

Consumption-based Asset Pricing with Loss Aversion*

Marianne Andries[†]
Toulouse School of Economics

September, 2012

Abstract

I incorporate loss aversion in a consumption-based asset pricing model with recursive preferences and solve for asset prices in closed-form. I find loss aversion increases expected returns substantially relative to the standard recursive utility model. This feature of my model improves the ability to match moments on asset prices. Further, I find loss aversion induces important nonlinearities into the expected excess returns as a function of the exposure to the consumption shocks. In particular, the elasticities of expected returns with respect to the exposure to the consumption shocks are greater for assets with smaller exposures to the shocks, thus generating interesting predictions for the cross-section of returns. I provide empirical evidence supporting this outcome. The model with loss aversion correctly predicts both a negative premium for skewness and a security market line, the excess returns as a function of the exposure to market risk, flatter than the CAPM.

Introduction

Loss-averse agents value consumption outcomes relative to a reference point, and losses relative to the reference create more disutility than comparable gains. I add such loss aversion features to a preference model with recursive utility, in which the value of the consumption stream depends on current consumption and next period's value for future consumption. I suppose agents are loss averse and thus suffer additional disutility if the realization of next period's value disappoints (i.e., falls below their expectation). My model of loss aversion allows me to find tractable solutions to the consumption-based asset pricing model with homogeneous agents.

Loss aversion has a “first-order risk aversion” impact: the certainty equivalents of small gambles around the reference point depend on first-order volatility terms in contrast to the second order

*I want to thank my committee chairs, Lars Peter Hansen and Pietro Veronesi, and my committee members, John C. Heaton, Emir Kamenica, Ralph Koijen. Also for their comments and advice, I want to thank Thomas Chaney, Nicolas Coeurdacier, John Cochrane, George Constantinides, Andrea Frazzini, Xavier Gabaix, Valentin Haddad, Ron Kaniel, Botond Koszegi, Junghoon Lee, Nan Li, Erik Loualiche and David Sraer.

[†]Contact: TSE, Toulouse, France. Tel: +33 5 61 12 87 70. Email: marianne.andries@tse-fr.eu.

terms of smooth utility models. The smaller the volatility, the more dominant these first order terms are: agents appear more risk averse for small gambles than for large ones, in line with evidence from the micro and experimental literature. Accordingly, I find loss aversion in the preferences has a first-versus-second order impact on asset prices, so that, compared to the standard recursive utility model, expected returns are substantially higher (level effect), even more so for assets with small rather than large underlying risk (cross-sectional effect).

Consider first the cross-sectional effect. The loss aversion specification induces important nonlinearities in the expected excess returns: the price of risk, represented by the elasticities of expected returns with respect to the exposure to the consumption shocks, varies with the exposures to the shocks, in contrast to the standard recursive utility model, which yields a constant pricing of risk across assets, in the cases I consider. My model with loss aversion thus generates novel predictions for the cross section of returns, which differentiate it from the standard recursive utility model. Two well-known results in finance provide empirical support for my model. First, Black, Jensen, and Scholes (1972) and more extensively Frazzini and Pedersen (2010) show the asset returns line (the excess returns as a function of beta, the exposure to market risk) is persistently flatter over time than the CAPM, for a wide class of assets (U.S. equities, 20 global equity markets, Treasury bonds, corporate bonds, and futures). Second, Harvey and Siddique (2000) show assets with the same volatility but different skewness in their returns distributions yield different expected returns: they find a negative premium for skewness. My model with loss aversion offers a novel theoretic explanation for these results.

Consider now the level effect. With loss aversion, my model generates higher expected excess returns and lower risk-free rates than in the standard recursive utility model. The recursive utility model, which allows one to disentangle the risk aversion and the intertemporal elasticity of substitution, is central to the consumption-based asset pricing literature, notably the long-run risk models (e.g., Bansal and Yaron (2004); Hansen, Heaton, and Li (2008); Bansal, Kiku, and Yaron (2007, 2009)). However, its calibration using moments on asset returns requires high levels of risk aversion. The level effect my model with loss aversion generates allows me to improve on such calibration exercises.

Beyond the contribution of developing a fully tractable consumption-based asset pricing model with loss aversion, my analysis of the cross-sectional risk-price elasticities, as well as the impact of loss aversion on the security market line relative to the CAPM, is novel to the behavioral finance and the asset pricing literature.

Previous papers analyze the impact on asset prices of preferences with loss aversion (e.g., Bernartzi and Thaler (1995); Barberis et al. (2001); Yogo (2008); Barberis and Huang (2009)). I add to this literature by defining a new model of preferences with loss aversion that allows me to solve the asset pricing model with recursive utility in a tractable way. The advantage of using recursive preferences in consumption-based asset pricing models is well established, and combining behavioral models and recursive utility gives rise to interesting results. Other authors have adopted this approach. Routledge and Zin (2010) present a model of generalized disappointment aversion, an extension of the disappointment aversion of Gul (1991). They analyze the asset pricing implications of Epstein-Zin preferences with generalized disappointment aversion and obtain closed-form solutions and interesting results in a simple two-state Markov economy. Bonomo et al. (2011) extend the analysis to a four-state Markov adapted from Bansal, Kiku, and Yaron (2007). They match first and second moments on the market returns and risk-free rate, predictability patterns, and autocorrelations, for realistic parameters. The tractable features of my model allow me to find closed-form solutions for more general economies, to extend the analysis to the cross-section of returns, and to analyze and derive solutions for various novel reference-point models, while remaining close in spirit to disappointment aversion. Barberis and Huang (2009) use a recursive utility model with loss aversion narrowly framed on the stock market returns and find closed-form solutions for both partial and general equilibria. My model differs from theirs in two crucial ways. First, I make the more conservative choice of not opting for narrow framing on financial risks, which makes the results I obtain all the more robust. Second, Barberis and Huang (2009) choose the constant risk free rate as the reference point for market returns. In contrast, to better reflect the empirical evidence on the reference point, I model it as endogenously determined as an expectation.

The rest of the paper is organized as follows: In section 1, I model loss aversion in a recursive model of preferences. In section 2, I analyze the consumption-based asset pricing model and obtain tractable solutions for the model of preferences with loss aversion. I then analyze the asset pricing implications of the model. The predictions of the model are brought to the data in section 3.

1 Preferences with Loss Aversion

I define a new model of preferences that display loss aversion, with a reference point endogenously specified as an expectation of the future utility of consumption. I focus on CRRA preferences and

a log-linear specification, which allows me to obtain closed-form solutions when adapted to the consumption-based asset pricing model with unit intertemporal elasticity of substitution.

For illustrative purposes, I start with a two-period model in section 1.1. In section 1.2, I extend the loss aversion specification to the multi-period, recursive utility model, and I fully describe my choice of preferences. In section 1.3, I derive the Euler Equation corresponding to my model of preferences.

1.1 Two-Period Model

At period $t = 1$, the agent receives consumption C , the level of which is uncertain at period $t = 0$.

The standard CRRA model for this two-period setting is:

$$U_0 = \mathbb{E} \left(\frac{C^{1-\gamma}}{1-\gamma} \mid \mathcal{I}_0 \right),$$

where \mathcal{I}_0 is the information set at time $t = 0$ and $\gamma > 1$ is the coefficient of risk aversion.

I modify the standard model by adding loss aversion around a reference point, which I define later as the agent's endogenous expectation for next-period consumption (see Eq. (4)). The reference point depends on the time $t = 0$ distribution for time $t = 1$ consumption, and is noted $\mathcal{R}(C)$. The two-period model is now given by:

$$U_0 = \mathbb{E} (U(C, \mathcal{R}(C)) \mid \mathcal{I}_0),$$

and in Figure 1, I illustrate how the modified utility from consumption $U(C, \mathcal{R}(C))$ incorporates loss aversion into the standard CRRA model.

Because loss averse agents dislike losses more than they value gains, the modified utility function displays a kink at the reference point, with a steeper slope below the reference than above. As a modeling choice, the utility function is unchanged from the standard CRRA model with risk aversion γ above the reference point. Below the reference point, the loss aversion specification results in a decrease in utility relative to the standard model.

The decrease in utility below the reference is determined by the sharpness of the kink. The more loss averse the agent, the sharper the kink in the preferences. I therefore define a loss aversion coefficient $\alpha \in [0, 1)$, where $1 - \alpha$ determines the ratio of the right-hand slope to the left-hand slope. In the limit case $\alpha = 0$, the agent displays no loss aversion (the ratio of the slopes is exactly one) and the model reverts to the standard CRRA model. As α increases, so does the sharpness

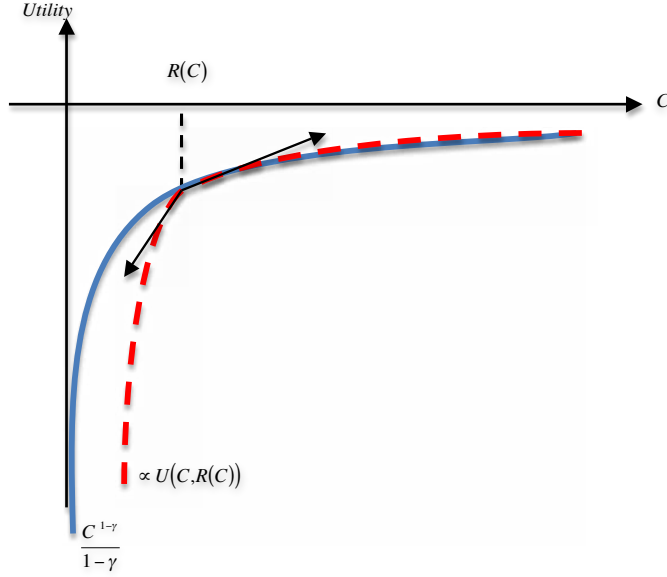


Figure 1: Loss Aversion in the Two-period Model

of the kink at the reference point.¹

To ensure tractability in the asset pricing model, I choose to maintain the homogeneous CRRA specification below the reference point.

Proposition 1 *If preferences $U(C, \mathcal{R}(C))$ satisfy:*

- 1) *preferences are continuous*
- 2) *preferences display a kink at a reference point $\mathcal{R}(C)$, with a right-hand to left-hand slope ratio equals to $1 - \alpha$ with $\alpha \in [0, 1)$*
- 3) *preferences are homogeneous CRRA above and below the reference point,*

Then:

$$U(C, \mathcal{R}(C)) = \begin{cases} a \frac{C^{1-\gamma}}{1-\gamma} & \text{for } C \leq \mathcal{R}(C) \\ b \frac{C^{1-\gamma}}{1-\gamma} & \text{for } C \geq \mathcal{R}(C) \end{cases},$$

with $\frac{b}{a} = \frac{1-\gamma}{1-\bar{\gamma}} (\mathcal{R}(C))^{\gamma-\bar{\gamma}}$ and $\frac{1-\gamma}{1-\bar{\gamma}} = 1 - \alpha$.

Without loss of generality, I can set $b = 1$ or $a = 1$. As I discuss below, I model the reference point $\mathcal{R}(C)$ as an expectation of future consumption outcomes, and it is thus endogenously determined by the agent's optimal consumption choice. Because the agent is loss averse for outcomes

¹Using micro evidence, Kahneman and Tversky (1979) estimate the ratio of the slopes at 1/2.25, which corresponds to $\alpha = 0.55$, and I present several quantitative results with this value. This estimation concerns loss aversion on individual gambles, and is therefore mainly illustrative in the context on a representative agent with loss aversion on total wealth.

below the reference point, choosing a consumption path that results in a low reference point rather than a high reference point at period $t+1$, thus decreasing the probability of disappointment, could be in her best interest. In such a case, the agent would sometimes reject first-order dominating outcomes. Some empirical evidence exists regarding such behavior.² However, in the context of asset pricing, first-order stochastic dominance should be preserved to avoid direct violations of the no-arbitrage condition.

Consequently, I ensure, in my model of preferences, the expected utility U_0 is increasing in $\mathcal{R}(C)$. This is satisfied when $a = 1$ and:

$$U(C, \mathcal{R}(C)) = \frac{1}{1 - \bar{\gamma}} \begin{cases} C^{1 - \bar{\gamma}} & \text{for } C \leq \mathcal{R}(C) \\ C^{1 - \gamma} \times \underbrace{(\mathcal{R}(C))^{\gamma - \bar{\gamma}}}_{\text{scaling factor}} & \text{for } C \geq \mathcal{R}(C) \end{cases} . \quad (1)$$

In that regard, I follow Kahneman and Tversky (1979), in which direct violation of dominance is prevented in the first stage of editing.

The ratio of the slopes above and below the reference point is given by

$$\frac{1 - \gamma}{1 - \bar{\gamma}} = 1 - \alpha .$$

This equation makes explicit $\bar{\gamma}$ as an increasing function of both γ and α , with $\bar{\gamma} \geq \gamma$. In my model, the curvature is stronger, and the agent is more risk-averse below the reference point than above. This is to be contrasted with the prospect theory model of Kahneman and Tversky (1979), in which agents display loss aversion in their preferences, with risk aversion above and risk seeking below the reference point. Agents have been documented to display risk-seeking below the reference point in the context of narrow-framing, in which gambles are evaluated independently from other sources of risk. This evidence does not contradict my model, in which agents display loss aversion over the total value of consumption.

1.2 Multi-Period Model, Recursive Utility

I now consider a multi-period model with consumption stream $\{C_t\}$.

As in the model of Epstein and Zin (1989), at each period t , the agent's valuation for the future consumption stream is given by V_t , which is defined recursively as:

$$V_t = \left((1 - \beta) C_t^{1 - \rho} + \beta (h(V_{t+1}))^{1 - \rho} \right)^{\frac{1}{1 - \rho}} ,$$

²Frederick and Loewenstein (1999) consider cases in which a prisoner is better off not trying for parole in order to avoid being disappointed. Gneezy, List, and Wu (2006) observe cases in which an agent chooses a worst outcome for certain rather than a lottery outcome. See also Akerlov and Dickens (1982) and Matthey (2010).

with $\rho > 0$ the inverse of the EIS (elasticity of intertemporal substitution) and $0 < \beta < 1$ the discount factor (with $-\log \beta$ the rate of time discount).

The period $t = 1$ consumption of the two-period model is replaced by next-period value V_{t+1} , which is uncertain at time t , and impacts current value V_t via a standard CRRA model:

$$h(V_{t+1}) = \left(\mathbb{E}_t \left(V_{t+1}^{1-\gamma} \right) \right)^{\frac{1}{1-\gamma}},$$

where $\gamma > 1$ is the coefficient of risk aversion.

I modify h by introducing loss aversion around a reference point, similarly to the two-period model of section (1.1). At each period t , the reference point depends on the conditional distribution for next period value V_{t+1} , and is noted $\mathcal{R}_t(V_{t+1})$. I obtain:

$$h(V_{t+1}) = \left\{ \mathbb{E}_t [U(V_{t+1}, \mathcal{R}_t(V_{t+1}))] \right\}^{\frac{1}{1-\bar{\gamma}}},$$

where

$$U(V_{t+1}, \mathcal{R}_t(V_{t+1})) = \begin{cases} V_{t+1}^{1-\bar{\gamma}} & \text{for } V_{t+1} \leq \mathcal{R}_t(V_{t+1}) \\ V_{t+1}^{1-\gamma} \times \underbrace{(\mathcal{R}_t(V_{t+1}))^{\gamma-\bar{\gamma}}}_{\text{scaling factor}} & \text{for } V_{t+1} \geq \mathcal{R}_t(V_{t+1}) \end{cases}, \quad (2)$$

and

$$\frac{1-\gamma}{1-\bar{\gamma}} = 1-\alpha. \quad (3)$$

Eq. (2) is the multi-period extension to the two-period model of Eq. (1).

As in the two-period model, loss aversion is represented by one coefficient, $\alpha \in [0, 1)$ which determines the sharpness of the kink in the preferences, with a ratio of slopes given by Eq. (3). As before, this relation makes explicit $\bar{\gamma}$ as an increasing function of both γ and α , with $\bar{\gamma} \geq \gamma$. When $\alpha = 0$, the agent displays no loss aversion and my model reverts to the standard recursive utility model. When $\alpha > 0$, the agent is loss averse and expects at time t to experience additional disutility at time $t + 1$ if the value of the future consumption stream V_{t+1} is disappointing, that is, falls below her time t reference point $\mathcal{R}_t(V_{t+1})$.

Notice I did not include loss aversion on the contemporaneous consumption C_t . The one-period discount rate is sufficiently low that most of the value in V_t comes from the second term in V_{t+1} and not from the first term in C_t . Simplifying the model by restricting the loss aversion specification to the second term in V_{t+1} is a valid choice.

Further, I did not include loss aversion over changes in the reference point $\mathcal{R}_t(V_{t+1})$. Adding loss aversion over changes in the news about future outcomes, and thus over changes in the reference point is left for future research.

Reference Point

In line with the benchmark model of Koszegi and Rabin (2006), I define a reference point endogenously determined by the agent's expectation of outcomes. As a modeling choice, I opt for a log-linear specification for the reference point: in my model, the agent is disappointed and registers additional disutility from loss aversion when $\log V \leq \mathbb{E}(\log V)$.

The log-linear specification for the reference point is a natural choice for the consumption-based asset pricing model with unit intertemporal elasticity of substitution. However, the model can be analyzed with other choices of the reference point as an expectation. In particular, the predictions of my model are largely unchanged by the more general CRRA model of $\mathcal{R}(V) = (\mathbb{E}(V^{1-\psi} | \mathcal{I}_0))^{\frac{1}{1-\psi}}$, with $\psi \geq 0$. I derive the solutions for this model and compare them to the log-linear case corresponding to $\psi = 1$ in the online Appendix C.³

There is ample empirical evidence for a reference point as an expectation (see for example Sprenger (2010), Crawford and Meng (2011), Pope and Schweitzer (2011), Abeler et al. (2011), Card and Dahl (2011) and Gill and Prowse (2012)), but none regarding which expectation model is most relevant. Consequently, Koszegi and Rabin (2006) model the reference point as stochastic. My choice of a deterministic reference point simplifies the model greatly. Allowing for uncertainty on the reference point is left for future research.

In the multi-period framework, the agent updates her reference point as an expectation when new information about future outcomes becomes available. However, the manner with which the agent updates the reference point is a modelling choice.

For most of the asset pricing analysis I present, I suppose the agent fully updates her reference point at each period, such that the reference point at time t is an expectation of outcomes at time $t + 1$ given the information \mathcal{I}_t :

$$\mathcal{R}_t(V_{t+1}) = \exp[\mathbb{E}(\log V_{t+1} | \mathcal{I}_t)] .$$

In section 2.3.2, I consider a more general, but less tractable, model in which the agent's reference point at time t depends on past expectations of the period $t + 1$ outcomes. Her reference point adjusts slowly as a weighted average of current and past expectations as in:

$$\mathcal{R}_t(V_{t+1}) = \left(\prod_{n=0}^T (\exp \mathbb{E}(\log V_{t+1} | \mathcal{I}_{t-n}))^{\xi^n} \right)^{\frac{1}{\sum_0^T \xi^n}} , \quad (4)$$

³http://home.uchicago.edu/mandries/lossaversion_appendix.pdf.

where $\xi \in [0, 1)$ and T is the number of past periods impacting the reference point. The case $\xi = 0$ reverts to the model where the reference point is fully updated at each period and only current expectations matter.⁴ When $\xi > 0$, the agent gradually upgrades the reference point following positive shocks to the consumption process and thus the risk of disappointment diminishes. Conversely, the reference point is gradually downgraded in a recession and thus the risk of disappointment increases. This mechanism introduces some counter-cyclicality in the pricing of risk, even when the consumption process has constant volatility.⁵

Characteristics of the Model

Combining the model of Eq. (2) and the modeling choice for the reference point of Eq. (4):

$$\begin{aligned} V_t &= \left((1 - \beta) C_t^{1-\rho} + \beta (h(V_{t+1}))^{1-\rho} \right)^{\frac{1}{1-\rho}} \\ h(V_{t+1}) &= \mathbb{E}_t \left(\overline{V}_{t+1}^{1-\bar{\gamma}} \right)^{\frac{1}{1-\bar{\gamma}}} \\ \log \overline{V}_{t+1} &= \log V_{t+1} - \alpha \max \left(0, \log V_{t+1} - \frac{\sum_{n=0}^T \xi^n \mathbb{E}_{t-n}(\log V_{t+1})}{\sum_{n=0}^T \xi^n} \right). \end{aligned} \quad (5)$$

Proposition 2 *h has the following properties:*⁶

- 1) *if the outcome V_{t+1} is certain, $h(V_{t+1}) = V_{t+1}$*
- 2) *h is increasing (first-order stochastic dominance)*
- 3) *h is concave (second-order stochastic dominance)*
- 4) *h is homogeneous of degree one (and therefore V_t is homogeneous of degree one in (C_t, V_{t+1}))*

These characteristics of my model allow me to use most of the results from Epstein and Zin (1989), notably the uniqueness of the solution to the optimization problem. The concavity in the preferences justifies the use of first-order conditions at the optimum, such as the Euler Equation. Because at time t , V_t is increasing in V_{t+1} (first-order stochastic dominance), the agent simultaneously optimizes the current value V_t and the continuation value V_{t+1} , and my model of preferences is time consistent.⁷

⁴Dillenberger and Rozen (2011) argue for a history-dependent risk attitude (past disappointments and elation have an impact on risk aversion), which would support a model of “sticky” updating of the reference point, and $\xi > 0$. On the other hand, price-dividend ratios are not well predicted in the data by past consumption growth (which is also a critique of all habit models), which tends to suggest the degree of “stickiness” ξ must remain small.

⁵In contrast to models in which time-varying risk aversion is exogenously enforced (see the habit model of Campbell and Cochrane (1999), as well as Barberis, Huang, and Santos (2001) and Yogo (2008)), counter-cyclical risk prices endogenously obtain in my model with “sticky” updating of the reference point.

⁶Proof of these properties is provided in Appendix A.

⁷Proposition 2 remains valid when the reference point is specified in the more general CRRA framework as $\mathcal{R}(V) = (\mathbb{E}(V^{1-\psi} | \mathcal{I}_0))^{\frac{1}{1-\psi}}$, with $\psi \geq 0$.

This is a discrete time model in which the length of time intervals can greatly influence the impact of loss aversion. Indeed, for any time period T , the probability that the agent experiences some loss aversion increases with the frequency of the model. For a given coefficient of loss aversion α , the agent would refuse to take any form of risk at the continuous time limit. It might be more realistic, however, for the agent not to allow herself to be greatly affected by small high frequency losses, and thus for the coefficient of loss aversion α to decrease with the frequency of the model, and for the continuous time limit to remain well behaved. Loss aversion models in continuous time are left for future research.

My model of loss aversion is similar in spirit to the disappointment aversion model. However, I explicitly define the reference point as an expectation, whereas, in the disappointment aversion model, it is the solution to a recursive problem. This greatly simplifies the solutions to the asset pricing model, while yielding similar quantitative results, in the model with full updating of the reference point. It also allows for great flexibility and the analysis of models such as the one with “sticky” updating ($\xi > 0$) in the reference point.

1.3 Stochastic Discount Factor

I now turn to the asset pricing implications of the model. At time t , all uncertain returns R_{t+1} must satisfy the Euler Equation:

$$\mathbb{E}_t [R_{t+1} S_{t,t+1}] = 1, \quad (6)$$

where $S_{t,t+1}$ is the stochastic discount factor between time t and $t + 1$.

Suppose $\xi = 0$.⁸

Proposition 3 For $V_{t+1} < \mathcal{R}_t(V_{t+1})$:

$$S_{t,t+1}^- = \underbrace{\beta \left(\frac{V_{t+1}}{h(V_{t+1})} \right)^{\rho-\bar{\gamma}} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho}}_{\text{standard recursive utility model}} \left(1 + \alpha \frac{\mathbb{E}_t \left(\mathbf{1}_{V_{t+1} \geq \mathcal{R}_t(V_{t+1})} \overline{V_{t+1}}^{1-\bar{\gamma}} \right)}{\overline{V_{t+1}}^{1-\bar{\gamma}}} \right).$$

For $V_{t+1} > \mathcal{R}_t(V_{t+1})$:

$$S_{t,t+1}^+ = \underbrace{\beta \left(\frac{V_{t+1}}{h(V_{t+1})} \right)^{\rho-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho}}_{\text{standard recursive utility model}} \left(\frac{\mathcal{R}_t(V_{t+1})}{h(V_{t+1})} \right)^{\gamma-\bar{\gamma}} \left((1 - \alpha) + \alpha \frac{\mathbb{E}_t \left(\mathbf{1}_{V_{t+1} \geq \mathcal{R}_t(V_{t+1})} \overline{V_{t+1}}^{1-\bar{\gamma}} \right)}{\overline{V_{t+1}}^{1-\bar{\gamma}}} \right).$$

⁸The details of the derivation for both $\xi = 0$ and $\xi > 0$ are in Appendix A. The case $\xi > 0$ is analyzed in the online Appendix E, http://home.uchicago.edu/mandries/lossaversion_appendix.pdf.

The first terms in the stochastic discount factor are those of the standard recursive utility model, with risk aversion $\bar{\gamma}$ below the reference point and risk aversion γ above the reference point. As in the standard recursive utility model, the covariations of cash-flows with the consumption growth and with the shocks to the value function determine prices. Shocks to all future realizations of consumption have an immediate impact on the value function. The recursive utility specification thus allows the pricing of such shocks. In contrast, in the expected utility CRRA model, the covariations with the immediate consumption shock only determine prices.

Note that if $\alpha = 0$, the stochastic discount factor reverts to the standard model with risk aversion γ .

At the reference point $V_{t+1} = \mathcal{R}_t(V_{t+1})$,

$$\frac{S_{t,t+1}^+}{S_{t,t+1}^-} = 1 - \alpha \frac{\mathcal{R}_t(V_{t+1})}{\mathcal{R}_t(V_{t+1}) + \alpha \mathbb{E}_t \left(\mathbf{1}_{V_{t+1} \geq \mathcal{R}_t(V_{t+1})} \bar{V}_{t+1}^{1-\bar{\gamma}} \right)} \leq 1 .$$

Because of the kink in the preferences due to loss aversion, the stochastic discount factor is discontinuous at the reference point, when $\alpha > 0$. The starkly different pricing effects I obtain for the model with loss aversion in section 2 mostly derive from this discontinuity.

2 Risk Pricing with Loss Aversion

I assume all agents have identical preferences with loss aversion, given by Eq. (5), and they differ only in their wealth.⁹ Because preferences are homothetic, the representative agent assumption is justified.

As a special case of the multi-period model of section 1.2, I start with a simple expected utility framework in section 2.1. I find the loss aversion specification has (i) a level effect: the expected excess returns are higher and the risk-free rate is lower than in the standard model; and (ii) a cross-sectional effect: depending on the exposures to the consumption shocks, the impact of loss aversion is more or less intense.

However, the quantitative implications of the expected utility model do not allow for a correct calibration of asset pricing moments. I therefore solve for asset prices in the model with both recursive utility and loss aversion in sections 2.2 and 2.3.

⁹Discussing the possible impact of heterogeneity in preferences is not in the scope of this paper, but would be worth exploring. The equilibrium existence, representative agent, and PDE solutions of Duffie and Lyons (1992) and Skiadas and Schroder (1999) cannot be used because the preferences are not continuously differentiable in the interior domain.

2.1 Expected Utility Model

I start with the special case $\rho = \gamma = 1$, and $\xi = 0$, of the multi-period model of Eq. (5), and I suppose the optimal consumption follows the process:

$$\log C_{t+1} - \log C_t = \mu_c + \sigma_c w_{t+1} ,$$

where $\{w_t\}$ is iid $\mathcal{N}(0, 1)$.¹⁰ Preferences are thus given by the expected utility model:

$$U_t = \log C_t + \mathbb{E}_t \left(\sum_{\tau=1}^{\infty} \beta^\tau (\log C_{t+\tau} - \alpha \max(0, \log C_{t+\tau} - \mathbb{E}_{t+\tau-1}(\log C_{t+\tau}))) \right) . \quad (7)$$

Define the value function V_t as $\log V_t = (1 - \beta) U_t$, for all t , and write $\log C = c$ and $\log V = v$.

Proposition 4 *The unique solution for the value function has a closed-form solution given by:*¹¹

$$v_t - c_t = \frac{\beta}{1 - \beta} \left(\mu_c - \alpha \frac{\sigma_c}{\sqrt{2\pi}} \right) . \quad (8)$$

Loss aversion has a clear dampening effect on the value of the consumption stream. Higher amounts of risk in the consumption process amplify the impact of loss aversion, so that, in contrast with the standard expected utility model, the value function varies with the volatility σ_c . With loss aversion $\alpha = 0.55$ (as in Kahneman and Tversky (1979)), the log value-to-consumption ratio is about 75% of the initial value of the standard model.

Consider an asset with time $t + 1$ return R_{t+1} , which is uncertain at time t and follows the log-normal process

$$\log R_{t+1} = \left(\bar{r} - \frac{1}{2} |\sigma_R|^2 - \frac{1}{2} |\widetilde{\sigma}_R|^2 \right) + \sigma_R w_{t+1} + \widetilde{\sigma}_R \widetilde{w}_{t+1} , \quad (9)$$

where $\{w_{t+1}\}$ are the shocks to the consumption process, $\{\widetilde{w}_{t+1}\}$ are independent shocks, and \bar{r} is the log expected return of the asset.¹²

The covariations of asset returns with the consumption shocks determine how “risky” the asset is and thus the expected returns the agent requires. Applying the Euler Equation of Eq. (6) to

¹⁰In all the empirical results I present in this section, I use the quarterly data (1947 to 2010) on the seasonally adjusted aggregate consumption of non-durables and services from the National Income and Product Accounts (NIPA) to estimate μ_c and σ_c .

¹¹Proof is given in Appendix B

¹²I choose to model the returns directly as log-normal to obtain closed-form solutions on the expected returns and risk-price elasticities as functions of the exposure to the consumption shocks. Another choice would be to model the asset’s cash-flows, rather than the returns, as log-normal. Such a modeling choice would generate returns with close to log-normal distributions and would yield numerical results in line with the closed-form solutions of my model.

the returns of Eq. (9) yields \bar{r} as a function of σ_R . Increasing the exposure of the log returns to the log-consumption shocks has a price, which is reflected in a change in the log expected returns. The risk-price elasticities, given by $rp(\sigma_R) = \partial \bar{r}(\sigma_R) / \partial \sigma_R$, measure such changes, and therefore quantify the pricing of risk in the model.

Proposition 5 *The risk-free rate, expected excess returns and risk-price elasticities are given by:*¹³

$$rf = -\log \beta + \mu_c - \frac{1}{2}\sigma_c^2 - \log \left[1 + \alpha \left(\frac{1}{2} - \Phi(-\sigma_c) \right) \right], \quad (10)$$

$$\bar{r}(\sigma_R) - rf = \sigma_c \sigma_R + \log \left[1 + \alpha \left(\frac{1}{2} - \Phi(-\sigma_c) \right) \right] - \log \left[1 + \alpha \left(\frac{1}{2} - \Phi(\sigma_R - \sigma_c) \right) \right], \quad (11)$$

and

$$rp(\sigma_R) = \sigma_c + \frac{\alpha}{\sqrt{2\pi}} \frac{\exp\left(-\frac{1}{2}(\sigma_R - \sigma_c)^2\right)}{1 + \alpha \left(\frac{1}{2} - \Phi(\sigma_R - \sigma_c) \right)}, \quad (12)$$

where Φ is the cumulative normal function.

The first three terms in Eq. (10) are those of the standard recursive utility model and the usual comparative statics obtain. The risk-free rate is (i) increasing in the mean consumption growth μ_c (when the expected consumption growth is high, agents are less inclined to save); (ii) decreasing in β (with a lower rate of time discount, the agents are more willing to substitute between immediate and future consumption and thus to save); and (iv) decreasing in the amount of risk in consumption.

Loss aversion results in an additional precautionary savings term that lowers the risk-free rate and amplifies its sensitivity to the amount of risk in the consumption process. Nonetheless, the calibration of the risk-free rate is dominated by the choice of the discount rate β , and the impact of loss aversion is somewhat small: loss aversion with $\alpha = 0.55$ reduces the annual risk-free rate from 2.3% to 1.9%, for a choice of $\beta = (0.999)^{\frac{1}{4}}$.

In both Eq. (11) and Eq. (12), the first term corresponds to the standard log-utility model, which yields a linear relation between returns and risk, and thus a constant pricing of risk, equal to σ_c , the volatility of the consumption process.

In addition, loss aversion has, first, a level effect on prices: it unambiguously increases the expected excess returns that the agent requires for a given amount of risk. Second, the additional

¹³Proof is given in Appendix B

terms due to loss aversion break down the linear relation between returns and risk, resulting in a cross-sectional effect on asset prices.

For $|\sigma_R|$ large, the pricing of risk is approximately unchanged from the standard model with $rp(\sigma_R) \approx \sigma_c$: loss aversion has virtually no impact on the pricing of risk for assets that carry large risks. On the other hand, for $|\sigma_R|$ very small, a first-order approximation yields:

$$rp(\sigma_R) \approx \frac{\alpha}{\sqrt{2\pi}} + \sigma_c \left(1 - \frac{\alpha^2}{2\pi}\right) + \frac{\alpha^2}{2\pi} \sigma_R,$$

where I take σ_c as approximately zero, with same order of magnitude as $|\sigma_R|$.¹⁴ Even for moderately loss averse agents, the constant term $\alpha/\sqrt{2\pi}$ dominates over the first-order terms in σ_R and σ_c , which reflects the “first-order risk aversion” characteristic of preferences with kinks.¹⁵ Loss aversion has a large, first-order, impact on the expected returns of assets that carry small risks.

The qualitative implications of loss aversion, with both a level and a cross-sectional impact on the pricing of risk, are well illustrated in the expected utility model. Quantitatively, however, this model cannot explain the asset pricing moments we observe. In particular, because of the low covariation between aggregate consumption and market returns, the model generates an equity premium of 0.65% annually, when $\alpha = 0.55$.¹⁶ In the next section, I analyse the asset pricing implications of loss aversion, when combined with the recursive utility model which yields realistic moments in the distributions of prices, as evidenced by the long-run risk literature.

2.2 Recursive Utility with Loss Aversion

I suppose the representative agent has recursive preferences with loss aversion as in Eq. (5), with full updating of the reference point. Following the methodology of Hansen, Heaton, Lee, and Roussanov (2007), the model is first solved in closed-form for a unit elasticity of intertemporal substitution (case $\rho = 1$).¹⁷ A first-order Taylor expansion around $\rho = 1$ allows me to analyze the model for $\rho \neq 1$, and I show in the online Appendix B¹⁸ that the asset pricing predictions of the

¹⁴Empirically, the aggregate consumption has very low volatility and this is a valid approximation.

¹⁵Since all terms decrease with the model’s frequency except for the constant term due to loss aversion, the solution for the pricing of risk highlights the sensitivity to frequency of my discrete time model. Calibrating the model at different frequencies would yield different values for α , which further highlights that the choice of $\alpha = 0.55$, as in Kahneman and Tversky (1979), is mostly illustrative.

¹⁶The equity premium reaches 1.15%, when α is pushed to one, relative to 6.09% in the data, for the 1926-2011 period.

¹⁷This is not an additional restriction due to loss aversion. In the standard recursive utility model also, closed-form solutions only obtain when $\rho = 1$.

¹⁸http://home.uchicago.edu/mandries/lossaversion_appendix.pdf

model are robust to small changes around $\rho = 1$.¹⁹

Write $\log C = c$, $\log V = v$, $\log \bar{V} = \bar{v}$. When $\rho = 1$ and $\xi = 0$, the model of Eq. (5) becomes:

$$\begin{aligned} v_t &= (1 - \beta) c_t + \frac{\beta}{1 - \bar{\gamma}} \log \mathbb{E}_t [\exp(1 - \bar{\gamma}) \bar{v}_{t+1}] \\ \bar{v}_{t+1} &= v_{t+1} - \alpha \max(0, v_{t+1} - \mathbb{E}_t(v_{t+1})) \end{aligned} \quad (13)$$

Because \bar{v} is increasing in v , this recursive problem trivially follows Blackwell conditions, and thus admits a unique solution.

I suppose the optimal consumption follows a log-normal process with time-varying drift, standard to the long-run risk literature:

$$\begin{aligned} \log C_{t+1} - \log C_t &= \mu_c + \phi_c X_t + \Sigma_c W_{t+1} \\ X_{t+1} &= AX_t + \Sigma_X W_{t+1} \end{aligned} \quad (14)$$

where $\{W_t\}$ is a two-dimension vector of shocks, iid $\mathcal{N}(0, I)$, and A is contracting (all eigen values have module strictly less than one): the state variable $\{X_t\}$ has stationary distribution with mean zero.

Proposition 6 *The unique solution for the value function v is:*²⁰

$$v_t - c_t = \mu_v + \phi_v X_t, \quad (15)$$

where

$$\phi_v = \beta \phi_c (I - \beta A)^{-1},$$

and μ_v is a decreasing function of α .

The solution for ϕ_v shows the log-value-to-consumption ratio is pro-cyclical: above average in good times ($\phi_c X_t > 0$) and below average in bad times ($\phi_c X_t < 0$). The dependence on the time varying $\{X_t\}$ is increasing in the persistence of the consumption growth drift, and decreasing

¹⁹There is some debate concerning the value of the elasticity of intertemporal substitution. Both the long-run risk model of Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007, 2009) and the disaster model of Barro et al. (2011) require $EIS \geq 1$ to explain the equity returns. A large number of papers (Hansen and Singleton (1982), Attanasio and Weber (1989), Beaudry and van Wincoop (1996), Vissing-Jorgensen (2002), Attanasio and Vissing-Jorgensen (2003), Mulligan (2004), Gruber (2006), Guvenen (2006), Hansen, Heaton, Lee, and Roussanov (2007), Enggelhardt and Kumar (2008)) argue the data supports $EIS \geq 1$. On the other hand, Hall (1988), Campbell (1999), and more recently Beeler and Campbell (2009) argue for small values of elasticity of intertemporal substitution ($EIS < 1$).

²⁰The details of the calculation are in the online Appendix A, http://home.uchicago.edu/mandries/lossaversion_appendix.pdf.

in the rate of time discount (increasing in β).²¹ The dependence in the state variable $\{X_t\}$ is unchanged from the standard recursive utility model.

The mean value-to-consumption ratio μ_v is increasing in the mean consumption growth μ_c , decreasing in the rate of time discount (increasing in β), decreasing in both the risk aversion γ and the underlying risk in the consumption process given by $|\Sigma_c|$ and $|\Sigma_X|$, and decreasing in α , the loss aversion coefficient. I find loss aversion lowers μ_v below the levels of the standard recursive utility model with either γ , or $\bar{\gamma}$, even though the agent has risk aversion $\gamma \leq \bar{\gamma}$ on the non-disappointing outcomes. The discontinuity in the marginal utility, due to the kink in the preferences, results in agents that are particularly averse to taking small risks around the reference point, and thus display an effective risk aversion that is higher than both γ , the risk aversion above the reference, and $\bar{\gamma}$, the risk aversion below the reference, in the valuation of the consumption stream.

Proposition 7 *The risk-free rate has a closed-form solution $r f_t = r f_t(\alpha)$, which is strictly decreasing in the loss aversion coefficient α .*²²

*As a second-order approximation around $\phi_v \Sigma_X = 0$, and $\Sigma_c = 0$.*²³

$$r f_t \approx -\log \beta + \mu_c + \phi_c X_t - \frac{1}{2} |\Sigma_c|^2 + (1 - \gamma) (\Sigma_c + \phi_v \Sigma_X) \Sigma'_c \quad (16)$$

$$- \alpha \underbrace{\left\{ \frac{1}{\sqrt{2\pi}} \frac{\Sigma_c (\Sigma_c + \phi_v \Sigma_X)'}{|\Sigma_c + \phi_v \Sigma_X|} \left(1 - \frac{1}{2\sqrt{2\pi}} \alpha \frac{\Sigma_c (\Sigma_c + \phi_v \Sigma_X)'}{|\Sigma_c + \phi_v \Sigma_X|} \right) + \frac{1}{2} \alpha \left(1 - \frac{1}{\pi} \right) (\bar{\gamma} - 1) (\Sigma_c + \phi_v \Sigma_X) \Sigma'_c \right\}}_{\text{loss aversion terms}}.$$

The first four terms are those of the standard expected utility model (see Eq. (10)), and the earlier comparative statics obtain. Because of time-varying in the drift of consumption, the risk-free rate is pro-cyclical. It is also decreasing in both the risk aversion γ and the risk of consumption, immediate ($|\Sigma_c|$) and long-term ($|\phi_v \Sigma_X|$), due to the additional precautionary savings term $(1 - \gamma) (\Sigma_c + \phi_v \Sigma_X) \Sigma'_c$ of the standard recursive utility model.

Loss aversion lowers the risk-free rate and amplifies its sensitivity to both the risk aversion and the risk of consumption. Its impact is displayed in Figure 2. Observe the risk-free rate in the model with loss aversion is lower than in the standard recursive utility model, with either risk aversion γ or high risk aversion $\bar{\gamma}$: the discontinuity in the stochastic discount factor results in the

²¹Shocks to $\{X_t\}$ impact next-period consumption the most (with impact $\phi_c \Sigma_X W_t$) and the impact slowly fades over time (with impact $\phi_c A^\tau \Sigma_X W_t$ after τ periods). The cumulative impact on all the future realizations of consumption is immediately reflected in the present value of the future consumption stream, the value function V_t , through the term $\phi_v X_t$ with $\phi_v = \beta \phi_c \sum_0^\infty \beta^i A^i = \beta \phi_c (I - \beta A)^{-1}$.

²²The details of the calculation are in the online Appendix A, http://home.uchicago.edu/mandries/lossaversion_appendix.pdf.

²³Empirically, the aggregate consumption growth is a low volatility process, and this approximation is justified.

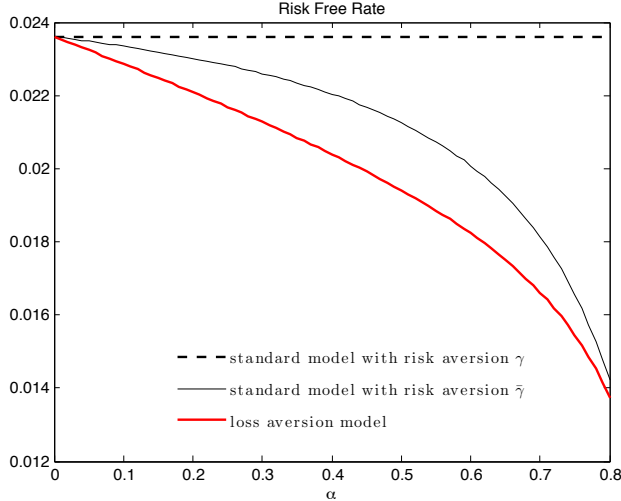


Figure 2: Risk-Free Rate

The annual risk-free rates with and without loss aversion (standard recursive utility model with risk aversions γ and $\bar{\gamma}$) are plotted as functions of the coefficient of loss aversion α . $\bar{\gamma}$ increases with α as in Eq. (3): $\bar{\gamma} = \gamma + \frac{\alpha}{1-\alpha}(\gamma - 1)$. Because the dependence on the state variable $\{X_t\}$ is the same with and without loss aversion, I plot the risk-free rates for $X_t = \mathbb{E}(X_t) = 0$. I use the parameters from Hansen, Heaton, and Li (2008) for the consumption process of Eq. (14) and $\beta = 0.999$, $\gamma = 10$.

only first-order term, $\frac{\alpha}{\sqrt{2\pi}} \frac{\Sigma_c(\Sigma_c + \phi_v \Sigma_X)'}{|\Sigma_c + \phi_v \Sigma_X|}$, in the risk-free rate of Eq. (16), which dominates over the standard precautionary savings term.

The standard recursive utility model tends to overvalue the risk-free rate. As a result, the model with loss aversion improves on the calibration of the risk-free rate, even when compared to the standard recursive utility model with high risk aversion $\bar{\gamma}$.²⁴

In Figure 3, I display the expected excess returns and risk-price elasticities of assets with log-normal returns as in Eq. (9), and exposures Σ_R to the consumption shocks.²⁵ These graphs illustrate the fundamental differences for asset pricing between the model with loss aversion and the standard recursive utility model. As in the expected utility framework, loss aversion has (1) a level effect: the expected excess returns for assets that covary positively with the consumption shocks are increased by the loss aversion specification; and (2) a cross-sectional effect: the risk-price elasticities decrease sharply between small exposures and large exposures (in absolute value) to the consumption shocks.

I also find the risk-price elasticities are higher for negative exposure to the consumption shocks

²⁴As long as $\gamma \leq 25$, using the parameters of Hansen, Heaton, and Li (2008).

²⁵ $\log R_{t+1} = \left(\bar{r}_t - \frac{1}{2} |\Sigma_R|^2 - \frac{1}{2} |\widetilde{\Sigma}_R|^2 \right) + \Sigma_R W_{t+1} + \widetilde{\Sigma}_R \widetilde{W}_{t+1}$, where $\{W_{t+1}\}$ are the shocks to the consumption process, $\{\widetilde{W}_{t+1}\}$ are independent shocks.

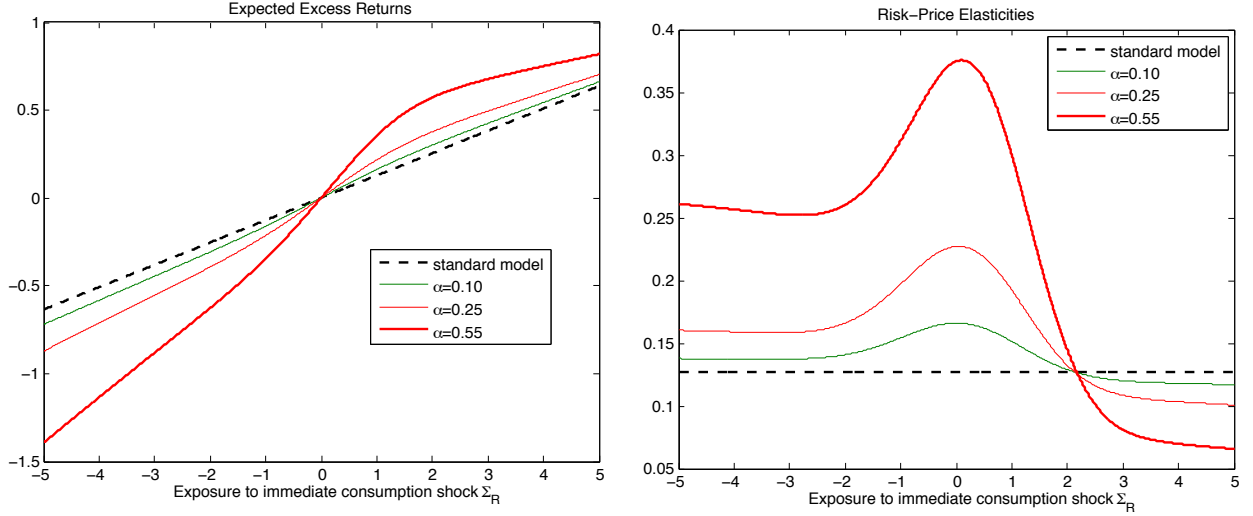


Figure 3: Asset Prices with Constant Volatility

The two graphs display the expected excess returns and the risk-price elasticities for assets with exposure to the immediate consumption shock $[\Sigma_R \ 0] W_{t+1}$ in the model with constant volatility, for various values of α , the coefficient of loss aversion. The case $\alpha = 0$ reverts to the standard recursive utility model. The graphs display the same characteristic shapes for assets that vary in their exposures to the second shock $[0 \ \Sigma_R] W_{t+1}$. I use the parameters from Hansen, Heaton, and Li (2008) for the consumption process of Eq. (14) and $\beta = 0.999$, $\gamma = 10$.

(hedges) than for positive ones. Hedges generate positive returns when the shocks are negative and the agent is disappointed, and are thus mostly priced in a model with high risk aversion $\bar{\gamma} \geq \gamma$. In contrast, assets with positive exposure to the consumption shocks generate positive returns when the agent is not disappointed, and are thus mostly priced in a model with risk aversion γ , thereby resulting in lower risk-price elasticities. This feature would extend to option prices, with higher implied volatilities on the put options than on the call options.

The closed-form solution for the expected returns is not conducive to direct interpretation. To better understand how these effects arise, I therefore analyze the returns behavior at the asymptotes ($|\Sigma_R| \rightarrow +\infty$) and around zero.

Proposition 8 *At the asymptotes:*²⁶

$$\bar{r}_t(\Sigma_R) - rft \approx_{|\Sigma_R| \rightarrow +\infty} (\gamma \Sigma_c + (\gamma - 1) \phi_v \Sigma_X) \Sigma'_R \quad (17)$$

$$- \log \left[\underbrace{\left[\begin{array}{l} \Phi \left(-\frac{(\phi_v \Sigma_X + \Sigma_c) \Sigma'_R}{|(\phi_v \Sigma_X + \Sigma_c)|} \right) \exp(-\alpha (\bar{\gamma} - 1) (\phi_v \Sigma_X + \Sigma_c) \Sigma'_R) \\ + \alpha \Phi(-(\gamma - 1) |\phi_v \Sigma_X + \Sigma_c|) \exp((\gamma - 1) (\phi_v \Sigma_X + \Sigma_c) \Sigma'_R) \end{array} \right]}_{\text{loss aversion term}} \right].$$

²⁶The details of the calculation are in the online Appendix A, http://home.uchicago.edu/mandries/lossaversion_appendix.pdf.

Around zero, as a second-order approximation.²⁷

$$\bar{r}_t(\Sigma_R) - rf_t \approx_{|\Sigma_R| \approx 0} (\gamma \Sigma_c + (\gamma - 1) \phi_v \Sigma_X) \Sigma'_R \quad (18)$$

$$+ \alpha \underbrace{\left[\begin{aligned} & \left(\frac{1}{\sqrt{2\pi}} \frac{\Sigma_R(\phi_v \Sigma_X + \Sigma_c)'}{|\phi_v \Sigma_X + \Sigma_c|} \left(1 - \frac{\alpha}{\sqrt{2\pi}} \frac{\Sigma_c(\phi_v \Sigma_X + \Sigma_c)'}{|\phi_v \Sigma_X + \Sigma_c|} \right) \right. \\ & \left. + \frac{1}{2} \alpha (\bar{\gamma} - 1) \left(1 - \frac{1}{\pi} \right) \Sigma_R(\phi_v \Sigma_X + \Sigma_c)' + \frac{1}{4\pi} \alpha \left(\frac{\Sigma_R(\phi_v \Sigma_X + \Sigma_c)'}{|\phi_v \Sigma_X + \Sigma_c|} \right)^2 \right]}_{\text{loss aversion terms}} \end{aligned} \right].$$

In both Eq. (17) and Eq. (18), the first term corresponds to the standard recursive utility model, which yields a linear relation between returns and risk, and thus a constant pricing of risk, equal to $\gamma \Sigma_c + (\gamma - 1) \phi_v \Sigma_X$, and therefore increasing in the coefficient of risk aversion γ , in the level of risk (given by $|\Sigma_c|$ and $|\Sigma_X|$), in the persistence of the consumption process, and in β .²⁸

The extra terms due to loss aversion introduce important non-linearities in the relation between the log expected returns and the log exposure to consumption shocks, and thus variations in the pricing of risk.

Below the reference point, the agent behaves as in the standard model with risk aversion $\bar{\gamma}$ and, accordingly, I find $\bar{r}_t(\Sigma_R) - rf_t \sim (\bar{\gamma} \Sigma_c + (\bar{\gamma} - 1) \phi_v \Sigma_X) \Sigma'_R$ when $(\phi_v \Sigma_X + \Sigma_c) \Sigma'_R \rightarrow -\infty$. Far and above the reference point, I find the direct contribution to the value function of the reference point dominates.²⁹ The log-utility reference point model yields $\bar{r}_t(\Sigma_R) - rf_t \sim \Sigma_c \Sigma'_R$ when $(\phi_v \Sigma_X + \Sigma_c) \Sigma'_R \rightarrow +\infty$.³⁰ Notice, on either asymptotes, the kink in the preferences due to loss aversion has no direct impact on the pricing of risk.

In contrast, for $\Sigma_R \approx 0$ in Eq. (18), the clearly dominating constant term $\frac{\alpha}{\sqrt{2\pi}} \frac{\Sigma_R(\phi_v \Sigma_X + \Sigma_c)'}{|\phi_v \Sigma_X + \Sigma_c|}$ reflects the “first-order risk aversion” characteristic of preferences with kinks. Notice this term does not depend on the risk aversion γ nor on the volatility $|\phi_v \Sigma_X + \Sigma_c|$. As in the expected utility model, loss aversion has a large, first-order, impact on the expected returns of assets that carry small risks, particularly when the risk aversion and the consumption risk are low, thus resulting in the hump shape of Figure 3: the risk-price elasticities for small exposures to the consumption shocks are above both asymptotes.³¹ In particular, they are above the risk-price elasticities of the

²⁷I am taking $|\Sigma_c|$ as approximately zero, with same order of magnitude as $|\Sigma_R|$. Empirically, the aggregate consumption has very low volatility and this is a valid approximation.

²⁸The first term, $\gamma \Sigma_c$, is identical to the expected utility CRRA model with risk aversion γ . The additional term, $(\gamma - 1) \phi_v \Sigma_X$, comes from the recursive specification, and reflects the pricing of the long-run consumption shocks.

²⁹Above the reference point, the agent behaves as in the standard model with risk aversion γ , with a scaling factor that depends on the the reference point.

³⁰Choosing another reference point model has a direct impact on the right-hand asymptote, as I show in on-line Appendix C, http://home.uchicago.edu/mandries/lossaversion_appendix.pdf. However, this barely affects the range of empirically reasonable assets.

³¹Using the parameters of Hansen, Heaton, and Li (2008) for the aggregate consumption, the hump-shape persists for risk aversion coefficients up to $\gamma = 25$.

standard recursive utility model with risk aversion $\bar{\gamma}$.

The pricing of risk in this model has striking empirical implications as I show in section (3), but it is constant in time, as can be seen in Eq. (17) and Eq. (18). I analyze models with dynamic pricing of risk in the next section.³²

2.3 Dynamic Risk Pricing with Loss Aversion

In section 2.3.1, I replicate the analysis of the recursive utility model with loss aversion and full updating of the reference point, but with time varying volatility in the consumption process. In section 2.3.2, I present tractable solutions for the model with “sticky” updating, $\xi > 0$.

2.3.1 Risk Prices with Stochastic Volatility

As before, I suppose the representative agent has preferences with recursive utility and loss aversion, with full updating of the reference point ($\xi = 0$) and unit elasticity of intertemporal substitution ($\rho = 1$), as in Eq. (13). This time, however, I let both the drift and the volatility of the optimal consumption process be time varying:

$$\begin{aligned} \log C_{t+1} - \log C_t &= \mu_c + \phi_c X_t + \sigma_t \Sigma_c W_{t+1} \\ X_{t+1} &= AX_t + \sigma_t \Sigma_X W_{t+1} \\ \sigma_{t+1} &= (1 - a) + a\sigma_t + \Sigma_\sigma W_{t+1} , \end{aligned} \tag{19}$$

where $\{W_t\}$ is a three-dimension vector of shocks, iid $\mathcal{N}(0, I)$, and Eq. (19) is the stochastic volatility equivalent of Eq. (14). Both the immediate consumption shocks $\{\sigma_t \Sigma_c W_{t+1}\}$, and the long-run consumption shocks $\{\sigma_t \Sigma_X W_{t+1}\}$, now have time varying volatility, which is affected by the iid shocks $\{\Sigma_\sigma W_{t+1}\}$. To simplify the model, volatility shocks are modeled as independent from expected consumption shocks: $\Sigma_\sigma \Sigma_X' = \Sigma_\sigma \Sigma_c' = 0$. A and a are contracting (all eigen values have module strictly less than one): both state variables have stationary distributions, with mean zero for $\{X_t\}$ and mean one for the scalar $\{\sigma_t\}$.

Proposition 9 *When the consumption process is smooth (Σ_c , Σ_X and Σ_σ close to zero), as we observe in the data, the unique solution for the value function v has closed-form approximation.³³*

$$v_t - c_t \approx \mu_v + \phi_v X_t + \phi_{v,\sigma} \sigma_t + \phi_{v,\sigma^2} \sigma_t^2 . \tag{20}$$

³²The need for asset pricing models with a counter-cyclical price of risk is illustrated in Melino and Yang (2003). In this paper, the authors show that in a two-state economy, the empirical pricing kernel that matches asset prices displays a higher price of risk in the bad state.

³³See the online Appendix D, http://home.uchicago.edu/mandries/lossaversion_appendix.pdf.

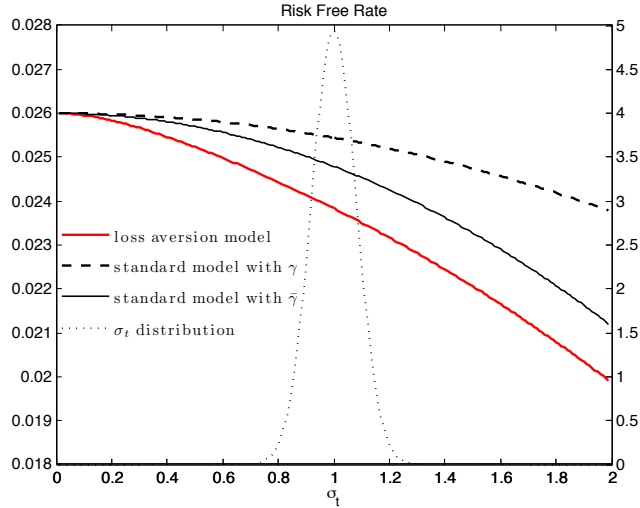


Figure 4: Risk-Free Rate

The annual risk-free rate with and without loss aversion (standard recursive utility model for risk aversion γ and $\bar{\gamma}$) are plotted on the left axis. $\bar{\gamma}$ increases with α as in Eq. (3): $\bar{\gamma} = \gamma + \frac{\alpha}{1-\alpha}(\gamma - 1)$. Because the dependence on the state variable $\{X_t\}$ is the same with and without loss aversion, I plot the risk-free rates for $X_t = \mathbb{E}(X_t) = 0$. On the right axis, I plot the distribution of σ_t . I use the parameters from Hansen, Lee, Polson, and Yae (2011) for the consumption process of Eq. (19) and $\beta = 0.999$, $\gamma = 10$, $\alpha = 0.55$.

where

$$\phi_v = \beta \phi_c (I - \beta A)^{-1},$$

and μ_v , $\phi_{v,\sigma}$ and ϕ_{v,σ^2} are functions of α .

As in the constant volatility case, I find μ_v is a decreasing function of α , and the value function is lower than in the standard recursive utility model, with either risk aversion γ or $\bar{\gamma}$. The value function varies with both the drift of consumption, as before, and with the volatility of consumption. I find $|\phi_{v,\sigma}|$ and $|\phi_{v,\sigma^2}|$ are increasing (i) in β ; (ii) in the persistence of the volatility process a ; (iii) in the risk aversion coefficient γ ; and (iv) in the volatility of the consumption process given by $|\Sigma_c|$, $|\Sigma_X|$, and $|\Sigma_\sigma|$. Further, I find the impact of changes in volatility is stronger, and thus the pro-cyclical variations in the value function are greater, in the model with loss aversion than in the standard recursive utility model.

This result extends to the risk-free rate, as can be observed in Figure 4: loss aversion results in a risk-free rate that is more strongly pro-cyclical, and below the levels of the standard recursive utility model with either γ or $\bar{\gamma}$.

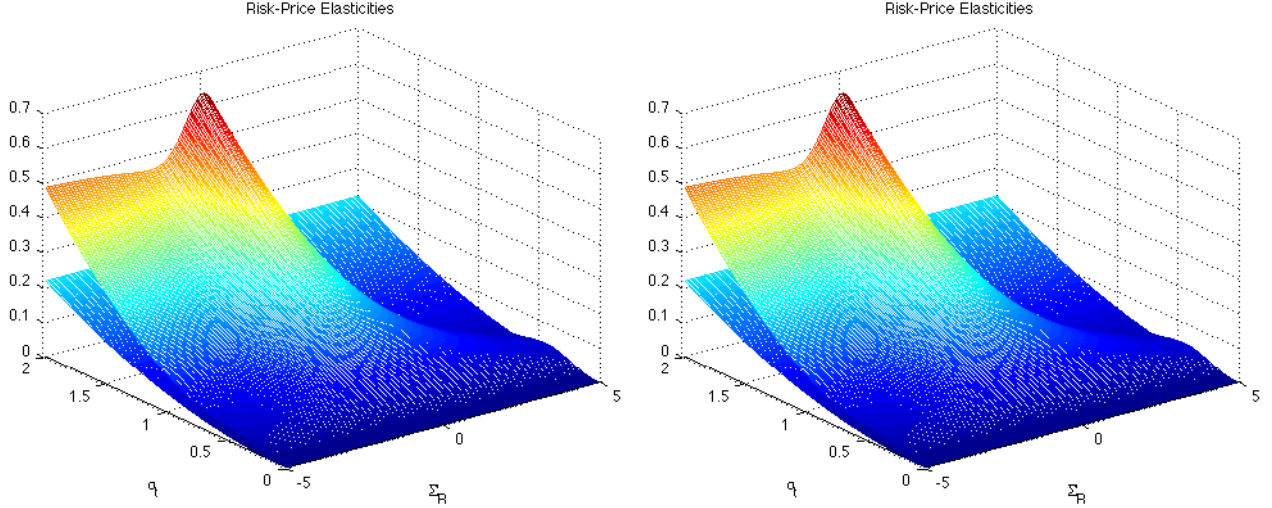


Figure 5: Asset Prices with Time-varying Volatility- Immediate Consumption Shock

The two graphs display the risk-price elasticities for an exposure $[\sigma_t \Sigma_R \ 0 \ 0] W_{t+1}$, for the loss aversion model with loss aversion $\alpha = 0.55$ and the standard recursive utility model with risk aversion γ (the plane in the first graph and the lower dotted line in the second graph) and $\bar{\gamma}$ (the higher dotted line in the second graph). $\bar{\gamma}$ increases with α as in Eq. (3): $\bar{\gamma} = \gamma + \frac{\alpha}{1-\alpha}(\gamma - 1)$. The second graph displays the three cases, $\sigma_t \approx 0$, $\sigma_t = 1$ (mean value), and $\sigma_t = 2$. The graphs display the same characteristic shapes for assets that vary in their exposures to the second consumption shock, $[0 \ \sigma_t \Sigma_R \ 0] W_{t+1}$. I use the parameters from Hansen, Lee, Polson, and Yae (2011) for the consumption process of Eq. (19) and $\beta = 0.999$, $\gamma = 10$, $\alpha = 0.55$.

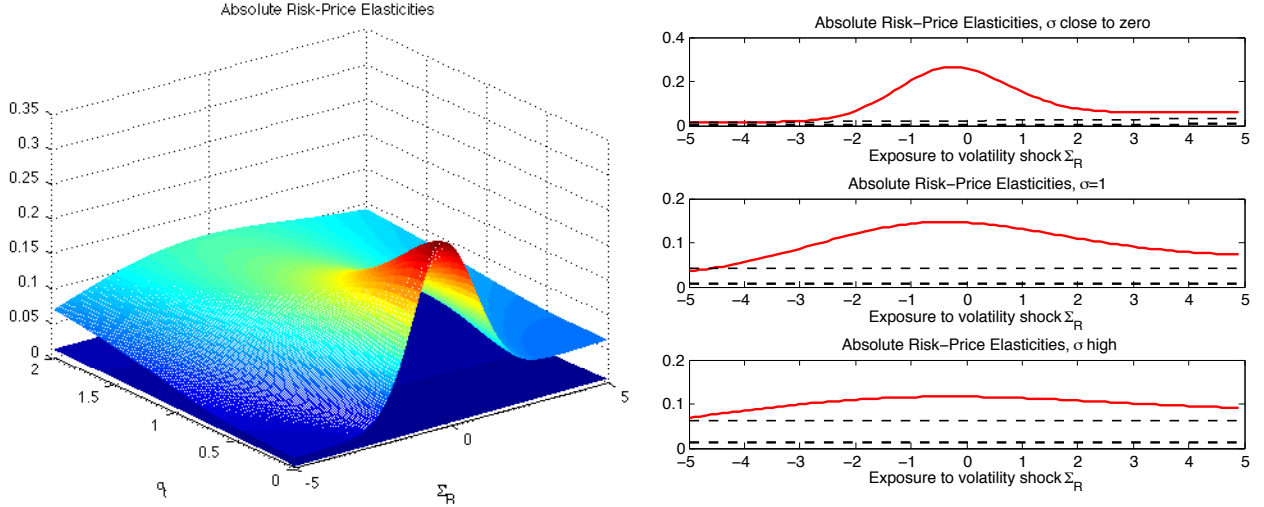


Figure 6: Asset Prices with Time-varying Volatility- Volatility Shock

The two graphs display the absolute value of the risk-price elasticities for an exposure $[0 \ 0 \ \Sigma_R] W_{t+1}$ (or $-RP_t([0 \ 0 \ \Sigma_R])$), for the loss aversion model with loss aversion $\alpha = 0.55$ and the standard recursive utility model with risk aversion γ (the plane in the 1st graph and the lower dotted line in the second graph) and $\bar{\gamma}$ (the higher dotted line in the 2d graph). $\bar{\gamma}$ increases with α as in Eq. (3): $\bar{\gamma} = \gamma + \frac{\alpha}{1-\alpha}(\gamma - 1)$. The second graph displays the three cases, $\sigma_t \approx 0$, $\sigma_t = 1$ (mean value), and $\sigma_t = 2$. I use the parameters from Hansen, Lee, Polson, and Yae (2011) for the consumption process of Eq. (19) and $\beta = 0.999$, $\gamma = 10$, $\alpha = 0.55$.

In Figure 5 and Figure 6, I display the risk-price elasticities of assets with conditionally log-normal returns and exposures $\Sigma_{R,t}$ to the consumption shocks.³⁴ The results derived in the constant volatility model extend to the model with stochastic volatility: the risk-price elasticities display asymmetrical bell shapes and both a level effect and a cross-sectional effect obtain.³⁵

Compared to the standard recursive utility model, the pricing dynamics are impacted by two channels of influence. The first derives from the amplification in the value function's variations and generates additional counter-cyclical in the pricing of risk. The second derives from the dampening effect of higher consumption volatilities on the first-order impact of loss aversion (assets are not priced as close to the kink when the underlying volatility is high), and generates less counter-cyclical in the pricing of risk. I find the first channel dominates for the pricing of assets that vary in their exposures to the consumption shocks, as in Figure 5: for small exposures to the consumption shocks, the pricing of risk increases with σ_t at a faster rate, and is thus more strongly counter-cyclical in the model with loss aversion than in the standard recursive utility model. In contrast, I find the second channel can dominate for assets that vary in their exposures to the volatility shocks, as in Figure 6, depending on the risk aversion and loss aversion coefficients. For $\gamma \leq 20$ and $\alpha = 0.55$, my loss aversion model results in a pricing of risk for the volatility shocks that is pro-cyclical, for assets with small exposures to the shocks, and counter-cyclical for assets with large exposures to the shocks. These striking predictions of my model can be contrasted with those of the standard recursive utility model, in which the risk-price elasticities are counter-cyclical for all the shocks. Exploring their empirical application is left for future research.

2.3.2 History Dependence in the Updating of the Reference Point

I now consider the model with history dependence in the updating of the reference point. I restrict the analysis to the constant consumption volatility case of Eq. (14), which allows me to disentangle the time variations due to the history dependence in the reference point from those due to time-varying consumption volatilities. I solve the model with unit intertemporal elasticity

³⁴I model the exposure $\{\Sigma_{R,t}\}$ as $\Sigma_{R,t} = \Sigma_R \begin{pmatrix} \sigma_t & 0 \\ 0 & \sigma_t \\ 0 & 1 \end{pmatrix}$, such that the aggregate volatility of the asset returns has same time dependence as the consumption risk.

³⁵The risk-price elasticities are negative and the asymmetry is reversed, with a higher right-hand side asymptote, for the volatility shocks, which impact the value function negatively.

of substitution ($EIS = 1$), and time dependence on the past two periods ($T = 1$):

$$\begin{aligned}
v_t &= (1 - \beta) c_t + \frac{\beta}{1 - \bar{\gamma}} \log \mathbb{E}_t [\exp(1 - \bar{\gamma}) \bar{v}_{t+1}] \\
\bar{v}_{t+1} &= v_{t+1} - \alpha \max\left(0, v_{t+1} - \tilde{\mathbb{E}}_{\xi,t}(v_{t+1})\right) \\
\tilde{\mathbb{E}}_{\xi,t}(v_{t+1}) &= \frac{\mathbb{E}_t(v_{t+1}) + \xi \mathbb{E}_{t-1}(v_{t+1})}{1 + \xi}.
\end{aligned} \tag{21}$$

Proposition 10 *The unique solution for the value function v is:*³⁶

$$v_t - c_t = \mu_{v,s} + \phi_v X_t + f\left(\frac{\xi}{1 + \xi} \Sigma_s W_t\right), \tag{22}$$

where ϕ_v and Σ_s are independent from α and ξ , in contrast to $\mu_{v,s}$ and f , and f has the following properties:

- 1) f is decreasing
- 2) $f(0) = 0$
- 3) f converges to a constant in $-\infty$
- 4) $f(x) \sim_{+\infty} -\alpha\beta x$.

W_t , the shocks to consumption between $t - 1$ and t , affect the time t value function through two channels, one increasing, $\phi_v X_t$, and one decreasing, $f\left(\frac{\xi}{1 + \xi} \Sigma_s W_t\right)$. I find the first channel dominates (the value function increases following positive shocks to consumption), and Eq. (22) can be rewritten as $v_t - c_{t-1} = g(X_{t-1}, W_t)$, with g increasing in both X_{t-1} and W_t .

The pricing of risk is time varying and, in Figure 7, I display the risk-free rate and the expected excess returns of a risky asset, as they vary with the past consumption shocks.

Like the value function, the risk-free rate can be written as $g_{r_f}(X_{t-1}, W_t)$, with g_{r_f} increasing in both terms. When $X_t = 0$, strictly positive and strictly negative shocks to consumption ($W_t \neq 0$) both decrease the probability of being close to the reference point, and thus diminish the impact of loss aversion. For this reason, I find the risk-free rate is at a local minimum when $W_t = 0$.

Following negative shocks to consumption, the probability of disappointment increases. Following positive shocks to consumption, the probability of disappointment decreases. The model with history dependence in the reference point thus yields mostly counter-cyclical expected excess returns. Non-zero shocks, however, decrease the probability of being close to the reference point, and thus diminish the impact of loss aversion. I find this effect dominates over the increase in the probability of disappointment following unusually large and negative shocks (more than two

³⁶The details of the calculation are in the online Appendix E, http://home.uchicago.edu/mandries/lossaversion_appendix.pdf.

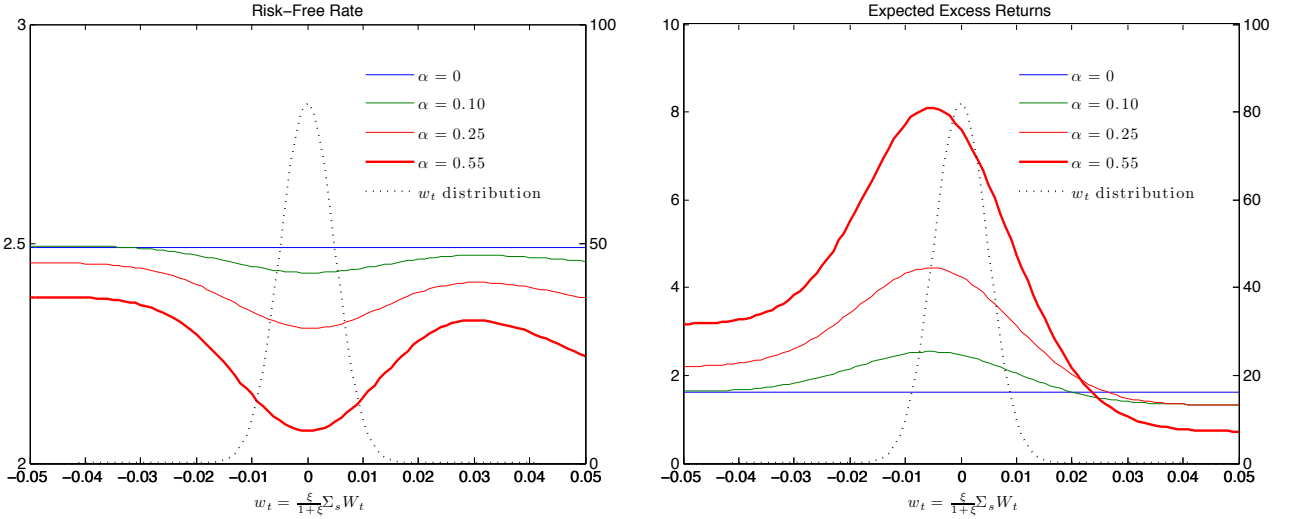


Figure 7: Asset Prices: History-dependent Reference Point

The two graphs display the annual (in %) risk-free rate and the expected excess returns of an asset with same characteristics (volatility and skewness) as the market portfolio, for various values of α , the coefficient of loss aversion (the case $\alpha = 0$ reverts to the standard recursive utility model), as functions of the past shock $w_t = \frac{\xi}{1+\xi} \sum_s W_t$. For the risk-free rate, I assume $X_t = \mathbb{E}(X_t) = 0$. I use the parameters from Hansen, Heaton, and Li (2008) for the consumption process of Eq. (14) and $\beta = 0.999$, $\gamma = 5$, $\xi = 0.5$. Other values of ξ yield similar asset pricing results.

standard deviations below zero), thus resulting in lower expected excess returns.³⁷ The analysis of empirical evidence regarding such effects, which differ from those of other models with history dependent preferences, such as the habit formation model, is left for future research.³⁸

In both dynamics models, with stochastic volatility or with “sticky” updating of the reference point, loss aversion has non-trivial implications for the pricing of risk. The counter-cyclical influence on pricing is occasionally reversed when assets are priced further and further away from the reference point. This feature is unique to the model with loss aversion and would not obtain in either the standard recursive utility model or in habit formation-type models.

3 Empirics

In this section, I bring my model to the data and find strong support for the recursive utility model with loss aversion. In section 3.1, I evaluate the risk-free rate, the value premium, and the equity premium and I find my model improves on the calibration of the standard recursive utility

³⁷For the parameters used in Figure 7

³⁸The empirical analysis of this model is complicated by the fact that large negative shocks to consumption are usually associated with changes in volatility.

model with both risk aversion γ and high risk aversion $\bar{\gamma}$. In sections 3.2 and 3.3, I show how the cross-sectional effect of the model with loss aversion can offer a novel theoretical justification for two important results of the empirical finance literature: the negative premium for skewness and the security market line (the excess returns as a function of beta, the exposure to market risk) flatter than the CAPM.³⁹

3.1 Asset Returns

To quantitatively analyze the asset returns in the model with loss aversion, I use the results of Hansen, Heaton, and Li (2008) for the consumption process of Eq. (14) and the recursive utility model with unit elasticity of intertemporal substitution ($\rho = 1$). The state variable $\{X_t\}$ is explicitly determined by consumption and earnings, which are assumed cointegrated.⁴⁰ The loadings on the shocks $\{W_t\}$ of any dividend process are obtained directly from the macro data, and are not influenced by the modeling choice for preferences. They can therefore be used to contrast the implications for asset returns of the standard recursive utility model and those of the model with loss aversion.⁴¹

In Table 1, observe loss aversion improves on the calibration of the risk-free rate, all the more so if the coefficient of risk aversion remains reasonably low.⁴² To match the historical level on the risk-free rate, the model with loss aversion still requires a low rate of time discount (β close to one). In the rest of the calibrations, I use $\beta = (0.999)^{\frac{1}{4}}$, which generates reasonable levels for the risk-free rate.

In Table 2, and Table 3, I display the long-run value premium, and equity premium, for the standard recursive utility model with risk aversion γ and $\bar{\gamma}$, and for the model with loss aversion.⁴³ The model with loss aversion $\alpha = 0.55$ can explain the value premium for a risk aversion coefficient

³⁹The empirical methods are described in online Appendix ??, http://home.uchicago.edu/mandries/lossaversion_appendix.pdf.

⁴⁰Both variables have quarterly time series (1947 to 2010) taken from the National Income and Product Accounts (NIPA). Consumption is the seasonally adjusted aggregate consumption of non-durables and services. Corporate earnings are converted to real terms using the implicit price deflator for non-durables and services.

⁴¹In contrast, in Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007, 2009), the state variable $\{X_t\}$ is a hidden variable and its evolution, as well as the loadings of the asset returns on the shocks, are chosen to match moments on both consumption and asset returns. The calibration is thus partly tailored to fit the standard recursive utility model.

⁴²Risk aversion must be pushed all the way up to $\gamma = 25$ for the model with loss aversion $\alpha = 0.55$ to yield a risk-free rate that is no longer lower than in the standard model with high risk aversion $\bar{\gamma}$ (but still improves on the standard model with risk aversion γ).

⁴³The value premium is calculated as the difference in long-run returns between the portfolio with the highest book-to-market ratio (value portfolio) and the portfolio with the lowest book-to-market ratio (growth portfolio) using five portfolios sorted on book-to-market ratios as in Fama and French (1992). As documented in Bansal, Dittmar, and Lundblad (2005) as well as in Hansen, Heaton, and Li (2008), value stocks have a higher covariance with long-run consumption than growth stocks, thus justifying the higher returns they yield.

	model with loss aversion			standard model	
	$\alpha = 0.10$	$\alpha = 0.25$	$\alpha = 0.55$	risk aversion γ	risk aversion $\bar{\gamma} _{\alpha=0.55}$
	$\gamma = 5$				
$\beta = (0.99)^{\frac{1}{4}}$	3.02%	2.91%	2.67%	3.10%	2.97%
$\beta = (0.999)^{\frac{1}{4}}$	2.12%	2.01%	1.77%	2.19%	2.07%
	$\gamma = 15$				
$\beta = (0.99)^{\frac{1}{4}}$	2.76%	2.63%	2.29%	2.84%	2.39%
$\beta = (0.999)^{\frac{1}{4}}$	1.85%	1.72%	1.38%	1.93%	1.49%
Risk free rate in the data (1947-2010) = 1.14%					
(From CRSP 30-day-Treasury-bill returns)					

Table 1: Risk-Free Rate

	model with loss aversion			standard model	
	$\alpha = 0.10$	$\alpha = 0.25$	$\alpha = 0.55$	risk aversion γ	risk aversion $\bar{\gamma} _{\alpha=0.55}$
$\gamma = 3$	1.45%	2.68%	5.20%	0.65%	1.37%
$\gamma = 5$	2.04%	3.29%	5.98%	1.23%	2.66%
$\gamma = 10$	3.53%	4.85%	8.05%	2.70%	5.92%
Value Premium in the data (1947-2010) = 4.22%					
(From the five Fama-French portfolios sorted on book-to-market in Kenneth French's website)					

Table 2: Value Premium

of $\gamma = 3$ (and $\bar{\gamma} = 5.5$), compared to $\gamma = 15$ in the standard model. Loss aversion also improves on the calibration of the equity premium even when compared to the standard recursive utility model with high risk aversion $\bar{\gamma}$.⁴⁴ The model with loss aversion $\alpha = 0.55$ explains 35% of the historical equity premium when $\gamma = 10$. For this level of risk aversion, the standard recursive utility model explains only 10% of the equity premium, and the standard model with high risk aversion $\bar{\gamma}$ explains 25% of the equity premium.

The covariation between the market returns and the shocks to aggregate consumption, both immediate and long-term, is too low in the data to generate the equity premium at reasonable levels of risk aversion, even with loss aversion. Increasing the frequency of the consumption process would increase the implied equity premium values, both through the loss aversion specification,

⁴⁴The relative impact of loss aversion is all the more salient if the coefficient of risk aversion remains reasonably low ($\gamma \leq 20$). For $\gamma \geq 25$, the model with loss aversion $\alpha = 0.55$ no longer improves on the standard model with high risk aversion $\bar{\gamma}$ (but still improves on the standard model with risk aversion γ).

	model with loss aversion			standard model	
	$\alpha = 0.10$	$\alpha = 0.25$	$\alpha = 0.55$	risk aversion γ	risk aversion $\tilde{\gamma} _{\alpha=0.55}$
$\gamma = 10$	0.94%	1.29%	2.14%	0.72%	1.57%
$\gamma = 15$	1.34%	1.72%	2.74%	1.11%	2.44%
$\gamma = 20$	1.74%	2.16%	3.39%	1.50%	3.31%
Equity Premium in the data (1947-2010) = 6.09%					
(From CRSP value-weighted portfolio returns minus CRSP 30-day-Treasury-bill returns)					

Table 3: Equity Premium

as noted in Benartzi and Thaler (1995), and through the persistence of the consumption drift, as in Bansal, Kiku, and Yaron (2009). Because the relevant macro-data is available for quarterly frequency, I limit the analysis to the empirical set up of Hansen, Heaton, and Li (2008), while keeping in mind that higher frequencies would improve the empirical fit of my model. Adapting the empirical set-up to stockholders' consumption, as in Malloy et al. (2009), would also improve on the calibration of the equity premium.

This calibration exercise does not allow one to separate a model with loss aversion α and risk aversion γ from a model with standard recursive utility and risk aversion $\tilde{\gamma} > \bar{\gamma} = \gamma + \frac{\alpha}{1-\alpha} (\gamma - 1)$. In the next sections, I turn to the truly differentiating feature of my model, the cross-sectional effect of loss aversion.

3.2 Negative Premium for Skewness

I show loss aversion offers a novel theoretic justification for the negative premium for skewness that obtains in the data.⁴⁵ In my consumption-based asset pricing framework, the assets' skewnesses are the ones implied by the loadings on the aggregate shocks (co-skewnesses).

Consider a cross-section of assets with log-normal returns $\{R_{i,t+1}\}$ as in Eq. (9), and positive loadings $\{\Sigma_{R,i}\}$ on the aggregate shocks to consumption, ordered such that $|\Sigma_{R,1}| \leq |\Sigma_{R,2}| \leq \dots \leq |\Sigma_{R,N}|$. The same ordering applies for the aggregate volatilities, $\sigma_1 \leq \sigma_2 \leq \dots \leq \sigma_N$, and co-skewnesses, $s_1 \leq s_2 \leq \dots \leq s_N$. To isolate the impact of skewness on the expected returns, I wish to compare assets with same aggregate volatility. Choose a reference asset i_0 . In the cross-section of assets I consider, assets $\{i \leq i_0\}$ have lower, and assets $\{i \geq i_0\}$ have higher, aggregate volatilities $\{\sigma_i\}$ and skewnesses $\{s_i\}$ than the reference asset. I lever-up assets $\{i \leq i_0\}$, and lever-

⁴⁵For same volatility, assets with lower skewness in the returns distribution yield higher expected returns than assets with higher skewness in the returns distribution.

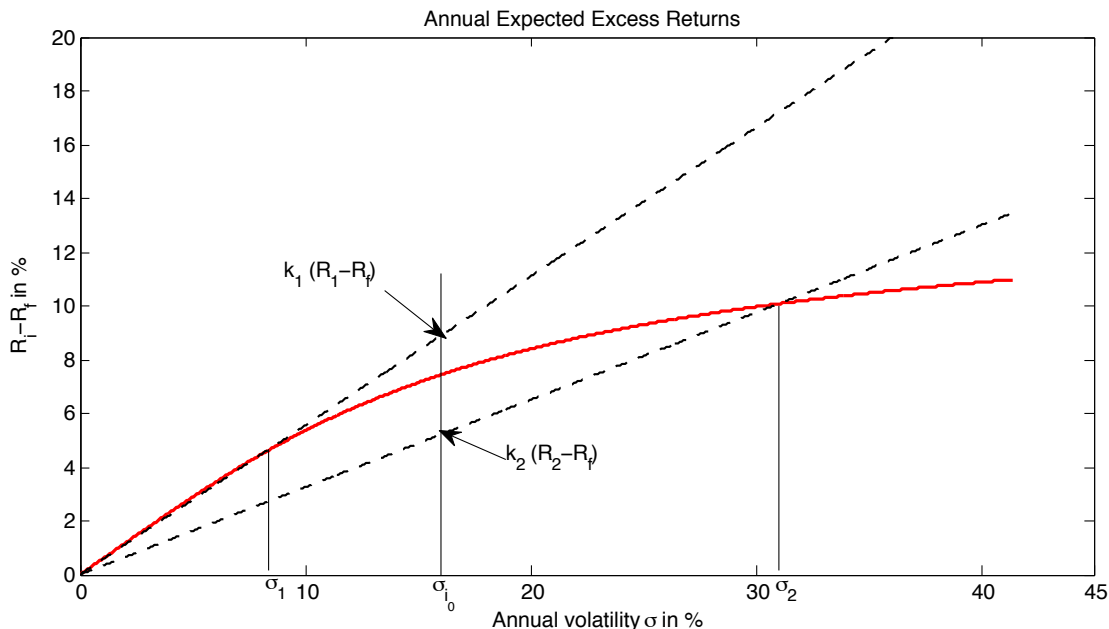


Figure 8: Leveraging on the Cross-Section of Assets

down assets $\{i \geq i_0\}$, so as to obtain a cross-section of assets with the same aggregate volatility σ_{i_0} , but expected excess returns $\{k_i(R_i - R_f)\}$ and aggregate skewnesses $\{s_i\}$ that vary in the cross-section.⁴⁶

In Figure 8, I display the expected excess returns $\{R_i - R_f\}$ as they vary with the aggregate volatilities $\{\sigma_i\}$, and I illustrate the leverage methodology described above, in the loss aversion model with $\alpha = 0.55$.⁴⁷ I lever-up asset 1 with $(\sigma_1 = 8\%) < (\sigma_{i_0} = 16\%)$ (and $k_1 = 2$), and lever-down asset 2 with $(\sigma_2 = 32\%) > (\sigma_{i_0} = 16\%)$ (and $k_2 = 0.5$). The returns $k_1(R_{1,t+1} - R_{f,t})$, $k_2(R_{2,t+1} - R_{f,t})$, and $R_{i_0,t+1} - R_{f,t}$ have same aggregate volatility but different co-skewnesses $s_1 < s_{i_0} < s_2$. Because of the concave relation between returns and risk in the model with loss aversion, I obtain $k_1(R_1 - R_f) > R_{i_0} - R_f > k_2(R_2 - R_f)$, and thus a negative premium for skewness. With a reference asset similar to the market portfolio (same volatility and skewness), the model results in a negative premium for skewness equal to 17% of the equity premium.⁴⁸

In a vastly different portfolio choice model, with the cumulative prospect theory of Tversky-

⁴⁶When leveraging asset i by a factor k_i , I modify its aggregate volatility and expected excess returns linearly to $k_i\sigma_i$ and $k_i(R_i - R_f)$, while keeping its aggregate skewness s_i unchanged.

⁴⁷For a cross-section of assets with loadings $[\Sigma_i \ 0]$, with Σ_i positive and increasing in i . I use the quarterly parameters from Hansen, Heaton, and Li (2008) with $\gamma = 5$, $\beta = (0.999)^{\frac{1}{4}}$.

⁴⁸Harvey and Siddique (2000) find a higher measure for the negative premium for skewness, but they use a different measure for co-skewness, and a shorter time series.

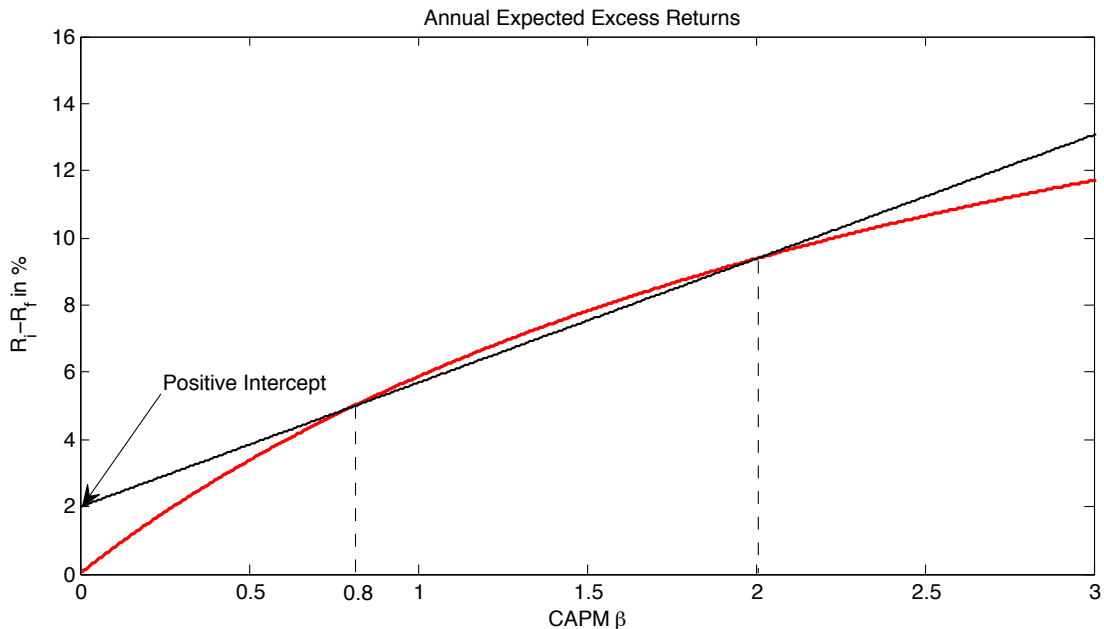


Figure 9: Prediction for the fit of the CAPM

Kahneman (1992), Barberis and Huang (2008) also point out the relation between loss aversion and the pricing of skewness in asset returns. Other papers (see Harvey and Siddique (2000)) directly modify the stochastic discount factor so as to generate the negative premium for skewness. In contrast, my consumption-based asset pricing model with loss aversion stems from preferences justified by the micro and experimental literature.

3.3 Prediction for CAPM Alphas

Black, Jensen, and Scholes (1972) point out the security market line (the excess returns as a function of beta, the exposure to market risk) for U.S. stocks is too flat relative to the CAPM model, and has a non-zero intercept of about 1.5% annually. Frazzini and Pedersen (2010) replicate this empirical result for a wider class of assets (U.S. equities, 20 global equity markets, Treasury bonds, corporate bonds, and futures), accounting for value, size, momentum, and liquidity risk.

To conduct a meaningful analysis of the CAPM, in my consumption-based asset pricing framework, I assume the shocks to the market returns are perfectly correlated with the immediate consumption shocks.⁴⁹ I consider a cross-section of assets ordered by their positive loadings on the consumption shocks ($0 \leq \Sigma_{R,1} \leq \Sigma_{R,2} \leq \dots \leq \Sigma_{R,N}$), as in section 3.2. The same ordering

⁴⁹The model with $\rho = 1$ yields constant wealth-to-consumption ratios. This is therefore equivalent to supposing the returns on wealth are perfectly correlated with the returns of the market portfolio.

	Small β Portfolio	Market Portfolio	High β Portfolio
Expected Excess Returns	7.50%	9.05%	15.34%
CAPM β	0.8	1	2
CAPM α	0.32%	0	-2.77%
Skewness	1.4	1.8	4.2
CAPM Intercept = 2.3% (= 25% of Equity premium)			
CAPM Intercept in Frazzini and Pedersen (2010) = 2.7% (= 45% of Equity premium)			

Table 4: CAPM intercept

Value-Weighted Portfolios Sorted on CAPM β										
	Low β	2	3	4	5	6	7	8	9	High β
β	0.75	0.86	0.97	1.1	1.2	1.3	1.4	1.5	1.6	1.8
Skewness	3.7	2.5	3.2	4.2	4.5	5.4	4.7	4.9	4.8	5.6

Table 5: Quarterly Skewness Results

applies for the aggregate volatilities, $\sigma_1 \leq \sigma_2 \leq \dots \leq \sigma_N$, and CAPM betas, $\beta_1 \leq \beta_2 \leq \dots \leq \beta_N$. For this cross-section of assets, the expected excess returns increase with the CAPM betas in a non-linear way, in my model with loss aversion, as illustrated in Figure 9 for $\alpha = 0.55$. I find a strictly positive intercept when fitting a line between the returns of $\beta_i = 0.8$ and the returns of $\beta_i = 2$, and in Table 4, I provide quantitative measures for the fit of the CAPM.⁵⁰ The loss aversion model can explain more than half of the positive intercept found in Frazzini and Pedersen (2010).

The results of Table 4 crucially depend on the fact that, in the theoretical cross-section of assets I consider, the CAPM betas and the co-skewnesses of the returns distributions increase simultaneously, with the loadings on the aggregate shock. I find, using quarterly returns on all U.S. equities in CRSP (1926-2009) in Table (5), the 10 value-weighted portfolios sorted on CAPM betas yield a correlation between CAPM beta and skewness of 82%, which validates the choice of the log-normal model of returns of Eq. (9) in my cross-sectional analysis.⁵¹

The model with loss aversion justifies, qualitatively and quantitatively, a security market line

⁵⁰Using quarterly returns on all U.S. equities in CRSP (1926-2009), 10 value-weighted portfolios sorted on their CAPM betas have CAPM betas between 0.75 and 1.8. I use the quarterly parameters from Hansen, Heaton, and Li (2008) with $\gamma = 5$, $\beta = (0.999)^{\frac{1}{4}}$.

⁵¹The are measures of skewness and not aggregate skewness however.

flatter than the CAPM. Other models in the literature rely on external constraints and heterogeneous agents to obtain the desired results on the security market line (e.g., Black (1972, 1992); Brennan (1971); Frazzini and Pedersen (2010), Hong and Sraer (2012)). My model offers a novel justification for this central issue in financial economics.

Conclusion

In this paper, I incorporate loss aversion features in a recursive model of preferences and find tractable solutions to the consumption-based asset pricing model with homogeneous agents. The model with loss aversion generates risk-price elasticities that vary with the exposure to the consumption shocks (cross-sectional effect) and that are generally higher than in the standard recursive utility model (level effect). The level effect my model with loss aversion generates allows me to match or improve on calibration exercises that use asset returns moments. More striking, I find the empirical evidence regarding the security market line relative to the CAPM and regarding the negative premium for skewness provide strong support for my model with loss aversion.

Appendix

A Properties of the Value Function

Let's show the value function is both increasing and concave in (C_t, V_{t+1}) .

Rewrite:

$$\log \bar{V}_{t+1} = \left(1 - \alpha \mathbf{1}_{v_{t+1} \geq \tilde{\mathbb{E}}_{\xi, t} v_{t+1}}\right) \log V_{t+1} + \alpha \mathbf{1}_{v_{t+1} \geq \tilde{\mathbb{E}}_{\xi, t} v_{t+1}} \left(\frac{1 - \xi}{1 - \xi^{T+1}}\right) \sum_{n=0}^T \xi^n \mathbb{E}_{t-n} (\log V_{t+1}) ,$$

which, for $\alpha < 1$ and $\xi < 1$, makes explicit \bar{V}_{t+1} as an increasing function of V_{t+1} , and thus h increasing.

Further

$$f(x, y) = \left[(1 - \beta) x^{1-\rho} + \beta g(y)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

is concave if g is concave. By Cauchy-Schwarz inequality, $g(Y) = \mathbb{E} \left(k(Y)^{1-\tilde{\gamma}} \right)^{\frac{1}{1-\tilde{\gamma}}}$ is concave if k is concave. I just need to prove that $k(Y) = Y \exp \left[-\alpha \max \left(0, y - \tilde{\mathbb{E}}_{\xi, t} y \right) \right]$ is concave. This is fairly straightforward.

Let's now derive the Euler Equation, and the stochastic discount factor, for the case with full updating of the reference point, $\xi = 0$.⁵² For all returns R_{t+1} , and $\delta \ll 1$, :

$$V_t(C_t + \delta) - V_t(C_t) = \delta(1 - \beta)C_t^{-\rho}V_t^\rho,$$

$$\begin{aligned} \overline{V}_{t+1}(C_{t+1} + \delta R_{t+1}) - \overline{V}_{t+1}(C_{t+1}) &= (1 - \alpha \mathbf{1}_{v_{t+1} \geq \mathbb{E}_t v_{t+1}}) \frac{\overline{V}_{t+1}}{V_{t+1}} \delta R_{t+1} dV_{t+1} / dC_{t+1} \\ &\quad + \overline{V}_{t+1} \alpha \mathbf{1}_{v_{t+1} \geq \mathbb{E}_t v_{t+1}} \mathbb{E}_t \left(\frac{1}{\overline{V}_{t+1}} \delta R_{t+1} dV_{t+1} / dC_{t+1} \right), \end{aligned}$$

and, the first order condition of the optimum consumption path is:

$$\mathbb{E}_t [R_{t+1} S_{t,t+1}] = \frac{V_t(C_{t+1} + \delta R_{t+1}) - V_t(C_{t+1})}{V_t(C_t + \delta) - V_t(C_t)} = 1,$$

so that

$$\begin{aligned} \mathbb{E}_t [R_{t+1} S_{t,t+1}] &= \beta (h(V_{t+1}))^{\bar{\gamma} - \rho} \mathbb{E}_t \left(\overline{V}_{t+1}^{-1 - \bar{\gamma}} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} V_{t+1}^{\rho - 1} (1 - \alpha \mathbf{1}_{v_{t+1} \geq \mathbb{E}_t v_{t+1}}) R_{t+1} \right) \\ &\quad + \beta (h(V_{t+1}))^{\bar{\gamma} - \rho} \mathbb{E}_t \left(\alpha \mathbf{1}_{v_{t+1} \geq \mathbb{E}_t v_{t+1}} \overline{V}_{t+1}^{-1 - \bar{\gamma}} \right) \mathbb{E}_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\rho} V_{t+1}^{\rho - 1} R_{t+1} \right), \end{aligned}$$

and

$$\begin{aligned} S_{t,t+1} &= \beta \left(\frac{V_{t+1}}{h(V_{t+1})} \right)^{\rho - \bar{\gamma}} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{\overline{V}_{t+1}}{V_{t+1}} \right)^{1 - \bar{\gamma}} \times \\ &\quad \left((1 - \alpha \mathbf{1}_{v_{t+1} \geq \mathbb{E}_t v_{t+1}}) + \alpha \frac{\mathbb{E}_t \left(\mathbf{1}_{v_{t+1} \geq \mathbb{E}_t v_{t+1}} \overline{V}_{t+1}^{-1 - \bar{\gamma}} \right)}{\overline{V}_{t+1}^{-1 - \bar{\gamma}}} \right). \end{aligned}$$

QED.

B Expected Utility Model with Loss aversion

In the model with expected utility and iid consumption growth, $\rho = \gamma = \bar{\gamma} = 1$, and $v_{t+1} - \mathbb{E}_t(v_{t+1}) = c_{t+1} - \mathbb{E}_t(c_{t+1})$. The stochastic discount factor is thus:

$$S_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left(1 + \frac{\alpha}{2} - \alpha \mathbf{1}_{c_{t+1} \geq \mathbb{E}_t c_{t+1}} \right).$$

⁵²The case $\xi > 0$ is analyzed in the online Appendix E, http://home.uchicago.edu/mandries/lossaversion_appendix.pdf.

The risk free rate $r_f = -\log \mathbb{E}_t (S_{t,t+1})$ is:

$$\begin{aligned} r_f &= -\log \beta + \mu_c - \log \left[\left(1 + \frac{\alpha}{2}\right) \mathbb{E}_t (\exp(-\sigma_c w_{t+1}) - \alpha \mathbb{E}_t (\mathbf{1}_{w \geq 0} \exp(-\sigma_c w_{t+1})) \right] \\ &= -\log \beta + \mu_c - \frac{1}{2} \sigma_c^2 - \log \left(1 + \frac{\alpha}{2} - \alpha \Phi(-\sigma_c)\right) . \end{aligned}$$

Assets with returns

$$\log R_{t+1} = \left(\bar{r} - \frac{1}{2} |\sigma_R|^2 - \frac{1}{2} |\widetilde{\sigma}_R|^2 \right) + \sigma_R w_{t+1} + \widetilde{\sigma}_R \widetilde{w}_{t+1}$$

have expected excess returns:

$$\bar{r} - r_f = -\log \mathbb{E}_t \left(\exp \left(\sigma_R w_{t+1} - \frac{1}{2} |\sigma_R|^2 \right) S_{t,t+1} \right) - \log \mathbb{E}_t (S_{t,t+1}) .$$

Therefore:

$$\begin{aligned} \bar{r} - r_f &= \frac{1}{2} |\sigma_R|^2 - \log \left[\left(1 + \frac{\alpha}{2}\right) \mathbb{E}_t (\exp((\sigma_R - \sigma_c) w_{t+1}) - \alpha \mathbb{E}_t (\mathbf{1}_{w \geq 0} \exp((\sigma_R - \sigma_c) w_{t+1})) \right] \\ &\quad + \log \left[\left(1 + \frac{\alpha}{2}\right) \mathbb{E}_t (\exp(-\sigma_c w_{t+1}) - \alpha \mathbb{E}_t (\mathbf{1}_{w \geq 0} \exp(-\sigma_c w_{t+1})) \right] , \end{aligned}$$

and

$$\bar{r} - r_f = \sigma_R \sigma_c - \log \left(1 + \frac{\alpha}{2} - \alpha \Phi(\sigma_R - \sigma_c)\right) + \log \left(1 + \frac{\alpha}{2} - \alpha \Phi(-\sigma_c)\right) .$$

QED.

References

- Abeler, J., Falk, A., Goette, L., Huffman, D. (2011), “Reference Points and Effort Provision,” *American Economic Review*, 101, 470–92.
- Akerlof, G., Dickens, W. (1982). “The Economic Consequences of Cognitive Dissonance,” *American Economic Review*, 72, 307–19.
- Attanasio, O., Vissing-Jorgensen, A. (2003), “Stock-Market Participation, Intertemporal Substitution, and Risk-Aversion,” *American Economic Review, Papers and Proceedings*, 383–391.
- Attanasio, O., Weber, G. (1989), “Intertemporal Substitution, Risk Aversion and the Euler Equation for Consumption,” *Economic Journal* 99, 59–73.
- Bansal, R., Dittmar, R., Lundblad, C. (2005), “Consumption, dividends, and the cross-section of equity returns,” *Journal of Finance* 60, 1639–1672.
- Bansal, R., Kiku, D., Yaron, A. (2007), “Risks for the Long Run: Estimation and Inference,” Working paper.
- Bansal, R., Kiku, D., Yaron, A. (2009), “An Empirical Evaluation of the Long-Run Risks Model for Asset Prices,” NBER Working Papers 15504
- Bansal, R., Yaron, A. (2004), “Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles,” *Journal of Finance* 59, 1481–1509.
- Barberis, N., Huang, M. (2008), “Stocks as Lotteries: The Implications of Probability Weighting for Security Prices.” *American Economic Review*, 98, 2066–2100.
- Barberis, N., Huang, M. (2009), “Preferences with Frames: A New Utility Specification that Allows for the Framing of Risks”, *Journal of Economic Dynamics and Control* 33, 1555-1576.
- Barberis, N., Huang, M., Santos, T. (2001), “Prospect theory and asset prices,” *Quarterly Journal of Economics* 116, 1–53.
- Barberis, N., Huang, M., Thaler, R. (2006) “Individual preferences, monetary gambles, and stock market participation: a case for narrow framing,” *American Economic Review* 96, 1069–1090.

- Barro, R., Nakamura, E., Steinsson, J., Ursúa, J. (2010), "Crises and Recoveries in an Empirical Model of Consumption Disasters," NBER Working Papers 15920.
- Beaudry, P., van Wincoop, E. (1996), "The Intertemporal Elasticity of Substitution: An Exploration Using A Panel of State Data," *Economica* 63, 495–512.
- Beeler, J., Campbell, J. (2009), "The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment," NBER Working paper 14788.
- Benartzi, S, Thaler, R. (1995), "Myopic Loss Aversion and the Equity Premium Puzzle," *Quarterly Journal of Economics* 110, 73–92.
- Black, F. (1972), "Capital market equilibrium with restricted borrowing," *Journal of business* 45, 3, pp. 444-455.
- Black, F. (1992), "Beta and Return," *The Journal of Portfolio Management*, 20, pp. 8-18.
- Black, F., M.C. Jensen, and M. Scholes (1972), "The Capital Asset Pricing Model: Some Empirical Tests," In Michael C. Jensen (ed.), *Studies in the Theory of Capital Markets*, New York, pp. 79-121.
- Bonomo, M., Garcia, R., Meddahi, N., Tédongap, R. (2010) "Generalized Disappointment Aversion, Long Run Volatility Risk and Asset Prices," TSE Working Papers 10-187, Toulouse School of Economics (TSE).
- Brennan, M.J., (1971), "Capital market equilibrium with divergent borrowing and lending rates," *Journal of Financial and Quantitative Analysis* 6, 1197-1205.
- Campbell, J. (1999), "Asset prices, Consumption and the Business Cycle," *Handbook of Macroeconomics*, Volume 1, pp. 1231-1303; John B. Taylor and Michael Woodford, editors.
- Campbell, J., Cochrane, J. (1999), "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy* 107, 205– 255.
- Card, D., Dahl, G. (2011), "Family Violence and Football: The Effect of Unexpected Emotional Cues on Violent Behavior," *Quarterly Journal of Economics*, 126, 103-143.

- Chabi-Yo, F., Leisen, D., Renault, E. (2007), "Implications of Asymmetry Risk for Portfolio Analysis and Asset Pricing," working paper.
- Crawford, V., Meng, J. (2008), "New York City Cabdrivers' Labor Supply Revisited: Reference-Dependence Preferences with Rational-Expectations Targets for Hours and Income," University of California at San Diego, Economics Working Paper Series 557104, Department of Economics, UC San Diego.
- Dillenberger, D., Rozen, K. (2010), "Disappointment Cycles," PIER Working Paper Archive 10-028, Penn Institute for Economic Research, Department of Economics, University of Pennsylvania.
- Duffie, D., Lyons, P. (1992), "PDE solutions of stochastic differential utility," *Journal of Mathematical Economics* 21 (1992) 577-606.
- Eil, D., Lien, J. (2011), "Staying Ahead and Getting Even: Evidence from Decisions of Online Poker Experts", working paper
- Engelhardt, G., Kumar, A. (2008), "The Elasticity of Intertemporal Substitution: New Evidence from 401(k) Participation," Working paper 0812, Federal Reserve Bank of Dallas.
- Epstein, L., Zin, S. (1989), "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical perspective," *Econometrica* 57, 937-969.
- Fama, E., French, K. (1992), "The Cross-Section of Expected Stock Returns," *Journal of Finance* 59, 427-465.
- Frazzini, A., Pedersen, L. (2010), "Betting Against Beta," NBER Working Papers 16601.
- Frederick, S., Loewenstein, G. (1999), "Hedonic Adaptation," in "Well being: The foundations of hedonic psychology", pp. 302-329; Diener, E., Kahneman, D., Schwarz, N., editors.
- Gill, D., Prowse, V. (2012), "A Structural Analysis of Disappointment Aversion in a Real Effort Competition," *American Economic Review*, 102, 469-503.
- Gneezy, U., List, J., Wu, G. (2006) "The Uncertainty Effect: When a Risky Prospect Is Valued Less Than Its Worst Possible Outcome," *The Quarterly Journal of Economics*, 121, 1283-1309.

- Gruber, J. (2006), “A Tax-Based Estimate of the Elasticity of Intertemporal Substitution,” NBER Working paper 11945.
- Gul, F. (1991), “A Theory of Disappointment Aversion,” *Econometrica*, 59, 667–86.
- Guvenen, F. (2006), “Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective,” *Journal of Monetary Economics* 53, 1451–1472.
- Hall, R. (1988), “Intertemporal Substitution in Consumption,” *Journal of Political Economy* 96, 339–357.
- Hansen, L., Lee, J., Polson, N., Yae, S. (2011), “Assessing the Recursive Utility Model,” working paper.
- Hansen, L., Heaton, J., Li, N. (2008), “Consumption strikes back?,” *Journal of Political Economy*, 116, 260-302.
- Hansen, L., Heaton, J., Lee, J., Roussanov, N. (2007), “Intertemporal Substitution and Risk Aversion,” *Handbook of Econometrics* 6.
- Hansen, L., Singleton, K. (1982), “Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models,” *Econometrica* 63, 767–804.
- Harvey, C., Siddique, A. (2000), “Conditional skewness in asset pricing tests,” *Journal of Finance* 55, 1263-1295.
- Hong, H., Sraer, D. (2011), “Speculative Betas,” working paper.
- Kahneman, D., Tversky, A. (1979), “Prospect theory: an analysis of decision under risk,” *Econometrica* 47, 263–291.
- Koszegi, B., Rabin, M. (2006), “A Model of Reference-Dependent Preferences,” *Quarterly Journal of Economics*, 121, 1133–65.
- Koszegi, B., Rabin, M. (2007), “Reference-dependent risk attitudes,” *American Economic Review* 97, 1047–1073.

- Koszegi, B., Rabin, M. (2009), "Reference-Dependent Consumption Plans," *American Economic Review*, 99, 909-936.
- Malloy, C., Moskowitz, T., Vissing-Jorgensen, A. (2009), "Long-Run Stockholder Consumption Risk and Asset Returns," *Journal of Finance*, 64, 2427-2479.
- Matthey, A. (2010), "The Influence of Priming on Reference States," *Games*, MDPI, Open Access Journal, 1, 34-52.
- Melino, A., Yang, A. (2003), "State dependent preferences can explain the equity premium puzzle," *Review of Economic Dynamics* 6, 806–830.
- Mulligan, C. (2004), "What do Aggregate Consumption Euler Equations Say about the Capital Income Tax Burden?," NBER Working Papers 10262.
- Pastor, L., Stambaugh, R. (2003), "Liquidity risk and expected stock returns," *Journal of Political Economy* 111, 642—685.
- Pope, D., Schweitzer, M. (2011), "Is Tiger Woods Loss Averse? Persistent Bias in the Face of Experience, Competition, and High Stakes," *American Economic Review*, 101, 129–157.
- Post, T., van den Assem, M., Baltussen, G., Thaler, R. (2008), "Deal or No Deal? Decision Making under Risk in a Large-Payoff Game Show," *American Economic Review*, 98, 38–71.
- Rabin (2000), "Risk Aversion and Expected-Utility Theory: A Calibration Theorem," *Econometrica*, 68, 1281-1292.
- Routledge, B., Zin, S. (2010), "Generalized Disappointment Aversion and Asset Prices," *Journal of Finance*, 65, 1303-1332.
- Schroder, M., Skiadas, C. (1999), "Optimal Consumption and Portfolio Selection with Stochastic Differential Utility," *Journal of Economic Theory* 89, 68-126.
- Sprenger, C. (2010), "An Endowment Effect for Risk: Experimental Tests of Stochastic Reference Points," working paper.
- Vissing-Jorgensen, A. (2002), "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," *Journal of Political Economy* 110, 825–853.

Tversky, A., Kahneman, D. (1992), “Advances in Prospect Theory: Cumulative Representation of Uncertainty.” *Journal of Risk and uncertainty*, 5, 297–323.

Yogo, M. (2008), “Asset Prices Under Habit Formation and Reference-Dependent Preferences,” *Journal of Business & Economic Statistics*, American Statistical Association, 26, 131-143.