QUARTERLY OF APPLIED MATHEMATICS
VOLUME LXXI, NUMBER 3
SEPTEMBER 2013, PAGES 501–508
S 0033-569X(2013)01289-X
Article electronically published on May 16, 2013

CONTINUOUS DEPENDENCE ON THE REACTION TERMS IN POROUS CONVECTION WITH SURFACE REACTIONS

By

NICOLA L. SCOTT (*Department of Mathematics, University of Durham, DH1 3LE, United Kingdom*)

AND

B. STRAUGHAN (*Department of Mathematics, University of Durham, DH1 3LE, United Kingdom*)

Abstract. We investigate continuous dependence of the solution on the coefficients of the reaction terms for the problem where convection in a saturated porous medium is primarily due to chemical reactions on the boundary. Such boundary reaction terms are not obviously controllable by the usual arguments involving integrals of the functions themselves over the interior domain. Continuous dependence is established for a porous medium of Brinkman type in a general three-dimensional setting.

1. Introduction. In a recent paper, Postelnicu [24] has investigated the transition to convective motion of a fluid in a saturated porous material when the saturating fluid is nonisothermal and contains a chemical which may react. The chemical reactions are at the boundary of the domain, in the Postelnicu [24] case, at the lower boundary of an infinite horizontal plane layer. When forcing terms are on the boundary of the region it is not obvious that continuous dependence on the reaction coefficients is possible and it is to this goal that we address this work.

The general question of continuous dependence on modelling, or structural stability as it is alternatively known, is one of great importance in continuum mechanics and in the field of partial differential equations, as explained in great detail by Hirsch & Smale [4]. Knops & Payne [6] showed the importance of continuous dependence on modelling in elasticity in a fundamental paper, and sharpened some of their results in Knops & Payne [7]. This aspect of continuous dependence on modelling was further analysed by Payne [15, 16, 17], and more recently analyses of structural stability have been shown to be of major importance in porous media; cf. Aulisa *et al.* [1], Celebi *et al.* [2], Franchi & Straughan [3], Hoang & Ibragimov [5], Lin & Payne [8, 9, 10], Liu [11], Liu *et al.* [12, 13],

Received August 18, 2011.
2010 *Mathematics Subject Classification.* Primary 35B30, 76S05, 76V05.
This work was supported in part by a grant from the Leverhulme Trust, “Tipping points: mathematics, metaphors and meanings”, and in part by a DTA award from EPSRC.
E-mail address: `n.l.scott@durham.ac.uk`
E-mail address: `brian.straughan@durham.ac.uk`

Ouyang & Yang [14], Payne *et al.* [18], Payne & Straughan [19, 20, 21, 22], Rionero & Vergori [25], Straughan & Hutter [28], Straughan [27], Ugurlu [29], Wang & Lin [30], and further references may be found in Chapter 2 of Straughan [26].

In this paper we demonstrate continuous dependence on the reaction coefficients for a model of convection in a porous medium when the chemical reactions are taking place at the boundary. Due to the fact that boundary terms are not easy to control and the system of equations involved is fully nonlinear, the analysis presented herein is of necessity, nontrivial.

2. The boundary-initial value problem. The porous medium is assumed to occupy a bounded region Ω in $\mathbb{R}^3$ with boundary Γ sufficiently smooth to allow application of the divergence theorem. The basic variables are velocity, v_i, temperature, T, concentration, C, and pressure, p. Then, without loss of generality for the class of problem under investigation here, the equations of motion are

$$\begin{aligned} &v_i - \Delta v_i = -p_{,i} + g_i T + \tilde{g}_i C, \\ &v_{i,i} = 0, \\ &T_{,t} + v_i T_{,i} = \Delta T, \\ &C_{,t} + v_i C_{,i} = \Delta C, \end{aligned} \tag{2.1}$$

on $\Omega \times (0, \mathcal{T})$, for some $\mathcal{T} < \infty$, where Δ is the Laplace operator, g_i, $\tilde{g}_i$ are gravity terms with $|\mathbf{g}| \le 1$, $|\tilde{\mathbf{g}}| \le 1$, and standard indicial notation is employed throughout. Equations $(2.1)_3$ and $(2.1)_4$ are transport equations for temperature and concentration, $(2.1)_2$ expresses incompressibility of the saturating fluid, and $(2.1)_1$ is the Brinkman equation; cf. Straughan [26], Chapter 1.

The boundary conditions are

$$v_i = 0, \;\; \frac{\partial T}{\partial n} = AC, \;\; \frac{\partial C}{\partial n} = -BC, \tag{2.2}$$

on $\Gamma \times [0, \mathcal{T})$, where A, B are positive constants, and where $\partial/\partial n$ denotes the unit normal derivative pointing out of Γ. The initial conditions to be satisfied are

$$T(\mathbf{x}, 0) = T_0(\mathbf{x}), \;\; C(\mathbf{x}, 0) = C_0(\mathbf{x}), \tag{2.3}$$

where T_0 and C_0 are prescribed functions. Let the boundary-initial value problem comprised of (2.1)–(2.3) be denoted by $\mathcal{P}$.

Before establishing continuous dependence on A and B it is necessary to establish some auxiliary results and some *a priori* estimates for T and C.

3. A priori estimates. Let ψ be a function defined on $\bar{\Omega}$ and let f_i be a function defined on Γ with

$$f_i n_i \ge f_0 > 0, \qquad \text{on } \Gamma,$$

where n_i is the unit outward normal to Γ and f_0 is a constant. For example, if Ω is star shaped with respect to an interior origin, then we may select $f_i = x_i$. We employ a Rellich-like identity, cf. Payne & Weinberger [23], to derive a bound for the $L^2(\Gamma)$ norm of ψ.

By integration and use of the divergence theorem,

$$\begin{aligned}
f_0 \oint_\Gamma \psi^2 dA &\le \oint_\Gamma f_i n_i \psi^2 dA \\
&= \int_\Omega (f_i\psi^2)_{,i} dx \\
&= \int_\Omega f_{i,i}\psi^2 dx + 2\int_\Omega f_i \psi \psi_{,i} dx.
\end{aligned} \tag{3.1}$$

Suppose now that $f_{i,i} \le m_1$ in Ω, $|f_i| \le m_2$ in Ω, m_1, m_2 positive constants; for example, if $f_i = x_i$, then $f_{i,i} = 3$ and $|f_i|$ is bounded by the geometry of Ω. Then, using the arithmetic-geometric mean inequality for a constant $\alpha > 0$ at our disposal,

$$2\int_\Omega f_i\psi\psi_{,i} dx \le \frac{m_2}{\alpha}\int_\Omega \psi^2 dx + \alpha m_2 \int_\Omega \psi_{,i}\psi_{,i} dx. \tag{3.2}$$

Upon employing (3.2) in (3.1) we derive the bound

$$f_0 \oint_\Gamma \psi^2 dA \le \Big(m_1 + \frac{m_2}{\alpha}\Big)\int_\Omega \psi^2 dx + \alpha m_2 \int_\Omega \psi_{,i}\psi_{,i} dx. \tag{3.3}$$

To determine a bound for C consider for $p \in \mathbb{N}$,

$$\begin{aligned}
\frac{d}{dt}\int_\Omega C^{2p} dx &= 2p\int_\Omega C^{2p-1} C_{,t} dx \\
&= 2p \int_\Omega C^{2p-1}(\Delta C - v_i C_{,i}) dx \\
&= -2p(2p-1)\int_\Omega C^{2p-2}|\nabla C|^2 dx - 2pB\oint_\Gamma C^{2p} dA \\
&\le 0.
\end{aligned}$$

Upon integration one finds

$$\Big(\int_\Omega C^{2p} dx\Big)^{\frac{1}{2p}} \le \Big(\int_\Omega C_0^{2p} dx\Big)^{\frac{1}{2p}}. \tag{3.4}$$

We now let $p \to \infty$ in (3.4) and we find

$$\sup_{\Omega\times[0,\mathcal{T}]} |C| \le \max_{\bar\Omega} |C_0| \equiv C_m. \tag{3.5}$$

It transpires that we require a further bound, namely for $\| T \|_4$, where $\| \cdot \|_4$ is the norm in $L^4(\Omega)$. We employ the notation $\| \cdot \|$ and $(\cdot,\cdot)$ to denote the norm and inner product on $L^2(\Omega)$. To derive this bound we observe that

$$\begin{aligned}
\frac{d}{dt}\frac{1}{4}\int_\Omega T^4 dx &= \int_\Omega T^3 T_{,t} dx \\
&= \int_\Omega T^3(\Delta T - v_i T_{,i}) dx \\
&= -3\int_\Omega T^2 T_{,i}T_{,i} dx + \oint_\Gamma T^3 \frac{\partial T}{\partial n} dA \\
&= -\frac{3}{4}\int_\Omega (T^2)_{,i}(T^2)_{,i} dx + A\oint_\Gamma T^3 C dA.
\end{aligned} \tag{3.6}$$

With the aid of Young's inequality we have

$$\oint_\Gamma T^3 C dA \le \frac{3}{4}\oint_\Gamma T^4 dA + \frac{1}{4}\oint_\Gamma C^4 dA, \tag{3.7}$$

and then employing inequality (3.3),

$$\oint_\Omega T^4 dA \le \left(\frac{m_1}{f_0} + \frac{m_2}{f_0\alpha}\right)\int_\Omega T^4 dx + \frac{\alpha m_2}{f_0}\int_\Omega |\nabla T^2|^2 dx. \tag{3.8}$$

Use of (3.7) and (3.8) in (3.6) yields

$$\begin{aligned}\frac{d}{dt}\frac{1}{4}\int_\Omega T^4 dx \le & -\frac{3}{4}\int_\Omega |\nabla T^2|^2 dx + \frac{A}{4}\oint_\Gamma C^4 dA \\ & + \frac{3A}{4}\left(\frac{m_1}{f_0} + \frac{m_2}{f_0\alpha}\right)\int_\Omega T^4 dx + \frac{3A\alpha m_2}{4f_0}\int_\Omega |\nabla T^2|^2 dx.\end{aligned} \tag{3.9}$$

Now pick $\alpha = f_0/(m_2 A)$, also use estimate (3.5), and define D and D_1 by

$$D = \frac{AC_m^4}{4}|\Gamma|, \quad D_1 = 4D,$$

with $|\Gamma|$ being the measure of Γ. Set $F(t) = \int_\Omega T^4 dx$, and put

$$\lambda = \frac{3m_1 A}{f_0} + \frac{3m_2^2 A^2}{f_0^2},$$

and then from (3.9) we may show that

$$F' \le D_1 + \lambda F, \tag{3.10}$$

where $F' = dF/dt$. Upon integration of (3.10) we find

$$F(t) \le F(0)e^{\lambda t} + \frac{D_1}{\lambda}e^{\lambda t}, \tag{3.11}$$

and putting $D_2^4 = (F(0) + D_1\lambda^{-1})\exp[\lambda\mathcal{T}]$, from (3.11) we see that

$$\| T(t) \|_4 \le D_2. \tag{3.12}$$

4. Continuous dependence on the boundary reaction terms. Now let (v_i^1, T^1, C^1, p^1) and (v_i^2, T^2, C^2, p^2) be solutions to $\mathcal{P}$ for the same initial data but for boundary coefficients (A_1, B_1) and (A_2, B_2) in (2.2), respectively. Define the quantities u_i, π, θ, ϕ, a and b by

$$\begin{aligned} u_i = v_i^1 - v_i^2, \quad & \pi = p^1 - p^2, \quad \theta = T^1 - T^2, \\ \phi = C^1 - C^2, \quad & a = A_1 - A_2, \quad b = B_1 - B_2. \end{aligned}$$

Then, one finds that (u_i, θ, ϕ, π) satisfies the boundary-initial value problem

$$\begin{aligned} u_i - \Delta u_i &= -\pi_{,i} + g_i\theta + \tilde{g}_i\phi, \\ u_{i,i} &= 0, \\ \theta_{,t} + v_i^1\theta_{,i} + u_i T_{,i}^2 &= \Delta\theta, \\ \phi_{,t} + v_i^1\phi_{,i} + u_i C_{,i}^2 &= \Delta\phi, \end{aligned} \tag{4.1}$$

in $\Omega \times (0, \mathcal{T})$, together with

$$u_i = 0, \ \ \frac{\partial \theta}{\partial n} = aC_1 + A_2\phi, \ \ \frac{\partial \phi}{\partial n} = -(bC_1 + B_2\phi),$$

on $\Gamma \times [0, \mathcal{T}]$, and

$$\theta(\mathbf{x}, 0) = 0, \ \ \phi(\mathbf{x}, 0) = 0, \ \ \mathbf{x} \in \Omega.$$

First, multiply $(4.1)_1$ by u_i and integrate over Ω to derive

$$\begin{aligned} \| \mathbf{u} \|^2 + \| \nabla \mathbf{u} \|^2 &= (g_i\theta, u_i) + (\tilde{g}_i\phi, u_i) \\ &\leq \| \theta \|^2 + \frac{1}{4} \| \mathbf{u} \|^2 + \| \phi \|^2 + \frac{1}{4} \| \mathbf{u} \|^2, \end{aligned}$$

where the arithmetic-geometric mean inequality has been employed. Thus, we see that

$$\frac{1}{2} \| \mathbf{u} \|^2 + \| \nabla \mathbf{u} \|^2 \leq \| \theta \|^2 + \| \phi \|^2 . \tag{4.2}$$

Next, we multiply $(4.1)_3$ by θ, $(4.1)_4$ by ϕ, and integrate by parts using the boundary conditions to find

$$\begin{aligned} \frac{d}{dt}\frac{1}{2} \| \theta \|^2 &= -(u_i T^2_{,i}, \theta) + (\theta, \Delta\theta) \\ &= (u_i T^2, \theta_{,i}) - \| \nabla \theta \|^2 + a \oint_\Gamma C_1 \theta dA + A_2 \oint_\Gamma \theta\phi dA \end{aligned} \tag{4.3}$$

and

$$\frac{d}{dt}\frac{1}{2} \| \phi \|^2 = (u_i C^2, \phi_{,i}) - \| \nabla \phi \|^2 - b \oint_\Gamma C_1 \phi dA - B_2 \oint_\Gamma \phi^2 dA. \tag{4.4}$$

To handle the cubic term on the right of (4.4) we use (3.5) and the Cauchy-Schwarz and arithmetic-geometric mean inequalities to find, for $\gamma > 0$ at our disposal,

$$(u_i C^2, \phi_{,i}) \leq C_m \| \mathbf{u} \| \| \nabla \phi \| \leq \frac{{C_m}^2}{2\gamma} \| \mathbf{u} \|^2 + \frac{\gamma}{2} \| \nabla \phi \|^2 . \tag{4.5}$$

Now, insert (4.5) in (4.4) and use the arithmetic-geometric mean inequality on the C,ϕ boundary term, then use also (4.2), to obtain

$$\begin{aligned} \frac{d}{dt}\frac{1}{2} \| \phi \|^2 \leq \frac{{C_m}^2}{\gamma}(\| \theta \|^2 + \| \phi \|^2) - \left(1 - \frac{\gamma}{2}\right) \| \nabla \phi \|^2 \\ - \left(B_2 - \frac{\beta}{2}\right) \oint_\Gamma \phi^2 dA + \frac{b^2}{2\beta} \oint_\Gamma {C_1}^2 dA, \end{aligned} \tag{4.6}$$

where $\beta > 0$ is a constant at our disposal.

To deal with the cubic term in (4.3) we use the Cauchy-Schwarz inequality as follows:

$$(u_i T^2, \theta_{,i}) \leq \| \nabla \theta \| \left(\int_\Omega u_i u_i {T_2}^2 dx \right)^{\frac{1}{2}} \leq \| \nabla \theta \| \| \mathbf{u} \|_4 \| T_2 \|_4 .$$

Next, employ the Sobolev inequality $\| \mathbf{u} \|_4 \leq \hat{c}_1 \| \nabla \mathbf{u} \|$ and estimates (4.2) and (3.12) to see that

$$(u_i T^2, \theta_{,i}) \leq \hat{c}_1 \| \nabla \theta \| (\| \theta \|^2 + \| \phi \|^2)^{\frac{1}{2}} D_2. \tag{4.7}$$

We employ inequality (4.7) in (4.3) and further use the arithmetic-geometric mean inequality with weights δ, $\epsilon > 0$, to find

$$\begin{aligned}\frac{d}{dt}\frac{1}{2}\parallel \theta \parallel^2 \leq \hat{c_1} D_2 \parallel \nabla\theta \parallel (\parallel \theta \parallel^2 + \parallel \phi \parallel^2)^{\frac{1}{2}} - \parallel \nabla\theta \parallel^2 + \frac{a^2}{2\delta}\oint_\Gamma {C_1}^2 dA \\ + \frac{1}{2}(\delta + A_2\epsilon)\oint_\Gamma \theta^2 dA + \frac{A_2}{2\epsilon}\oint_\Gamma \phi^2 dA.\end{aligned} \tag{4.8}$$

Now, add (4.6) and (4.8), and employ inequalities (3.3) and (3.5), to find

$$\begin{aligned}\frac{d}{dt}\frac{1}{2}(\parallel \theta \parallel^2 + \parallel \phi \parallel^2) \leq \left(\frac{{C_m}^2}{\gamma} + \hat{c_1} D_2 \frac{\xi}{2}\right)(\parallel \theta \parallel^2 + \parallel \phi \parallel^2) - \left(1 - \frac{\gamma}{2}\right) \parallel \nabla\phi \parallel^2 \\ - \left(1 - \frac{\hat{c_1} D_2}{2\xi} - \left(\frac{\delta}{2} + \frac{A_2\epsilon}{2}\right)\alpha_2 m_2\right) \parallel \nabla\theta \parallel^2 \\ - \left(B_2 - \frac{\beta}{2} - \frac{A_2}{2\epsilon}\right)\oint_\Gamma \phi^2 dA + \left(\frac{a^2}{2\delta} + \frac{b^2}{2\beta}\right){C_m}^2|\Gamma| \\ + \frac{1}{2f_0}(\delta + A_2\epsilon)\left(m_1 + \frac{m_2}{\alpha_2}\right) \parallel \theta \parallel^2 .\end{aligned} \tag{4.9}$$

We now select $\gamma = 2$, $\beta = B_2$, $\epsilon = A_2/B_2$, so that the coefficients of the second and fourth terms on the right of (4.9) are zero. Then pick $\xi = \hat{c_1} D_2$, $\delta = 1$, and $\alpha_2 = m_2/(1 + {A_2}^2/B_2)$ in order that the coefficient of the third term is likewise zero. Finally, select

$$\begin{aligned}\gamma_1 &= \max\left\{1, \frac{1}{B_2}\right\}, \\ \gamma_2 &= \gamma_1 {C_m}^2|\Gamma|, \\ \gamma_3 &= {C_m}^2 + {\hat{c_1}}^2 D_2^2 + \frac{1}{f_0}\left(1 + \frac{{A_2}^2}{B_2}\right)\left(m_1 + 1 + \frac{{A_2}^2}{B_2}\right).\end{aligned}$$

Now define $G(t) = \parallel \theta \parallel^2 + \parallel \phi \parallel^2$ and then from (4.9) we observe that

$$\frac{dG}{dt} \leq \gamma_2(a^2 + b^2) + \gamma_3 G.$$

This inequality integrates to find

$$G(t) \leq \frac{\gamma_2}{\gamma_3} e^{\gamma_3 t}(a^2 + b^2). \tag{4.10}$$

Inequality (4.10) demonstrates continuous dependence on the coefficients A and B in the measures $\parallel \theta \parallel$ and $\parallel \phi \parallel$.

Thanks to inequality (4.2) we also have continuous dependence in the measures $\parallel \mathbf{u} \parallel$ and $\parallel \nabla\mathbf{u} \parallel$.

References

[1] E. Aulisa, L. Bloshanskaya, L. Hoang, and A. Ibragimov. Analysis of generalized Forchheimer flows of compressible fluids in porous media. *J. Math. Phys.*, 50:103102, 2009. MR2573128 (2010k:76110)

[2] A. O. Celebi, V. K. Kalantarov, and D. Ugurlu. On continuous dependence on coefficients of the Brinkman-Forchheimer equations. *Appl. Math. Letters*, 19:801–807, 2006. MR2232258 (2007e:35222)

[3] F. Franchi and B. Straughan. Continuous dependence and decay for the Forchheimer equations. *Proc. Roy. Soc. London A*, 459:3195–3202, 2003. MR2027361 (2004j:35121)

[4] M. W. Hirsch and S. Smale. *Differential equations, dynamical systems, and linear algebra*. Academic Press, New York, 1974. MR0486784 (58:6484)

[5] L. Hoang and A. Ibragimov. Structural stability of generalized Forchheimer equations for compressible fluids in porous media. *Nonlinearity*, 24:1–41, 2011. MR2739829 (2011j:76169)

[6] R. J. Knops and L. E. Payne. Continuous data dependence for the equations of classical elastodynamics. *Proc. Camb. Phil. Soc.*, 66:481–491, 1969. MR0270604 (42:5492)

[7] R. J. Knops and L. E. Payne. Improved estimates for continuous data dependence in linear elastodynamics. *Math. Proc. Camb. Phil. Soc.*, 103:535–559, 1988. MR932678 (89d:73008)

[8] C. Lin and L. E. Payne. Structural stability for a Brinkman fluid. *Math. Meth. Appl. Sci.*, 30:567–578, 2007. MR2298682 (2008d:35166)

[9] C. Lin and L. E. Payne. Structural stability for the Brinkman equations of flow in double diffusive convection. *J. Math. Anal. Appl.*, 325:1479–1490, 2007. MR2275033 (2007i:35190)

[10] C. Lin and L. E. Payne. Continuous dependence on the Soret coefficient for double diffusive convection in Darcy flow. *J. Math. Anal. Appl.*, 342:311–325, 2008. MR2440798 (2009g:76146)

[11] Y. Liu. Convergence and continuous dependence for the Brinkman-Forchheimer equations. *Math. Computer Modelling*, 49:1401–1415, 2009. MR2508355 (2010b:76121)

[12] Y. Liu, Y. Du, and C. Lin. Convergence results for Forchheimer's equations for fluid flow in porous media. *J. Math. Fluid Mech.*, 12:576–593, 2010. MR2749444

[13] Y. Liu, Y. Du, and C. H. Lin. Convergence and continuous dependence results for the Brinkman equations. *Applied Mathematics and Computation*, 215:4443–4455, 2010. MR2596122

[14] Y. Ouyang and L. Yang. A note on the existence of a global attractor for the Brinkman-Forchheimer equations. *Nonlinear Analysis, Theory, Methods and Applications*, 70:2054–2059, 2009. MR2492141 (2010b:35374)

[15] L. E. Payne. On geometric and modeling perturbations in partial differential equation. In R. J. Knops and A. A. Lacey, editors, *Proceedings of the LMS Symposium on Non-Classical Continuum Mechanics*, pages 108–128, Cambridge University Press. Cambridge, 1987. MR926500 (89b:35011)

[16] L. E. Payne. On stabilizing ill-posed problems against errors in geometry and modeling. In H. Engel and C. W. Groetsch, editors, *Proceedings of the Conference on Inverse and Ill-posed Problems: Strobhl*, pages 399–416, Academic Press. New York, 1987. MR1020326 (90i:35277)

[17] L. E. Payne. Continuous dependence on geometry with applications in continuum mechanics. In G. A. C. Graham and S. K. Malik, editors, *Continuum Mechanics and its Applications*, pages 877–890, Hemisphere Publ. Co. Washington, DC, 1989. MR1051694 (91f:35038)

[18] L. E. Payne, J. C. Song, and B. Straughan. Continuous dependence and convergence results for Brinkman and Forchheimer models with variable viscosity. *Proc. Roy. Soc. London A*, 455:2173–2190, 1999. MR1702738 (2000e:76120)

[19] L. E. Payne and B. Straughan. Stability in the initial-time geometry problem for the Brinkman and Darcy equations of flow in porous media. *J. Math. Pures et Appl.*, 75:225–271, 1996. MR1387521 (97d:35177)

[20] L. E. Payne and B. Straughan. Analysis of the boundary condition at the interface between a viscous fluid and a porous medium and related modelling questions. *J. Math. Pures et Appl.*, 77:317–354, 1998. MR1623387 (99c:35200)

[21] L. E. Payne and B. Straughan. Structural stability for the Darcy equations of flow in porous media. *Proc. Roy. Soc. London A*, 454:1691–1698, 1998. MR1631980 (99d:76093)

[22] L. E. Payne and B. Straughan. Convergence and continuous dependence for the Brinkman-Forchheimer equations. *Stud. Appl. Math.*, 102:419–439, 1999. MR1684989 (2000a:76165)

[23] L. E. Payne and H. F. Weinberger. New bounds for solutions of second order elliptic partial differential equations. *Pacific J. Math.*, 8:551–573, 1958. MR0104047 (21:2809)

[24] A. Postelnicu. Onset of convection in a horizontal porous layer driven by catalytic surface reaction on the lower wall. *Int. J. Heat and Mass Transfer*, 52:2466–2470, 2009.

[25] S. Rionero and L. Vergori. Long-time behaviour of fluid motions in porous media according to the Brinkman model. *Acta Mechanica*, 210:221–240, 2010.

[26] B. Straughan. *Stability and wave motion in porous media*. Applied Mathematical Sciences. Springer, New York, 2008. MR2433781 (2009e:76202)

[27] B. Straughan. Continuous dependence on the heat source in resonant porous penetrative convection. *Studies in Applied Mathematics*, 127:302–314, 2011. MR2852484

[28] B. Straughan and K. Hutter. A priori bounds and structural stability for double diffusive convection incorporating the Soret effect. *Proc. Roy. Soc. London A*, 455:767–777, 1999. MR1701018

(2000c:76088)

[29] D. Ugurlu. On the existence of a global attractor for the Brinkman-Forchheimer equations. *Nonlinear Analysis, Theory, Methods and Applications*, 68:1986–1992, 2008. MR2388758 (2009a:37170)

[30] B. Wang and S. Lin. Existence of global attractors for the three-dimensional Brinkman-Forchheimer equation. *Math. Meth. Appl. Sci.*, 31:1479–1495, 2008. MR2427000 (2009h:37162)