

CONTINGENCY SPACES AND MEASURES IN CLASSICAL AND INSTRUMENTAL CONDITIONING¹

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The contingency between conditional and unconditional stimuli in classical conditioning paradigms, and between responses and consequences in instrumental conditioning paradigms, is analyzed. The results are represented in two- and three-dimensional spaces in which points correspond to procedures, or procedures and outcomes. Traditional statistical and psychological measures of association are applied to data in classical conditioning. Root mean square contingency, \emptyset , is proposed as a measure of contingency characterizing classical conditioning effects at asymptote. In instrumental training procedures, traditional measures of association are inappropriate, since one degree of freedom—response probability—is yielded to the subject. Further analysis of instrumental contingencies yields a surprising result. The well established "Matching Law" in free-operant concurrent schedules subsumes the "Probability Matching" finding of mathematical learning theory, and both are equivalent to zero contingency between responses and consequences.

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I. INTRODUCTION

A series of seminal papers by Rescorla (1967, 1968, 1969) argued for the importance of the contingency between conditional and unconditional stimuli in classical conditioning paradigms. The term "contingency" has not been quantitatively defined in the literature. Rather, positive contingencies, in which the conditional stimulus (CS) is a reliable predictor of the unconditional stimulus (US), have

been distinguished from negative contingencies, in which the CS reliably predicts the *absence* of the US. The lack of a contingency is then the intermediate condition in which the CS and US are completely uncorrelated ("truly random control"). Rescorla argued that excitatory conditioning results from positive contingencies and inhibitory conditioning from negative contingencies, so that only the non-contingent control procedure is appropriate for assessing the size and direction of the training effects. Much experimental work following Rescorla's lead investigated parameters of the non-contingent condition. Several researchers found it produced no differential conditioning (Rescorla, 1968; Bull and Overmier, 1968; Davis and McIntyre, 1969; Ayres and Quincy, 1970). Others found strong generalized effects under certain conditions (Seligman, 1968; Seligman, Maier, and Solomon, 1971); some transitory conditioning to the CS has also been reported (Kremer and Kamin, 1971; Quincy, 1971; Benedict and Ayres, 1972).

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These studies were procedurally similar, in that CS and US presentations were scheduled independently, but differed with respect to the frequency of CS and US events as well as several other parameters, including the dependent measures. One purpose of the present paper is to examine the range of possible procedural variations in contingency manipulations in classical conditioning and to relate some data to an appropriate metric.

In instrumental conditioning paradigms, the term contingency has had a more varied interpretation. For example, early definitions of the relation between a response and a positive reinforcer often referred to the stimulus consequence as "contingent upon" the response. While it was clear that the intent of such a usage was "follows immediately after", it was seldom explicit what consequences, if any, were "contingent upon" *not* responding. More recent work sought to break the contingency implicit in positive schedules of reinforcement by scheduling "free" reinforcers independently of responses, or for non-response periods (*e.g.*, Neuringer, 1969, 1970; Rescorla and Skucy, 1969). In aversive conditioning procedures, a variety of meanings is also evident. For example, the role of the contingency between a response and a shock occupied experimental attention from the outset of work on punishment (*e.g.*, Estes, 1944; Azrin, 1956). Yet, while investigators agreed on what constituted a contingent punishment procedure, they often differed on the appropriate non-contingent control. Frequently, non-contingent delivery of a stimulus meant delivery without the experimenter knowing whether a response was occurring or not. Indeed, the non-contingent delivery has even included a stricture on some minimum delay elapsing between a preceding response and the "non-contingent" shock (*e.g.*, Estes, 1944; Hunt and Brady, 1955). More recent work compared strict contingent punishment with non-contingent controls that equate response and non-response shock probabilities (Church, 1969; Church, Wooten, and Mathews, 1970; Gibbon, 1967).

In avoidance conditioning, contingency variations have received less explicit experimental attention, but work on shock density (Herrnstein and Hineline, 1966) and on "free" shock (Jones, 1969; Sidman, Herrnstein, and Conrad, 1957) is in the spirit of the non-contingent control investigations of other paradigms.

The present paper defines contingency in a relatively narrow statistical context and explores its implications with an eye to parallels or asymmetries between classical and instrumental training procedures. Classical conditioning paradigms are considered first, and an argument is advanced for a metric specification of the degree of contingency that results in the association statistic, \emptyset , as a measure of the control exerted by the training contingencies. Instrumental conditioning paradigms are then examined, and their contingencies are shown to differ fundamentally from those in classical conditioning. This difference results from the different degrees of freedom in the two paradigms. In the classical conditioning case, contingencies are completely controlled by the experimenter, while in the instrumental case, the contingency itself is in some sense a dependent variable. This distinction is mirrored in the metric representation.

In instrumental conditioning, as in classical conditioning, non-contingent procedures are the appropriate conditions for assessing the importance of contingencies. Contingent procedures in the instrumental case, however, are not uniquely specified by the experimenter and several cases of general interest, the so-called "Matching Laws", are analyzed for a fundamental property. It is argued that both probability matching and free-operant matching are basically the same, and are equivalent to a statement of zero statistical association between alternatives and consequences.

II. CLASSICAL CONTINGENCIES

Rescorla's (1968) adaptation of the classical pairing procedure consisted of scheduling shock with some probability in the presence of the CS and never in its absence (\sim CS). This contingent group was then compared with another, the truly random control group, for which shock probability was the same in the presence and absence of the conditional stimulus. Possible procedures of this sort are specified by plotting US probabilities in the presence and absence of the signal against each other. The resulting contingency square is shown in Figure 1. Such representations have appeared several times in recent literature (Catania, 1971; Church, 1969; Gibbon, Berryman, and Thompson, 1970; Seligman *et al.*, 1971). US probability in the presence of the

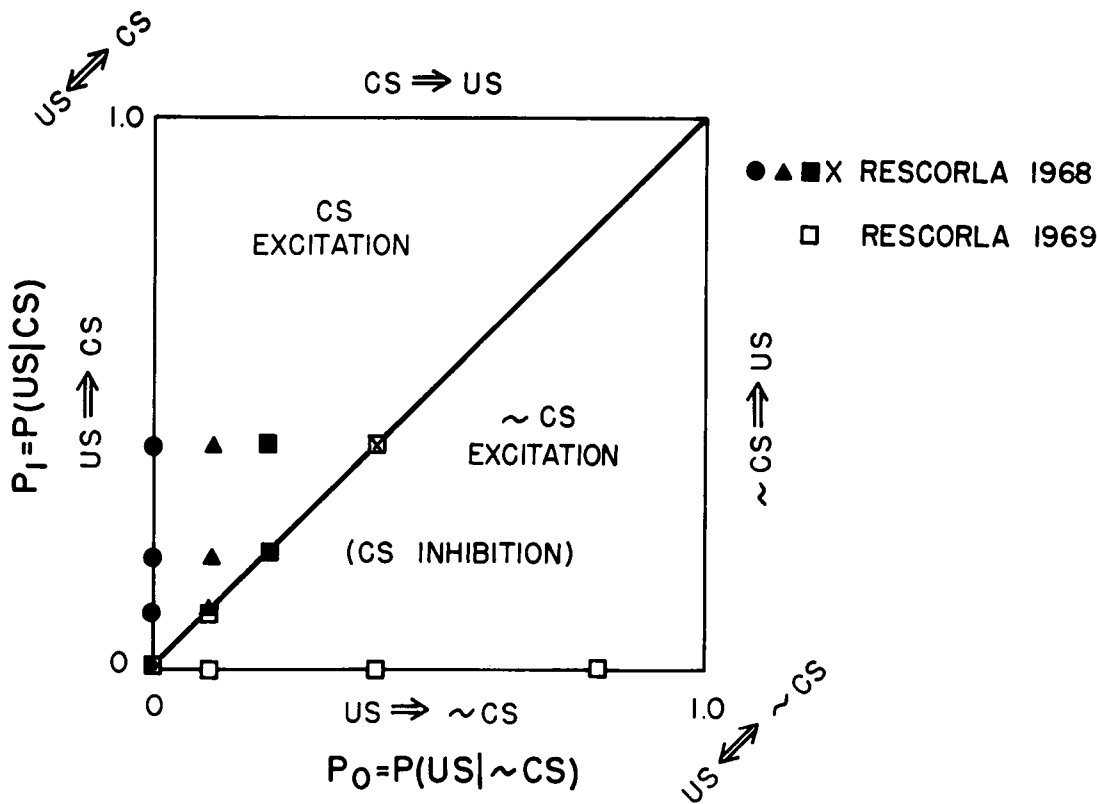


Fig. 1. Contingency square presenting possible combinations of conditional probability of unconditioned stimulus (US) delivery in the presence or absence of the conditioned stimulus (CS). Non-contingent procedures are represented along the diagonal.

CS, $P_1 = P(US|CS)$, is plotted on the vertical axis and US probability in $\sim CS$, P_0 , is on the horizontal axis. The diagonal passing through the origin is the locus of all non-contingent procedures, in which these two probabilities are equal. The upper-left corner represents the traditional pairing procedure, in which the US is always delivered in the CS and never in $\sim CS$, and the left edge represents the class of partial reinforcement procedures in which the US occurs only in the presence of the CS, but with probability less than one. The distinction between partial and consistent pairing procedures at the edges and corners of the square has a parallel in the logic of implication. The upper-left corner represents an "if and only if", or double implication. The CS is a necessary and sufficient condition for the US. The left edge, the locus of traditional partial schedules, represents the "US implies CS" case. The upper edge represents the "CS implies US" implication. This second implication represents another case of partial sched-

ules that has not been investigated. Along this upper edge of the square, CS implies US but USs occur with some probability in $\sim CS$ also.

The filled points represent Rescorla's (1968) training procedures, which yielded increasing suppression in a Conditioned Emotional Response (CER) paradigm with increasing distance from the diagonal. Along the diagonal, differential conditioning was not observed. The finding of no conditioning for non-contingent procedures is not universal. These procedures evidently produce considerable conditioning to the entire stimulus complex ("learned helplessness": Seligman, 1970; Seligman *et al.*, 1971). When differential conditioning to a CS is observed here, the effects are generally short-lived, and appear to depend on early contingent samples of the training sequence (Quincy, 1971; Benedict and Ayres, 1972). Our concern here will be with asymptotic conditioning levels.

The formal symmetry of this space suggests that conditioning to the CS produced by pro-

cedures represented above the diagonal might be matched by conditioning to \sim CS below the diagonal. Thus, excitatory conditioning to the CS might equally well be described as inhibitory conditioning to \sim CS, and *vice versa* for the space below the diagonal. This formal symmetry, of course, does not guarantee symmetric contingency effects. Stimulus dynamism (Kamin, 1965), "salience" (Wagner, 1969; Rescorla and Wagner, 1972), "preparedness" (Seligman, 1970), as well as signal discriminability and choice of dependent measure, may all be expected to modulate the manner in which contingency exerts control.

An ideal theoretical account would specify the mechanism whereby P_1 and P_0 combine to produce varying degrees of association between conditional and unconditional stimuli. A promising model has been proposed by Rescorla and Wagner (1972). A somewhat less ambitious goal is to specify contingent pro-

cedures that produce equivalent asymptotic levels of conditioning. Such procedures would define "iso-contingency contours" along which some measure of conditioning effects remains constant. To approach that goal, we consider below several candidates for a contingency metric. It will be seen that data exist in sufficient quantity to discriminate among some alternatives. Before pursuing that development, a more concrete specification of trials and probabilities is required.

A. TRIAL AND PROBABILITY DEFINITIONS

Consider the trial sequence diagrammed in Figure 2A. Each CS terminates with a brief shock and two of the four "intertrial intervals" (\sim CSs) do also. Such a sequence might occur, for example, for the schedule point $P_1 = 1$, $P_0 = 0.5$. A procedure of this sort is close to the original CER paradigm and seems a natural way to specify trials and probabilities.

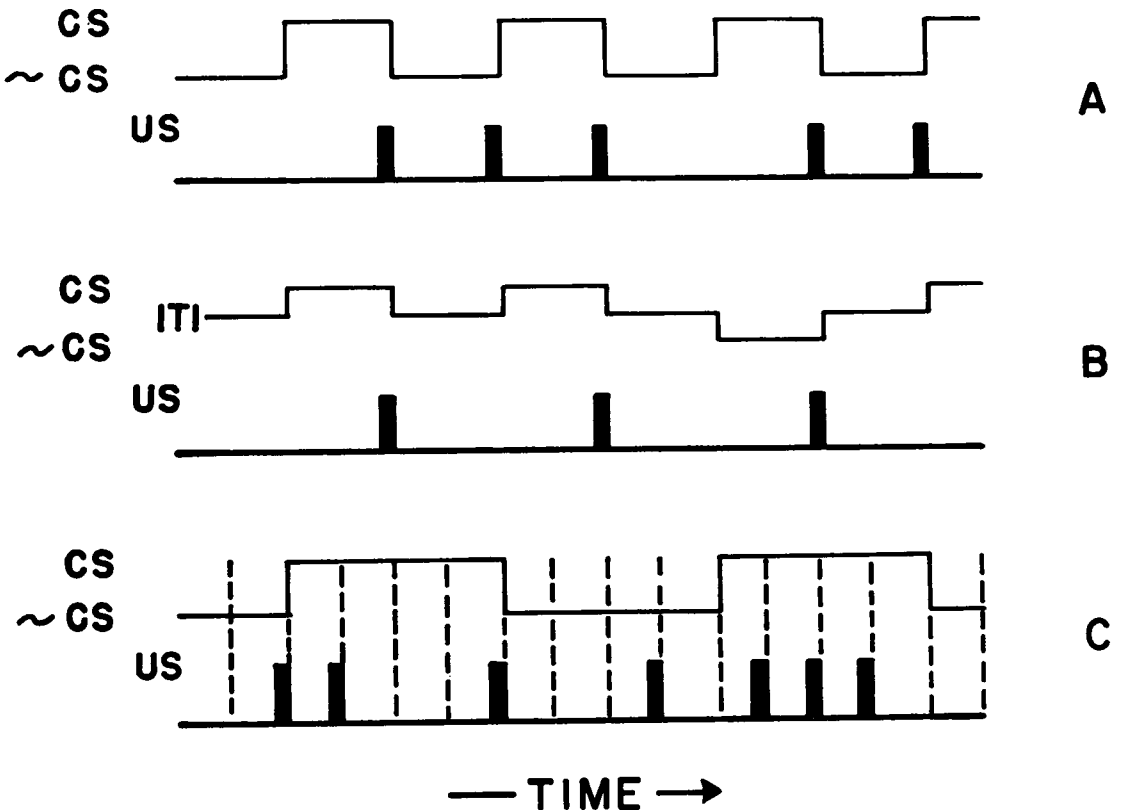


Fig. 2. Trial specification for three possible procedures. In A, CS and \sim CS alternate and shocks are scheduled to occur at the end of some proportion of each kind of trial. In B, a "true" intertrial interval separates successive trials, so that CS and \sim CS trials are not forced to alternate. In C, the time axis has been divided into discrete intervals to accommodate the delivery of multiple shocks in a given CS or \sim CS period. The discrete trial unit time for this analysis must be less than or equal to the shortest CS duration, or intershock time.

A somewhat more flexible alternative results from inserting a third stimulus condition at the end of each CS and \sim CS trial. The third condition would then act as a "true" intertrial interval. This procedure is indicated in Figure 2B. Note that the "on-off" specification for CS and \sim CS is an arbitrary convention. Functional CS and \sim CS trials may be cued by any (preferably very different) stimulus conditions. Equal length CS and \sim CS trials have been presented in Figure 2 simply for convenience. We will later consider unequal durations.

The simple paradigm of Figure 2A is not Rescorla's trial and probability specification for the interior-point procedures of Figure 1. Instead, CS and \sim CS durations were divided into smaller time periods (2 min), each of which met the shock probabilities indicated by the points in Figure 1. This situation is represented schematically in Figure 2C. Each CS and \sim CS period contains several "trials" in a row. A trial specification of this sort, which is not differentially cued, poses difficulties. For example, the same sequence of shocks might be produced by P_1 and P_0 values that were reduced by $\frac{1}{2}$ if the trial "unit" times were also reduced by $\frac{1}{2}$. Indeed, any number of units is possible so long as they are small enough to restrict the number of shocks within a trial unit to one. We will consider later how this arbitrary specification affects the analysis of contingency metrics. For the present purposes, the specification of Figure 1 is unique only up to a multiplicative constant; *i.e.*, the probability values P_1 and P_0 have a unique ratio only (as do the trial frequencies).

B. CONTINGENCY MEASURES

One approach to the metric problem regards the level of conditioning as a function of the discriminability of the difference between CS and \sim CS conditions. In the present context, this discrimination is between the two shock rates or probabilities. On this view, Weber fractions based upon the two shock probabilities might be an appropriate measure. Considerable latitude must be recognized regarding which of several alternative Weber fractions is appropriate. For example, if we set $\Delta P = P_1 - P_0 = (1 - P_0) - (1 - P_1) = Q_0 - Q_1$, then Weber fractions based on delivery probabilities are given by $\Delta P/P_i (i=1,0)$, and Weber fractions based on omission probabilities are given by $\Delta P/Q_i$, where $Q_i = 1 - P_i$.

An alternative is to regard the level of conditioning as a function of the statistical association between the conditional and unconditional stimuli. For example, some measure of the correlation between CS and US would be appropriate on this view. In either case, any contingency measure is referable to the basic contingency table for absolute frequencies of the four possible joint events:

Table I

	\sim US	US	Σ	
CS	a	b	a + b	$P_1 = b/(a + b)$
\sim CS	c	d	c + d	$P_0 = d/(c + d)$
			n	

Trials are assumed to continue over a sufficiently large number of sessions to produce asymptotic conditioning. Thus, if the above cross classification refers to the frequencies per session, continued training amounts to multiplying all entries by the (large) number of sessions. If there exists a measure of contingency that corresponds to asymptotic conditioning levels, then the measure must be invariant over multiplication of the contingency table by a constant. The measures considered here have this property.

Several traditional measures also have the additional property (first suggested by Yule, 1912, discussed by Kendall and Stuart, 1967, pp. 546-546) that they are invariant with multiplication of any row or column by a constant. This property is a very strong one, and measures of association possessing it are *not* feasible candidates for organizing contingency effects. The reason is that such measures regard all values along the edges of the contingency square as equivalent. Consider points on the left edge of the square. They have values of d in Table I equal to zero, since the US is delivered only in the presence of the CS. But then all such tables may be obtained one from another by multiplying the US column by an appropriate constant. Thus, a measure of contingency that is invariant with column multiplication would not be able to describe graded effects obtainable along this edge of the square. Rescorla (1968) found partial reinforcement effects along this edge. In any case, one would expect *a priori* the level of conditioning to decrease as the extinction con-

dition ($P_1 = P_0 = 0$) is approached. Thus, a measure that hopes to describe these effects must not obey Yule's invariance property.

This invariance property is shared by many traditional measures of association between two binary variables. For example, Yule's coefficient of association, Q , and coefficient of colligation, Y , are disqualified by this property, as are some more recent measures, notably Goodman and Kruskal's measures of predictive association, λ_a , λ_b , λ (Goodman and Kruskal, 1954, 1959). A similar exclusion applies to functions of the cross ratio, bc/ad , and functions of the ratio coefficient, P_0/P_1 (Edwards, 1963).

Exclusion of the ratio coefficient means that Weber fractions based on delivery probabilities are also excluded, since these Weber fractions may be represented as functions of P_0/P_1 . Weber fractions based on delivery or omission probabilities define lines in the contingency square radiating from the origin or the upper-right corner. Thus, if Weber fractions were an appropriate contingency measure, then at least two different fractions would be required to describe data that showed graded effects along all the edges of the square.

A classical measure that is not invariant over row or column multiplication is the familiar phi coefficient (root mean square contingency), $\phi = \sqrt{\chi^2/n}$. ϕ has two important properties: (1) it attains its maximum of unity only at the opposing maximally contingent corners of the contingency square, and thus is a potential measure for intermittency effects along any edge of the square: and, (2) if trials for CS and \sim CS have different frequencies in the sense of Figure 2C, then ϕ is sensitive to changes in the "On/Off" duration ratio (Stein, Sidman, and Brady, 1958). Note that for trial "units" of duration t (Figure 2C), if the duration of CS and \sim CS are T_1 , T_0 , respectively, then $T_1/t = a + b$ and $T_0/t = c + d$. With these conventions and restricting P_1 , $P_0 \neq 0, 1$, ϕ may be written²,

$$\phi = \frac{P_1 - P_0}{\sqrt{(1 - P_0 + (1 - P_1)T_1/T_0)(P_1 + P_0T_0/T_1)}} \quad (1)$$

²Equation (1) may be obtained from the standard frequency formulation for χ^2 : $\chi^2 = [ad - bc]^2 / [(a + b)(c + d)(b + d)(a + c)]$, by dividing numerator and denominator by $[(a + b)(c + d)]^2$ and rearranging terms, noting that $T_1/T_0 = (a + b)/(c + d)$.

In the above form, it may be shown that for constant T_1/T_0 ratios, ϕ is monotone increasing with increasing P_1 and with decreasing P_0 . These properties hold for Rescorla's findings for suppression (Figure 1), and his data have been replotted in Figure 3 as a function of ϕ . The data represent suppression ratios obtained on the first day of extinction in a CER paradigm after training at the schedule values indicated in Figure 1. The relation seemed sufficiently linear to warrant the straight line function³.

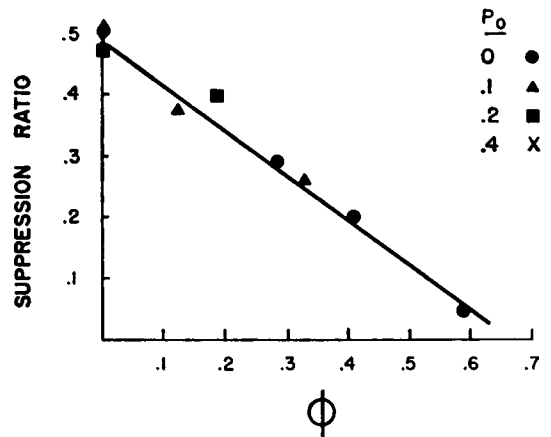


Fig. 3. Suppression ratio as a function of ϕ for Rescorla's (1968) study. The suppression ratio is response rate in the CS, divided by response rate in the CS plus response rate in the \sim CS. When these two rates are equal (no suppression) the suppression ratio is 0.5. Note that 0.5 is approximately the value for all noncontingent points ($\phi = 0$). Increasing suppression is indicated by smaller suppression ratio values as ϕ increases.

Another reason for choosing ϕ is that it has a chance of describing "On/Off" (T_1/T_0) effects. Stein *et al.* (1958) found a strong inverse relation between amount of suppression and T_1/T_0 . Their experimental paradigm was as diagrammed in Figure 2A, except that shock was scheduled at the end of every CS and never occurred in \sim CS. Under these circumstances, the trial unit time is unspecified. Another complication is introduced by temporal

³The linear fit is intended to describe these data over the intermediate range of ϕ values. It does not imply a model. Clearly, the rate of approach to asymptote and the level of suppression at asymptote may be a function of several parameters, independent of ϕ values. Evidently, the present choice of parameters produces linearity of suppression with ϕ .

discriminations of CS onset and shock onset at the end of the CS that are occasionally evident in their data. Nevertheless, a rough approach to specifying \emptyset for these data is possible, and seems worthwhile in the absence of more recent data on the temporal parameters of CER. The trial unit time and P_1 may be specified by using the smallest CS or \sim CS duration in the subject's preceding training history, and defining shock probability as $P_1 = (\text{smallest preceding } T_1 \text{ or } T_0) / (\text{current } T_1)$. Such a specification regards large CS or \sim CS durations as successive trials-in-a-row, where each trial lasts only as long as the shortest CS or \sim CS trial the subject has ever experienced. For example, the first pair of T_1, T_0 values that a subject is exposed to is associated with $P_1 = 1.0$, when $T_1 \leq T_0$. Since shocks never occur in \sim CS, P_0 is always zero. For this case, equation (1) becomes,

$$\emptyset = \sqrt{\frac{P_1}{1 + (1 - P_1)T_1/T_0}} \quad (2)$$

Evidently $\emptyset \rightarrow 0$ as $(T_1/T_0) \rightarrow \infty$, and \emptyset attains the maximum, $\emptyset = \sqrt{P_1}$ as $(T_1/T_0) \rightarrow 0$. These

limits are in the appropriate direction for the On/Off ratio effect. The Stein, Sidman, and Brady data have been recalculated in suppression-ratio form and are plotted against \emptyset in Figure 4. Despite the considerable variance, there is a clear trend toward smaller suppression ratios with increasing \emptyset . Unfortunately, under our construction of the trial unit, increasing relative duration, T_1/T_0 , generally also decreases P_1 , so that the observed trend is not uniquely ascribable to the T_1/T_0 variable.

An assessment of the \emptyset coefficient for a description of temporal parameters requires an experimental paradigm with a "true" inter-trial interval bracketing each CS and \sim CS trial (as described in Figure 2B). The T_1/T_0 variable is then the ratio of trial frequencies $(a + b)/(c + d)$, and may be experimentally manipulated.

C. INTERPRETATION OF \emptyset^2

Mean square contingency, \emptyset^2 , has suffered from obscurity of interpretation in the classical statistical literature. In the present context, \emptyset

STEIN, SIDMAN, & BRADY 1958

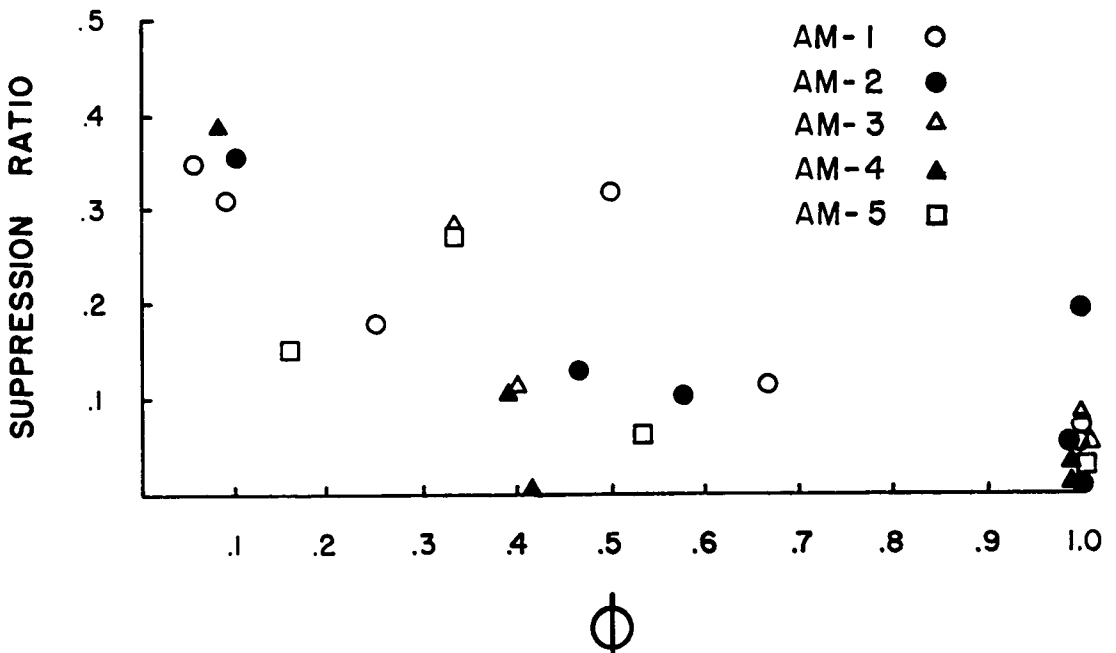


Fig. 4. Suppression ratio as a function of \emptyset for data from Stein, Sidman, and Brady's (1958) study. Their data were reported in the form of the ratio of rate in the signal to rate out of the signal, and have been recalculated in suppression ratio form to make them comparable to the data in Figure 3.

is sensitive to two features of associative learning paradigms that are essential for a description of the effects: both the relative and absolute values of the shock schedules in the two signals appear to be important to the associative learning that results. However, while \emptyset is sensitive to these features, it is not at all clear why behavior ought to be sensitive to \emptyset . That is, it is not clear whether some more fundamental relation is implied by the linearity in Figure 3.

We do not have a detailed description of how behavior makes contact with the shock schedules, but an approach to this problem is implicit in the following probabilistic interpretation of \emptyset^2 . Goodman and Kruskal (1954) noted that a proportional prediction scheme for predicting the second classification in a two-way polytomy reduces to \emptyset^2 in the two-by-two case. Their prediction scheme may be reinterpreted in terms of the classical conditioning paradigm. Imagine that in the contingency table a trial is chosen at random from the population of trials, and subjects guess whether it is shocked or unshocked when:

- (1) they are unaware of which sort of trial (CS or \sim CS) it is, or
- (2) they know which sort of trial it is.

Sequentially, case (1) is a guess about shock on the next trial or "in general", and case (2) is a guess about shock on the present trial. The proportional prediction scheme predicts shock in case (1) with probability, $(b + d)/n$, and in case (2) with probabilities, P_1 for CS and P_0 for \sim CS trials. Goodman and Kruskal show that this proportional prediction scheme implies that the relative reduction in errors of prediction going from case 1 to case 2 is \emptyset^2 .

If this interpretation is taken seriously in the CER situation, it means that the suppression ratio reflects the relative reduction in error probability produced by the CS. However, it is not clear how errors in prediction might contact behavior, even if one construes "prediction" of shock as "anticipation" or "fear" of shock. The \emptyset^2 interpretation suggests that it is reduction in uncertainty that counts, rather than increases in P_1 or decreases in P_0 *per se*.

Possibly, the two kinds of incorrect anticipations produce two different kinds of "cost" to the subject. Failing to anticipate shock when it does occur may result in a functional increase in aversiveness. Lockard's (1963) and

Badia and Culbertson's (1972) data on preference for warned shock support such a view. In fact, the mechanism underlying conditioned suppression is often assumed to be preparatory, or an anticipatory fear reaction incompatible with ongoing behavior. This latter view also suggests a cost interpretation for the second kind of prediction error: anticipation of shock when it is not presented. If anticipation of shock effectively interrupts responding on the appetitive baseline schedule, then incorrect anticipations reduce positive reinforcement density, so to say, unnecessarily. Both Stein's *et al.* (1958) "reinforcements missed" analysis, and Lyon's (1964) data, implicate appetitive reinforcement density as an important parameter of conditioned suppression.

These two sorts of errors evidently combine to produce orderly relative suppression data as a function of \emptyset . The strong generalized suppression found by Seligman (1968, 1969) for the non-contingent procedures suggests that an attentional process may be involved. Non-contingent shock may produce considerable overall suppression, but no differential suppression, since attending to the trial cues does not reduce anticipation errors. Suppression is distributed differentially between the two stimuli only when attending to these stimuli lowers the frequency of "unnecessarily missed" reinforcements and "unanticipated" shocks.

In sum, \emptyset , or some monotone function of \emptyset , appears a reasonable measure of the power of the training contingencies at asymptote in the conditioned suppression paradigm. Other measures fail to describe partial reinforcement effects and temporal duration effects. Although the \emptyset interpretation in the latter case is somewhat strained, sharp experimental tests are easily constructed by scheduling "true" inter-trial intervals following CS and \sim CS trials. Such an experimental arrangement is of interest in its own right because it allows control of trial frequency independently of P_1 and P_0 .

III. INSTRUMENTAL CONTINGENCIES

Instrumental contingencies arrange for relations between responses and consequences that share some features, and not others, of the relations between conditional and unconditional stimuli in classical conditioning. In the classical case, the contingency is between two stimuli, both under experimental control,

whereas in the instrumental case, the experimenter loses one degree of freedom to the subject. This fundamental difference reappears in the following treatment in different forms. It leads to two different ways of specifying instrumental contingencies: one is "ratio-like", and parallels the contingency specifications above; the other is "interval-like", and leads to a different metric specification.

A. CONDITIONAL PROBABILITIES:
CONTINGENCY SQUARE

The conditional probability specification parallels that described above (Figure 1) except that the role of CS and ~CS is replaced by response alternatives. Figure 5 presents the contingency square for an aversive stimulus (Catania, 1971; Church, 1969; Gibbon *et al.*, 1970). The ordinate represents the probability of shock given a response (R), the abscissa the probability of shock given no response (~R). The strict or double implication points at the upper-left and lower-right corners represent traditional punishment and avoidance pro-

cedures respectively. Under strict punishment, a response is the necessary and sufficient condition for shock, whereas under strict avoidance, the lack of a response is the necessary and sufficient condition for shock.

The partial schedules parallel those for classical conditioning. Partial punishment schedules, in which shocks imply responses but not all responses are shocked, are represented on the left edge. Partial avoidance, in which shock trials mean that no response has occurred but not all such trials are shocked, is represented on the lower edge. Partial punishment procedures do not seem to have been investigated in discrete trials, but a graded effect is well documented in free-operant work (*e.g.*, Azrin and Holz, 1966). The schedule on the upper edge might be thought of as partial punishment of not responding: responses always produce shocks (R implies S), but not-responding is also shocked occasionally. Gibbon (1967) studied the two extremes of this condition, and the extinction point, at which shock is never delivered (filled circles in

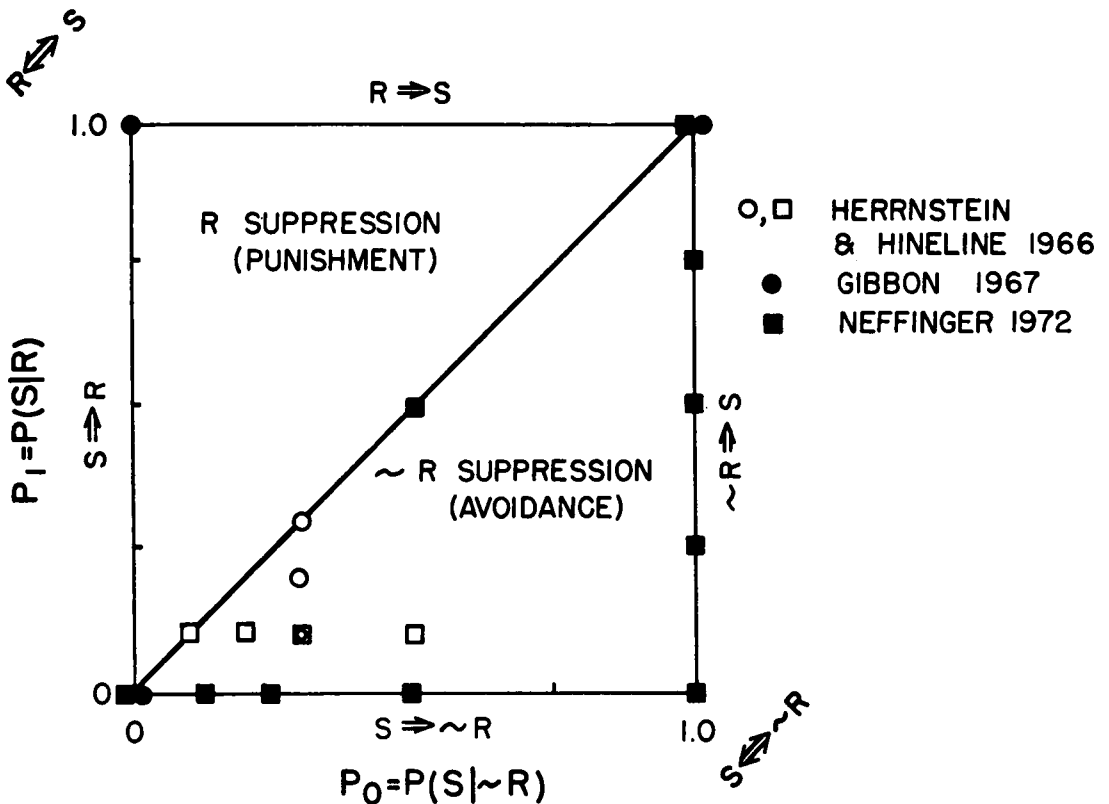


Fig. 5. Contingency square for presentation of an aversive stimulus.

Figure 5). Two different trial signals cued the two conditions on the upper edge, and responding was supported by an appetitive baseline in intertrial intervals. In both kinds of trials, responses terminated the trial and produced a shock ($P_1 = 1.0$). For strict punishment, not-responding resulted in trial termination without shock ($P_0 = 0$), while on non-contingent trials (upper-right corner) shocks occurred at the end of every trial whether the subject responded or not ($P_0 = 1.0$). The result was that responding was more frequent in the non-contingent condition. When shocks were inevitable, subjects responded more than when they were avoidable, and so it seems likely that graded effects may be obtained for intermediate points along this edge also.

Data from partial contingencies have not been collected in discrete trials, although uncued procedures in the "punishment-like" upper-left half of the square are beginning to receive experimental attention (Kadden, 1971; Kop and Snapper, 1970).

"Avoidance-like" procedures in the lower half of the square have received somewhat more attention. Graded contingency effects have been found for free-operant procedures that correspond roughly to interior points. Herrnstein and Hineline (1966) studied rats on schedules comparable to those indicated by the open circles and squares. P_0 values represent the probability of shock delivery at 2-sec intervals when no response was made. The first post-shock response changed this probability to P_1 . Subsequent responses had no effect until the next shock, which changed shock probability back to P_0 . Their experiment focussed on shock density effects, and discrete trials were not explicitly scheduled. However, the placement in Figure 5 is unique up to a multiplicative constant, as with the probability specifications in the classical case (Figure 2C). Under the non-contingent procedures along the diagonal, responding eventually ceased and response rate for contingent procedures increased with increasing distance from the diagonal. Orderly graded effects are probably obtainable from these interior "avoidance-like" points, with response strength increasing with proximity to the strict avoidance point.

The procedures represented by diagonal points do not invariably make responding cease. Studies of non-contingent shock in the free-operant literature suggest that some be-

havior may be controlled by shock density alone (Byrd, 1969; Kelleher, Riddle, and Cook, 1963; McKearney, 1968, 1969; Morse, Mead, and Kelleher, 1967; Stretch, Orloff, and Dalrymple, 1968). One subject in Gibbon's (1967) punishment study maintained responding under the non-contingent condition. Also, recent work by Neffinger (1972) in this laboratory showed that a substantial proportion of subjects trained under the maximal avoidance contingency maintained responding when studied subsequently under non-contingent procedures. Analyzing behavior not under contingency control is beyond the scope of this paper, but it should be clear that the procedures outlined provide a means for studying that control, because they allow parametric variation in non-contingent frequency of the unconditioned stimulus.

Neffinger also studied procedures represented by points on the right and bottom edges in Figure 5. After considerable training, subjects that showed control by the degree of contingency alone (subjects *not* showing the non-associative responding under the control of shock density) eventually ceased to respond as non-contingent conditions were approached. For the bottom edge, most of the drop in responding took place close to the zero point (traditional extinction). The right edge (punishment of avoidance), produced more variability across subjects, but after sufficient exposure most subjects showed a substantial drop only close to the 100% punishment condition. Thus, for those subjects sensitive to a change in contingency, most of the control exerted by the conditional probability of shock occurs close to the non-contingent values. The drop as traditional extinction is approached is somewhat steeper than the drop produced by punishment. This asymmetry is consonant with a discrimination argument proposed below for data of this sort.

If one substitutes food for shock in this scheme, the corresponding contingency square has continuous reinforcement in the upper-left corner, and "food avoidance" in the lower-right corner. The latter condition recently received attention (Williams and Williams, 1969) in assessing stimulus-stimulus contingencies in autoshaping (Brown and Jenkins, 1968). The partial schedules represented by the left edge then correspond to random-ratio food delivery schedules (*e.g.*, Sidley and

Schoenfeld, 1964). Partial contingencies other than random-ratio schedules seem not to have been studied in the appetitive case. In terms of the logic of the contingency analysis, they would appear to repay investigation.

In both appetitive and aversive instrumental situations, measures of behavior corresponding to measures of partial contingencies are not readily available. The reason is fundamental and derives from the loss of one degree of freedom to the subject. In the classical contingency case, a measure of partial contingency such as \emptyset , required a specification of the On/Off ratio, or the frequency of CS to \sim CS trials. In the instrumental case, the corresponding parameter is the ratio of response to non-response trials, and so a measure such as \emptyset is calculable only on a *post hoc* basis. One cannot then argue that partial contingencies control behavior when behavior in turn defines the degree of contingency. This fact is not apparent in the conditional probability specification (Figures 1 and 5), since only two degrees of freedom, both experimentally controlled, are represented there. The next section develops a more general representation of contingencies that includes this additional degree of freedom.

B. ABSOLUTE PROBABILITIES: CONTINGENCY TETRAHEDRON

The more general representation arises from considering absolute rather than conditional probabilities. Absolute probabilities are represented in Table II below. For the moment, S

Table II

	\sim S	S	Σ
R	P_{10}	P_{11}	$P_{10} + P_{11} = P(\mathbf{R})$
\sim R	P_{00}	P_{01}	$P_{00} + P_{01} = P(\sim\mathbf{R})$
Σ	$P(\sim\mathbf{S})$	$P(\mathbf{S})$	$1.0 = P(\mathbf{R}) + P(\sim\mathbf{R})$

$$P_1 = P(\mathbf{S}|\mathbf{R}) = \frac{P_{11}}{P(\mathbf{R})}$$

$$P_0 = P(\mathbf{S}|\sim\mathbf{R}) = \frac{P_{01}}{P(\sim\mathbf{R})}$$

is intended to represent presentation of a "biologically relevant" stimulus, either positive or negative (e.g., food or shock). Cell entries represent long-term asymptotic training levels and are estimated by the frequencies of the conjoint events divided by the (presumably

very large) number of trials over the course of many sessions of training. For example, the strict avoidance contingency maintains $p_{00} = p_{11} = 0$, and subjects distribute response or non-response alternatives by changing the two remaining probability values. An efficient avoidance performance has p_{10} close to 1.0. A discrete-trial analog of continuous reinforcement is represented by maintaining $p_{01} = p_{10} = 0$, and of course in this case, subjects would be expected to maintain p_{11} close to 1.0. Both the strict implications may be represented by *either* the absolute probability specification of Table II, or the contingency-square space. For "interval-like" schedules of reinforcement, however, the absolute probability values are the appropriate representation, and the contingency square is inadequate. Consider a discrete-trial analog of a variable-interval (VI) schedule, arranging, say, one reinforced response for every 12 trials on the average. If trials were scheduled at very short intervals and occupied about 5 sec each, then such a schedule would correspond to a VI 1-min free-operant schedule. The "unlimited hold" feature of such schedules ensures that as long as one response per minute is made, the total number of reinforcers in a given amount of time or, in our case, a given number of trials, remains constant.⁴ That means that p_{11} is fixed by the schedule. In this example $p_{11} = 1/12$. Not responding is never reinforced, so that $p_{01} = 0$, and again subjects distribute response alternatives between the remaining two probability values. Thus, interval schedules on this construction are analogous to holding absolute probabilities constant. Note that the conditional probabilities may be obtained from the absolute probabilities, as indicated below the table. Note also that the dependent variable, response probability, is simply the sum of the two absolute probabilities for the top row.

The table entries must perforce sum to unity, since one of the four joint events must

⁴Strictly speaking, VI schedules in practice rarely hold reinforcements per unit time constant. Generally, VI timing stops once reinforcement is available so that if response rate is low, availability time can represent a substantial contribution to the schedule and lower the overall reinforcement rate in a session. The assumption that VI schedules hold reinforcements per unit time constant or, for this construction, absolute probabilities constant, is an approximation.

occur on any trial. This restriction constrains contingency representations to three degrees of freedom. In the classical case, all three are experimenter controlled; in the present case, two degrees of freedom are available to the experimenter and one is the dependent variable, $P(R)$. The conditional probabilities represent one of several ways of specifying two of the three dimensions available in any contingency table. A more general representation, which subsumes the conditional probability specification, is obtained by considering the space of all possible values of the four absolute probabilities under the constraint that they sum to one. This space is represented by an equilateral tetrahedron (three-dimensional simplex).

Feinberg and Gilbert (1970) studied this space in a statistical context; the reader is referred to their work for a discussion of the relevant mathematics (see also Bush, 1955, for a two-dimensional representation in a different context). The four absolute probabilities are related to the conditional probabilities in ways that may be illustrated by the geometry of the tetrahedral space. In Figure 6, two tetrahedra are shown. In the left figure, the p_{ij} coordinate system is shown, with each vertex representing some $p_{ij} = 1$ and the opposing triangular

face representing $p_{ij} = 0$. These tetrahedra are easily constructed from tri-coordinate paper (Style 359-32, Keuffel and Esser Co., New York, New York) for each face. On the right the $P_0, P_1, P(R)$ coordinate system is indicated. P_0 is represented by angular displacement between the rear face ($p_{01} = 0 = P_0$) and the right face ($p_{10} = 0, P_0 = 1$). Similarly, P_1 is represented by angular displacement between the bottom face ($p_{11} = 0 = P_1$) and the left-front face ($p_{10} = 0, P_1 = 1$). Plane sections have been drawn in the tetrahedron corresponding to $P_0 = 1/4$, and $P_1 = 1/2$. The variable, $P(R)$, corresponds to orthogonal displacement along the line connecting the midpoint of the lower-left edge and the midpoint of the upper-right edge. Plane sections corresponding to $P(R) \neq 0, 1$ form rectangles in the tetrahedron with horizontal and vertical axes representing P_0, P_1 respectively. The contingency square in the tetrahedron becomes a rectangle with sides in the ratio $P(\sim R)/P(R)$. The rectangle corresponding to $P(R) = 2/3$ is drawn in the figure. The maximally contingent corners of the contingency square become edges in the tetrahedron. For example, in the instrumental conditioning paradigms for negative reinforcement, the strict punishment contingency is represented by the upper-left edge ($p_{01} = 0 =$

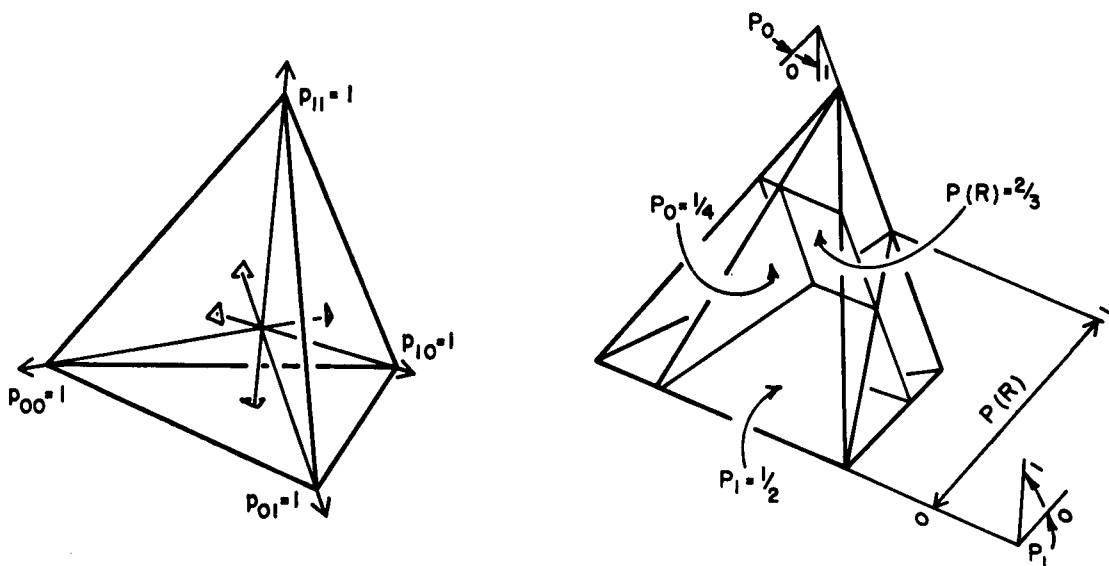


Fig. 6. Tetrahedra representing all possible combinations of absolute probabilities in a contingency table. The left tetrahedron shows the absolute probability coordinates, p_{ij} . The right tetrahedron shows the $P_1, P_0, P(R)$ coordinate system. Planes in the tetrahedron show some specific values of these three coordinates. The plane sections within the tetrahedron are represented as opaque, thus occluding, for example, the lower-rear edge of the tetrahedron, which is shown on the left figure.

p_{10}) and the maximal avoidance contingency is represented by the bottom right front edge ($p_{11} = 0 = p_{00}$).

1. Surface of Independence

A case of special interest is the locus of points corresponding to non-contingent procedures. In the contingency square, the non-contingent cases are represented by points along the $P_1 = P_0$ diagonal. The generalization of this locus for the tetrahedron is the surface generated by this diagonal as response probability moves from 0 to 1. In Figure 7, the non-contingent surface is shown along with the rectangle corresponding to a response probability of $1/4$. Response probabilities less than this value are thus represented below this plane toward the front edge and response probabilities higher than $1/4$ are represented above and behind the plane. For example, subjects that are sensitive to a break in contingency and stop responding under non-contingent procedures produce data falling on the left lower-front edge. Subjects responding to the density of reinforcement under non-contingent presentation produce points internal to the edges on the surface of independence. This surface provides a geometric representation of a fact often noted in studies of avoidance. When subjects are responding maximally it is difficult for a discrimination between contingent and non-contingent conditions to occur.

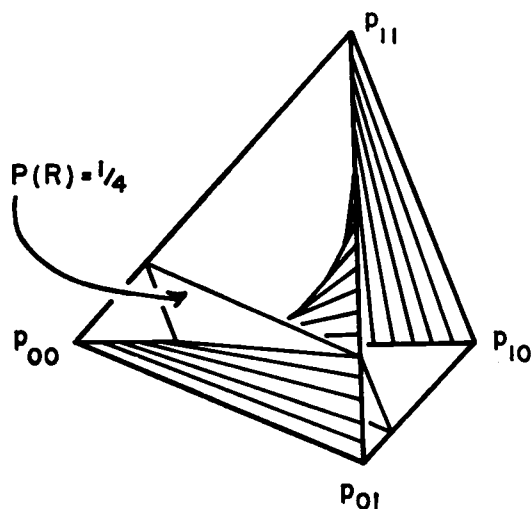


Fig. 7. Surface of independence in the contingency tetrahedron. The rulings shown are those generated by the non-contingent diagonal in the rectangles corresponding to the contingency square.

The frequency of shock is so low that subjects, so to say, never "discover" whether or not shock is scheduled for not responding. This is represented in the tetrahedral volume by the fact that when p_{10} is close to 1.0, the strict avoidance edge is close to the non-contingent surface. Thus, the distance between contingent and non-contingent points in the tetrahedron has an analog in the discriminability of the difference between reinforcement conditions under contingent and non-contingent presentations. At the response extremes of no responding or responding on every trial, there is no difference between these conditions. There is a sense, then, in which a subject's response probability specifies the "distance" between the conditions experienced under contingent and non-contingent procedures. The next section develops three different surfaces in this space that correspond to three different modes of behavior that might result from different contingencies.

2. Matching Surfaces

(a) *Conditional matching.* A recent proposal for organizing effects attributable to shock density variations in avoidance conditioning (Gibbon, 1972) regards subjects as discriminating whether or not responding results in a "worthwhile" improvement in shock density. The complete theoretical proposal is more detailed than suits our present purpose, but a special case of unbiased discrimination is instructive as a contrasting approach to the so-called "Matching Laws" considered next. The theory treats response strength as directly related to the ease with which response-produced and non-response-produced conditions may be discriminated. Suppose that subjects maintain "in memory" a sample of intershock times generated by sequences of response-terminated trials. Under the maximal avoidance contingency, when shocks are delivered only for not responding, such sequences are approximately geometrically distributed, with a mean proportional to the inverse of the probability of failing to respond. When this value is small, intershock times are quite long. Intershock times produced by a non-response series are equal to the time between the end of one trial and the end of the next, as long as $P_0 = 1.0$ (maximal avoidance). If the shock delivery schedule is relaxed by reducing P_0 , intershock intervals produced by not responding become

geometrically distributed, with a mean proportional to the inverse of P_0 . An idealized conceptualization regards a subject as sampling from these two distributions and responding at its maximum avoidance probability whenever intershock time for not responding is shorter than that for responding. As long as P_1 is maintained equal to zero, P_0 must be very low before the non-response intershock interval distribution approaches the intershock times produced by responding (for good avoidance performances). Thus, the theory predicts that behavior should be insensitive to P_0 until P_0 is close to zero. Similarly, when P_0 is maintained at 1.0 and punishment at the end of response trials is introduced, the discrimination hypothesis regards subjects as sampling from the intershock interval distribution on response trials, which is now geometric and inversely proportional (approximately) to P_1 . P_1 , similarly, may be expected to exert its effect most strongly close to 1.0, since it is in this range that the distribution of intershock times for responding approaches that for not responding. These two features of the discrimination hypothesis are consonant with Neffinger's data described earlier. It may be readily shown that when $0 < (P_0, P_1) < 1$, the discrimination of improvement in shock density associated with responding is given approximately by relative conditional probability. That is, if subjects respond whenever the response sample for intershock time is longer than the nonresponse sample, then response probability is approximately

$$P(R) = \frac{P_0}{P_0 + P_1}. \quad (3)$$

But then,

$$\frac{P_1}{P_0} = \frac{P(\sim R)}{P(R)}, \quad (4)$$

and, by the definition of P_{11} , we have

$$P_{11} = P_{01}. \quad (5)$$

Thus, matching of relative conditional probability in the aversive case is equivalent to matching absolute shock rates for the two response alternatives. If subjects adjust responding to the discriminability of improvement in intershock intervals without asymmetries in response preferences, they match absolute punishment frequencies for the two response alternatives.

If reinforcements were substituted for shocks in this scheme, and the more favorable interreinforcement time governed responding, then the role of P_0 and P_1 would be interchanged in Equation (3). This results in a different relation among the absolute probabilities, and it is readily shown that in this case, response probability is given by

$$P(R) = \frac{\sqrt{P_{11}}}{\sqrt{P_{11}} + \sqrt{P_{01}}}. \quad (6)$$

The square root relation, Equation (6), follows directly from reversing P_0 and P_1 in the preceding argument (see also Herrnstein, 1970⁵).

In sum, a discrimination mechanism based on interreinforcement intervals produces matching of relative conditional probabilities. For the aversive case, this means equality of absolute reinforcement rates for the response alternatives, and in the appetitive case, it means matching of the relative square roots of absolute reinforcement rates. The two relations define surfaces in the tetrahedron, which are shown on the left of Figure 8. The equal-shock-rate relation (Equation 5) is the plane bisecting the rising front edge of the tetrahedron, and the square root relation (Equation 6) is the (singly ruled) surface indicated by the rays from the lower rear edge. The two surfaces intersect along the non-contingent diagonal for the contingency square corresponding to a probability of response of $1/2$. This is because the interreinforcement interval distributions producing response indifference must be completely indiscriminable, and this occurs only when $P_0 = P_1$.

(b) *Probability matching.* Conditional matching does not appear to result from appetitive discrete trial experiments when reinforcement is scheduled in the "ratio-like" fashion prescribed by the P_0, P_1 specification. Rather, subjects appear to maximize reinforcement density by choosing the more favorable alternative nearly all the time. However, an

⁵Herrnstein's derivation of Equation 6 for the appetitive case is correct for *conditional* reward probabilities only. He mistakenly describes this relation as appropriate to the probability matching phenomenon in the mathematical learning literature. As will be seen later, that paradigm controls absolute rather than conditional probability of reinforcement.

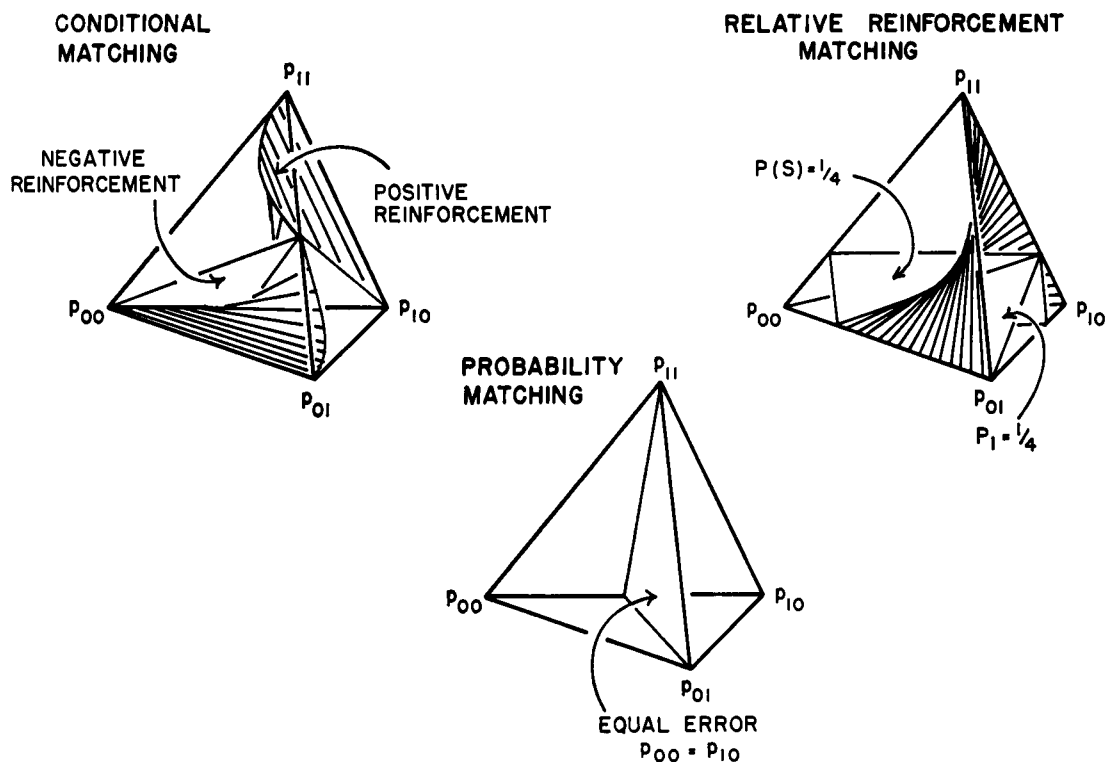


Fig. 8. Surfaces corresponding to three different kinds of matching. On the left, surfaces for the negative reinforcement and positive reinforcement cases are those generated by a discrimination mechanism, which equates response probability to the relative conditional reinforcement probability. The center tetrahedron shows the plane corresponding to probability matching on choice trials with a correction procedure. The right-hand tetrahedron shows the surface of independence corresponding to relative reinforcement matching in concurrent free-operant experiments. The rulings on this surface are those generated by the diagonal in the rectangular section corresponding to reinforcement probability. (Note: Negative reinforcement refers to avoidance-punishment, Figure 5.)

important phenomenon emerges from procedures that include a "correction".

The intent of the correction is to ensure that subjects experience all scheduled reinforcements for the less-frequently reinforced alternative. The early result with these procedures was that response probability to a given alternative approximately equalled the reinforcement probability scheduled for that alternative. The phenomenon originally arose in the context of mathematical theories of learning (e.g., Bush and Mosteller, 1955, pp. 310-328; Estes, 1959). The early data (Brunswik, 1939; Bush and Wilson, 1956; Lauer and Estes, 1954) were not very comprehensive, and later workers questioned the probability matching phenomenon in some contexts. Bitterman and his colleagues maintain that probability matching with correction procedures does occur in the fish (Behrend and Bitterman, 1961), does not in the rat (Bitterman, Wo-

dinsky, and Candland, 1958), and occurs under only some circumstances in the pigeon (Graf, Bullock, and Bitterman, 1964). Shimp (1966, 1973) has shown that probability matching for the pigeon depends heavily on the amount of training and on the kind of correction procedure used.

We will consider first the traditional procedure in which reinforcement is available on every trial, and correction responses are forced to the reinforced alternative when the choice response was "incorrect". A good example is provided by the experiment on probability matching in the fish by Behrend and Bitterman (1961) or by the "guidance" procedure with pigeons studied by Graf *et al.* (1964). In Table III below, choice trials and correction trials are presented separately. Table entries refer to absolute frequencies pooled over some considerable period of training. The lower row has been labelled R_0 to indicate that in

Table III

Choice Trials		Correction Trials			
	~S	S			
R ₁	a	b	R ₁	0	c
R ₀	c	d	R ₀	0	a

n

these experiments, an explicit two-alternative response situation is generally used, rather than the non-response alternative used in the aversive conditioning studies. The entries for correction trials are strictly determined by the choice behavior because every unreinforced choice is followed by a reinforced correction. For example, unreinforced choices of R₁ (a many) are all followed by a reinforced forced choice of R₀ and appear in the R₀, S cell of the correction trials table. The conditional probabilities P₀, P₁ for choice trials are not fixed here. Instead, the total number of reinforcements (n) and the proportion of this total eventually delivered for each alternative are fixed. Letting this proportion be denoted by π = (b + c)/n, the probability matching relation is the finding that the asymptotic probability of R₁ on choice trials equals π. Equivalently,

$$P(R_1) = \frac{a + b}{n} = \pi$$

or

$$\frac{a}{n} = p_{10} = p_{00} = \frac{c}{n}. \tag{7}$$

Thus, the "probability" matching relation is an "error" matching relation. Subjects distribute R₀, R₁ alternatives in such a way that their unreinforced frequencies are equal. This condition defines a plane in the contingency tetrahedron shown in the center panel of Figure 8.

Later variants of this procedure (Graf *et al.*, 1964; Shimp, 1966) allowed repetitive errors on correction trials. The "correct" response was not forced and so the contingency table for correction trials contained non-zero entries in the ~S column. We will consider a variant of this sort later. In either case, response probability is defined in these experiments for choice trials only, and so the equal-error relation is the appropriate description for probability matching with both kinds of correction.

The surfaces corresponding to conditional matching and error matching differ fundamentally. The subjects' control of the degree of freedom corresponding to response probability results in different matching surfaces when the conditioning paradigms control the remaining two degrees of freedom in different ways. In the conditional matching case, P₀ and P₁ are under experimenter control, while in the probability matching case, π and n are under experimenter control.

(c) *Relative reinforcement matching.* The free-operant concurrent experiment has established still another sort of matching: matching of relative response rate on two operanda to relative reinforcement rate on those operanda. The finding requires that variable-interval reinforcements be scheduled on at least one of the operanda, but otherwise it is quite general (Herrnstein, 1970). To analyze this phenomenon in contingency terms we must regard the session as divided into small periods of time (as in Figure 2C) and cross-classify those intervals containing reinforcement or no reinforcement with responding to one or the other of the operanda or not responding. A cross-classification of this sort results in the 3 × 2 contingency Table IV below. The entries represent frequencies divided by total number of "trials" produced by the assumption of a discrete time trial-unit, so that they are estimates of long-term probabilities of the joint events. The VI schedules ensure that all scheduled reinforcers are eventually received by the subject as long as response rates are not very close to zero. This means that the joint event probabilities for S and R₀ or R₁ (p'_{01} or p'_{11}) are held constant by the experimenter.

Table IV

	~S ≡ S ₀	S ≡ S ₁
~(R ₀ or R ₁)	p'_{20}	p'_{21}
R ₁	p'_{10}	p'_{11}
R ₀	p'_{00}	p'_{01}

The cross-classification assumes that the session may be regarded as a long sequence of "trials-in-a-row" with no intertrial intervals. This assumption may be less critical in the present case than in the classical paradigm (Figure 2C) because our purpose here is to obtain from Table IV a conditional table for

R_1 and R_0 entries alone, ignoring nonresponse occurrences⁶. Thus, any trial definition sufficiently short to prohibit multiple responses will suffice. Shorter trial definitions simply increase the nonresponse row in Table IV and do not alter the conditional probabilities defined by

$$p_{ij} = P(R_i S_j | R_0 \text{ or } R_1) = \frac{P'_{ij}}{\sum_{k,l=0}^1 P'_{kl}}, \quad i, j, = 0, 1, \quad (8)$$

where $R_i S_j$ denotes the joint occurrence of response R_i and consequence S_j . Then, we may collapse the 3×2 table into a 2×2 table as in Table II, by identifying p_{ij} as the conditional probability of response i and consequence j given a response on one of the two operanda. The relative rate of response is then

$$\frac{\bar{R}_1}{\bar{R}_0 + \bar{R}_1} = \frac{p'_{10} + p'_{11}}{\sum_{i,j=0}^1 p'_{ij}} = p_{10} + p_{11} = P(R_1 | R_0 \text{ or } R_1). \quad (9)$$

Similarly, the relative rate of reinforcement for R_1 is

$$\frac{\bar{R}_1 S_1}{\bar{R}_0 S_1 + \bar{R}_1 S_1} = \frac{p'_{11}}{p'_{01} + p'_{11}} = \frac{p_{11}}{p_{01} + p_{11}}. \quad (10)$$

The well-established matching relation equates these two relative rates. In terms of the probabilities, p_{ij} ,

$$p_{10} + p_{11} = \frac{p_{11}}{p_{01} + p_{11}} \quad (11)$$

or

$$[P(R_1)] [P(S_1)] = P(R_1 S_1). \quad (12)$$

But the probability form of the relation, Equation 12, is the definition of independence between response and reinforcement variables!

⁶This formulation ignores "time matching" (e.g., Killeen, 1972; Rachlin, 1973), since non-response occurrences are ignored. Time matching may be derived from the present formulation with additional assumptions about non-responding. For our present purposes, however, interest centers on the correspondence between free-operant and probability matching, and thus the time matching development is not pursued here.

Subjects distribute responses across alternatives in such a way as to produce no statistical association between their behavior and its consequences. Thus, there is a sense in which the "Law of Effect" is the "Law of No Effect" or zero contingency between responding and reinforcement. Rachlin (1971) argued a similar point, but from *a priori* grounds unrelated to the independence argument.

This seemingly paradoxical equivalence generates more surprise at first blush than it deserves on consideration. The matching relation is an asymptotic one and requires averaging over a large sample. Subsamples of contingency tables obtained for small segments of a session are likely to show some degree of contingency attributable to local rates of reinforcement.

It is often pointed out that in concurrent schedules, time spent responding to one of the two operanda increases the probability of reinforcement on the other. Thus, the matching relation may result from subject's responding so as to "track" local changes in reinforcement probability (Shimp, 1969). A mechanism of this sort has an analogue in the probabilistic interpretation of \varnothing^2 outlined for the classical conditioning situation. Recall that \varnothing^2 was interpreted as a relative reduction in error probability going from the *a priori* to the *a posteriori* situation. $\varnothing^2 = 0$ when the overall "error" probability, $1 - P(S_1)$, equals the conditional error probabilities, $1 - P_i$, $i = 0, 1$. Or, equivalently, $\varnothing^2 = 0$ when $P(S_1) = P_0 = P_1 = P(R_1 S_1) / P(R_1)$, which is equivalent to the independence relation, Equation 12. Thus, a mechanism that avoids the alternative with the momentarily higher error probability results in the asymptotic equivalence of conditional and overall error rates, or statistical independence.

In the right-hand tetrahedron of Figure 8, the surface of independence corresponding to relative reinforcement matching is generated in a way that exhibits the other set of rulings not shown in Figure 7. Analogously to $P(R)$, $P(S_1)$ values correspond to rectangular plane sections orthogonal to the line connecting the midpoints of the front rising edge ($p_{10} = 0 = p_{00}$, or $P(S_1) = 1$) and the rear bottom edge ($p_{01} = 0 = p_{11}$, or $P(S_1) = 0$). The condition that $P(S_1)$ equal P_1 means that the plane corresponding to P_1 intersects the $P(S_1)$ rectangle along the lower-left to upper-right diagonal. The surface generated by these diagonals along

with the planes corresponding to $P(S_1) = P_1 = \frac{1}{4}$ is drawn in the figure.

Comparing the left and right tetrahedra of Figure 8 shows that the surfaces corresponding to conditional matching for positive reinforcement and relative reinforcement matching are not very different at several points in the tetrahedral volume. At probability of response values near either 0, 1, or 0.5, a discrimination between these two rules for behavior would be difficult to make. It is only for $P(R)$ values near 0.25 and 0.75 that the conditional matching surface for positive reinforcement deviates substantially from the independence surface. Thus, it is at these response levels that a discrimination between the two kinds of matching may be made.

3. Probability Matching Equals Relative Reinforcement Matching

As indicated above, the discrete-trial correction procedure results in equality of the absolute error probabilities ($p_{00} = p_{10}$) on choice trials. In the free-operant concurrent situation, matching results in equality of the conditional error probabilities ($1 - P_0 = 1 - P_1$). We believe that the distinction is superficial rather than fundamental, and results from different data-reporting conventions, rather than different kinds of behavior. The argument is simple but important, and rests upon a similarity between procedures in the two situations. The correction procedure has an analogue in the unlimited hold feature of VI schedules. Once reinforcement is assigned, it is held until collected by the subject. The two situations differ in data-reporting conventions. In the probability matching experiment, response probability is reported for

choice trials only, while in the free-operant situation, there is no distinction between choice and correction trials.

We wish to propose a discrete-trial analogue of the concurrent schedule in which it is reasonable to suppose that relative reinforcement matching would be obtained. Suppose that trials are cued by the onset of choice stimuli and end with responses. Reinforcement is available on only some proportion of the trials, and once available, is held until collected by the appropriate response. If inter-trial intervals are set very short, the central difference between such a procedure and the discretized time analysis of the concurrent situation (Table IV) is the assumption of a response on every trial.

Nevin (1969, Experiment I) studied a paradigm of this sort, but with a 6-sec intertrial interval, and found matching of response probability to relative reinforcement. Shimp (1966, Experiment III) studied a situation close to this with a zero intertrial interval, but with an observing response required to initiate choice trials. Again, matching to relative reinforcement was confirmed.

We may analyze this paradigm from the probability matching approach by distinguishing three sorts of trials in Table V below. Trials in which reinforcement is first available, correction trials in which reinforcement is available and was available but not collected on the preceding trial, and choice trials in which reinforcement is not available are indicated separately.

The two left tables are the same as those for the probability matching experiment (Table III) with the addition of unreinforced correction responses, a' and c' . The probability

Table V

Choice	Correction	Choice												
S ₀ S ₁	S ₀ S ₁	S ₀ S ₁												
R ₁ <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 2px;">a</td><td style="padding: 2px;">b</td></tr><tr><td style="padding: 2px;">c</td><td style="padding: 2px;">d</td></tr></table>	a	b	c	d	R ₁ <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 2px;">a'</td><td style="padding: 2px;">c</td></tr><tr><td style="padding: 2px;">c'</td><td style="padding: 2px;">a</td></tr></table>	a'	c	c'	a	R ₁ <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 2px;">a''</td><td style="padding: 2px;">0</td></tr><tr><td style="padding: 2px;">c''</td><td style="padding: 2px;">0</td></tr></table>	a''	0	c''	0
a	b													
c	d													
a'	c													
c'	a													
a''	0													
c''	0													
R ₀ <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 2px;">c</td><td style="padding: 2px;">d</td></tr></table>	c	d	R ₀ <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 2px;">c'</td><td style="padding: 2px;">a</td></tr></table>	c'	a	R ₀ <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 2px;">c''</td><td style="padding: 2px;">0</td></tr></table>	c''	0						
c	d													
c'	a													
c''	0													
	n													
$n = a + b + c + d,$														
$\pi = \frac{b + c}{n}$														
 reinforcement available		 reinforcement not available												

Table VI

	S ₀	S ₁	Σ
R ₁	A	nπ = b + c	A + nπ
R ₀	$\left(\frac{1-\pi}{\pi}\right)A$	n(1-π) = a + d	(1-π) $\left(\frac{A}{\pi} + n\right)$
Σ	$\frac{A}{\pi}$	n	$\frac{A}{\pi} + n$

matching finding has been restricted in the preceding analysis to trials when reinforcement was available, with the result that errors in the choice table were approximately equal. The extension of the probability matching performance to all three tables constitutes our proposal for the correspondence between probability matching and relative reinforcement matching. If subjects match their probability of response to $\pi = (b + c)/n$, and if they do so for all three tables, then pooling the three kinds of trials into a single contingency table results in Table VI above, where $A = a + a' + a''$. Response probability, $P(R_1)$, is of course equal to π in the pooled table, but π now represents the proportion of the total reinforcements assigned and collected for R_1 . That is, dividing all entries by the total, $A/\pi + n$, one obtains the asymptotic relative frequencies p_{ij} for the joint events, and

$$P(R_1) = \pi = \frac{p_{11}}{p_{01} + p_{11}},$$

which is the relative reinforcement matching or independence relation, Equation 10.

On this construction then, the unlimited hold property of variable-interval schedules represents a sort of generalized correction procedure in which corrections are not cued and not forced. It seems likely that both these features, as well as unreinforced choice trials, are important to matching. For example, Shimp (1966, Experiment II) found that pigeons maximized when correction trials were differentially cued. When reinforcement was scheduled for all choice trials, but corrections were not differentially cued, behavior was intermediate between matching and maximizing (Experiment I). Under these conditions, while corrections are not explicitly cued, non-reinforcement may become a cue for switching. Only when correction trials were completely indiscriminable from choice trials (Experiment III) was matching obtained.

That relative-reinforcement matching and

probability matching are equivalent on this construction represents a marriage of sorts between discrete-trials and free-operant procedures. The free-operant case generalizes the discrete-trial case by allowing three response alternatives and unreinforced trials. That relative-reinforcement matching subsumes probability matching is thus possibly not surprising. But that they are both equal to a zero contingency between behavior and consequences remains puzzling. The arguments for a continual adjustment away from dependence towards independence, outlined above, leave us with some residual uneasiness. Certainly the details of such a dynamic equilibrium deserve study.

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