Continuous Network Coding in Wireless Relay Networks

Wei Pu
University of Science and
Technology of China
Hefei, Anhui, China
puwei@ustc.edu

Chong Luo, Shipeng Li
Microsoft Research Asia
Beijing, China
{cluo, spli}@microsoft.com

Chang Wen Chen
State Univ. of New York at Buffalo
Buffalo, NY 14260 USA
chencw@buffalo.edu

***Abstract*—Network coding has recently been applied to wireless networks and has achieved some initial success. Researches in wireless network coding have been mostly focusing on utilizing the broadcast nature of the wireless networks. In this paper, we propose a novel network coding framework for wireless relay networks that also takes into consideration the fading and error prone nature of the wireless networks. First, we extend the traditional network coding in lossless networks which operates on 0-1 bits, to a new framework which defines network coding on the posterior probability of each bit. This new framework allows an imperfect decode-recode process at a relay node and avoids possible error propagation when a hard decision is made at the relay node. It implicitly integrates decode-and-forward and estimate-and-forward strategies for wireless network coding to address the technical issues of channel fading and transmission errors. The proposed approach is validated through both theoretical analysis and extensive simulations. Both analysis and simulation confirm that this new framework is able to achieve significant gain over traditional network coding. This new framework also enables the introduction of adaptive scheme into network coding. We demonstrate a basic adaptation scheme and present some preliminary experimental results. The proposed adaptive scheme will lay down an essential foundation in this emerging field of wireless network coding in order to address issues related to link heterogeneity.**

Keywords – adaptivity, cooperative diversity, low-density parity-check codes, network coding, relay networks.

I. INTRODUCTION

Network coding is first introduced in [2]. It extends the traditional routing paradigm by allowing interior network nodes to perform coding operations rather than just routing. It has been shown in [2] and [4] that network coding can help a communication system achieve multicast capacity in lossless networks. Network Coding has recently been applied to wireless networks [6][10].

A. Network Coding in Wireless Networks with Physical-Layer Broadcast

The application of network coding in wireless networks is facilitated by the broadcast nature of wireless medium. The best studied case is a three-node topology where two terminals wish to exchange mutual information through an intermediate relay. By applying network coding and utilizing broadcast capability at the relay, Wu *et al.* [6] demonstrated that the time it takes to exchange a message can be reduced from four time slots of naïve transmission to three. Katti *et al.* [12] extended this idea to multiple terminals and implemented a practical protocol which uses overhearing and opportunistic packet combining. Their work also showed remarkable throughput increase brought by network coding and physical-layer broadcast.

Other related research topics include analog or physical network coding which utilize wireless interference for network coding [13] [17].

B. Cooperative Diversity to Combat Fading

The broadcast nature provides many benefits for wireless networking. However, one major problem of the wireless medium is the channel fading effects. To combat this dominant channel impairment in wireless communications, researchers have explored varies diversities and recognized user cooperative diversity as one of the most effective ways.

Laneman *et al.* [5] exploited cooperative diversity using a collection of distributed antennas belonging to multiple terminals, each with its own information to transmit. They examined several cooperation strategies, including amplify-and-forward, decode-and-forward, selection relaying, and incremental relaying. They showed that except for decode-and-forward, all the other schemes achieve a diversity of two in case of two terminals. Chen *et al.* [10] claimed that spatial diversity is better explored if combined with network coding. They examined a distributed antenna system (DAS) with and without network coding. Their results show that network coding leads to significant reduction of outage probability.

For large systems, distributed channel coding can be combined with network coding to further improve system performance [7][11]. Both space diversity and coded diversity [8] have been exploited by these joint channel-network coding schemes. They use distributed capacity achieving codes to form a unified framework which treats channel codes as a part of network codes.

We believe that joint channel-network coding is an effective way to combat channel fading. However, one major limitation of the current approach is that the traditional network coding defined on Galois finite field requires the relay nodes to fully decode the received information. The schemes discussed earlier all assume error-free decoding at relay nodes, which is virtually impossible under current wireless environments. Without this assumption, however, relay nodes will introduce additional errors in the decode-recode operation, and these errors will affect the belief propagation decoding process at destination node.

C. Soft Information Utilization

Fortunately, this problem can be relatively easily resolved when soft information relaying is introduced in wireless network coding. Specifically, instead of making a hard decision on the decoded information symbols, the relay can calculate and forward the soft information, such as posterior probability or logarithm likelihood ratio (LLR), for each bit. Sneesens *et al.* [9] indicated that soft decode-and-forward relay can be seen as an analog signal-to-noise ratio enhancer. Similarly, Li *et al.* [14] proposed distributed turbo code with soft relaying for single-source single-destination topology. Their results showed that their proposed scheme can effectively mitigate error propagation caused by erroneous decoding at the relay. Woo *et al.* [20] implement practical systems that use soft information to help packet recovery.

Recently, Yang *et al.* [16] proposed a belief propagation approach for network coding over AWGN channels. Although this parallel work is similar to our framework, there are two fundamental differences. First, in [16], network coding at the relay is performed in a bit-by-bit manner while we generate new parity bits using new codeword. It has been show in [5] that, bit-by-bit coding (repetition coding) cannot fully explore channel capacity. Second, [16] studies AWGN channels while we aim at fading channels. In fact, for AWGN channel model, very high performance channel codes can be designed without soft information relaying [15]. However, for fading channels, designing channel codes that works well in all channel implementations is unrealistic. The proposed continuous network coding offers a new approach to solve this classical problem.

D. Summary of Our Work and Paper Organization

The first major innovation of the proposed research lies in the development of a new approach to perform network coding in fading environment that can fully utilize the independently observed information at the relay node even the relay cannot successfully decode the source information. We extend the traditional network coding in lossless network which operates on 0-1 bits to a new framework which defines network coding on the posterior probability of each bit. Because the posterior probabilities are continuous numbers, we name this new framework as continuous network coding (CNC). This framework allows an imperfect decode-recode process at a relay node and avoids the possible error propagation introduced by the hard decision at the relay node. It explicitly integrates the decode-and-forward and estimate-and-forward strategies [21] to deal with channel fading. Significant gain over traditional network coding has been achieved with this CNC approach.

The second major innovation lies in the development of an adaptive framework based on the proposed CNC (ACNC). We demonstrate the adaptive CNC with simple adaptive bit loading. Our adaptive scheme will lay down an essential foundation in the field of wireless network coding in order to address issues related to link heterogeneity.

The rest of this paper is organized as follows. Section II describes the system model of wireless relay networks under consideration. In Section III, we propose the new framework of continuous network coding and provide theoretical analysis for this new framework. Section IV shows the simulation results to confirm the advantages of the proposed scheme. Section V extends CNC to ACNC to allow the wireless network coding to accommodate time varying and error prone wireless relay links. Section VI concludes this paper with summary and possible future research directions.

II. SYSTEM MODEL

A. Network Topology

The system of interest is a two-hop wireless relay network consisting of two sources, one relay, and one destination, as depicted in Fig. 1.

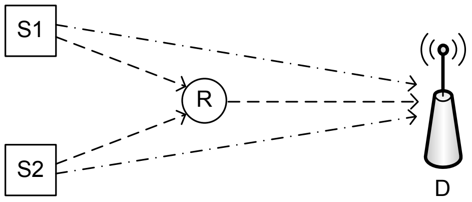


Figure 1. Two-hop wireless relay network of interest.

All terminals employ the half-duplex transmission. The channel is divided into several orthogonal sub-channels. Without lose of generality, we assume time-division medium access (TDMA) in this research, i.e. each node transmits in different time slots. Fig. 2 depicts the time slot allocation between the relay and two sources.

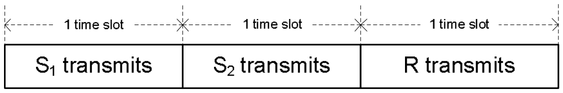


Figure 2. Time slot allocation for transmission.

Each source independently generates information bits, and protects them with LDPC codes [1]. To simplify our symbol notation, we map the information alphabet from {0, 1} to {+1, -1}, i.e. 0→+1 and 1→-1. It is easy to verify that the XOR operation in LDPC encoding on {0, 1} corresponds to multiplication on {+1, -1}. The coded information bits are denoted by $U = \{u_1, u_2 ... u_n\}$, $u_i \in \{\pm 1\}$. They are modulated with binary phase-shift keying (BPSK) modulation.

It should be noted that when a source broadcasts data packets, both relay and destination notes are able to hear. The main functionality of the relay is to provide diversity to combat channel fading.

B. *channel model*

All the channels in Fig. 1 are assumed to be quasi-static Rayleigh fading channels with additive Gaussian noise. We model the channel as:

$$\mathbf{Y}_m = h_m \sqrt{E_{s,m}} \mathbf{X}_m + \mathbf{w}_m \tag{1}$$

where $\mathbf{Y}_m$ are received samples after matched filter, $\mathbf{X}_m$ are transmitted symbols after modulation, $E_{s,m}$ is the symbol energy, h_m is the fading gain, $\mathbf{w}_m$ is the channel noise, and m is the block index. We normalize h_m with $E[h_m^2]=1$. So the power spectrum density of $\mathbf{w}_m$ also considers the affect of path-loss.

The fading gain h_m is modeled as a Rayleigh distributed variable. It remains constant during one codeword length, and changes independently from one packet to another. Such a channel is also referred to as quasi-static channel. $\mathbf{w}_m$ is additive Gaussian white noise (AGWN), with zero mean and one side power density spectrum N_0.

III. CONTINUOUS NETWORK CODING

We employ distributed LDPC code as the joint channel-network code. The differentiator of this framework is that the relay performs continuous network coding instead of network coding on the bits after hard decision.

In this section, we first define the XOR operation on posterior probability, and then provide a system view of our approach. Finally, we present theoretical analysis for CNC.

A. *XOR over Posterior Probability*

In BPSK modulation, we first need to map the sign-magnitude representation to posterior probability based representation. We define:

$$z_i := 2\Pr\{u_i = 1 \mid y_i\} - 1 \in [-1,1] \tag{2.1}$$

Assuming that the fading coefficient can be perfectly estimated by the receiver, then the posterior probability is:

$$\Pr\{u_i = 1 \mid y_i\} = \frac{e^{-\frac{(y_i-\sqrt{E_s})^2}{2\sigma^2}}}{e^{-\frac{(y_i-\sqrt{E_s})^2}{2\sigma^2}} + e^{-\frac{(y_i+\sqrt{E_s})^2}{2\sigma^2}}} = \frac{e^{\frac{\sqrt{E_s}}{\sigma^2}y_i}}{e^{\frac{\sqrt{E_s}}{\sigma^2}y_i} + e^{-\frac{\sqrt{E_s}}{\sigma^2}y_i}} \tag{2.2}$$

Then, z_i can be simply expressed as:

$$z_i = \frac{e^{\frac{\sqrt{E_s}}{\sigma^2}y_i} - e^{-\frac{\sqrt{E_s}}{\sigma^2}y_i}}{e^{\frac{\sqrt{E_s}}{\sigma^2}y_i} + e^{-\frac{\sqrt{E_s}}{\sigma^2}y_i}} = \tanh\left(\frac{\sqrt{E_s}}{\sigma^2} y_i\right) \tag{2.3}$$

From above equations, when the value of z_i calculated from the received symbol y_i is close to 1, the corresponding binary information bit u_i has high probability to be 1; when z_i is close to -1, u_i is likely to be -1. Therefore, in error prone wireless networks, z_i can be seen as a generalization of ± 1 in the continuous valued information transmission. To emphasis this generalization, we name z_i as *soft bit*, and explicitly name ± 1 as *hard bit.*

At the relay node, the decode-and-forward strategy tries to recover the original bits with an LDPC decoder. Although this process can correct a certain number of errors, it cannot correct all the errors particularly when the channel suffers deep fading. Recoding operation will likely propagate these newly introduced errors into relay's output symbols. This may significantly decrease the probability of successful decoding at the receiver.

CNC adopts a different scheme: CNC attempts to retain the information carried by y_i, and relay the information to the destination. Since the destination has another coded packet received from an independent channel, it is more likely that the destination can recover the information bits.

The important problem is how to encode two bit sequences without knowing the exact value of each bit. We propose a novel definition of XOR over posterior probability to solve this problem.

Theorem 1: For a sequence of k soft bits z_i, $i = 1,2...k$, which are calculated from received symbols y_i using (2.3). Let $u = u_1 \oplus u_2 \oplus \cdots \oplus u_k$. Then, the posterior probability $\Pr\{u = 1 \mid y_1,...,y_k\}$ can be calculated as follows:

$$\Pr\{u = 1 \mid y_1,...,y_k\} = \frac{1+\prod_{i=1}^{k} z_i}{2} \tag{3}$$

The proof of Theorem 1 is straightforward using (2.1) and induction on k.

We then define XOR over posterior probability as:

$$z = \oplus_{i=1}^{k} z_i := 2\Pr\{u = 1 \mid y_1,...,y_k\} - 1 \tag{4.1}$$

Using Theorem 1, we have:

$$z = \prod_{i=1}^{k} z_i \tag{4.2}$$

Note that when channel approaches to lossless, i.e. $u_i = y_i$, then:

$$z_i = \lim_{\sigma \to 0^+} \tanh\left(\frac{\sqrt{E_s}}{\sigma^2} y_i\right) = \lim_{\sigma \to 0^+} \tanh\left(\frac{\sqrt{E_s}}{\sigma^2} u_i\right) = u_i$$

$$\longrightarrow z = \prod_{i=1}^{k} z_i = \prod_{i=1}^{k} u_i = \oplus_{i=1}^{k} u_i \tag{5}$$

Equation (5) illustrates that traditional channel-network coding can be considered as a special case in the unified framework of CNC.

B. CNC Framework

Fig. 3 presents the block diagram of CNC framework. B_{Si} ($i=1,2$) are the original information bits from source S_i. U_{Si} is the corresponding coded bits after LDPC encoder. Then, the modulated signal X_{Si} is sent out and is received by both relay R and destination D . The received signals at R and D are denoted Y_{SiR} and Y_{SiD} respectively.

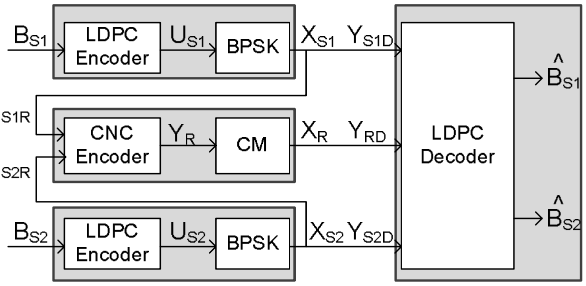


Figure 3. Block diagram of CNC framework.

The relay receives signals from both sources and generates network coded message Y_R using the CNC encoder. The CNC encoder (Fig. 4) is composed of a soft-in soft-out LDPC decoder and a network encoder. The network encoder is similar to standard LDPC encoder except that the XOR operation is defined over *soft bits* as explained in the above subsection. Here, we generate additional parity check bits on the concatenated messages of Y'_{S1R} and Y'_{S2R} , where Y'_{S1R} and Y'_{S2R} are the decoded posterior probability (mapped to [-1, 1] using (2.1)) of the received packets Y_{S1R} and Y_{S2R} . Comparing with the bit-by-bit XOR operation, this scheme can better exploit the capability of network coding.

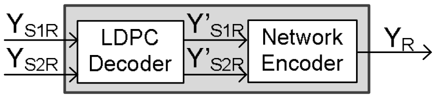


Figure 4. Block diagram of CNC Encoder.

The elements of Y_R are continuous values in the range of [-1, 1] and cannot be modulated by BPSK. Hence, we choose to use analog phase modulation to transmit these continuous values. We name this modulation mechanism as continuous modulation (CM).

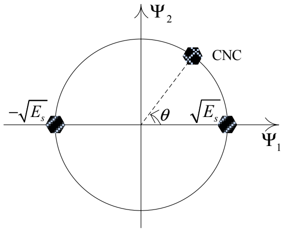


Figure 5. Constellation of CM

Fig. 5 shows the constellation of CM. In this figure, Ψ1, Ψ2 are the two orthogonal base signals in QPSK. The phase θ is determined by the following equation, in which y is an element of Y_R :

$$\theta = \frac{1-y}{2}\pi \tag{5}$$

The destination receives three signals: Y_{S1D} , Y_{S2D} and Y_{RD} . It passes them to LDPC decoder in order to obtain the original codeword of the two sources. The LDPC decoder needs to know each bit's LLR to start its iterative decoding process. It is straight forward to obtain the LLR for bits in Y_{S1D} , Y_{S2D} . For bits in Y_{RD} , their LLR can be expressed as:

$$\begin{aligned} LLR &= \log\frac{\Pr\{u=1 \mid y_{rd}\}}{\Pr\{u=-1 \mid y_{rd}\}} \\ &= \log\frac{\int_{-1}^{1} p(u=1 \mid y_r)p(y_r \mid y_{rd})p(y_r)dy_r}{\int_{-1}^{1} p(u=-1 \mid y_r)p(y_r \mid y_{rd})p(y_r)dy_r} \\ &= \log\frac{\int_{-1}^{1} (1+y_r)p(y_r \mid y_{rd})p(y_r)dy_r}{\int_{-1}^{1} (1-y_r)p(y_r \mid y_{rd})p(y_r)dy_r} \end{aligned} \tag{6}$$

(6) can be reduced by Gaussian approximation. From [3], we know that Y_R can be modeled using Gaussian approximation as:

$$y_r = \tanh((hu+w)/2) \tag{7}$$

where w is zero mean Gaussian distributed variable with variance σ^2 and $h=\sigma^2/2$. Using this model, (6) can be expressed as:

$$\log\frac{\int_{-1}^{1}\exp(-\frac{(h_{rd}y_r-y_{rd})^2}{2\sigma_{rd}^2})\frac{\exp(-(2\tanh^{-1}(y_r)-\sigma^2/2)^2/2\sigma^2)}{1-y_r}dy_r}{\int_{-1}^{1}\exp(-\frac{(h_{rd}y_r-y_{rd})^2}{2\sigma_{rd}^2})\frac{\exp(-(2\tanh^{-1}(y_r)+\sigma^2/2)^2/2\sigma^2)}{1+y_r}dy_r} \tag{8}$$

where h_{rd} and σ_{rd}^2 are equivalent fading parameter and noise variance seeing from Y_R at the relay to destination. The above expression with integration is too complex for a decoder to implement. To reduce computation complexity, we approximate it using a linear model. It is true that using more accurate non-linear model will improve the decoder's performance. However, we find that linear model is adequate and works well.

Once LLR is obtained, the destination executes the standard LDPC decoding process, and solves B_{S1} , B_{S2} .

C. Theoretical Analysis

In this section, we analyze the performance of CNC and compare it with two traditional network coding (TNC) schemes – TNC-DF and TNC-HRD. We show that CNC constantly outperforms these two relay strategies.

First, let us define TNC-DF and TNC-HRD:

1) TNC-DF: DF is short for decode-recode-forward. Specifically, relay node first decodes its received messages and then perform traditional network coding.

2) TNC-HRD: HRD stands for hard decision. Relay node does not decode its received message. It simply maps the received message into 0-1 stream via direct detection, i.e. for BPSK modulation, if the received signal is '+', it is mapped to +1, otherwise, it is mapped to -1. Then discrete network coding is done on ± 1.

Using the results in [18], the pairwise error probability (PEP) at the receiver after decoding can be expressed as:

$$\begin{aligned}&P(d = d_1 + d_2 + d_3 \mid h_{S1,D}, h_{S2,D}, h_{R,D}) \\ &\approx Q(\sqrt{2d_1 SNR_{S1,D} + 2d_2 SNR_{S2,D} + 2d_3 SNR_{R,D}})\end{aligned} \tag{9}$$

where d_1 and d_2 are codes' minimum distance of S_1 and S_2. d_3 is the minimum distance of the code at the relay node. $h_{S1,D}, h_{S2,D}, h_{RD}$ are channel fading parameters. $SNR_{S1,D}$, $SNR_{S2,D}$, and $SNR_{R,D}$ are the corresponding signal to noise ratios observed by the destination. In the proposed scenario, the above expression is only an approximation because y_{RD} is not strictly Gaussian random variable due to the imperfect decoding operation at the relay node. However, from the above formula, if we fix those variables that are the same for CNC, TNC-DF and TNC-HRD, we can see that the PEP is determined by $SNR_{R,D}$ — the received SNR at the receiver from the relay node. One step further, because fading coefficient h_{rd} and power spectrum density of w_{rd} are equal for the three schemes, we will have $y_{rd} = h_{rd} y_r + w_{wr}$. Then, $SNR_{R,D}$ is determined by y_r's probability distribution. Therefore, in the following analysis, we can focus on analyzing the relay node's output signal y_r's probability distribution. As explained in [3], we can assume that all-one codeword is transmitted in the following analysis. All transmitted symbols are normalized to be ± 1.

Belief propagation decoding iteratively updates each bit's confidence level (LLR or posterior probability). This suggests that we can define a new metric called weighted BER (WBER) that measures the average confidence level for incorrectly decoded bits at the relay node. We name it WBER because it coincides with the definition of BER for TNC-DF and TNC-HRD cases.

Definition: Weighted Bit Error Rate (WBER).

Assume that the message map have probability density function $p(x)$, x is either discrete or continuous. For discrete x, $p(x)$ is expressed with delta function. As we have assumed that all transmitted symbols from the source are +1, we define WBER as:

$$WBER = \frac{\int_{-\infty}^{0} |x| p(x) dx}{\int_{-\infty}^{+\infty} |x| p(x) dx} \tag{10}$$

The denominator is used for normalization. From the above definition, we can see that when x is binary value from ± 1, we have:

$$\begin{aligned}WBER &= \frac{\int_{-\infty}^{0} |x| (p_{-1}\delta(x+1) + p_1 \delta(x-1)) dx}{\int_{-\infty}^{+\infty} |x| (p_{-1}\delta(x+1) + p_1 \delta(x-1)) dx} \\ &= \frac{p_{-1}}{p_{-1} + p_1} = p_{-1} = BER\end{aligned} \tag{11}$$

This means that WBER is compatible with the definition of BER when dealing with discrete symbol. In the following discussion, we use WBER to compare the performance of CNC, TNC-DF and TNC-HRD.

We first discuss the proposed CNC scheme. At the relay node, CNC operation can be expressed as follows:

$$\mathbf{y}_R = [\mathbf{y}'_{S1R} \quad \mathbf{y}'_{S2R}]\underline{\mathbf{G}} \tag{12.1}$$

where $\underline{\mathbf{G}}$ is the generation matrix. Note that although the LDPC code's parity check matrix is sparse, the code generation matrix $\underline{\mathbf{G}}$ is generally dense. A dense generator matrix will diminish the benefits achieved by soft information relaying. In implementation, there are some options to mitigate this problem. For example, we can adopt a special ensemble of LDPC code, called low-density generator matrix (LDGM) codes [22] whose generator matrix $\underline{\mathbf{G}}$ is also sparse matrix. The row vectors $\mathbf{y}'_{SiR}$, *i*=1, 2, is the posterior probability after LDPC decoding at the relay node (mapped to [-1, 1] using (2.1)). Matrix multiplication operation in (12.1) is performed using (4.1). Then, network coding operation can be described as:

$$x_i = \oplus_{j=1}^{n_i} y_j \tag{12.2}$$

where n_i is the total number of 1's in the i^th^ column of $\underline{\mathbf{G}}$, y_j is a scalar value coming from either $\mathbf{y}'_{S1R}$ or $\mathbf{y}'_{S2R}$, depending on the corresponding bit's position in $\underline{\mathbf{G}}$. x_i's probability distribution can be calculate using density evolution. Here, we simplify this calculation using Gaussian approximation which was first introduced in [3] as an effective technique to approximate messages' distribution passing between check nodes and variable nodes for belief propagation decoder. From [3], we assume that the LDPC decoder's output LLR value is Gaussian distributed. At the relay node, let the LLR value corresponding to y_j be m_j, then $y_j = \tanh(m_j/2)$ and we model m_j as follows:

$$m_j \sim N\left(\frac{2}{\sigma_j^2}, \frac{4}{\sigma_j^2}\right) = N(\overline{m}_j, 2\overline{m}_j) \tag{12.3}$$

k=1 or 2. (12.2) can be expressed as:

$$x_i = \prod_{j=1}^{n_i} \tanh(\frac{m_j}{2}) \qquad (12.4)$$

To apply Gaussian approximation for x_i, we introduce an auxiliary variable 'u_i' defined as follows:

$$u_i \sim N(\bar{u}_i, 2\bar{u}_i) \qquad (12.5)$$

Then we have:

$$\tanh(\frac{u_i}{2}) = \prod_{j=1}^{n_i} \tanh(\frac{m_j}{2}) \qquad (12.6)$$

By taking expectation on each side of (12.5), we have:

$$\begin{aligned} E[\tanh(\frac{u_i}{2})] &= E[\prod_{j=1}^{n_i} \tanh(\frac{m_j}{2})] = \prod_{j=1}^{n_i} E[\tanh(\frac{m_j}{2})] \\ &\cong (E[\tanh(\frac{m_{S1}}{2})]E[\tanh(\frac{m_{S2}}{2})])^{\bar{n}/2} \end{aligned} \qquad (12.7)$$

In the last step of derivation above, we assume that, on average, there are $\bar{n}$ elements that are equal to 1 in each column of $\underline{\mathbf{G}}$ and these 1's are uniformly distributed. Then, as in [3], we can find $\bar{u}_i$ by solving the following equation:

$$E[\tanh(\frac{u_i}{2})] = \frac{1}{\sqrt{4\pi\bar{u}_i}} \int_{-\infty}^{+\infty} \tanh(\frac{u_i}{2}) e^{-\frac{(u_i-\bar{u}_i)^2}{4\bar{u}_i}} du_i \qquad (12.8)$$

With $\bar{u}_i$, output signal's WBER after CNC can be calculated as: (Assume all +1 codeword is transmitted.)

$$WBER_{CNC} = \frac{\int_{-\infty}^{0} \left|\tanh(\frac{u_i}{2})\right| e^{-\frac{(u_i-\bar{u}_i)^2}{4\bar{u}_i}} du_i}{\int_{-\infty}^{+\infty} \left|\tanh(\frac{u_i}{2})\right| e^{-\frac{(u_i-\bar{u}_i)^2}{4\bar{u}_i}} du_i} \qquad (12.9)$$

For TNC-DF, using Gaussian approximation and with similar derivation, we can get:

$$WBER_{DF} = \frac{1}{\sqrt{4\pi\bar{u}_i}} \int_{-\infty}^{0} e^{-\frac{(u_i-\bar{u}_i)^2}{4\bar{u}_i}} du_i \qquad (13)$$

Now, we show how to calculate output SNR at the relay node under TNC-HRD scheme. Using BPSK modulation, BER after hard decision can be expressed as [18]:

$$p_e = Q(\sqrt{SNR}) \qquad (14.1)$$

where SNR is the input signal to noise ratio. For DNC-HRD, network coding operation can be expressed as:

$$\mathbf{x} = [\hat{\mathbf{y}}_{S1} \quad \hat{\mathbf{y}}_{S2}]\underline{\mathbf{G}} \qquad (14.2)$$

where $\hat{y}_{Si} \in \{\pm 1\}$, i=1 or 2, is $\mathbf{y}_{Sk}$ after hard decision. Therefore, we have:

$$x_i = \prod_{j=1}^{n_i} \hat{y}_j \qquad (14.3)$$

where $\hat{y}_j$ is chosen from either $\hat{\mathbf{y}}_1$ or $\hat{\mathbf{y}}_2$, depending on the corresponding bit's position in $\underline{\mathbf{G}}$. Assume that $\hat{y}_j$'s bit error probability is $p_{e,j}$. With simple derivation on n_i, we can easily get the error probability relation between $\hat{y}_j$ and x_i as follows:

$$\begin{aligned} p_e^{HRD} &= \frac{1 - \prod_{j=1}^{n_i}(1 - 2p_{e,j})}{2} \xrightarrow{p_{e,j} \to 0} \sum_{j=1}^{n_i} p_{e,j} \\ &\cong \frac{p_{e,S1} + p_{e,S2}}{2} \bar{n} \end{aligned} \qquad (14.4)$$

The last step is derived based on the same reason as explained after (12.7). Then WBER for TNC-HRD is:

$$WBER_{HRD} = p_e^{HRD} \qquad (14.5)$$

Finally, let's discuss the relationship between the LLR value m_j after LDPC decoding and the input signal to noise ratio. Finding the exact expression for this relationship requires the knowledge of weight enumerating function (WEF) of LDPC code. In fact, even finding the minimum distance for LDPC is proved to be NP-hard [19]. So, in this research, we focus on comparing the asymptotic behavior of CNC and TNC-DF when SNR becomes high. For linear block code, the coding gain can be approximated by $10\log(R_c d_{\min})dB$ [14][18]. At the relay node, the code rate R_c is explicitly contained in SNR_{in} and SNR_{out}, the decoder's input/output SNR can be described as:

$$SNR_{out} \approx SNR_{in} d_{\min} \qquad (15.1)$$

Using (12.3), we have

$$\bar{m}_j \approx 2 SNR_{in} d_{\min} \qquad (15.2)$$

TABLE I. Parameters Definition.

N	Information block-length. N=4000
αN	Average number of '1' in $\underline{\mathbf{G}}$'s column. $\alpha N = 7$
$d_{\min}$	Code's free distance. $d_{\min}$=10

Now, we show one numerical example to visualize our analysis result. In this example, we assume that the link qualities from S1→R and S2→R are equal. Table 1 is the parameter definition used in this numerical illustration. The parameter settings are conservative because LDPC code can be designed with much larger minimum distance and the information block-length can be larger in practice. Even with these parameter settings, we can see that CNC constantly outperforms TNC-HRD and TNC-DF. We know that (15.1) is only an asymptotic approximation, not practical value. In practice, our simulation result in next section will show that under practical parameter settings and general interested SNR region, CNC performs best among the three schemes.

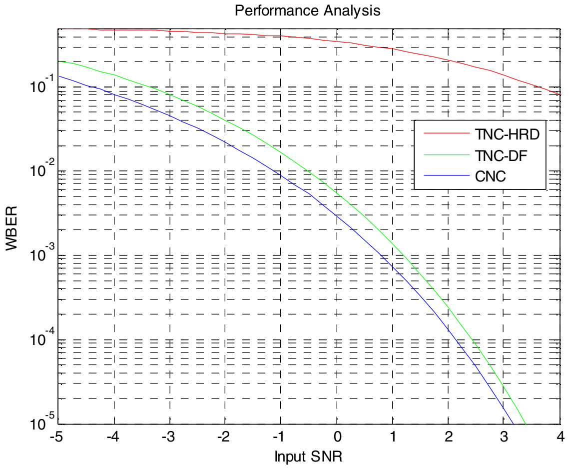


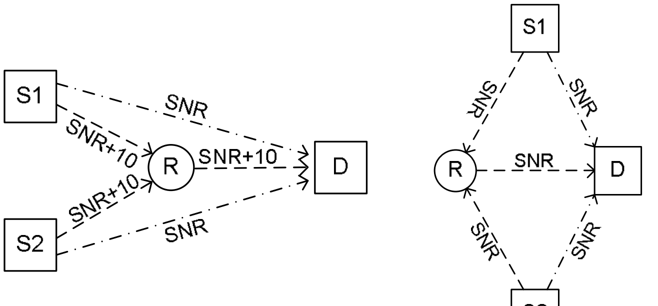
Figure 6. BER performance of the three schemes.

IV. SIMULATION EXPERIMENTS

We have analyzed in theory that performing network coding over soft bits at the relay has the potential of obtaining smaller bit error rate. In this section, we validate this conclusion through extensive simulation experiments.

A. Simulation Methodology

We conduct simulation experiments on two different configurations as shown in Fig. 7 for the general two-hop wireless relay networks shown in Fig. 1. The difference between them is the SNR settings on each link. In Fig. 7(a), the relay has better channel condition to the destination as well as to the two sources. The gain from the two-hop relaying route is apparent. In Fig. 7(b), the relay has the same channel condition to the destination as two sources.



(a) The relay has better connection to the destination than the sources

(b) All the links has the same channel condition

Figure 7. Topology settings for simulation experiments

The following experiments assume mobile relay, this assumption is true when the relay is also a mobile user and cooperation is performed between users. Although not shown in this paper due to page limit, we also conducted experiments on infrastructural relay (IR), i.e. there is a strong LOS path between relay and destination. We model this link as AWGN channel without fading (an extreme case of Rician fading). Experimental results show that we can draw similar conclusion from IR scenarios as we do from mobile relay.

In a single test run, the two sources each randomly generate one information block, which is then LDPC coded. When these coded messages are received and resolved by the destination using the proposed transmission scheme or the reference schemes, we compute the bit error rate (BER). Such test run is repeated for thousands of times for each setting and the average BER is obtained.

B. Reference Schemes

We compare our proposed CNC with three other schemes:

1) *No relay:* Communication is carried out via point to point fashion between source nodes and destination node.
2) *TNC-DF:* See C in last section.
3) *TNC-HRD:* See C in last section.

To ensure a fair comparison, we normalize bit energies to the unit value. The number of original information bits is 3000, 1500 bits from each source. The overall bits communicated in the system are 6000 bits. In the no relay simulation, each source uses (3000, 1500) LDPC code. In all other simulations that have relay node, we use (2000, 1500) LDPC code at each source and use (2000, 4000) code at the relay node. (2000, 4000) means that at the relay node, it calculates 2000 additional parity symbols from the 4000 received symbols. The system's effective code rates in all these systems are 1/2.

C. Results

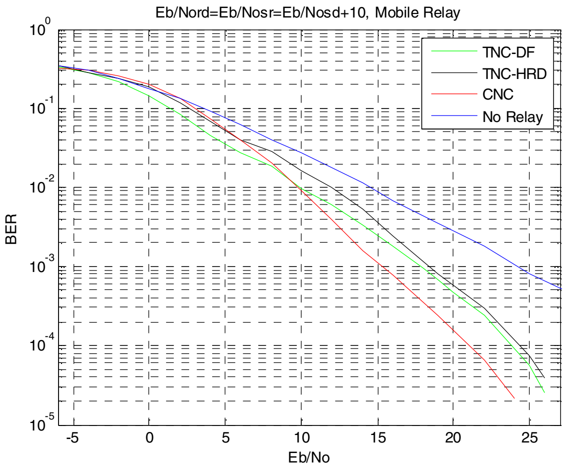


Figure 8. BER performance under setting shown in Fig. 7a

For all the results presented in this section, the x-axis is the E_b / N_0 (in dB) of link S→D and y-axis is the bit error rate (BER).

Fig. 8 presents the results when the relay has better channel condition to the destination than the two sources (as in Fig. 7a). We have the following observations:

1) Our CNC scheme is constantly better than no-relay and TNC-HRD. When the BER is 10^{-3}, our scheme outperforms TNC-HRD scheme by about 4dB and outperforms no-relay scheme much larger than 8dB.
2) CNC is better than TNC-DF when the channel condition is moderate or good, while it slightly underperforms TNC-DF in low SNR region (less than 2dB in this experiment). The reason is that, when the channel condition grows bad, the magnitude of a larger proportion of the decoded soft bits y_R is small. Recoding on these bits will make the resulting coded soft bits around zero, which means that the benefit of soft relaying is greatly diminished. Although TNC-DF scheme cannot successfully decode the whole message either, it at least provides some correct information to the decoder. However, we point out that this low SNR region is practically not important because of the poor error performance (BER>10^{-2}).

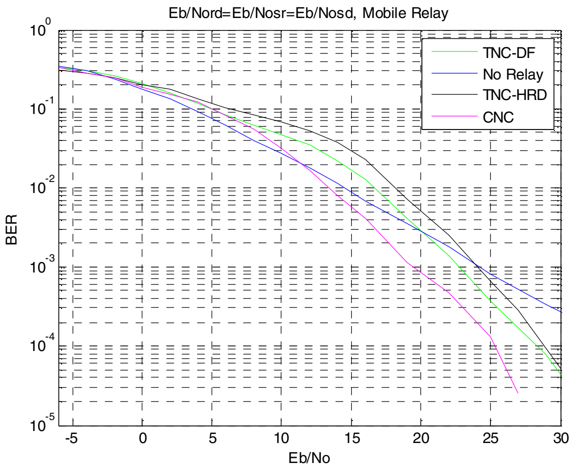


Figure 9. BER performance under setting shown in Fig. 7b

Fig. 9 presents the results when the relay has the same channel condition to the destination as the two sources (as in Fig. 7b). The results are similar to that obtained in the previous setting, except that no-relay scheme outperforms CNC in low SNR region (less than 2dB in this experiment). The reason is that, when the channel condition is bad, the received signal at the receiver from the relay is around zero, although this won't introduce error (which makes CNC better than TNC-DF and TNC-HRD), it also does not convey much additional valuable information, i.e. those energy allocated at the relay is wasted. So, CNC underperforms no relay scheme under this SNR region.

V. ADAPTIVE FRAMEWORK FOR CNC

From the experiments, we observe that on the one hand, when the direct link condition is poor, more LDPC parity bits need to be allocated to guarantee single-hop transmission. On the other, when the direct link condition is good, over protection with too many parity bits does not improve single-hop transmission quality. In this section, we propose an adaptive framework for CNC, and present our preliminary results on a simple adaptation mechanism: bit loading.

By adjusting the bits allocated to relay, the system trades off error-correction capacity between the direct link and the relay link. For clarity, we fix the SNR of S→D and R→D at 15dB, and vary the SNR of S→R from 0dB, 10dB, 20dB to 35dB. We compare three bit allocation schemes:

1) Heavy channel protected scheme: (2250, 1500) LDPC codes at the source, and 1500 parity bits at the relay;
2) Moderate channel protected scheme: (2000, 1500) LDPC codes at the source, and 2000 parity bits at the relay;
3) Light channel protected scheme: (1750, 1500) LDPC codes at the source, and 2500 parity bits at the relay.

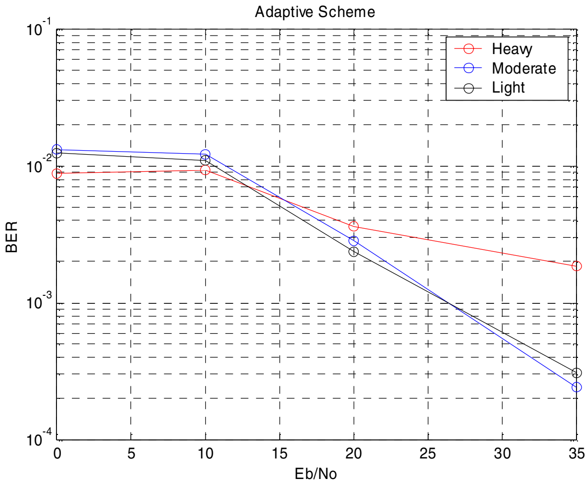


Figure 10. BER of three comparing schemes in different channel conditions

Fig. 10 shows the results. These results are consistent with our analysis:

1) When the channel condition is poor (SNR less than 12dB in this experiment), the heavy channel protected scheme outperforms the others, because it guarantee single-hop transmission.
2) When the channel condition is moderate (SNR between 12 dB and 32dB), allocating more bits to exploit space diversity achieves better performance.
3) When the channel condition is good (SNR greater than 32dB), even more bits can be allocated to the relay node to fully exploit the space diversity.

These observations provide useful hints to a system on when and how to adjust bit allocation between sources and the relay. Note that the adaptive capability profits from our flexible CNC design, which can generate arbitrary number of parity bits at the relay.

VI. CONCLUSION AND FUTURE DIRECTIONS

A. Concluding Remarks

In this research, we have developed a new framework called continuous network coding, or CNC, for wireless relay networks. CNC is fundamentally different from traditional network coding in that CNC is defined on the continuous value of posterior probability. By computing and relaying the soft information of each bit, the relay node avoids possible error propagation in the decode-recode process at relay node. This desired property results in significant performance gain over traditional network coding for wireless relay networks.

We have demonstrated this novel framework by integrating CNC into distributed LDPC coding. Such a framework takes advantage of both space and coded diversity in multi-hop transmission, and fully exploits the potential of network coding to achieve high throughput. We have also developed an adaptive CNC which allows transmitting nodes to adapt to the current channel conditions. Preliminary results on adaptive bit loading scheme are encouraging.

B. Future Directions

The proposed CNC is able to provide novel solutions to the serious problems encountered in wireless relay networks. However, possible enhancements over current scheme are currently under investigation. First, more rigorous theoretical analysis is very much desired to investigate the relationship between CNC, decode-and-forward strategy, and estimate-and-forward strategy. Second, some relaxation over ideal assumptions, e.g., imperfect estimation of channel fading, should be studied. Third, Performance analysis under other channel model is needed. We assume CNC is under quasi-static flat fading channel with BPSK modulation. We can certainly extend the CNC framework to frequency selective fading channels and non-BPSK modulation technologies. Finally, we believe that a joint design of CNC and resource allocation, including channel, code rate, transmission power and modulation constellation, will further improve the overall communication quality in wireless relay networks.

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