

## Contact Instability of the Direct Drive Robot When Constrained by a Rigid Environment

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### ABSTRACT

Robot manipulations require mechanical interaction with the environment (i.e., the object being manipulated) which can constrain the robot endpoint in some or all directions. A sufficient condition for the stability of robot manipulators in constrained maneuvers is derived in the work presented here. Attention is focused on the class of direct drive robots whose rigid links dominate the robot's dynamic behavior; compared to the robot, the environment is assumed to be infinitely rigid. The stability of the manipulator-environment system is investigated, and a bound for stable manipulation is determined. This bound is verified experimentally on the Minnesota direct drive robot.

### 1. INTRODUCTION

Both fast motion in unconstrained space and mechanical interaction with the environment (i.e., the object being manipulated) are required in most robotic manufacturing tasks. Robotic assembly is an example of such a task where the robot must follow a trajectory when unconstrained by the environment, but, during the insertion process, the robot must comply with the environmental constraints. Robotic deburring (Kazerooni et al. 1986b) is another example of such a task where the interaction force<sup>1</sup> must be accommodated rather than resisted.

Whitney (1987) suggested two methods for implementing compliant motion. The first method controls force and position in a nonconflicting way (Mason 1981, Mills and Goldenberg 1989, Paul and Shimano 1976, Raibert and Craig 1981, Whitney 1977): force is commanded along those directions constrained by the environment, while position is commanded along those directions in which the manipulator is free to move. The second method develops a relationship between the interaction force and the manipulator position (Hogan 1985 and 1987, Kazerooni 1986a, 1989b, Kazerooni and Tsay 1988, Salisbury 1980): by controlling the manipulator position and specifying its relationship with the interaction force, a designer ensures that the manipulator can maneuver in a constrained space while maintaining appropriate contact force. This paper analyzes the stability of the robot-environment system when the second method is employed to control robot compliancy. A sufficient condition for system stability is derived analytically and verified experimentally on a direct drive robot arm. References 5, 11, 12, and 13 give some preliminary results on the stability of robotic constrained maneuvers. An and Hollerbach (1987) and Colgate (1988) discuss stability analysis for linear, single-degree-of-freedom systems.

### 2. UNCONSTRAINED ANALYSIS

The dynamic behavior of direct drive robots with  $n$  degrees of freedom is expressed by equation 1:

$$M(\theta) \ddot{\theta} + C(\theta, \dot{\theta}) = \tau - J^T f \quad (1)$$

where  $\ddot{\theta}$ ,  $\dot{\theta}$ ,  $\theta$  are vectors containing the joints' accelerations, velocities, and positions;  $M(\theta)$  is the inertia matrix;  $C(\theta, \dot{\theta})$  is the vector representing the coriolis, centrifugal, and gravity forces;  $\tau$  is the vector of joint torques;  $J^T$  is the Jacobian transpose matrix; and  $f$  is the vector of external forces applied at the robot endpoint (Hollerbach 1980). Trajectory control of the manipulator is performed by a digital implementation of a feedforward torque controller, which torque is given by:

$$\tau = K_p (\theta_d - \theta) + K_v (\dot{\theta}_d - \dot{\theta}) + \dot{M}(\theta_d) \ddot{\theta}_d + \dot{C}(\theta_d, \dot{\theta}_d) \quad (2)$$

where  $\tau$  is the vector of joint torques;  $(\theta_d - \theta)$  is the error between the command position,  $\theta_d$ , and the actual position,  $\theta$ , and  $(\dot{\theta}_d - \dot{\theta})$  is the error between the respective velocities;  $K_p$  is an  $n \times n$  matrix containing the position gains;  $K_v$  is an  $n \times n$  matrix containing the velocity gains;  $\dot{M}(\theta_d)$  and  $\dot{C}(\theta_d, \dot{\theta}_d)$ , which can be found experimentally or analytically, are educated guesses for  $M(\theta)$  and  $C(\theta, \dot{\theta})$ . The nonlinear feedforward terms,  $\dot{M}(\theta_d)$  and  $\dot{C}(\theta_d, \dot{\theta}_d)$ , cancel the nonlinear effects of  $M(\theta)$  and  $C(\theta, \dot{\theta})$  in the robot's dynamics and result in a nearly uncoupled linear system (Spong and Vidyasagar, 1985). In feedforward torque control, the robot trajectory is specified in joint coordinates, and the joint positions, velocities, and accelerations for a given trajectory are computed and stored before the trajectory is executed. The  $\text{Kin}(\cdot)$  operator in the diagram represents the forward kinematics, while  $\text{Kin}^{-1}(\cdot)$  represents the inverse kinematics. When the trajectory is specified in Cartesian space as a function of time,  $e(t)$ , inverse kinematics and numerical differentiation transform it into the joint space,  $\theta_d(t)$ .

$$\theta_d(t) = \text{kin}^{-1} [ e(t) ] \quad (3)$$

$$\dot{\theta}(t) = \text{kin} [ \dot{\theta}(t) ] \quad (4)$$

The nonlinear control law of equation 2 is chosen because the inertia and coriolis terms can be computed and stored. During the trajectory execution, these terms are added to the error terms to compute the joint torques. The chief advantage of this approach is

<sup>1</sup>In this article, "force" implies force and torque, and "position" implies position and orientation.



$\|H^*(f)\|_{r2}$  can be replaced with  $\sigma_{\max}(Q) \|f\|_{r2}$  where  $\sigma_{\max}(Q)$  is the maximum singular value<sup>3</sup> of  $Q$  and  $Q$  is a matrix whose entry  $(i,j)$  is given by  $(Q)_{ij} = \sup_{\omega} |H(j\omega)_{ij}|$ . The closed loop system of Figure 1 is  $L_2$  stable ( $f$  is  $L_2$  bounded) if inequality 12 is satisfied over the commanded trajectory.

$$\sigma_{\max}(J^{-T} K_p J^{-1}) \sigma_{\max}(Q) < 1 \quad (12)$$

$$\sigma_{\max}(Q) < \frac{1}{\sigma_{\max}(J^{-T} K_p J^{-1})} \text{ (over the commanded trajectory)} \quad (13)$$

Equivalently, one can satisfy inequality 14.

$$\sigma_{\max}(Q) < \text{infimum of } \sigma_{\min}(J K_p^{-1} J^T) \text{ (over the commanded trajectory)} \quad (14)$$

The minimum singular value of  $(J K_p^{-1} J^T)$  must be calculated at each point in the commanded trajectory. The infimum is the lowest of all the minimum singular values. The gain of  $H$ , expressed in terms of  $\sigma_{\max}(Q)$ , must be chosen to be smaller than this infimum. From inequality 14, the stability region will approach zero when the robot maneuvers near a singular point ( $\det(J) \rightarrow 0$ ) and/or when the position gains approach infinity. Both cases are representative of "infinite stiffness" for the robot; the first is due to the robot configuration, while the second is due to the tracking controller. The above stability condition has been extended to robots with unstructured models (Kazerooni and Tsay 1988).

#### 4. EXPERIMENTAL RESULTS

To evaluate the nonlinear stability condition, a compliance controller is implemented on a direct-drive, three-degree-of-freedom robot (Kazerooni and Kim 1988 and 1989a). The University of Minnesota direct drive robot (Figure 2) uses a four-bar linkage and is statically balanced without any counter weights. As a result of the elimination of the gravity forces, smaller actuators and, thus, smaller amplifiers were chosen to drive this robot. The motors yield acceleration of 5g at the endpoint without overheating. High torque, low speed motors power the robot; specifically, the motors are neodymium (NdFeB) magnet AC brushless synchronous motors. Due to the high magnetic field strength (maximum energy products: 35 MGOe) of the rare earth NdFeB magnets, the motors have high torque-to-weight ratios. Pancake-type resolvers are used as position and velocity sensors. The peak torque of motors 1, 2, and 3 are 118 Nm, 78 Nm, and 58 Nm, respectively. The robot links are made of graphite-epoxy composite material. A microcomputer, which hosts a 4-node parallel processor with a PC/AT bus interface, is the main robot controller. Each node is an independent 32-bit processor with local memory and communication links to the other nodes in the system.

Because this robot has no gears, frictional losses are small and the manipulator can be modeled by equation 1. For a sufficient stability condition, if the condition is satisfied, stability is guaranteed; however, if the condition is violated, no conclusion can be made. Two experiments are performed to demonstrate that inequality 14 is a sufficient condition for stability: one experiment in which the satisfaction of the condition leads to a stable maneuver and one experiment in which parameters for an unstable maneuver violate the condition. In the first experiment, an  $H$  is designed such that inequality 14 is satisfied and it is shown, through experiment, that the system is stable. In the second experiment, it is shown that an  $H$  which destabilizes the system also violates inequality 14.

A reinforced aluminum wall is mounted vertically in the robot workspace, as shown in Figure 2, to simulate a stiff environment. Motor 2 is mechanically locked while motors 1 and 3 maneuver the robot endpoint horizontally. A piezoelectric force sensor is mounted on the manipulator endpoint to measure contact forces.

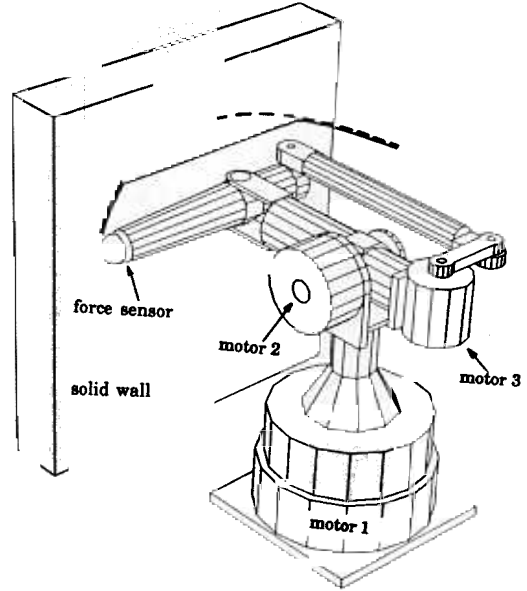


Figure 2: The dashed line is the desired endpoint trajectory while the solid line is the actual trajectory.

Since the experiments are all two-dimensional,  $H$  is a  $2 \times 2$  matrix operating on contact forces which are normal and tangential to the wall. (The endpoint force measurements are resolved into the global coordinate frame.) In these experiments, only the compliancy in the direction normal to the wall is supplemented, so the following form of  $H$  is chosen:

$$H = \begin{bmatrix} \frac{H_0}{Ts+1} & 0 \\ 0 & 0 \end{bmatrix} \quad (15)$$

where  $T$  is an empirical constant and filters high frequency noise in the force measurement.  $T$  is fixed at 0.05 for all the experiments. The function  $r(t)$ , shown in Figure 2 by the dashed line, is the trajectory assigned to the robot. Since  $H$  has only one non-zero member, then  $\sigma_{\max}(Q)$  will be the maximum value of the magnitude of  $H_0/(Tj\omega + 1)$ . The maximum value of  $H$  is  $H_0$  and occurs at DC ( $\omega=0$ ).

In the first experiment, it is shown that if inequality 14 is satisfied for the maneuver shown in Figure 2, then the robot can have stable interaction with the environment.  $H_0$  is chosen to be .0003 so  $H(s)$  is smaller than  $\sigma_{\min}(J K_p^{-1} J^T)$  for all configurations within the maneuver. Figure 3 shows the experimental values of the normal contact force. Stable contact is indicated by the absence of undamped oscillations in the normal force.

In the second experiment,  $H_0$  is set to .0015. Figure 4 shows the experimental normal contact force as a function of time. The contact force oscillates throughout the maneuver, indicating that the compliance controller is unstable. Comparison with the singular value plots of  $J K_p^{-1} J^T$  shows that  $H_0$  exceeds the lower bound on  $\sigma_{\min}(J K_p^{-1} J^T)$ ; hence, the stability condition is violated. Since inequality 14 is a sufficient condition for stability, violation of this condition does not lead to any conclusion. The system can be stable even if the stability condition is not satisfied as in Figure 5, which shows the experimental contact forces when  $H_0=0.0005$ .

<sup>3</sup> The maximum singular value of a matrix  $H$ ,  $\sigma_{\max}(H)$  is defined as:

$$\sigma_{\max}(H) = \max \frac{|Hz|}{|z|}, \quad z \neq 0$$

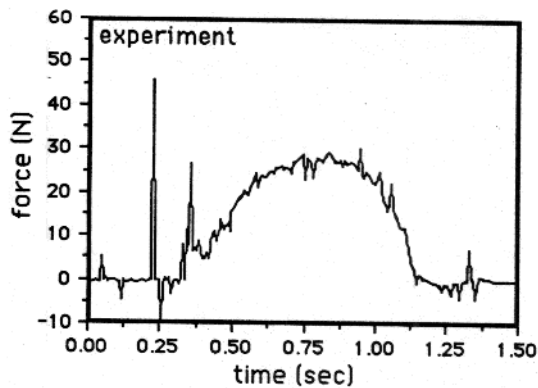


Figure 3:  $H_0 = 0.0003$  satisfies the stability condition.

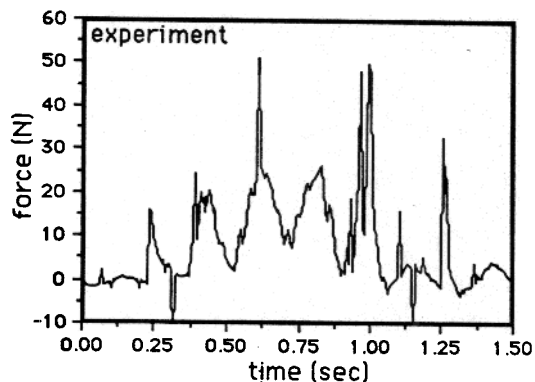


Figure 4:  $H_0 = 0.0015$  does not satisfy the stability condition.

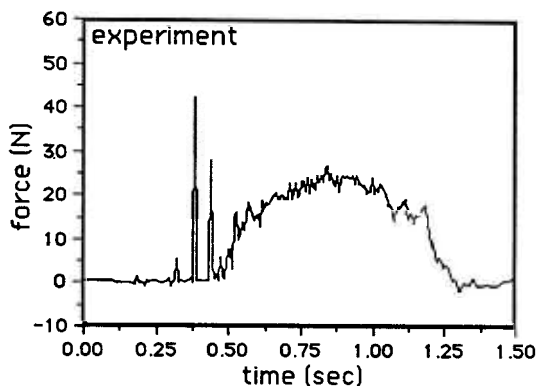


Figure 5:  $H_0 = 0.0005$  violates the stability condition; however, the system is stable.

## 5. SUMMARY AND CONCLUSION

An architecture for compliance control of direct drive robot manipulators is presented. The control approach allows not only tracking the input command vector in unconstrained space, but also compliancy in constrained maneuvers. A bound for the global stability of the robot and infinitely stiff environment, taken together, is derived. The stability region theoretically will approach zero when the robot maneuvers near its singular point and/or the position gain approaches infinity. Both cases are representative of infinite stiffness for the robot. Through both simulation and experimentation, the sufficiency of this condition is demonstrated.

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