

Contaminant Transport Modelling Through Landfill Liners

Kumar, R. Praveen

e-mail: praveen.rachakonda@unisa.edu.au

Dodagoudar, G.R.¹

*Associate Professor
e-mail: goudar@iitm.ac.in*

Quadri, S.S.²

*Associate Professor
e-mail: ssquadri@bvb.edu*

Centre for Environmental Risk Assessment and Remediation, University of South Australia, South Australia 5095

¹Department of Civil Engineering, IIT Madras, Chennai

²Department of Civil Engineering, BVB College of Engg. and Technology, Hubli

ABSTRACT

In the framework of meshfree methods, in this study a methodology is developed based on radial point interpolation method (RPIM). This methodology is applied to a one-dimensional (1D) contaminant transport in the saturated porous media. The 1D form of advection-dispersion equation involving reactive contaminant is considered. The Galerkin weak form of the governing equation is formulated using 1D meshfree shape functions constructed using thin plate spline radial basis functions. MATLAB code is developed to obtain the numerical solution. Numerical examples are presented to illustrate the applicability of the proposed RPIM and the results are compared with those obtained from the analytical and finite element solutions. The RPIM has generated results with no oscillations and they are insensitive to Peclet constraints. In order to test the practical applicability and performance of the RPIM, a case study of contaminant transport through landfill liner is presented. A good agreement is obtained between the results of the RPIM and the field investigation data.

1. INTRODUCTION

A frequent use of land for the disposal of a wide variety of domestic and industrial wastes accentuates the importance of contaminant transport modelling in the porous media. In the last two decades, the need for solving complex problems related to contaminant transport in geoenvironmental engineering in general and landfill engineering in particular has led to the development and use of advanced numerical methods of modelling and analysis. For solving transport problems having simplified boundary conditions and geometries, exact analytical solutions are available in the literature. While the historical response of the system could be inferred from the available field records, there is no way of computing the future behaviour of the full-scale system unless it is actually subjected to real conditions. A full-scale experiment would be prohibitively costly and time consuming. The only feasible recourse therefore is to construct a model, which reasonably portray the behaviour of the full-scale system and simulate the relevant physical parameters, and describes the overall significant characteristics of the transport phenomena.

One outstanding feature of the numerical modelling approach is the capacity for predictions to be made well into the future, thereby demonstrating the likely impacts

of current practice on the environment and future generations. The most frequently used solution techniques can be divided into six broad categories, namely analytic, finite layer, boundary element, finite difference, finite element and finite volume. Finite element techniques provide the opportunity for modelling problems with complex geometries, complicated flow patterns, heterogeneity and nonlinearity. They have some difficulties when dealing with problems where there are high advective velocities, low dispersivities and/or high contrast in dispersivity.

In recent years, a group of new methods called meshfree methods, such as the element free Galerkin method, the smooth particle hydrodynamics, the reproducing kernel particle method, the radial point interpolation method and others, have been developed (Praveen Kumar 2008); whose main aim is to eliminate the structure of the mesh and construct the approximate solutions for the discrete equation entirely in terms of a set of nodes. Among all the meshfree methods, the radial point interpolation method (RPIM) has been successfully used for solving numerous boundary-value problems related to various fields of study.

In this study, a methodology is developed for modelling one-dimensional advection-dispersion equation involving first-order degradation through the saturated porous media

using the RPIM. In this paper, thin plate spline radial basis functions (TPS-RBFs) are made use of in the analysis. MATLAB code is developed for modelling the contaminant migration using the RPIM. Results of the RPIM are compared with analytical solutions, finite element results and field investigation data and found to be satisfactory.

2. RADIAL POINT INTERPOLATION METHOD

The development of numerical methods for simulating complex flow and mass transport problems is of major importance in view of the numerous applications in many different areas of applied science and engineering. The Radial Point Interpolation Method (RPIM) is a meshfree method developed using the Galerkin weak form and the radial basis shape functions that are constructed based only on a group of nodes arbitrarily distributed in a local support domain by means of interpolation. For solving integrals in the weak form formed due to the Galerkin approximation procedure, a background mesh is used. Since the RPIM shape functions satisfy the Kronecker delta function property, the enforcement of essential boundary conditions is simple in the RPIM as in the case of finite element method (FEM).

Governing Equation

The one-dimensional (1D) form of the governing equation for contaminant migration through the saturated porous media is expressed as

$$\left(1 + \frac{\rho_d}{n} K_d\right) \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - \left(\frac{u}{n}\right) \frac{\partial C}{\partial x} - \alpha C \dots (1)$$

The governing Eq. (1) must be complemented with initial and boundary conditions. They are:

$$C(x, 0) = C_i \quad \forall x \in \Omega \dots (2a)$$

$$C(0, t) = C_0 \quad \text{on } \Gamma_S \dots (2b)$$

$$\frac{\partial C}{\partial x} n_s = g \quad \text{on } \Gamma_E \dots (2c)$$

where n is the porosity of the soil, ρ_d is the bulk density of the soil [ML⁻³], C is the concentration of contaminant [ML⁻³], D is the hydrodynamic dispersion coefficient [L²T⁻¹], u is the discharge velocity [LT⁻¹], K_d is the distribution coefficient [L³M⁻¹], α is the decay coefficient [T⁻¹], C_0 and g are the concentration of contaminant at the source [ML⁻³] and the concentration gradient at the exit boundary respectively, n_s is the unit normal to domain Ω and, Γ_S and Γ_E are the portions of boundary Γ where the source concentration and the concentration gradient are prescribed. The complete formulation of RPIM as applied to contaminat transport modelling is given by Praveen Kumar (2008).

3. NUMERICAL EXAMPLES

For the numerical examples, it is assumed that the porous medium in which the contaminants move is homogenous

and the source of contaminant is continuous. To illustrate the applicability of the RPIM, two examples dealing with the processes occurring in the porous media during the transport are considered and the findings are compared with the analytical solutions and finite element results. In the RPIM, a linear basis function is used for constructing the shape functions and for the TPS-RBFs, the shape parameter $q = 1.25$ is taken.

Validation with Published Data: Advection-Dispersion

This numerical example deals with the modelling of plume developed due to advection-dispersion process. Boztosun & Charafi (2002) considered a one-dimensional advection-diffusion problem with initial condition $C(x, 0) = 0$ and boundary conditions $C(x, t) = 300$ units at $x = 0$ and $\partial C(x, t)/\partial x = 0$ at $x = 6$ units. The proposed RPIM is applied to this problem. The parameters considered in the analysis are: seepage velocity (v) = 1 unit and 6 units, coefficient of hydrodynamic diffusion (D) = 1. The analytical solution given by Ogata & Banks (1961) is:

$$C = \frac{C_0}{2} \left[\operatorname{erfc} \left(\frac{x - vt}{2\sqrt{Dt}} \right) + \exp \left(\frac{vx}{D} \right) \operatorname{erfc} \left(\frac{x + vt}{2\sqrt{Dt}} \right) \right] \dots (3)$$

The problem domain [0, 6] is discretised into 21 cells and the shape parameter q is taken as 1.25. A comparison between the results obtained from the RPIM and Boztosun & Charafi (2002) with that of the analytical solution at different time steps is shown in Figures 1 and 2. From figures, it is noted that both the RPIM and analytical results, as well the results from Boztosun and Charafi model are in good agreement.

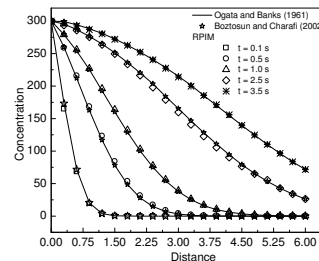


Fig. 1: Comparison of RPIM and Boztosun and Charafi Model Results with Analytical Solution for $D = 1$ unit and $v = 1$ unit (Advection-dispersion)

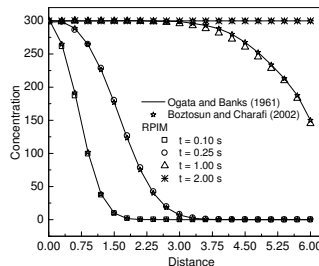


Fig. 2: Comparison of RPIM and Boztosun and Charafi Model Results with Analytical Solution for $D = 1$ unit and $v = 6$ units (Advection-Dispersion)

Advection Dominant Transport

This example presents the case of a contaminant transport for which advection is highly dominant [Peclet number (P_e) = 20 and 200]. The RPIM is applied for a column of horizontal length of 50 cm with initial condition $C(x, 0) = 0$ and boundary conditions $C(x, t) = 1$ mg/L at $x = 0$ cm and at $x = 50$ cm. The parameters considered in the analysis are: $u = 1.0 \times 10^{-2}$ cm/s; $n = 0.368$ and dispersivity of soil = 0.1 cm and 0.01cm for the two Peclet numbers, respectively. The problem domain [0 – 50 cm] is divided into 26 uniformly spaced meshfree nodes with 25 cells. The simulation has been carried out for 20 minutes with a time step of 1 minute.

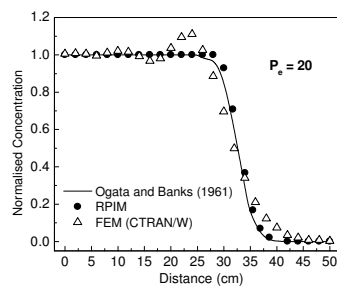


Fig. 3: Comparison of Results of FEM and RPIM with Analytical Solution for $P_e = 20$ (Advection dominant transport)

A finite element package, CTRAN/W (2007) is also used for solving this example problem and the results are compared with that of the RPIM. In the finite element analysis, domain is discretised into 25 elements with 26 nodes. A comparison between the results obtained from the RPIM and FEM with that of the analytical solution at different P_e is shown in Figures 3 and 4. The results obtained from the proposed RPIM for advection dominated transport problem are in good agreement with the analytical solution. Thus, it ensures that the present model is free from numerical oscillations and insensitive to Peclet constraints.

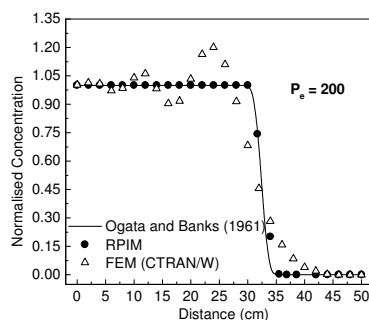


Fig. 4: Comparison of Results of FEM and RPIM with Analytical Solution for $P_e = 200$ (Advection Dominant Transport)

4. CASE STUDY: ENGINEERED LANDFILL

Modern landfills and lagoons widely utilise composite liners but unfortunately there is a paucity of published field

investigations that have examined contaminant transport through the complete geosynthetic composite liner system (Rowe et al. 2004). The understanding of factors associated with design for contaminant transport mitigation, selection of materials and long-term performance of composite liners has improved considerably over the last two decades. The importance of some of these factors can be best illustrated with reference to actual case records.

One case study of contaminant transport through landfill liners is used to demonstrate the practical applicability and performance of the proposed RPIM. The case study considered is: The Confederation road landfill.

The Confederation Road Landfill

A case study of the Confederation Road landfill reported in Quigley & Rowe (1986) has been considered. The site consists of ~ 7.5 m of domestic solid waste, with a 0.5 m clay cover, overlies homogeneous, massive grey silty clay. The waste projects about 2 m above the surrounding ground surface and was placed in a borrow trench excavated about 5.5 m into the original clay. The calculated effective stresses incorporate the effect of a slight regional downward gradient ($i = 0.25$) which creates a downward average linearised groundwater velocity of ~ 0.0024 m/year.

The in-situ vertical effective stress (σ'_v) indicated that the clay is overconsolidated by about 90 kPa with a slight increase near the clay/waste interface where the moisture content decreases slightly from 23 to 21%. The desiccated crust is highly fissured in the upper 4 m of the brown oxidised clay and much less fissured from 4 to 6.5 m in the lower grey portion of the crust. At the interface, which is probably within 1 m of the base of the desiccated crust, no fissures are observed in the 50 or more boreholes drilled at the site.

The wastes included in the original published report are: sodium chloride, heavy metals, dissolved organic carbon and isotopes. In regard to the hydraulic conductivity of a contaminated clay liner, an interdisciplinary study was published by Quigley et al. (1987) appears to be one of the very few scientific field studies performed. The study employed municipal solid waste (MSW) leachate as the permeant and it confirms that the leachate did not increase the hydraulic conductivity of inactive barrier clay. The hydraulic conductivity profiles show approximately constant values except within about 20 cm of the clay/water interface where a slight decrease is observed.

This case study example is used for modelling of contaminant transport through the clay liner using the RPIM. In RPIM, the domain [0, 3 m] is divided into 11 uniformly spaced nodes with 10 cells. The parameters used in the analysis are given in Table 1. For modelling contaminant migration through the landfill liner, it was assumed that the influent concentration was constant at C_0 and the boundary is located at the infinitely thick layer of

the porous medium. In Figures 5 and 6, the spatial solute distribution curves obtained from the RPIM are compared with the pore fluid concentrations of chloride and sodium obtained from the field investigation. From the figures it is observed that the results are agreeing well with the field data. This confirms the practical applicability of the RPIM.

Table 1: Data Used for Case Study of the Confederation Road landfill

Parameter	Value	
	Chloride	Sodium
Seepage velocity (m/year)	0.0024	0.0024
Length of the reach (m)	3.0	3.0
Dispersion coefficient (m ² /year)	0.02	0.012
Retardation coefficient	1.0	1.46
Total duration of simulation (years)	15	15
Time step (Δt) (year)	0.5	0.5
Number of divisions in length direction	10	10
Pore water concentration (mol/m ³)	34	24

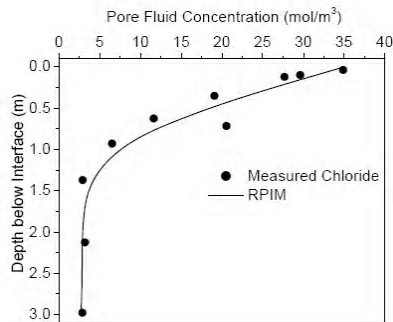


Fig. 5: Pore Fluid Concentrations of Cl⁻ in Clay Below Waste After 15 Years: RPIM and Field Data

5. CONCLUSIONS

The RPIM with polynomial reproduction and its numerical implementation for modelling one-dimensional contaminant transport through the saturated porous media are presented in the paper. It is noted that the enforcement of essential boundary conditions in the RPIM is similar as in the conventional finite element method. Numerical results obtained from the MATLAB program, developed for the RPIM are compared with the analytical and finite element results for two types of one-dimensional contaminant transport processes that occur in the saturated porous media. The results of the RPIM agree very well with those obtained by the analytical solutions, thus ensures the accurate formulation of the RPIM for contaminant transport modelling. The proposed RPIM generated results with no oscillations and thus RPIM can be used in the case of highly advective flow systems. The practical applicability of the RPIM is demonstrated with one case study of contaminant transport through landfill liners. The developed program can be modified accordingly for

analysing two- and three-dimensional cases and problems involving heterogeneity, fractured media, nonlinear isotherms and multi-species transport.

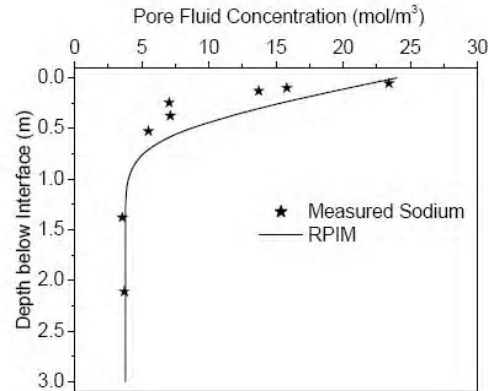


Fig. 6: Pore Fluid Concentrations of Na⁺ in Clay Below Waste After 15 Years: RPIM and Field Data

REFERENCES

- Boztosun, I. and Charafi, A. (2002). An analysis of the linear advection-diffusion equation using mesh-free and mesh-dependent methods. *Eng. Anal. Bound. Elem.*, 26(10), 889-895.
- GeoSlope International Ltd. (2007), *Transport modelling with CTRAN/W 2007: an engineering methodology*, Student Version 7.02, Alberta, Canada.
- Ogata, A. and Banks, R. B. (1961). A solution of the differential equation of longitudinal dispersion in porous media. *USGS, Professional Paper: 411-A, Reston, VA.*
- Praveen Kumar, R. (2008). Modelling of 1d, 2d and 3d contaminant transport through saturated and unsaturated porous media using meshfree techniques. *Ph. D. Thesis, Department of Civil Engineering, IIT Madras, Chennai, India.*
- Quigley, R. M. and Rowe, R. K. (1986). Leachate migration through clay below a domestic waste landfill, Sarnia, Ontario, Canada: chemical interpretation and modelling philosophies. *Hazardous and Industrial Solid Waste Testing and Disposal, Sixth Volume*, ASTM STP, Lorenzen, D., Conway, R.A., Jackson, L.P., Hamza, A., Perket, C.L., Lacy, W.J. (eds.), 933, 93-103.
- Quigley, R. M., Fernandez, F., Yanful, E., Helgason, T., Margaritis, A. and Whitby, J. L. (1987). Hydraulic conductivity of contaminated natural clay directly beneath a domestic landfill. *Can. Geotech. J.*, 24(3), 377-383.
- Rowe, R. K., Quigley, R. M., Brachman, R. W. I. and Booker, J. R. (2004). *Barrier Systems for Waste Disposal*. E & FN Spon Press, London.