

**Contingent Capital: Valuation and Risk Implications  
Under Alternative Conversion Mechanisms**

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## ABSTRACT

# Contingent Capital: Valuation and Risk Implications Under Alternative Conversion Mechanisms

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Several proposals for enhancing the stability of the financial system include requirements that banks hold some form of contingent capital, meaning equity that becomes available to a bank in the event of a crisis or financial distress. Specific proposals vary in their choice of conversion trigger and conversion mechanism, and have inspired extensive scrutiny regarding their effectivity in avoiding costly public rescues and bail-outs and potential adverse effects on market dynamics. While allowing banks to leverage and gain a higher return on their equity capital during the upturns in financial markets, contingent capital provides an *automatic* mechanism to reduce debt and raise the loss bearing capital cushion during the downturns and market crashes; therefore, making it possible to achieve stability and robustness in the financial sector, without reducing efficiency and competitiveness of the banking system with higher regulatory capital requirements.

However, many researchers have raised concerns regarding unintended consequences and implications of such instruments for market dynamics. Death spirals in the stock price near the conversion, possibility of profitable stock or book manipulations by either the investors or the issuer, the marketability and demand for such hybrid instruments, contagion and systemic risks arising from the hedging strategies of the investors and higher risk taking

incentives for issuers are among such concerns. Though substantial, many of such issues are addressed through a prudent design of the trigger and conversion mechanism.

In the following chapters, we develop multiple models for pricing and analysis of contingent capital under different conversion mechanisms. In [Chapter 2](#) we analyze the case of contingent capital with a capital-ratio trigger and partial and on-going conversion. The capital ratio we use is based on accounting or book value to approximate the regulatory ratios that determine capital requirements for banks. The conversion process is partial and on-going in the sense that each time a bank's capital ratio reaches the minimum threshold, just enough debt is converted to equity to meet the capital requirement, so long as the contingent capital has not been depleted.

In [Chapter 3](#) we simplify the design to all-at-once conversion however we perform the analysis through a much richer model which incorporates tail risk in terms of jumps, endogenous optimal default policy and debt rollover. We also investigate the case of bail-in debt, where at default the original shareholders are wiped out and the converted investors take control of the firm. In the case of contingent convertibles the conversion trigger is assumed as a contractual term specified by market value of assets. For bail-in debt the trigger is where the original shareholders optimally default. We study incentives of shareholders to change the capital structure and how CoCo's affect risk incentives.

Several researchers have advocated use of a market based trigger which is forward looking, continuously updated and readily available, while some others have raised concerns regarding unintended consequences of a market based trigger. In [Chapter 4](#) we investigate one of these issues, namely the existence and uniqueness of equilibrium when the conversion trigger is based on the stock price.

**keywords:** contingent capital, bail-in debt, market discipline, financial regulation, contingent convertible

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# Introduction

**C**ONTINGENT capital in the form of debt that converts to equity when a bank faces financial distress has been proposed as a mechanism to enhance financial stability and avoid costly government rescues. Variants of this idea differ in the choice of trigger for the activation of contingent capital and in how the capital is held before a triggering event. The Dodd-Frank act calls for regulators to study the potential effectiveness of contingent capital, and specific definitions for triggering events are put forward in a consultative document issued by the Basel Committee on Banking Supervision [5]. The European Commission and the Financial Stability Board (FSB) are also investigating mechanism that convert debt into equity based on regulators' discretion in the context of bank resolution or contractual terms of issued debt. Such discussions are generally in regard to systemically important financial institutions, since automatic provision of capital to a troubled bank can potentially prevent contagion of risk through the system during liquidity crises. According to William Dudley, President of Federal Reserve Bank of New York

*“[contingent capital] has the potential to be more efficient because the capital arrives as equity only in the bad states of the world when it is needed. It also has the benefit of improving incentives by creating two-way risk for bank managements and shareholders. If the bank encounters difficulties, triggering conversion, shareholders would be automatically and immediately diluted. This would create strong incentives for bank managements to manage not only for good outcomes on the upside of the boom, but also against bad outcomes on the downside.”*

The cyclical nature of capital requirements is an important issue in crisis regulation. The problem arises from the fact that banks tend to need more capital when it is hardest to obtain it. Allowing banks to issue contingent capital instruments that can boost their capital cushion against losses during a recession can be a more efficient way to achieve stability in the financial market as compared to the more costly alternative of requiring banks to hold more equity at all times. This idea has also gained increasing support as an effective way to reduce the need for bail-outs and government rescues during a crisis.

Contingent Convertibles (CoCos) and bail-in are among such forms of debt that convert to equity as a firm's assets lose value. They are points on a continuum of such securities differing primarily along two dimensions. One dimension is the level of the trigger for conversion from debt to equity, with CoCos (often classified as *going-concern* contingent capital) converting as a firm nears, but has not yet reached, financial distress, and bail-in (often classified as *gone-concern* contingent capital) converting at the point of non-viability. The second dimension along which these types of securities vary is their conversion ratio and its impact on the original shareholders: the conversion of CoCos dilutes the original shareholders, but a bail-in is accompanied by a reorganization that wipes out the original shareholders – an infinite dilution. Thus, these forms of contingent capital offer firms and regulators two levers in their design, the conversion trigger and the dilution ratio.

## 1.1 Literature Review

Flannery [31] proposed reverse convertible debentures (called contingent capital certificates in Flannery [32]) that would convert from debt to equity based on a bank's stock price. His proposal uses a capital ratio based on the market value of the bank's equity and the book value of its debt. Kashyap, Rajan, and Stein [44] proposed a "lock box" to hold bank funds that would be released in the event of a crisis; in this proposal, the trigger is a systemic event, and not a risk of bankruptcy at an individual institution. McDonald [56] and the Squam Lake Working Group [73] propose contingent capital with a trigger that depends on the health of both an individual bank and the banking system as a whole. The convertible securities designed by the U.S. Treasury for its Capital Assistance Program may be viewed as a type of contingent capital in which banks hold the option to convert preferred shares to common equity and find it advantageous to do so if their share price drops sufficiently low; this contract is studied in Glasserman and Wang [33].

Other designs have been proposed in Bolton and Samama [10], Duffie [25], McDonald [56], Pennacchi, Vermaelen, and Wolf [65], Squam Lake Working Group [73], and Sundaresan and Wang [74]; see Calomiris and Herring [17] or Pazarbasioglu et al. [62], for an overview and comparison. Calello and Ervin [16] outline a bail-in proposal; see Basel Committee on Banking Supervision [6] for a recent regulatory update. Von Furstenberg [79] discusses design features of CoCo's from a marketability point of view.

Among recent alternatives to the mechanisms considered in these papers, Duffie [25] proposes mandatory rights offerings by banks facing financial distress, McAndrews [55] proposes a combination of a rights offering and convertible debt, and Pennacchi, Vermaelen, and Wolff [65] suggest bundling contingent capital with buyback options for equity holders. Brennan and de Longevialle [12] estimate the overall potential size of the contingent capital market at one trillion dollars and discuss investor perspectives on some alternative features.



Alternative proposals for the design of contingent capital have led to work on valuation. McDonald [56] prices contingent capital with a dual trigger through joint simulation of a bank's stock price and a market index. Pennacchi [64] compares several cases by simulation in a jump-diffusion model of a bank's assets. Albul, Jaffee, and Tchitsyi [2] obtain closed-form pricing expressions under the assumption that all debt has infinite maturity and that the conversion trigger is defined by a threshold level of assets. Hilscher and Raviv [41] also uses an asset-level trigger in a tractable structural model and study how design of contingent capital can eliminate risk-shifting incentives of stockholders. Madan and Schoutens [53] incorporate bid-ask spreads in a firm's liabilities. Von Furstenberg [78] builds a binomial tree for the evolution of a bank's capital ratio. Sundaresan and Wang [74] show that setting the conversion trigger at a level of the stock price may result in multiple solutions or no solution for the market price of the stock and convertible debt, raising questions about the viability of contracts designed with market-based triggers; related points are made in a more general context in Bond, Goldstein, and Prescott [11]. Koziol and Lawrenz [46] note that contingent capital can increase incentives for risk-taking by making bankruptcy more remote. Barucci and Viva [4] study the optimal capital structure of a bank issuing perpetual contingent capital and conclude that there is a significant net gain in terms of reduction in the bankruptcy costs and the coupon of straight debt, even though the convertible debt may require a high spread. Berg and Kaserer [7] analyze CoCo bonds issued by Lloyds, Rabobank and Credit Suisse and point out that shareholders can impose part of losses on CoCo investors once losses exceed a certain amount and potentially CoCo bonds can exacerbate risk taking incentives of shareholders and debt overhang problem. They also propose a conversion mechanism which can eliminate such opposite effects. Culp [22] studies contingent capital in the context of corporate finance and concludes that contingent capital can help companies reduce their overall cost of capital by limiting the costs of financial distress and providing more cost-effective management of regulatory capital. Maes and Schoutens [52] discuss counterparty risk, contagion and systemic risk and death-spiral issues

arising from the hedging strategies of the investors. Corcuera et al. [21] look at the problem of pricing CoCo bonds where the underlying risky asset follows an exponential Lévy process incorporating jumps and heavy tails. Metzler and Reesor [54] analyze CoCo bonds in a Merton-type structural model and point out that the rule for determining the conversion price is the single most important feature of the contingent capital contract. Tsyplakov and Powers [75] incorporate endogenous equity capital addition to de-lever and reduce the likelihood of CoCo conversion in a dynamic continuous time pricing model and argue that properly designed CoCo bonds can encourage the bank raise equity capital to avoid an automatic conversion, and as such, CoCos can address the too big to fail problem. Himmelberg and Tsyplakov [42] investigate incentive effects of CoCo bonds and conclude that, if properly designed, CoCo bonds can encourage banks to maintain high capital ratios and firms would preemptively raise equity to avoid the dilutive consequences of automatic conversion.

## 1.2 Complications in Design of Contingent Capital

Many researchers have raised concerns regarding unintended consequences and implications of such instruments for market dynamics. Death spirals in the stock price near the conversion, possibility of profitable stock or book manipulations by either the investors or the issuer, the marketability and demand for such hybrid instruments, contagion and systemic risks arising from the hedging strategies of the investors and higher risk taking incentives for issuers are among such concerns. Across one dimension, many of these issues are related to what an effective conversion trigger is and in particular if such trigger should be based on the market data or accounting measures.

Existing regulatory capital requirements for banks are based primarily on *book values*. Under Basel rules, banks must maintain regulatory capital equal to at least 8% of their risk-weighted assets. U.S. banks also face an overall capital-to-assets constraint with a minimum

of 3% and a threshold of 5% to qualify as “well capitalized.” All of these ratios are based on regulatory accounting measures of debt and capital rather than the market price of a bank’s stock. Existing issuances to date — the contingent core capital (“CoCo”) bonds issued by Lloyd’s Banking Group in November 2009, mortgage lender Yorkshire Building Society in December 2009 and Credit Suisse in February 2011, and the principal write-down bonds issued by Rabobank in March 2010 — all use triggers based on regulatory capital ratios and not market prices. However, book values are backward looking, lag behind market events, are subject to accounting manipulations and may convert too late.

Flannery [31, 32] and Pennachi et al. [65] advocate the use of market data because it is continuously updated, forward-looking, and less vulnerable to accounting manipulation, while noting concerns that market values could potentially be manipulated to trigger conversion. The results of Sundaresan and Wang [74] show that defining an internally consistent market-based trigger can be problematic. Prescott [67] also illustrates the potential pitfalls of using a market-price trigger in contingent capital in terms of multiple equilibria and nonexistence results.

In some circumstances, a conversion mechanism based on the stock price can lead to a *death spiral*, in which the dilution of the existing shareholders’ claims that would occur in a conversion lowers the stock price, leading to more dilution and even further drop in the stock price. Similar concerns are raised in the context of floating-priced convertibles by Hillion and Vermaelen [40]. The Squam Lake Working Group [73] suggests that to avoid this problem each dollar of debt must convert into a fixed quantity of equity shares, rather than a fixed value of equity. If the number of shares issued at conversion is fixed then dilution of existing shareholders and the impact of dilution on the stock price will not exceed a certain amount. Pennacchi et al. [65] point out that a death spiral in the stock price transfers wealth from shareholders to CC investors but the solvency of the bank is not affected. However, managers will be reluctant to issue such instruments that puts the

stockholders at risk of massive dilution.

Another critical issue in the design of conversion mechanism is the possibility of *stock price manipulation*, resulting from the fact that a short position in the stock can be covered by the shares provided by the issuer after conversion. With a trigger based on the stock price or market capitalization of the firm, a speculator can buy the contingent convertible, short-sell the stock to push the price down, trigger conversion, and profit from the gain on the converted shares when the stock returns to its correct level above the trigger price. McDonald [56] argues that manipulation is more profitable with a fixed dollar conversion instead of a fixed share conversion, and asserts that conversion at a premium price would make manipulation less likely and easier to detect. Flannery [31] points out that large financial firms whose shares trade in deep markets are the intended issuers of CoCos and making the trigger apply to a trailing average of share price can mitigate market manipulation for such firms.

### 1.3 Outline

In Chapter 2 we analyze the case of contingent capital with a capital-ratio trigger and partial and on-going conversion. The capital ratio we use is based on accounting or book values to approximate the regulatory ratios that determine capital requirements for banks. The conversion process is partial and on-going in the sense that each time a bank's capital ratio reaches the minimum threshold, just enough debt is converted to equity to meet the capital requirement, so long as the contingent capital has not been depleted. We derive closed-form expressions for the market value of such securities when the firm's asset value is modeled as geometric Brownian motion, and from these we get formulas for the fair yield spread on the convertible debt. A key step in the analysis is an explicit expression for the fraction of equity held by the original shareholders and the fraction held by converted investors in the contingent capital.

In [Chapter 3](#) we develop a capital structure model combining endogenous default, debt rollover, and jumps; these features are essential in examining how changes in capital structure to include CoCos or bail-in debt change incentives for equity holders. Our formulation includes firm-specific and market-wide tail risk in the form of two types of jumps and leads to a tractable jump-diffusion model of the firm's income and asset value. The firm's liabilities include insured deposits and senior and subordinated debt, as well as convertible debt. We derive closed-form expressions to value the firm and its liabilities, and we use these to investigate how CoCos affect debt overhang, asset substitution, the firm's ability to absorb losses, the sensitivity of equity holders to various types of risk, and how these properties interact with the firm's debt maturity profile, the tax treatment of CoCo coupons, and the pricing of deposit insurance. We examine the effects of varying the two main design features of CoCos, the conversion trigger and the conversion ratio, and we compare the effects of CoCos with the effects of reduced bankruptcy costs through orderly resolution. Across a wide set of considerations, we find that CoCos generally have positive incentive effects when the conversion trigger is not set too low. The need to roll over debt, the debt tax shield, and tail risk in the firm's income and asset value have particular impact on the effects of CoCos. We also identify a phenomenon of debt-induced collapse that occurs when a firm issues CoCos and then takes on excessive additional debt: the added debt burden can induce equity holders to raise their default barrier above the conversion trigger, effectively changing CoCos to junior straight debt; equity value experiences a sudden drop at the point at which this occurs. Finally, we calibrate the model to past data on the largest U.S. bank holding companies to see what impact CoCos might have had on the financial crisis. We use the calibration to gauge the increase in loss absorbing capacity and the reduction in debt overhang costs resulting from CoCos. We also time approximate conversion dates for high and low conversion triggers.

Many proposals of contingent capital have advocated using a conversion trigger based on market value of shares. However, as Sundaresan and Wang [\[74\]](#) point out, equity and

contingent capital are claims on the same assets, and their prices must be determined simultaneously. Market prices of shares will adjust to reflect the imminence of conversion. With a market based trigger, this adjustment may delay or precipitate conversion. Such circular feedback between prices and the conversion event can create multiple equilibria or no equilibrium. In [Chapter 4](#) we analyze existence and uniqueness of equilibrium in a continuous-time setting where conversion trigger of contingent capital is based on the stock price.

Like most of the cited papers, we take a structural approach to modeling and valuation. Reduced-form credit risk models of the type in Duffie and Singleton [\[27\]](#) and Jarrow and Turnbull [\[43\]](#) could potentially be used for pricing and hedging CoCos, but they are less well-suited to capturing incentive effects. A limitation of many structural models, including ours, is that they do not incorporate asymmetric information between shareholders and creditors. In [Chapter 3](#) this is partly mitigated by the inclusion of jumps in asset value, which could reflect a sudden release of information, as in Duffie and Lando [\[26\]](#).

## 2

## Contingent Capital With A Capital-Ratio Trigger and Partial Conversion

**E**XISTING regulatory capital requirements for banks are based primarily on *book values*. In this chapter we develop a model to study contingent capital in the form of debt that converts to equity based on a capital-ratio trigger. The bank is required to hold a minimum ratio of equity to total assets (equivalently, it faces an upper bound on leverage); if its asset value drops too low, part of its debt converts to equity in order to maintain the required capital ratio. Our setting is thus similar to Flannery's [31, 32], though he compares the market value of equity to the book value of debt.

Under Basel rules, banks must maintain regulatory capital equal to at least 8% of their risk-weighted assets. U.S. banks also face an overall capital-to-assets constraint with a minimum of 3% and a threshold of 5% to qualify as “well capitalized.” All of these ratios are based on regulatory accounting measures of debt and capital rather than the market price

of a bank's stock. Existing issuances to date — the contingent core capital (“CoCo”) bonds issued by Lloyd's Banking Group in November 2009, mortgage lender Yorkshire Building Society in December 2009 and Credit Suisse in February 2011, and the principal write-down bonds issued by Rabobank in March 2010 — all use triggers based on regulatory capital ratios and not market prices. Flannery [31, 32] and Pennachi et al. [65] advocate the use of market data because it is continuously updated, forward-looking, and less vulnerable to accounting manipulation, while noting concerns that market values could potentially be manipulated to trigger conversion. The results of Sundaresan and Wang [74] show that defining an internally consistent market-based trigger can be problematic. As there are good arguments for both market-value and book-value triggers, both types of securities merit investigation; as the two require somewhat different analysis, in this chapter we limit ourselves to book-value capital ratios.

A distinguishing feature of our analysis in this chapter is that we model partial and on-going conversion of contingent capital as a bank's capital ratio declines, consistent with Flannery's [31] original proposal. (Acharya et al. [1, p.166] call this *progressive* conversion.) Previous models have relied on the assumption that convertible debt is converted in its entirety as soon as a threshold is hit. Instead, we assume just enough conversion takes place to maintain the minimum capital ratio required, leading to a process of continuous conversion. This partial conversion process lends itself to a somewhat larger tranche of convertible debt than all-at-once conversion would, and it makes the full tranche truly contingent, with each layer converted only as needed. With all-at-once conversion, most of the debt is converted too early (or too late).

Partial conversion has important implications for investors: as contingent capital converts to equity, bond holders become shareholders and thus share in any costs or benefits to shareholders of subsequent conversion. We will show that increasing the minimum capital requirement has the effect of slowing conversion and thus shifts more of the dilution cost



from conversion to investors who became shareholders through earlier conversion of debt. A higher capital ratio can therefore benefit the original shareholders if the loss in asset value is sufficiently large; the value of the convertible debt need not be monotone in the required capital ratio.

We undertake our valuation in a structural model, starting from the firm's assets. The firm's capital structure is comprised of senior (unconvertible) debt, contingent capital, and equity. Market values of debt and equity are determined, as usual, by viewing these as claims on the assets; but the book value of debt is calculated by discounting future coupon and principal payments at the yield at which the bond was issued, consistent with accounting rules. We use the resulting book values in our capital ratio. The market and book values of debt must agree at issuance and at maturity, and we incorporate this constraint in our analysis to fix the coupon rates. In our framework, investors in contingent capital hold claims on four types of payments: coupons on unconverted debt, the remaining principal on convertible debt, dividends earned through debt converted to equity, and the value of this equity at the maturity of the debt. We value the contingent capital as the sum of the values of these payments.

Once the contingent capital is exhausted, we assume that a failure to meet the minimum capital requirement results in a seizure and liquidation by regulators. Liquidation occurs prior to bankruptcy in the sense that a bank has positive equity when it first breaches its capital ratio. We incorporate potential liquidation costs for shareholders and also for bond holders in our valuation. Indeed, these costs have a significant impact on our valuations, as does asset volatility. Asset volatility affects both the likelihood of conversion of debt to equity and the upside potential of equity following conversion.

## 2.1 Outline

The rest of this chapter is organized as follows. [Section 3.3](#) presents our model of the firm and the conversion of debt to equity, and [Section 2.3](#) examines how equity is allocated between converted shareholders and the original shareholders as the value of the firm's assets evolve. [Section 2.4](#) introduces dividends. [Section 2.5](#) details the cashflows paid to investors in the firm's senior debt, contingent capital, and equity, and [Section 2.6](#) presents explicit expressions for the values of these cashflows. [Section 2.7](#) closes the model by solving for the coupons on the two types of debt to equate market and book values at issuance; from these we get the yield spread on contingent capital. [Section 2.8](#) extends the model to distinguish between market and book value of assets. [Section 2.9](#) illustrates our results through numerical examples. Detailed calculations leading to our valuation formulas are deferred to appendices.

## 2.2 Model of the Firm

Our model of the firm (or bank) builds on a long line of research on capital structure that includes Merton [[57](#)], Black and Cox [[9](#)], Leland [[50](#)], and numerous subsequent papers. This approach starts by modeling the dynamics of a firm's assets and then prices debt and equity as claims on those assets. In Merton [[57](#)], the firm defaults at the maturity of the debt if its asset value is less than the face value of the debt. In Black and Cox [[9](#)], bankruptcy occurs when asset value drops to an exogenous reorganization boundary, and in Leland [[50](#)], the time of default is chosen strategically by shareholders. In our setting, we will need to provide a corresponding prescription for the conversion of contingent capital to equity, as well as specifying a trigger for liquidation of the firm. We interpret the liquidation event as resulting from seizure by regulators when the firm is unable to sustain its capital requirement which, by design, occurs prior to a traditional bankruptcy event.

Our starting point is a stochastic process  $V_t$  that models the book value of the firm's assets; this process drives the required level of capital in our model, just as accounting-based measure of asset value drive capital requirements in practice. For tractability, we take  $V_t$  to be geometric Brownian motion,

$$\frac{dV_t}{V_t} = (r - \delta) dt + \sigma dW_t, \quad (2.1)$$

where  $W$  is a standard Brownian motion, and  $\delta$  is a constant payout rate to the firm's security holders. In [Section 2.2.1](#), we calculate book values for senior and convertible debt; subtracting the book value of debt from the book value of assets leaves  $Q_t$ , the book value of shareholder's equity which is our measure of capital. (In practice, regulatory capital also includes certain debt instruments not captured in our model.) Our minimum capital requirement is expressed as a lower bound on  $Q_t/V_t$ .

We use these book values to model capital requirements and the conversion of debt to equity. But for valuation, we need to calculate market values: we take the market value of a security to be the expected discounted value of cash flows received by investors, irrespective of book values. In the basic version of our model, we assume that the market value of the firm's assets equals the book value  $V_t$  — in other words, we assume the bank uses mark-to-market accounting for its assets.<sup>1</sup> In the more general version of our model (introduced in [Section 2.8](#)), we represent market and book values of assets through correlated geometric Brownian motions, thus allowing an imperfect relationship between the two and creating some uncertainty about how much market value will be realized when a liquidation is triggered by a book-value-based capital ratio.

In either version of the model, we calculate market values for senior and convertible debt

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<sup>1</sup>This would be the case under Financial Accounting Standard 157. Even prior to this proposed rule, using data from 2001–2005, Calomiris and Nissim [\[20\]](#) report that for many bank assets (in contrast to those of non-financial firms) book value is indeed close to fair value.

as contingent claims on the market value of assets. We pin down the market values of these contingent claims with the constraint that market and book values of debt must coincide at issuance and at maturity: when debt is issued, its book value is recorded at its selling price (market value), and when it matures its book value and market value equal the final payment of principal and interest. In short, we use the book value of assets to drive the conversion of contingent capital and we use the market value of assets to drive the valuation of contingent capital. Keeping track of these two notions of value is essential to pricing securities that depend on an accounting-based trigger.

Our model entails several idealizations and simplifications. We assume that capital ratios can be observed continuously; in practice, regulatory capital is calculated quarterly, but large banks routinely calculate internal “economic capital” on a daily basis, so the necessary information could in principle be monitored for regulatory purposes to trigger conversion. A limitation of our model is that it does not allow for jumps in asset value — a large jump could potentially wipe out all the contingent capital and leave the firm bankrupt. This type of event is beyond the scope of our model.

### 2.2.1 Debt

The firm issues ordinary senior debt as well as junior convertible debt. Both types of debt are issued at time zero and mature at time  $T > 0$ . The senior debt has a face or par value of  $D$  (due at time  $T$ ) and a continuous coupon rate of  $c_2$ , meaning that it pays  $c_2 D$  per unit of time. The debt is issued at a price of  $D_0$ . From an accounting perspective, the effective interest rate for the debt is the discount rate  $d_2$  that equates the cash raised ( $D_0$ ) to the present value of future payments promised on the debt; i.e., the value of  $d_2$  that solves

$$D_0 = De^{-d_2 T} + \int_0^T c_2 D e^{-d_2 s} ds = D \left[ e^{-d_2 T} \left( 1 - \frac{c_2}{d_2} \right) + \frac{c_2}{d_2} \right].$$

The book value of the debt at any intermediate date  $t$ ,  $0 < t < T$ , is then

$$D_t = D \left[ e^{-d_2(T-t)} \left( 1 - \frac{c_2}{d_2} \right) + \frac{c_2}{d_2} \right] \quad (2.2)$$

if the firm has not yet failed. In other words, throughout the life of the debt, book value is calculated by discounting remaining payments at the effective interest rate at which the debt was originally issued.

In the absence of any other type of debt, we would model default as occurring the first time the value of the firm's assets fall below the boundary defined by  $D_t$ ,  $0 \leq t \leq T$ . This is an instance of the mechanism used in Black and Cox [9], though they use an exponential boundary, which corresponds to setting  $c_2 = 0$ . The boundary in Black and Cox [9] is often interpreted as a protective debt covenant, and that interpretation could be applied here. In the case of a regulated bank, which is our focus, the boundary will serve to define a minimum capital requirement the bank must maintain, rather than a privately negotiated covenant. The capital requirement will set the liquidation boundary higher (by the amount of the required capital buffer) than the default boundary (2.2). The bank is seized by regulators before bankruptcy if the capital requirement is not maintained.

Next we introduce convertible debt with a face value of  $B$ , a continuous coupon rate  $c_1$ , and maturity  $T$ , issued at time zero at a price of  $B_0$ . The assumption that all of the debt has the same maturity  $T$  is a simplifying idealization. The effective interest rate  $d_1$  equates  $B_0$  to the present value of the promised payments of coupon and principal,

$$B_0 = B e^{-d_1 T} + \int_0^T c_1 B e^{-d_1 s} ds = B \left[ e^{-d_1 T} \left( 1 - \frac{c_1}{d_1} \right) + \frac{c_1}{d_1} \right].$$

As part of the original contingent capital issuance converts to equity, the remaining principal decreases, but we apply the same effective interest rate  $d_1$  to calculate the book value of the debt outstanding. If the remaining principal at time  $t$  is  $\tilde{B}_t$ , then the book value at time  $t$

is

$$B_t = \tilde{B}_t \left[ e^{-d_1(T-t)} \left( 1 - \frac{c_1}{d_1} \right) + \frac{c_1}{d_1} \right]. \quad (2.3)$$

We take up the conversion mechanism that determines  $\tilde{B}_t$  in the next subsection.

Equations (2.2) and (2.3) take the coupon rates  $c_1$  and  $c_2$  as given. As part of our analysis, we will solve for the values of  $c_1$  and  $c_2$  that make the values of the two types of debt consistent with the overall value of the firm. In particular, we will choose  $c_1$  and  $c_2$  to ensure that the initial values  $B_0$  and  $D_0$  are consistent with market values of debt given the face amounts  $B$  and  $D$  and the dynamics of the firm's asset value.

### 2.2.2 Conversion From Debt to Equity

We denote by  $V_t$  the book value of the firm's assets at time  $t$ . Subtracting the firm's debt from its assets at time  $t$  leaves

$$Q_t = V_t - B_t - D_t; \quad (2.4)$$

we refer to  $Q_t$  as capital, shareholder's equity, or simply equity, but it should be interpreted as a book value or regulatory measure and not as the market value of equity, because (2.2) and (2.3) are accounting based measures of debt. Indeed, the goal of our analysis is to calculate market values based on the contractual terms of the contingent capital.

The firm is required to maintain a capital ratio of at least  $\alpha$ ,  $0 < \alpha < 1$ , which imposes the constraint

$$Q_t \geq \alpha V_t \quad \text{or} \quad (1 - \alpha)V_t \geq B_t + D_t.$$

For example, to model a bank that is required to hold capital equal to 5% of assets, we would set  $\alpha = 0.05$ .<sup>2</sup> As  $V$  fluctuates, a bank could be in danger of violating this requirement;

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<sup>2</sup>We can model a capital requirement tied to risk-weighted assets, rather than total assets, by adjusting the

<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="border-right: 1px solid black; border-bottom: 1px solid black;">Assets</th> <th style="border-bottom: 1px solid black;">Liabilities</th> </tr> </thead> <tbody> <tr> <td style="border-right: 1px solid black;">V = 100</td> <td>D = 60</td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td>B = 30</td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td>Q = 10</td> </tr> </tbody> </table> <p style="text-align: center;">(a)</p>	Assets	Liabilities	V = 100	D = 60		B = 30		Q = 10	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="border-right: 1px solid black; border-bottom: 1px solid black;">Assets</th> <th style="border-bottom: 1px solid black;">Liabilities</th> </tr> </thead> <tbody> <tr> <td style="border-right: 1px solid black;">V = 95</td> <td>D = 60</td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td>B = 30</td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td>Q = 5</td> </tr> </tbody> </table> <p style="text-align: center;">(b)</p>	Assets	Liabilities	V = 95	D = 60		B = 30		Q = 5	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="border-right: 1px solid black; border-bottom: 1px solid black;">Assets</th> <th style="border-bottom: 1px solid black;">Liabilities</th> </tr> </thead> <tbody> <tr> <td style="border-right: 1px solid black;">V = 95</td> <td>D = 60</td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td>B = 25.5</td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td>Q = 9.5</td> </tr> </tbody> </table> <p style="text-align: center;">(c)</p>	Assets	Liabilities	V = 95	D = 60		B = 25.5		Q = 9.5
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**Figure 2.1:** (a) Initial balance sheet with a 10% capital ratio satisfied; (b) after a drop in asset value; (c) after conversion of debt to equity restoring the 10% capital ratio.

the contingent capital converts from debt to equity (decreasing  $B_t$  and increasing  $Q_t$ ) to maintain the constraint as long as possible. Flannery [31] introduced this mechanism using the market value of equity, rather than regulatory capital, to drive conversion.

Before formalizing the conversion mechanism in our model, we consider the example in Figure 2.1. Part (a) of the figure shows an initial balance sheet with 100 in assets, 60 in senior debt and 30 in convertible debt, leaving 10 in shareholder's equity. For simplicity, we consider a minimum capital requirement of 10%, which is just met in (a). In (b), the firm's assets drop to a value of 95; the loss of 5 is absorbed by equity. To meet the capital requirement, the firm converts 4.5 of convertible debt to equity to arrive at the balance sheet in (c), which again just meets the capital requirement.

In our model,  $V$  evolves continuously in time with continuous paths, and we will derive the process of minimal conversion under which conversion takes place precisely at those times  $t$  at which  $Q_t = \alpha V_t$ ; i.e., times at which  $(1 - \alpha)V_t = B_t + D_t$ . We will assume throughout that the bank is initially well capitalized in the sense that  $Q_0 > \alpha V_0$ .

In terms of the amount  $\tilde{B}_t$  of principal remaining (not converted) at time  $t$ , the capital value of  $\alpha$ . The average ratio of risk-weighted assets to total assets over all FDIC banks was 70–75% during 2003–2010, so a capital requirement of 8% of risk-weighted assets could be approximated by a requirement of 5–6% of total assets. For the largest bank holding companies, the asset ratio is 40–60%, corresponding to a lower value of  $\alpha$ . The adjustment in  $\alpha$  could be tailored to a specific institution based on its mix of assets.

constraint is

$$(1 - \alpha)V_t \geq \tilde{B}_t \left[ e^{-d_1(T-t)} \left(1 - \frac{c_1}{d_1}\right) + \frac{c_1}{d_1} \right] + D \left[ e^{-d_2(T-t)} \left(1 - \frac{c_2}{d_2}\right) + \frac{c_2}{d_2} \right]. \quad (2.5)$$

Once the contingent capital is exhausted, the constraint becomes  $(1 - \alpha)V_t \geq D_t$ . Let  $\tau_b$  denote the first time  $(1 - \alpha)V_t = D_t$ , at which point the firm is seized by regulators. Define  $L_t$  by setting

$$(1 - \alpha)L_t = \max_{0 \leq s \leq t} \left\{ \left( B + \frac{D \left[ e^{-d_2(T-s)} \left(1 - \frac{c_2}{d_2}\right) + \frac{c_2}{d_2} \right] - (1 - \alpha)V_s}{\left[ e^{-d_1(T-s)} \left(1 - \frac{c_1}{d_1}\right) + \frac{c_1}{d_1} \right]} \right)^+ \right\}. \quad (2.6)$$

Then we show below that  $(1 - \alpha)L_t$  is the cumulative amount of principal converted up to time  $t$ . More precisely, we claim that if we set  $\tilde{B}_t = B - (1 - \alpha)L_t$ , then (2.5) is satisfied for all  $t \in [0, \tau_b]$ , and  $(1 - \alpha)L_t$  is the least amount of conversion that meets this condition.

Equation (2.6) simplifies when both kinds of debt have constant book value. This holds when the debt is issued at par (i.e.,  $B_0 = B$  and  $D_0 = D$ ) so the coupon rates coincide with the effective interest rates, meaning that  $c_1 = d_1$  and  $c_2 = d_2$ . In this case, equation (2.6) simplifies to

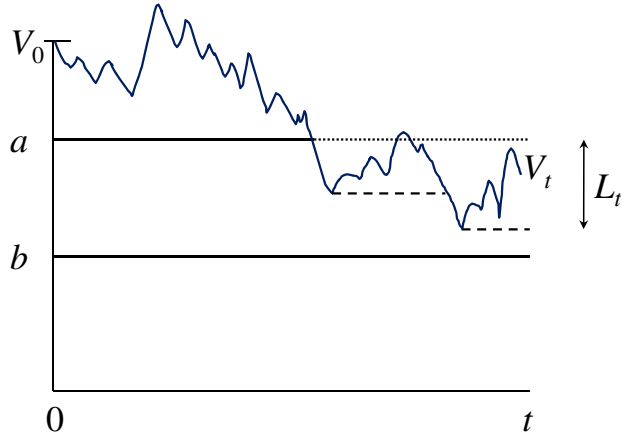
$$(1 - \alpha)L_t = \left( B + D - (1 - \alpha) \min_{0 \leq s \leq t} V_s \right)^+. \quad (2.7)$$

The conversion process in this case becomes easier to visualize if we introduce two thresholds

$$a = \frac{B + D}{1 - \alpha}, \quad b = \frac{D}{1 - \alpha}. \quad (2.8)$$

Under our standing assumption that the capital constraint is satisfied at time zero,  $V_0 > a$ . Conversion starts when  $V$  first hits  $a$ . Subsequently, at each instant at which  $V$  hits a level lower than any previously reached, additional contingent capital is converted to satisfy the constraint. Once  $V$  hits  $b$  (which happens at  $\tau_b$ ), the contingent capital has been fully





**Figure 2.2:** Illustration of the conversion process. Conversion begins when  $V$  reaches the upper boundary  $a$ . The total amount converted to time  $t$  is  $(1 - \alpha)L_t$ , where  $L_t$  is the distance from the running minimum of  $V$  to  $a$ , capped at  $a - b$ .

converted. See Figure 2.2. The process  $L$  is given by

$$L_t = \min \left\{ \left( a - \min_{0 \leq s \leq t} V_s \right)^+, a - b \right\}, \quad \text{for all } t \in [0, T]. \quad (2.9)$$

The width  $a - b$  is  $(1 - \alpha)$  times the face value  $B$  of contingent capital. A similarly tractable case holds when the two types of debt pay no coupon and have the same effective interest rate – that is, when  $c_1 = c_2 = 0$  and  $d_1 = d_2 = d$ .

We formalize the conversion mechanism in the following result, in which we view (2.6) as a mapping from a path of  $V$  to a path of  $L$ :

**Proposition 2.2.1.** *Let  $D$ ,  $B$ ,  $c_1$ ,  $c_2$ ,  $d_1$ , and  $d_2$  be given. The function  $\{L_t, t \in [0, \tau_b]\}$  defined by applying (2.6) to  $\{V_t, t \in [0, \tau_b]\}$  is the only function with the following properties:*

- (i)  $L$  is increasing and continuous with  $L_0 = 0$ ;
- (ii)  $V_t - (B - (1 - \alpha)L_t)(e^{-d_1(T-t)}(1 - \frac{c_1}{d_1}) + \frac{c_1}{d_1}) - D_t \geq \alpha V_t$  for all  $t \in [0, \tau_b]$ ;
- (iii)  $L$  increases only when equality holds in (ii).

Any function satisfying (i) and (ii) is greater than or equal to  $L$  on  $[0, \tau_b]$ .

Condition (i) is natural for the process of cumulative conversion. Condition (ii) states that conversion occurs to preserve the required capital ratio until  $\tau_b$  when the contingent capital is exhausted. Condition (iii) states that conversion occurs only as needed – when the firm is at its minimum capital requirement. The result follows from the standard reflection mapping (as in Harrison [38, p.21]) applied to the function

$$V_t - \frac{1}{1 - \alpha} \left( B[e^{-d_1(T-t)}(1 - \frac{c_1}{d_1}) + \frac{c_1}{d_1}] + D_t \right).$$

The proposition determines  $L$  only up to the time  $\tau_b$  when the contingent capital has been fully converted. Using (2.6) or the special case in (2.9), we can conveniently extend the definition of  $L$  to the interval  $[0, T]$ , even if  $\tau_b < T$ .

### 2.3 Equity Allocation

We will value the contingent capital bond by calculating the expected present value of the payments to the holder of the security. The payments include coupons (paid continuously in proportion to the unconverted debt), any remaining principal at maturity, a fraction of the firm's equity earned through conversion, and dividends paid on a fraction of equity. From the analysis in the previous section, we can determine how much of the contingent capital remains unconverted at each point in time. To value the equity component as the bond converts, we need to analyze what fraction of the firm's equity is held by investors who were converted from contingent capital holders to equity holders. We limit ourselves to the case  $c_1 = d_1$  and  $c_2 = d_2$  which, as explained in the previous section, equates book value to remaining face value for both kinds of debt.

To motivate the analysis that follows, consider again the example of [Figure 2.1](#). Suppose, for simplicity, that the firm starts with 10 shares outstanding. By writing down 4.5 in convertible debt in (c), the firm automatically adds 4.5 to equity, but how the total equity is apportioned to the prior and new shareholders depends on how many new shares are issued in exchange for the converted debt. We introduce a conversion ratio  $q > 0$ , which is the book value of equity received by the contingent capital investors for each dollar of face value of debt converted. If  $q = 1$ , then in (c), the converted investors need to get 4.5 in book value of equity. This is accomplished by issuing them 9 shares, since they then own a fraction  $9/(10 + 9)$  of the firm, and  $9/19$ ths of the total equity of 9.5 is indeed 4.5. If  $q = 2$ , then they should get 180 shares: this gives them a fraction  $180/(10 + 180)$  of the total equity of 9.5 for a book value of 9, which is indeed twice the book value of the debt they gave up. The dilution leaves the original shareholders with 0.5 in book value, or  $1/18$ th of the total equity. The conversion ratio  $q$  has no effect on the total amount of equity, but it determines how the equity is divided between the original and converted shareholders. We need to keep track of this allocation to determine the market value of the convertible debt. Book value of equity is not, by itself, a direct measure of market value; but the proportions of book value of equity held by the two types of investors determine how cash flows are allocated, and the market value of the contingent capital is the expected discounted value of all cash flows received by the investors in these securities.

We will derive an expression for the amount of equity held at any time by the original equity investors. As a lead-in to the continuous-time setting, we consider a discrete-time formulation with a discrete transition over a small interval  $\Delta t$  and write  $V_{t+\Delta t} = V_t + \Delta V_t$ . Suppose (as in [Figure 2.1a](#)) that the firm is just at the capital ratio boundary at time  $t$  and it suffers an asset loss  $\Delta V_t < 0$ . From (2.9) (and [Proposition 2.2.1](#)), we know that  $L$  increases when  $V$  reaches a new minimum and  $\Delta L_t = -\Delta V_t$ . The resulting amount of equity

following conversion is given by

$$Q_{t+\Delta t} = Q_t + \Delta V_t + (1 - \alpha)\Delta L_t = Q_t + \alpha\Delta V_t,$$

the minimal amount of additional equity required to preserve the capital ratio (as in Fig. 2.1c).

Let  $Q^o$  denote the amount (book value) of equity held by the original shareholders, and let  $\pi_t = Q_t^o/Q_t$  denote the fraction of equity they own. Suppose the conversion at time  $t$  is the first to occur, so that the equity is fully held by the original shareholders just before conversion and  $Q_t^o = Q_t$ . Then

$$Q_{t+\Delta t}^o = Q_t^o + \Delta V_t - (q - 1)(1 - \alpha)\Delta L_t.$$

In other words, the original shareholders absorb the full loss  $\Delta V_t$  in asset value, and they lose an amount  $(q - 1)(1 - \alpha)\Delta L_t$  to the new shareholders as a result of the conversion. More generally, if the original shareholders own a fraction  $\pi_t$  of the equity at time  $t$ , then they absorb a fraction  $\pi_t$  of the losses, and we have

$$Q_{t+\Delta t}^o = Q_t^o + \pi_t (\Delta V_t - (q - 1)(1 - \alpha)\Delta L_t). \quad (2.10)$$

To formulate a precise result, we work directly in continuous time. We defined  $Q_t$  in (2.4). Under our constant book-value condition  $c_1 = d_1$ ,  $c_2 = d_2$ , (2.4) becomes

$$Q_t = V_t - [B - (1 - \alpha)L_t] - D, \quad (2.11)$$

and the expression

$$dQ_t = dV_t + (1 - \alpha)dL_t \quad (2.12)$$

is well-defined because  $V$  is geometric Brownian motion and  $L$  has increasing paths. We introduce the process  $Q^o$  by setting

$$dQ_t^o = \frac{Q_t^o}{Q_t} (dV_t - (q-1)(1-\alpha)dL_t), \quad 0 \leq t \leq \tau_b, \quad (2.13)$$

with initial condition  $Q_0^o = Q_0$ . We interpret  $Q^o$  as the equity held by the original shareholders: equation (2.13) says that the change in their equity is their share of the change in asset value plus their share of the transfer to new shareholders upon conversion. Using (2.12) to write this equation as

$$\frac{dQ_t^o}{Q_t^o} = \frac{dQ_t}{Q_t} - q(1-\alpha)\frac{dL_t}{Q_t} \quad (2.14)$$

offers the following interpretation: the percentage change in the book value of equity held by the original shareholders  $dQ_t^o/Q_t^o$  equals the overall percentage change  $dQ_t/Q_t$  so long as no conversion occurs; at an instant of conversion, the percentage change in book value held by the original shareholders is reduced by the fraction of equity transferred to the new shareholders. (In Figure 2.1, (2.14) describes the transition from (a) to (c) with  $q = 1$ ,  $Q_t^o = Q_t = 10$ ,  $dQ_t = -0.5$ ,  $dQ_t^o = -5$ , and the converted amount  $(1-\alpha)dL_t = 4.5$ .) Because  $dL_t = 0$  for  $t > \tau_b$ , based on (2.14) we extend  $Q_t^o$  beyond  $\tau_b$  if  $\tau_b < T$  by setting

$$Q_t^o = (Q_t/Q_{\tau_b})Q_{\tau_b}^o, \quad t \in (\tau_b, T], \quad (2.15)$$

so the fraction  $Q_t^o/Q_t$  does not change in  $[\tau_b, T]$ . The following result confirms that these definitions are meaningful and that they lead to an explicit solution.

**Theorem 2.3.1.** *Suppose  $B_t \equiv B$  and  $D_t \equiv D > 0$  for  $t \in [0, T]$ . Then (2.14) and (2.15) have exactly one solution, and it is given by*

$$Q_t^o = Q_t \left( \frac{a - L_t}{a} \right)^{(q\frac{1-\alpha}{\alpha})}, \quad 0 \leq t \leq T. \quad (2.16)$$

Consequently, the fraction of equity held by the original shareholders at time  $t$  is given by

$$\pi_t = \left( \frac{a - L_t}{a} \right)^{\left( q \frac{1-\alpha}{\alpha} \right)} = \left( 1 - \frac{(1-\alpha)L_t}{B+D} \right)^{\left( q \frac{1-\alpha}{\alpha} \right)} = \left( \min \left\{ 1, \frac{(1-\alpha) \min_{0 \leq s \leq t} V_s}{B+D} \right\} \right)^{\left( q \frac{1-\alpha}{\alpha} \right)}, \quad (2.17)$$

for  $t \in [0, T]$ .

A remarkable feature of (2.17) is that the fraction of equity held by the original shareholders at any time  $t$  depends only on the minimum asset value reached up to time  $t$ . Different paths of  $V$  may produce very different paths for the conversion process and may result in different terminal values for equity; and yet, if they reach the same minimum asset value, they leave the original shareholders owning the same fraction of the firm. The total amount of contingent capital converted to time  $t$  is  $(1-\alpha)L_t$ , and it is interesting that the dependence of  $\pi_t$  on this amount is nonlinear yet explicit.

We note some properties of (2.17). If  $L_t = 0$  (i.e., if  $V$  never reaches the capital-ratio trigger  $a = (B+D)/(1-\alpha)$  in  $[0, t]$ ), then  $\pi_t = 1$ , reflecting the fact that no conversion has occurred. If  $L_t = b - a$  (i.e., if  $V$  reaches the lower boundary  $b = D/(1-\alpha)$  at which the required capital ratio can no longer be sustained), the contingent capital is fully exhausted, but the original shareholders are not wiped out; they own a fraction

$$\left( \frac{b}{a} \right)^{\left( q \frac{1-\alpha}{\alpha} \right)} = \left( \frac{D}{B+D} \right)^{\left( q \frac{1-\alpha}{\alpha} \right)} \quad (2.18)$$

of the remaining equity  $V_t - D = \alpha D/(1-\alpha)$ . The following result records the dependence of  $\pi_t$  on the minimum ratio  $\alpha$ :

**Corollary 2.3.2.** *The proportion  $\pi_t$  of equity owned by the original shareholders is an increasing function of  $\alpha$  with  $\min_{0 \leq s \leq t} V_s$  held fixed if*

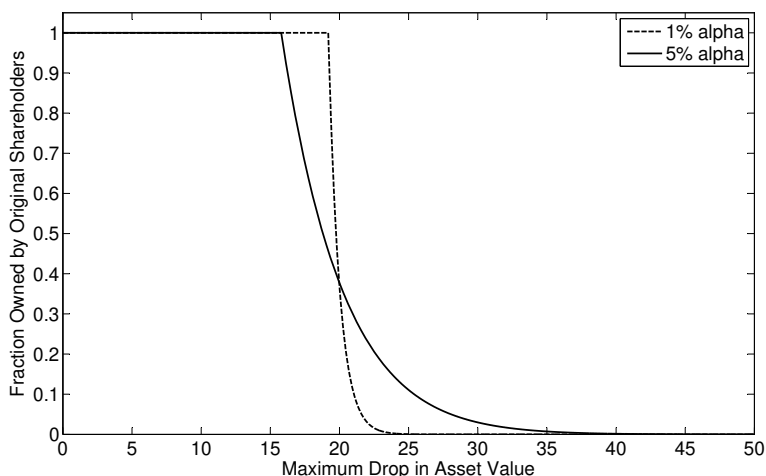
$$\min_{0 \leq s \leq t} V_s < \frac{\exp(-\alpha)}{1-\alpha} (B+D); \quad (2.19)$$

*it is decreasing in  $\alpha$  if the opposite inequality holds.*

This result is easily established by differentiating the third expression for  $\pi_t$  given in (2.17). We interpret the corollary as stating, perhaps surprisingly, that a higher required capital ratio ultimately protects the original shareholders: if the loss in asset value is sufficiently large, the original shareholders keep a higher fraction of the firm under a higher (and thus more stringent) capital ratio  $\alpha$ . Moreover, the total amount of shareholder equity  $Q_t$  is itself an increasing function of  $\alpha$ ; this follows from (2.9) and (2.11).

To interpret the condition in the corollary, recall that conversion of debt to equity begins when asset value reaches  $a = (B + D)/(1 - \alpha)$ . For small  $\alpha$ ,  $\exp(-\alpha) \approx 1$ , so the threshold in (2.19) is nearly the same as the trigger for conversion. Thus, at higher  $\alpha$ , conversion is triggered sooner (resulting in a lower  $\pi$ ), but if asset value continues to decline, a higher  $\alpha$  results in a higher fraction of equity held by the original shareholders.

This phenomenon is illustrated in Figure 2.3 for a firm with  $D = 50$ ,  $B = 30$ , and initial asset value  $V_0 = 100$ . The figure plots  $\pi_t$  against the maximum loss in asset value,  $V_0 - \min_{0 \leq s \leq t} V_s$  for two different values of  $\alpha$ . Conversion begins when the loss in value reaches  $V_0 - (B + D)/(1 - \alpha)$ , which is approximately 15.8 with  $\alpha = 0.05$  and 19.2 with  $\alpha = 0.01$ . The higher capital ratio triggers conversion sooner; however, once conversion begins at the smaller value of  $\alpha$ , the two curves quickly cross. Indeed, from the corollary we know that once the loss exceeds  $V_0 - \exp(-.01)(B + D)/(1 - .01) \approx 20$ , any capital ratio greater than 1% keeps a higher fraction of equity with the original shareholders.



**Figure 2.3:** Comparison of the fraction  $\pi_t$  held by the original shareholders as a function of the maximum loss in asset value up to time  $t$ , for two values of the capital ratio  $\alpha$ .

## 2.4 Dividends and Debt Service Payments

As is standard in much of the capital structure literature (e.g., Leland and Toft [51]), we will assume that the firm's assets generate cash at a rate proportional to their value (in our setting, book value), and these cashflows are used to service the firm's debt and to pay dividends to shareholders. If the firm pays out a constant fraction  $\delta \in (0, 1)$  of its asset value, then from time  $t$  to  $t + dt$ , the cashflow available will be  $\delta V_t dt$ .

With a coupon rate of  $c_2$  and a face value of  $D$ , the senior debt requires payments at rate  $c_2 D$  prior to maturity. Interest on debt is tax deductible, and we model this as in, e.g., Leland [50] and Leland and Toft [51]: if the firm's marginal tax rate is  $\kappa \in (0, 1)$ , it incurs an after-tax cost rate of  $(1 - \kappa)c_2 D$  in servicing the senior debt. We could apply different marginal tax rates  $\kappa_1, \kappa_2$  to the two types of debt<sup>3</sup> to get after-tax coupon rates  $(1 - \kappa_i)c_i$ ,  $i = 1, 2$ ; for simplicity, we use a common value  $\kappa$ . The outstanding convertible debt at time

<sup>3</sup>It is unclear if coupons on contingent capital would be tax deductible under the current tax code in the U.S. because the conversion feature may make the debt too equity-like. This possibility could be modeled by taking  $\kappa_1 = 0$ . But tax rules could also be changed if regulators sought to create incentives for banks to hold more of their debt in the form of contingent capital.



$t$  is  $B - (1 - \alpha)L_t$ , requiring an after-tax payment at rate  $c_1(1 - \kappa)[B - (1 - \alpha)L_t]$ .

The difference

$$\delta V_t - (1 - \kappa)(c_1(B - (1 - \alpha)L_t) + c_2D)$$

between the rate at which cash is generated and the rate at which it is paid to debt holders is the rate at which dividends are paid to shareholders, whenever this difference is positive. When the difference is negative, the firm is generating insufficient cash to service its debt. As is customary, we interpret a negative dividend as the issuance of a small amount of new equity, which brings cash into the firm. This cash is immediately paid out to the debt holders, so the issuance has no impact on the total amount of capital in the firm.

We will assume, in fact, that the new equity is issued to existing shareholders (as in a rights offering) and that the original and converted shareholders participate in equal proportions. Thus, the proportion  $\pi_t$  of the firm owned by the original shareholders is unchanged. The new shareholders then receive a net cashflow at rate

$$(1 - \pi_t)(\delta V_t - (1 - \kappa)(c_1(B - (1 - \alpha)L_t) + c_2D)), \quad (2.20)$$

regardless of whether this is positive (in which case it is a dividend) or negative (in which case it is the cost of raising equity). We will need to incorporate this stream of payments into our overall valuation of the contingent capital.

Two parameter ranges for the coupon and payout rates merit special mention. We know that as long as the firm has not exhausted its convertible debt, it can maintain the minimum capital ratio by converting debt into equity; that is, it can maintain the bound

$$(1 - \alpha)V_t \geq B - (1 - \alpha)L_t + D,$$

with equality holding at the instants of conversion. It follows that, if

$$(1 - \alpha)\delta > (1 - \kappa) \max\{c_1, c_2\}$$

the firm always generates enough cash to service its debt, and shareholders always earn a dividend. In contrast, if

$$(1 - \alpha)\delta < (1 - \kappa) \min\{c_1, c_2\}$$

then the firm will stop paying a dividend — and will start issuing small amounts of equity — in advance of any debt converting to equity.

## 2.5 Decomposition of Payments on Convertible and Senior Debt

In this section, we decompose the payments to holders of the convertible debt into a principal payment, coupon payments, dividends on converted equity and a terminal equity payment. We decompose payments on the senior debt contingent on the firm's ability to maintain the required capital ratio. These decompositions prepare the way for the valuations in the next section.

The horizon for the valuation is the smaller of the debt maturity  $T$  and the time  $\tau_b$  at which  $V$  first hits  $b = D/(1 - \alpha)$ . At  $\tau_b$ , the firm has exhausted its contingent capital and can no longer sustain the required capital ratio; as before, we assume the firm is then seized by regulators and liquidated.<sup>4</sup> The firm still has equity at this point, but not enough to meet the capital requirement. To capture the possible loss in value from seizure, we assume that shareholders recover a random fraction  $X_1 \in [0, 1]$  of the equity value at  $\tau_b$ , the remaining fraction  $1 - X_1$  representing a deadweight cost. (An alternative loss mechanism is

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<sup>4</sup>An alternative interpretation is that the firm undergoes a distressed sale, so the full value of the assets is not recovered, but the equity holders need not be wiped out.

the delayed recapitalization used in Peura and Keppo [66].) Similarly, we apply a random recovery fraction of  $X_2 \in [0, 1]$  to senior debt. We assume that  $X_1$  and  $X_2$  are independent of  $V$  but not of each other. Indeed, to enforce absolute priority of debt over equity, we need  $P(X_2 = 1 | X_1 > 0) = 1$ . Independence between  $(X_1, X_2)$  and  $V$  will imply that only the expected recovery rates  $R_i = E[X_i]$ ,  $i = 1, 2$ , enter into our valuations. These can satisfy  $R_1 > 0$  and  $R_2 < 1$  without violating absolute priority. As just one illustration, any  $0 \leq R_1 \leq R_2 \leq 1$  can be realized as expected recovery rates while satisfying absolute priority by assigning to  $(X_1, X_2)$  the outcomes  $(1, 1)$ ,  $(0, 1)$ , and  $(0, 0)$  with probabilities  $R_1$ ,  $R_2 - R_1$ , and  $1 - R_2$ , respectively.

### 2.5.1 Convertible Debt

We use  $r > 0$  to denote a fixed (risk-free) interest rate at which to discount all payoffs for valuation. The discounted payoffs of the components of the convertible debt are as follows:

- *principal payment at maturity:*

$$e^{-rT} (B - (1 - \alpha)L_T) \quad (2.21)$$

- *earned coupon:*

$$\int_0^T e^{-rs} c_1 (B - (1 - \alpha)L_s) ds \quad (2.22)$$

- *equity earned through conversion:*

$$e^{-rT} (1 - \pi_T) (V_T - [(B - (1 - \alpha)L_T) + D]) \mathbf{1}_{\{\tau_b > T\}} + e^{-r\tau_b} (1 - \pi_{\tau_b}) X_1 \alpha V_{\tau_b} \mathbf{1}_{\{\tau_b \leq T\}} \quad (2.23)$$

- *net dividends:*

$$\int_0^{\min\{T, \tau_b\}} e^{-rt} (1 - \pi_t) (\delta V_t - (1 - \kappa) (c_1 (B - (1 - \alpha)L_t) + c_2 D)) dt \quad (2.24)$$

In (2.21),  $(1 - \alpha)L_T$  is the total amount of debt converted to equity, so  $B - (1 - \alpha)L_T$  is the remaining principal at maturity. Similarly, in (2.22),  $B - (1 - \alpha)L_s$  is the remaining principal at time  $s$ , and multiplying this expression by  $c_1$  yields the rate at which the holders of the bond earn coupons.

Equation (2.23) breaks down the claim on equity into two parts, depending on whether liquidation occurs before the maturity of the debt. In the first term,  $\tau_b > T$  so the firm survives throughout the interval  $[0, T]$ . The market value of the firm's total equity at  $T$  is the difference

$$V_T - [(B - (1 - \alpha)L_T) + D] \quad (2.25)$$

between the value of the firm's assets and the principal payments on the two kinds of debt. Here we invoke our assumption (relaxed in Section 2.8), that asset value is marked to market so that (2.25) is the cash paid to equity holders after retiring all debt if the assets are sold at  $T$ . A fraction  $(1 - \pi_T)$  of this residual value goes to the new shareholders — those who acquired an equity stake through conversion of the contingent capital. In the second case in (2.23), the firm is seized and liquidated at time  $\tau_b$  when the contingent capital is exhausted. At this instant, the firm just meets its capital requirement, so the residual market value is  $\alpha V_{\tau_b}$ . A fraction  $X_1$  of this is recovered by shareholders upon liquidation, and a fraction  $(1 - \pi_{\tau_b})$  of the recovered value goes to the new shareholders.

Finally, the integrand in (2.24) is the discounted value of the net dividend rate in (2.20) paid to the converted shareholders at time  $t$ . To value the contingent capital, we will need to calculate the expectations of (2.21)–(2.24).

### 2.5.2 Senior Debt

The payments on the senior debt can be decomposed similarly but more simply into principal and coupon payments. We again distinguish the cases  $\tau_b \leq T$  and  $\tau_b > T$ , the first case corresponding to seizure and liquidation of the firm. The discounted payoffs to senior debtholders are as follows:

- *earned coupon:*

$$\int_0^{\min\{\tau_b, T\}} c_2 D e^{-rs} ds \quad (2.26)$$

- *principal:*

$$(e^{-rT} \mathbf{1}_{\{\tau_b > T\}} + X_2 e^{-r\tau_b} \mathbf{1}_{\{\tau_b < T\}}) D \quad (2.27)$$

In equation (2.26), coupons are paid until either the maturity of the debt at time  $T$  or the liquidation at  $\tau_b$ . In (2.27), the principal payment is reduced from the original face value of  $D$  to  $X_2 D$  in the case of liquidation, reflecting a random recovery fraction of  $X_2$  for the senior debt and the possibility of a deadweight cost of seizure and liquidation. If  $X_2 \equiv 1$ , the senior debt would be entirely riskless.

## 2.6 Valuation

To calculate expectations of (2.21)–(2.27), we posit that the dynamics of the book value of the firm's assets are given by (2.1). Equivalently, we have, with  $\mu = r - \delta - \sigma^2/2$ ,

$$V_t = V_0 \exp \{ \mu t + \sigma W_t \}. \quad (2.28)$$

We are assuming that the firm's assets are marked to market, so that  $V$  also represents the market value of the firm's assets; we drop this assumption in [Section 2.8](#). In writing the drift in (2.1) as  $r - \delta$ , we are implicitly specifying the dynamics of  $V$  under a risk-neutral pricing measure that we will use to take expectations in (2.21)–(2.26). Mathematically, this is by no means necessary — we could use any constant drift, including one that incorporates a risk premium, and modify our valuation formulas accordingly.

### 2.6.1 A Partial Transform

Inspection of the discounted payoffs in (2.21)–(2.26) and the proportion  $\pi_t$  in (2.17) indicates that the key remaining step for valuation is taking expectations involving powers of  $V$  and its running minimum with the running minimum restricted to an interval. We therefore undertake a preliminary calculation of a general such expression which we will then use to value the various payments.

Set

$$\widetilde{W}_t = \log(V_t/V_0) \quad \text{and} \quad \widetilde{m}_t = \min_{0 \leq s \leq t} \widetilde{W}_s; \quad (2.29)$$

then  $\widetilde{W}$  is a Brownian motion with drift  $\mu$  and diffusion coefficient  $\sigma$ . Let

$$\begin{aligned} H(t, v, k, y) &= H_{\mu, \sigma}(t, v, k, y) \\ &= \mathbf{E} \left[ \exp \left( v \widetilde{W}_t + k \widetilde{m}_t \right) \mathbf{1} \{ \widetilde{m}_t \leq y \} \right], \quad t, k \geq 0, v, y \in (-\infty, \infty). \end{aligned} \quad (2.30)$$

The function  $H$  depends on the parameters  $\mu$  and  $\sigma$  through the processes  $\widetilde{W}$  and  $\widetilde{m}$ ; as these parameters remain fixed, we suppress this dependence and write simply  $H(t, v, k, y)$  in referring to the function. The function is given explicitly in the following result.

**Proposition 2.6.1.** *The function  $H$  in (2.30) evaluates to*

$$H(t, v, k, y) = \exp(\mu vt + v^2 \sigma^2 t / 2) h(t, k, y), \quad (2.31)$$

with

$$h(t, k, y) = \frac{2\theta}{2\theta + k\sigma^2} e^{ky + 2y\theta/\sigma^2} \Phi\left(\frac{y + t\theta}{\sigma\sqrt{t}}\right) + \frac{2\theta + 2k\sigma^2}{2\theta + k\sigma^2} e^{k\theta t + k^2\sigma^2 t/2} \Phi\left(\frac{y - (\theta + k\sigma^2)t}{\sigma\sqrt{t}}\right), \quad (2.32)$$

where  $\theta = \mu + v\sigma^2$ , and  $\Phi$  is the standard normal distribution function.

With  $y = 0$ , (2.30) defines the joint Laplace transform of  $\widetilde{W}_t$  and  $-\widetilde{m}_t$ , and in this sense the general case in (2.30) defines a partial transform. In our application of the formula,  $y$  will always take the value  $\log(a/V_0)$  or  $\log(b/V_0)$ , corresponding to the asset levels at which conversion of contingent capital starts and ends. In several cases, we need to take the difference of values of  $H$  at these two values of  $y$  with other arguments held fixed, so it will be convenient to define

$$\Delta H(t, v, k) = H(t, v, k, \log(a/V_0)) - H(t, v, k, \log(b/V_0)). \quad (2.33)$$

## 2.6.2 Principal and Coupon Payments

The discounted expected value of the principal payment on the convertible debt is the expected value of equation (2.21) and is given by

$$e^{-rT} (B - (1 - \alpha)\mathbf{E}[L_T]). \quad (2.34)$$

Thus, to value the principal payment it suffices to find the expectation of  $L_T$ .

**Proposition 2.6.2.** *The expected present value of the contingent capital's principal payment is (2.34), where*

$$\mathbf{E}[L_t] = aH(t, 0, 0, \log(a/V_0)) - bH(t, 0, 0, \log(b/V_0)) - V_0\Delta H(t, 0, 1).$$

This expression evaluates to

$$\begin{aligned} \mathbf{E}[L_t] = & a\Phi(\delta_{a1}^-) - b\Phi(\delta_{b1}^-) + \frac{2V_0(\mu + \sigma^2)}{(2\mu + \sigma^2)} e^{t\mu + \frac{t\sigma^2}{2}} (\Phi(\delta_{a2}^-) - \Phi(\delta_{b2}^-)) \\ & + \frac{\sigma^2}{(2\mu + \sigma^2)} \left( a \left( \frac{a}{V_0} \right)^{\frac{2\mu}{\sigma^2}} \Phi(\delta_{a1}^+) - b \left( \frac{b}{V_0} \right)^{\frac{2\mu}{\sigma^2}} \Phi(\delta_{b1}^+) \right) \end{aligned} \quad (2.35)$$

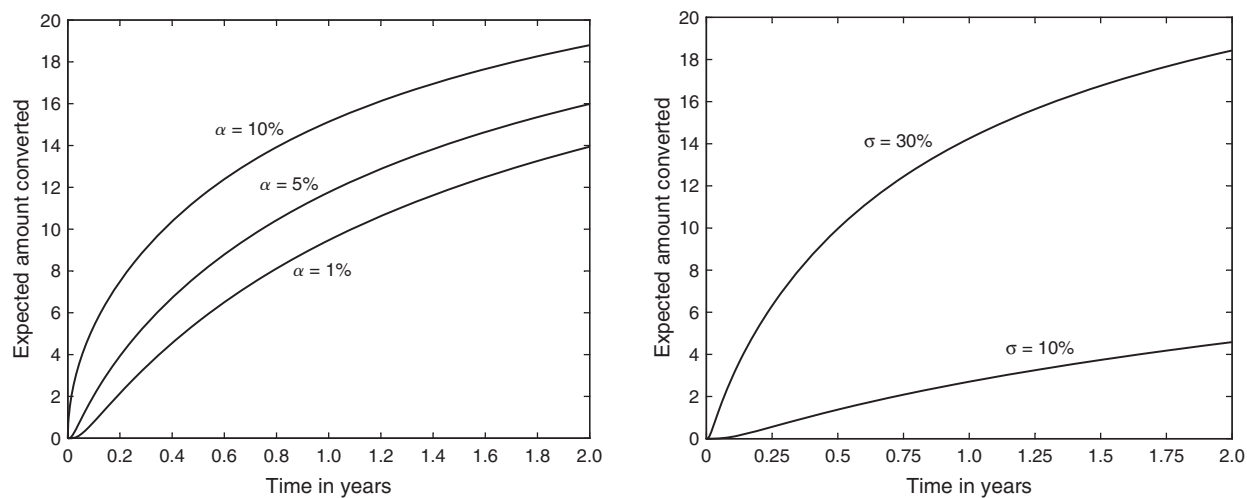
where

$$\begin{aligned} \delta_{a1}^\pm &= \frac{\pm t\mu + \log(\frac{a}{V_0})}{\sigma\sqrt{t}} & \delta_{a2}^\pm &= \frac{t(\mu + \sigma^2) \pm \log(\frac{a}{V_0})}{\sigma\sqrt{t}} \\ \delta_{b1}^\pm &= \frac{\pm t\mu + \log(\frac{b}{V_0})}{\sigma\sqrt{t}} & \delta_{b2}^\pm &= \frac{t(\mu + \sigma^2) \pm \log(\frac{b}{V_0})}{\sigma\sqrt{t}} \end{aligned}$$

Figure 2.4 plots the expected amount of contingent capital converted by time  $t$ , namely  $(1 - \alpha)\mathbf{E}[L_t]$ , over a two-year horizon for various levels of  $\alpha$  and  $\sigma$ . Recall that  $\mathbf{E}[L_t]$  depends on  $\alpha$  through the boundaries  $a$  and  $b$  of the conversion band. The figure uses  $V_0 = 100$  with  $D = 60$ ,  $B = 30$ ,  $r = 5\%$ , and  $\delta = 3\%$ . The left panel fixes  $\sigma$  at 25%, and the right panel fixes  $\alpha$  at 5%. The curves show qualitatively different behavior near time zero: when the initial asset level is far from the conversion trigger (either because  $\alpha$  is small or because  $\sigma$  is small), the expected amount converted is nearly flat for small  $t$ ; the curves are steeper when the conversion trigger is closer.

The expected present value of the contingent capital coupon payments (2.22) is given by





**Figure 2.4:** Comparison of  $(1 - \alpha)\mathbf{E}[L_t]$ , the expected amount of contingent capital converted by time  $t$  for different values of the capital ratio  $\alpha$  (left) and the asset volatility  $\sigma$  (right).

$$B \frac{c_1}{r} (1 - e^{-rT}) - c_1 (1 - \alpha) \int_0^T e^{-rt} \mathbf{E}[L_t] dt. \quad (2.36)$$

We do not have a simple expression for the integral in (2.36); however, because  $\mathbf{E}[L_t]$  is smooth and monotone, the integral can be accurately approximated by replacing it with a sum.

### 2.6.3 Equity Earned Through Conversion

We turn now to (2.23), which gives the discounted terminal value of the equity acquired by the contingent capital investors through the process of conversion. We value separately the two terms in (2.23), the first corresponding to the firm surviving until  $T$ , the second corresponding to seizure and liquidation before  $T$ .

**Proposition 2.6.3.** *The value of the converted equity stake in the event of survival (the first*

term in (2.23)) is given by  $\exp(-rT)$  times

$$\begin{aligned} & V_0 \Delta H(T, 1, 0) - V_0 \left( \frac{V_0}{a} \right)^{q(1-\alpha)/\alpha} \Delta H(T, 1, q(1-\alpha)/\alpha) \\ & - V_0(1-\alpha) \Delta H(T, 0, 1) + V_0(1-\alpha) \left( \frac{V_0}{a} \right)^{q(1-\alpha)/\alpha} \Delta H(T, 0, 1 + q(1-\alpha)/\alpha). \end{aligned} \quad (2.37)$$

In the event of seizure and liquidation (the second term in (2.23)), the value of the converted equity stake is with  $R_1 = \mathbf{E}[X_1]$  and  $\theta_1 = \sqrt{\mu^2 + 2\sigma^2 r}$ ,

$$R_1 \alpha b \left( 1 - \left( \frac{b}{a} \right)^{q(1-\alpha)/\alpha} \right) \left( \frac{b}{V_0} \right)^{(\mu-\theta_1)/\sigma^2} e^{-rT} H(T, (\theta_1 - \mu)/\sigma^2, 0, \log(b/V_0)). \quad (2.38)$$

#### 2.6.4 Net Dividends

As discussed in Section 2.4, the difference between the total payout rate  $\delta V_t$  and debt service payments creates a dividend stream for equity holders, a fraction  $1 - \pi_t$  of which flows to investors who originally held convertible debt, as in (2.24). Taking the expected value of this expression, we get

$$\begin{aligned} & \mathbf{E} \left[ \int_0^{\min\{T, \tau_b\}} e^{-rt} (1 - \pi_t) (\delta V_t - (1 - \kappa) (c_1 (B - (1 - \alpha)L_t) + c_2 D)) dt \right] \\ & = \int_0^T e^{-rt} \mathbf{E} [(1 - \pi_t) (\delta V_t - (1 - \kappa) (c_1 (B - (1 - \alpha)L_t) + c_2 D)) \mathbf{1}\{\tau_b > t\}] dt \end{aligned} \quad (2.39)$$

The expectation inside the integral can be evaluated in closed form:

**Proposition 2.6.4.** *The expected net rate at which the contingent capital investors earn*

dividends (i.e., the expectation on the right side of (2.39)) is given by

$$\begin{aligned} & \delta V_0 \Delta H(t, 1, 0) - (1 - \alpha)(1 - \kappa)(c_2 - c_1)b \Delta H(t, 0, 0) - (1 - \alpha)(1 - \kappa)c_1 V_0 \Delta H(t, 0, 1) \\ & - (1 - \alpha)(1 - \kappa)c_1 V_0 \Delta H(t, 0, 1 + q(1 - \alpha)/\alpha) - \left(\frac{V_0}{a}\right)^{q(1 - \alpha)/\alpha} \times \\ & [\delta V_0 \Delta H(t, 1, q(1 - \alpha)/\alpha) - (1 - \alpha)(1 - \kappa)(c_2 - c_1)b \Delta H(t, 0, q(1 - \alpha)/\alpha)]. \end{aligned} \quad (2.40)$$

The present value of the cumulative dividends is the time-integral of this expression, which is easily and accurately approximated by a sum over a discrete set of dates.

It is also evident from this expression that the effect of the marginal tax rate  $\kappa$  is simply to replace each original coupon rate  $c_i$  with  $(1 - \kappa)c_i$ . The formula remains valid if we replace  $(1 - \kappa)c_i$  with  $(1 - \kappa_i)c_i$  to allow different levels of tax-deductibility of the two types of coupons.

### 2.6.5 Senior Debt

The expected value of the coupon payments (2.26) is given by

$$\begin{aligned} \mathbf{E} \left[ \int_0^{\min\{\tau_b, T\}} c_2 D e^{-rs} ds \right] &= D \frac{c_2}{r} (1 - \mathbf{E}[\exp\{-r \min\{\tau_b, T\}\}]) \\ &= D \frac{c_2}{r} (1 - e^{-rT} \mathbb{P}(\tau_b > T) - \mathbf{E}[e^{-r\tau_b} \mathbf{1}_{\{\tau_b \leq T\}}]). \end{aligned} \quad (2.41)$$

Similarly, the discounted expected value of the principal payment (2.27) is given by

$$D \mathbf{E} [e^{-rT} \mathbf{1}_{\{\tau_b > T\}} + X_2 e^{-r\tau_b} \mathbf{1}_{\{\tau_b < T\}}] = D (e^{-rT} \mathbb{P}(\tau_b > T) + R_2 \mathbf{E} [e^{-r\tau_b} \mathbf{1}_{\{\tau_b \leq T\}}]). \quad (2.42)$$

The probability  $\mathbb{P}(\tau_b > T)$  coincides with  $\mathbb{P}(\tilde{m}_T > \log(b/V_0))$ , which can be evaluated directly using equation (A.2) in the appendix; the expectation  $\mathbf{E}[\exp(-r\tau_b) \mathbf{1}_{\{\tau_b \leq T\}}]$  is evaluated

explicitly in equation (A.6) of the appendix. With these substitutions, the total discounted expected value of the senior debt becomes

$$D \frac{c_2}{r} + D \left(1 - \frac{c_2}{r}\right) e^{-rT} \left[ \Phi \left( \frac{\mu T - \log\left(\frac{b}{V_0}\right)}{\sigma \sqrt{T}} \right) - \left(\frac{b}{V_0}\right)^{\frac{2\mu}{\sigma^2}} \Phi \left( \frac{\mu T + \log\left(\frac{b}{V_0}\right)}{\sigma \sqrt{T}} \right) \right] \\ + D \left(R_2 - \frac{c_2}{r}\right) \left[ \left(\frac{b}{V_0}\right)^{\frac{\mu - \theta_1}{\sigma^2}} \Phi \left( \frac{\log\left(\frac{b}{V_0}\right) - \theta_1 T}{\sigma \sqrt{T}} \right) + \left(\frac{b}{V_0}\right)^{\frac{\mu + \theta_1}{\sigma^2}} \Phi \left( \frac{\log\left(\frac{b}{V_0}\right) + \theta_1 T}{\sigma \sqrt{T}} \right) \right],$$

where, as before,  $\theta_1$  is the square root of  $2\sigma^2 r + \mu^2$ . The following result values the senior debt using the function  $H$ :

**Proposition 2.6.5.** *The value of the senior debt, including both coupon payments (2.26) and principal (2.27), is given, with  $\theta_1 = \sqrt{2\sigma^2 r + \mu}$ , by*

$$D \frac{c_2}{r} + D \left(1 - \frac{c_2}{r}\right) e^{-rT} (1 - H(T, 0, 0, \log(b/V_0))) \\ + D \left(R_2 - \frac{c_2}{r}\right) \left(\frac{b}{V_0}\right)^{\frac{\mu - \theta_1}{\sigma^2}} e^{-rT} H(T, (\theta_1 - \mu)/\sigma^2, 0, \log(b/V_0)),$$

## 2.7 Closing the Model: Market Yields

In our calculations, we have assumed that both the senior debt and the convertible debt are sold at par at time zero; this leads to constant book values (for the unconverted principal), (2.9), and the resulting tractability. In Section 2.6, we have calculated market prices for senior and convertible debt, with coupon rates assumed given. For our model to be internally consistent, we need the market prices we calculate at time zero to coincide with our assumption that the bonds sell at par. We now show that this is indeed possible and that it determines the coupon rates for both types of debt.

For the senior debt, equating the expected discounted value of the coupon and principal

calculated in Section 2.6.5 to the face value  $D$  yields the coupon rate

$$c_2 = r \left( 1 + \frac{(1 - R_2) \mathbf{E} [e^{-r\tau_b} \mathbf{1}_{\{\tau_b \leq T\}}]}{1 - e^{-rT} \mathbb{P}(\tau_b > T) - R_2 \mathbf{E} [e^{-r\tau_b} \mathbf{1}_{\{\tau_b \leq T\}}]} \right).$$

The probability and expectation in this expression are evaluated in Appendix A.1.6, thus allowing direct evaluation of  $c_2$ . If  $R_2 = 1$ , the coupon rate  $c_2$  reduces to  $r$ : under our assumption that the firm is seized and liquidated when it violates its capital requirement — before insolvency — the senior debt is riskless if there is no loss of value at liquidation.

Similarly, for the convertible debt, equating our valuation (the sum of the expectations of (2.21)–(2.24)) with the face value  $B$  yields the coupon rate

$$c_1 = \frac{B - A_1 - A_3 - A_4}{A_2 + A_5},$$

where  $A_1$  is the expected principal in (2.34),

$$A_2 = \frac{B}{r} (1 - e^{-rT}) - (1 - \alpha) \int_0^T e^{-rt} \mathbf{E}[L_t] dt,$$

from (2.36),  $A_3$  is the expected terminal equity value (the sum of (2.38) and  $\exp(-rT)$  times (2.37)), and

$$A_4 = \mathbf{E} \left[ \int_0^{\min\{T, \tau_b\}} e^{-rt} (1 - \pi_t) (\delta V_t - (1 - \kappa) c_2 D) dt \right]$$

and

$$A_5 = \mathbf{E} \left[ \int_0^{\min\{T, \tau_b\}} e^{-rt} (1 - \pi_t) (1 - \kappa) (B - (1 - \alpha) L_t) dt \right]$$

come from the net dividends in (2.39). The results in Section 2.6 yield explicit expressions for  $A_1$ – $A_5$  and thus for the coupon rate  $c_1$ .

We view these expressions as the key practical contribution of our analysis. Given the

characteristics of the firm — its asset volatility and the face value of its senior and convertible debt — these equations give the coupon rates required by the market. For debt issued at par, the coupon rate equals the yield; so, more generally, we interpret these rates as the yields required by the market for the two types of debt. These equations are therefore useful in gauging the yield required by investors in contingent capital as compensation for bearing the risk that the debt they hold converts to equity.

## 2.8 Distinguishing Market and Book Values of Assets

To this point, we have assumed that the bank's assets are marked to market so that  $V$  represents the market value of assets as well as their book value. We now extend the model to capture a stochastic relation between the two. We use  $A_t$  to denote the market value of assets. Our key assumption is that while the market and book values of assets may differ, they are sufficiently aligned to agree on whether a bank is solvent. If the bank were liquidated at time  $t$ , debt holders would be due  $B_t + D$ , so the bank is solvent if its assets have at least this value. Our condition, then, is that  $A_t > B_t + D$  whenever  $V_t > B_t + D$ . To model this relationship, we introduce a second geometric Brownian motion  $U$ ,

$$U_t = U_0 \exp \{ \theta_u t + \sigma_u W'_t \},$$

with  $W'$  and  $W$  (the original Brownian motion driving  $V$ ) having instantaneous correlation  $\rho$ . We model  $A_t$  as satisfying

$$A_t - B_t - D = U_t(V_t - B_t - D). \quad (2.43)$$

The process  $U$  can be roughly interpreted as a market-to-book ratio, but whereas  $V_t - B_t - D$  is the book value of equity,  $A_t - B_t - D$  is the difference between the market value of assets

and the book value of debt. A natural choice in this setting would be to take  $\theta_u = -\sigma_u^2/2$ , so that  $E[U_t]$  is constant, but we need not limit ourselves to this case.

In this extension of our basic model, conversion from debt to equity is still governed by book value  $V$ , just as before; but the value received by equity holders at either the maturity date  $T$  or at a seizure at  $\tau_b$  now depends on the market value  $A$ . Accordingly, we modify (2.23) by replacing  $V_T$  with  $A_T$  and  $V_{\tau_b}$  with  $A_{\tau_b}$ . This case remains tractable under the parameter restriction  $2\sigma^2\gamma \geq \mu^2$ , where

$$\gamma = -\theta_u + \sigma_u\mu\rho/\sigma - \frac{1}{2}\sigma_u^2(1 - \rho^2) + r.$$

In Proposition 2.6.3, (2.37) becomes

$$\begin{aligned} & \vartheta\Delta H(T, 1 + \sigma_u\frac{\rho}{\sigma}, 0) - \vartheta\left(\frac{V_0}{a}\right)^{q(1-\alpha)/\alpha} \Delta H(T, 1 + \sigma_u\frac{\rho}{\sigma}, q(1-\alpha)/\alpha) \\ & - \vartheta(1-\alpha)\Delta H(T, \sigma_u\frac{\rho}{\sigma}, 1) + \vartheta(1-\alpha)\left(\frac{V_0}{a}\right)^{q(1-\alpha)/\alpha} \Delta H(T, \sigma_u\frac{\rho}{\sigma}, 1 + q(1-\alpha)/\alpha), \end{aligned}$$

with  $\vartheta = V_0U_0 \exp((r - \gamma)T)$ , and (2.38) becomes

$$\begin{aligned} & R_1U_0\alpha b \left(1 - \left(\frac{b}{a}\right)^{q\frac{1-\alpha}{\alpha}}\right) \times \\ & \left[ \left(\frac{b}{V_0}\right)^{\frac{\rho\sigma_u/\sigma + \mu - \theta_2}{\sigma^2}} \Phi\left(\frac{\log(\frac{b}{V_0}) - \theta_2T}{\sigma\sqrt{T}}\right) + \left(\frac{b}{V_0}\right)^{\frac{\rho\sigma_u/\sigma + \mu + \theta_2}{\sigma^2}} \Phi\left(\frac{\log(\frac{b}{V_0}) + \theta_2T}{\sigma\sqrt{T}}\right) \right]. \end{aligned}$$

These expressions are derived through a minor modification of the proof of Proposition 2.6.3 after making the substitution in (2.43).

With the condition that  $\theta_u = -\sigma_u^2/2$ , this extension introduces two new parameters, the “book-to-market volatility”  $\sigma_u$  and correlation  $\rho$ , as well as the initial value  $A_0$ . Though not directly observable, these parameters could be calibrated using market values of a firm’s

		I	II
Debt over assets ratio	$D/V_0$	90%	
Capital adequacy ratio	$\alpha$	4%	
Risk free rate	$r$	5%	0.5%
Volatility of asset returns	$\sigma$	8%	16%
Debt maturity	$T$	1.5	
Fractional payout of assets	$\delta$	3%	1.5%
Tax rate	$\kappa$	30%	
Recovery rate for equity	$R_1$	30%	
Recovery rate for senior debt	$R_2$	95%	

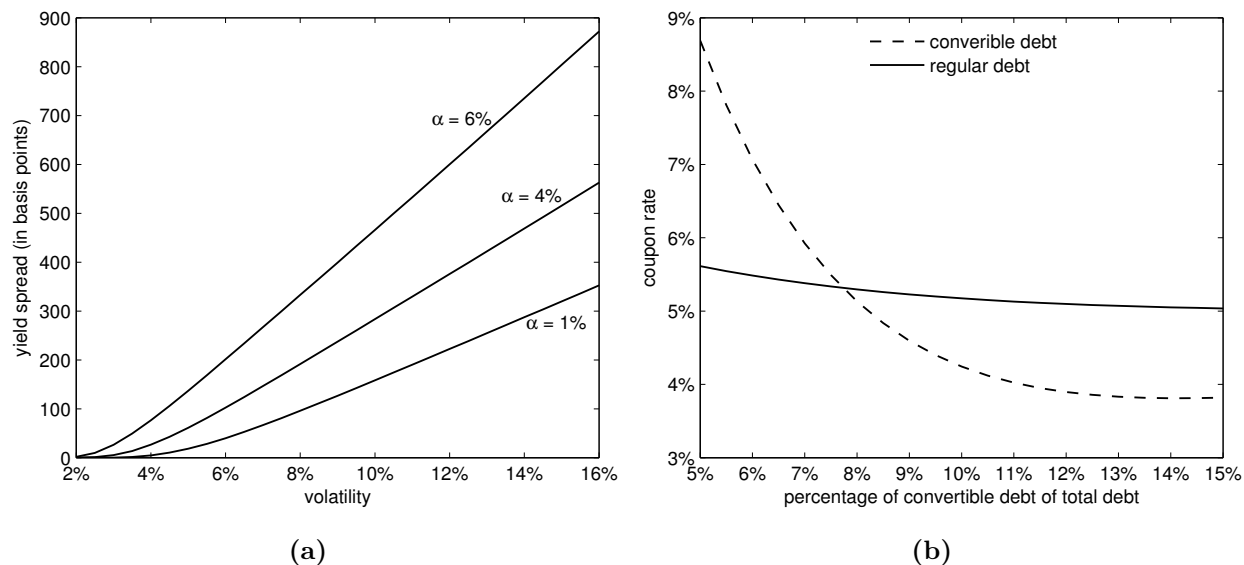
**Table 2.1:** Parameters for base case (I) and modified scenario (II).

debt and equity and book values from financial statements. As our model already has several parameters, in the numerical examples of the next section we limit ourselves to the basic model in which  $A_t = V_t$ .

## 2.9 Example

In this section, we use numerical examples to investigate how the yields derived from our model change with parameter inputs and how the introduction of convertible debt influences the spread on senior debt. Table 2.1 shows the parameter values we use. The first set (I) is our base case and is intended to be representative of the end of 2006, before the financial crisis, based on data for the twenty largest (by assets) banks in the U.S. The parameter modifications indicated under II are intended to be representative of 2009. In both cases, we consider a bank with 90% debt that is required to maintain a minimum capital ratio of 4% of assets (which corresponds to 8% of risk-weighted assets for a bank whose assets have an average risk weight of 50%). The maturity  $T$  approximates the weighted average maturity of debt for large banks, using a six-month maturity for deposits. The base case has a relatively low asset volatility of 8% (see, e.g., the estimates in Nikolova [61]), a payout rate of 3% (reflecting both interest payments and dividends) and a risk-free rate of 5%, which is very





**Figure 2.5:** (a) Sensitivity of senior debt to volatility and minimum capital ratio  $\alpha$  in the absence of convertible debt. (b) Coupon rates at different magnitudes of convertible debt as a percentage of total debt.

close to the average Treasury rate at the end of 2006, when the Treasury yield curve was quite flat.

We begin with nonconvertible debt only. Recall that the coupon rate is set to price the bond at par, so the coupon and yield are equal. Figure 2.5a shows the yield spreads we obtain with our assumed recovery rate  $R_2$  of 95%. With a 100% recovery rate, the debt would be riskless. The potential loss of 5% takes effect only in case of seizure by regulators; this occurs at a positive capital ratio, when the bank's assets still exceed the value of its debt, so the loss reflects a liquidation cost. Our base case of  $\alpha = 4\%$  and  $\sigma = 8\%$  produces a spread of 1.9%. As expected, the figure shows that the spread increases if we increase  $\alpha$  or  $\sigma$ , as each of these changes increases the likelihood of seizure and thus of a loss from liquidation.

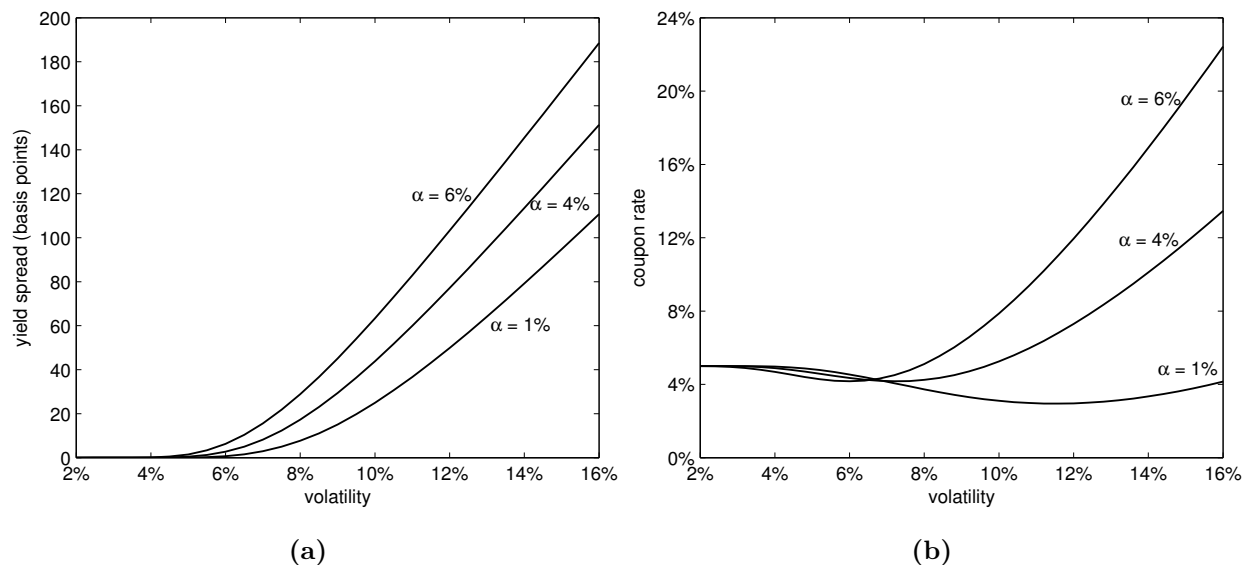
Figure 2.5b illustrates the effect of introducing convertible debt to the balance sheet.

The total amount of debt (regular debt and convertible debt) is fixed at 90% of total asset value. We change the proportion of convertible debt from 5% to 15% of the total debt. The graph shows the required coupon rates for both the senior and the convertible debt. The coupon rate on the convertible debt depends on the loss incurred by shareholders at seizure and liquidation; we assume a 30% recovery rate, meaning that 70% of the remaining equity value at seizure is lost through liquidation.

The first observation is that the coupon rate on the senior debt decreases when the proportion of convertible debt increases. The contingent capital works as a cushion against liquidation; therefore, with the same recovery rate, the senior debt suffers lower liquidation costs because of the reduced likelihood of seizure, and this translates to a lower compensating coupon rate.

With only a small amount of convertible debt, the required coupon on this debt is high, and this can be understood as follows. With a thin layer of convertible debt on the balance sheet, the probability of liquidation does not change much, and if the asset level hits the conversion trigger it is very likely that the full layer of contingent capital will be converted and the liquidation boundary will be reached, leaving little chance for the converted investors to benefit from the potential upside to equity. Indeed, they are likely to incur the 30% liquidation cost to equity shortly after conversion.

However, the coupon rate decreases quickly as we thicken the layer of convertible debt. Indeed, when  $q = 1$  and convertible debt makes up more than 7.8% of total debt it earns a lower coupon than the senior debt; and at more than 8% of the total debt, its coupon drops below the risk-free rate. This pattern results from the potential upside of the equity the contingent capital investors earn through conversion. Conversion occurs precisely when the book value of equity is low, so, conditional on survival, the contingent capital investors can benefit substantially from an increase in equity value. Increasing the proportion of

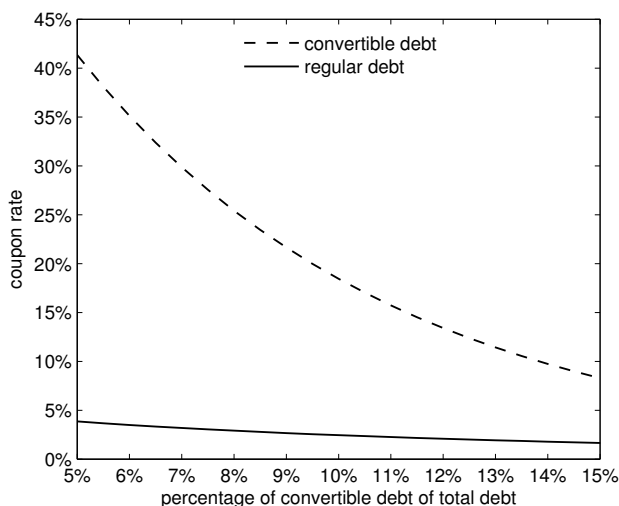


**Figure 2.6:** (a) Sensitivity of senior debt to volatility and minimum capital ratio  $\alpha$  when 10% of debt is replaced with convertible debt. (b) Coupon rates for convertible debt.

contingent capital widens the interval between the conversion trigger and the liquidation trigger and increases the likelihood of an upside gain through conversion to equity. Lowering  $q$  to 0.8 reduces the upside gain from conversion and thus requires a higher coupon to keep the convertible debt priced at par.

Figure 2.6a reproduces Figure 2.5a but now with convertible debt making up 10% of total debt. The figure shows that the coupon rate for senior debt is now much less sensitive to volatility; for example, at  $\alpha = 6\%$  and a volatility of 16% the spread does not exceed 200 basis points whereas without contingent capital it was over 800 basis points. This clearly shows the effect of the protection provided by the convertible debt.

Figure 2.6b shows the required coupon rate for convertible debt at different values of volatility and  $\alpha$ . The graphs are more complicated and non-monotonic in this case. This reflects the hybrid nature of the contingent capital, with both equity-like and debt-like behavior. Volatility has an adverse effect on debt and a favorable effect on equity. What we



**Figure 2.7:** Coupon rates at different magnitudes of convertible debt with parameter set II.

observe in Figure 2.6b is the trade-off between these two effects.

Next we consider parameter set II of Table 2.1, based roughly on conditions in 2009. Volatility is much higher, the risk-free rate is much lower, and we have cut the payout rate  $\delta$  to reflect lower dividend rates. Figure 2.7 shows the resulting coupon rate on senior and convertible debt, and it shows that in these new market conditions, the fair coupon rate on the convertible debt is dramatically higher. Indeed, with these parameters, the geometric Brownian motion that models assets has a negative risk-neutral drift ( $r - \delta - \sigma^2/2 = -0.0228$ ) and a high volatility, implying a higher chance of liquidation. As the debt sells at par, higher liquidation probabilities must be compensated with higher coupon rates. Increasing the size of the convertible debt decreases the required coupon rate; but, in contrast to the previous parameter set, even at 10% convertible debt we observe very high coupon rates. We see this as reflecting the necessity of issuing contingent capital in advance of a crisis; in an environment of high volatility, investors will demand a much higher coupon unless the overall level of leverage is substantially reduced. The problem is diminished with a wider tranche of contingent capital, which provides a buffer for the senior debt and yields for the

convertible debt in the range of 5–10%.

## 2.10 Concluding Remarks

We have developed a model to value contingent capital in the form of debt that converts to equity. The key distinguishing features of our analysis are that we formulate a capital-ratio trigger and we model partial and on-going conversion. Our capital-ratio trigger approximates a regulatory capital requirement by using book values for debt and equity. Our partial conversion process allows just enough debt to convert to equity to maintain the required ratio until the contingent capital is fully exhausted. We derive closed-form expressions for yield spreads by adding a consistency requirement that market and book values of debt agree at issuance and at maturity.

Our numerical examples indicate that the fair yield for contingent capital in our model is quite sensitive to some of the model's inputs — in particular, to the size of the convertible tranche, to the volatility of the firm's assets, and to recovery rates in the event that the firm breaches its minimum capital requirement and is seized by regulators. This sensitivity — particularly to asset volatility and recovery rates, which are not directly observable and are difficult to estimate — as well as the overall complexity of the product, could present obstacles to generating the investor demand that would be needed for widespread issuance of contingent convertible bonds.

## 3

## Contingent Convertibles, Bail-In Debt, and Tail Risk in Optimal Endogenous Default Setting

**D**EBT rollover, endogenous default, and tail risk are essential features in examining how changes in capital structure to include CoCos or bail-in debt change incentives for equity holders. In this chapter we develop a model of the capital structure of a financial firm that includes CoCos or bail-in debt along with insured deposits, senior debt, and subordinated debt. Importantly, bankruptcy in our model is endogenous, as in Leland [50] and Leland and Toft [51], meaning that it results from the optimal decision of shareholders to exercise their option to surrender the firm's assets to the creditors. We are thus interested in how the two levers in the design of contingent capital affect the incentives for shareholders to invest additional capital in the firm, and how the levers affect the shareholders' incentives to take on different types of risk in investing the firm's assets. Our model incorporates debt rollover, and a central theme of our analysis is that the shareholders' incentives are strongly influenced by this feature — the cost of debt rollover can motivate shareholders to reduce the

firm's leverage and the riskiness of its assets. Also crucial to our analysis is the inclusion of both jumps and diffusion in asset value. We interpret the diffusive risk as the ordinary level of volatility in the firm's business, which is readily observable by a regulator. In contrast, the jumps capture the firm's ability to take on high-yielding tail risk that is much harder to measure if jumps are rare. Among the questions we examine is how replacing straight debt with convertible debt affects the attractiveness of the two types of risk to equity holders.

We obtain explicit expressions to value all pieces of the capital structure by building on results of Chen and Kou [18] and Cai, Chen, and Wan [14]. Using the valuation formulas, we investigate the effect of varying key model parameters including the trigger level for conversion of debt to equity, the dilution ratio at conversion, the mix and average maturity of different types of debt, bankruptcy costs, deposit insurance premiums, the tax-deductibility of interest payments on CoCos, and the riskiness of the firm's investments. We investigate how these parameters affect debt and equity values, the timing of bankruptcy, the risk-sensitivity of equity, the propensity for asset substitution, and the extent of debt overhang as an obstacle to raising capital <sup>1</sup>.

### 3.1 Summary of Main Results

We can draw some conclusions theoretically, whereas others are illustrated through comparative statics. Details are provided in later sections, but we highlight some key observations as follows:

- (i). So long as the trigger level and the conversion ratio are designed to ensure that con-

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<sup>1</sup>Pelger [63] independently develops a model applying results of Chen and Kou [18] to analyze contingent convertible bonds. Among the most important differences between his work and ours are that his model does not distinguish firm-specific and market-wide jumps with different recovery rates for the two cases, nor does he consider the case of bail-in debt or any calibration to market data.

version occurs prior to endogenous bankruptcy, the precise values of the trigger and the conversion ratio have no effect on the timing of bankruptcy or the asset level at which it occurs. This simple but important observation will underlie several other implications of the model. Conversion in our model is triggered when asset value falls below a specified level. The asset level at which shareholders optimally surrender the firm is insensitive to the conversion level and conversion ratio, so long as the conversion level is higher than the resulting bankruptcy level. If conversion precedes bankruptcy, the optimal bankruptcy level is the level for the post-conversion firm, which does not depend on the conversion trigger or ratio.

- (ii). CoCos can reduce default risk, as we explain below. In so doing, they reduce the cost of rolling over straight debt as it matures, and this increases dividends available to equity holders. This effect, together with a desire to avoid unfavorable conversion, can lead equity holders to prefer less risky assets.
- (iii). Replacing some straight debt with CoCos has several effects.
  - This replacement reduces the value of the debt tax shield if CoCo coupons are not tax deductible. Even assuming tax-deductibility of CoCo coupons, this replacement will reduce the tax shield through the eventual conversion of CoCos to equity, unless the CoCo coupon is so high as to offset this effect.
  - This replacement lowers the endogenous default barrier and thus increases the firm's ability to sustain a loss in asset value. It thus reduces bankruptcy costs. The reduction in bankruptcy costs and the reduction in the tax shield have opposite effects on total firm value, but we find that the reduction in bankruptcy cost is greater in our numerical examples.
  - Reducing bankruptcy costs lowers the cost of rolling over the remaining straight debt; thus, replacing some straight debt with convertible debt *can increase the value of equity*, which we interpret as a reduction in the firm's cost of capital. The



benefit to shareholders of replacing straight debt with convertible debt increases as asset value decreases.

- (iv). We also consider the effects of increasing firm size by issuing CoCos while keep other forms of debt fixed.
- If the size of the additional CoCo issue is sufficiently large, the increased coupon payments may make it optimal for shareholders to default prior to conversion resulting in greater value destruction at bankruptcy through the increase in the firm's assets and a phenomenon of debt-induced collapse discussed below. We mainly focus on the case (which should be more typical) that the optimal default barrier is lower than the conversion trigger.
  - So long as this holds, the default barrier is unchanged and the default risk decreases because the distance to default increases with the value of additional assets.
  - The reduced default risk lowers the cost of rolling straight debt which increases the value of equity. If CoCo coupons are tax-deductible, this further increases equity value, lowering the cost of equity capital.
- (v). For completeness, we also consider the effect of replacing some equity with CoCos, though this case is of less interest in practice. If CoCo coupons are tax-deductible, and if the substitution is not so large as to drive the default barrier above the conversion level, then equity holders capture all the value of the increased tax shield with no change in the firm's default risk. However, this replacement can also induce the equity holders to prefer less risky assets in order to preserve the funding advantage provided by unconverted CoCos through the tax shield.
- (vi). CoCos can mitigate the debt overhang problem, creating two incentives for new equity investment as the firm's asset value approaches the conversion trigger. If the CoCo

coupons are tax deductible, it is optimal for the shareholders to invest in the firm to prevent conversion and preserve the tax shield. Also, assuming the number of shares issued to CoCo investors at conversion is fixed, the value of the equity issued to CoCo investors is largest at the conversion trigger, so the incentive for the shareholders to stave off conversion through additional investment is greatest just above this point.

- (vii). CoCos affect asset substitution — the tendency of equity holders to prefer riskier assets after issuing debt — in several ways. As already noted, lowering the cost of rolling over straight debt provides an incentive for equity holders to take on less risk, and this incentive can be increased by the presence of CoCos, particularly in the presence of a tax shield. However, CoCos can also create incentives for equity holders to increase exposure to tail risk (i.e., downward jumps in asset value) because the cost (to shareholders) of conversion is lower if it occurs at a lower asset value.
- (viii). As bond investors, holders of CoCos may be unwilling or unable to hold equity following conversion and may therefore receive less than full market value in a forced sale of shares. Anticipating this outcome, they would demand a lower price at the time of their initial investment in CoCos. This effect reduces but, in our examples, does not eliminate the attractiveness to shareholders of replacing some straight debt with CoCos.
- (ix). In the pure bail-in case, conversion of debt to equity occurs just as the firm would otherwise declare bankruptcy and the original shareholders are wiped out. We assume that the bail-in avoids the deadweight costs of bankruptcy. Although they are wiped out at bail-in, the original shareholders benefit from replacing straight debt with bail-in debt because the reduction in bankruptcy costs lowers the cost of debt service. The benefit to shareholders of such a replacement increases as asset value decreases.
- (x). Our model identifies a phenomenon of “debt-induced collapse” specific to a setting with convertible debt and endogenous default. The phenomenon occurs when a firm issues CoCos and then takes on excessive additional debt. If sufficiently extreme, the

additional debt will induce equity holders to default prior to conversion, effectively changing CoCos to junior straight debt. At the point at which this occurs, equity value experiences a sudden drop as the value of the conversion feature to equity holders is eliminated. Avoiding this phenomenon requires setting the conversion trigger to be unambiguous about whether conversion will occur prior to bankruptcy (as with CoCos) or only at bankruptcy (the bail-in case).

- (xi). Considering a regulator's perspective, we have already noted that the level of the conversion trigger has no direct effect on the timing of bankruptcy, so long as the conversion trigger remains above the endogenous default barrier. Nevertheless, the regulator can have indirect influence through CoCos. A higher trigger creates a greater incentive for equity holders to invest additional capital in the firm earlier and can reduce incentives to increase the riskiness of the assets; but a lower trigger creates a greater incentive for equity holders to voluntarily replace some straight debt with convertible debt.
- (xii). Charging deposit insurance in proportion to all of the firm's debt, including CoCos, reduces some of the positive incentives resulting from CoCos, just as the tax-deductibility of CoCo coupons increases some of these positive incentives.
- (xiii). We have calibrated our model to bank balance sheet and stock price data during 2004Q1–2011Q3 for 17 of the 19 largest U.S. bank holding companies. (We exclude Ally because it is privately held and MetLife because it is predominantly an insurance company.) We use the calibration to gauge how much CoCos would have increased banks' ability to sustain losses during the crisis. We also use the calibration to measure debt overhang costs and find that CoCos with a high trigger would have created positive incentives for additional investment in 2008–2009 for most of the banks. Based on the calibration, we time the conversion of CoCos with high and low triggers for each bank and identify which banks would not have triggered conversion.

Our model integrates endogenous default, debt rollover, and jumps into the valuation of contingent capital. We see these features as essential to examining the incentive effects of including CoCos: to understand, for example, if CoCos help overcome debt overhang and motivate equity holders to invest in the firm, we need a model in which equity holders exercise their option to abandon the firm optimally; otherwise, apparent incentive effects from CoCos may simply reflect inconsistent modeling of the decisions of equity holders. Moreover, these incentives are strongly influenced by the need to roll debt, and many of our conclusions would be missing in a model with a single maturity date for debt. Our analysis combines these features with a rich capital structure that includes several types of liabilities with a variety of maturity profiles. On the asset side we capture tail risk through two types of jumps, and we combine these features while achieving tractability in our valuations.

Pennacchi [64] highlights the importance of jumps in valuing contingent convertibles and uses a jump-diffusion model of assets, as we do, but his work differs from ours in several respects. He foregoes tractability, instead using simulation for valuation, and incorporates stochastic interest rates in his valuations. All debt in his model shares a fixed maturity, and default is determined exogenously through a mechanism similar to that of Black and Cox [9]. As already noted, endogenous default and the rollover of maturing debt are key features of our analysis. We combine deposits, straight debt, and CoCos, each with its own maturity profile. Pennacchi's [64] simulation model does not include a debt tax shield or bankruptcy costs; these features are important to our conclusions. Our model also adds alternative assumptions about deposit insurance premiums and allows a potential loss at conversion as CoCo investors with a preference for debt are forced to sell unwanted equity shares at a discount. Moreover, our model identifies the phenomenon of "debt-induced collapse" discussed above, which can be observed only in a model with both contingent convertibles and endogenous default. Also, our model distinguishes firm-specific and market-wide jumps; we capture fire-sale effects by imposing a lower recovery rate on assets when default occurs at a market-wide jump.

Like most of other works in the context of contingent capital, we take a structural approach to modeling and valuation. Reduced-form credit risk models of the type in Duffie and Singleton [27] and Jarrow and Turnbull [43] could potentially be used for pricing and hedging CoCos, but they are less well-suited to capturing incentive effects. A limitation of many structural models, including ours, is that they do not incorporate asymmetric information between shareholders and creditors. This is partly mitigated by the inclusion of jumps in asset value, which could reflect a sudden release of information, as in Duffie and Lando [26].

## 3.2 Outline

We formulate the model in Section 3.3 and develop our valuation method in Section 3.4. Section 3.5 explores the effects of replacing debt or equity with CoCos, with particular focus on the change in the cost of equity capital. Section 3.6 examines the effect of CoCos on the problem of debt overhang, and Section 3.7 examines their effect on asset substitution and risk sensitivity. Section 3.8 explains debt-induced collapse. Section 3.9 contrasts resolution authority with contingent capital and differentiates recovery rates for defaults at firm-specific jumps and market-wide jumps. Section 3.10 calibrates our model to data from individual banks through the financial crisis. Technical results are collected in an appendix.

## 3.3 The Model

### 3.3.1 Firm Asset Value

Much as in Merton [57], Black and Cox [9], Leland [50], Leland and Toft [51], and Goldstein, Ju, and Leland [34], consider a firm generating cash through its investments and operations

continuously at rate  $\{\delta_t, t \geq 0\}$ . The income flow  $\{\delta_t\}$  is exposed to firm-specific and market jump risk, with dynamics given by

$$\frac{d\delta_t}{\delta_{t-}} = \tilde{\mu}dt + \tilde{\sigma}d\tilde{W}_t + d \left( \sum_{i=1}^{\tilde{N}_t^f} (\tilde{Y}_i^f - 1) \right) + d \left( \sum_{i=1}^{\tilde{N}_t^m} (\tilde{Y}_i^m - 1) \right). \quad (3.1)$$

Here,  $\tilde{\mu}$  and  $\tilde{\sigma}$  are constants,  $\{\tilde{W}_t, t \geq 0\}$  is a standard Brownian motion, and we write  $\delta_{t-}$  to indicate the value just prior to a possible jump at time  $t$ . The last two terms reflect two types of jumps, the first we interpret as firm-specific and the second as market-wide. These are driven by Poisson processes  $\{\tilde{N}_t^f, t \geq 0\}$  and  $\{\tilde{N}_t^m, t \geq 0\}$  with intensities  $\tilde{\lambda}_f$  and  $\tilde{\lambda}_m$ . The jump sizes  $\{\tilde{Y}_i^f, i = 1, 2, \dots\}$  and  $\{\tilde{Y}_i^m, j = 1, 2, \dots\}$ , and  $\tilde{N}^f$ ,  $\tilde{N}^m$ , and  $\tilde{W}$  are all independent of each other. Since we are mainly concerned with the impact of downside shocks to the firm's business, we assume that the  $\tilde{Y}_i^f$  and  $\tilde{Y}_i^m$  are all less than 1. We can represent the common distribution of the  $\tilde{Y}_i^f$  and the common distribution of the  $\tilde{Y}_i^m$  by setting  $\tilde{Z}_f := -\log(\tilde{Y}^f)$  and  $\tilde{Z}_m := -\log(\tilde{Y}^m)$  and positing, for tractability, that these have exponential distributions,

$$f_{\tilde{Z}_f}(z) = \tilde{\eta}_f e^{-\tilde{\eta}_f z} \quad \text{and} \quad f_{\tilde{Z}_m}(z) = \tilde{\eta}_m e^{-\tilde{\eta}_m z}, \quad z \geq 0, \quad (3.2)$$

for some  $\tilde{\eta}_f, \tilde{\eta}_m > 0$ . In addition, we assume a constant risk-free interest rate  $r$ .

In a rational expectations framework with a representative agent having HARA utility, the equilibrium price of any claim on the future income of the firm can be shown to be the expectation of the discounted payoff of the claim under a ‘‘risk-neutral’’ probability measure  $\mathbb{Q}$ ; see Naik and Lee [60] and Kou [45] for a detailed justification of this assertion. The value of the firm's assets is the present value of the future cash flows they generate,

$$V_t = \mathbb{E}^{\mathbb{Q}} \left[ \int_t^{\infty} e^{-r(u-t)} \delta_u du \middle| \delta_t \right],$$

for all  $t \geq 0$ . Following Naik and Lee [60] and Kou [45], we can easily show that  $\delta := V_t/\delta_t$  is a constant and  $V_t$  evolves as a jump-diffusion process

$$\frac{dV_t}{V_{t-}} = (r - \delta) dt + \sigma dW_t + d \left( \sum_{i=1}^{N_t^f} (Y_i^f - 1) \right) + d \left( \sum_{j=1}^{N_t^m} (Y_j^m - 1) \right) - (\lambda_m + \lambda_f)\xi dt \quad (3.3)$$

with the parameter  $\xi < 0$  given by

$$\xi = \frac{\lambda_f}{\lambda_m + \lambda_f} \cdot \frac{\eta_f}{\eta_f + 1} + \frac{\lambda_m}{\lambda_m + \lambda_f} \cdot \frac{\eta_m}{\eta_m + 1} - 1.$$

Under  $\mathbb{Q}$ ,  $\{W_t\}$  in (3.3) is a standard Brownian motion and  $\{N_t^f\}$  and  $\{N_t^m\}$  are two Poisson processes with intensities  $\lambda_f$  and  $\lambda_m$ . The distributions of the jump sizes  $Y_i^f$  and  $Y_i^m$  have the same form as before, but now with parameters  $\eta_f$  and  $\eta_m$ . Kou [45] gives explicit expressions for the parameters in (3.3) in terms of the parameters in (3.1). We will value pieces of the firm's capital structure as contingent claims on the asset value process  $V$ , taking expectations under  $\mathbb{Q}$  and using the dynamics in (3.3).

### 3.3.2 The Capital Structure

The firm finances its assets by issuing four kinds of liabilities: insured deposits, senior and junior debt, contingent capital, and equity. We detail these in order of seniority.

#### A. Insured Deposits

Insured deposits have no contractual maturity and are subject to withdrawal at any time. We model this by assigning to each deposit a randomly distributed lifetime; for tractability, we take this lifetime to be exponentially distributed (as in Leland and Toft [51]) with a mean of  $1/m_1$ . More explicitly, the firm issues new accounts with a par value of  $p_1 dt$  at

every moment  $(t, t + dt)$  for all  $t \geq 0$ . In each subsequent interval  $(t + s, t + s + ds)$ ,  $s \geq 0$ , a fraction

$$\phi_1(s) = m_1 e^{-m_1 s} ds$$

of the initial deposit  $p_1 dt$  is withdrawn.

This specification generates a stationary profile for the firm's insured deposits. At any moment  $t > 0$ , the total par value of outstanding deposits is given by

$$\int_t^{+\infty} \left( \int_{-\infty}^t p_1 \phi_1(s - u) du \right) ds = \frac{p_1}{m_1} =: P_1,$$

and this remains constant until the default of the firm. Deposits earn interest at rate  $c_1$ , so the total interest paid on deposits in each interval  $(t, t + dt)$  is  $c_1 P_1 dt$ .

At bankruptcy, depositors have the most senior claim on the firm's assets. If these assets are insufficient to repay depositors, government insurance makes up the difference, so depositors are guaranteed repayment at par. Prior to bankruptcy, the firm pays premiums for deposit insurance at rate  $\varphi$ . The insurance premium may be fairly priced — exactly offsetting the expected payout from the insurance fund — but need not be. We also examine the implications of making insurance premiums proportional to all the firm's debt (consistent with rules adopted by the FDIC in 2011), and not just deposits.

## B. Senior and Subordinated Debt

In addition to deposits, the firm issues unsecured senior and subordinated debt. The costs and consequences of debt rollover are important to our analysis,<sup>2</sup> so we use the exponential

<sup>2</sup>Short-term debt addresses problems of asymmetric information and monitoring, as discussed in Calomiris and Kahn [19], Diamond and Rajan [24], and Gorton and Pennacchi [36]. But discussions of the financial crisis, including Brunnermeier [13], Krishnamurthy [47], and Shin [72], have highlighted the role of short-term financing and the resulting rollover risk (e.g., He and Xiong [39]).



maturity framework for these instruments as well. The firm continuously issues two classes of straight bonds, senior and subordinated, with respective par values  $p_2 dt$  and  $p_3 dt$  in  $(t, t + dt)$  for all  $t \geq 0$ . The maturities of the newly issued bonds are exponentially distributed; that is, a portion

$$\phi_i(s) = m_i e^{-m_i s} ds,$$

of the total amount  $p_i dt$ ,  $i = 2, 3$ , matures during the time interval  $(t + s, t + s + ds)$ , for all  $s \geq 0$ . As long as the firm is not in default, the par values of the outstanding senior and subordinated debt remains constant at levels  $P_i := p_i / m_i$ ,  $i = 2, 3$ . In effect, we assume the firm manages its debt issuance to target a fixed maturity profile and fixed levels of various types of debt.

Senior and subordinated debt pay coupons at rates  $c_i$ ,  $i = 2, 3$ , respectively. The total coupon payment on these bonds is then  $(c_2 P_2 + c_3 P_3) dt$  in each interval  $(t, t + dt)$ , up to the default of the firm.

Upon default, we assume that a fraction  $(1 - \alpha)$ ,  $0 \leq \alpha \leq 1$ , of the firm's asset value is lost to bankruptcy and liquidation costs. If we let  $V$  denote asset value at the moment of bankruptcy and thus  $\alpha V$  the value just after bankruptcy, then repaying depositors leaves  $(\alpha V - P_1)^+$ . Senior bond holders are repaid to the extent that the remaining funds suffice, so they get  $P_2 \wedge (\alpha V - P_1)^+$ , and the junior bond holders similarly get  $P_3 \wedge (\alpha V - P_1 - P_2)^+$ .

This discussion presupposes recovery at par value, in the sense that the bond holders have a claim of  $P_i$ ,  $i = 2, 3$ , in bankruptcy, as is the case in practice. Alternative modeling assumptions used in the literature include recovery at market value or at a fraction of an otherwise equivalent Treasury note; see Duffie and Singleton [27], Jarrow and Turnbull [43], and Lando [49]. The differences in these conventions are relatively minor and would not change our conclusions qualitatively.

### C. Contingent Convertibles

We use the same basic framework to model the issuance and maturity of CoCos as we use for other types of debt. In all cases, we would retain tractability if we replaced the assumption of exponential maturity profile with consols, but, as already noted, debt rollover is an important part of our analysis, so we use finite-maturity debt. We denote by  $P_4$  the par value of CoCos outstanding, which remains constant prior to conversion or default and pays a continuous coupon at rate  $c_4$ . The mean maturity is  $1/m_4$ , and new debt is issued at rate  $p_4$ .

Conversion of CoCos from debt to equity is triggered when the value of the firm's assets fall below an exogenously specified threshold  $V_c$ . Thus, conversion occurs at

$$\tau_c = \inf\{t \geq 0 : V_t \leq V_c\},$$

and we assume the trigger  $V_c$  is lower than the initial asset level  $V_0$ . (Because earnings  $\delta V_t$  are proportional to asset value, the trigger could equivalently be based on earnings, as posited in Koziol and Lawrenz [46].) At the instant of conversion, the CoCo liability is erased and CoCo investors receive  $\Delta$  shares of the firm's equity for every dollar of principal, for a total of  $\Delta P_4$  shares. We normalize the number of shares to 1 prior to conversion. Thus, following conversion, the CoCo investors own a fraction  $\Delta P_4 / (1 + \Delta P_4)$  of the firm. In the bail-in case,  $\Delta = \infty$ , so the original shareholders are wiped out and the converted investors take control of the firm. We think of the parameters  $(V_c, \Delta)$  as part of the contractual terms of the convertible debt and examine the consequences of varying these parameters.<sup>3</sup>

<sup>3</sup>We do not distinguish between contractual and statutory conversion. Under the former, conversion is an explicit contractual feature of the debt. The statutory case refers to conversion imposed on otherwise standard debt at the discretion of a regulator granted explicit legal authority to force such a conversion.

### 3.3.3 Endogenous Default

The firm has two types of cash inflows and three types of cash outflows. The inflows are the income stream  $\delta_t dt = \delta V_t dt$  and the proceeds from new bond issuance  $b_t dt$ , where  $b_t$  is the total market value of bonds issued at time  $t$ . The cash outflows are the after-tax coupon payments, the principal due  $(p_1 + p_2 + p_3 + p_4) dt$  on maturing debt, and insurance premiums  $\varphi P_1 dt$  or, more generally,  $\varphi P dt$  for some assessment base  $P$ . The firm has a marginal tax rate of  $\kappa$ , and we assume that interest payments on deposits and straight debt are tax-deductible. Thus, the after-tax coupon payment rate is given by  $A_t = (1 - \kappa)(c_1 P_1 + c_2 P_2 + c_3 P_3 + c_4 P_4)$  or  $A_t = (1 - \kappa)(c_1 P_1 + c_2 P_2 + c_3 P_3) + c_4 P_4$ , depending on whether or not coupon payments on CoCos are also tax deductible.

Let  $\bar{p}$  denote the total rate of issuance (and retirement) of par value of debt, just as  $b_t$  denotes the total rate of issuance measured at market value. We have  $\bar{p} = p_1 + p_2 + p_3 + p_4$  prior to conversion of any CoCos and  $\bar{p} = p_1 + p_2 + p_3$  after conversion. Whenever

$$b_t + \delta V_t > A_t + \bar{p} + \varphi P_1 \quad (3.4)$$

the firm has a net inflow of cash, which is distributed to equity holders as a dividend flow. When the inequality is reversed, the firm faces a cash shortfall. The equity holders then face a choice between making further investments in the firm — in which case they invest just enough to make up the shortfall — or abandoning the firm and declaring bankruptcy. Bankruptcy then occurs at

$$\tau_b = \inf\{t \geq 0 : V_t \leq V_b^*\},$$

the first time the asset level is at or below  $V_b^*$ , with  $V_b^*$  chosen optimally by the equity holders. In fact, it would be more accurate to say that  $V_b^*$  is determined simultaneously with  $b_t$ , because the market value of debt depends on the timing of default, just as the firm's

ability to raise cash through new debt influences the timing of default.

In Section 3.4 and the appendix, we derive explicit expressions for all the firm's liabilities, including the (before-conversion) equity value  $E^{\text{BC}}(V; V_b)$  as a function of the current asset value  $V$  and the default barrier  $V_b$ . In determining the optimal threshold at which to default, the equity holders seek to maximize the value of equity. They solve

$$\max_{V_b} E^{\text{BC}}(V; V_b)$$

subject to the limited liability constraint

$$E^{\text{BC}}(V'; V_b) \geq 0, \quad \text{for all } V' \geq V_b.$$

If the solution  $V_b^*$  to this problem is below the conversion trigger  $V_c$ , then the same default barrier solves the corresponding problem for the post-conversion equity value  $E^{\text{PC}}(V; V_b)$ :

$$\max_{V_b} E^{\text{PC}}(V; V_b)$$

subject to the limited liability constraint

$$E^{\text{PC}}(V'; V_b) \geq 0, \quad \text{for all } V' \geq V_b.$$

### 3.4 Valuing the Firm's Liabilities

We value the firm's liabilities by discounting their cashflows and taking expectations under the risk neutral probability  $\mathbb{Q}$ .

### A. Insured Deposits

In the presence of deposit insurance, repayment is guaranteed, but the timing of repayment may be accelerated by the default of the firm. To value a unit of deposit at time  $t$ , to be held on deposit until time  $t + T$ , we discount the interest earned over the interval  $[t, (t + T) \wedge \tau_b]$  and the principal received at  $(t + T) \wedge \tau_b$  to get a market value of

$$\begin{aligned} b_1(V_t; T) &= \mathbb{E}^{\mathbb{Q}} \left[ \int_t^{(t+T) \wedge \tau_b} c_1 e^{-r(u-t)} du + e^{-r(T \wedge (\tau_b - t))} \middle| V_t \right] \\ &= \frac{c_1}{r} + \left(1 - \frac{c_1}{r}\right) \mathbb{E}^{\mathbb{Q}} \left[ e^{-r(T \wedge (\tau_b - t))} \middle| V_t \right]. \end{aligned}$$

To simplify notation, we will henceforth take  $t = 0$  and omit the conditional expectation given  $V_t$ , though it should be understood that the value of each liability is a function of the current value  $V$  of the firm's assets.

Recall that we take the deposit lifetimes  $T$  to be exponentially distributed with density  $m_1 \exp(-m_1 T)$ , and the total amount in deposits is  $P_1$ ; the total market value of deposits then evaluates to

$$\begin{aligned} B_1(V) &= P_1 \int_0^\infty b_1(V; T) m_1 e^{-m_1 T} dT \\ &= \frac{c_1}{r} P_1 + m_1 P_1 \left(1 - \frac{c_1}{r}\right) \left( \frac{1}{m_1 + r} + \left( \frac{1}{m_1} - \frac{1}{m_1 + r} \right) \mathbb{E}^{\mathbb{Q}} \left[ e^{-(m_1 + r)\tau_b} \right] \right). \end{aligned}$$

The key to the valuation is thus the transform of the default time  $\tau_b$ , which is given explicitly by Cai and Kou [15] and Cai, Chen, and Wan [14].

The asset value remaining just after bankruptcy is  $\alpha V_{\tau_b}$ , and if this amount is less than the total deposits  $P_1$ , the difference is covered by deposit insurance. The market value of

this guarantee is therefore given by

$$\mathbb{E}^{\mathbb{Q}} \left[ e^{-r\tau_b} (P_1 - \alpha V_{\tau_b}) \mathbf{1}_{\{\alpha V_{\tau_b} < P_1\}} \right].$$

The firm pays a premium for deposit insurance at rate  $\varphi$ . If this rate is charged on the deposit base  $P_1$ , the cost of insurance is

$$\mathbb{E}^{\mathbb{Q}} \left[ \int_0^{\tau_b} \varphi P_1 e^{-rs} ds \right].$$

We also consider an alternative in which the premium is applied to all of the firm's debt, for which we replace  $P_1$  with  $P_1 + P_2 + P_3 + P_4$ .

## B. Senior and Subordinated Debt

We follow a similar approach to valuing straight debt, first considering a bond with a face value of 1 and a time-to-maturity  $T$ . The value of a senior bond with these terms is as follows:

$$\begin{aligned} b_2(V; T) &= \mathbb{E}^{\mathbb{Q}} \left[ e^{-rT} \mathbf{1}_{\{\tau_b > T\}} \right] \quad (\text{principal payment if no default}) \\ &+ \mathbb{E}^{\mathbb{Q}} \left[ e^{-r\tau_b} \mathbf{1}_{\{\tau_b \leq T\}} \mathbf{1}_{\{\alpha V_{\tau_b} \geq P_1 + P_2\}} \right] \quad (\text{payment at default, full recovery}) \\ &+ \mathbb{E}^{\mathbb{Q}} \left[ e^{-r\tau_b} \mathbf{1}_{\{\tau_b \leq T\}} \frac{\alpha V_{\tau_b} - P_1}{P_2} \mathbf{1}_{\{P_1 \leq \alpha V_{\tau_b} < P_1 + P_2\}} \right] \quad (\text{partial recovery}) \\ &+ \mathbb{E}^{\mathbb{Q}} \left[ \int_0^{\tau_b \wedge T} c_2 e^{-r(u-t)} du \right] \quad (\text{coupon payments}) \end{aligned} \quad (3.5)$$

In this expression,  $\{\tau_b \leq T\}$  is the event that default occurs before the bond matures, and  $\alpha V_{\tau_b}$  gives the value of the firm's assets just after default. If  $\alpha V_{\tau_b}$  exceeds  $P_1 + P_2$ , the senior bonds are repaid in full; if  $P_1 \leq \alpha V_{\tau_b} < P_1 + P_2$ , then each dollar of face value of senior debt recovers  $(V_{\tau_b} - P_1)/P_2$ .

Proceeding as we did for the value of the deposits, we calculate the total market value of senior debt to be

$$\begin{aligned}
B_2(V) &= P_2 \int_0^\infty b_2(V; T) e^{-m_2 T} dT \\
&= P_2 \left[ \left(1 - \frac{c_2}{r}\right) \cdot \frac{m_2}{m_2 + r} + \frac{c_2}{r} \right] \cdot \mathbb{E}^{\mathbb{Q}} [1 - e^{-(m_2+r)\tau_b}] \\
&+ P_2 \mathbb{E}^{\mathbb{Q}} \left[ e^{-(m_2+r)\tau_b} \left( \mathbf{1}_{\{\alpha V_{\tau_b} \geq P_1 + P_2\}} + \frac{\alpha V_{\tau_b} - P_1}{P_2} \mathbf{1}_{\{P_1 \leq \alpha V_{\tau_b} < P_1 + P_2\}} \right) \right]. \quad (3.6)
\end{aligned}$$

The expectation in (3.6) is evaluated in the appendix.

By exactly the same argument, the total market value of the subordinated debt is

$$\begin{aligned}
B_3(V) &= P_3 \left[ \left(1 - \frac{c_3}{r}\right) \cdot \frac{m_3}{m_3 + r} + \frac{c_3}{r} \right] \cdot \mathbb{E}^{\mathbb{Q}} [1 - e^{-(m_3+r)\tau_b}] \\
&+ P_3 \mathbb{E}^{\mathbb{Q}} \left[ e^{-(m_3+r)\tau_b} \left( \mathbf{1}_{\{\alpha V_{\tau_b} \geq P_1 + P_2 + P_3\}} + \frac{\alpha V_{\tau_b} - P_1 - P_2}{P_3} \mathbf{1}_{\{P_1 + P_2 \leq \alpha V_{\tau_b} < P_1 + P_2 + P_3\}} \right) \right].
\end{aligned}$$

### C. Contingent Convertibles

The market value of a CoCo combines the value of its coupons, its principal, and its potential conversion to equity.<sup>4</sup> Fix a default barrier  $V_b$ , and suppose that  $V_c > V_b$ , so that bankruptcy cannot occur prior to conversion. A CoCo with maturity  $T$  and unit face value has market value

$$b_4(V; T) = \mathbb{E}^{\mathbb{Q}} [e^{-rT} \mathbf{1}_{\{\tau_c > T\}}] + \mathbb{E}^{\mathbb{Q}} \left[ \int_0^{T \wedge \tau_c} c_4 e^{-rs} ds \right] + \frac{\Delta}{1 + \Delta P_4} \mathbb{E}^{\mathbb{Q}} [e^{-r\tau_c} E^{\text{PC}}(V_{\tau_c}) \mathbf{1}_{\{\tau_c < T\}}],$$

where  $E^{\text{PC}}(V)$  is the post-conversion value of equity at an asset value of  $V$ . At conversion, the CoCo investors collectively receive  $\Delta P_4$  shares of equity for each share outstanding,

<sup>4</sup>In applying the same pricing measure with and without CoCos, we are implicitly assuming that the impact of CoCos is not so great as to change the market's stochastic discount factor.

giving them a fraction  $\Delta P_4/(1 + \Delta P_4)$  of the firm, and dividing this by  $P_4$  yields the amount that goes to a CoCo with a face value of 1. The total market value of CoCos outstanding is then

$$\begin{aligned} B_4(V) &= P_4 \int_0^{+\infty} b_4(V; T) m_4 e^{-m_4 T} dT \\ &= P_4 \left[ \left(1 - \frac{c_4}{r}\right) \frac{m_4}{m_4 + r} + \frac{c_4}{r} \right] (1 - \mathbb{E} [e^{-(r+m_4)\tau_c}]) \\ &\quad + \frac{\Delta P_4}{1 + \Delta P_4} \mathbb{E} [e^{-(r+m_4)\tau_c} E^{\text{PC}}(V_{\tau_c})]. \end{aligned}$$

If  $V_c \leq V_b$ , so that conversion does not occur prior to bankruptcy, a similar calculation yields

$$\begin{aligned} B_4(V) &= P_4 \left[ \left(1 - \frac{c_4}{r}\right) \frac{m_4}{m_4 + r} + \frac{c_4}{r} \right] (1 - \mathbb{E} [e^{-(r+m_4)\tau_b}]) + P_4 \mathbb{E} [e^{-(r+m_4)\tau_b}] \\ &\quad + \mathbb{E}^{\mathbb{Q}} \left[ e^{-(r+m_4)\tau_b} \left( (\alpha V_{\tau_b} - P_1 - P_2 - P_3) \mathbf{1}_{\{P_1+P_2+P_3 \leq \alpha V_{\tau_b} < P_1+P_2+P_3+P_4\}} \right. \right. \\ &\quad \left. \left. + P_4 \mathbf{1}_{\{\alpha V_{\tau_b} \geq P_1+P_2+P_3+P_4\}} \right) \right] \end{aligned}$$

It remains to calculate  $E^{\text{PC}}(V)$ , the post-conversion equity value with a default barrier of  $V_b$ . We derive this value by calculating total firm value and subtracting the value of debt. After conversion, total firm value is given by

$$\begin{aligned} F^{\text{PC}}(V) &= V + \underbrace{\mathbb{E}^{\mathbb{Q}} \left[ \int_0^{\tau_b} \kappa (c_1 P_1 + c_2 P_2 + c_3 P_3) e^{-rs} ds \right]}_{\text{tax benefits}} - \underbrace{\mathbb{E}^{\mathbb{Q}} [e^{-r\tau_b} (1 - \alpha) V_{\tau_b}]}_{\text{bankruptcy costs}} \quad (3.7) \\ &\quad + \underbrace{\mathbb{E}^{\mathbb{Q}} [e^{-r\tau_b} (P_1 - \alpha V_{\tau_b}) \mathbf{1}_{\{\alpha V_{\tau_b} \leq P_1\}}]}_{\text{deposit insurance}} - \underbrace{\mathbb{E}^{\mathbb{Q}} \left[ \int_0^{\tau_b} \varphi P_1 e^{-rs} ds \right]}_{\text{insurance premiums}} \\ &= V + \left[ (c_1 P_1 + c_2 P_2 + c_3 P_3) \frac{\kappa}{r} - P_1 \frac{\varphi}{r} \right] (1 - \mathbb{E}^{\mathbb{Q}} [e^{-r\tau_b}]) \\ &\quad + \mathbb{E}^{\mathbb{Q}} [e^{-r\tau_b} (P_1 - \alpha V_{\tau_b}) \mathbf{1}_{\{\alpha V_{\tau_b} \leq P_1\}}] - \mathbb{E}^{\mathbb{Q}} [e^{-r\tau_b} (1 - \alpha) V_{\tau_b}] \quad (3.8) \end{aligned}$$



The post-conversion equity value (for a given barrier  $V_b$ ) is then given by

$$E^{\text{PC}}(V) \equiv E^{\text{PC}}(V; V_b) = F^{\text{PC}}(V) - B_1(V) - B_2(V) - B_3(V),$$

the value that remains after subtracting deposits and senior and subordinated debt from total firm value. The expression we need to value the CoCos,

$$\mathbb{E}^{\mathbb{Q}} \left[ e^{-r\tau_c} E^{\text{PC}}(V_{\tau_c}) \mathbf{1}_{\{\tau_c < T\}} \right], \quad (3.9)$$

is evaluated using the method in the appendix.

#### D. Equity Value Before Conversion

Just as we did for the post-conversion value of equity, we calculate the value before conversion by first calculating total firm value before conversion,  $F^{\text{BC}}(V)$ , given a default barrier  $V_b$ . To the expression above for the post-conversion value, we need to add the tax shield on CoCos,

$$\mathbb{E}^{\mathbb{Q}} \left[ \int_0^{\tau_c \wedge \tau_b} \kappa c_4 e^{-ru} du \right],$$

if the CoCo coupons are tax-deductible. Thus,

$$F^{\text{BC}}(V) = F^{\text{PC}}(V) + \begin{cases} c_4 P_4 \frac{\kappa}{r} (1 - \mathbb{E}^{\mathbb{Q}} [e^{-r\tau_b}]), & \text{if } V_c \leq V_b; \\ c_4 P_4 \frac{\kappa}{r} (1 - \mathbb{E}^{\mathbb{Q}} [e^{-r\tau_c}]), & \text{if } V_c > V_b. \end{cases}$$

If insurance premiums are charged in proportion to all debt, and not just deposits, then conversion of CoCos also reduces premium payments, and we need to add

$$\mathbb{E}^{\mathbb{Q}} \left[ \int_0^{\tau_c \wedge \tau_b} \varphi P_4 e^{-rs} ds \right]$$

to  $F^{\text{PC}}(V)$  to get  $F^{\text{BC}}(V)$ . The market value of the firm's equity before conversion is then given by

$$E^{\text{BC}}(V) = F^{\text{BC}}(V) - B_1(V) - B_2(V) - B_3(V) - B_4(V). \quad (3.10)$$

Both  $F^{\text{BC}}(V)$  and  $E^{\text{BC}}(V)$  admit closed-form expressions.

We thus have explicit expressions for the market values of all pieces of the firm's capital structure, for a given default barrier  $V_b$ . To complete the valuation, we endogenize default, letting equity holders choose  $V_b$  optimally, as discussed in [Section 3.3.3](#).

To solve this type of problem in a pure-diffusion model without CoCos, Leland [\[50\]](#) and Leland and Toft [\[51\]](#) use a smooth-pasting principle. In our setting, this becomes

$$\left. \frac{\partial E^{\text{BC}}(V)}{\partial V} \right|_{V=V_b^*} = 0, \quad (3.11)$$

with the understanding that  $E^{\text{BC}}(V) = E^{\text{PC}}(V)$  if  $V \leq V_c$ . We take the solution to [\(3.11\)](#) to be the optimal barrier  $V_b^*$ . Chen and Kou [\[18\]](#) justify this approach in a jump-diffusion model without CoCos (see also Kyprianou and Surya [\[48\]](#)).<sup>5</sup> This calculation has to be done numerically, so we do not have explicit expressions for  $V_b^*$  or for the value of equity using  $V_b^*$ .<sup>6</sup>

<sup>5</sup>Décamps and Villeneuve [\[23\]](#) show that the situation is more complex, and the resulting equilibrium unknown in general, when equity holders may deviate from a stated default rule and creditors anticipate this possibility. The added generality would be needed to incorporate strategic behavior on the part of creditors and the possibility that creditors might force the firm into bankruptcy.

<sup>6</sup>The detailed verification of the smooth pasting condition in our setting has been outlined in notes provided by Professor Nan Chen.

### 3.5 Changes in Capital Structure

We can use the valuation results of the previous section to investigate the effects of changes in the firm's liability structure. We focus in particular on the perspective of the equity holders and the impact of issuing CoCos.

For our base case, we use the parameters in [Table 3.1](#). The firm initially funds 100 in assets with par values of 40 in deposits, 30 in senior debt, 15 in subordinated debt, and 15 in equity or a combination of equity and CoCos. The proceeds of issuing new debt may be used to scale up the firm's assets, to pay down another form of debt, or to buy back equity. We consider all three cases. Under any change in capital structure, we recompute the optimal default barrier and recompute the value of the firm and its liabilities.

The process of rolling debt is important to our analysis, so we briefly describe how this works. Under our exponential maturity assumption, old debt is continuously maturing and new debt is continuously issued. Within each debt category, the coupon and the total par value outstanding remain constant; but while debt matures at par value, it is issued at *market* value. If the par value is greater, the difference is a cash shortfall that needs to be paid out by the firm; if the market value is greater, the difference generates additional cash for the firm. We refer to these as rollover costs — a negative cost in the first case, a positive cost in the second — and treat them the same way we treat coupon payments. Rollover costs will change as the firm's asset value changes, becoming more negative as asset value declines, the firm gets closer to default, and the market value of its debt decreases. Rollover costs thus capture the increased yield demanded of riskier firms.

Parameter		Value
initial asset value	$V_0$	100
debt principal	$(P_1, P_2, P_3)$	(40, 30, 15)
risk free rate	$r$	6%
volatility	$\sigma$	8%
payout rate	$\delta$	1%
tax rate	$\kappa$	35%
firm specific jump intensity	$\lambda_f$	.2
market jump intensity	$\lambda_m$	.05
firm specific jump exponent	$\eta_f$	4
market jump exponent	$\eta_m$	3
coupon rates	$(c_1, c_2, c_3, c_4)$	$(r, r + 3\%, r + 3\%, r)$
deposits insurance premium rate	$\varphi$	1%
contingent capital principal	$P_4$	1 or 5
maturity profile exponent – base case	$(m_1, m_2, m_3, m_4)$	(1, 1/4, 1/4, 1/4)
maturity profile exponent – long maturity	$(m_1, m_2, m_3, m_4)$	(1, 1/16, 1/16, 1/16) or (1, 1/25, 1/25, 1/25)
conversion trigger	$V_c$	75 (in most cases)
conversion loss (if applied)		20% of value of shares

**Table 3.1:** Base case parameters. Asset returns have a total volatility (combining jumps and diffusion) of 20.6% and overall drift rate of 3.3%. In the base case, the number of shares  $\Delta$  issued at conversion is set such that if conversion happens at exactly  $V_c$ , the market value of shares delivered is the same as the face value of the converted debt.

### 3.5.1 Replacing Straight Debt with CoCos

We begin by replacing some straight debt — either senior or subordinated — with CoCos. We assume that the conversion trigger  $V_c$  is higher than the bankruptcy barrier  $V_b^*$ , so bankruptcy cannot precede conversion. This holds in our base case. As noted before, so long as the trigger is above the barrier, it has no effect on the barrier; in other words,  $\partial V_b^*/\partial V_c = 0$  whenever  $V_b^* > V_c$ .

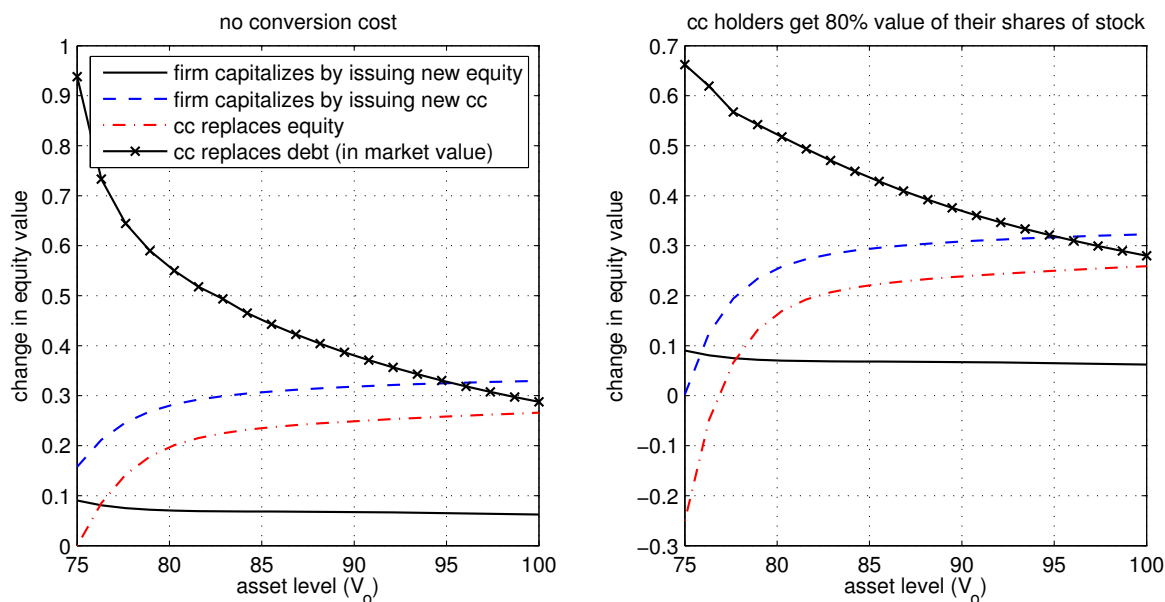
We can walk through the consequences of the substitution as follows.

- If coupon payments on CoCos are not tax deductible, then replacing straight debt with CoCos has the immediate effect of reducing firm value by reducing the value of the tax

shield. Even if CoCo coupons are tax deductible, this benefit ends at conversion, so, other things being equal, the substitution still has the immediate impact of reducing firm value; see (3.7)–(3.8). The reduction in firm value has the direct effect of lowering the value of equity.

- However after conversion the firm will have less debt outstanding and lower debt service payments (coupons and rollover costs) than it would without the substitution of CoCos for straight debt. With lower debt service, more of the cash  $\delta V_t dt$  generated by the firm's assets flows to equity holders in dividends. This reduces the default barrier  $V_b^*$ , which extends the life of the firm, reduces the bankruptcy cost  $\mathbb{E}[e^{-r\tau_b}(1-\alpha)V_{\tau_b}]$ , and thus increases firm value in (3.7)–(3.8).
- We thus have two opposite effects on firm value: the reduced tax shield from CoCos reduces firm value, but the reduced default probability and bankruptcy cost increases firm value. In our numerical examples, we find that the second effect dominates over a wide range of parameter values, so that the net effect of replacing straight debt with CoCos is to increase firm value.
- Part of this increase in firm value is captured by senior and subordinated debt holders because the reduced bankruptcy risk increases the value of the debt; see (3.6), for example. Part of the increase is also captured by equity holders: the increased debt value reduces rollover costs which increases the flow of dividends. Thus, *equity holders have a positive incentive to issue CoCos.*

This conclusion contrasts with that of Albul et al. [2], who find that equity holders would never voluntarily replace straight debt with contingent convertibles. In their model, straight debt has infinite maturity and is never rolled. As a result, all of the benefit of reduced bankruptcy costs from CoCos is captured by debt holders, indeed leaving no incentive for equity holders. This difference highlights the importance of debt rollover in influencing



**Figure 3.1:** Change in equity value resulting from various changes in capital structure. In the right hand figure the CoCo holders dump their shares in the market following the conversion and as a result lose 20% value of their shares due price impact and transaction fees.

incentives for equity holders, an effect we return to at several points.

The line marked with crosses in the left panel of Figure 3.1 shows the increase in equity value resulting from a substitution of one unit (market value) of CoCos for one unit (market value) of straight debt, plotted against the value of the firm's asset value. The conversion level  $V_c$  is 75. Despite the dilutive effect of conversion, the benefit to equity holders of the substitution is greatest just above the conversion level and decreases as asset level increases. This follows from the fact that the benefit to equity holders derives from the reduction in bankruptcy costs, which is greater at lower asset values. We will discuss the other curves in the left panel shortly.

The right panel of Figure 3.1 incorporates a friction in the conversion of debt to equity. To this point, we have valued each security as the expected present value of its cash flows.

In practice, the markets for debt and equity are segmented, and some bond investors may be unwilling (or unable under an investment mandate) to own equity. Such investors would value CoCos at less than their present value, and this effect could well move the price at which the market clears, given the comparatively small pool of investors focused on hybrid securities.

To capture this effect, we suppose that the equity received by CoCo investors at conversion is valued at 80% of market value. For example, we can think of CoCo investors as dumping their shares at a discount, with the discount reflecting a market impact that is only temporary and therefore does not affect the original equity holders. CoCo investors anticipate that they will not receive the full value of equity at conversion and thus discount the price of CoCos up front. This makes CoCos more expensive for the firm as a source of funding. The line marked with crosses in the right panel shows the benefit to equity holders of the same substitution examined in the left panel. As one would expect, the benefit is substantially reduced near the conversion trigger of 75 (comparing the two panels); at higher asset values, the difference between the cases vanishes, with the crossed lines in both panels near 0.3 at an asset level of 100. This discussion can be summarized as follows: *Segmentation between debt and equity investors that creates a friction in conversion reduces the benefit of issuing CoCos; this effect is partially offset by lowering the conversion trigger.*

### 3.5.2 Increasing the Balance Sheet with CoCos

We now consider the effects of issuing CoCos without an offsetting reduction in any other liabilities. The proceeds from issuing CoCos are used to scale up the firm's investments. The consequences of this change are as follows:

- Because the post-conversion debt outstanding is unchanged, the endogenous default

barrier  $V_b^*$  is unchanged, so long as the optimal barrier is below the conversion trigger  $V_c$ .

- In this case, the risk of default decreases because an increase in assets moves the firm farther from the default barrier. The reduction in bankruptcy costs increases firm value and the value of straight debt. The additional tax shield from issuing CoCos (assuming their coupons are tax-deductible) further increases firm value.
- Shareholders benefit from the increase in firm value combined with the decrease in rollover costs for straight debt and the increase in cash generated from the larger asset base. These benefits work in the opposite direction of the increase in coupon payments required for the new CoCos.
- With a sufficiently large CoCo issue, it becomes optimal for equity holders to default prior to conversion. At this point, the firm moves, in effect, from one equilibrium to another, and the increased default risk works against the increase in firm, debt, and equity value. We return to this in [Section 3.8](#).

The dashed line in each panel of [Figure 3.1](#) shows the benefit to shareholders of issuing a unit of new CoCos with  $V_b^* < V_c$ . The benefit is lower on the right in the presence of a conversion friction. Whereas the incentive for debt substitution decreases with asset value, the incentive for issuing new CoCos increases with asset value.

### 3.5.3 Replacing Equity with CoCos

We include this case for completeness. The firm issues CoCos and the proceeds are used to buy back equity; the cash from the buy-back is part of the benefit received by equity holders. If the conversion trigger is above the default barrier, the post-conversion firm is unaffected because the CoCos that replaced the equity convert to equity; thus, the timing of default is



unchanged. The value of straight debt is therefore unaffected by the replacement, and there are no changes in rollover costs of straight debt. If the coupon on CoCos is tax deductible, this benefit from the newly issued CoCos is entirely captured by the equity holders. The total benefit to equity holders (the buy-back cash and the tax shield) is illustrated by the dash-dot line in each panel of [Figure 3.1](#). In the right panel, the benefit is negative at asset levels near the conversion trigger because the conversion friction increases the cost of issuing CoCos and decreases the cash received in the buy back.

Finally, the solid line in the figures shows the net benefit to shareholders of a unit increase in their equity investment. The benefit is positive because the additional equity reduces default risk and thus lowers rollover costs, a phenomenon not observed in a static model of capital structure. Thus, the negative effect of debt overhang does not overwhelm the potential value of additional investment. We further develop this point in [Section 3.6](#).

#### 3.5.4 The Bail-In Case

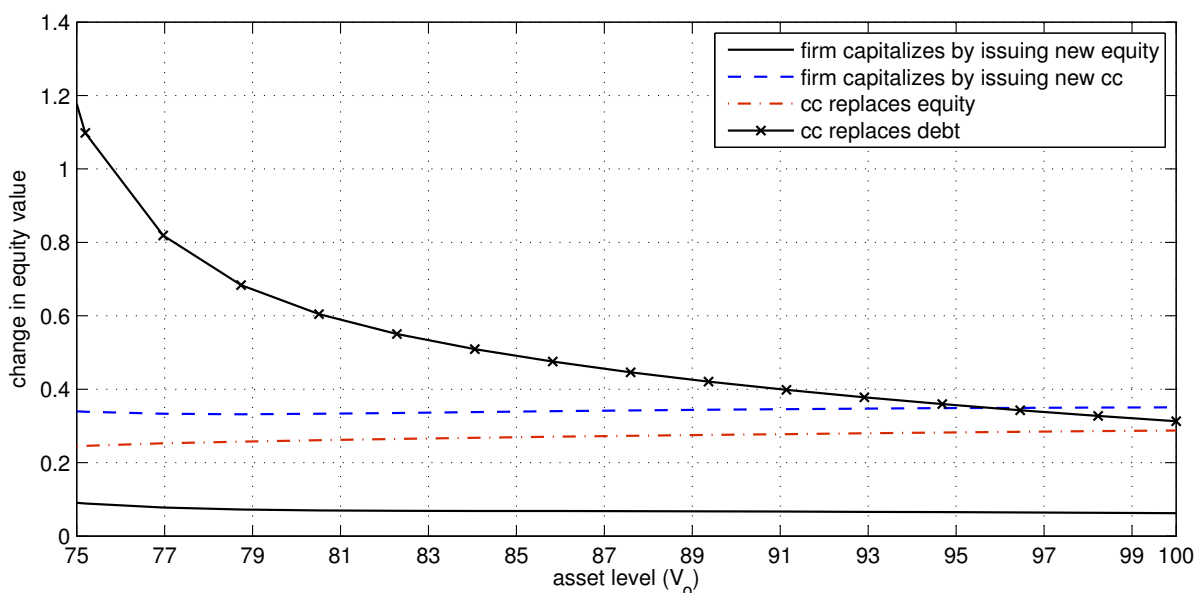
We model bail-in debt by taking  $\Delta = \infty$ , meaning that the original shareholders are infinitely diluted and thus wiped out. Also, there is no exogenous trigger level  $V_c$  in the bail-in case; instead, conversion occurs at the moment that the original equity holders declare bankruptcy. The convertible debt converts to equity, the converted investors become the sole shareholders, and they then determine a new default barrier endogenously. We assume that the bail-in avoids all bankruptcy costs, but the key assumption is that the asset recovery rate at bail-in is greater than the recovery rate  $\alpha$  at default.

[Figure 3.2](#) illustrates the same comparisons made in the left panel of [Figure 3.1](#). The main observation is that the incentive to issue convertible debt is greater in [Figure 3.2](#) than in [Figure 3.1](#). This is primarily due to the lowering of the conversion threshold — the trigger

is 75 in Figure 3.1 whereas the bail-in point is a bit below 70 in Figure 3.2. As long as conversion occurs before bankruptcy, the level of the conversion threshold has no effect on firm value or the value of straight debt. It does affect how value is apportioned between equity holders and investors in the convertible debt.

### 3.6 Debt Overhang and Investment Incentives

In most capital structure models, equity holders are least motivated to invest in a firm precisely when the firm most needs additional equity. For a firm near bankruptcy, much of the value of an additional equity investment is captured by debt holders as the additional equity increases the market value of the debt by reducing the chances of bankruptcy. This is a problem of debt overhang (Myers [59]), and it presents a significant obstacle to recapitalizing ailing banks. Duffie [25] has proposed mandatory rights offerings as a mechanism to compel



**Figure 3.2:** Change in equity value resulting from various changes in capital structure with bail-in debt.

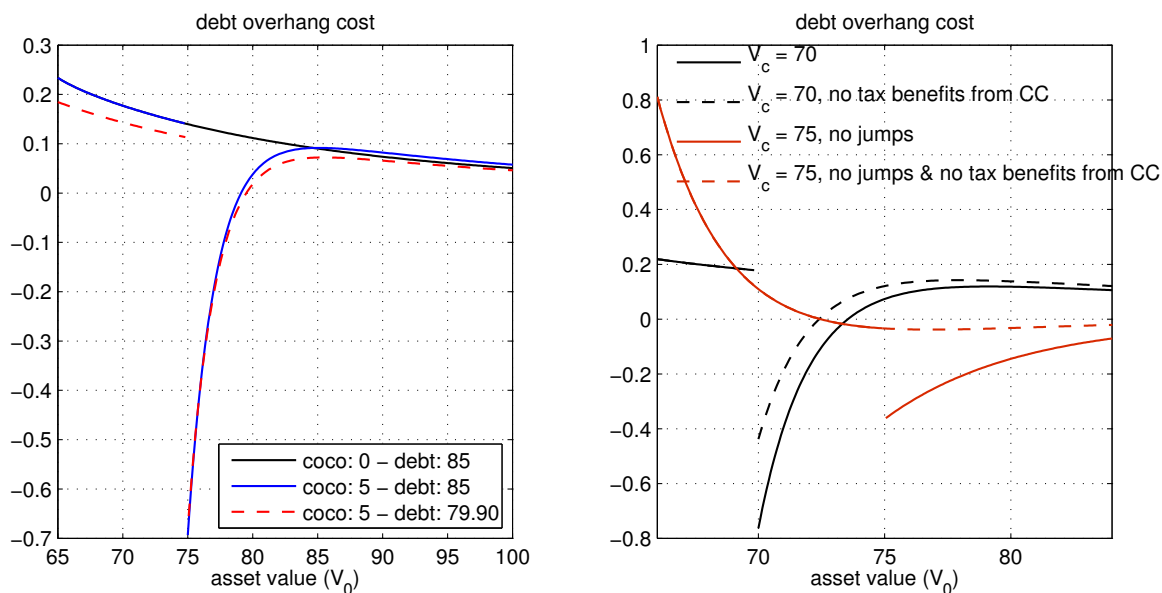
investment. Here we examine the effect of CoCos on investment incentives.

The phenomenon of debt overhang is easiest to see in a static model, viewing equity as a call option on the assets of a firm with a strike price equal to the face value of debt, as in Merton [57]. At a low asset value, where the option is deep out-of-the-money, the option delta is close to zero: a unit increase in asset value produces much less than a unit increase in option value, so equity holders have no incentive to invest. Indeed, in this static model, the net benefit of investment is always negative.

At least three features distinguish our setting from the simple static model. First, the reduction in rollover costs that follows from safer debt means that equity holders have the potential to derive some benefit from an increase in their investment. Second, the dilutive effects of CoCo conversion creates an incentive for shareholders to invest to prevent conversion. Third, if CoCo coupons are tax deductible, shareholders have an added incentive to invest in the firm near the conversion trigger to avoid the loss of this tax benefit.

Figure 3.3 shows the cost to equity holders of an additional investment of 1 in various scenarios. Negative costs are benefits. For this example, we use the longer maturities for debt in Table 3.1, as the overhang problem is more acute in this case. This is illustrated by the solid black line in the left panel, which shows the overhang cost is positive throughout the range of asset values displayed.

The solid blue line and the dashed line show the overhang cost after the firm has issued CoCos. The blue line corresponds to replacing equity with CoCos, and the dashed line corresponds to replacing straight debt with CoCos. As we move from right to left, tracing a decline in asset value toward the conversion threshold  $V_c = 75$ , we see a dramatic increase in the benefit (negative cost) to equity holders of an additional investment. In other words, *the presence of CoCos creates a strong incentive for equity holders to invest in the firm to*



**Figure 3.3:** Net cost to shareholders of increasing the firm's asset by 1. Negative costs are gains. The figures show that CoCos and tail risk create a strong incentive for additional investment by equity holders near the conversion trigger.

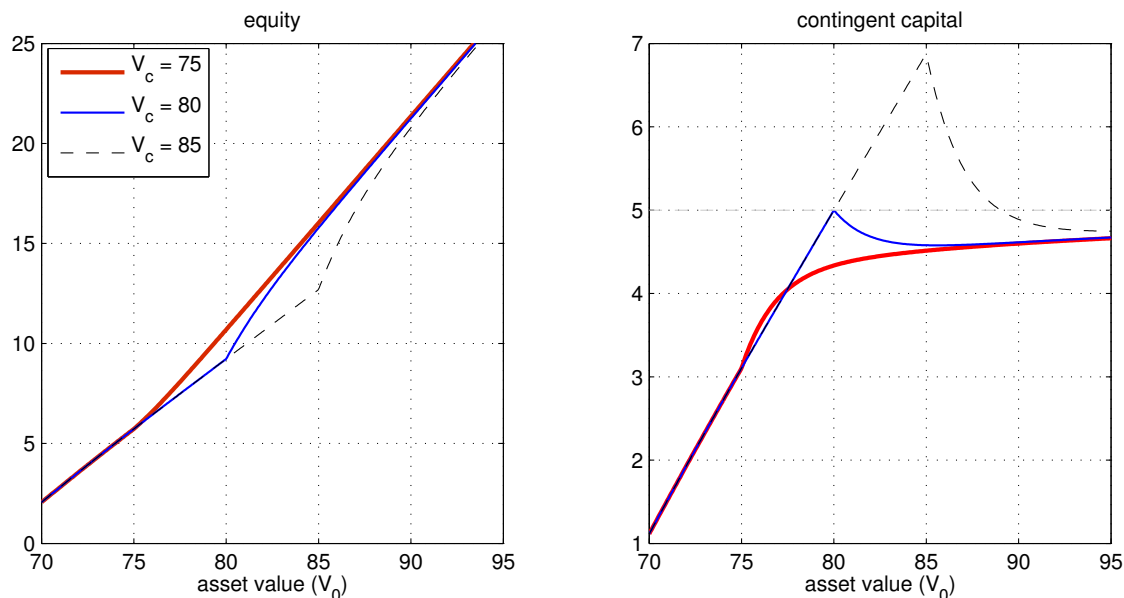
*avoid conversion.* After conversion (below an asset level of 75), the overhang cost reverts to its level in a firm without CoCos.

The right panel of Figure 3.3 provides further insight into the investment incentive illustrated in the left panel. If we lower the conversion trigger from 75 to 70, we see from the solid black line that the investment incentive becomes greatest at 70, as expected, where it is a bit greater than the greatest value in the left figure. Removing the tax-deductibility of CoCo coupons yields the dashed black line, which shows that the investment incentive is reduced but not eliminated. In the solid red line, we have returned the conversion trigger to 75 but removed the jumps from the asset process. This eliminates close to half the incentive for investment, compared to the left panel. Removing both the tax shield on CoCos and jumps in asset value eliminates almost all the investment incentive, as indicated by the dashed red line.

The tax effect is immediate: the tax shield increases the value to shareholders of avoiding the conversion of CoCos and thus creates a greater incentive for investment. The jump effect requires some explanation. Recall that the conversion ratio  $\Delta$  is set so that the market value of the shares into which the CoCos convert equals the face value of the converted debt *if conversion occurs at an asset level of  $V_c$* . If a downward jump takes  $V_t$  from a level above the trigger  $V_c$  to a level below it, then conversion occurs at an asset level lower than  $V_c$ , and the market value of the equity granted to CoCo investors is less than the face value of the debt. Equity holders thus prefer conversion following a jump to conversion at the trigger; indeed, conversion right at the trigger is the worst conversion outcome for equity holders, and this creates an incentive for investment as asset value approaches the trigger. The equity holders would prefer to delay conversion and, in effect, bet on converting at a jump rather than right at the trigger. This suggests that CoCos may create an incentive for equity investors to take on further tail risk, an issue we investigate in the next section.

We close this section by examining the value of equity and CoCos across the conversion trigger. In [Figure 3.4](#), we fix the conversion ratio  $\Delta$  at the “fair” value for a CoCo size of 5 and a conversion trigger of  $V_c = 80$ ; with  $\Delta = .108$ , the market value of the  $5\Delta$  shares issued to the CoCo investors equals the par value of 5 if the asset value  $V$  equals  $V_c$ . We keep  $\Delta$  fixed at this level and vary  $V_c$  so that the value received at the trigger is either greater ( $V_c = 85$ ) or smaller ( $V_c = 75$ ).

The figures indicate that there is no instantaneous value transfer across the conversion trigger, a property introduced by Sundaresan and Wang [\[74\]](#). In other words, the values are continuous across the trigger. There does appear to be a kink at conversion — a discontinuity in the derivative. The higher sensitivity of equity to asset value above  $V_c$  is consistent with higher leverage. Conversion that is less attractive to CoCo investors (a lower  $V_c$  with the same  $\Delta$ ) produces a small equity sensitivity above the trigger, suggesting that this may also reduce risk-taking incentives; we examine this in greater detail in the next section.



**Figure 3.4:** Equity and CoCo values are continuous functions of asset level — there is no abrupt value transfer at conversion. The three curves use the same conversion ratio  $\Delta$ , set here so that the value of the equity held by CoCo investors just after conversion equals the par value of the CoCos if conversion occurs at an asset level of 80. With conversion at 85, CoCo investors get more than the par value in equity; with conversion at 75, they get less the par value.

The right panel shows an interesting pattern for CoCo value near the conversion trigger. When conversion grants shares at a discount price advantageous to CoCo investors (the dashed line), CoCo value increases as asset value decreases toward the trigger. We see a similar but less pronounced increase in the blue line, in which conversion is at par. The red line reflects conversion at a premium price per share, and in this case CoCo value declines steadily.

A parallel pattern has been observed for contingent capital with a stock price trigger, and the possibility that the CoCo value would increase as the stock price decreases has raised concerns about potential market manipulation and a downward spiral as CoCo investors

short the stock to try to trigger conversion. This concern is not directly applicable to our setting, as investors have little chance of moving asset value. Nevertheless, the two panels of [Figure 3.4](#) suggest that conversion at a small premium (the solid red line) yields lower volatility in CoCo and equity value around the conversion trigger.

Calomiris and Herring [\[17\]](#) argue that CoCos should be designed so that they convert at a ratio punitive to shareholders and also so that their yield spreads widen as they approach conversion, providing a signal to the market of the firm's condition (as has been discussed for subordinated debt — see Evanoff and Wall [\[28\]](#), Flannery [\[30\]](#), and Hancock and Kwast [\[37\]](#)). The right panel of [Figure 3.4](#) shows that these objectives are incompatible: for spreads to widen near conversion, the conversion should be at small premium over the market value of the shares earned.

### 3.7 Asset Substitution and Risk Sensitivity

We reviewed the problem of debt overhang in the previous section in Merton's [\[57\]](#) model, which views equity as a call option on the firm's assets. The same model predicts that equity value increases with the volatility of the firm's assets, giving equity holders an incentive to increase the riskiness of the firm's investments after they have secured funding from creditors. In this section, we examine this phenomenon in our dynamic model, focusing on how CoCos change the incentives. Related questions of risk-shifting incentives are studied in Albul et al. [\[2\]](#), Hilscher and Raviv [\[41\]](#), Koziol and Lawrenz [\[46\]](#), and Pennacchi [\[64\]](#) with contingent capital and in Bhanot and Mello [\[8\]](#) for debt with rating triggers. Morellec [\[58\]](#) studies the impact of asset liquidity on debt capacity.

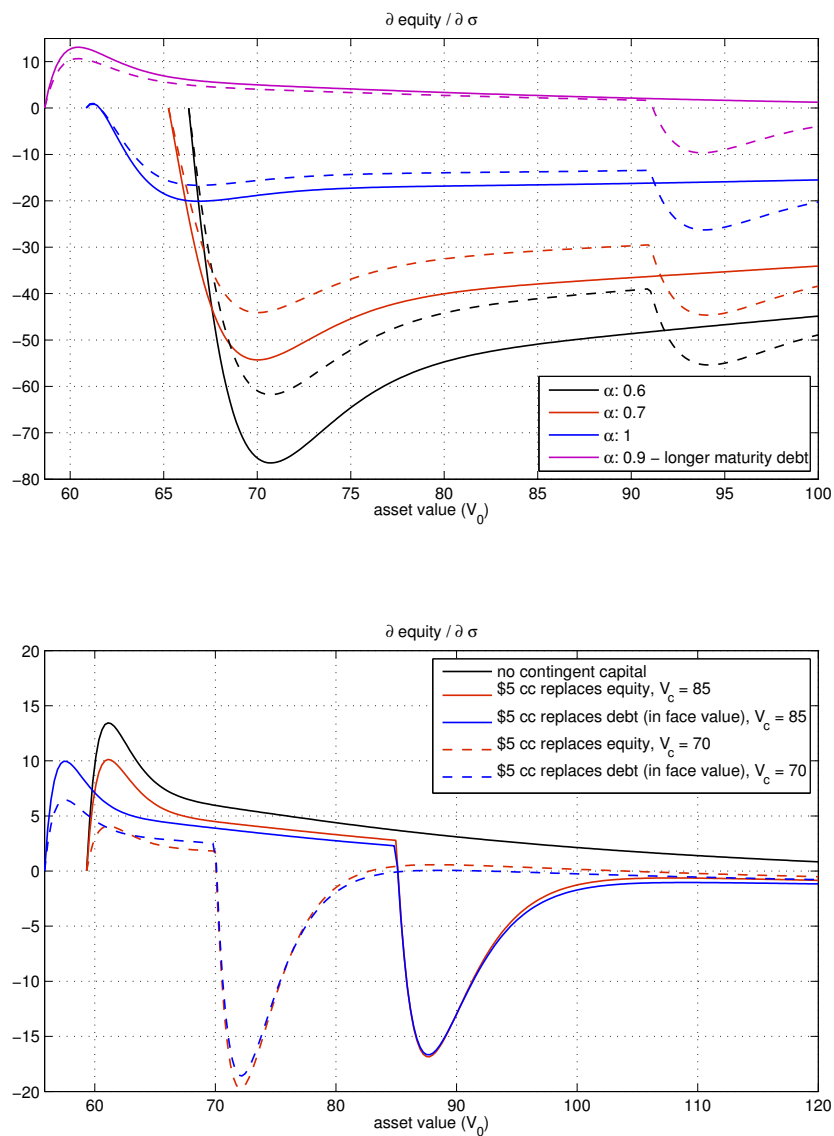
We can summarize our main observations as follows. Because of the need to roll maturing debt, equity holders do not necessarily prefer more volatile assets in a dynamic model; longer

debt maturity makes riskier assets more attractive to equity holders. Even when equity value does increase with asset volatility, CoCos can mitigate or entirely offset this effect, in part because equity holders are motivated to avoid conversion. In some cases, CoCos can make tail risk more attractive to equity holders even while making diffusive risk less attractive.

To illustrate these points, we start with the lower panel of [Figure 3.5](#), which shows the sensitivity of equity to diffusive volatility as a function of asset value. The solid black line corresponds to a firm with no contingent capital — the sensitivity of equity to  $\sigma$  is positive throughout the range and peaks just above the default barrier. As the firm nears bankruptcy, the equity holders are motivated to take on extra risk in a last-ditch effort at recovery.

We see a very different pattern in the two blue lines, corresponding to a firm in which some straight debt has been replaced with CoCos, and the two red lines, based on replacing some equity with CoCos. In both cases, the solid line is based on a conversion trigger of 85, and the dashed line uses a trigger of 70. This gives us four combinations of capital structure and trigger level. In all four, the sensitivity is *negative* at high asset values and turns sharply negative as asset value decreases toward the conversion boundary before becoming slight positive just above the trigger, where equity holders would prefer to gamble to avoid conversion. After conversion, the pattern naturally follows that of a firm without CoCos. The key implication of the figure is that CoCos decrease, and even reverse, the incentive for the shareholders to increase the riskiness of the firm's assets.





**Figure 3.5:** Sensitivity of equity value to diffusive volatility  $\sigma$ . With longer maturity debt, equity holders have a positive risk-shifting incentive. CoCos tend to reverse this incentive.

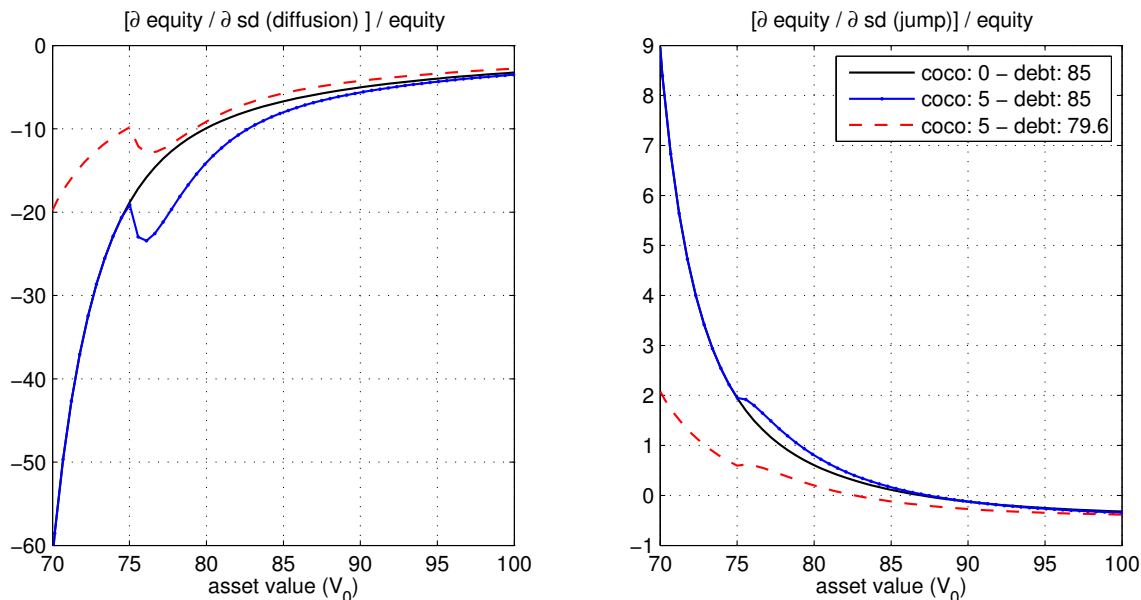
The top half illustrates the effect of debt maturity and bankruptcy costs on the risk-shifting incentive. In each pair of lines, the dashed line has the same level of deposits and straight debt as the solid line but it also has CoCos. Considering first the solid lines, we

see that with long-maturity debt, the risk-shifting incentive is positive, even at rather high recovery rate of  $\alpha = 90\%$ . In contrast, with shorter maturity debt, the sensitivity is nearly always negative, even with a recovery rate of 100% — i.e., with no bankruptcy costs. Thus, debt maturity and not bankruptcy cost is the main driver of the sign of the risk-sensitivity. CoCos therefore have a greater effect on the risk-shifting incentive when the rest of the firm's debt has longer average maturity. The impact of CoCos is not very sensitive to the recovery rate  $\alpha$ .

Figures 3.6 and 3.7 illustrate similar comparisons but with the sensitivity at each asset level normalized by the value of equity at that asset level; we interpret this as measuring the risk-shifting incentive per dollar of equity. Also, the figures compare sensitivities to diffusive volatility on the left with sensitivity to tail risk, as measured by  $1/\eta_f$ , on the right. Figure 3.7 uses a longer average maturity of debt than Figure 3.6.

The left panels of Figures 3.6 and 3.7 are consistent with what we saw in Figure 3.5 for the unnormalized sensitivities: with longer maturity debt, CoCos reverse the risk-shifting incentive; with shorter maturity debt, equity holders already have an incentive to reduce risk, particularly at low asset values, and CoCos make the risk sensitivity more negative.

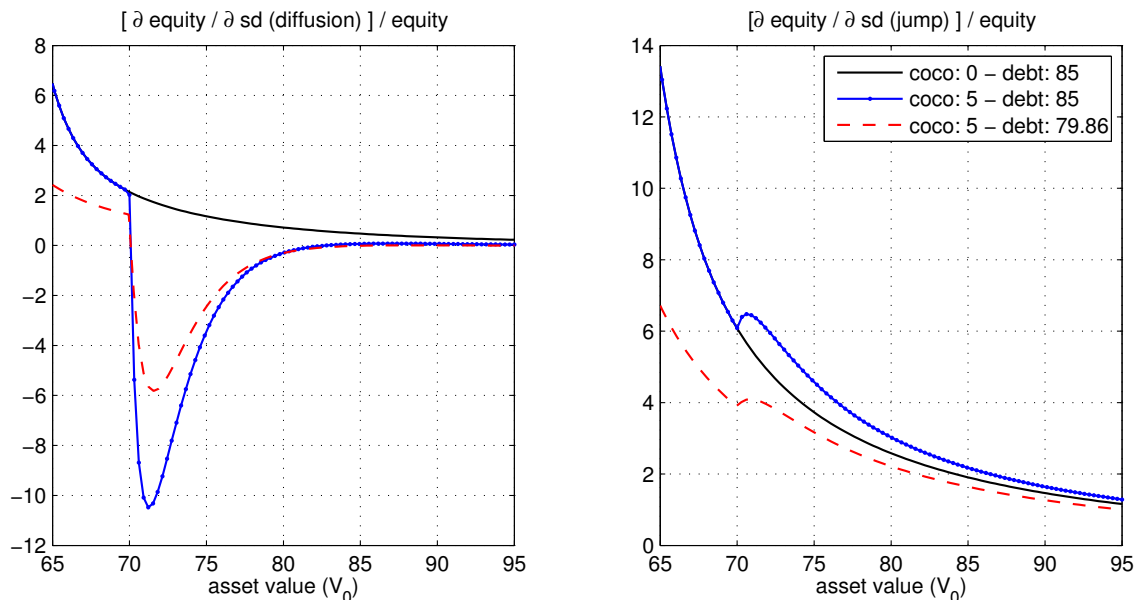
The right panels add new information by showing sensitivity to tail risk. In both Figures 3.6 and 3.7, equity holders have a positive incentive to add tail risk, particularly with long maturity debt, but also with short maturity debt at low asset levels. Indeed, the incentive becomes very large in both cases as asset value falls. Increasing the size of the firm's balance sheet by adding CoCos leads to a modest increase in this incentive above the conversion trigger. Replacing some straight debt with CoCos reduces the incentive to take on tail risk but does not reverse it. Related comparisons are examined in Albul et al. [2] and Pennacchi [64]. Pennacchi's [64] conclusions appear to be consistent with ours, though modeling differences make a direct comparison difficult; the conclusions in Albul et al. [2]



**Figure 3.6:** Sensitivity of equity value to diffusive volatility and jump risk in assets.

are quite different, given the absence of jumps and debt rollover in their framework.

The patterns in our results can be understood, at least in part, from the asset dynamics in (3.3); in particular, whereas the diffusive volatility  $\sigma$  plays no role in the (risk-neutral) drift, increasing the jump parameter  $1/\eta_f$  increases the drift. In effect, the firm earns a higher continuous yield on its assets by taking on greater tail risk. This has the potential to generate additional dividends for shareholders, though the additional yield needs to be balanced against increased rollover costs resulting from increased default risk. In addition to generating a higher yield, jump risk is attractive to shareholders because the cost of conversion is lower if it takes place at a lower asset value than at the conversion trigger. Moreover, shareholders are indifferent between a bankruptcy at an asset value below their default barrier or right at their barrier, so they are motivated to earn the higher yield from tail risk without bearing all of the downside consequences.



**Figure 3.7:** Same comparisons as Figure 3.6 but with with longer average maturity. In all plots, at the same asset level the dashed line corresponds to a larger distance to default due to less outstanding regular debt.

### 3.8 Debt-Induced Collapse

At several points in our discussion we have qualified our remarks with the condition that conversion precedes bankruptcy — in other words, that the conversion trigger  $V_c$  is above the endogenous bankruptcy boundary  $V_b$ . We now examine this condition in greater detail, highlighting a phenomenon of debt-induced collapse in equity value: an increase in the firm's debt drives its bankruptcy level  $V_b$  higher; if the increase is sufficiently extreme to drive the bankruptcy level above the conversion trigger, then just at the point at which  $V_b$  crosses  $V_c$  — where the CoCos become junior debt — equity value experiences a sharp decline. No comparable phenomenon can occur in the absence of CoCos.

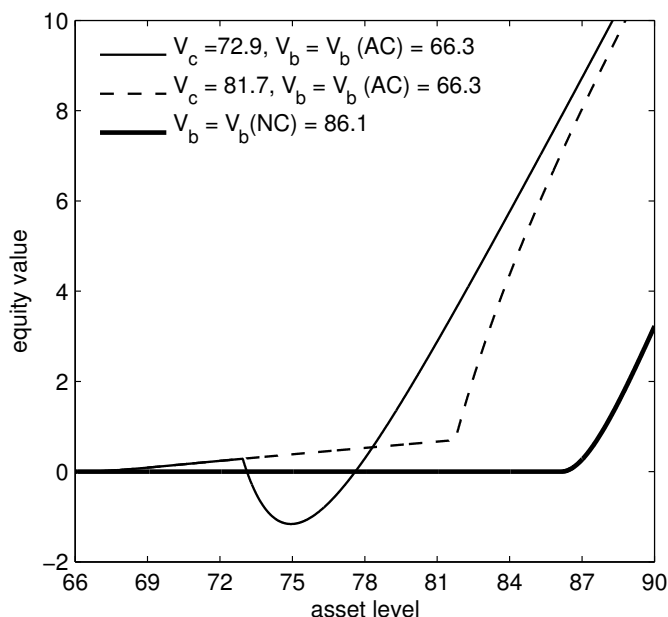
To explain this phenomenon, we introduce two other firms that are identical to the

original firm except that in one the CoCos have already been converted, and in the other the conversion feature has been removed so the CoCos will never convert. Call these the AC and NC firms, respectively. The equity holders of the NC firm (which has no convertible debt) set an optimal default barrier  $V_b(NC)$ . If  $V_b(NC) \geq V_c$ , then  $V_b(NC)$  is a feasible choice of default barrier for the original firm (and makes equity values for the two firms identical). It is feasible in the sense that if the original firm chose  $V_b = V_b(NC)$ , then equity value would be positive prior to default and equal to zero at the time of default. These assertions follow from the fact that equity value in the original firm would equal equity value in the NC firm if  $V_b = V_b(NC) \geq V_c$ , because conversion would never precede default under this condition.

To illustrate, we consider an example. The heavy solid line in [Figure 3.8](#) shows equity value for the NC firm. The optimal default barrier  $V_b(NC)$  is at 86.1, and the NC equity value and its derivative are equal to zero at this point. If the conversion trigger  $V_c$  is below 86.1 (two cases are considered in the figure), then this is a feasible default level — and a feasible equity value — for the original firm.

Denote by  $V_b(AC)$  the optimal default barrier for the AC firm. We always have  $V_b(AC) \leq V_b(NC)$  because the NC firm has all the debt of the AC firm plus additional debt. Suppose  $V_b(AC) < V_c$ . Below the conversion trigger  $V_c$ , the original firm is identical to the AC firm, so for asset values below  $V_c$  the only possible choice of default barrier for the original firm is  $V_b(AC)$ . However, this choice may not be feasible for the original firm because it potentially produces negative equity values at higher asset levels. But equity holders would default rather than accept negative value; the inconsistency in such a case would indicate that  $V_b(AC)$  would not be a feasible choice of default barrier for the original firm.

Both cases are illustrated in [Figure 3.8](#). The dashed line corresponds to a conversion trigger of  $V_c = 81.7$ , where the kink occurs. The equity holders choose  $V_b = V_b(AC) = 66.3$  as their default barrier, conversion occurs prior to default, equity value is always nonnegative,



**Figure 3.8:** Candidate equity value as a function of asset value in three scenarios. The heavy solid line reflects default at  $V_b(NC) = 86.1$ , prior to conversion. The other two lines reflect default at  $V_b(AC) = 66.3$  with two different conversion triggers. With  $V_c = 72.9$ , equity becomes negative so  $V_b(AC)$  is infeasible and default occurs at  $V_b(NC)$ . With  $V_c = 81.7$ , default at  $V_b(AC)$  is feasible, and it is optimal because it yields higher equity than  $V_b(NC)$ .

and it is characterized by the smooth pasting condition at the default barrier. However, at a conversion trigger of  $V_c = 72.9$ , an attempt to choose  $V_b = V_b(AC) = 66.3$  as the default barrier would result in negative equity at a higher asset value, which means that equity holders would actually default near 78; but this choice would then change the entire path of equity value, meaning that  $V_b = V_b(AC)$  fails to be internally consistent — it is not a feasible choice. The only feasible default barrier for the original firm is then  $V_b = V_b(NC)$ .

Now consider the implications of having the two candidate solutions  $V_b(AC)$  and  $V_b(NC)$  for optimal default barrier of the original firm. With a conversion trigger of  $V_c = 81.7$ , the optimal default barrier is  $V_b = V_b(AC) = 66.3$ , conversion occurs prior to default, and equity

value follows the dashed line. But if we lower  $V_c$ , we eventually get to a point (somewhere before  $V_c = 72.9$ ) at which  $V_c(AC)$  becomes infeasible and the optimal default barrier jumps to  $V_b(NC) = 86.1$ . This jump up in default barrier is accompanied by a sudden drop in equity value.

The economic explanation is that lowering the conversion trigger eventually has the effect of changing the CoCos into junior straight debt — debt with no ability to absorb losses. In an equilibrium in which conversion occurs prior to bankruptcy, equity holders derive greater benefit from the presence of the convertible debt and are thus more willing to continue to sustain the firm at times when the inequality in (3.4) is reversed. In an equilibrium in which bankruptcy precedes conversion, the convertibility feature has no value to equity holders, equity holders have less incentive to sustain the firm, and equity value drops.

Although we have described this phenomenon through a change in  $V_c$ , a similar and more significant pattern holds if  $V_c$  is fixed but the firm increases its debt, whether straight debt or CoCos. Consider an increase in straight debt. This moves both  $V_b(AC)$  and  $V_b(NC)$  to the right in the figure, which has the same effect as moving  $V_c$  to the left.<sup>7</sup> Eventually, the additional debt service becomes so great that equity holders become unwilling to sustain the firm all the way down to  $V_b(AC)$  and instead commit to abandoning the firm at  $V_b(NC)$ , prior to conversion. The equity holders thus effectively eliminate the conversion feature of the CoCos, and at the point at which this happens, equity experiences a sudden drop given by the vertical distance between the dashed line and the heavy solid line in the figure.

We view this scenario as a real phenomenon, and one that is possible only with convertible debt and endogenous default. The implications are as follows: the conversion trigger for CoCos needs to be sufficiently high to ensure unambiguously that conversion will take place prior to default; the firm's capital structure needs to be managed to ensure that this continues

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<sup>7</sup>If the additional debt is convertible,  $V_b(AC)$  will not change, but equity will become negative somewhere above the conversion boundary, as in Figure 3.8.

to hold if the firm takes on more debt. A switch from conversion prior to bankruptcy to bankruptcy prior to conversion is accompanied by a sharp drop in equity value as the value of the conversion feature is lost.

### 3.9 Orderly Resolution Versus Contingent Capital

Resolution authority and contingent capital can be viewed as complementary tools in avoiding financial crises: whereas the objective of replacing straight debt with CoCos is to reduce the likelihood of a bank failure, the objective of orderly resolution is to reduce the costs and negative externalities of a failure. These tools are also complementary in the sense that orderly resolution includes the option of a bail-in mechanism in which equity holders are wiped out and creditors are forced to take some losses and accept repayment in the form of equity in a reorganized entity.<sup>8</sup>

#### 3.9.1 Varying the Recovery Rate

In our model, the impact of orderly resolution is reflected, in a reduced-form manner, by the parameter  $\alpha$ , which measures the recovery rate on assets in the event of default: an ideal and seamless resolution would have  $\alpha = 100\%$ . We have used  $\alpha = 50\%$  as part of our base case parameter set. In this section, we examine the relative impact of CoCos and orderly resolution by exploring how much  $\alpha$  would need to be increased to achieve the same effect as a CoCo issue of a given size. We examine this trade-off for a few different performance measures as  $\alpha$  and CoCo size vary.

Before doing so, we comment briefly on the relationship between our reduced-form model

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<sup>8</sup>See, for example, Resolution Strategy Overview, FDIC Office of Complex Financial Institutions, January 25, 2012, [http://www.fdic.gov/about/srac/2012/2012-01-25\\_resolution-strategy.pdf](http://www.fdic.gov/about/srac/2012/2012-01-25_resolution-strategy.pdf).



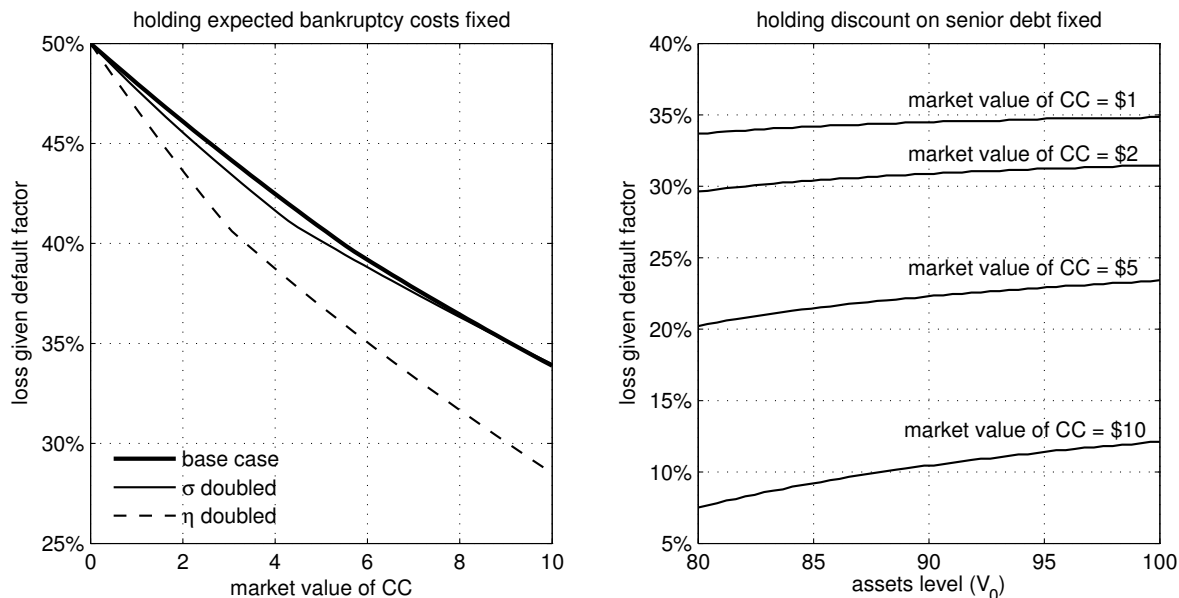
and the complexities of unwinding a large financial institution. For this, we draw on an analysis by the FDIC [29] of how the failure of Lehman Brothers would have been managed had Title II of the Dodd-Frank act been in effect at the time. The report highlights (p.6) three elements of the FDIC's resolution authority that are particularly relevant to our reduced-form bankruptcy costs: supporting an orderly liquidation that maintains asset values, the ability to continue key operations, and the ability to transfer contracts to preserve value.<sup>9</sup> According to the report, the Chapter 11 reorganization plan filed on January 25, 2011, estimates a 21.4% recovery rate for senior unsecured creditors. The report further concludes that an FDIC resolution would have produced a 97% recover rate for senior unsecured creditors. These recovery rates are not directly comparable to our  $\alpha$ , because  $\alpha$  is a recovery rate on assets, not debt, which must be lower. To arrive at a 97% recovery rate on senior debt under an orderly resolution, the report estimates that Lehman's problem assets would have experienced a loss in the range of 60–80%, and that its \$210 billion in total assets would have suffered \$40 billion in losses, for a recovery rate of 81%, eliminating \$35 billion in equity and subordinated debt. To achieve the 21.4% recovery rate on senior unsecured debt in the Chapter 11 filing, a similar calculation shows that the recovery rate on assets would have to be less than 17.8%. Although none of these values corresponds directly to our  $\alpha$ , they suggest an aspiration for a very substantial increase in the recovery rate, from something in the vicinity of 20% to something closer to 80-100%.<sup>10</sup>

In Figure 3.9, we vary the loss given default factor,  $1 - \alpha$ , to achieve the same result as a CoCo issue. In the left panel, we hold expected bankruptcy costs fixed. In our base case (the heavy solid line) we see, for example, that replacing 10% of debt with CoCos achieves the same reduction in expected bankruptcy costs as reducing the loss given default from 50% to

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<sup>9</sup>The report also highlights advance resolution planning and prompt distributions to creditors based on anticipated recoveries.

<sup>10</sup>Valukas [76], pp.202–209, observes that Lehman's assets were in principle reported at fair value but that there was public skepticism about Lehman's marks on its illiquid assets. The loss in asset value at bankruptcy may therefore combine a correction in valuation with costs more directly connected to financial distress; the two effects are difficult to disentangle.

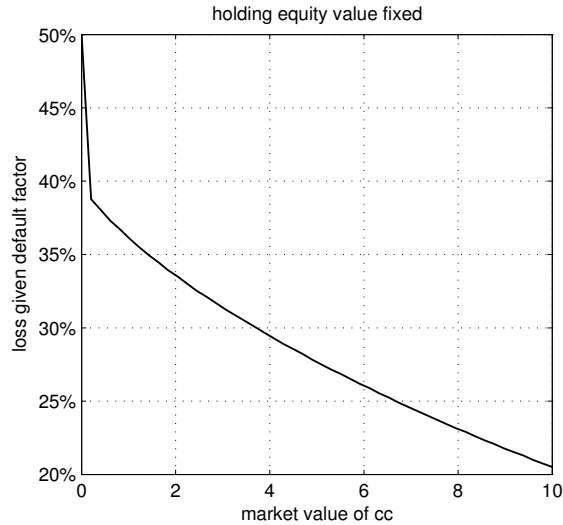


**Figure 3.9:** The left panel shows how much the loss given default would have to decrease to achieve the same expected bankruptcy costs as replacing straight debt with CoCos. The heavy solid line is our base case, and the other two lines double either  $\sigma$  or  $\eta$ . In the right panel, we show the corresponding trade-off holding the discount on senior debt fixed.

around 34%. The other two curves on the left show the same comparison with either  $\sigma$  or  $\eta$  doubled. Reducing the severity of the jumps (increasing  $\eta$ ) makes CoCos relatively more effective as measured by the equivalent reduction in  $1 - \alpha$ .

The right panel shows a similar comparison holding the discount on senior debt constant. The four lines correspond to replacing senior debt with CoCo issues of varying sizes. Asset value increases as we move from left to right. The largest CoCo size has the same effect on the yield of senior debt as reducing the loss given default to 7.5–12.5% from the base case value of 50%.

Finally, we consider the impact on equity. Figure 3.10 shows the reduction in loss given



**Figure 3.10:** The figure shows how much the loss given default would have to decrease to achieve the same increase in equity value as replacing straight debt with CoCos.

default required to achieve the same increase in equity value as a CoCo issuance of the indicated size. The sharp decline near the left end of the curve is due to deposit insurance: equity value is nearly insensitive to the loss given default until the recovery exceeds the deposits. The figure indicates that replacing straight debt with CoCos in an amount equal to 10% of assets has the same effect on equity value as reducing the loss given default from 50% to just over 20%.

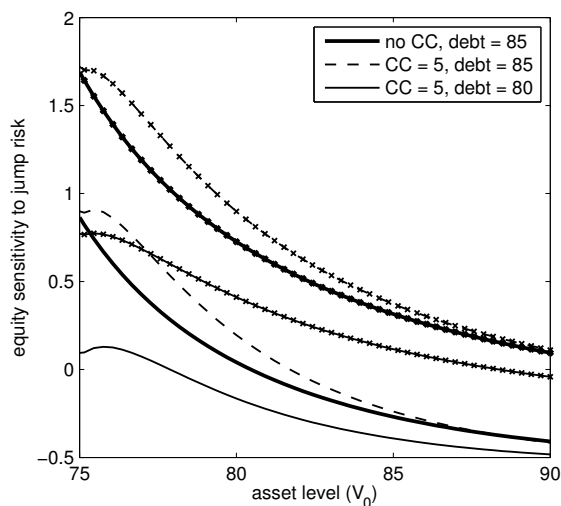
Focusing on the impact on debt and equity, these comparisons suggest that replacing approximately 10% of debt with CoCos has a similar effect as increasing the recovery rate  $\alpha$  from 50% to roughly 80–90%, for parameter values similar to our base case. This improvement is substantial, though it falls short of the objective of a seamless resolution with nearly 100% recovery.

### 3.9.2 Market-Wide Jumps and Systemic Effects

Our model distinguishes firm-specific jumps and market-wide jumps, with the interpretation that market-wide jumps are rarer but more severe. The analysis in the appendix further allows us to differentiate recovery rates for defaults triggered by the two types of jumps. Default at a market-wide jump is likely to have spill-over effects: if many banks suffer losses simultaneously, many may need to liquidate assets simultaneously, further depressing prices. We model this systemic effect through a lower recovery rate for defaults that occur at market-wide jumps.

We revisit the comparison on the right side of [Figure 3.5](#) from this perspective. To achieve a more pronounced separation between systemic and ordinary defaults, we set the baseline recovery rate to 30% at market-wide jumps and 70% otherwise. [Figure 3.11](#) compares the sensitivity of equity value to market-wide jump risk and to firm-level jump risk for three different combinations of straight debt and CoCos. In all cases, the sensitivities are positive at low asset values, reflecting an incentive for equity holders to take on additional jump risk in this setting. The sensitivity to market-wide jump risk is consistently higher than the sensitivity to firm-specific jump risk. Replacing straight debt with CoCos reduces the attractiveness of jump risk to equity holders, as measured by the sensitivities, even making the sensitivity to firm-specific jump risk negative. However, the gap between the two sensitivities is not affected. Indeed, as discussed in [Section 3.6](#), equity holders prefer CoCo conversion to occur at a low asset level rather than near the trigger level, and this creates an incentive to take on tail risk.

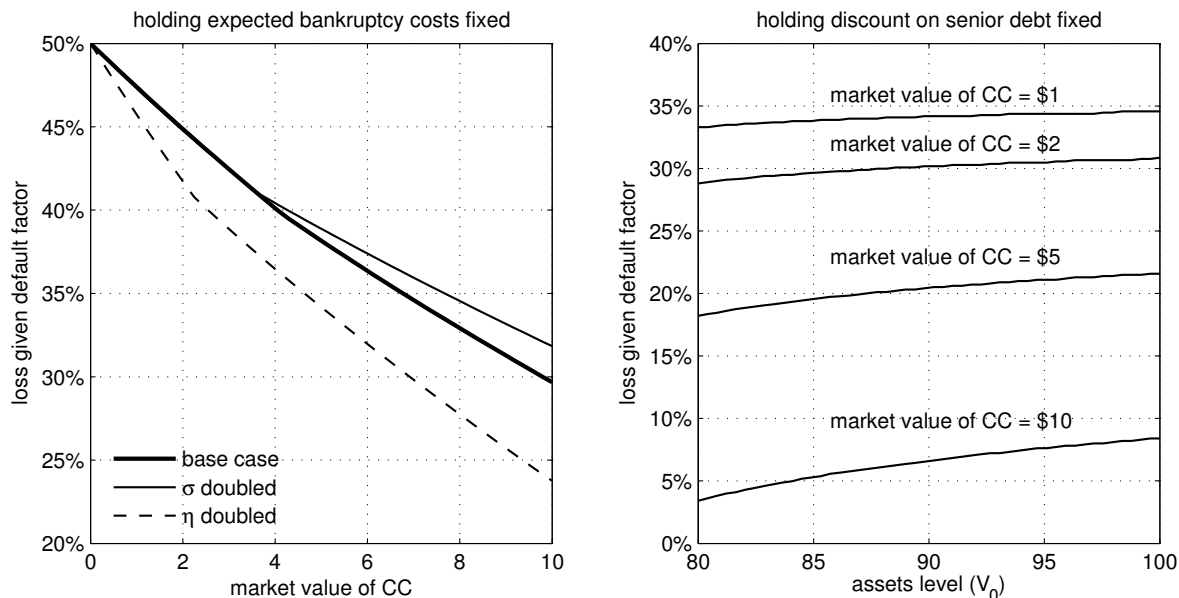
Next, we re-examine the trade-off between CoCos and resolution authority. We suppose that resolution authority can improve the recovery rate when default is due to diffusive risk or a firm-specific jump, but that it cannot offset the fire-sale effects of a market-wide decline in asset values. The setting is the same as that of [Figure 3.9](#), except that the recovery rate



**Figure 3.11:** The figure revisits the example on the right side of Figure 3.5 with a lower recovery rate for defaults that occur at market-wide jumps.

at market-wide jumps is fixed at 30% and we vary only the recovery rate for other types of default, starting at the base case value of 50%. The results are shown in Figure 3.12. As one might expect, CoCos translate to a greater improvement in recovery in this setting because the improvement applies in only a subset of cases. However, the change compared to Figure 3.9 is minor because defaults due to market-wide jumps are rare for the parameters in our base case.

A simple variant of our model would calculate bankruptcy costs from a regulator's perspective using a lower  $\alpha$  than the recovery rate used by shareholders and creditors, with the interpretation that these greater costs reflect negative externalities from the failure of a financial institution. Taking this idea a step further, one could try to develop an extension in which the regulator sets CoCo requirements to get shareholders to internalize these externalities. The model in Van den Heuvel [77] is potentially useful in formulating a regulatory objective.



**Figure 3.12:** The figure revisits the example of Figure 3.9 with the restriction that resolution authority does not affect the recovery rate for defaults that occur at a market-wide jump.

### 3.10 Calibration to Bank Data Through the Crisis

In this section, we calibrate our model to specific banks. We focus on the years leading up to and during the financial crisis, with the objective of gauging what impact CoCos might have had, had they been issued in advance of the crisis. We examine the increase in the banks' ability to absorb losses, relative to the amount of straight debt replaced with CoCos, and we calculate the reduction in debt overhang costs as an indication of whether CoCos would have created greater incentives for equity holders to inject private capital at various points in time.

As candidates for our calibration, we chose the 19 bank holding companies (the largest 19 at the time) that underwent the Supervisory Capital Assessment Program (SCAP) in 2009.

From this list, we removed MetLife because banking is a small part of its overall business, and we removed GMAC (now Ally) because it is privately held. The banks are listed in Table 3.2, in order of asset value in 2009.

We obtain quarterly balance sheet information from each bank holding company's quarterly 10-Q/10-K S.E.C. filings from 2004 through the third quarter of 2011, except in the case of American Express, for which we begin in 2006 because of a large spin-off in 2005. Several of the firms became bank holding companies late in our time window, so Y-9 reports would not be available throughout the period. Also, the Y-9 reports contain less information about debt maturities and interest expenses than the quarterly reports. We group all debt into three categories — deposits, short-term debt, and long-term debt — in this order of seniority. We do not separate subordinated debt from other long-term debt because of difficulties in doing so consistently and reliably. The distinction would not have much effect on our calculations. We calculate average debt maturity within each category using information provided in annual reports. We calculate total dividends and interest payments to get a total payout rate.

We interpolate values within each quarter, using values from the beginning of the quarter and the beginning of the subsequent quarter; this gives us values at a weekly frequency and avoids abrupt changes at the end of each quarter. For debt maturities, we interpolate between annual reports.

Our model is driven by asset value, but asset value is not observable. So, we fit our model using balance sheet and market information and then use the model to infer asset value or a model-defined proxy for asset value. In more detail, at each week we use the interpolated values to determine the bank's debt profile, dividends, and interest. As the risk-free rate, we use the Treasury yield corresponding to the weighted average maturity of each bank's debt.

Jump parameters are difficult to estimate, particularly for rare jumps as contemplated by our model. For the calibrations, we simplify the model to a single type of jump and choose from a finite set of values for the jump rate  $\lambda$  and the mean jump size  $1/\eta$ . For each  $(\lambda, \eta)$ , we calibrate a value for the diffusive volatility  $\sigma$  iteratively as follows. Given a starting value for  $\sigma$ , we can numerically invert our model's formula for equity at each point in time (using the market value of equity at each point in time) to get an implied market value for the assets. We then calculate the annualized sample standard deviation of the implied asset log returns, excluding returns of magnitude greater than  $3.3\sigma$ , which we treat as jumps, and compare it with  $\sigma$ . We adjust  $\sigma$  up or down depending on whether the standard deviation is larger or smaller than  $\sigma$ , proceeding iteratively until the values match. At that point, we have found a path of underlying assets that reproduces the market value of equity with an internally consistent level of asset volatility, for a fixed  $(\lambda, \eta)$ .

We repeat this procedure over a grid of  $(\lambda, \eta)$  values. We limit  $\lambda$  to 0.1 or 0.3; for  $\eta$ , we consider integer values between 5 and 10, but if the best fit occurs at the boundary we extend the range to ensure that does not improve the fit. We choose from the set of  $(\lambda, \eta, \sigma)$  values by comparing model implied debt prices with market data of traded debt from the Fixed Income Securities Database and TRACE databases. We add up the total principal of traded debt and total market price paid in those transactions. Their ratio gives an average discount rate that the market applies to the debt. We calculate the corresponding model implied average discount for each  $(\lambda, \eta, \sigma)$  using quarterly balance sheet data for the principal of debt outstanding and the model implied prices. The interest payments are already matched through our choice of coupon rates, so we choose the  $(\lambda, \eta, \sigma)$  that comes closest to matching the discount on the principal as our calibrated parameters. The parameters for the 17 banks are reported in [Table 3.2](#).

The results of applying this procedure to the banks are illustrated in [Figures 3.13](#) and [3.14](#), respectively, for Bank of America (the largest bank) and SunTrust (one of the two banks



Bank Holding Company	Parameters			Conversion Date	
	$\lambda$	$\eta$	$\sigma$	50%	75%
Bank of America Corp	0.1	5	4.1%	Jan-09	
JPMorgan Chase & Co.	0.1	8	4.4%		
Citigroup Inc.	0.1	9	3.9%	Nov-08	
Wells Fargo & Company	0.1	5	4.7%		
Goldman Sachs Group, Inc.	0.1	5	3.8%	Nov-08	
Morgan Stanley	0.1	8	4.2%	Sep-08	Dec-08
PNC Financial Services	0.3	8	7.0%	Nov-08	Jan-09
U.S. Bancorp	0.3	5	5.5%	Jan-09	
Bank of New York Mellon Corp.	0.3	6	7.3%	Oct-08	
SunTrust Banks, Inc.	0.3	9	4.1%	Apr-08	Jan-09
Capital One Financial Corp.	0.3	7	7.9%	Jun-08	Jan-09
BB&T Corporation	0.3	6	5.3%	Jun-08	
Regions Financial Corporation	0.3	8	4.7%	Jun-08	Jan-09
State Street Corporation	0.3	5	7.4%	Oct-08	
American Express Company	0.3	8	8.6%		
Fifth Third Bancorp	0.3	5	6.3%	Jan-08	Jun-08
KeyCorp	0.3	8	4.2%	Nov-07	Nov-08

**Table 3.2:** The table shows the calibrated parameter values ( $\lambda, \eta, \sigma$ ) for each bank holding company. The last two columns show the months in which CoCo conversion would have been triggered, according to the calibration, assuming CoCos made up 10% of debt. The 50% and 75% dilution ratios correspond to higher and lower triggers, respectively.

in the middle of the list). The top panel of each figure displays the market capitalization (the dashed line, using the right scale), asset values from quarterly reports (the piecewise constant line, using the left scale), and the calibrated asset value (the solid line, using the left scale). We have undertaken the same procedure for every bank in Table 3.2.

Given the path of asset value and all the other model parameters, we can calculate model-implied quantities. As a first step, we calculate the endogenous bankruptcy level  $V_b$  based on the bank's debt profile at each point in time. We can also undertake a counterfactual experiment in which part of the debt is replaced with CoCos and recalculate the default boundary. We take CoCos to be 10% of total debt, keeping the relative proportions of other types of debt unchanged. Recall that the default boundary does not depend on the CoCo conversion trigger or conversion ratio, as long as the trigger is above the default boundary,

so we do not need to specify values for these features to determine  $V_b$ .

In the second panel of each of Figures 3.13 and 3.14, we show the endogenous default boundaries calculated from the model, with and without CoCos. The boundaries are displayed together with the calibrated asset values, which are repeated from the top panel, to illustrate the distance to default. The boundaries are not flat because we calculate a different default boundary at each point in time, given the capital structure at that time. The gap between the two default boundaries measures the increase in loss absorption capacity that results from replacing 10% of total debt with CoCos.

Table 3.3 provides more detailed information at four points in time. Under each date, the value on the left is the ratio of increased loss absorption to the market value of CoCos. A ratio of 1 indicates that a dollar of CoCos absorbs a dollar of additional losses; a ratio greater or smaller than 1 indicates a greater or smaller degree of loss absorption. The second entry under each date is the distance to default as a percentage of asset value. Comparing a single institution at different points in time, the pattern that emerges is that the loss absorption ratio tends to be greater when the firm is closer to default. The pattern does not hold across institutions because there are too many other differences in their balance sheets besides the distance to default.

The design and market value of the CoCos depends on two contractual features, the trigger  $V_c$  and the conversion price  $\Delta$ . By the definition of  $\Delta$ , the fraction of total equity held by CoCo investors just after conversion is  $\Delta P_4 / (1 + \Delta P_4)$ , where  $P_4$  is the face value of CoCos issued. We choose  $\Delta$  so that this ratio is either 50% or 75%, and we refer to this as the dilution ratio. We then set the conversion level  $V_c$  so that if conversion were to occur exactly at  $V_t = V_c$ , the market value of the equity CoCo investors would receive would equal the face value  $P_4$  of the CoCos: conversion at  $V_t = V_c$  implies neither a premium nor a discount. In order that the equity value received be equal to  $P_4$  at both 50% and 75%

	Jan-2006		Jan-2007		Jan-2008		Jan-2009	
Bank of America Corp	1.47	7%	1.43	8%	1.63	5%	1.54	3%
JPMorgan Chase & Co.	1.29	6%	1.29	6%	1.49	5%	1.50	5%
Citigroup Inc.	1.34	7%	1.32	6%	1.42	4%	-	2%
Wells Fargo & Company	1.11	19%	1.06	22%	1.44	9%	1.60	5%
Goldman Sachs Group, Inc.	1.35	4%	1.41	5%	1.52	4%	-	4%
Morgan Stanley	1.43	4%	1.38	4%	1.50	5%	-	5%
PNC Financial Services	1.17	19%	1.11	21%	1.29	14%	-	8%
U.S. Bancorp	0.95	32%	0.98	32%	1.11	24%	1.17	18%
Bank of New York Mellon	1.15	24%	1.06	28%	1.04	28%	0.80	17%
SunTrust Banks, Inc.	0.91	21%	0.87	22%	0.91	16%	-	8%
Capital One Financial Corp.	0.93	29%	0.92	26%	0.97	16%	-	12%
BB&T Corporation	1.03	25%	1.03	23%	0.97	14%	-	9%
Regions Financial Corp.	0.90	24%	0.89	19%	0.87	12%	-	4%
State Street Corporation	1.33	18%	1.25	20%	1.07	24%	-	11%
American Express Company	1.15	38%	1.13	36%	1.26	28%	1.50	18%
Fifth Third Bancorp	0.89	26%	0.77	31%	-	17%	-	6%
KeyCorp	1.11	17%	1.01	20%	-	10%	-	5%
mean	1.15	18.81%	1.11	19.23%	1.23	13.73%	1.35	8.15%
median	1.15	19.32%	1.06	20.52%	1.26	13.80%	1.50	5.81%

**Table 3.3:** Under each date the left column shows the ratio of the increase in loss absorption (the change in the default boundary after CoCo issuance) to CoCo size (as measured by market value). The right column is the distance to default (without CoCos) as a percentage of asset level. The dilution ratio is 50%.

dilution ratios, the higher dilution ratio must coincide with a lower conversion trigger. The results in Table 3.3 are based on a 50% dilution ratio, but the corresponding results with 75% dilution are virtually identical.

The last two columns of Table 3.2 report the month in which the model calibrations predict each of the banks would have triggered conversion of CoCos with a high trigger (50% dilution ratio) and a low trigger (75% dilution ratio). In each case, the CoCo size is equal to 10% of the bank's total debt. The calibrations predict that all the banks except JPMorgan Chase, Wells Fargo, and American Express would have crossed the high conversion trigger sometime between November 2007 and January 2009; seven of the banks would have crossed the lower conversion trigger as well.

Next, we consider debt overhang costs. For each bank in each week, we calculate the size of the equity investment required to increase assets by 1%. From this we subtract the net increase in equity value, which we calculate by taking the value of equity just after the investment (as calculated by the model) and subtracting the value of equity just before the investment (as observed in the data). This is our measure of debt overhang cost: if it is positive, it measures how much less equity holders get from their investment than they put in. A negative cost indicates a net benefit to investment.

Table 3.4 presents more detailed information at three dates prior to key points in the financial crisis: one month before the announcement of JP Morgan's acquisition of Bear Stearns; one month before final approval of the acquisition; and one month before the Lehman bankruptcy. For each date, the table shows the debt overhang cost without CoCos and with high-trigger CoCos; the third column under each date shows the distance to the conversion boundary as a percentage of asset value. Interestingly, several of the largest banks show significantly negative debt overhang costs even without CoCos. Recall from Section 3.6 that this is possible in a model with debt rollover, though not with a single (finite or infinite) debt maturity. Greater asset value implies greater bankruptcy costs and reducing these costs may partly explain the motivation for shareholders to increase their investments in the largest firms. Also, if the market perceives a too-big-to-fail guarantee for the largest banks that is absent from our model, then the model's shareholders may see the largest banks as overly leveraged relative to the market's perception.

We focus on comparisons between columns of the table — a single firm under different conditions — rather than comparisons across rows. With few exceptions, the effect of the CoCos is to lower the debt overhang cost, and the impact is often substantial. The effect depends on the interaction of several factors, including leverage, debt maturity, and the risk-free rate, which enters into the risk-neutral drift. The largest reductions in debt overhang cost generally coincide with a small distance to conversion, and, in most cases in which a

	Feb-2008			Apr-2008			Aug-2008		
Bank of America Corp	-29%	-32%	6%	-26%	-30%	5%	-28%	-42%	3%
JPMorgan Chase & Co.	-75%	-51%	5%	-43%	-41%	5%	-93%	-60%	3%
Citigroup Inc.	-42%	-53%	3%	-24%	-45%	2%	-54%	-65%	2%
Wells Fargo & Company	-35%	-23%	8%	-33%	-20%	8%	-33%	-21%	7%
Goldman Sachs Group	-51%	-45%	2%	-33%	-42%	2%	-53%	-54%	2%
Morgan Stanley	21%	-42%	1%	21%	-36%	1%	-20%	-58%	2%
PNC Financial Services	-11%	-16%	7%	-7%	-12%	8%	-10%	-12%	8%
U.S. Bancorp	4%	4%	13%	5%	5%	13%	5%	5%	11%
Bank of New York Mellon	-3%	-2%	17%	-1%	0%	14%	6%	4%	8%
SunTrust Banks, Inc.	-2%	-20%	2%	5%	-	-	9%	-	-
Capital One Financial	-4%	-28%	3%	4%	-34%	2%	6%	-	-
BB&T Corporation	2%	-11%	4%	4%	-12%	4%	6%	-60%	1%
Regions Financial Corp.	-7%	-24%	3%	-8%	-42%	2%	-9%	-	-
State Street Corporation	2%	2%	11%	5%	-1%	6%	0%	-11%	5%
American Express Co.	-12%	-13%	20%	-7%	-10%	20%	-10%	-12%	17%
Fifth Third Bancorp	12%	-79%	0%	17%	-	-	19%	-	-
KeyCorp	-6%	-137%	0%	-1%	-	-	5%	-	-

**Table 3.4:** Under each date, the first column is the debt overhang cost as a percentage of the increase in assets with no CoCos. The second column quotes the same value when 10% of debt is replaced with CoCos and CoCo investors receive 50% of equity at conversion. The third column is the distance to conversion as the percentage of assets. The dates correspond to one month before announcement and final approval of acquisition of Bear Stearns by JPMorgan and one month before the Lehman bankruptcy. A table entry is blank if the corresponding date is later than the CoCo conversion date for the corresponding bank.

bank draws closer to the conversion boundary over time, the resulting reduction in debt overhang cost becomes greater. The values in the table are for 50% dilution. The pattern with 75% is similar, but the decrease in the debt overhang cost is smaller in that case because the distance from the conversion trigger is greater. For illustration, [Figure 3.15](#) plots the conversion boundaries calibrated for SunTrust at the two dilution ratios, and the lower figure plots debt overhang costs with and without CoCos. The CoCos create a positive incentive for investment as the asset level approaches the conversion boundary.

The magnitudes of the quantities reported in these tables and figures are subject to the many limitations and simplifications of our model and calibration. We see these results as

providing a useful additional perspective on the comparative statics of earlier sections of this chapter; the directional effects and the comparisons over time should be more informative than the precise numerical values. These calibrations and our exploration of counterfactual scenarios, though hypothetical, shed light on how CoCo issuance in advance of the financial crisis might have affected loss absorption capacity, incentives for additional equity investment, and how the choice of conversion trigger and dilution ratio might have determined the timing of conversion.

### 3.11 Concluding Remarks

The key contribution our analysis lies in combining endogenous default, debt rollover, and jumps and diffusion in income and asset value to analyze the incentive effects of contingent convertibles and bail-in debt. Through debt rollover, shareholders capture some of the benefits (in the form of lower bankruptcy costs) from reduced asset riskiness and lower leverage — benefits that would otherwise accrue solely to creditors. These features shape many of the incentives we consider, as do the tax treatment of CoCos, the deposit insurance base, and tail risk. The phenomenon of debt-induced collapse, which is observable only when CoCos are combined with endogenous default, points to the need to set the conversion trigger sufficiently high so that conversion unambiguously precedes bankruptcy, or else to use bail-in debt structured to avoid bankruptcy costs. Our calibrations suggest that CoCos could have had a significant impact on the largest U.S. bank holding companies in the lead up to the financial crisis.

Our analysis does not include asymmetric information, nor does it directly incorporate agency issues; both considerations are potentially relevant to the incentives questions we investigate. Some important practical considerations, such as the size of the investor base for CoCos, the behavior of stock and bond prices near the trigger, and the complexity of these

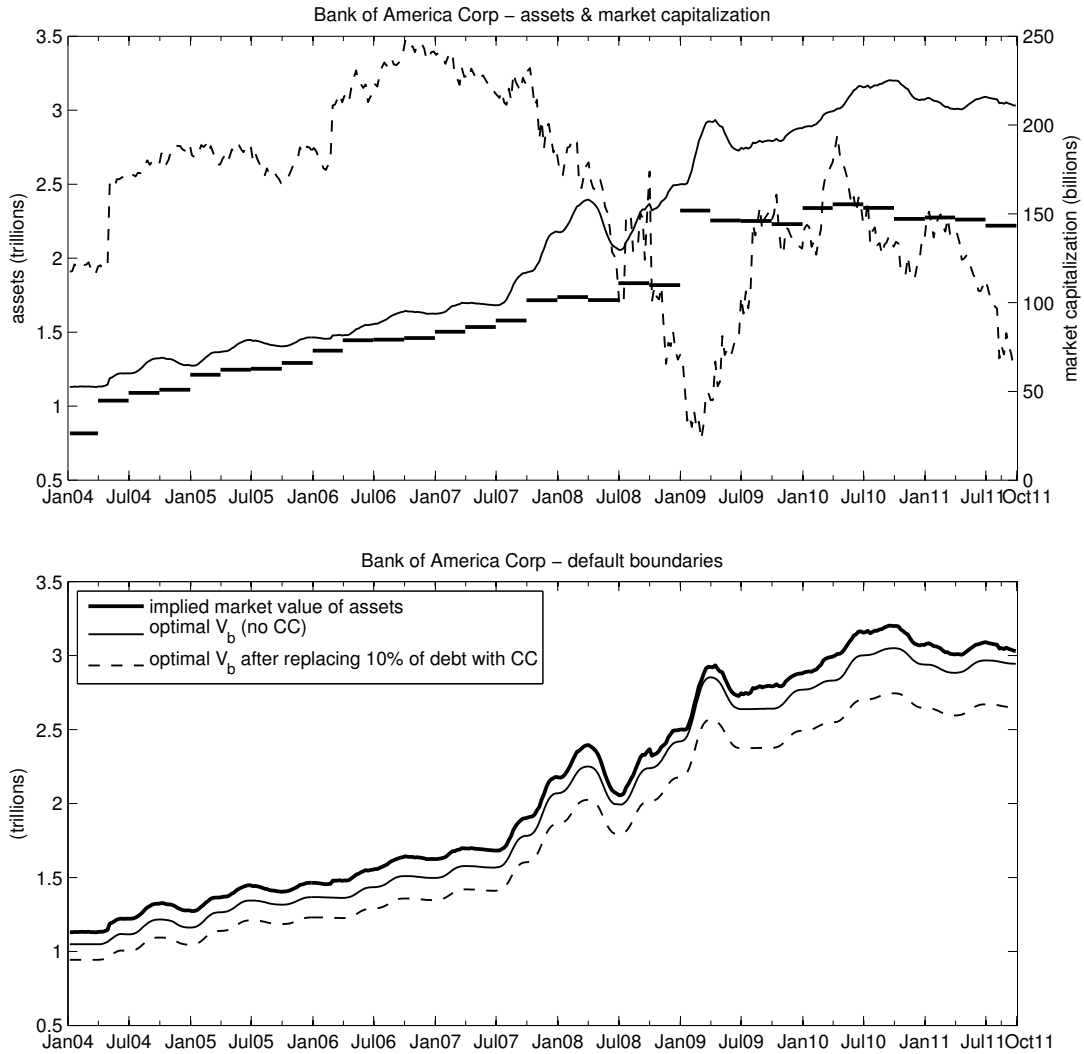


Figure 3.13: Calibration results for Bank of America.

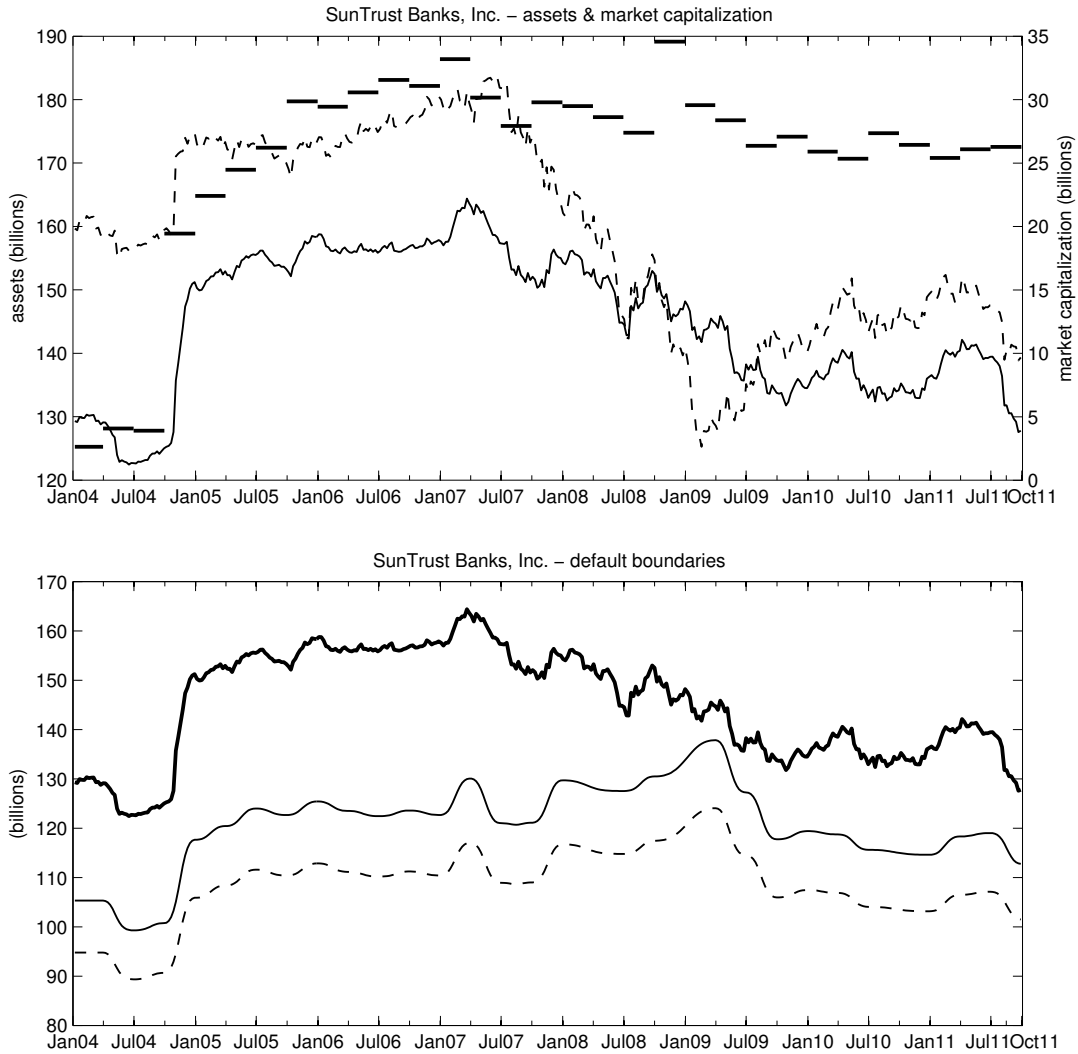
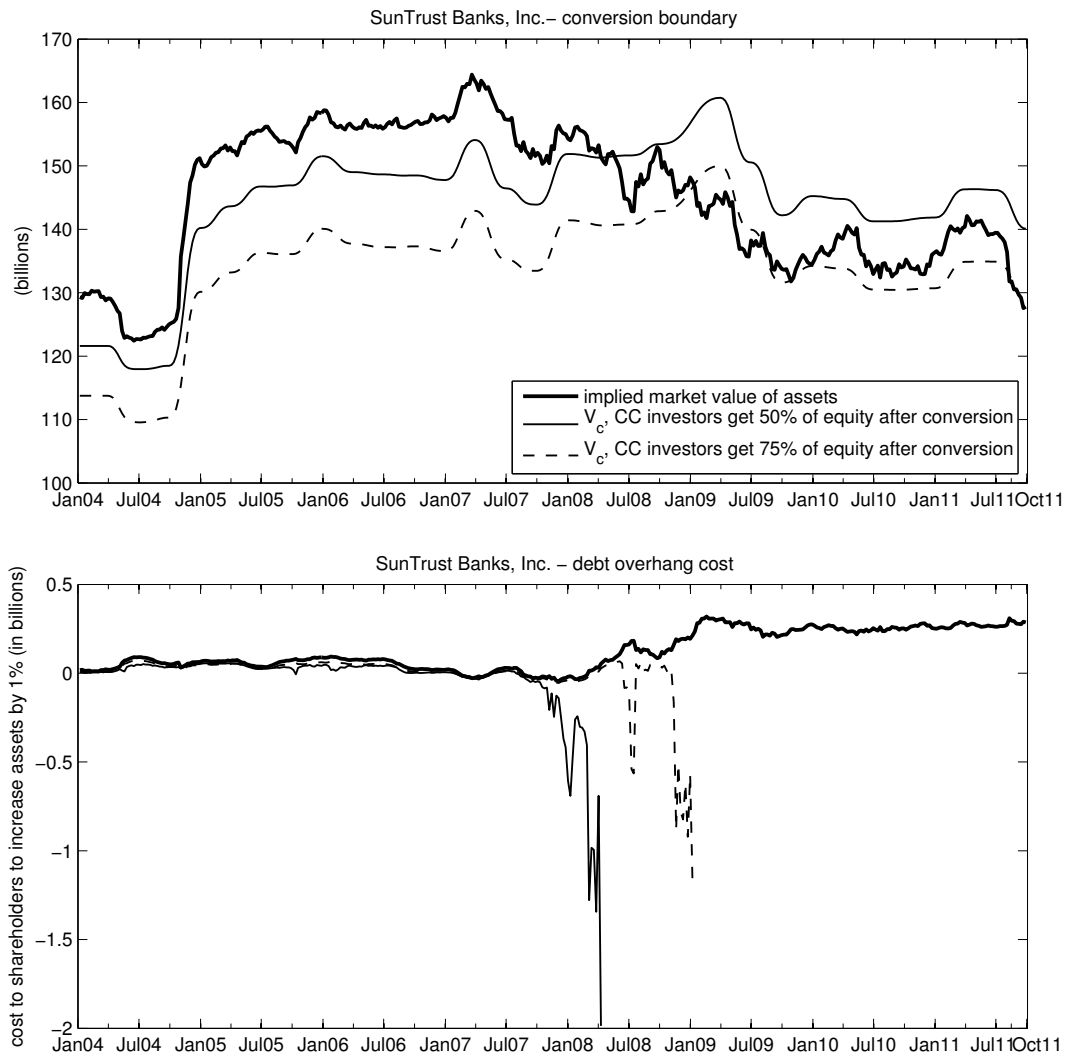


Figure 3.14: Calibration results for SunTrust.





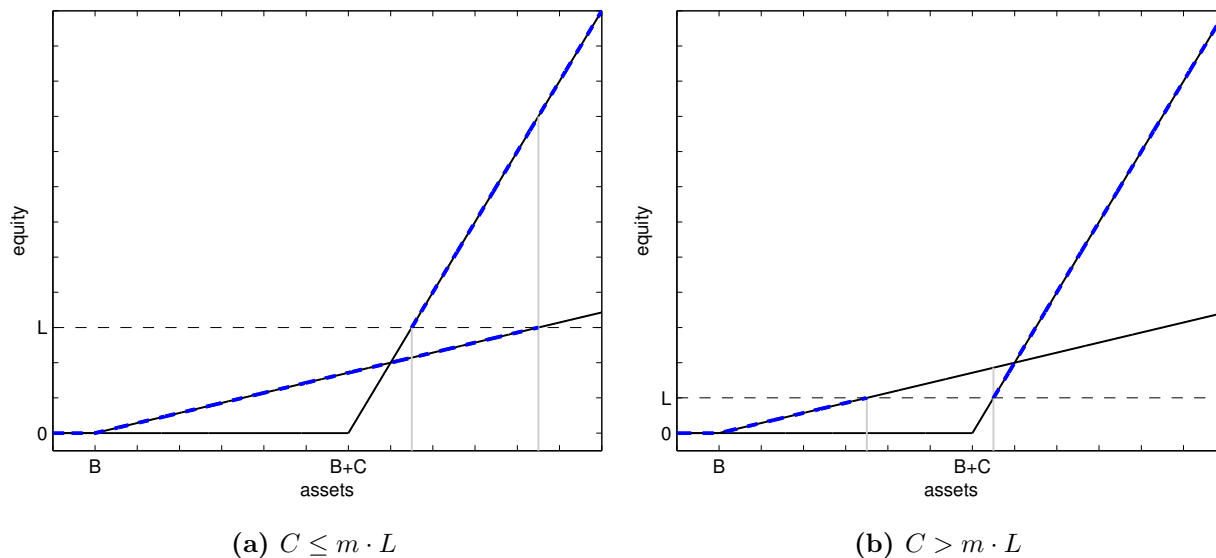
**Figure 3.15:** The top figure shows calibrated conversion boundaries for SunTrust at 50% and 75% dilution. The lower figure shows debt overhang costs without CoCos (heavy solid line) and with CoCos at 50% (thin solid line) and 75% dilution (dashed line).

instruments are also outside the model. The analysis provided here should nevertheless help inform the discussion of the merits and potential shortcomings of CoCos and other hybrid capital instruments.

## 4

Existence and Uniqueness of Equilibrium with  
Stock Price Trigger

STOCK prices are forward looking, continuously updated and readily available. Therefore, many proposals of contingent capital have advocated using a conversion trigger based on market value of shares. However, as Sundaresan and Wang [74] point out, equity and contingent capital are claims on the same assets, and their prices must be determined simultaneously. Market prices of shares will adjust to reflect the imminence of conversion. With a market based trigger, this adjustment may delay or precipitate conversion. Such circular feedback between prices and the conversion event can create multiple equilibria or no equilibrium. Sundaresan and Wang [74] show that for a unique equilibrium to exist, it is necessary that mandatory conversion must not result in any value transfers between equity and CC holders; in their model this may be achieved if contingent capital earns floating rate coupons equal to the risk-free rate. Another possibility is that the stock prices adjust in advance to reflect the impact of conversion on the claims so that there will be no value



**Figure 4.1:** Equilibrium problem at the maturity: (a) for  $A \in [B+C+L, B+(1+m) \cdot L]$  there are multiple solutions, (b) no feasible solutions exists for  $A \in [B+(1+m) \cdot L, B+C+L]$

transfer at the actual conversion. Such a situation is similar to results of Bond, Goldstein, and Prescott [11] which observe that if corrective actions by an economic agent is based on market prices, then prices adjust to reflect this.

## 4.1 Static Case

In order to understand the equilibrium problem we look at payoffs in a finite maturity model. Assume that senior debt and contingent capital have face values of respectively  $B$  and  $C$ . Convergence trigger is when the stock price hits  $L$  and upon conversion  $m$  new shares are issued to CC investors (total initial shares are normalized to 1). At the maturity, if CC does not convert the share price is  $(A - B - C)^+$  where  $A$  is value of the assets. If CC converts, each share is worth  $(A - B)^+ / (1 + m)$ . An equilibrium is a mapping from the assets ( $A$ ) to

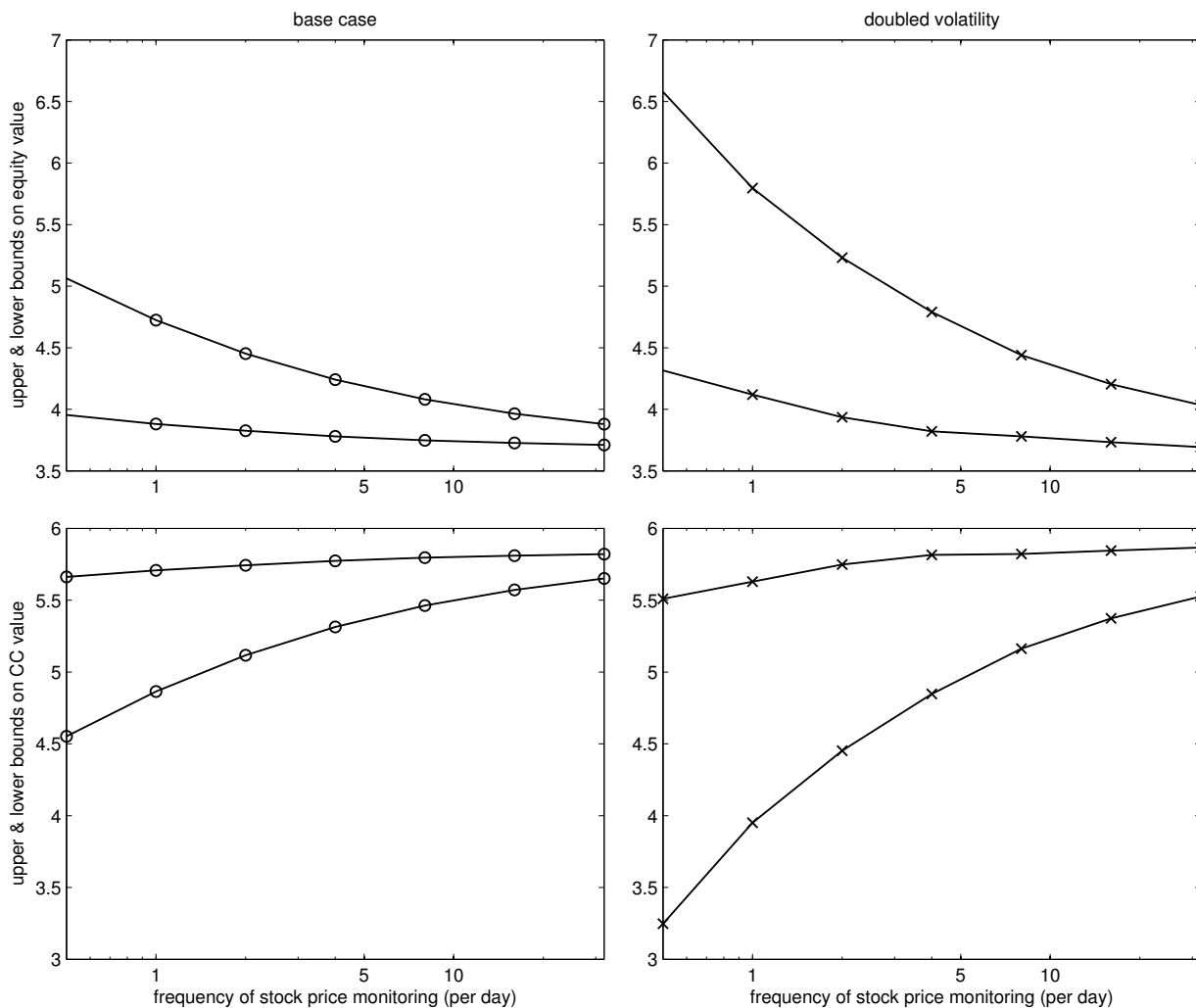
the stock price ( $S$ ) where

$$S = \mathbf{1}\{S > L\} \cdot (A - B - C)^+ + \mathbf{1}\{S \leq L\} \cdot \frac{(A - B)^+}{1 + m}$$

A feasible solution without conversion is obtained if  $(A - B - C)^+ > L$ ; whereas  $(A - B)^+ / (1 + m) \leq L$  results in a solution with conversion. Figure 4.1 shows how these conditions can lead to multiple equilibria or no equilibrium. If  $C \leq m \cdot L$ , for  $A \in [B + C + L, B + (1 + m) \cdot L]$  both conditions are valid and there are multiple solutions; whereas  $C > m \cdot L$  leads to no feasible solutions for  $A \in [B + (1 + m) \cdot L, B + C + L]$ , as neither of the conditions are valid in this range.

It is important to note that in a dynamic model, if uniqueness of equilibrium is established before the maturity then the singularity at the maturity is significant only if that state can be reached without being preceded by conversion with a positive probability. A result of our analysis (Lemma 4.3.2) suggests that under any equilibrium process the conversion occurs at or before the first time *post*-conversion stock price hits the the conversion trigger. In terms of Figure 4.1a this implies that although there are multiple solutions for  $A \in [B + C + L, B + (1 + m) \cdot L]$ , CC is already converted for  $A$  strictly less than  $B + (1 + m) \cdot L$ , and probability of assets hitting  $B + (1 + m) \cdot L$  right at the maturity is zero.

In the rest of this chapter we investigate the equilibrium problem in a continuous time setting. As Figure 4.2 shows, in the discrete-time setting of Sundaresan and Wang [74], using binomial tree models, there is a range of possible equilibrium values for equity and contingent capital before the maturity. However, the range shrinks as the number of steps increase, suggesting that in the continuous time limit there is a unique equilibrium, resulting from the adjustment in the stock price.



**Figure 4.2:** Convergence of upper and lower bounds on equity and contingent capital in a binomial tree model, and impact of volatility on range of equilibrium prices.

## 4.2 Model

Suppose that asset value is a jump-diffusion process,

$$A_t = A_0 \exp \left( [r_t - \delta_t - \lambda\mu + \sigma^2/2] t + \sigma W_t \right) \prod_{n=1}^{N_t} Y_n, \quad 0 \leq t \leq T,$$

where  $\delta_t$  is the *fixed* payout rate of the assets,  $N$  is a Poisson process with rate  $\lambda$ ,  $\{Y, Y_1, Y_2, \dots\}$  are i.i.d. positive jump sizes,  $\mu = \mathbb{E}[Y] - 1$ , and  $N$ , the jump sizes, and the Brownian motion  $W$  are mutually independent. Assuming  $\mathbb{E}[Y] < \infty$  ensures that  $\mathbb{E}[A_t] < \infty$  for all  $t \in [0, T]$ .

Senior debt has face value  $B$  and contingent capital has face value  $C$ ; both mature at  $T$ , and earn coupons at *fixed* rates of respectively  $c_t^1$  and  $c_t^2$ . The difference between the payout of the assets and the coupon payments is paid to shareholders as dividends. Bankruptcy can occur only at  $T$  (as in a Merton [57] model). Investors are risk-neutral. For simplicity, we take the risk-free rate to be zero. The trigger for conversion is  $L > 0$ ; at conversion, CC holders get  $m$  shares.

Unless otherwise indicated, we assume all processes are right-continuous with left limit (RCLL). Thus, if there is a jump at  $t$ ,  $A_t$  denotes the asset value just after the jump, and  $A_{t-}$  denotes the value just before the jump; more precisely, it is the limit of  $A_s$  as  $s$  approaches  $t$  from the left.

**Definition 4.2.1.** *An equilibrium stock price is any RCLL process  $\{S_t, 0 \leq t \leq T\}$  satisfying*

$$S_T = \mathbf{1}\{\tau^S > T\} (A_T - B - C)^+ + \mathbf{1}\{\tau^S \leq T\} (A_T - B)^+ / (1 + m)$$

and, for all  $t \in [0, T]$ ,

$$S_t = \mathbb{E}_t \left[ \int_t^T \left( \mathbf{1}\{\tau^S > v\} (\delta_v A_v - c_v^1 B - c_v^2 C) + \mathbf{1}\{\tau^S \leq v\} \frac{1}{1 + m} (\delta_v A_v - c_v^1 B) \right) dv \right. \\ \left. + \mathbf{1}\{\tau^S > T\} (A_T - B - C)^+ + \mathbf{1}\{\tau^S \leq T\} (A_T - B)^+ / (1 + m) \right] \quad (4.1)$$

where  $\mathbb{E}_t$  denotes conditional expectation given the history  $\{(W_u, N_u, Y_1, \dots, Y_{N_u}), 0 \leq u \leq t\}$  and

$$\tau^S = \inf \{0 \leq t \leq T : S_t \leq L\}$$

is the first time  $S$  reaches  $[0, L]$ .

Take  $\tau^S = \infty$  if  $S$  never reaches this set in  $[0, T]$ , and apply the same convention to all hitting times introduced below. The right-continuity of  $S$  implies that  $S_{\tau^S} \leq L$ .

For each equilibrium stock price (as defined above), there is an associated CC price process  $\{C_t, 0 \leq t \leq T\}$ ,

$$C_t = \mathbb{E}_t \left[ \int_t^T \left( \mathbf{1}\{\tau^S > v\} c_v^2 C dv + \mathbf{1}\{\tau^S \leq v\} \frac{m}{1+m} (\delta_v A_v - c_v^1 B) \right) dv \right. \\ \left. + \mathbf{1}\{\tau^S > T\} \min\{(A_T - B)^+, C\} + \mathbf{1}\{\tau^S \leq T\} m(A_T - B)^+ / (1+m) \right] \quad (4.2)$$

It is worth noting here that in the diffusion only case, the two processes

$$S_t + \int_0^t \left( \mathbf{1}\{\tau^S > v\} (\delta_v A_v - c_v^1 B - c_v^2 C) + \mathbf{1}\{\tau^S \leq v\} \frac{1}{1+m} (\delta_v A_v - c_v^1 B) \right) dv$$

and

$$C_t + \int_0^t \left( \mathbf{1}\{\tau^S > v\} c_v^2 C dv + \mathbf{1}\{\tau^S \leq v\} \frac{m}{1+m} (\delta_v A_v - c_v^1 B) \right) dv$$

are martingales with respect to Brownian motion and therefore automatically continuous. Consequently  $S_t$  and  $C_t$  are continuous processes. Thus, an equilibrium stock price process automatically satisfies the Sundaresan-Wang no-value-transfer principle and, in particular,

$$P(C(\tau^S-) = mL) = P(C(\tau^S+) = mL) = 1$$

for any equilibrium stock price process  $S$ .

Define

$$U_t = \frac{1}{1+m} \mathbb{E}_t \left[ \int_t^T (\delta_v A_v - c_v^1 B) dv + (A_T - B)^+ \right] \quad (4.3)$$



We interpret  $U$  as the stock price process if conversion took place at time zero. Let

$$\tau^U = \inf \{0 \leq t \leq T : U_t \leq L\}$$

be the first time  $U$  hits  $[0, L]$ . If  $U_0 \leq L$ , then  $\tau^U = 0$ . By definition (and right-continuity),  $U_{\tau^U} \leq L$ .

### 4.3 Technical Results

**Lemma 4.3.1.** *If  $S$  is an equilibrium stock price, then  $P(\tau^U \leq \tau^S) = 1$ .*

*Proof.* Let  $S$  be an equilibrium stock price. Then, on the event  $\{\tau^S < T\}$ ,

$$S_{\tau^S} = \frac{1}{1+m} \mathbb{E}_{\tau^S} \left[ \int_t^T (\delta_v A_v - c_v^1 B) dv + (A_T - B)^+ \right] = U_{\tau^S}.$$

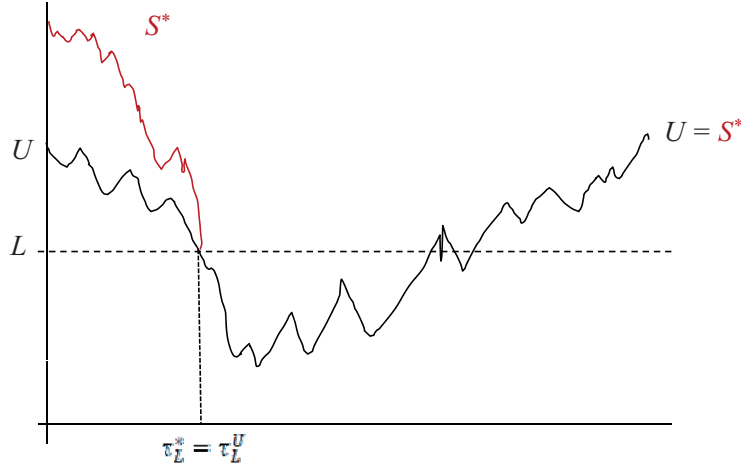
But, by right continuity,  $S_{\tau^S} \leq L$ , so  $U_{\tau^S} \leq L$ , which implies  $\tau^U \leq \tau^S$ . ■

**Lemma 4.3.2.** *If  $S$  is an equilibrium stock price, then  $P(\tau^U \geq \tau^S) = 1$ .*

We defer the proof of above lemma until the end.

**Proposition 4.3.3.** *There is at most one equilibrium stock price process.*

*Proof.* Let  $S^i$ ,  $i = 1, 2$ , be equilibrium stock price processes, and let  $\tau^i$ ,  $i = 1, 2$ , be their hitting times for  $[0, L]$ . From Lemmas 4.3.1 and 4.3.2 we have  $P(\tau^i = \tau^U) = 1$ , so, for all



**Figure 4.3:** Construction of equilibrium:  $S_t^*$  converts the first time the post-conversion price  $U_t$  hits the trigger  $L$ . No equilibrium stock price can hit  $L$  earlier than that. After conversion  $S_t = U_t$ .

$t \in [0, T]$ ,

$$\begin{aligned}
 S_t^1 &= \mathbb{E}_t \left[ \int_t^T \left( \mathbf{1} \{ \tau^U > v \} (\delta_v A_v - c_v^1 B - c_v^2 C) + \mathbf{1} \{ \tau^U \leq v \} \frac{1}{1+m} (\delta_v A_v - c_v^1 B) \right) dv \right. \\
 &\quad \left. + \mathbf{1} \{ \tau^U > T \} (A_T - B - C)^+ + \mathbf{1} \{ \tau^U \leq T \} (A_T - B)^+ / (1+m) \right] \\
 &= S_t^2
 \end{aligned}$$

■

Define

$$\begin{aligned}
 S_t^* &= \mathbb{E}_t \left[ \int_t^T \left( \mathbf{1} \{ \tau^U > v \} (\delta_v A_v - c_v^1 B - c_v^2 C) + \mathbf{1} \{ \tau^U \leq v \} \frac{1}{1+m} (\delta_v A_v - c_v^1 B) \right) dv \right. \\
 &\quad \left. + \mathbf{1} \{ \tau^U > T \} (A_T - B - C)^+ + \mathbf{1} \{ \tau^U \leq T \} (A_T - B)^+ / (1+m) \right]
 \end{aligned}$$

noting that the trigger here is defined by  $U$ . Also, let  $\tau^*$  be the hitting time for  $S^*$  to  $[0, L]$ .

**Lemma 4.3.4.**  $P(\tau^* \leq \tau^U) = 1$ .

*Proof.* If  $\tau^U > T$ , then  $\tau^U = \infty$  and the required inequality holds. On the event  $\{\tau^U \leq T\}$ ,

$$S_{\tau^U}^* = \frac{1}{1+m} \mathbb{E}_{\tau^U} \left[ \int_{\tau^U}^T (\delta_v A_v - c_v^1 B) dv + (A_T - B)^+ \right] = U_{\tau^U} \leq L$$

so  $\tau^* \leq \tau^U$ . ■

In the following, [Lemma 4.3.5](#) and [Proposition 4.3.6](#) assume that there are no coupons or dividends (i.e.  $\delta_t = 0, c_t^i = 0$ ).

**Lemma 4.3.5.** *If  $C \leq mL$ , then  $P(\tau^U \leq \tau^*) = 1$ .*

*Proof.* We claim that if  $C \leq mL$ , then  $S_T^* \geq U_T$ . There are two cases. If  $A_T - B \geq C(1+m)/m$ , then

$$A_T - B - C \geq (A_T - B)/(1+m),$$

and  $S_T^* \geq U_T$ ; if  $A_T - B < C(1+m)/m$ , then  $U_T < C/m \leq L$ , so  $S_T^* = U_T$ . Taking conditional expectations, we get  $S_t^* \geq U_t$  for all  $t \in [0, T]$ . But if  $S^*$  is always greater than or equal to  $U$ , it cannot reach  $[0, L]$  before  $U$  does. ■

**Proposition 4.3.6.** *Suppose  $U_0 > L$ . If  $C \leq mL$  then  $S^*$  is the unique equilibrium stock price, and if  $C > mL$  there is no equilibrium stock price. If  $U_0 \leq L$ , then  $S^*$  is always the unique equilibrium stock price.*

*Proof.* Consider the case  $U_0 > L$  and  $C \leq mL$ . From [Lemmas 4.3.4](#) and [4.3.5](#), we have  $\tau^* = \tau^U$ , so, for all  $t \in [0, T]$ ,

$$S_t^* = \mathbb{E}_t \left[ \mathbf{1}_{\{\tau^* > T\}} (A_T - B - C)^+ + \mathbf{1}_{\{\tau^* \leq T\}} (A_T - B)^+ / (1+m) \right]$$

which shows that  $S^*$  is an equilibrium and, by [Proposition 4.3.3](#), the only equilibrium. If  $U_0 \leq L$ , then  $\tau^U = \tau^* = 0$  and  $S^* \equiv U$  is trivially an equilibrium stock price process.

The remaining case is  $U_0 > L$  and  $C > mL$ . Suppose an equilibrium stock price process  $S$  exists. If  $S_0 \leq L$ , then  $\tau^S = 0$  and  $S_0 = U_0$ , but  $U_0 > L$ , resulting in a contradiction. Suppose  $S_0 > L$ . Conditional on  $N_T = 0$ ,  $A$  is simply geometric Brownian motion on  $[0, T]$ , so

$$P(A_t > a_t, 0 \leq t \leq T, \text{ and } A_T < L + B + C | N_T = 0) > 0,$$

with  $a_t$  the boundary defined in the proof of Lemma 4.3.2,  $a_T = B + (1 + m)L < L + B + C$ . Because  $P(N_T = 0) > 0$ , it follows that

$$P(A_t > a_t, 0 \leq t \leq T, \text{ and } A_T < L + B + C \text{ and } N_T = 0) > 0,$$

and, as a consequence,

$$P(\tau^S > T \text{ and } A_T < L + B + C) > 0.$$

But  $\tau^S > T$  implies  $S_T = (A_T - B - C)^+$ , and then  $A_T < L + B + C$  implies  $S_T < L$ , contradicting  $\tau^S > T$ . So, no equilibrium  $S$  is possible. ■

With coupons and dividends, Lemmas 4.3.1 and 4.3.2 impose that if there exist any equilibrium, it is unique and it will convert at  $\tau^U$ . Consequently, the constructed stock process  $S_t^*$  has to be the unique equilibrium stock process. A contradiction will occur if  $S_t^*$  crosses  $L$  before  $\tau_u$ . So we need  $P(S_t^* \leq L, U_t > L) = 0$  to avoid the contradiction. A sufficient condition is  $C \leq mL$  and  $\frac{m}{m+1}(\delta_t A_t - c_t^1 B) - c_t^2 C \geq 0$  for all  $t \in [0, T]$ , which guarantees  $P(S_t^* - U_t \geq 0) = 1$ .

*Proof of Lemma 4.3.2.* We will introduce two boundaries for the asset process  $A$  with the following interpretation: no equilibrium stock price can hit  $L$  until  $A$  hits the upper boundary; every equilibrium stock price must hit  $L$  before  $A$  hits the lower boundary.

In light of the Markov property of  $A$ , we can write  $U_t$  as

$$\frac{1}{1+m} \mathbb{E} \left[ \int_t^T (\delta_v A_v - c_v^1 B) dv + (A_T - B)^+ \middle| A_t \right]$$

in other words,  $U_t$  is a deterministic function of  $t$  and  $A_t$ ,  $U_t = g(t, A_t)$ . The function is strictly increasing in  $A_t$  and has range  $(0, \infty)$ .  $\sigma > 0$  allows to conclude that  $A_t$  has a  $C^\infty$  density with all derivatives vanishing at infinity (Sato [71, Proposition 28.3]), which ensures the function  $g(t, A_t)$  is a continuous and smooth function of its arguments (Rong [69]). Thus, we can then define, for each  $t \in [0, T]$ , an asset level  $a_t$  such that

$$g(t, a_t) = L.$$

Thus,  $U_t = L$  if and only if  $A_t = a_t$ , and  $U_t \leq L$  if and only if  $A_t \leq a_t$ . If  $U_0 > L$ ,  $\tau^U$  is the first time  $A$  is at or below the boundary  $\{a_t, 0 \leq t \leq T\}$ . Moreover,  $\{a_t, 0 \leq t \leq T\}$  is continuous in  $t$ . This is our upper boundary.

We introduce a second boundary by defining

$$V_t = \mathbb{E}_t \left[ \int_t^T \delta_v A_v dv + (A_T - B - C)^+ + (A_T - B)^+ / (1+m) \right].$$

Clearly,  $V_t \geq S_t$ , for any equilibrium stock price  $S_t$ . Again by the Markov property,  $V_t = h(t, A_t)$  for some function  $h$ . Following similar arguments as in the case of  $g(t, A_t)$ , for each  $t \in [0, T]$  there is  $b_t > 0$  at which  $h(t, b_t) = L$ . In fact,  $\{b_t, 0 \leq t \leq T\}$  is bounded from below by some constant  $b > 0$ , and then, for all  $t \in [0, T]$ .

$$A_t \leq b \Rightarrow V_t \leq L \Rightarrow S_t \leq L. \tag{4.4}$$

The constant  $b$  is our lower boundary.

For any  $t < T$  and any  $x \in (b, a_t)$ , consider the evolution of  $A$  on  $[t, T]$  given  $A_t = x$ . Write  $N(t, T]$  for the number of jumps in the interval  $(t, T]$ ; i.e.,

$$N(t, T] = N_T - N_t,$$

and note that  $N(t, T]$  is independent of all time- $t$  information. Conditional on  $N(t, T] = 0$  and  $A_t = x$ ,  $A$  evolves like geometric Brownian motion on  $[t, T]$  starting at  $x$ . It is then a standard property of Brownian motion that (conditional on  $A_t = x$  and  $N(t, T] = 0$ ) the probability that  $A$  reaches  $b$  before reaching the  $a$  boundary and before  $T$  is strictly positive. Now fix  $\epsilon > 0$  and let

$$\tau_\epsilon = \inf \{0 \leq t \leq T : A_t \leq a_t - \epsilon\}$$

denote the first time  $A$  drops  $\epsilon$  or more below the  $a$  boundary, and let  $\tau_a^+$  and  $\tau_b^+$  denote the first time in  $[\tau_\epsilon, T]$  that  $A$  reaches the boundaries  $a$  and  $b$ , respectively, taking each of these to be  $\infty$  if the corresponding boundary is never hit in  $[\tau_\epsilon, T]$ . (If  $A_{\tau_\epsilon} \leq b$ , then  $\tau_b^+ = \tau_\epsilon$ .) Then, by the strong Markov property of  $A$ ,

$$\begin{aligned} P(\tau_b^+ < \min \{\tau_a^+, T\}, N(\tau_\epsilon, T] = 0 | A_t, 0 \leq t \leq \tau_\epsilon) \\ = P(\tau_b^+ < \min \{\tau_a^+, T\}, N(\tau_\epsilon, T] = 0 | A_{\tau_\epsilon}) > 0, \end{aligned}$$

on the event  $\{\tau_\epsilon < T\}$ .

Next, we claim that for any equilibrium stock price  $S$ , the event

$$\{\tau_\epsilon < \tau^S, \tau_b^+ < \tau_a^+\} \cap \{N(\tau_\epsilon, T] = 0\} \tag{4.5}$$

has probability zero. To see why, observe from (4.4) that  $S_{\tau_b^+} \leq L$ , so on the event  $\{\tau_\epsilon < \tau^S, \tau_b^+ < \tau_a^+\}$ ,  $S$  must hit  $[0, L]$  for the first time sometime in  $(\tau_\epsilon, \tau_b^+]$ . But, on the event  $\{\tau_b^+ < \tau_a^+\}$ ,  $U$  is strictly below  $L$  (because  $A$  is strictly below the upper boundary)

throughout the interval  $[\tau_\epsilon, \tau_b^+]$ . If  $S$  is continuous on  $(\tau_\epsilon, \tau_b^+]$ , then the first time it hits  $[0, L]$ , it must in fact hit  $L$ . But  $S$  and  $U$  must coincide at  $\tau^S$ , so this would contradict the fact that  $U$  is strictly below  $L$ . Thus, the event  $\{\tau_\epsilon < \tau^S, \tau_b^+ < \tau_a^+\}$  is contained within the event that  $S$  has a discontinuity somewhere in  $(\tau_\epsilon, T]$ . However, on  $\{N(\tau_\epsilon, T] = 0\}$ ,  $S$  is continuous on  $(\tau_\epsilon, T]$  so the intersection in (4.5) has probability zero.

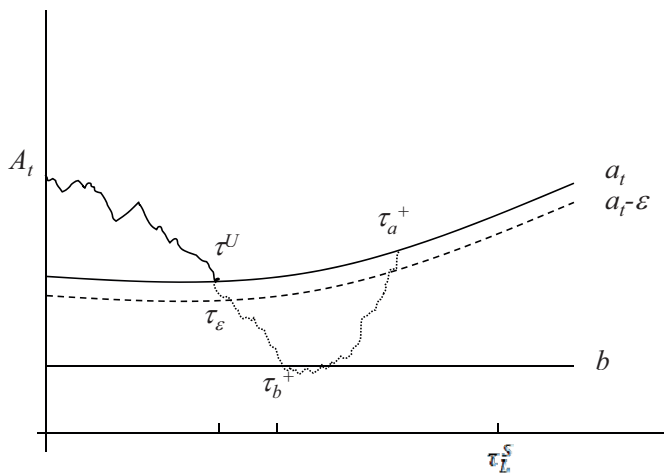
We now have

$$\begin{aligned}
 0 &= P(\tau_\epsilon < \tau^S, \tau_b^+ < \tau_a^+, N(\tau_\epsilon, T] = 0) \\
 &= \mathbb{E} [P(\tau_\epsilon < \tau^S, \tau_b^+ < \tau_a^+, N(\tau_\epsilon, T] = 0 | A_t, 0 \leq t \leq \min \{\tau_\epsilon, T\})] \\
 &= \mathbb{E} [\mathbf{1} \{\tau_\epsilon < \tau^S\} P(\tau_b^+ < \tau_a^+, N(\tau_\epsilon, T] = 0 | A_t, 0 \leq t \leq \min \{\tau_\epsilon, T\})] \\
 &= \mathbb{E} [\mathbf{1} \{\tau_\epsilon < \tau^S\} P(\tau_b^+ < \tau_a^+, N(\tau_\epsilon, T] = 0 | A_t, 0 \leq t \leq \tau_\epsilon < T)],
 \end{aligned}$$

the third equality using the fact that  $\mathbf{1} \{\tau_\epsilon < \tau^S\}$  is determined by (measurable with respect to)  $\{A_t, 0 \leq t \leq \min \{\tau_\epsilon, T\}\}$ , and the fourth equality using the fact that  $\tau_\epsilon < T$  on the event  $\{\tau_\epsilon < \tau^S\}$ . But as the conditional probability inside the expectation is strictly positive with probability one, the only way the expectation can be zero is if

$$P(\tau_\epsilon < \tau^S) = 0;$$

i.e.,  $P(\tau_\epsilon \geq \tau^S) = 1$ . But, with probability one,  $\tau^U = \inf_{\epsilon > 0} \tau_\epsilon$ , so  $P(\tau^U \geq \tau^S) = 1$  as well. ■



**Figure 4.4:**  $U_t = L$  on the  $a_t$  boundary (conversion possible);  $S_t < L$  below the  $b$  boundary (conversion necessary); An actual conversion time later than the first time  $U_t$  hits  $a_t$  leads to a contradiction.





# Appendix

## A.1 Technical Appendix for Chapter 2

### A.1.1 Equity Allocation

In this appendix, we prove [Theorem 2.3.1](#) under the more general assumption that  $V$  is any continuous semimartingale (as in Protter [\[68\]](#), p.44 and p.114). We first show that the expression for  $Q^o$  in [\(2.16\)](#) satisfies [\(2.14\)](#). For  $t \in [0, \min(\tau_b, T)]$ , we have  $Q_t > 0$ . By [\(2.4\)](#),  $Q$  is a continuous semimartingale and  $L$  is an increasing process, so we may take the differential of [\(2.16\)](#) to get

$$dQ_t^o = dQ_t \frac{Q_t^o}{Q_t} + Q_t d \left[ \left( \frac{a - L_t}{a} \right)^{(q \frac{1-\alpha}{\alpha})} \right],$$

and

$$d \left[ \left( \frac{a - L_t}{a} \right)^{(q \frac{1-\alpha}{\alpha})} \right] = -q \frac{1-\alpha}{\alpha} \frac{Q_t^o}{Q_t} \frac{dL_t}{a - L_t}. \quad (\text{A.1})$$

From part (iii) of Proposition 2.2.1, we know that if  $t$  is a point of increase (in the sense of Harrison [38], p.xvii) of  $L$ , then  $V_t - (B - (1 - \alpha)L_t) - D = \alpha V_t$ ; in other words,  $Q_t = \alpha V_t$ . This expression also gives  $V_t = a - L_t$ . Thus, we have

$$\frac{dL_t}{a - L_t} = \frac{dL_t}{V_t} = \frac{dL_t}{Q_t/\alpha}.$$

Making this substitution in (A.1) and rearranging terms, we get (2.14). If  $\tau_b < T$ , then for  $t \in (\tau_b, T]$ , we have  $L_t = L_{\tau_b}$  and (2.16) is consistent with (2.15). Thus,  $Q^o$  in (2.16) solves (2.14)–(2.15).

To prove uniqueness, we use Theorem 6 on p.194 of Protter [68], for which we rewrite (2.14)–(2.15) as

$$Q_t^o = Q_o + \int_0^t f(s, \omega, Q_s^o) dZ_s,$$

with  $Z_s = V_s + (q - 1)(1 - \alpha)L_s$  and

$$f(t, \omega, x) = \begin{cases} x/Q_t(\omega), & t \leq \tau_b(\omega); \\ (b/a)^{q(1-\alpha)/\alpha}, & t > \tau_b(\omega), \end{cases}$$

the second case giving  $Q_{\tau_b}^o/Q_{\tau_b}$ , as in (2.18). For each fixed  $x$ , the mapping  $(t, \omega) \rightarrow f(t, \omega, x)$  is continuous in  $t$  and adapted. For each fixed  $(t, \omega)$  and any real  $x, y$ ,

$$|f(t, \omega, x) - f(t, \omega, y)| \leq \frac{(1 - \alpha)}{\alpha D} \mathbf{1}_{\{s < \tau_b(\omega)\}} |x - y|,$$

because  $Q_t \geq \alpha D/(1 - \alpha)$  for  $t \in [0, \tau_b)$ . The conditions for Protter's [68] theorem are thus satisfied and uniqueness follows. The expressions in (2.17) for  $\pi$  follow directly from (2.14).

### A.1.2 Proof of Proposition 2.6.1

The main objective of this section is to prove Proposition 2.6.1. First, we recall that, at each  $t$ ,  $\widetilde{W}_t$  defined in (2.29) has a  $N(\mu t, \sigma^2 t)$  distribution, and  $\widetilde{m}_t$  has the following distribution and density (see, e.g., Harrison [38], p.14), for  $m \leq 0$ :

$$\mathbb{P}(\widetilde{m}_t \leq m) = \Phi\left(\frac{m - \mu t}{\sigma\sqrt{t}}\right) + \exp\left\{2\mu\frac{m}{\sigma^2}\right\} \Phi\left(\frac{m + \mu t}{\sigma\sqrt{t}}\right) \quad (\text{A.2})$$

$$f_{\widetilde{m}_t}^\mu(m) = \frac{2}{\sigma\sqrt{2\pi t}} \exp\left\{\frac{-1}{2\sigma^2 t}(m - \mu t)^2\right\} + 2\frac{\mu}{\sigma^2} \exp\left\{2\mu\frac{m}{\sigma^2}\right\} \Phi\left(\frac{m + \mu t}{\sigma\sqrt{t}}\right). \quad (\text{A.3})$$

Now let

$$h_\mu(t, k, y) = H(t, 0, k, y) = \int_{-\infty}^y e^{km} f_{\widetilde{m}_t}^\mu(m) dm.$$

Integration yields

$$h_\mu(t, k, y) = \frac{2\mu}{2\mu + k\sigma^2} e^{ky + 2y\mu/\sigma^2} \Phi\left(\frac{y + t\mu}{\sigma\sqrt{t}}\right) + \frac{2\mu + 2k\sigma^2}{2\mu + k\sigma^2} e^{k\mu t + k^2\sigma^2 t/2} \Phi\left(\frac{y - (\mu + k\sigma^2)t}{\sigma\sqrt{t}}\right).$$

We can now evaluate  $H$ . By the Girsanov theorem,

$$\mathbf{E}\left[e^{v\widetilde{W}_t + k\widetilde{m}_t} \mathbf{1}\{\widetilde{m}_t \leq y\}\right] = e^{v\mu t + v^2\sigma^2 t} \mathbf{E}_\theta\left[e^{k\widetilde{m}_t} \mathbf{1}\{\widetilde{m}_t \leq y\}\right],$$

the subscript  $\theta$  indicating that the expectation is taken with the drift of  $\widetilde{W}$  equal to  $\theta = \mu + v\sigma^2$  rather than  $\mu$ . The remaining expectation is given by  $h_\theta(t, k, y)$ .

### A.1.3 Proof of Proposition 2.6.2

We may write  $L_t = (a - V_0 \exp(\tilde{m}_t))^+ \wedge (a - b)$  as

$$\begin{aligned} L_t &= (a - b)\mathbf{1}\{\tilde{m}_T \leq \log(b/V_0)\} + (a - V_0 e^{\tilde{m}_T})\mathbf{1}\{\log(b/V_0) < \tilde{m}_T \leq \log(a/V_0)\} \\ &= a\mathbf{1}\{\tilde{m}_T \leq \log(a/V_0)\} - b\mathbf{1}\{\tilde{m}_T \leq \log(b/V_0)\} \\ &\quad - V_0 e^{\tilde{m}_T} \mathbf{1}\{\log(b/V_0) < \tilde{m}_T \leq \log(a/V_0)\}. \end{aligned}$$

The first expression in the proposition now follows from the definition of  $H$  in (2.30) and  $\Delta H$  in (2.33). The second expression follows by making the substitutions in (2.31) and simplifying terms.

### A.1.4 Proof of Proposition 2.6.3

We begin with the second part of the proposition, showing that (2.38) is the expectation of the second term in (2.23). By definition, we have  $V_{\tau_b} = b$ , and the fraction  $\pi_{\tau_b}$  is given in (2.18). With these substitutions, the second term in (2.23) simplifies to

$$X_1 \alpha b \left( 1 - \left( \frac{b}{a} \right)^{\left( q \frac{1-\alpha}{\alpha} \right)} \right) e^{-r\tau_b} \mathbf{1}\{\tau_b \leq T\}.$$

To calculate its expectation, we need to find  $\mathbf{E}[e^{-r\tau_b} \mathbf{1}\{\tau_b \leq T\}]$ . By the Girsanov theorem, this expectation coincides with

$$\mathbf{E}_\theta \left[ \exp \left\{ -r\tau_b + \frac{\mu - \theta}{\sigma^2} \widetilde{W}_{\tau_b} - \frac{\mu^2 - \theta^2}{2\sigma^2} \tau_b \right\} \mathbf{1}\{\tau_b \leq T\} \right] \quad (\text{A.4})$$

where  $\mathbf{E}_\theta$  indicates expectation with the drift of  $\widetilde{W}$  changed to  $\theta$ . This identity holds for any real  $\theta$ ; if we choose  $\theta = \theta_1$ , with  $\theta_1 = \sqrt{2\sigma^2 r + \mu^2}$ , then, recalling that  $\widetilde{W}_{\tau_b} = \log(b/V_0)$ ,

the expectation in (A.4) becomes

$$\left(\frac{b}{V_0}\right)^{\frac{\mu-\theta_1}{\sigma^2}} \mathbb{P}_{\theta_1}(\tau_b \leq T). \quad (\text{A.5})$$

Observing that  $\mathbb{P}_{\theta_1}(\tau_b \leq T) = \mathbb{P}_{\theta_1}(\tilde{m}_T \leq \log(b/V_0))$  and applying formula (A.2) with  $m$  replaced by  $\log(b/V_0)$  and  $\mu$  replaced by  $\theta_1$ , it follows that (A.5) is equal to

$$\left(\frac{b}{V_0}\right)^{\frac{\mu-\theta_1}{\sigma^2}} \Phi\left(\frac{\log(\frac{b}{V_0}) - \theta_1 T}{\sigma\sqrt{T}}\right) + \left(\frac{b}{V_0}\right)^{\frac{\mu+\theta_1}{\sigma^2}} \Phi\left(\frac{\log(\frac{b}{V_0}) + \theta_1 T}{\sigma\sqrt{T}}\right). \quad (\text{A.6})$$

Thus we have shown that

$$\begin{aligned} \mathbf{E} \left[ e^{-r\tau_b} (1 - \pi_{\tau_b}) X_1 \alpha V_{\tau_b} \mathbf{1}_{\{\tau_b \leq T\}} \right] &= R_1 \alpha b \left( 1 - \left(\frac{b}{a}\right)^{q\frac{1-\alpha}{\alpha}} \right) \times \\ &\quad \left[ \left(\frac{b}{V_0}\right)^{\frac{\mu-\theta_1}{\sigma^2}} \Phi\left(\frac{\log(\frac{b}{V_0}) - \theta_1 T}{\sigma\sqrt{T}}\right) + \left(\frac{b}{V_0}\right)^{\frac{\mu+\theta_1}{\sigma^2}} \Phi\left(\frac{\log(\frac{b}{V_0}) + \theta_1 T}{\sigma\sqrt{T}}\right) \right]. \end{aligned}$$

A further application of the Girsanov theorem yields

$$\begin{aligned} \mathbb{P}_{\theta_1}(\tau_b \leq T) &= \mathbf{E} \left[ \exp \left\{ \frac{\theta_1 - \mu}{\sigma^2} \tilde{W}_T + \frac{\mu^2 - \theta_1^2}{2\sigma^2} T \right\} \mathbf{1}_{\{\tau_b \leq T\}} \right] \\ &= e^{-rT} H(T, (\theta_1 - \mu)/\sigma^2, 0, \log(b/V_0)) \end{aligned}$$

and thus the expression in (2.38).

Next we turn to (2.37). On the event that the firm survives until the debt matures, the present value of the equity held by the contingent capital investors is given by the first term in (2.23). We can replace the indicator  $\mathbf{1}_{\{\tau_b > T\}}$  in this expression with  $\mathbf{1}_{\{\tilde{m}_T > \log(b/V_0)\}}$ ; and, if  $\tilde{m}_T > \log(a/V_0)$  then no debt was converted and  $(1 - \pi_T) = 0$ , so we may restrict the expectation to the event that  $\tilde{m}_T$  lies between  $\log(b/V_0)$  and  $\log(a/V_0)$ . Moreover, for  $\tilde{m}_T$  is

in this interval,  $L_T = a - V_0 \exp(\tilde{m}_T)$ . On this event, we therefore get (from (2.17))

$$\pi_T = \left(\frac{V_0}{a}\right)^{q\frac{1-\alpha}{\alpha}} e^{(q\frac{1-\alpha}{\alpha})\tilde{m}_T},$$

and  $B - (1 - \alpha)L_T + D = a(1 - \alpha) - (1 - \alpha)L_T = (1 - \alpha)V_0 e^{\tilde{m}_T}$ . Making these substitutions, the first term in (2.23) becomes  $\exp(-rT)$  times

$$\left(1 - \left(\frac{V_0}{a}\right)^{q\frac{1-\alpha}{\alpha}} e^{(q\frac{1-\alpha}{\alpha})\tilde{m}_T}\right) \left(V_0 e^{\tilde{W}_T} - (1 - \alpha)V_0 e^{\tilde{m}_T}\right) \mathbf{1} \left\{ \log\left(\frac{b}{V_0}\right) < \tilde{m}_T \leq \log\left(\frac{a}{V_0}\right) \right\}.$$

By expanding the product and taking the expectation we get four terms, each of the type that defines the function  $\Delta H$ , and this yields (2.37).

#### A.1.5 Proof of Proposition 2.6.4

If  $\tilde{m}_t \leq \log(b/V_0)$ , then  $\tau_b \leq t$  and if  $\tilde{m}_t > \log(a/V_0)$ , then  $\pi_t = 1$ . In addition for  $\tilde{m}_t$  in the interval  $[\log(b/V_0), \log(a/V_0)]$ ,  $L_t$  and  $\pi_t$  are respectively equal to  $a - V_0 e^{\tilde{m}_t}$  and  $(V_0 e^{\tilde{m}_t}/a)^{(q\frac{1-\alpha}{\alpha})}$ . It follows that  $(1 - \pi_t)(\delta V_t - (1 - \kappa)[c_1(B - (1 - \alpha)L_t) + c_2 D]) \mathbf{1}\{\tau_b > t\}$  equals

$$\left(1 - \left(\frac{V_0 e^{\tilde{m}_t}}{a}\right)^{(q\frac{1-\alpha}{\alpha})}\right) \left(\delta V_0 e^{\tilde{W}_t} - (1 - \kappa)(1 - \alpha)[(c_2 - c_1)b + c_1 V_0 e^{\tilde{m}_t}]\right) \times \mathbf{1} \left\{ \log\left(\frac{b}{V_0}\right) < \tilde{m}_t \leq \log\left(\frac{a}{V_0}\right) \right\}.$$

Here again the expectation is a linear combination of values of  $\Delta H$ , as given in (2.40).

### A.1.6 Proof of Proposition 2.6.5

We have  $\mathbb{P}(\tau_b > T) = 1 - \mathbb{P}(\tilde{m}_T \leq \log(b/V_0)) = 1 - H(T, 0, 0, \log(b/V_0))$ , and we showed that

$$\mathbf{E} [e^{-r\tau_b} \mathbf{1}\{\tau_b \leq T\}] = e^{-rT} H(T, (\theta_1 - \mu)/\sigma^2, 0, \log(b/V_0))$$

in the proof of Proposition 2.6.3. The result follows from making these substitutions in (2.41)–(2.42).

## A.2 Technical Appendix for Chapter 3

All the valuations used in the chapter reduce to expectations of certain functions of the asset value process  $V_t$  in (3.3) and the default time  $\tau_b$ . This appendix derives the necessary formulas. Our analysis builds on work on the hyperexponential jump-diffusion process in Cai et al. [14]. There is an extensive body of work on ruin probabilities and random walks that uses related techniques; see Asmussen and Albrecher [3] for a thorough treatment of the topic and extensive references.

Let  $X_t = \log(V_t)$  and write

$$X_t = X_0 + \mu t + \sigma W_t + \sum_{i=1}^{N_t} Y_i. \quad (\text{A.7})$$

Here,  $N_t = N_t^{(m)} + N_t^{(f)}$ , and the jump sizes  $Y_i$  are i.i.d. with density

$$f_Y(y) = q_f \eta_f e^{\eta_f y} \mathbf{1}_{\{y < 0\}} + q_m \eta_m e^{\eta_m y} \mathbf{1}_{\{y < 0\}}, \quad (\text{A.8})$$

where  $q_f = \Lambda_f / (\Lambda_f + \Lambda_m)$  and  $q_m = 1 - q_f$  are the probabilities of the two types of jumps.

This is a Lévy process with Lévy exponent

$$G(x) := \frac{1}{t} \log \mathbb{E}[\exp(xX_t)] = x\mu + \frac{1}{2}x^2\sigma^2 + (\Lambda_f + \Lambda_m) \left( \frac{q_f\eta_f}{\eta_f + x} + \frac{q_m\eta_m}{\eta_m + x} - 1 \right).$$

By some elementary calculus, it can be shown that for any given  $a > 0$ , the equation  $G(x) = a$  has four distinct real roots  $\beta$ ,  $-\gamma_1$ ,  $-\gamma_2$ , and  $-\gamma_3$ , where  $\beta, \gamma_j > 0$  for  $j = 1, 2, 3$ . All these roots are different from  $\eta_f$  and  $\eta_m$ . See Cai et al. [14].

Given a constant  $b$ , define  $\tau_b \equiv \inf \{t \geq 0 : X_t \leq b\}$ . The process  $X$  can reach or cross level  $b$  in three ways: without a jump at  $\tau_b$ , with a firm-specific jump at  $\tau_b$ , or with a market-wide jump at time  $\tau_b$ . Let  $J_0$ ,  $J_1$ , and  $J_2$  denote these three events. We need to consider the overshoot across level  $b$  in these three cases, so we define the events  $F_0 := \{X_{\tau_b} = b\} \cap J_0$ ,  $F_1 := \{X_{\tau_b} < b + y\} \cap J_1$ , and  $F_2 := \{X_{\tau_b} < b + y\} \cap J_2$  for some negative  $y$ . The pricing equations in Section 3.4 all reduce to evaluating quantities of the form

$$u_i(x) = \mathbb{E} [e^{-a\tau_b + \theta X_{\tau_b}} \mathbf{1}_{F_i} | X_0 = x], \quad i = 0, 1, 2,$$

where  $a \geq 0$  and  $\theta$  are constants.

Introduce a matrix

$$M = \begin{bmatrix} e^{-\gamma_1 b} & e^{-\gamma_1 b} \frac{\eta_f}{\eta_f - \gamma_1} & e^{-\gamma_1 b} \frac{\eta_m}{\eta_m - \gamma_1} \\ e^{-\gamma_2 b} & e^{-\gamma_2 b} \frac{\eta_f}{\eta_f - \gamma_2} & e^{-\gamma_2 b} \frac{\eta_m}{\eta_m - \gamma_2} \\ e^{-\gamma_3 b} & e^{-\gamma_3 b} \frac{\eta_f}{\eta_f - \gamma_3} & e^{-\gamma_3 b} \frac{\eta_m}{\eta_m - \gamma_3} \end{bmatrix}.$$

The matrix  $M$  is invertible because the roots  $\gamma_j$  are distinct. We can use it to express the functions  $u_i(x)$  explicitly:

**Theorem 1.** *Given  $a > 0$  and the negative roots  $-\gamma_j$ ,  $j = 1, 2, 3$ , of the algebraic equation*



$G(x) = a$ , let  $w(x) := (\exp(-\gamma_1 x), \exp(-\gamma_2 x), \exp(-\gamma_3 x))^\top$ . Then,

$$\begin{bmatrix} u_0(x) \\ u_1(x) \\ u_2(x) \end{bmatrix} = DM^{-1}w(x),$$

where

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{\theta b} \frac{\eta_f}{\eta_f + \theta} e^{(\theta + \eta_f)y} & 0 \\ 0 & 0 & e^{\theta b} \frac{\eta_f}{\eta_f + \theta} e^{(\theta + \eta_f)y} \end{bmatrix}.$$

*Proof.* Conditional on the event  $J_1$ , the memoryless property of the exponential distribution implies that  $b - X_{\tau_b}$  is exponentially distributed with mean  $1/\eta_f$ , independent of  $\eta_f$ . Therefore,

$$\begin{aligned} \mathbb{E}[\exp(-a\tau_b + \theta X_{\tau_b}) \mathbf{1}_{F_1} | X_0 = x] &= e^{\theta b} \mathbb{E}[\exp(-a\tau_b + \theta(X_{\tau_b} - b)) \mathbf{1}_{F_1} | X_0 = x] \\ &= e^{\theta b} \mathbb{E}[\exp(-a\tau_b) \mathbf{1}_{J_1} | X_0 = x] \frac{\eta_f}{\theta + \eta_f} e^{(\theta + \eta_f)y}. \end{aligned} \quad (\text{A.9})$$

Similarly, we have

$$\mathbb{E}[\exp(-a\tau_b + \theta X_{\tau_b}) \mathbf{1}_{F_2} | X_0 = x] = e^{lb} \mathbb{E}[\exp(-a\tau_b) \mathbf{1}_{J_2} | X_0 = x] \frac{\eta_m}{\theta + \eta_m} e^{(\theta + \eta_m)y}, \quad (\text{A.10})$$

and

$$\mathbb{E}[\exp(-a\tau_b + \theta X_{\tau_b}) \mathbf{1}_{F_0} | X_0 = x] = e^{lb} \mathbb{E}[\exp(-a\tau_b) \mathbf{1}_{J_0} | X_0 = x] \quad (\text{A.11})$$

Thus, we need to find

$$\mathbb{E} [e^{-a\tau_b + \theta X_{\tau_b}} \mathbf{1}_{J_i} | X_0 = x], \quad i = 1, 2, 3.$$

For any  $a > 0$  and any purely imaginary number  $l$  (i.e.,  $l = \sqrt{-1}c$  for some real  $c$ ),

$$M_t := \exp(-at + lX_t) - \exp(lX_0) - (G(l) - a) \int_0^t \exp(-as + lX_s) ds$$

is a zero-mean martingale. By the optional sampling theorem for martingales, we know that  $\mathbb{E}[M_{\tau_b} | X_0 = x] = 0$ , i.e.,

$$\begin{aligned} \mathbb{E}[\exp(-a\tau_b + lX_{\tau_b}) | X_0 = x] - e^{lx} \\ - (G(l) - a) \mathbb{E} \left[ \int_0^{\tau_b} \exp(-as + lX_s) ds | X_0 = x \right] = 0. \end{aligned} \quad (\text{A.12})$$

On the other hand, we can further decompose the first term on the right as

$$\begin{aligned} \mathbb{E}[\exp(-a\tau_b + lX_{\tau_b}) | X_0 = x] &= \sum_{i=1}^3 \mathbb{E}[\exp(-a\tau_b + lX_{\tau_b}) \mathbf{1}_{J_i} | X_0 = x] \\ &= e^{lb} \mathbb{E}[\exp(-a\tau_b) \mathbf{1}_{J_0} | X_0 = x] \\ &\quad + e^{lb} \frac{\eta_f}{\eta_f + \theta} \mathbb{E}[\exp(-a\tau_b) \mathbf{1}_{J_1} | X_0 = x] \\ &\quad + e^{lb} \frac{\eta_m}{\eta_m + \theta} \mathbb{E}[\exp(-a\tau_b) \mathbf{1}_{J_2} | X_0 = x] \end{aligned}$$

From (A.12) and (A.13), we know that

$$\begin{aligned} 0 &= \mathbb{E}[\exp(-a\tau_b) \mathbf{1}_{J_0} | X_0 = x] e^{lb} + e^{lb} \frac{\eta_f}{\eta_f + \theta} \mathbb{E}[\exp(-a\tau_b) \mathbf{1}_{J_1} | X_0 = x] \\ &\quad + e^{lb} \frac{\eta_m}{\eta_m + \theta} \mathbb{E}[\exp(-a\tau_b) \mathbf{1}_{J_2} | X_0 = x] - e^{lx} \\ &\quad - (G(l) - a) \mathbb{E} \left[ \int_0^{\tau_b} \exp(-as + lX_s) ds | X_0 = x \right]. \end{aligned} \quad (\text{A.13})$$

Denote the right side of (A.13) by  $h(l)$ . The equality (A.13) indicates that  $h(l) \equiv 0$  for

all imaginary  $l$ . Multiply  $h(l)$  by  $(l + \eta_m)(l + \eta_f)$  to obtain a new function

$$H(l) = h(l) \cdot (l + \eta_m) \cdot (l + \eta_f).$$

Then,  $H(l)$  is well-defined and analytic in the whole complex domain  $\mathbb{C}$ . By (A.13),  $H(l)$  equals zero whenever  $l$  is a purely imaginary. The identity theorem of analytic functions in the complex domain (Rudin[70], Theorem 10.18) then implies that  $H(l) \equiv 0$  for all  $l \in \mathbb{C}$ . Accordingly,  $h(l) = 0$  for all  $l \in \mathbb{C} \setminus \{-\eta_f, -\eta_m\}$ .

If we choose  $l = -\gamma_j$ ,  $j = 1, 2, 3$ , then  $G(l) = 0$ , and the equation  $h(l) = 0$  becomes

$$\begin{aligned} e^{-\gamma_j x} &= \mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_0}|X_0 = x]e^{-\gamma_j b} + e^{-\gamma_j b}\mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_1}|X_0 = x]\frac{\eta_f}{\eta_f - \gamma_j} \\ &\quad + e^{-\gamma_j b}\mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_2}|X_0 = x]\frac{\eta_m}{\eta_m - \gamma_j}, \end{aligned} \quad (\text{A.14})$$

for  $j = 1, 2, 3$ . This give us a system of three linear equations in the three unknown quantities  $\mathbb{E}[e^{-a\tau_b}\mathbf{1}_{J_i}|X_0 = x]$ ,  $i = 1, 2, 3$ . Using the solution to the linear equations in (A.9)–(A.11), we get  $\mathbb{E}[e^{-a\tau_b + \theta X_{\tau_b}}\mathbf{1}_{F_i}|X_0 = x]$ ,  $i = 1, 2, 3$ .  $\square$

Iterated expectations of the form  $\mathbb{E}[e^{-a_1\tau_c + \theta_1 X_{\tau_c}}\mathbb{E}[e^{-a_2(\tau_b - \tau_c) + \theta_2 X_{\tau_b}}\mathbf{1}_{F_i}|X_{\tau_c}]]$  can be evaluated the same way, and this is what we need for (3.9).

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