a high price firm in a TPE, $D_h(p_h)$.

$$D_h(p_h) = (1 - \mu(c_d^*))X(p_h)/n_d. \qquad \ldots(A.7)$$

Since μ is a monotonic non-decreasing function of c^* the lemma implies that $\mu(\bar{c}^*) \leqq \mu(c_d^*)$. Furthermore, proposition (6) states that $n_d > \bar{n}$. Hence, combining (6) and (7) we obtain:

$$D_h(p_h) < \bar{D}(p_H) < q(p_H). \qquad \ldots(A.8)$$

Equation (8) states that a high price firm in a TPE never generates enough demand to cover its cost, i.e. it always generates negative profit, which establishes the contradiction. ‖

Corollary. *If a perfectly competitive equilibrium exists, it is the only (Bertrand–Nash) equilibrium.*

Since $n_l < n_d$, it follows that either $n_d > \bar{n}$ or $n_d = \bar{n}$ for $n_d = n_l + 1$. However, the second case is impossible since if $n_d = n_l + 1 = \bar{n}$, then $D_h(\bar{p}) = q(\bar{p}) = D_i(\bar{p})$, which contradicts Lemma 2. ‖

Corollary. Combining propositions (3) and (6), we can conclude that *the number of firms in monopolistic competition, either with a single price or with price dispersion, is larger than the number of perfectly competitive firms.*

(iii) *Welfare.* Clearly, perfect competition dominates any other equilibria on surplus grounds. In general we cannot rank a monopoly, a monopolistically competitive and a TPE equilibria, when we use the welfare measure of total surplus minus cost of resources spent on information acquisition (search). However, if $p_h \leq p_{MC}$, the following Lemma holds.

Lemma 3. *If $p_h \leq p_{MC}$, then a TPE is welfare superior to a single price monopolistically competitive equilibrium.*

Proof. Consider a TPE and a single price monopolistically competitive equilibrium at p_{MC}, where $p_h \leq p_{MC}$. Since producers' surplus is zero in both equilibria, we have to compare only consumers' utilities. Now, uninformed consumers are better off in a TPE, since their worst random entry results in buying the product at $p_h \leq p_{MC}$. Informed consumers are better off in a TPE since they prefer the option of buying information to that of staying uninformed. Hence, every consumer is strictly better off in a TPE, provided $p_h \leq p_{MC}$, i.e. the gain in consumers' surplus due to lower prices more than compensates the loss of resources spent on information acquisition. ‖

4. CONCLUSION

We have shown how different monopolistically competitive equilibria may arise from consumers' imperfect information regarding different prices of a homogeneous commodity. Under such a structure, there is no longer one market place. Each firm may, in a sense, constitute a local market, and different consumers with different search costs may equilibrate the market at different prices. The nature of the differences in consumers' search costs determine what type of an equilibrium arises: a perfectly competitive, a monopolistically competitive, or a two-price equilibrium. If a perfectly competitive equilibrium exists, it is the unique (Bertrand–Nash) equilibrium).

For a monopolistically competitive equilibrium with a single price or for a TPE the " excess capacity " theorem holds without reservations. Since we are dealing with a homogeneous commodity, determining the optimal product set is not an issue.[14] Imperfect information gives rise to excessive entry, excessive fixed costs and non-exploitation of scale opportunities. Such equilibria may generate lower surplus than a monopoly, and they entail *more* firms than perfect competition.

Each model of equilibrium price dispersion due to imperfect information shares the feature that there are always some uninformed customers in the market. Ignorance may be maintained by assuming birth of ignorant and death of informed customers, or alternatively that there is slow diffusion of information relative to the lifetime of firms. So far no one has attempted a rigorous justification of the assumption of maintained ignorance. There is a need for a dynamic model which will take into account the possibility of a firm's acquiring a " price reputation " over time.

APPENDIX

THE UNIQUENESS OF THE PERFECTLY COMPETITIVE EQUILIBRIUM

To prove the uniqueness of a perfectly competitive equilibrium (if it exists) we only have to exclude the possibility of a TPE, since a monopolistically competitive equilibrium with

NOTES

1. For a sequential search version of this model, see Von Zur Muehlen (1976).

2. However, permitting consumers to take an action according to the outcome of a flipped coin will not change the results.

3. Though not completely clear from the exposition, it seems that in Diamond's (1971) description of a long run equilibrium consumers have the option of costlessly entering a store at random only once, and if deciding not to purchase but to enter another store, they incur a *positive* search cost. In equilibrium a consumer optimizes by entering a store at random only once and purchasing there. Such a decision rule can result from our model if we assume that the information (search) cost of each consumer is prohibitively large, so that his optimal decision is to stay uninformed and to enter a store at random.

4. We assume a Nash, non-cooperative game in prices, where each producer uses only pure strategies. Monopolistically competitive equilibria with price dispersion, which are generated under mixed strategies, are analysed by Butters (1977) and Shilony (1977).

5. A sufficient condition for the existence of a perfectly competitive equilibrium is:

$$\text{infimum}_{p \geq \bar{p}} AC^{-1}(p)/X(p) > (1 - \mu(0))AC^{-1}(\bar{p})/X(\bar{p}),$$

where $X(p)$ denotes a consumer's demand function and $1 - \mu(0)$ denotes the fraction of consumers with strictly positive search costs. (See Braverman–Guasch (1977)).

6. Sweezy (1939) assumes that if a firm cuts its price, it perceives its rivals to follow suit. On the other hand, if a firm raises its price, it conjectures that its rivals will not initiate any price changes. The perceived demand curve for the firm's product would be then more elastic for price increases than for price decreases (the opposite of Figure 2). In our model, if a firm changes its price it assumes in the Bertrand–Nash manner that other firms will not change their price in response.

7. Salop–Stiglitz (1975) derived a single price equilibrium at the monopoly price, u, while assuming positive uniform search density, *only* because they assumed a perfectly inelastic demand function up to u.

8. Since consumers are identical except for their search costs, it does not pay the monopoly to charge different prices in its different stores, as opposed to Salop's (1977) model of " Noisy Monopolist ".

9. The DD curve assumes that all firms move their prices together. Thus it ignores the gain in informed consumers due to being the lowest price store in the market, which is computed by the dd curve.

10. For a more general and more realistic interpretation of a Chamberlinian equilibrium, this assumption is not necessary (e.g. see Friedman (1977, p. 57), i.e. $DD \neq X(p)/n$. Hence, under the more general interpretation of DD, Proposition 3 does not hold for a Chamberlinian equilibrium.

11. A TPE is also consistent with existence of atoms of consumers at different search costs. In particular, it is consistent with an atom at the positive critical search cost, c^*, which divides consumers into informed and uninformed. This is so since we assume that if consumers are given the same expected utility from random selection as from purchasing information, they remain uninformed or choose an action according to the outcome of a flipped coin. Thus only a price rise will generate a discontinuous decline in the perceived demand of a high price deviant, due to the discontinuous loss of an atom of uninformed customers. Hence, this price increase is clearly a profit-decreasing move (see Braverman (1976, p. 83).

12. In Braverman (1976, p. 42), we prove that for every utility function, cost function, high price, industry size and distribution of low and high price firms, which satisfy reasonably weak conditions, there exists an infinite family of measures of search costs, and in particular, continuous densities, which support a TPE.

13. A high price firm maximizes profits with respect to its price. We can reformulate this problem in the following way: The high price producer chooses the amount X, that he will sell to one uninformed customer if he charges p. Each uninformed customer who enters his store will spend $Xp(X)$. The number of informed customers, μ, is affected by $p(X)$, given the low price, $\bar{p}$, the industry size, n, and the distribution of firms into low and high prices.

14. See Spence (1976) and Dixit–Stiglitz (1977) for discussion of the product selection under monopolistic competition.

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