Landau-Zener interferometry with superconducting qubits

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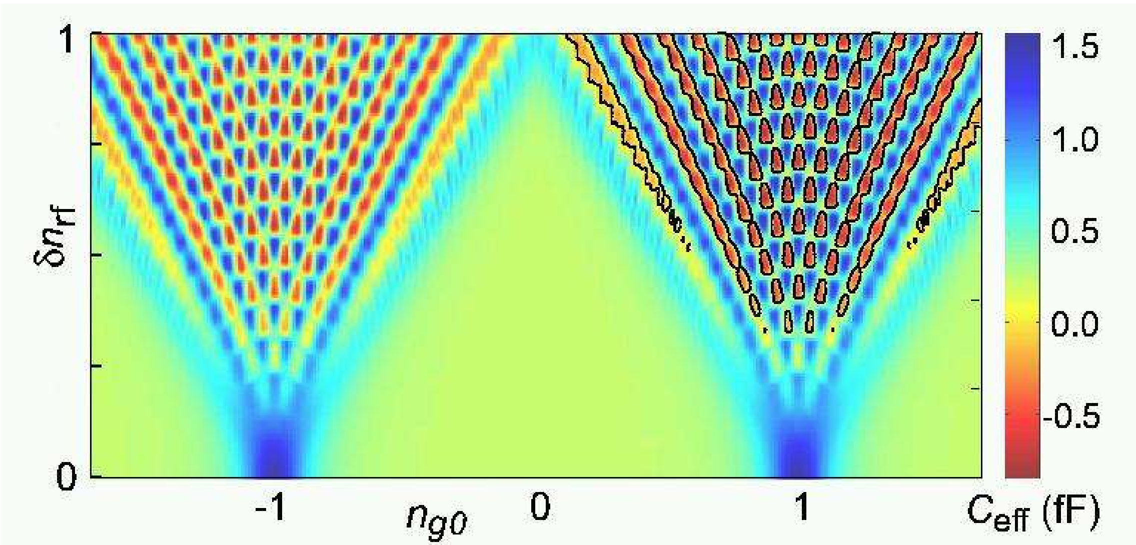


Thanks to
M. Feigel'man, R. Fazio

Outline

- Landau, Zener and others (1932)

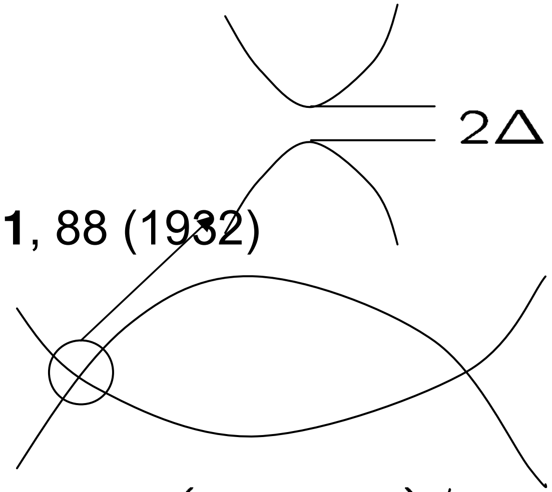
  (after Di Giacomo, Nikitin, Phys. Usp. 2005)

- superconducting qubits
- LZ interferometer
- Stokes phase
- continuous CSET quantum measurement
- data and discussion

Landau, Zener and others (1932)

atomic collisions

L. Landau, *Phys. Z. Sowjetunion* **1**, 88 (1932)



$$P = 2\zeta \equiv 4\pi \frac{\Delta^2}{v}$$

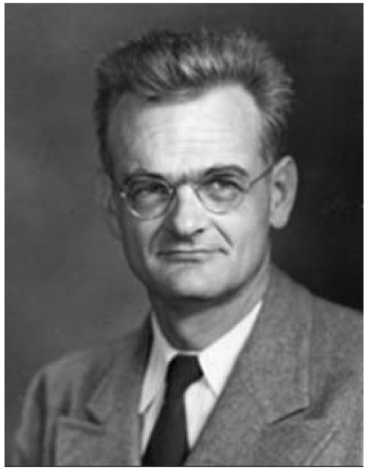
$$v = d(E_2 - E_1)/dt$$

sudden limit: $\zeta \ll 1$

L. Landau, *Phys. Z. Sowjetunion* **2**, 46 (1932)

$$P = C \exp(-\zeta)$$

adiabatic limit: $\zeta \gg 1$



C.Zener, *Proc. R. Soc. London Ser. A* **137**, 696 (1932)

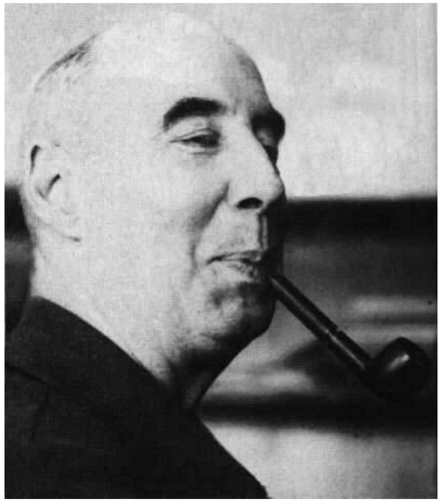
one crossing

$$P = \exp(-\zeta)$$

double crossing

$$P = 2e^{-\zeta}(1 - e^{-\zeta})$$

h vs. $\hbar$



E.C.G. Stueckelberg, *Helv. Phys. Acta* **5**, 369 (1932)

$$H(R) = T(R) + U(R)$$

semiclassical approximation, analytical continuation
of the WKB solution across *Stokes* lines

$$P = 4e^{-\zeta}(1 - e^{-\zeta}) \sin^2 \Phi_{St}$$

$\sin^2 \Phi_{St} \to 1/2$ => LZ expression

E. Majorana, *Nuovo Cimento* **9**, 43 (1932)

spin dynamics in a time-dependent magnetic field

$$H = \gamma \dot{B}_z t \hat{s}_z + \gamma B_x \hat{s}_x$$

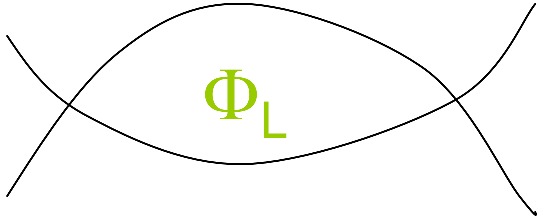
$$P = \exp\left(-\frac{\pi\gamma B_x^2}{\hbar \dot{B}_z}\right) \equiv \exp(-\zeta)$$

Phase sensitivity and interference

atomic collisions

L. Landau, *Phys. Z. Sowjetunion* **1**, 88 (1932)

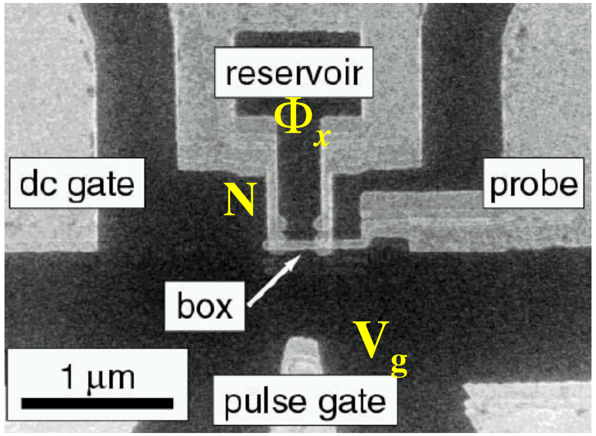
$$P_{WKB} = 4\zeta \cdot \sin^2 \Phi_L$$

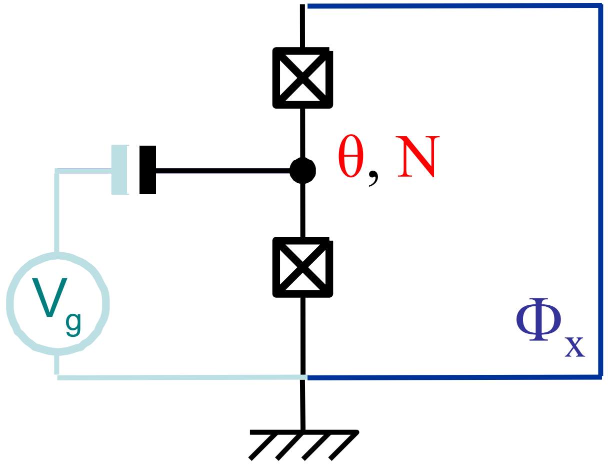


sudden limit: $\zeta \ll 1$

$$\Phi_L \gg 1 \Rightarrow \sin^2 \to 1/2$$

Josephson charge qubits





$$H = E_C \left(N - \frac{C_g V_g}{2e}\right)^2 - E_J \cos\left(\pi \frac{\Phi_x}{\Phi_0}\right) \cos\theta$$

tunable E_J

2 degrees of freedom
$[N, \theta] = -i$ charge and phase

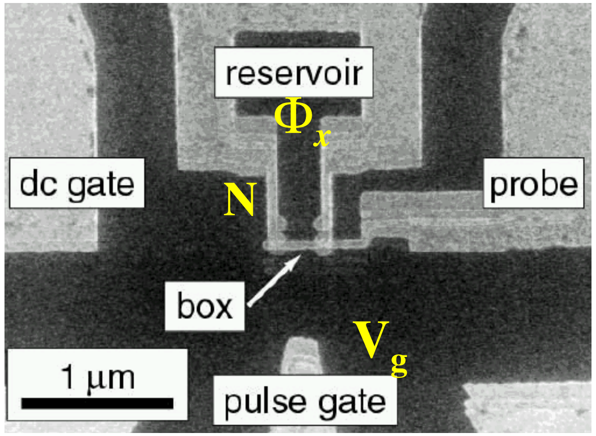
2 energy scales E_C , E_J

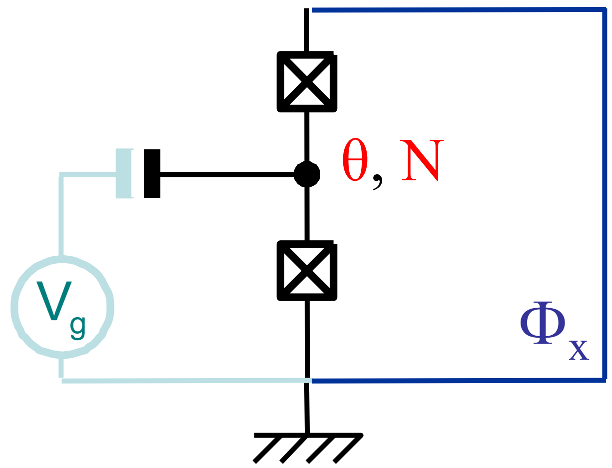
charging energy, Josephson coupling

2 control fields: V_g and Φ_x

gate voltage, flux

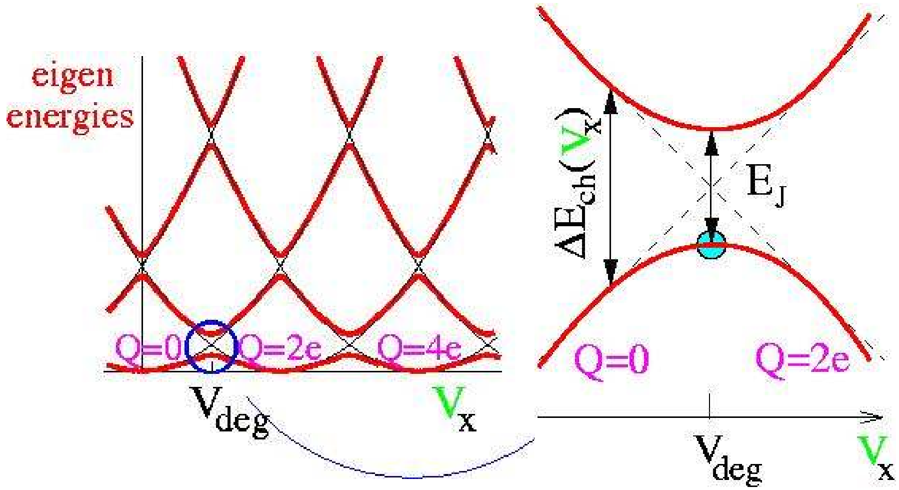
Josephson charge qubits





$$H = E_C\left(N - \frac{C_g V_g}{2e}\right)^2 - E_J \cos\left(\pi \frac{\Phi_x}{\Phi_0}\right)\cos\theta$$

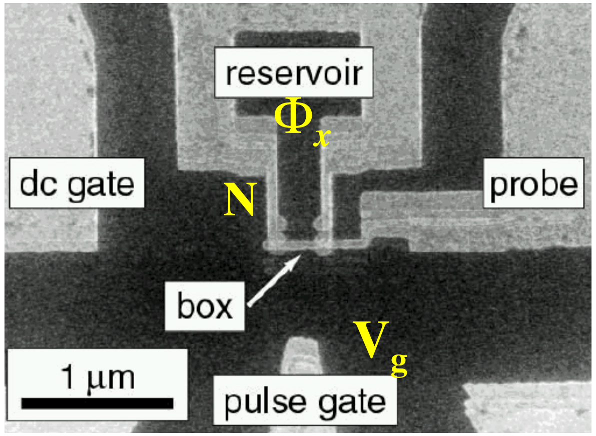
tunable E_J

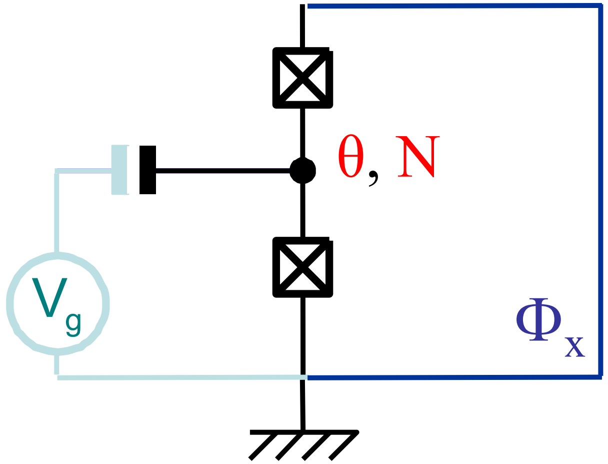


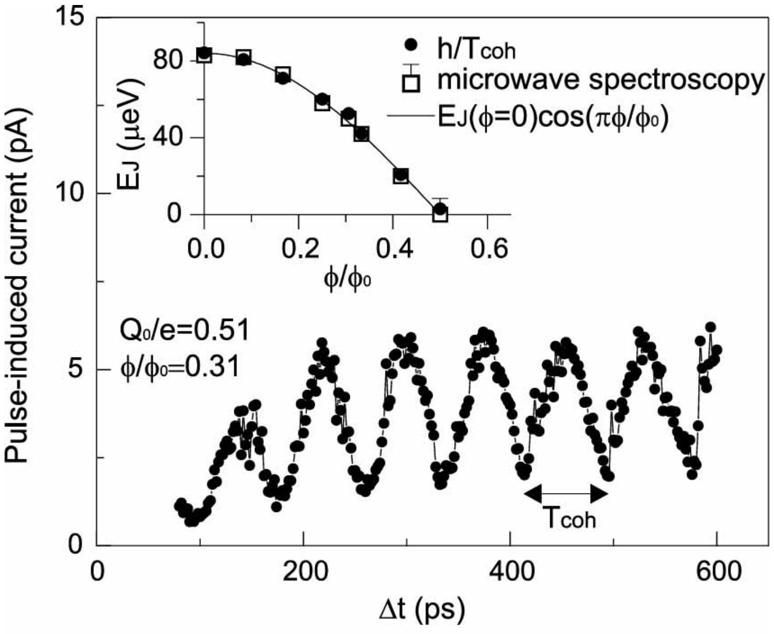
2 states only, e.g. for E_C » E_J

$$H = -\frac{1}{2}E_{ch}(V_g)\,\sigma_z - \frac{1}{2}E_J(\Phi_x)\,\sigma_x$$

Josephson charge qubits







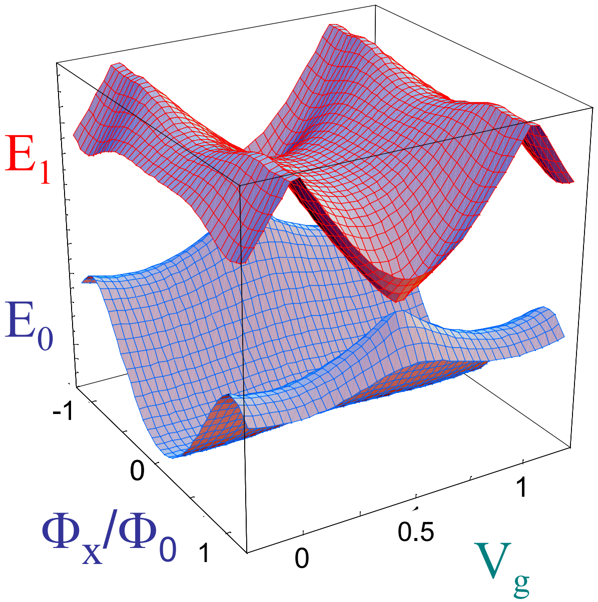
$$H = E_C\left(N - \frac{C_g V_g}{2e}\right)^2 - \underbrace{E_J \cos\left(\pi\frac{\Phi_x}{\Phi_0}\right)}_{\text{tunable } E_J} \cos\theta$$

tunable E_J

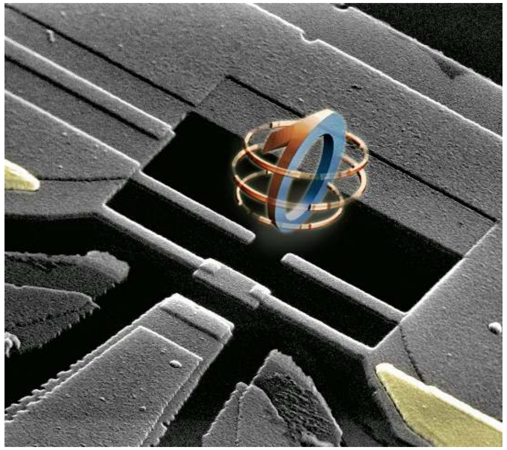
Charge-phase qubit

Vion et al. (Saclay)

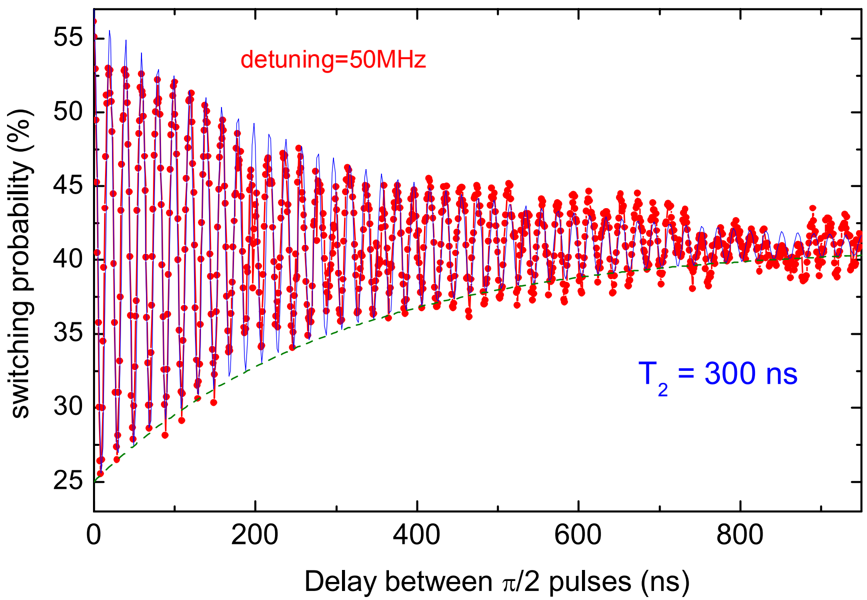
$$H = -\frac{1}{2} E_{ch}(V_g)\,\sigma_z - \frac{1}{2} E_J(\Phi_x)\,\sigma_x$$



Quantronium



operation at saddle point

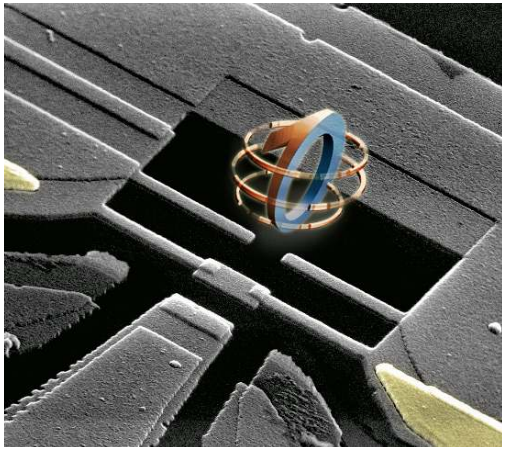


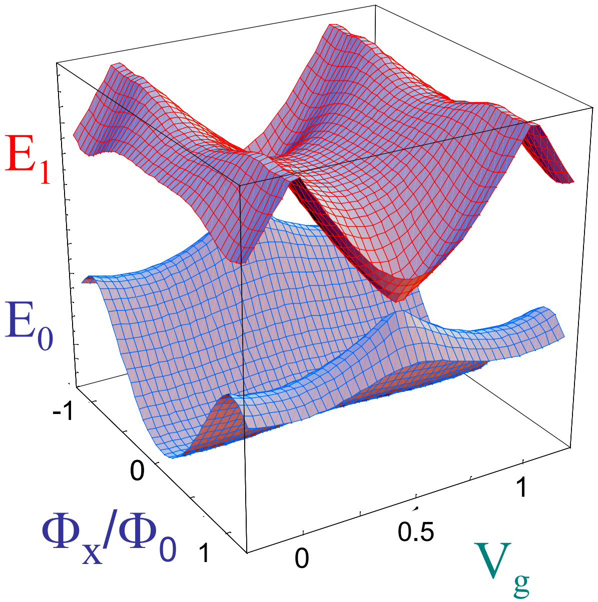
Charge-phase qubit

Vion et al. (Saclay)

$$H = -\frac{1}{2} E_{ch}(V_g)\,\sigma_z - \frac{1}{2} E_J(\Phi_x)\,\sigma_x$$

Quantronium

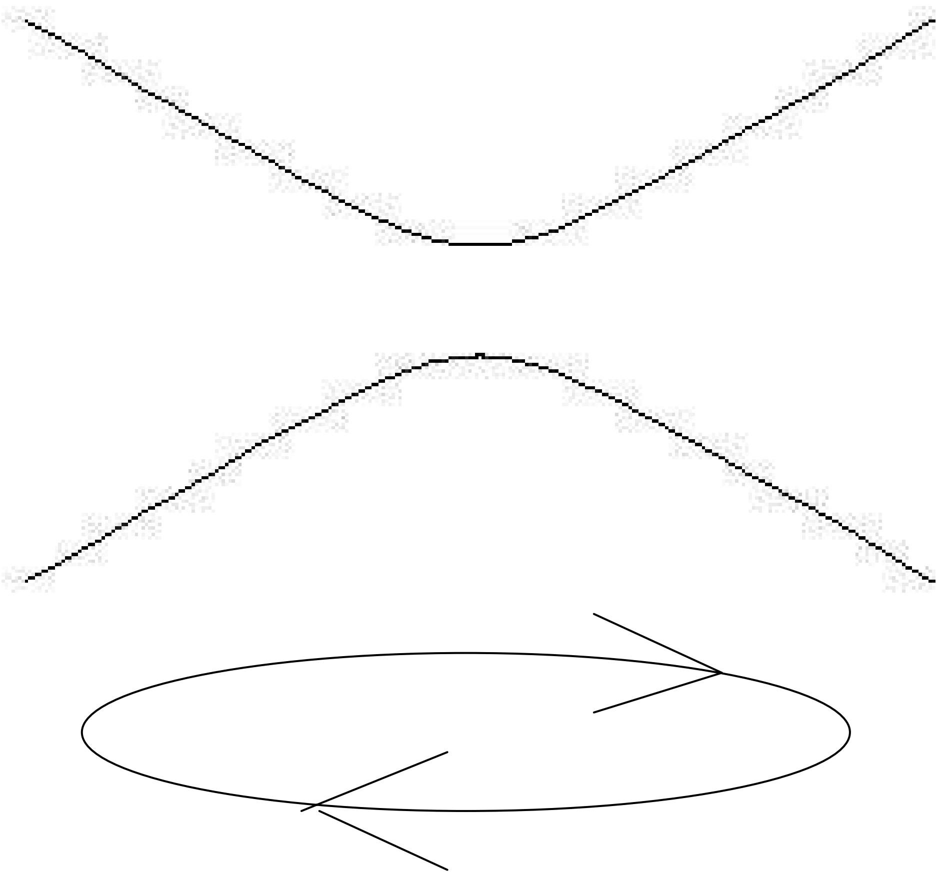




operation at saddle point

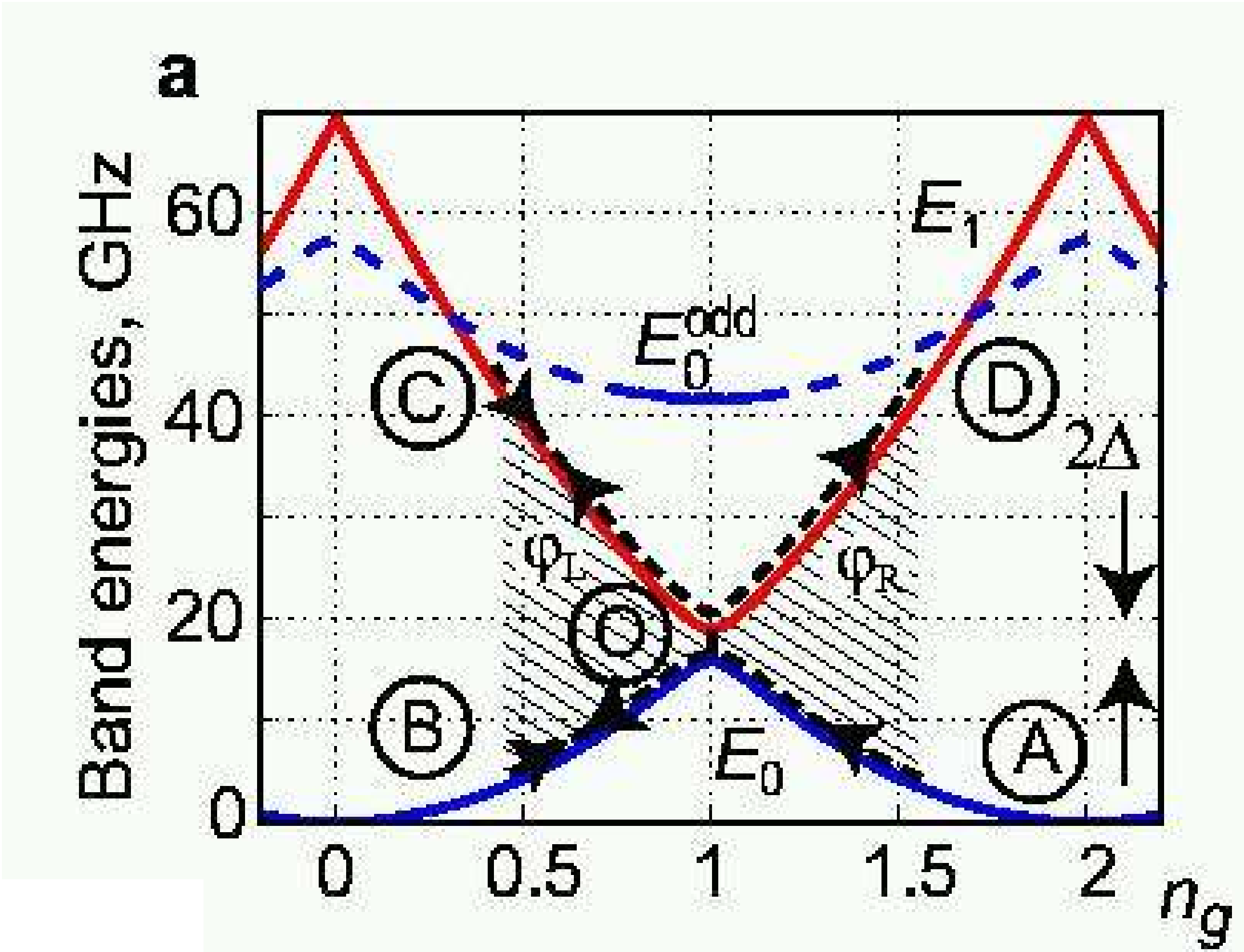
$$H = -\frac{1}{2} E_J(\Phi_{x0})\,\sigma_x - \frac{1}{2} \frac{\partial E_{ch}}{\partial V_g}\Big|_{V_{go}} \delta V_g\, \sigma_z - \frac{1}{4} \frac{\partial^2 E_J}{\partial \Phi_x^2}\Big|_{\Phi_{xo}} \delta\Phi_x^2\, \sigma_x$$

LZ interferometry



Henry, Lang `77
Lopez-Castillo et al. `92
Kayanuma 90‘s
Shytov, Ivanov, Feigelman `03

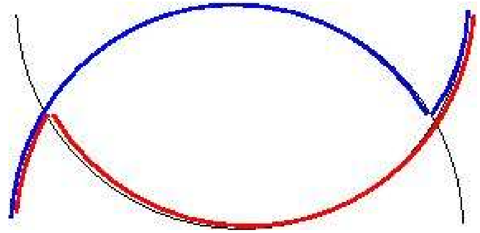
probe decoherence in sc. qubits

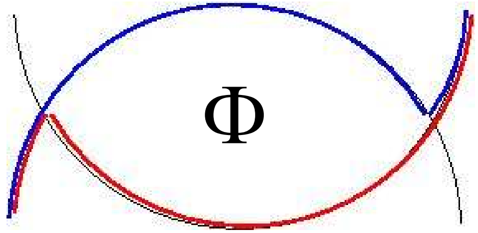


a
Band energies, GHz
0
20
40
60
0
0.5
1
1.5
2
n_g
E_1
E_0^{odd}
E_0
C
D
B
A
Q
2Δ
φ_L
φ_R

One period:

LZ: $$2P(1-P)$$





interference: $$2P(1-P)(1+\cos\Phi)$$

oscillating dependence on control parameters

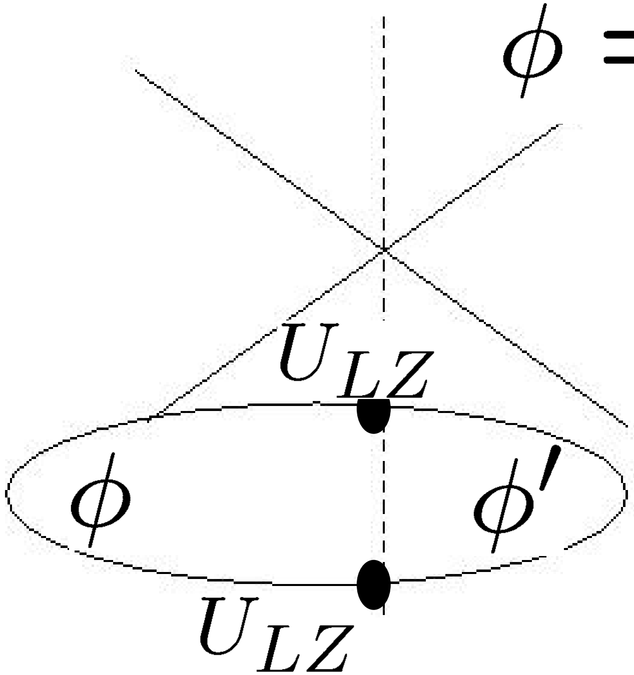
Multipass interferometer (Mach-Zehnder)

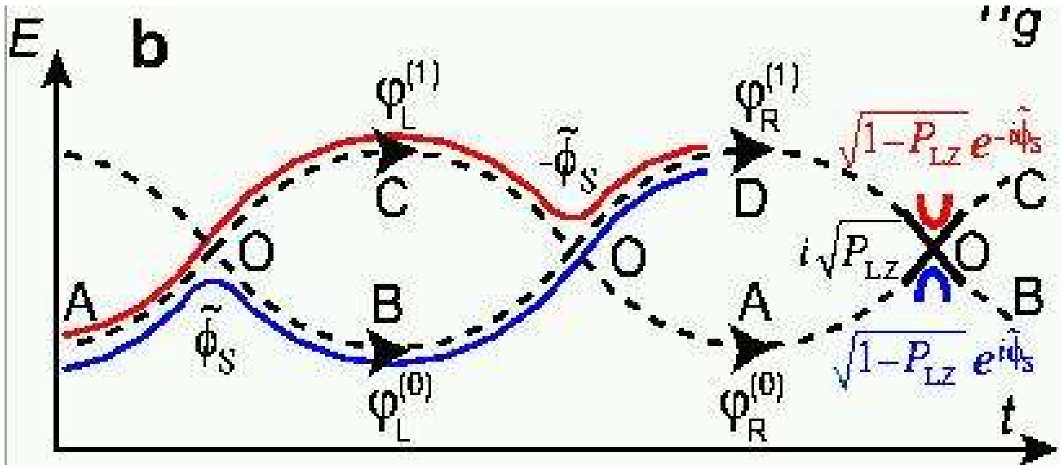
operation per period:

$$U = e^{-\frac{i}{2}\phi\hat{\sigma}_z} \cdot U_{LZ} \cdot e^{-\frac{i}{2}\phi'\hat{\sigma}_z} \cdot U_{LZ}$$

phases picked away from the crossing:

$$\phi = \frac{1}{\hbar}\int (E_1(t) - E_0(t))dt$$





in terms of spin-1/2 rotations:

Euler angles

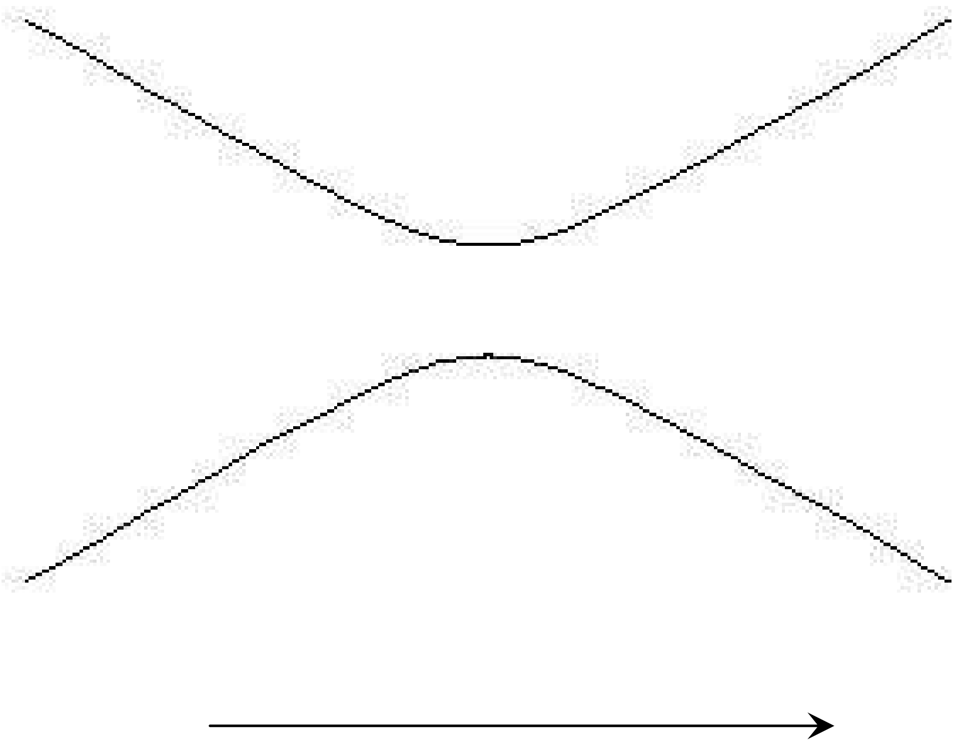
$$U_{LZ} = U_z(\chi)\ U_x(\alpha)\ U_z(\chi')$$

$$\ldots U_z(\phi)\quad \overbrace{U_z(\chi)U_x(\alpha)U_z(\chi')}\quad U_z(\phi')\times$$
$$\underbrace{U_z(\chi)U_x(\alpha)U_z(\chi')}\quad U_z(\phi)\quad \ldots$$

Stokes phase modifies ϕ, ϕ' : $\phi \longrightarrow \phi + \chi + \chi'$

$$\phi' \longrightarrow \phi' + \chi + \chi'$$

Scattering phase



phase = „dynamic“ + ...

Transformation at the crossing U_{LZ}

$$\begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix} \Rightarrow \begin{pmatrix} \sqrt{1-P_{\mathrm{LZ}}}\,e^{i\tilde{\phi}_S} & i\sqrt{P_{\mathrm{LZ}}} \\ i\sqrt{P_{\mathrm{LZ}}} & \sqrt{1-P_{\mathrm{LZ}}}\,e^{-i\tilde{\phi}_S} \end{pmatrix} \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix}$$

$$P_{LZ} = e^{-2\pi\delta}$$

adiabaticity parameter:

$$\delta = \Delta^2/v$$

Stokes phase:

$$\tilde{\phi}_S = \phi_S - \frac{\pi}{2} = -\frac{\pi}{4} + \arg\Gamma(1-i\delta) + \delta(\ln\delta - 1)$$

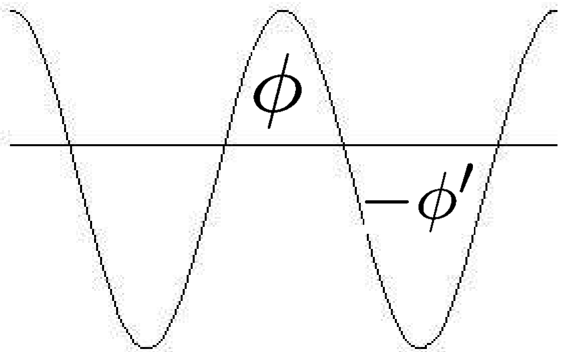
Many periods:

$$|\uparrow\rangle \rightarrow \left[U_z(\phi)U_x(\alpha)U_z(\phi')U_x(\alpha)\right]^N |\uparrow\rangle$$

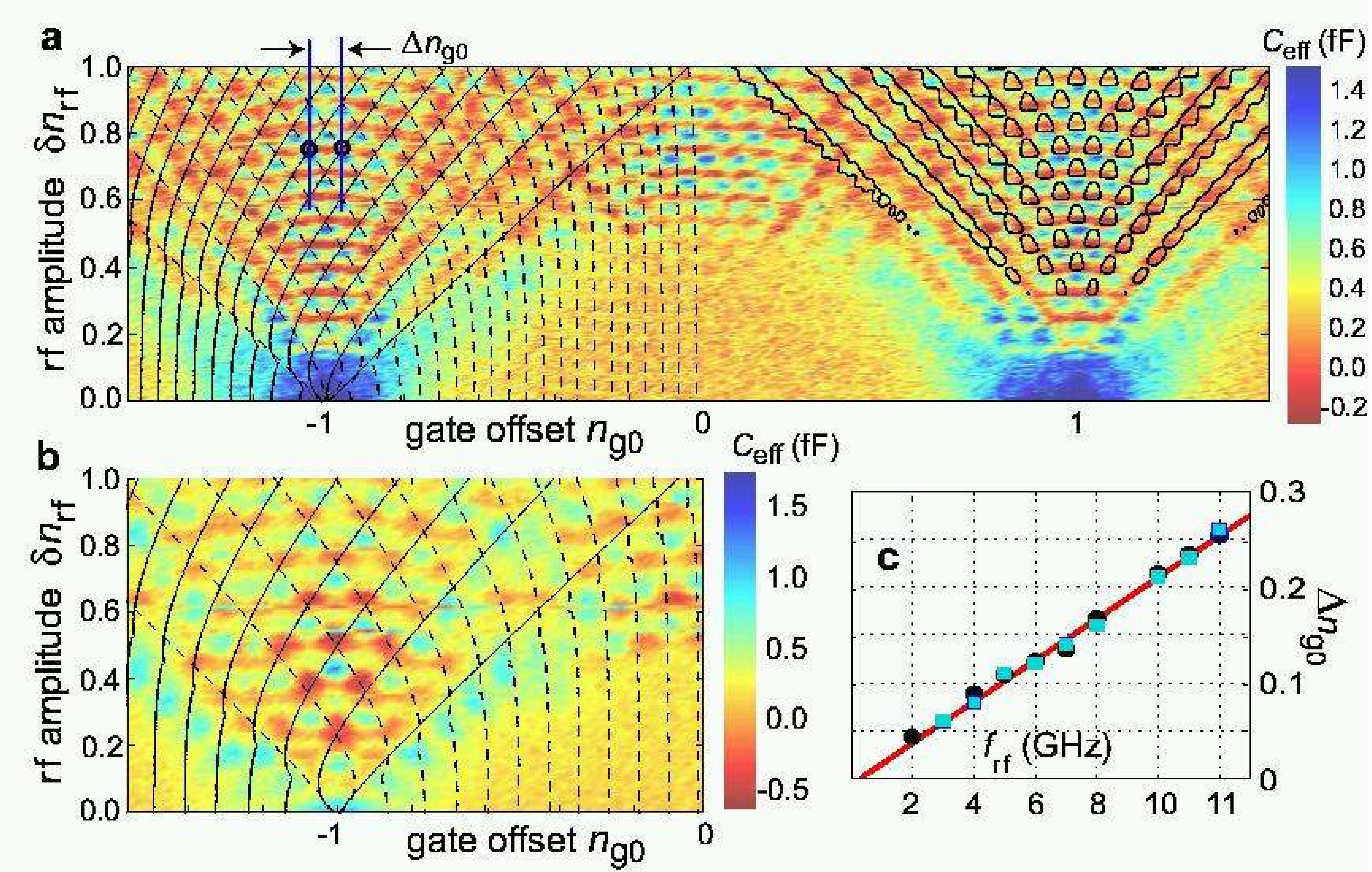
period 2π in ϕ and ϕ‘

ϕ, ϕ` =multiples of 2π – maxima

$\phi \equiv \phi' \equiv \pi \pmod{2\pi}$ - minima



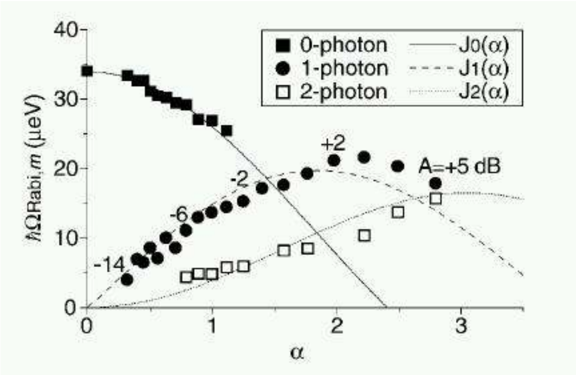
Note: $\phi - \phi' = 2\pi \frac{4E_C(1-n_{g0})}{\hbar\omega_{rf}}$ depends only on offset n_g, not rf *amplitude* !



a
Δn_{g0}
C_{eff} (fF)
rf amplitude δn_{rf}
gate offset n_{g0}
b
C_{eff} (fF)
rf amplitude δn_{rf}
gate offset n_{g0}
c
f_{rf} (GHz)
Δn_{g0}

Multiphoton resonances

$$H = \frac{1}{2}\begin{pmatrix} \varepsilon(t) & \Delta \\ \Delta & -\varepsilon(t) \end{pmatrix} \qquad \varepsilon(t) = n\omega + a\cos(\omega t)$$



in the rotating frame:

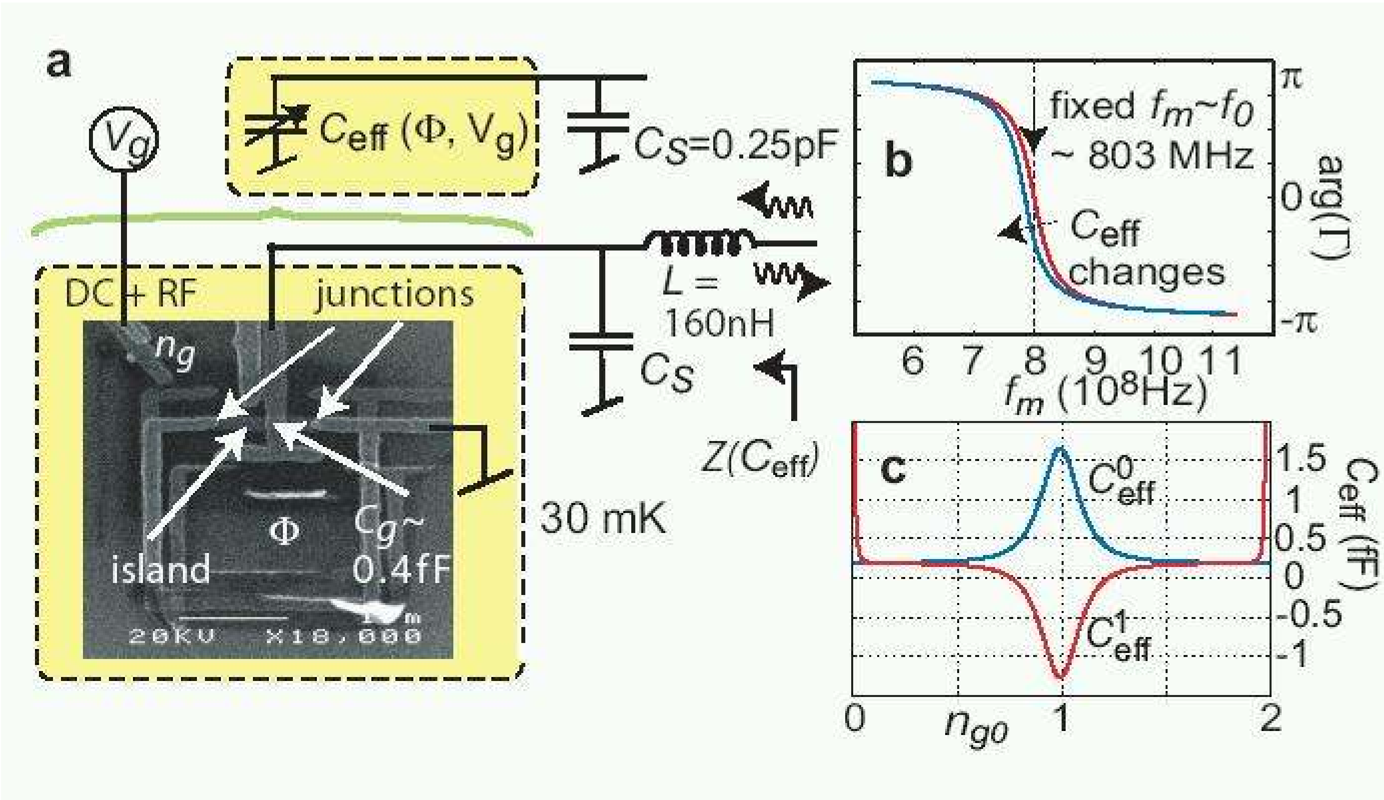
$$\tilde{\Delta} = \Delta e^{in\omega t} e^{i\frac{a}{\omega}\sin(\omega t)} \to \Delta J_n\left(\frac{a}{\omega}\right)$$

NMR

Bloch eqs. give stationary solution with population response $\propto$

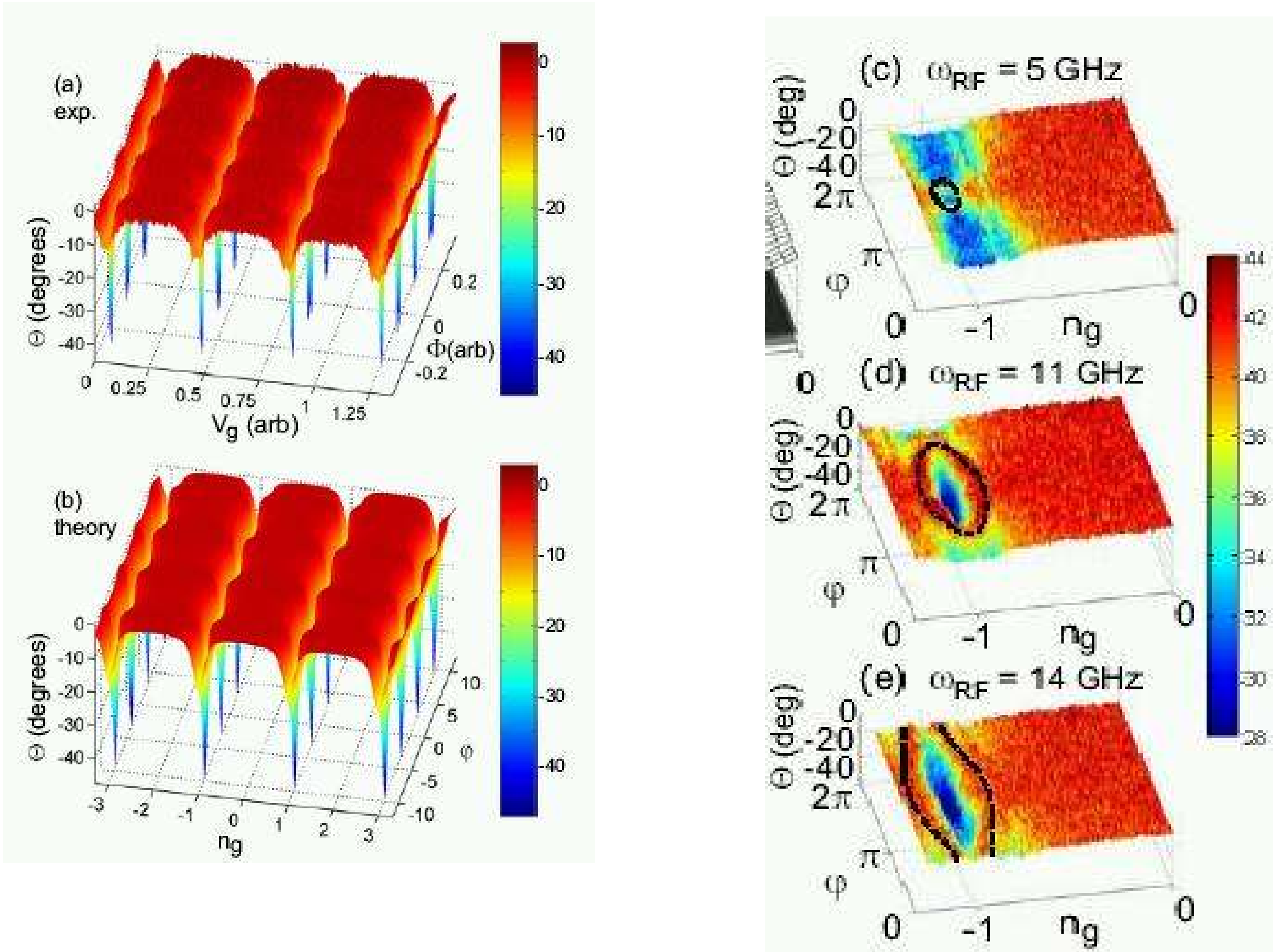
$$\frac{T_1 T_2 \tilde{\Delta}^2}{1 + T_1 T_2 \tilde{\Delta}^2 + (4E_C(n_{g0} - n_{g0}^{(n)})T_2)^2}$$

Setup and readout

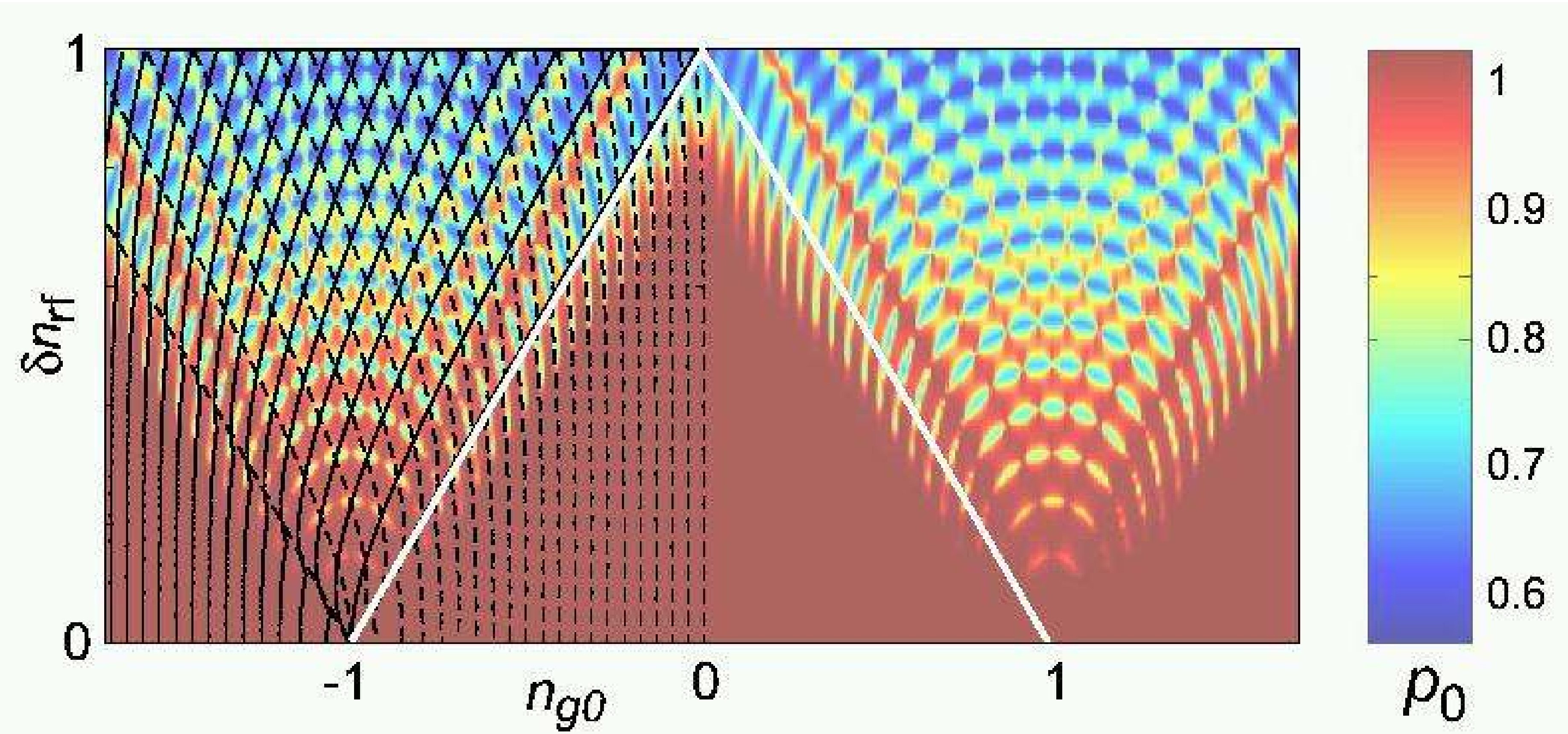


Γ - reflection coefficient

$$C_{eff} = -\frac{\partial^2 H_{qb}}{\partial V_g^2}$$


(a)
exp.
Θ (degrees)
0
-10
-20
-30
-40
0
0.25
0.5
0.75
1
1.25
V_g (arb)
Φ(arb)
0.2
0
-0.2
-10
-20
-30
-40
(b)
theory
Θ (degrees)
-3
-2
-1
0
1
2
3
n_g
φ
10
5
0
-5
-10
(c) ω_RF = 5 GHz
Θ (deg)
0
-20
-40
2π
π
φ
0
-1
n_g
0
(d) ω_RF = 11 GHz
(e) ω_RF = 14 GHz
44
42
40
38
36
34
32
30
28

Population response



What is monitored?

$$C_{eff} = \frac{\partial^2}{\partial V_g^2}(p_0 E_0 + p_1 E_1)$$

Read-out frequency f_m >> relax. rate: too weak

$$C_{eff} = p_0 C_0 + p_1 C_1$$

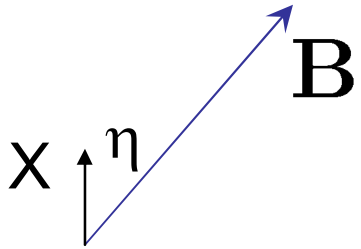
Fast relaxation => account for $\partial^2 p(V_g)/\partial V_g^2$
(too strong response)

In practice, they are comparable: f_m ~ relax. rate

Simulations of the Bloch equations

$$\frac{d}{dt}\mathbf{M} = -\mathbf{B} \times \mathbf{M} - \frac{1}{T_1}(\mathbf{M}_{||} - \mathbf{M}_{||}^{eq}) - \frac{1}{T_2}\mathbf{M}_{\perp}$$

$$\frac{1}{T_1} = \frac{\sin^2\eta}{2\hbar^2} S_X(\omega = \Delta E/\hbar)$$

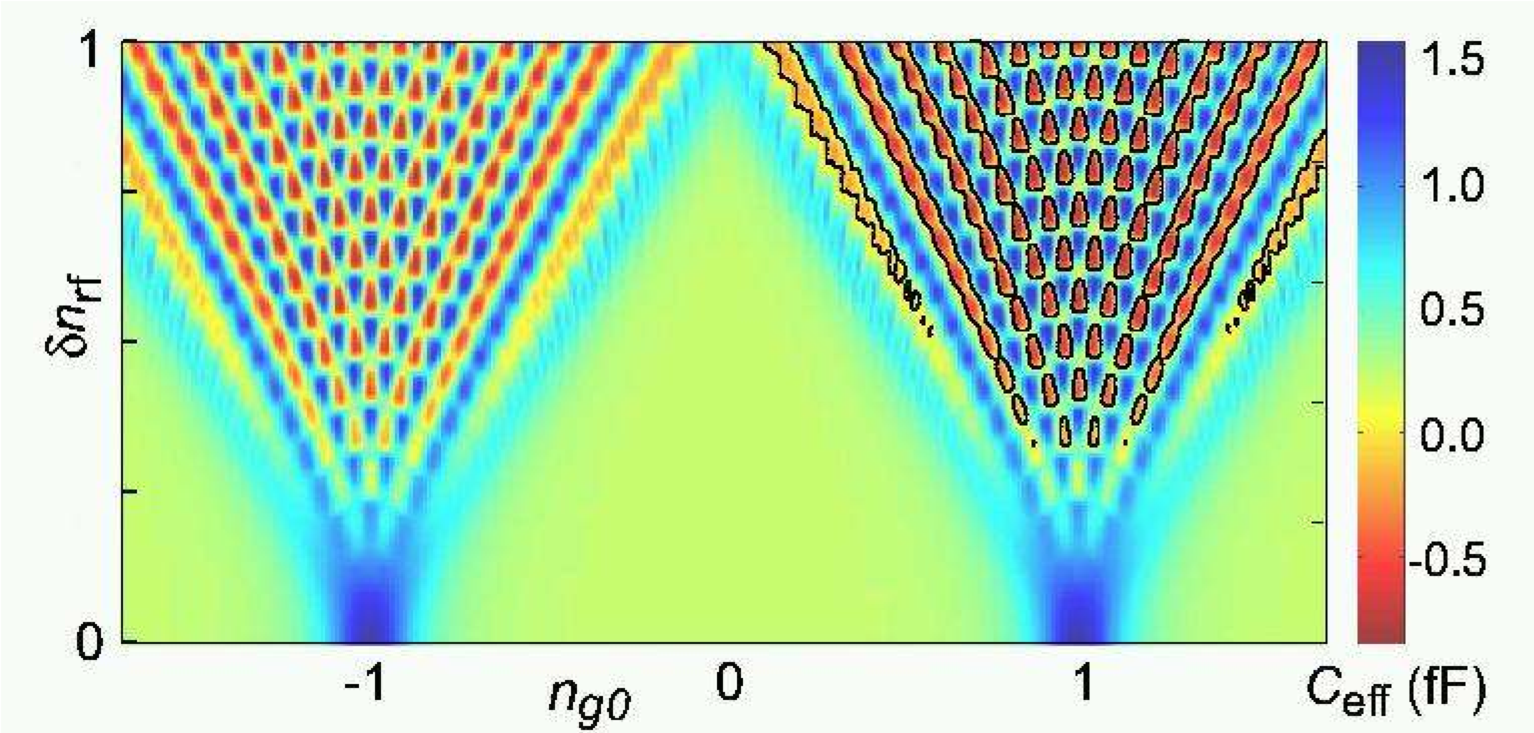


$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{\cos^2\eta}{2\hbar^2} S_X(\omega = 0)$$

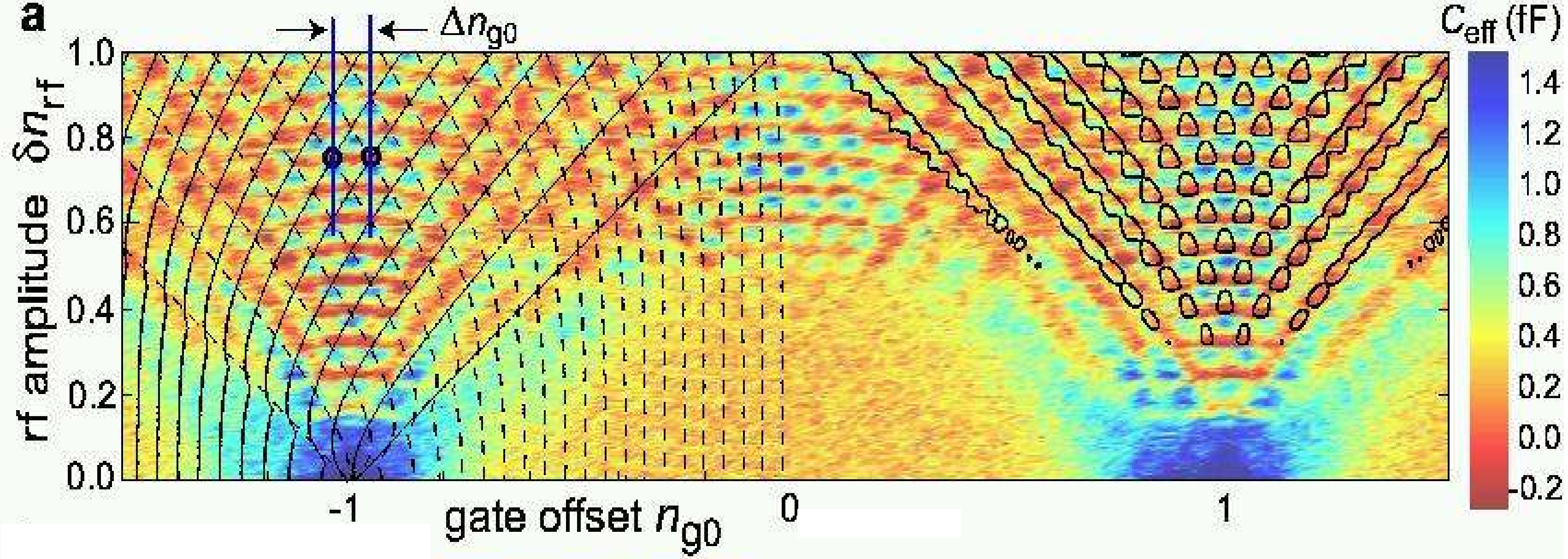
Applicable? More or less

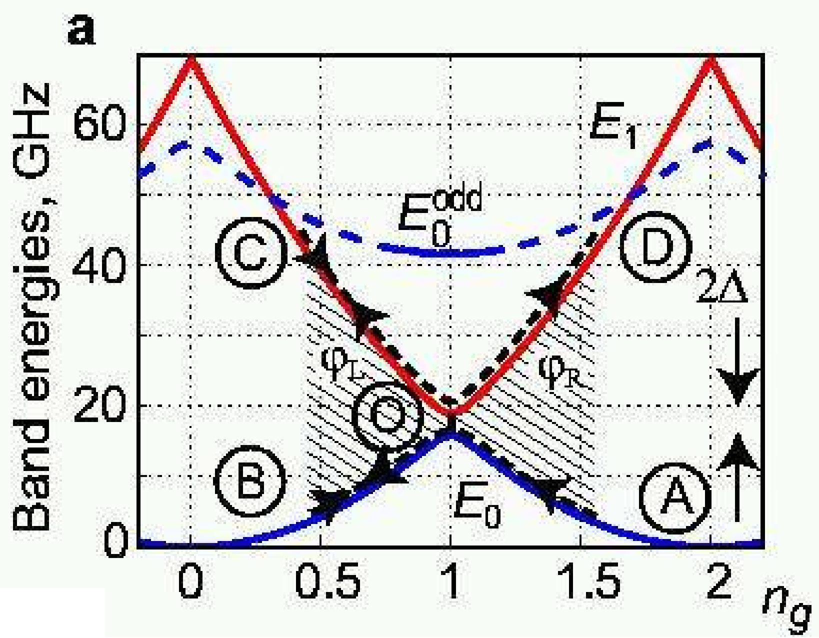
Bloch eqs. in discrete time
(cf. Shytov et al.)

Simulations of the Bloch equations

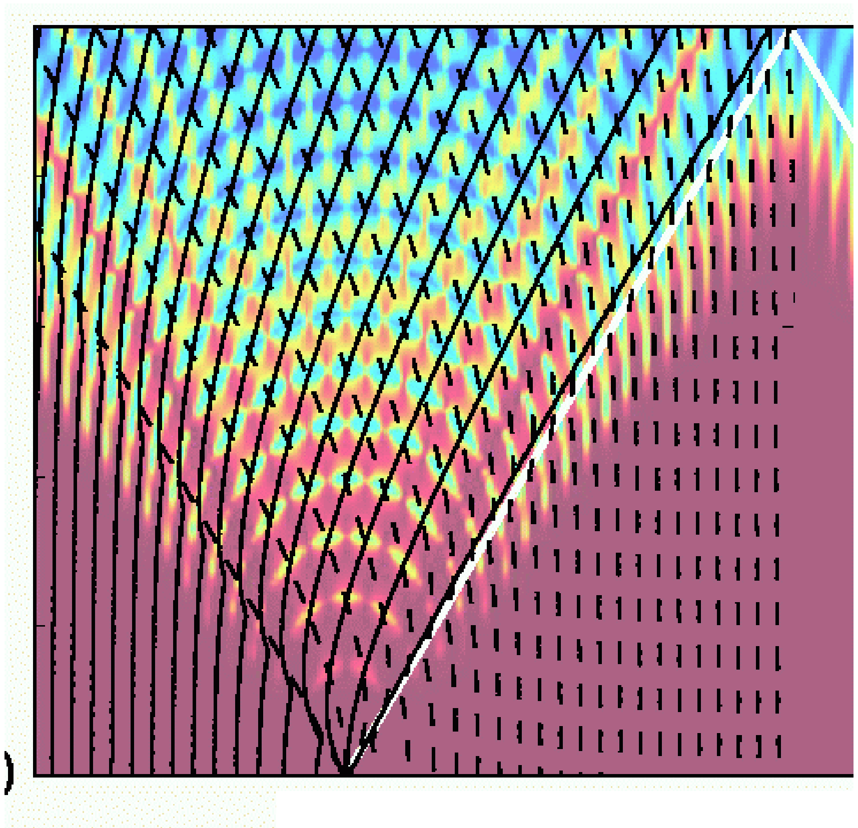


Stronger drive: quasiparticle states

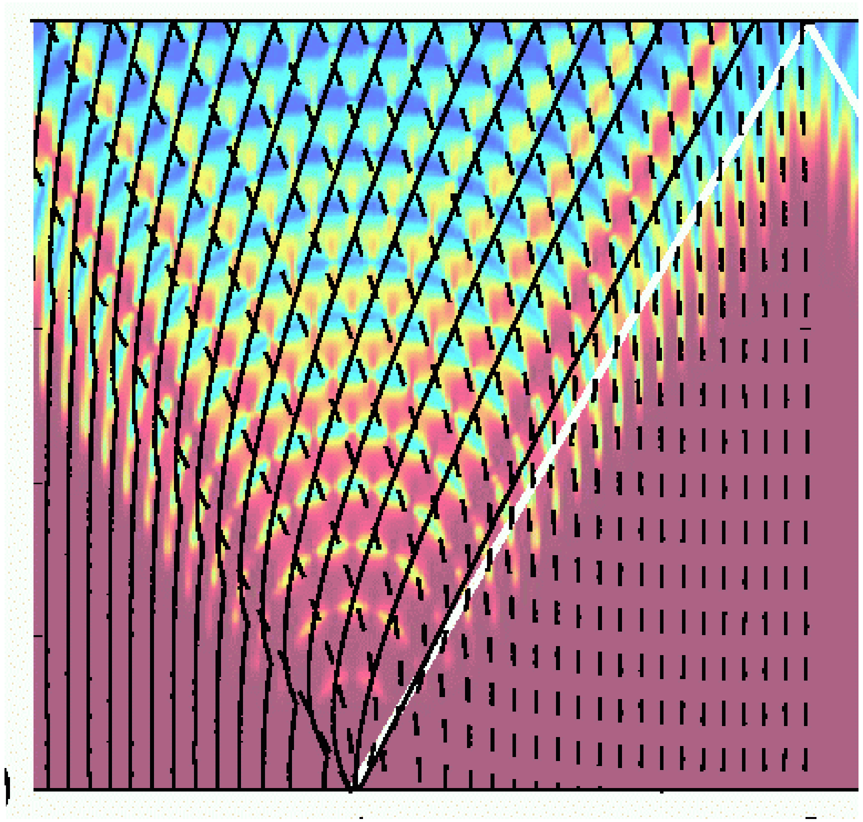




Stokes phase?

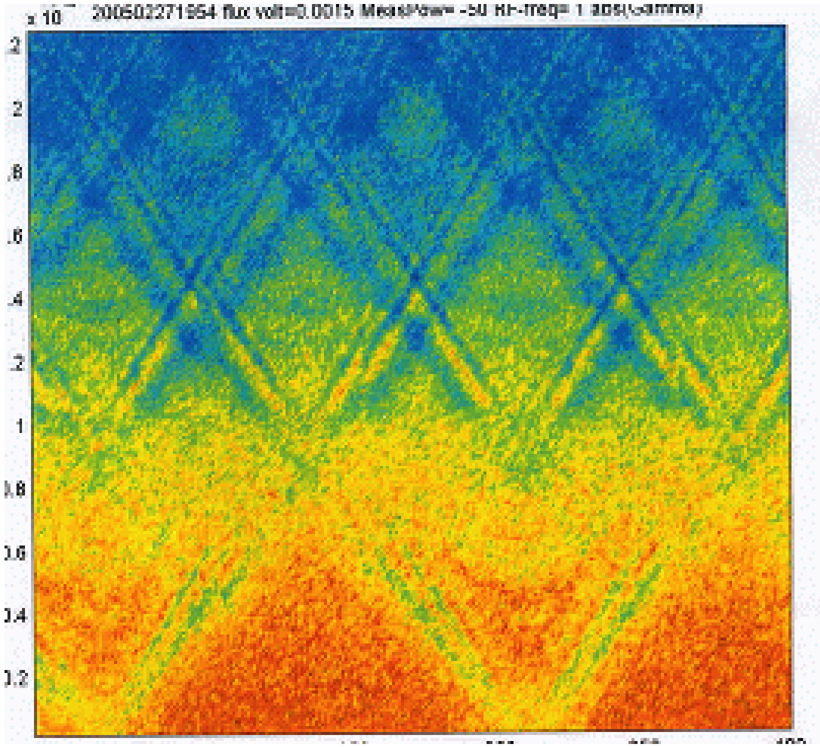


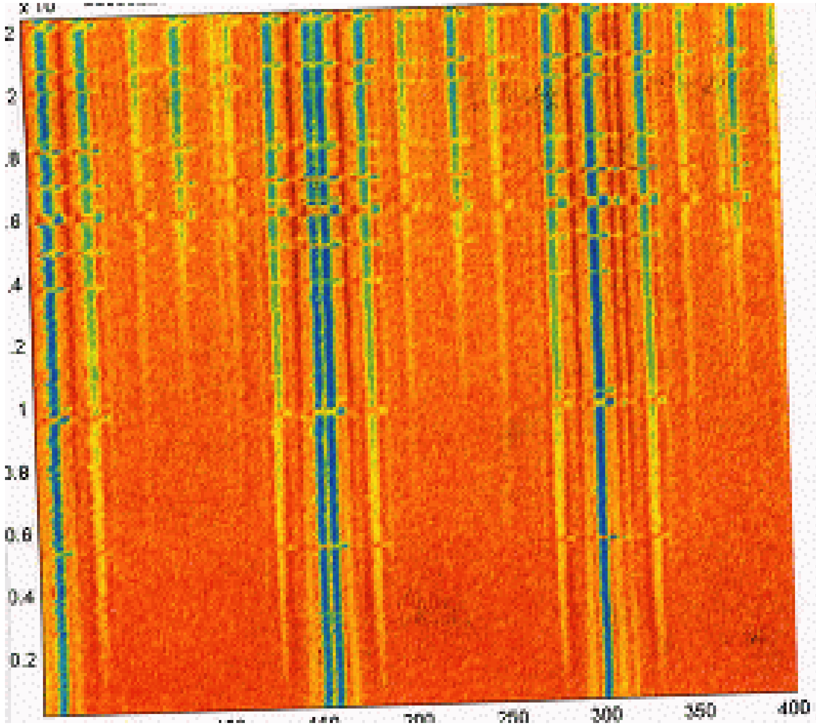
w/ Stokes



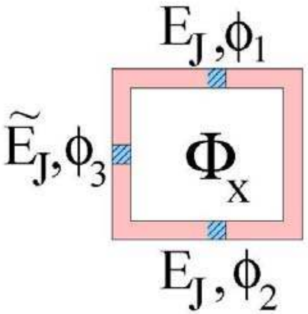
w/o Stokes

Variation with frequency





Josephson flux qubits

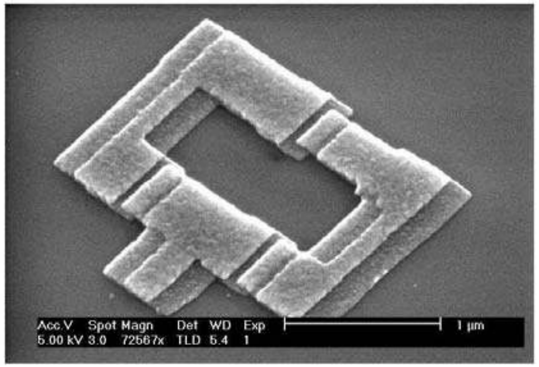


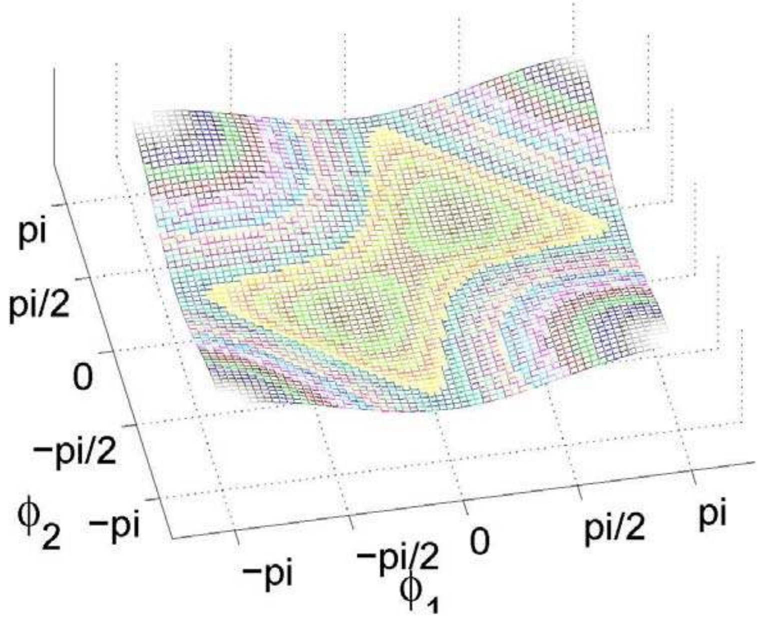
smaller loops
lower I_c

$\Rightarrow$ possibly longer coherence

$$U(\phi_1, \phi_2) = -E_\mathrm{J} \cos\phi_1 - E_\mathrm{J}\cos\phi_2 - \tilde{E}_\mathrm{J}\cos(\Phi_x - \phi_1 - \phi_2)$$

$$\Phi_x = \pi \qquad \tilde{E}_\mathrm{J} = 0.8 E_\mathrm{J}$$



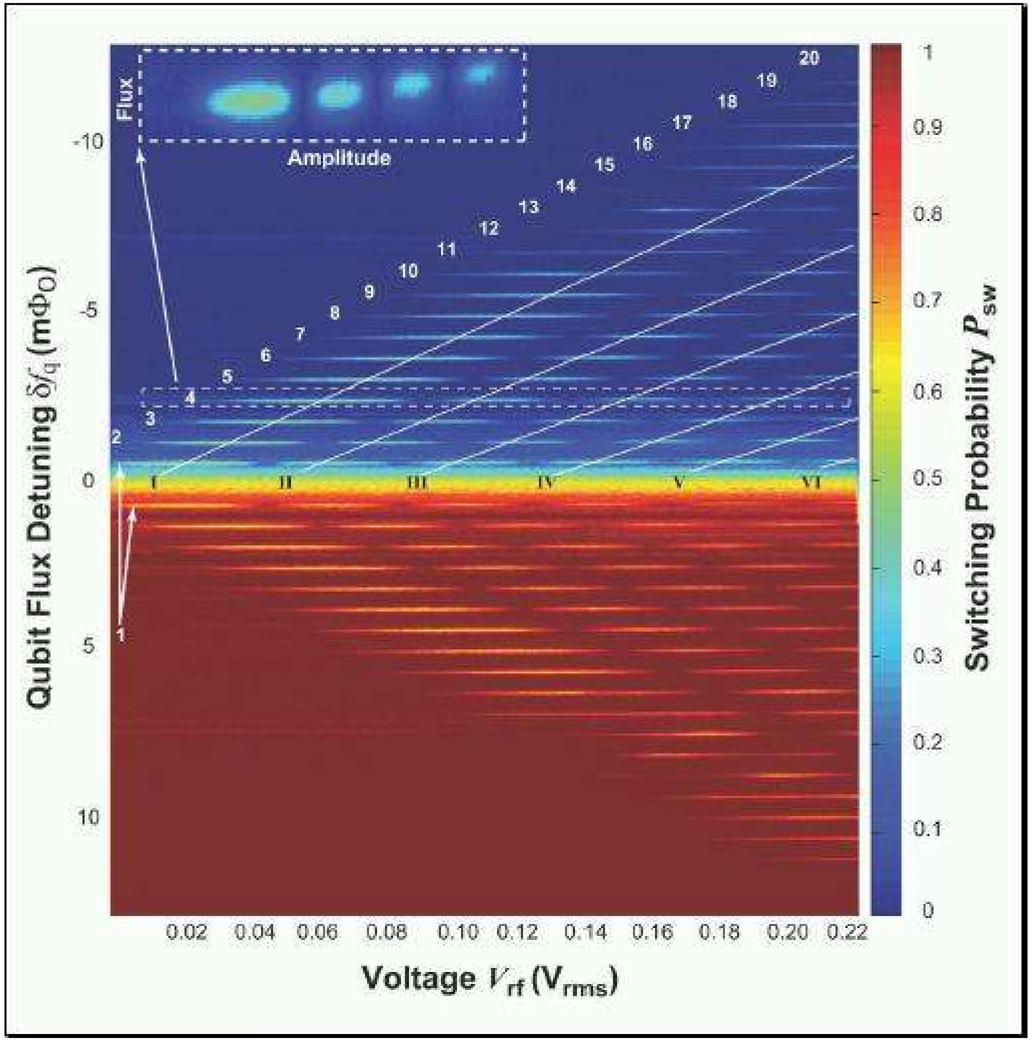


LZ interference in flux qubits

Oliver et al. (MIT)

low Δ

$$1 - P_{LZ} \ll 1$$



Some applications

- investigation of decoherence
- sensitive readout of charge or flux
- coherent manipulations