

1 **Contribution of Lattice Distortion to Solid Solution Strengthening in a**  
2 **Series of Refractory High Entropy Alloys**

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## 16 **Abstract**

17 We present an experimental approach for revealing the impact of lattice distortion on solid solution  
18 strengthening in a series of body-centered (bcc) Al-containing, refractory high entropy alloys from the  
19 Nb-Mo-Cr-Ti-Al system. By systematically varying the Nb and Cr content, a wide range of atomic  
20 size difference as a common measure for the lattice distortion was obtained. Single phase, bcc solid  
21 solutions were achieved by arc-melting and homogenization as well as verified by means of scanning  
22 electron microscopy (SEM) and X-ray diffraction (XRD). The atomic radii of the alloying elements  
23 for determination of atomic size difference were re-calculated on the basis of the mean atomic radii in  
24 and the chemical compositions of the solid solutions. Microhardness at room temperature correlates  
25 well with the deduced atomic size difference. Nevertheless, the mechanisms of microscopic slip lead  
26 to pronounced temperature dependence of mechanical strength. In order to account for this particular  
27 feature, we present a combined approach, using microhardness, nanoindentation and compression tests.  
28 The athermal proportion to the yield stress of the investigated equimolar alloys is revealed. These  
29 parameters support the universality of this aforementioned correlation. Hence, the pertinence of lattice  
30 distortion for solid solution strengthening in bcc high entropy alloys is proven.

## 31 **1. Introduction**

32 Since the introduction of high entropy alloys (HEAs) [1], the focus has especially been on their  
33 microstructural and mechanical properties [2]. Solid solution strengthening in such chemically  
34 complex alloys is of particular interest [3], as these systems typically exceed the constitutional range  
35 spanned by classical theories for the description of solid solution strengthening in dilute [4,5] and  
36 more concentrated [6] binary alloys. Accordingly, some efforts were made to develop models  
37 succeeding the current state of the description of solid solution strengthening [7–10]. The approach  
38 presented by Toda-Caraballo and Rivera-Díaz-del-Castillo [8] is based on the theory by Labusch [6]  
39 while it uses a generalization of the gradual changes of the lattice parameter and thus the misfit with  
40 composition in binary solid solution towards multi-component systems. In contrast, Varvenne, Luque  
41 and Curtin [10] presented a mechanistic, parameter-free and predictive theory for the yield strength of  
42 random alloys and face-centered cubic (fcc) HEAs, respectively, which was validated against data  
43 available for the CoCrFeMnNi system mainly contributed by Wu et al. [11]. The fundamental  
44 assumption is that dislocations move through an effective medium having average lattice parameter,  
45 shear modulus, stacking fault energy, etc. [10]. Strengthening accordingly arises from the interaction  
46 of dislocations with random, local concentration fluctuations around the average composition of the  
47 HEA [10]. This especially accounts for the fact that in case of HEAs solute and solvent cannot be  
48 distinguished due to their equal concentration. In conclusion of Ref. [10], alloys with maximized  
49 strength can be obtained by maximizing the concentration-weighted, mean-square misfit volume  
50 and/or the shear modulus. Assuming the validity of Vegard's law and using reasonable values for

51 individual atomic radii, the concentration-weighted, mean-square misfit volume which is used in this  
52 theory can directly be transformed into the more common atomic size difference  $\delta$  [10]:

$$\delta = \sqrt{\sum_i x_i \left(1 - \frac{r_i}{\bar{r}}\right)^2} \quad (1)$$

53 where  $x_i$  is the molar fraction of the alloying element,  $r_i$  is the atomic radius of the respective alloying  
54 element and  $\bar{r}$  is the average atomic radius of the alloy. An extension of this procedure was introduced  
55 by Okamoto et al. [9] by taking the individual mean-square atomic displacements of the alloying  
56 elements into account. They propose a linear relationship of the yield strength and the averaged mean  
57 square atomic displacement.

58 Strictly, the proposed theory by Varvenne, Luque and Curtin [10] does only apply to dislocation slip  
59 with low impact of Peierls stress as it is observed for fcc metals and alloys. Nevertheless, lattice  
60 distortion might significantly attribute to solid solution strengthening in bcc HEAs as well, and the  
61 simplicity and elegance of the atomic size difference approach makes it worth to be tested for  
62 correlation of  $\delta$  with the plastic yield of bcc HEAs. In this case, the contribution to solid solution  
63 strengthening might not be driven by the interaction of the elastic distortion field of dislocations and  
64 the one of solute atoms but by the interaction of dislocation cores with random, local concentration  
65 fluctuations as it is suggested by Rao et al. in Ref. [12]. Deviations from significant correlation might  
66 lead to the identification of (i) interesting model alloys with non-equimolar composition and/or (ii)  
67 additional contributions by short range ordering [13] as well as variations of local dislocation core  
68 structures [12] specific to certain alloys.

69 Recently, we presented a suitable processing route for obtaining an almost single phase, bcc HEA  
70 NbMoCrTiAl [14]. The basic design goal was to combine elements with high melting points (Nb, Mo)  
71 and, thus, high intrinsic creep strength with protective oxide scale forming elements (Cr, Al) while  
72 simultaneously reducing density (Al, Ti) by a proper choice of elements. Arc-melting and  
73 homogenization treatment at 1300 °C for 20 h lead to an almost single phase microstructure that  
74 possesses significant plasticity above 600 °C. Moreover, the formation of combined  $\langle 111 \rangle$ -/ $\langle 001 \rangle$ -  
75 fiber texture components with respect to the loading axis developing during compression testing at  
76 these temperatures reveal dislocation slip with  $\langle 111 \rangle$  Burgers vectors as it is expected for bcc  
77 materials. The alloy exhibits a reasonable high temperature strength. Furthermore, the capability of  
78 increasing oxidation resistance by adding Cr and Al has been verified for the present and related alloys  
79 [15–18].

80 In general, large differences in atomic size which are expected for NbMoCrTiAl as well as its ability  
81 to deform by dislocation slip makes it a favorable model system for studying the impact of the  
82 contribution of lattice distortion to solid solution strengthening in bcc HEAs. For this purpose, suitable  
83 changes in alloy composition and the according homogenization treatment for obtaining single phase

84 microstructure were established and are presented in this article. In order to quantitatively describe the  
85 lattice distortion in the present alloys, lattice parameters and, thus, interatomic spacings were  
86 determined by means of XRD. These data are used to re-calculate the individual atomic radii of the  
87 alloying elements with the coordination of a bcc lattice. Finally, the according atomic size difference is  
88 correlated with mechanical properties exhibited by the alloy series. In order to account for the  
89 temperature-dependent strength of bcc metals, a suitable combination of microhardness for screening  
90 at room temperature as well as nanoindentation up to 400 °C and compression tests for temperatures  
91 beyond 400 °C were exemplarily conducted. The relevance of the microhardness results at room  
92 temperature of the respective alloys is stressed.

## 93 2. Experimental

94 The alloys investigated in this work were manufactured from elemental bulk material. Nb, Mo, Cr, Ti  
95 and Al with a purity of 99.9 %, 99.96 %, 99 %, 99.8 %, and 99.9 %, respectively, were mixed in the  
96 desired concentrations and then melted under Ar atmosphere, using an AM/0.5 arc melting furnace by  
97 Edmund Bühler GmbH (Germany). The furnace chamber was set to a pressure of  $5 \cdot 10^{-2}$  mbar and  
98 alternately evacuated and flooded with Ar for three times, before pumping to a high vacuum of less  
99 than  $1 \cdot 10^{-4}$  mbar. The Ar pressure for arc melting was set to 600 mbar. A Zr getter was liquefied  
100 prior to each melting step to reduce residual oxygen content. The melted buttons were flipped and re-  
101 melted for at least five times and optionally cast into a water-cooled rod-shaped Cu mold with a  
102 diameter of 12 mm and length of 60 mm. Compositions of the equiatomic alloys in as-cast state were  
103 determined by inductively coupled plasma optical emission spectrometry (ICP-OES). O and N content  
104 were analyzed by means of carrier gas hot extraction analysis by a TC500 device (Leco Instrumente  
105 GmbH). Depending on the alloy composition, homogenization was performed under Ar atmosphere in  
106 a Gero HTRH 70-600/18 resistance tube furnace for 20 h at 1200, 1300, 1400 and 1500 °C. For this  
107 purpose, buttons and rods were cut into platelets with a thickness of 5 mm. The heating and cooling  
108 rates were 4.2 K/min. Subsequent to face-grinding, XRD analyses (step width for acquisition 0.01°,  
109 acquisition time of 384 s/step) were carried out on a D2 Phaser system by Bruker equipped with a  
110 Lynxeye line detector. The lattice parameter was determined by extrapolating towards  $\theta = 90^\circ$  using  
111 the following weighting function  $\frac{1}{2}(\cot^2\theta + \cot\theta \cdot \cos\theta)$  according to Nelson-Riley [19]. For  
112 microstructural investigations, the samples were prepared by a standard metallographic procedure.  
113 After grinding with a series of abrasive papers of decreasing grain size, the samples were polished,  
114 etched with NH<sub>3</sub>, H<sub>2</sub>O<sub>2</sub> and distilled water in a ratio of 1:1:1 at 50 °C for 5 min and finalized by a  
115 vibratory polishing step for 8 h, using a non-crystallizing oxide polishing suspension with pH = 9.8  
116 supplied by Struers. SEM and energy dispersive X-ray spectroscopy (EDX) analysis were performed  
117 on a Zeiss EVO50 system equipped with a Thermo Scientific EDX system. For each equiatomic alloy,

118 porosity was investigated in terms of area fraction which was determined from several binarized SEM  
119 micrographs. The investigated area was 1.2 mm<sup>2</sup>, respectively.

120 Mechanical properties were characterized in terms of yield stress and hardness. Yield stress  $\sigma_y$  during  
121 compression tests was determined for 0.2 % plastic strain at elevated temperatures in a range of  
122 400 °C up to 1200 °C at an initial engineering strain rate of 10<sup>-3</sup> s<sup>-1</sup>. The samples of (3 x 3 x 4.5) mm<sup>3</sup>  
123 were tested under vacuum in a Zwick Z100 electromechanical universal testing machine equipped  
124 with a vacuum furnace by Maytec. Strain was determined using strain gauges attached to the samples.  
125 Vickers hardness  $\mu H$  was investigated by a Qness Q10A+ microhardness tester with a load of 0.98 N.  
126 Mean values and standard deviations were obtained by analyzing 16 hardness indentations on each  
127 sample. Furthermore, a nanoindenter G200XP by Keysight Technologies GmbH equipped with a laser  
128 heating stage by SURFACE systems + technology GmbH was utilized to measure temperature-  
129 dependent nanohardness  $nH$  between room temperature and 400 °C at a constant indentation strain  
130 rate of 5 · 10<sup>-2</sup> s<sup>-1</sup>. Strain-rate sensitivity  $m = \left(\frac{\partial \ln \sigma_y}{\partial \ln \dot{\epsilon}}\right)_{\epsilon, T} = \left(\frac{\partial \ln \left(\frac{nH}{C}\right)}{\partial \ln \dot{\epsilon}}\right)_{\epsilon, T}$  [20,21] ( $\sigma_y$ : yield stress,  $\dot{\epsilon}$ : strain  
131 rate,  $T$ : temperature,  $nH$ : nanohardness,  $C$ : constraint factor, usually  $C \approx 3$  [22]) was analyzed from  
132 room temperature up to 300 °C by strain-rate jump tests with strain rates of 5 · 10<sup>-2</sup> s<sup>-1</sup>, 1.6 · 10<sup>-2</sup> s<sup>-1</sup>  
133 and 5 · 10<sup>-3</sup> s<sup>-1</sup>. Shear modulus  $G$  of selected alloys was calculated by  $G = \frac{1}{1-p} \cdot \rho \cdot v_T^2$  ( $p$ : porosity;  
134  $\rho$ : density;  $v_T$ : velocity of transverse ultrasonic waves). The experimentally determined density  $\rho$   
135 comprises an error that arises from the sample porosity  $p$  and is corrected by the factor  $\frac{1}{1-p}$  of the  
136 aforementioned equation. The density was determined by Archimedes principle,  $v_T$  was analyzed by  
137 ultrasonic phase spectroscopy on cubic samples with an edge length of 5 mm within the frequency  
138 range of 5 kHz to 10 MHz using two identical broadband ultrasonic wave transducers (Panametrics  
139 model V155) and a R3754A network analyzer by Advantest. Ref. [23] provides further experimental  
140 details of ultrasonic phase spectroscopy.

### 141 **3. Results and discussion**

#### 142 *Microstructure*

143 Prerequisite for the accurate assessment of solid solution strengthening is a homogenous  
144 microstructure of the alloys under investigation. In the following, this is exemplified with the three  
145 equimolar compositions which were manufactured for this work. All non-equiatomic compositions  
146 were treated in similar ways in order to ensure homogeneity of the solid solution. Starting point for the  
147 development was the aforementioned homogeneous, quinary, equimolar NbMoCrTiAl alloy which can  
148 be manufactured by arc-melting and subsequent homogenization treatment [14]. After heat-treatment  
149 of the as-cast state at 1300 °C for 20 h, an almost single phase microstructure as presented in Fig. 1b

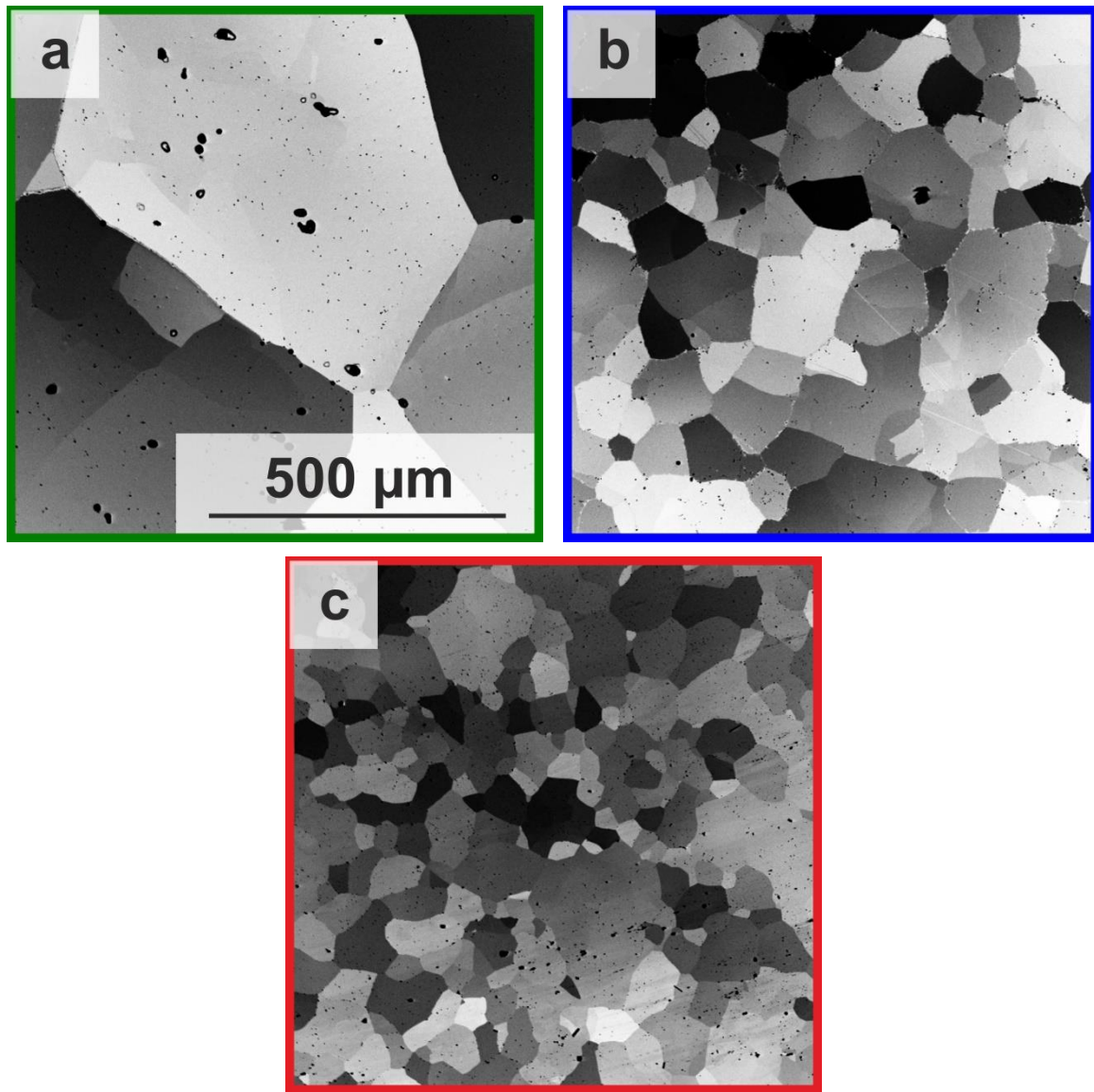
150 can be obtained for this alloy. The XRD pattern in Fig. 2b reveals the bcc crystal structure (W  
151 prototype, Strukturbericht A2).

152 During development of this alloy, several phases competing with the bcc solid solution were identified.  
153 Most importantly, the formation of the hexagonal  $\text{Cr}_2\text{Nb}$ -Laves phase ( $\text{MgZn}_2$  prototype,  
154 Strukturbericht C14) has to be suppressed for retaining single phase bcc. In NbMoCrTiAl, the Laves  
155 phase appears for homogenization heat-treatments below 1300 °C as well as during compression  
156 testing of as-homogenized material at 1200 °C. Moreover,  $(\text{Mo,Nb})_3\text{Al}$  ( $\text{Cr}_3\text{Si}$  prototype,  
157 Strukturbericht A15) forms during annealing at all temperatures tested for homogenization up to now<sup>1</sup>.  
158 Nevertheless, this phase appears only as a fringe of tiny dimension in the vicinity of some grain  
159 boundaries. For 1300 °C and 20 h, its volume fraction is below 0.3 vol% in total and can thus be  
160 considered to be not a relevant factor for the mechanical properties determined in this work. Some  
161 trace contents of  $(\text{Al,Cr})_3\text{Ti}$  ( $\text{Cu}_3\text{Au}$  prototype, Strukturbericht D0<sub>9</sub>) as well as B2 superstructure can  
162 occasionally be found for a variety of processing conditions but are as well believed to be not relevant  
163 for mechanical property determination. All aforementioned phases were identified based on global  
164 XRD patterns as well as locally by means of EBSD patterns.

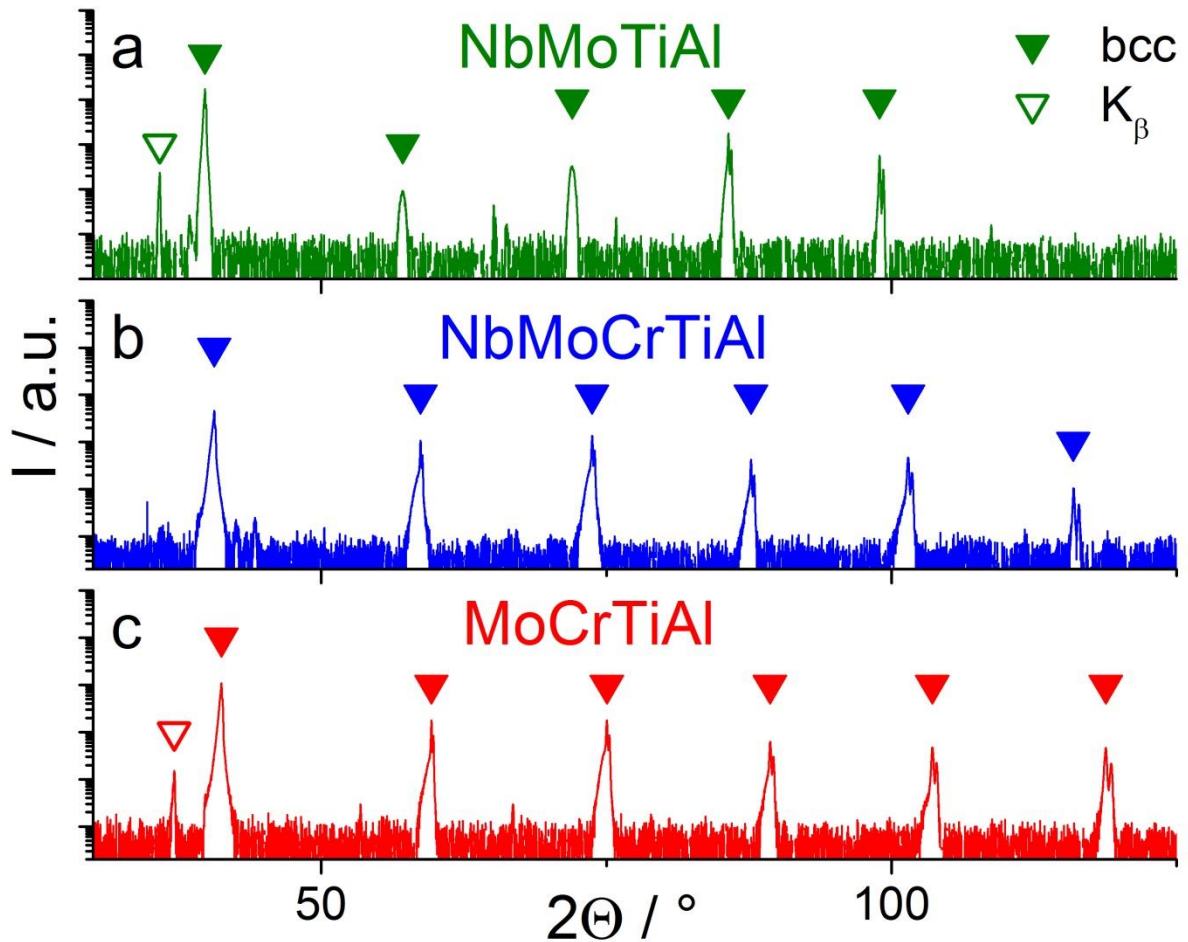
165 Since most of the competing phases are expected for varying alloy compositions derived from  
166 NbMoCrTiAl, only specific changes of alloying elements were conducted in this research. For the  
167 suppression of Laves phase formation, either Nb or Cr content was successively reduced towards the  
168 quaternary, equimolar alloys NbMoTiAl and MoCrTiAl. In case of NbMoTiAl, a significantly higher  
169 homogenization temperature was needed in order to achieve a homogeneous bcc solid solution,  
170 namely 1500 °C for 20 h. The according microstructure and the XRD pattern are presented in Figs. 1a  
171 and 2a, respectively. In contrast, annealing at only 1200 °C for 20 h is necessary to obtain single phase,  
172 bcc MoCrTiAl shown in Figs. 1c and 2c. The concentrations of Mo, Ti and Al were kept constant. It  
173 was expected that increasing Mo, Ti and Al in Nb-Mo-Cr-Ti-Al could favor the formation of  
174  $(\text{Mo,Nb})_3\text{Al}$  and  $(\text{Al,Cr})_3\text{Ti}$  while reducing the amount of these particular elements could lead to  
175 stabilization of  $\text{Cr}_2\text{Nb}$ .

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<sup>1</sup> This phase remained unidentified in our previous investigation [14].



176 Figure 1: Microstructure of alloys with equiatomic composition in homogenized condition, obtained  
177 by SEM using BSE contrast imaging (same magnification for all images): (a) NbMoTiAl annealed at  
178 1500 °C, (b) NbMoCrTiAl annealed at 1300 °C and (c) MoCrTiAl annealed at 1200 °C, each for 20 h.  
179 Certain porosity (black features within the grains) is typical for cast material (< 2 vol.% in all  
180 presented alloys) and was randomly distributed in both as-cast (not presented in this article) and  
181 as-homogenized state.



182 Figure 2: XRD patterns with logarithmic intensity scale of alloys with equiatomic composition in  
 183 homogenized condition: (a) NbMoTiAl annealed at 1500 °C, (b) NbMoCrTiAl annealed at 1300 °C  
 184 and (c) MoCrTiAl annealed at 1200 °C, each for 20 h. Peaks arising from residual Cu-K $\beta$  radiation are  
 185 indicated by open triangles  $\nabla$ .

186 Tab. 1 summarizes all investigated alloys, their compositions and the necessary heat-treatments to  
 187 obtain almost single phase microstructures with residual phases having a total volume fraction of less  
 188 than 0.5 vol%. In certain cases, higher homogenization temperatures were used as well in order to  
 189 exclude effects by the grain size of the material and possible influences by the homogenization  
 190 temperature (e. g. different extent of any type of ordering) on the mechanical response. For  
 191 NbMoCrTiAl, several batches were tested for assessment of the reproducibility.

192 Table 1: Composition in at.% of solid solutions in annealed condition determined by standard-related  
 193 EDX as well as lattice parameters of the bcc structure; O contents were exemplarily analyzed for  
 194 NbMoTiAl, NbMoCrTiAl and MoCrTiAl: 780 at.ppm, 312 at.ppm and 289 at.ppm, respectively. N  
 195 content was below the detection limit for all tested alloys.  $\blacklozenge$  highlights EDX on a different batch [14],  
 196 \* were used for re-calculation of the atomic radii.

$j$	nominal	annealing	$x_{i,j}$	$a_j^{\text{bcc}} / \text{nm}$
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	<b>composition</b>	<b>condition</b>	<b>Nb</b>	<b>Mo</b>	<b>Cr</b>	<b>Ti</b>	<b>Al</b>	
	NbMoTiAl	1400 °C, 20 h	0.251 ± 0.001	0.248 ± 0.002	-	0.253 ± 0.002	0.248 ± 0.001	
<b>1</b>		1500 °C, 20 h*	0.252 ± 0.001	0.249 ± 0.001	-	0.252 ± 0.001	0.247 ± 0.001	3.208 <sup>+0.018</sup> <sub>-0.007</sub>
<b>2</b>	NbMoCr <sub>0.25</sub> TiAl	1300 °C, 20 h*	0.259 ± 0.001	0.233 ± 0.001	0.058 ± 0.001	0.230 ± 0.001	0.220 ± 0.000	3.198 <sup>+0.002</sup> <sub>-0.058</sub>
<b>3</b>	NbMoCr <sub>0.5</sub> TiAl	1300 °C, 20 h*	0.234 ± 0.001	0.226 ± 0.003	0.099 ± 0.001	0.216 ± 0.001	0.225 ± 0.001	3.175 <sup>+0.018</sup> <sub>-0.013</sub>
		1400 °C, 20 h	0.231 ± 0.000	0.223 ± 0.001	0.101 ± 0.001	0.216 ± 0.000	0.228 ± 0.000	
<b>4</b>	NbMoCr <sub>0.75</sub> TiAl	1300 °C, 20 h*	0.234 ± 0.001	0.208 ± 0.003	0.157 ± 0.003	0.204 ± 0.001	0.197 ± 0.001	3.165 <sup>+0.035</sup> <sub>-0.005</sub>
<b>5</b>	NbMoCrTiAl	1300 °C, 20 h*	0.219 ± 0.001	0.201 ± 0.002	0.200 ± 0.002	0.192 ± 0.000	0.187 ± 0.001	3.149 <sup>+0.021</sup> <sub>-0.004</sub>
		1400 °C, 20 h	0.219 ± 0.001	0.197 ± 0.001	0.195 ± 0.001	0.191 ± 0.000	0.198 ± 0.000	
		1300 °C, 20 h♦	0.205 ± 0.001	0.195 ± 0.000	0.197 ± 0.001	0.205 ± 0.002	0.199 ± 0.001	
<b>6</b>	Nb <sub>0.75</sub> MoCrTiAl	1300 °C, 20 h*	0.173 ± 0.001	0.214 ± 0.001	0.208 ± 0.000	0.205 ± 0.000	0.200 ± 0.001	3.141 <sup>+0.048</sup> <sub>-0.011</sub>
<b>7</b>	Nb <sub>0.5</sub> MoCrTiAl	1300 °C, 20 h*	0.112 ± 0.000	0.232 ± 0.002	0.221 ± 0.001	0.215 ± 0.001	0.220 ± 0.000	3.127 <sup>+0.044</sup> <sub>-0.011</sub>
		1400 °C, 20 h	0.110 ± 0.000	0.228 ± 0.001	0.218 ± 0.001	0.218 ± 0.001	0.226 ± 0.002	
<b>8</b>	Nb <sub>0.25</sub> MoCrTiAl	1300 °C, 20 h*	0.065 ± 0.001	0.241 ± 0.002	0.238 ± 0.002	0.234 ± 0.001	0.222 ± 0.001	3.114 <sup>+0.006</sup> <sub>-0.009</sub>
<b>9</b>	MoCrTiAl	1200 °C, 20 h*	-	0.253 ± 0.001	0.248 ± 0.000	0.245 ± 0.001	0.254 ± 0.001	3.101 <sup>+0.018</sup> <sub>-0.006</sub>
		1300 °C, 20 h	-	0.254 ± 0.002	0.247 ± 0.002	0.242 ± 0.000	0.257 ± 0.001	

197

## 198 *Atomic radii of the alloying elements*

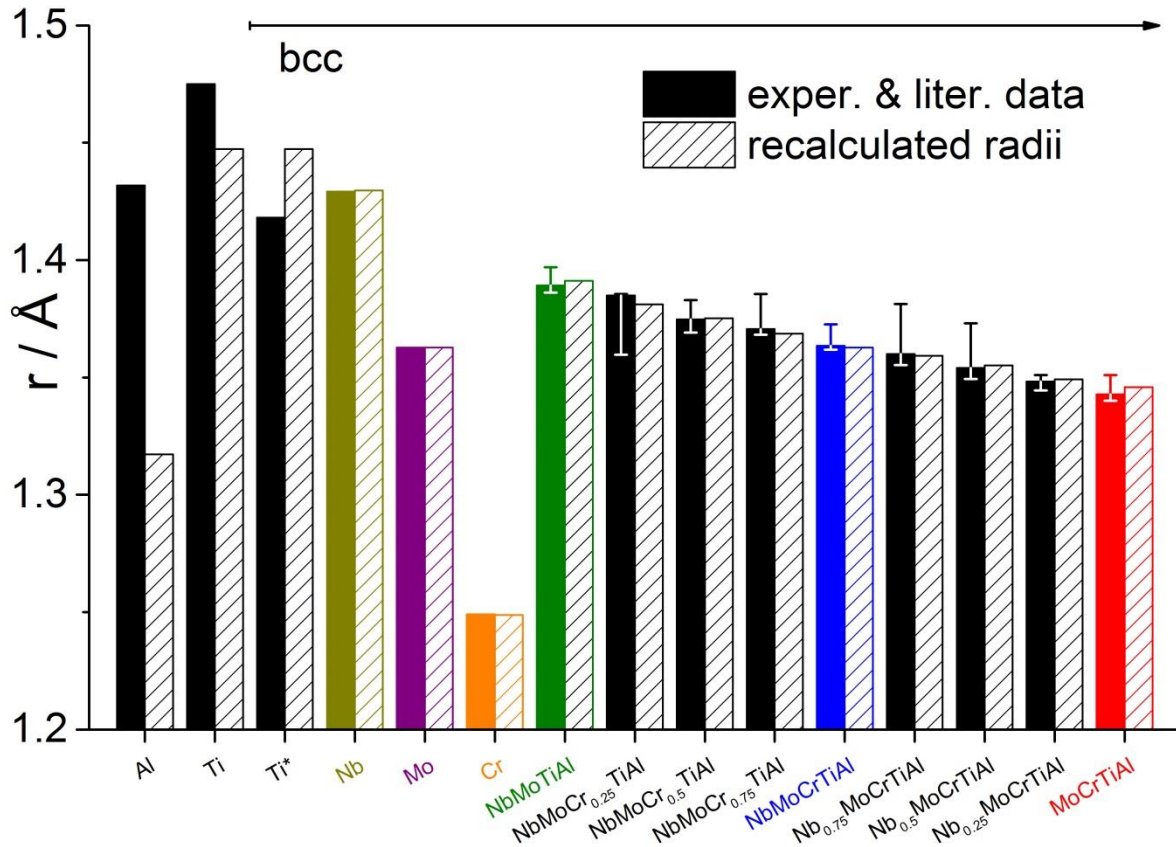
199 In principle, three degrees of complication of the description of the variation of the lattice in a HEA  
200 are possible. The easiest assumption is that the HEA has a defined lattice parameter without a  
201 variation on atomic length scale at all. This assumption is physically not meaningful and would lead to  
202 the conclusion that there is no hardening since the solute atoms do not have a distortion field. Further  
203 complication is possible by assuming an effective lattice parameter of the alloy as well as its variation  
204 caused by the individual size of the alloying elements – this variation might be used as a measure for  
205 the lattice distortion. Even more complex is a description where each species of alloying elements has  
206 an average size and an according variation. In this case, the exact size of an atom is determined by the  
207 actual environment of the atom (not by its species alone as it is assumed in the previous case). Despite  
208 the latter case being the most realistic description and providing several quantities for the lattice  
209 distortion, the second assumption already allows for a possible description of solid solution  
210 strengthening. Furthermore, the second case is rather easy to be assessed when the atomic radii of the  
211 alloying elements within the solid solution are known. As pointed out by Varvenne, Luque and  
212 Curtin [10], the assessment of atomic size difference based on the (tabulated) atomic radii of the pure  
213 elements might lead to arbitrary results when the pure elements crystallize in a different crystal  
214 structure as compared to the HEA solid solution. In such case, the binding state of the alloying  
215 element is different and accordingly a different contribution to the lattice distortion has to be expected.

216 In the present work, we try to obtain reasonable atomic radii by analyzing all obtained bcc alloys  
 217 (coordination number CN = 8). In the case of the crystal structure being constant, binding state  
 218 remains rather constant. Furthermore, only the refractory elements Nb and Cr are changed in their  
 219 concentrations which should further reduce the possibility of varying amounts of directed binding. The  
 220 situation might be different when Al or Ti content is changed over a wide composition range. For the  
 221 analyses of the data, an over-determined set of equations is formulated on the basis of: (i) the chemical  
 222 compositions  $x_{i,j}$  presented in Tab. 1, where  $i$  is the respective alloying element and  $j$  the respective  
 223 alloy; (ii) the mean atomic radius  $\bar{r}_j$  in the alloys obtained from  $\bar{r}_j = \sqrt{3}/4 \cdot a_j^{\text{bcc}}$  (CN = 8) where  $a_j^{\text{bcc}}$   
 224 are the lattice parameters presented in Tab. 1 which were determined by XRD. Moreover, the atomic  
 225 radii of the bcc crystallizing elements are included in the calculation as well (CN = 8). The  
 226 overdetermined set of equations reads thus

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ x_{\text{Nb},1} & x_{\text{Mo},1} & x_{\text{Cr},1} & x_{\text{Ti},1} & x_{\text{Al},1} \\ x_{\text{Nb},2} & x_{\text{Mo},2} & x_{\text{Cr},2} & x_{\text{Ti},2} & x_{\text{Al},2} \\ x_{\text{Nb},3} & x_{\text{Mo},3} & x_{\text{Cr},3} & x_{\text{Ti},3} & x_{\text{Al},3} \\ \vdots & & & & \end{pmatrix} \cdot \begin{pmatrix} r_{\text{Nb}} \\ r_{\text{Mo}} \\ r_{\text{Cr}} \\ r_{\text{Ti}} \\ r_{\text{Al}} \end{pmatrix} = \begin{pmatrix} r_{\text{Nb}} \\ r_{\text{Mo}} \\ r_{\text{Cr}} \\ \bar{r}_1 \\ \bar{r}_2 \\ \bar{r}_3 \\ \vdots \end{pmatrix} \quad (2)$$

227  
 228 and is solved with respect to  $r_{\text{Nb}}$ ,  $r_{\text{Mo}}$ ,  $r_{\text{Cr}}$ ,  $r_{\text{Ti}}$  and  $r_{\text{Al}}$  by a least-square procedure. The over-determined  
 229 set of equations is in principle a multi-component extension of Vegard's law. Fig. 3 shows the  
 230 comparison of the atomic radii of the elements (CN = 12 for Al and Ti; CN = 8 for Mo, Cr and Nb as  
 231 well as for an extrapolation for Ti designated as Ti\*), the mean radii in the alloys deduced from the  
 232 bcc lattice parameter (CN = 8) as well as the values obtained from solving the aforementioned over-  
 233 determined set of equations. Since Ti exhibits a bcc high temperature modification, a second atomic  
 234 radius for Ti based on the lattice parameter of the high temperature bcc phase (CN = 8) and  
 235 compensated by thermal expansion down to room temperature taken from Ref. [24] is included as a  
 236 reference as well. It is obvious, that the atomic radii of all elements which are deduced from non-bcc  
 237 crystal structures, namely Al and Ti, are quite different from the re-calculated atomic radii whereas the  
 238 radii of the bcc elements are well reflected by this approach. A good agreement of the re-calculated  
 239 atomic radii with the experimentally observed mean atomic radii in the alloys of the present  
 240 contribution is found. It is pointed out that the error indication for the experimental mean atomic radii  
 241 is conservatively chosen (taking the peak width into account) in all cases and might be significantly  
 242 smaller in reality. Thus, the description of alloys within the Nb-Mo-Cr-Ti-Al system on the basis of  
 243  $r_{\text{Nb}} = 1.4297 \text{ \AA}$ ,  $r_{\text{Mo}} = 1.3627 \text{ \AA}$ ,  $r_{\text{Cr}} = 1.2488 \text{ \AA}$ ,  $r_{\text{Ti}} = 1.4474 \text{ \AA}$  and  $r_{\text{Al}} = 1.3172 \text{ \AA}$  is rather good  
 244 and the linearity by Vegard's law seems to be reasonable – at least for variations of the Cr and Nb  
 245 concentration. Further details of the binding states in such alloys have to be investigated in future –

246 especially with respect to individual variation of the radii of the elements which will not be considered  
 247 for the further course of this article. Also, the actual misfit volume was not experimentally determined  
 248 within this work, but would describe the extent of lattice distortion in an even more accurate way.



249 Figure 3: Comparison of experimental and re-calculated atomic radii in the elements  $r_i$  as well as  
 250 investigated alloys  $\bar{r}_j$ . Literature data for Al, Ti (using CN = 12, respectively), Nb, Mo and Cr (using  
 251 CN = 8), respectively, represented by solid bars are all taken from [25], “Ti\*” designates an  
 252 extrapolation of the atomic radius of bcc Ti corrected by thermal expansion down to room temperature  
 253 (CN = 8) according to Ref. [24].

### 254 *Temperature-dependent mechanical properties*

255 Depending on the homologous temperature, bcc metals and substitutional solid solutions exhibit an  
 256 intrinsic temperature-dependent yield stress which can be divided into a low-temperature, a plateau  
 257 and a high-temperature regime, as exemplarily shown for Fe-3wt%Si in Refs. [26,27]. This is  
 258 schematically presented in Fig. 4a. At low temperatures, plastic deformation of bcc metals and alloys  
 259 is thought to be mediated by the thermally activated formation and movement of kink pairs. The  
 260 movement of kink pairs itself can be controlled by (i) either line-tension when the critical kink width  
 261 is very small and of the order of the dimension of the dislocation cores or the distance between minima  
 262 in the Peierls potential or (ii) by the elastic interaction of the kinks when the kink width is necessarily

263 high [28]. Consequently, strength increases considerably with (i) decreasing temperature and (ii)  
264 increasing strain rate. Above a certain temperature, usually referred to as “critical temperature” [29] or  
265 “knee-temperature” [21,28] a strength plateau is reached. In this temperature regime, strength becomes  
266 virtually strain-rate independent. The knee temperatures of the bcc principal elements of the present  
267 HEA system are about 130 °C for Nb, 275 °C for Mo and 300 °C for Cr [21,30]. For the investigated  
268 alloys a similar temperature range for the knee temperature is expected. When exceeding about  $0.4 \cdot T_m$ ,  
269 ( $T_m$ : melting temperature) diffusional processes lead to rapid decrease of the yield stress. Mechanical  
270 strength becomes strain-rate dependent again. Thus, for the assessment of solid solution strengthening  
271 in bcc alloys the strain-rate independent plateau strength, frequently referred as athermal strength,  
272 should be ideally investigated.

273 In order to account for temperature dependence of the mechanical properties, several mechanical tests  
274 were performed and are summarized in Figs. 4b and c, respectively. Compression tests performed  
275 from 400 °C up to 1200 °C on the equiatomic alloys in Fig. 4c reveal the transition from plateau-like  
276 yield stress to creep-controlled strength. As shown in previous investigations, the predominant  
277 deformation mechanism in NbMoCrTiAl is dislocation slip up to a deformation temperature of  
278 1200 °C [14]. Hence, this quinary alloy and its derivatives are well suited to discuss the temperature-  
279 dependent yield stress, also in the creep regime. Onset of the drop of strength is observed between  
280 600 °C (MoCrTiAl) and 1000 °C (NbMoCrTiAl).

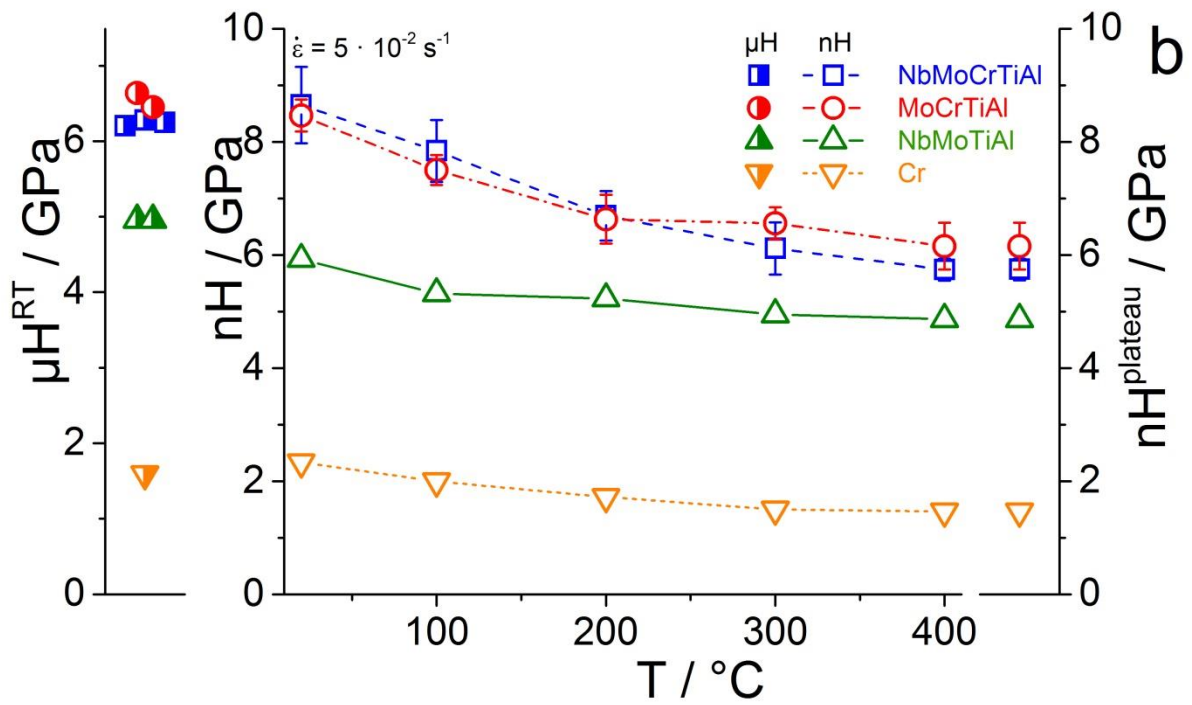
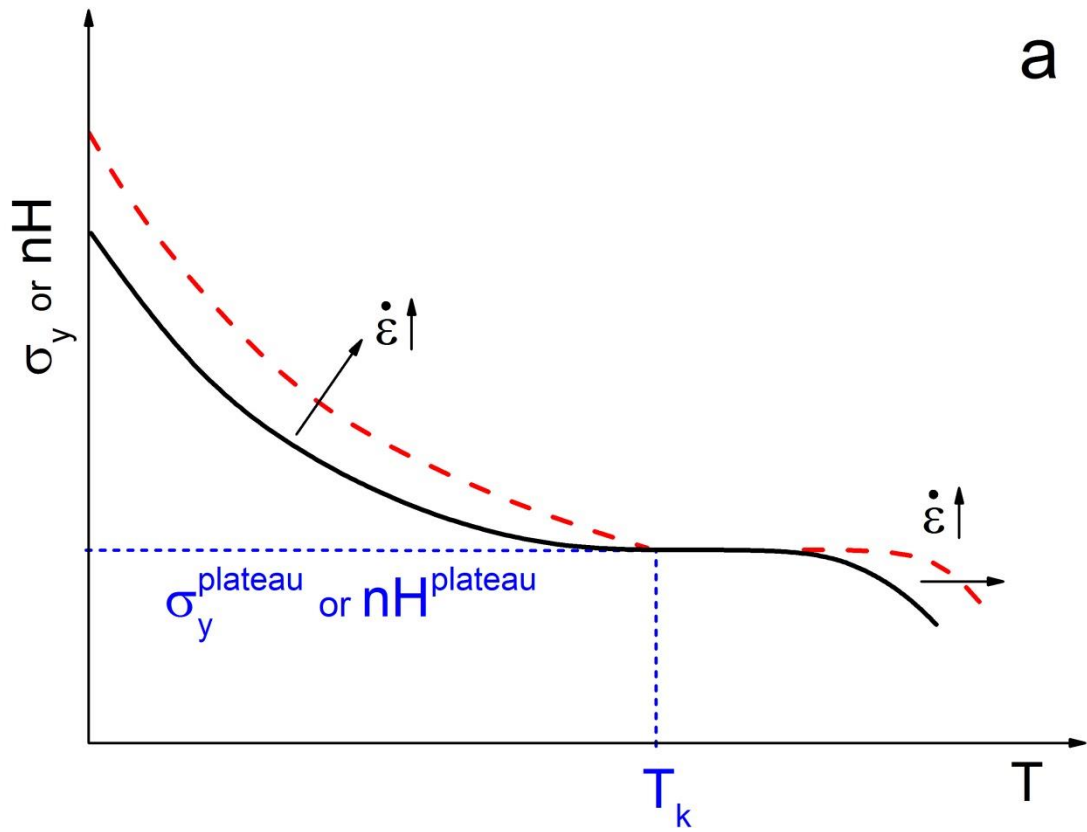
281 Due to microstructural features, NbMoCrTiAl and its derivatives do not reveal significant plasticity on  
282 a macroscopic scale at temperatures lower than 600 °C (NbMoTiAl, NbMoCrTiAl) and 400 °C  
283 (MoCrTiAl). As shown in Fig. 1 and discussed in Ref. [14], abnormal grain growth appears during  
284 homogenization – this might lead to increasing compatibility stresses between these large grains and  
285 the surrounding microstructure during compression testing. This is a possible reason for a limited  
286 ductility of the presented alloys. In addition, pores within the compression samples, developed during  
287 solidification after casting, further lead to an increase of internal stresses due to localization. Thus,  
288 crack formation is initiated although, macroscopically, yield stress has not been reached. Nevertheless,  
289 microhardness and nanoindentation reveal plasticity (without cracking) on a local scale. Fig. 5  
290 exemplarily shows slip traces of NbMoCr<sub>0.5</sub>TiAl in the vicinity of a microhardness indent at room  
291 temperature.

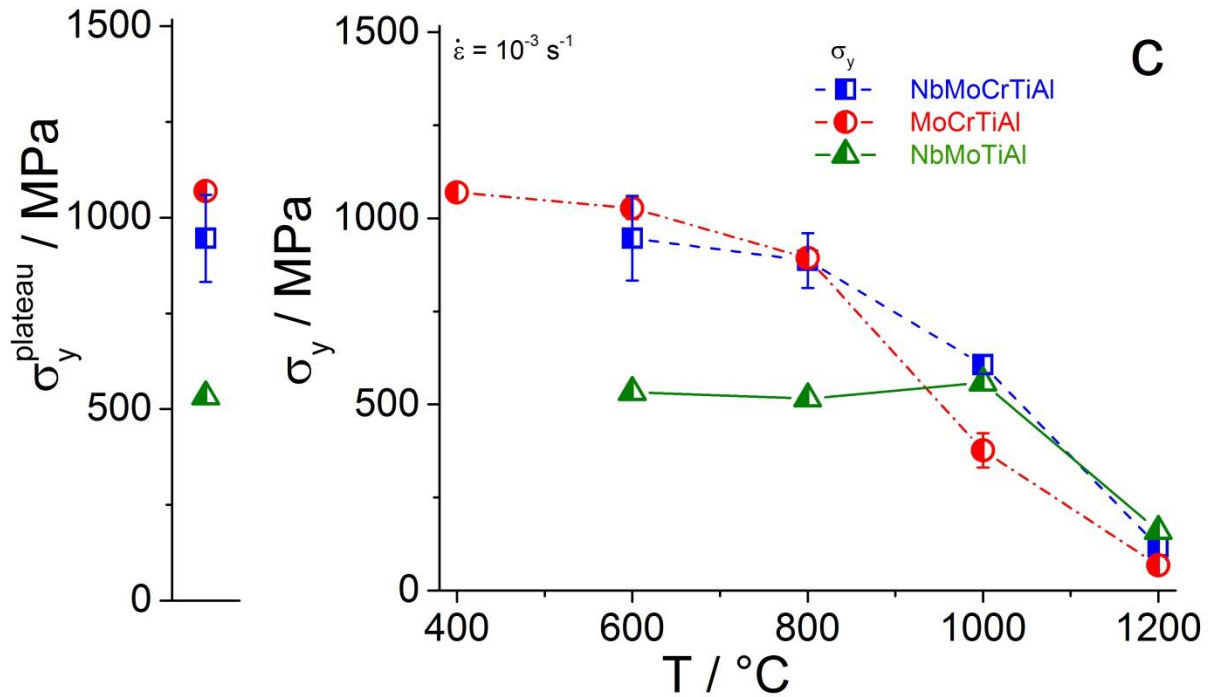
292 By using nanoindentation, a temperature range between RT and 400 °C could be covered in order to  
293 reveal the plateau region of the yield stress in Fig. 4b. Further details about the temperature-dependent  
294 nanohardness of pure Cr can be found elsewhere [21]. It has to be taken into account that indentation  
295 tests are typically affected by varying work hardening behavior due to higher plastic strains when  
296 compared to uniaxial tests for yield stress determination. Typically, hardness determined with a  
297 Berkovich indenter corresponds to a yield stress at about 7 % plastic strain [22]. Hence, contribution  
298 of work hardening to strengthening that occurs from indentation eventually varies with alloy

299 composition. Further alloy development with respect to ductility might provide chances to assess this  
300 in the plateau temperature range. In agreement with the observations from compression testing, the  
301 nanohardness data indicate a plateau of the mechanical strength at temperatures higher than  
302 approximately 300 °C for the quaternary and quinary alloys. The temperature-dependent strain-rate  
303 sensitivity was exemplarily investigated for NbMoCrTiAl in order to verify that the plateau hardness  
304 is indeed achieved.

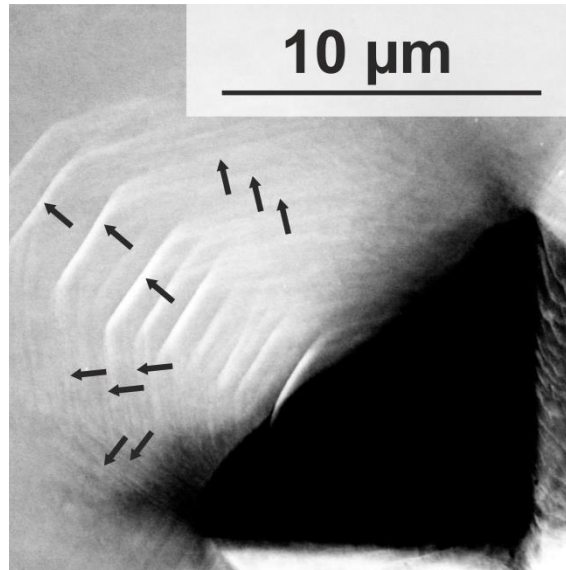
305 Compared to the rate-sensitive deformation behavior of pure bcc-structured elements in the low  
306 temperature regime, such as Cr with a strain-rate sensitivity of about  $m^{RT} = 0.08$  [21,29] at room  
307 temperature, NbMoCrTiAl shows an already, comparably low  $m^{RT} = 0.018 \pm 0.004$  at room  
308 temperature. Nevertheless, as is in the case of pure Cr [21,29], the strain-rate sensitivity of  
309 NbMoCrTiAl almost vanishes when approaching the knee temperature:  $m^{300\text{ °C}} = 0.007 \pm 0.003$ .  
310 Thus, clearly the plateau strength  $\sigma_y^{\text{plateau}}$  is reached in the temperature range between 300 °C and  
311 600 °C in NbMoCrTiAl. This is in agreement with our observations by compression tests at elevated  
312 temperatures. In accordance, the chosen values for  $nH^{\text{plateau}}$  and  $\sigma_y^{\text{plateau}}$  are included as separate  
313 charts in Figs. 4b and c, respectively.

314 The assessment of solid solution strengthening in the last part of the present article will be mainly  
315 based on microhardness measurements at room temperature  $\mu H^{RT}$  due to their simplicity and  
316 reproducibility.  $\mu H^{RT}$  does not significantly depend on alloy batch or varying homogenization  
317 parameters. Hence, it is independent of grain size, grain orientation and RT hardness is reproducible.  
318 As shown in Fig. 4b, strength and hardness at room temperature are already influenced by the  
319 thermally activated slip processes. Nevertheless, microhardness  $\mu H^{RT}$  at room temperature on the one  
320 hand and  $nH^{\text{plateau}}$  as well as  $\sigma_y^{\text{plateau}}$  on the other hand exhibit a very reasonable correlation (please  
321 compare Figs. 4b and c).





322 Figure 4: a) Scheme of the temperature-dependent yield stress of bcc metals and alloys. Experimental  
 323 results for the temperature dependence of the equiatomic alloys MoCrTiAl, NbMoTiAl and  
 324 NbMoCrTiAl: b) Microhardness  $\mu H^{RT}$  was obtained at room temperature. Multiple data points  
 325 represent  $\mu H^{RT}$  obtained for varying homogenization parameters and on different batches (please see  
 326 Tab. 1). Nanohardness  $nH$  was determined between room temperature and 400 °C in order to obtain  
 327 the plateau nanohardness  $nH^{plateau} = nH(400^{\circ}C)$ . c) Compression tests were performed at a  
 328 consistent strain rate of  $10^{-3}s^{-1}$  between 400 °C and 1200 °C depending on the macroscopic ductility  
 329 of the polycrystalline material in order to obtain the plateau yield stress  $\sigma_y^{plateau}$  equal to  $\sigma_y^{600^{\circ}C}$   
 330 (NbMoCrTiAl, NbMoTiAl) and  $\sigma_y^{400^{\circ}C}$  (MoCrTiAl). The connecting lines are guiding the eyes. Error  
 331 bars that do not exceed the symbol size are omitted.



332

333 Figure 5: Secondary electron (SE) contrast image of a Vickers indent (lower right corner) in  
 334 NbMoCr<sub>0.5</sub>TiAl at room temperature, revealing slip traces in various directions in the vicinity of the  
 335 indent; slip traces are highlighted by black arrows.

### 336 *Assessment of solid solution strengthening*

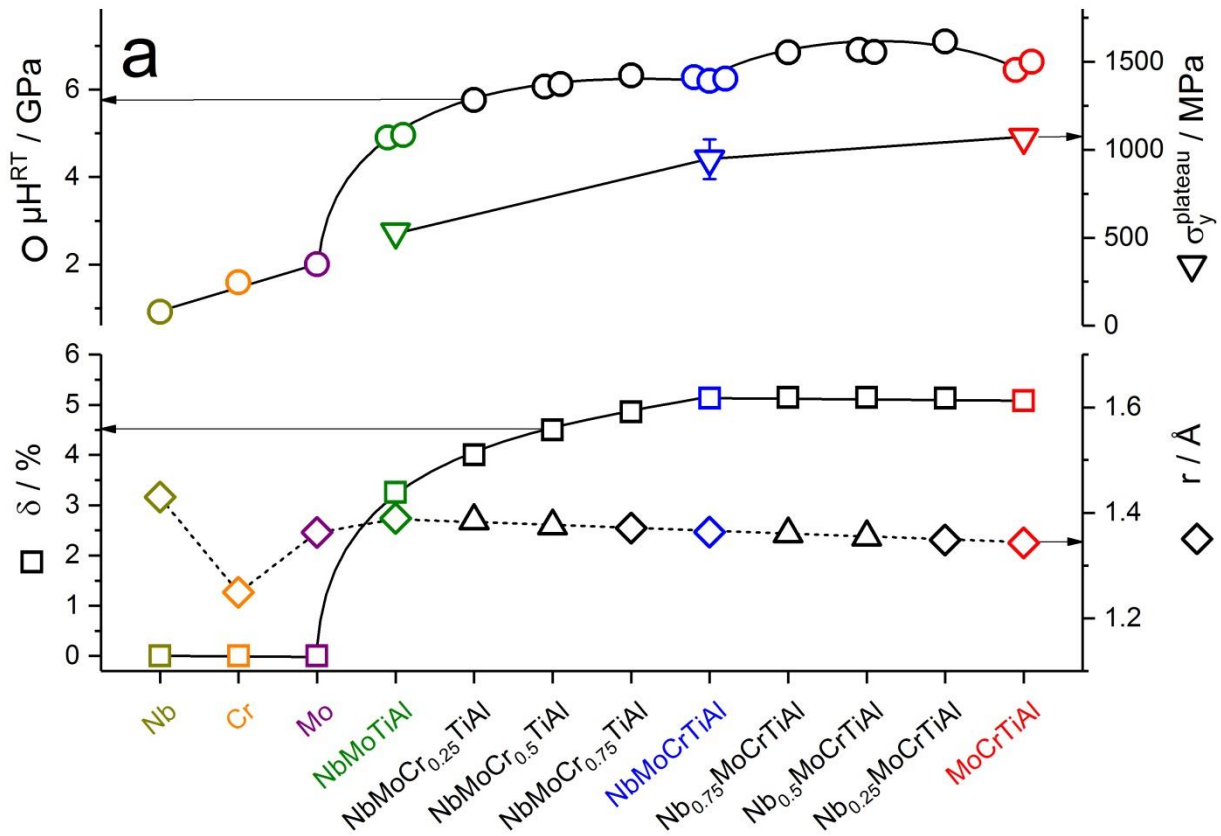
337 By using the atomic radii presented in Fig. 3, the atomic size difference  $\delta$  was calculated. While the  
 338 mean atomic radius, calculated from the lattice parameters presented in Tab. 1, is monotonically  
 339 decreasing with increasing Cr content from NbMoTiAl to NbMoCrTiAl and further decreases by  
 340 decreasing the Nb content from NbMoCrTiAl to MoCrTiAl, atomic size difference exhibits a different  
 341 trend. Fig. 6a shows that for alloys between NbMoTiAl and NbMoCrTiAl, monotonic increase of  $\delta$  is  
 342 observed, while it is almost constant for alloys between NbMoCrTiAl and MoCrTiAl. This is caused  
 343 by the significant difference of the size of Cr compared to the average atomic size of the alloy while  
 344 Nb exhibits a size similar to mean atomic radii of the prepared alloys and, thus, provides negligible  
 345 potential for additional lattice distortion. Nevertheless, all presented alloys based on Nb-Mo-Cr-Ti-Al  
 346 exhibit a  $\delta$  ranging from 3.25 to 5.15 %. Thus, a significant variation of lattice distortion for  
 347 investigating solid solution strengthening is provided. For reference, Cr, Mo and Nb without any  
 348 lattice distortion are investigated, too.

349 For the assessment of the effect of  $\delta$  on the mechanical properties, microhardness at room temperature  
 350  $\mu\text{H}^{\text{RT}}$  and plateau yield stress  $\sigma_y^{\text{plateau}}$  are plotted in a unified way in Fig. 6a. Varying homogenization  
 351 treatments as well as varying alloy batches (different data points in Fig. 6a, see Tab. 1 for details) do  
 352 not significantly alter microhardness of the alloys. For reference,  $\mu\text{H}^{\text{RT}}$  of Nb, Cr and Mo are  
 353 presented as well. Indeed, atomic size difference and microhardness seem to correlate. In the case of  
 354 the alloy series ranging from NbMoCrTiAl to MoCrTiAl where constant mechanical strength should  
 355 be expected, an excess hardness of about 0.8 to 0.9 GPa above the average microhardness of  
 356 NbMoCrTiAl to MoCrTiAl of 6.3 GPa is observed. Staying approximately constant from NbMoTiAl

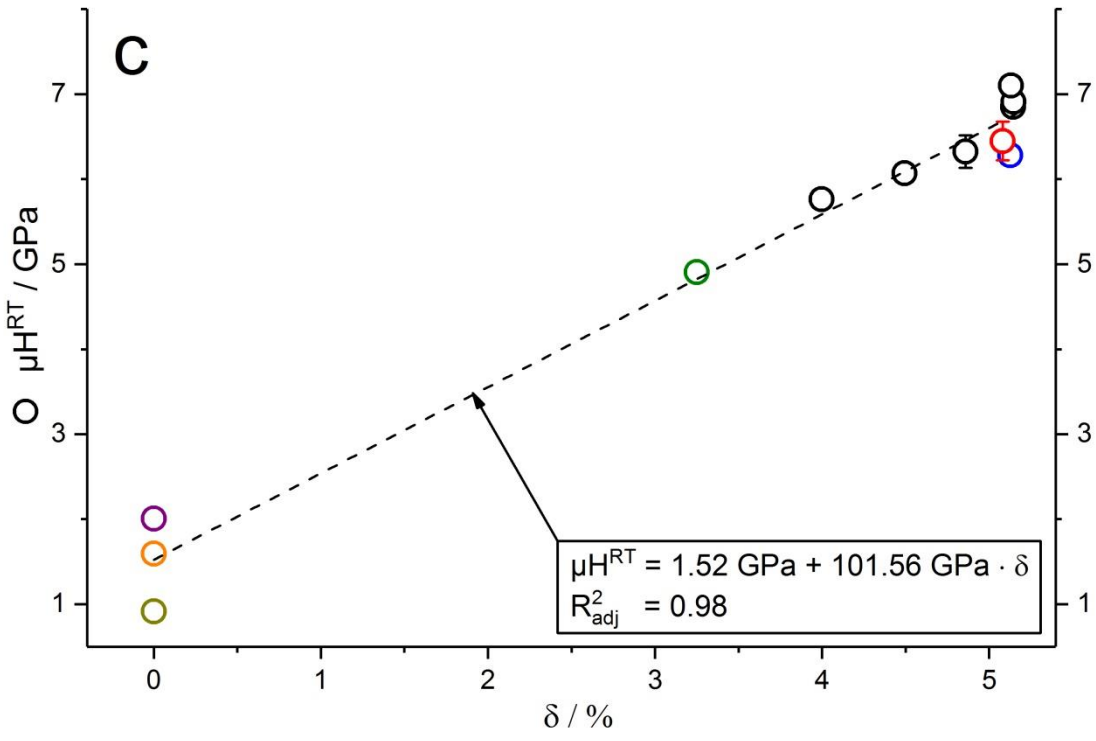
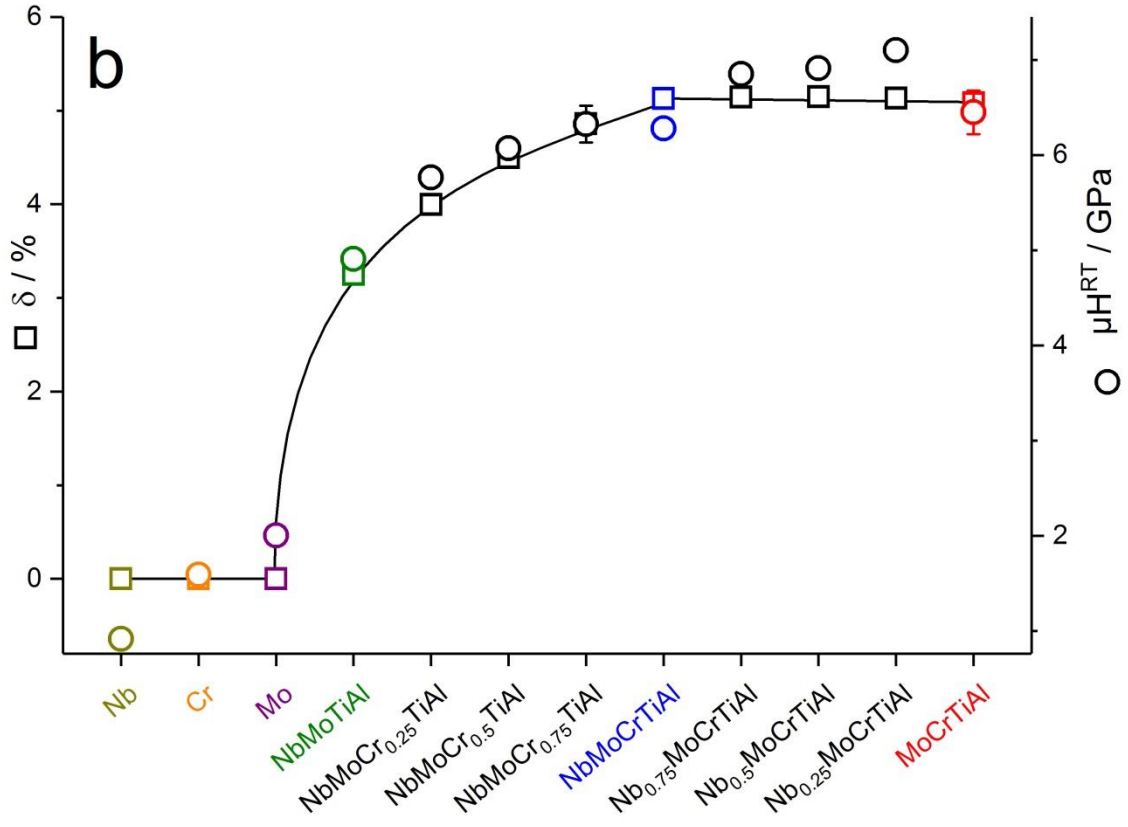


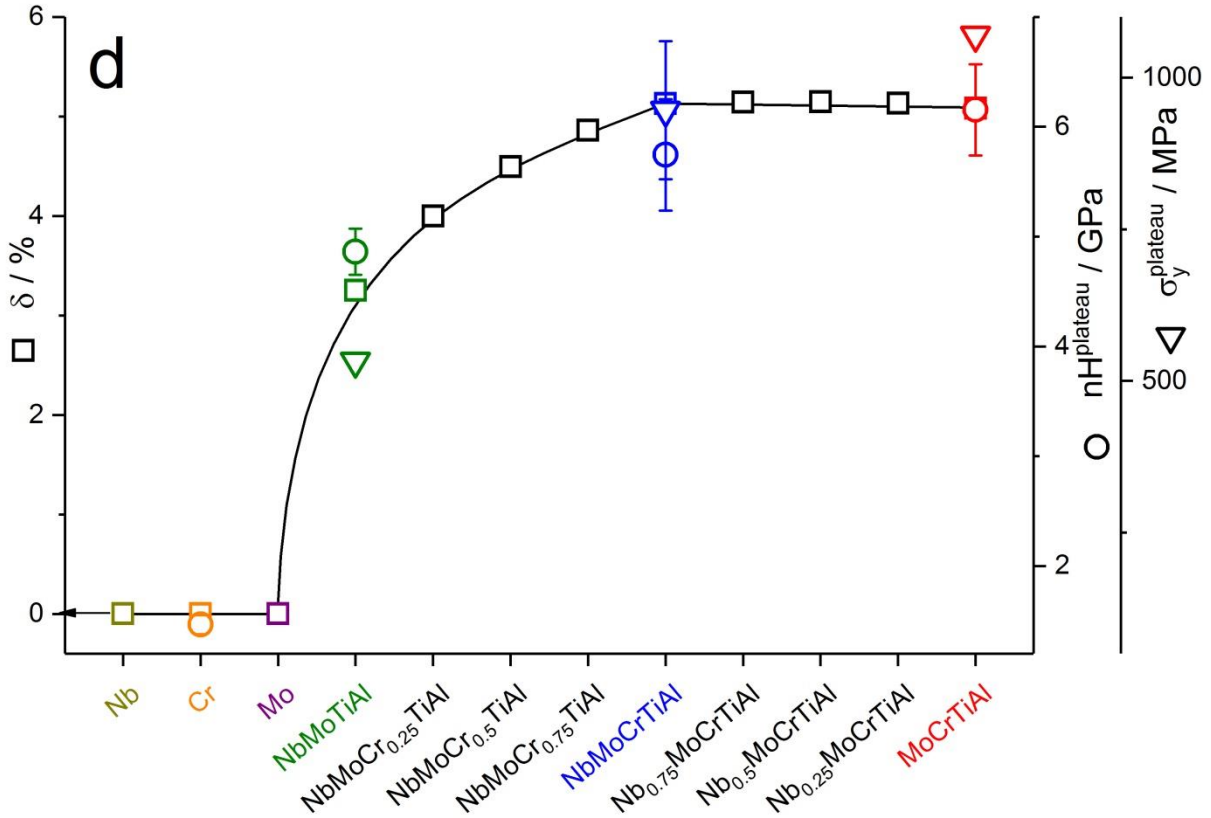
357 over NbMoCrTiAl towards MoCrTiAl, the gradual change of lattice parameter with composition  $\frac{1}{a} \frac{\partial a}{\partial x_i}$ ,  
 358 serving as a classical measure for the extent of lattice distortion in binary alloys [4], apparently does  
 359 not reflect the present evolution of microhardness.

360 Fig. 6b visualizes the determined correlation of  $\mu H^{RT} = 1.52 \text{ GPa} + 101.56 \text{ GPa} \cdot \delta$  with  $R_{\text{adj}}^2 =$   
 361  $0.98^2$ . As pointed out in the previous section,  $\mu H^{RT}$  is altered by a possibly different hardness  
 362 increase due to thermally activated slip among the alloy series. For this reason, Fig. 6c exemplarily  
 363 incorporates the plateau values for nanohardness  $nH^{\text{plateau}}$  as well as yield stress  $\sigma_y^{\text{plateau}}$  for the  
 364 quaternary and quinary alloys. Even then, a reasonable correlation of  $\delta$  and mechanical strength is  
 365 observed for the present HEA system.



<sup>2</sup>  $R_{\text{adj}}^2 = 1 - \left( \frac{(1-R^2) \cdot (n-1)}{n-k-1} \right)$  is a modified coefficient of determination  $R^2$  taking the degrees of freedom  $k$  and the number data points for data fitting  $n$  into account. The coefficient of determination  $R^2$  corresponds to the ratio of the covariance and the product of the standard deviations of the fitted data.





366 Figure 6: a) Assessment of atomic size  $r$  and atomic size difference  $\delta$  using re-calculated radii (lower  
 367 part of the figure) as well as of microhardness at room temperature  $\mu H^{RT}$  and plateau yield stress  
 368  $\sigma_y^{plateau}$  (upper part of the figure). Multiple microstructural conditions (higher homogenization  
 369 temperature, different batches) for various alloys are included here and do not reveal significant  
 370 variation. b) Correlation of  $\delta / \mu H^{RT}$  and according element / alloy composition. c)  $\mu H^{RT}$  evolution  
 371 with respect to  $\delta$  reveals linear correlation with  $\mu H^{RT} = 1.52 \text{ GPa} + 101.56 \text{ GPa} \cdot \delta$  and  $R_{adj}^2 = 0.98$ .  
 372 d) Correlation of  $\delta$  and  $\sigma_y^{plateau}$  as well as  $nH^{plateau}$  from Figs. 4b and c, respectively. For all diagrams,  
 373 error bars which do not exceed the symbol size are omitted. Color coding is consistent for all diagrams.

374 As already mentioned, the hardness of  $Nb_xMoCrTiAl$  alloys with  $x = 0.25, 0.5$  and  $0.75$  with almost  
 375 constant atomic size difference apparently exceeds the average hardness of  $NbMoCrTiAl$  and  
 376  $MoCrTiAl$ . In addition to the severely distorted lattice as a major contribution to solid solution  
 377 strengthening, additional factors might cause the exceeding microhardness of  $Nb_{0.25}MoCrTiAl$ ,  
 378  $Nb_{0.5}MoCrTiAl$  and especially  $Nb_{0.75}MoCrTiAl$ .

379 A change in shear modulus which correlates with the line energy of dislocations could influence  
 380 dislocation motion. The most obvious deviations from the correlation of  $\mu H^{RT}$  and  $\delta$  is observed for  
 381 the alloys with almost constant atomic size difference. Thus, the shear modulus  $G$  was determined for

382 Nb<sub>x</sub>MoCrTiAl with x = 0, 0.25, 0.5, 0.75, 1 and NbMoTiAl for comparison also in order to cover the  
 383 entire composition range of the investigated alloys. Shear moduli exhibit only minor changes from  
 384 about 52 GPa upto about 66 GPa and do not seem to provide a correlation with the observed room  
 385 temperature microhardness  $\mu\text{H}^{\text{RT}}$ . Hence, we assume that within the Nb-Mo-Cr-Ti-Al system with  
 386 varying Cr and Nb concentration, the shear modulus does not play a major role for solid solution  
 387 strengthening. Although no global superstructure was detected by XRD analyses, different types of  
 388 ordering might additionally strengthen some of the alloys.

389 Table 2: Relevant materials properties for the evaluation of composition-depending shear modulus G.

<i>j</i>	<b>nominal composition</b>	<b>annealing condition</b>	<b>porosity</b>	<b><math>\rho / \text{kg/m}^3</math></b>	<b><math>v_{\text{T}} / \text{m/s}</math></b>	<b>G / GPa</b>
1	NbMoTiAl	1500 °C, 20 h	0.0078 ± 0.0018	6637 ± 2	2940 ± 15	<b>57.8 ± 2.2</b>
5	NbMoCrTiAl	1300 °C, 20 h	0.0116 ± 0.0027	6594 ± 5	3150 ± 18	<b>66.2 ± 1.0</b>
6	Nb <sub>0.75</sub> MoCrTiAl	1300 °C, 20 h	0.0127 ± 0.0110	6501 ± 46	2990 ± 2	<b>58.9 ± 2.5</b>
7	Nb <sub>0.5</sub> MoCrTiAl	1300 °C, 20 h	0.0083 ± 0.0025	6420 ± 16	3130 ± 1	<b>63.4 ± 3.4</b>
8	Nb <sub>0.25</sub> MoCrTiAl	1300 °C, 20 h	0.0080 ± 0.0045	6245 ± 40	2880 ± 1	<b>52.2 ± 1.0</b>
9	MoCrTiAl	1200 °C, 20 h	0.014 ± 0.0070	6165 ± 57	2927 ± 29	<b>53.6 ± 0.5</b>

## 390 4. Conclusions

391 Providing almost single phase microstructure, dislocation mediated plasticity with  $\langle 111 \rangle$  Burgers  
 392 vectors and varying atomic size differences between 3.25 and 5.15 %, NbMoCrTiAl and its  
 393 derivatives are very well suited for investigating the impact of lattice distortion on the solid solution  
 394 strengthening in concentrated bcc alloys. In the present article, a correlation between the atomic size  
 395 difference  $\delta$  and room temperature microhardness  $\mu\text{H}^{\text{RT}}$ , namely  $\mu\text{H}^{\text{RT}} = 1.52 \text{ GPa} + 101.56 \text{ GPa} \cdot \delta$ ,  
 396 was found for a series of alloys Nb-Mo-Cr-Ti-Al including two quaternary and one quinary, equimolar  
 397 composition. The following conclusions can be drawn:

- 398 1. Bcc solid solutions were stabilized by appropriate heat treatments in all investigated alloys and  
 399 competing phases were accordingly suppressed when gradually reducing the concentration of  
 400 Nb and Cr.
- 401 2. Individual atomic radii of the alloying elements were determined by evaluation of  
 402 experimentally observed mean atomic radii of all investigated alloys.

- 403 3. Although  $\mu\text{H}^{\text{RT}}$  reasonably correlates with the atomic size difference  $\delta$ , the magnitude of  
404  $\mu\text{H}^{\text{RT}}$  is altered by the well-known feature of thermally activated dislocation slip in bcc metals  
405 and solid solutions. Thus, combined temperature-dependent mechanical characterization by  
406 both nanoindentation and compression tests was utilized to exemplarily assess the athermal  
407 strength of the equimolar alloys. Both,  $\sigma_y^{\text{plateau}}$  as well as  $n\text{H}^{\text{plateau}}$  further support the  
408 correlation with  $\delta$  found for  $\mu\text{H}^{\text{RT}}$ .
- 409 4. Variation of shear moduli remains small within the composition range of the investigated  
410 alloys and cannot account for the observed dependence of the hardness of the alloys.

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420 Infrastructure at Karlsruhe Institute of Technology (KIT, [www.kit.edu](http://www.kit.edu)).

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## 465 Captions

466 Table 1: Composition in at.% of solid solutions in annealed condition determined by standard-related  
467 EDX as well as lattice parameters of the bcc structure; O contents were exemplarily analyzed for  
468 NbMoTiAl, NbMoCrTiAl and MoCrTiAl: 780 at.ppm, 312 at.ppm and 289 at.ppm, respectively. N  
469 content was below the detection limit for all tested alloys. ♦ highlights EDX on a different batch [14],  
470 \* were used for re-calculation of the atomic radii.

471 Table 2: Relevant materials properties for the evaluation of composition-depending shear modulus G.

472 Figure 1: Microstructure of alloys with equiatomic composition in homogenized condition, obtained  
473 by SEM using BSE contrast imaging (same magnification for all images): (a) NbMoTiAl annealed at  
474 1500 °C, (b) NbMoCrTiAl annealed at 1300 °C and (c) MoCrTiAl annealed at 1200 °C, each for 20 h.  
475 Certain porosity (black features within the grains) is typical for cast material (< 2 vol.% in all  
476 presented alloys) and was randomly distributed in both as-cast (not presented in this article) and  
477 as-homogenized state.

478 Figure 2: XRD patterns with logarithmic intensity scale of alloys with equiatomic composition in  
479 homogenized condition: (a) NbMoTiAl annealed at 1500 °C, (b) NbMoCrTiAl annealed at 1300 °C  
480 and (c) MoCrTiAl annealed at 1200 °C, each for 20 h. Peaks arising from residual Cu-K $\beta$  radiation are  
481 indicated by open triangles  $\nabla$ .

482 Figure 3: Comparison of experimental and re-calculated atomic radii in the elements  $r_i$  as well as  
483 investigated alloys  $\bar{r}_j$ . Literature data for Al, Ti (using CN = 12, respectively), Nb, Mo and Cr (using  
484 CN = 8), respectively, represented by solid bars are all taken from [25], “Ti\*” designates an  
485 extrapolation of the atomic radius of bcc Ti corrected by thermal expansion down to room temperature  
486 (CN = 8) according to Ref. [24].

487 Figure 4: a) Scheme of the temperature-dependent yield stress of bcc metals and alloys. Experimental  
488 results for the temperature dependence of the equiatomic alloys MoCrTiAl, NbMoTiAl and  
489 NbMoCrTiAl: b) Microhardness  $\mu\text{H}^{\text{RT}}$  was obtained at room temperature. Multiple data points  
490 represent  $\mu\text{H}^{\text{RT}}$  obtained for varying homogenization parameters and on different batches (please see

491 Tab. 1). Nanohardness  $nH$  was determined between room temperature and  $400\text{ }^{\circ}\text{C}$  in order to obtain  
492 the plateau nanohardness  $nH^{\text{plateau}} = nH(400^{\circ}\text{C})$ . c) Compression tests were performed at a  
493 consistent strain rate of  $10^{-3}\text{ s}^{-1}$  between  $400\text{ }^{\circ}\text{C}$  and  $1200\text{ }^{\circ}\text{C}$  depending on the macroscopic ductility  
494 of the polycrystalline material in order to obtain the plateau yield stress  $\sigma_y^{\text{plateau}}$  equal to  $\sigma_y^{600^{\circ}\text{C}}$   
495 ( $\text{NbMoCrTiAl}$ ,  $\text{NbMoTiAl}$ ) and  $\sigma_y^{400^{\circ}\text{C}}$  ( $\text{MoCrTiAl}$ ). The connecting lines are guiding the eyes. Error  
496 bars that do not exceed the symbol size are omitted.

497 Figure 5: Secondary electron (SE) contrast image of a Vickers indent (lower right corner) in  
498  $\text{NbMoCr}_{0.5}\text{TiAl}$  at room temperature, revealing slip traces in various directions in the vicinity of the  
499 indent; slip traces are highlighted by black arrows.

500 Figure 6: a) Assessment of atomic size  $r$  and atomic size difference  $\delta$  using re-calculated radii (lower  
501 part of the figure) as well as of microhardness at room temperature  $\mu\text{H}^{\text{RT}}$  and plateau yield stress  
502  $\sigma_y^{\text{plateau}}$  (upper part of the figure). Multiple microstructural conditions (higher homogenization  
503 temperature, different batches) for various alloys are included here and do not reveal significant  
504 variation. b) Correlation of  $\delta / \mu\text{H}^{\text{RT}}$  and according element / alloy composition. c)  $\mu\text{H}^{\text{RT}}$  evolution  
505 with respect to  $\delta$  reveals linear correlation with  $\mu\text{H}^{\text{RT}} = 1.52\text{ GPa} + 101.56\text{ GPa} \cdot \delta$  and  $R_{\text{adj}}^2 = 0.98$ .  
506 d) Correlation of  $\delta$  and  $\sigma_y^{\text{plateau}}$  as well as  $nH^{\text{plateau}}$  from Figs. 4b and c, respectively. For all diagrams,  
507 error bars which do not exceed the symbol size are omitted. Color coding is consistent for all diagrams.