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CONTRIBUTION TO METHODS FOR CALCULATING THE FLOW ABOUT THIN LIFTING WINGS AT TRANSONIC SPEEDS - ANALYTIC EXPRESSIONS FOR THE FAR FIELD

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16. Abstract

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CONTRIBUTION TO METHODS FOR CALCULATING THE FLOW ABOUT THIN LIFTING WINGS AT TRANSONIC SPEEDS – ANALYTIC EXPRESSIONS FOR THE FAR FIELD

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SUMMARY

The problem of determining the small-disturbance flow about two-dimensional airfoils at transonic speeds has been successfully treated by Murman and Cole by the process of matching a numerical solution of the near field to analytic expressions for the far field. The three-dimensional problem, it would appear, can be treated in a similar way with the aid of algorithms adapted to high-speed and high-capacity computers. The far-field potential for both lifting and nonlifting three-dimensional wings at transonic speeds is developed herein for a subsonic free stream. This potential could be used for a three-dimensional-wing computation similar to the computation made by Murman and Cole for the two-dimensional wing.

INTRODUCTION

The problem of calculating the flow about two-dimensional airfoils at transonic speeds with a subsonic free-stream Mach number has been successfully treated by several authors using both time-dependent and steady-flow calculation techniques. The elliptic nature of the problem, which requires that the disturbances vanish at infinity, leads to the use of a large computational network together with large demands on computer storage unless special provision is made for satisfying the boundary condition at infinity. The numerical solution of a three-dimensional problem is even more demanding on computer storage and time requirements, and it is essential to restrict the computational network insofar as possible in order to obtain solutions with present-generation computers in a reasonable time.

Several techniques have been developed for satisfying the far-field boundary conditions in the finite-difference computations of the flow about two-dimensional airfoils. Sills (ref. 1) has taken the simplest approach by simply extending the computational network sufficiently far from the airfoil to satisfy effectively the boundary conditions in the far field. A large computational network is required with this approach, particularly for lifting airfoils at transonic speeds, since the disturbances created by the body decay slowly in the lateral direction. Magnus and Gallaher (ref. 2) limited the computational grid by transforming the far-field region into a finite region by an inversion. Matching the computational mesh of the near field with that of the far field presents a difficulty with this method. The transformation of the airfoil into a circle and a subsequent inversion has been used by Sells (ref. 3) and others to restrict the computational network to the interior of a circle. This technique leads to a nearly ideal computational system for two-dimensional flows, but the extension to three-dimensional flows is not evident. The method developed by Murman and Cole (ref. 4) provides a means for limiting the computational network and satisfying the far-field boundary conditions. Their approach is to match the numerical solution of the near field to an analytic representation of the far field to satisfy, in effect, the far-field boundary conditions. This technique can be extended to three-dimensional wings, for which an economy of grid points is a prime concern.

The Murman and Cole technique requires an analytic expression for the asymptotic form of the potential at infinity. A rigorous development of the asymptotic form of sub-subsonic two-dimensional flow solutions has been given by Ludford (ref. 5) as a series. The leading terms for the far-field behavior for the two-dimensional airfoil at transonic speeds are given by an integral representation for the nonlifting case in reference 4 and for the lifting case in reference 6. The purpose of the present paper is to develop the leading terms of the far-field representation for both lifting and nonlifting three-dimensional wings appropriate to a transonic flow with a subsonic free stream. The development is in the form of an integral representation similar to that of reference 4. The two-dimensional results are presented as a special case.

SYMBOLS

b	wing span
$F(\xi,\eta)$	wing thickness distribution
$\bar{i},\bar{j},\bar{k}$	unit vectors along X-, Y-, and Z-axes
L	Laplacian operator
M_∞	free-stream Mach number
n	inward normal direction
O	order symbol

R	magnitude of difference vector defined by equation (6) or (14)
S	surface
u,v,w	disturbance velocity component in x-, y-, and z-direction, respectively
V	volume
X,Y,Z	Cartesian axes
x,y,z	Cartesian coordinates
x,r,μ	cylindrical coordinates
$\beta = \sqrt{1 - M_\infty{}^2}$	
Γ	circulation
γ	ratio of specific heats
Δ	denotes difference between value on upper and lower surface of wing
ξ,η,ζ	dummy variables
ϕ	disturbance velocity potential
ψ	solution to Laplace equation
$\vartheta_1,\vartheta_2,\vartheta_3$	angles between shock normal and coordinate axes
$[\;]$	denotes jump values across shock

Subscripts:

D	shock discontinuity
I	interior region
P	singular point (x',y',z')

V trailing vortex sheet

W wing

1,2 upstream and downstream side of shock, respectively

A prime indicates a transformed space in which the coordinates transverse to the stream direction are stretched by the factor β. Single and double subscripts on ϕ and ψ denote first and second derivatives with respect to the indicated variables.

ANALYSIS

In the following development the transonic small-disturbance differential equation is put in the form of an integral equation (with the use of Green's theorem) which is suitable for representing the flow about three-dimensional wings at transonic speeds with a subsonic free stream. The leading terms of the asymptotic form of the equation at infinity are taken as the far-field representation.

Transonic Differential Equation

Let x, y, and z be Cartesian coordinates and let ϕ be the disturbance velocity potential that is the perturbation about the uniform free stream. The free stream is taken in the x-direction and the wing is in the XY-plane. The small-disturbance potential-flow equation applicable to transonic flows is

$$\beta^2\phi_{xx} + \phi_{yy} + \phi_{zz} = \frac{\gamma + 1}{2}\frac{\partial}{\partial x}\left(\phi_x{}^2\right) \tag{1}$$

where $\beta^2 = 1 - M_\infty{}^2$. The disturbance velocity components are $u = \phi_x$, $v = \phi_y$, and $w = \phi_z$. The left-hand side of equation (1) becomes the Laplacian operator for $\beta^2 > 0$ with the transformation $x' = x$, $y' = \beta y$, and $z' = \beta z$. In terms of x', y', and z',

$$L(\phi) = \phi_{x'x'} + \phi_{y'y'} + \phi_{z'z'} = \frac{\gamma + 1}{2\beta^2}\frac{\partial}{\partial x'}\left(\phi_{x'}{}^2\right) \tag{2}$$

In the next section an integral equation for the velocity potential ϕ is developed from equation (2) and subsequently transformed back to the unprimed coordinates.

Integral Equation

The symmetric form of Green's theorem for the two functions ϕ and ψ which are twice continuously differentiable in the volume V bounded by the surface S is

$$\int_V \left\{ \psi L(\phi) - \phi L(\psi) \right\} dV' = \int_S \left(\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n} \right) dS' \tag{3}$$

where n is the inward normal direction to S and where L is the Laplacian operator. Let

$$L(\psi) = \psi_{x'x'} + \psi_{y'y'} + \psi_{z'z'} = 0 \tag{4}$$

The fundamental solution of equation (4) is

$$\psi = \frac{1}{4\pi R} \tag{5}$$

where

$$R = \left\{ (x' - \xi')^2 + (y' - \eta')^2 + (z' - \zeta')^2 \right\}^{1/2} \tag{6}$$

This choice of ψ leads to source and doublet distributions in the integrals over the wing surface. The boundary surface S (fig. 1) includes (1) the surface at infinity, (2) the

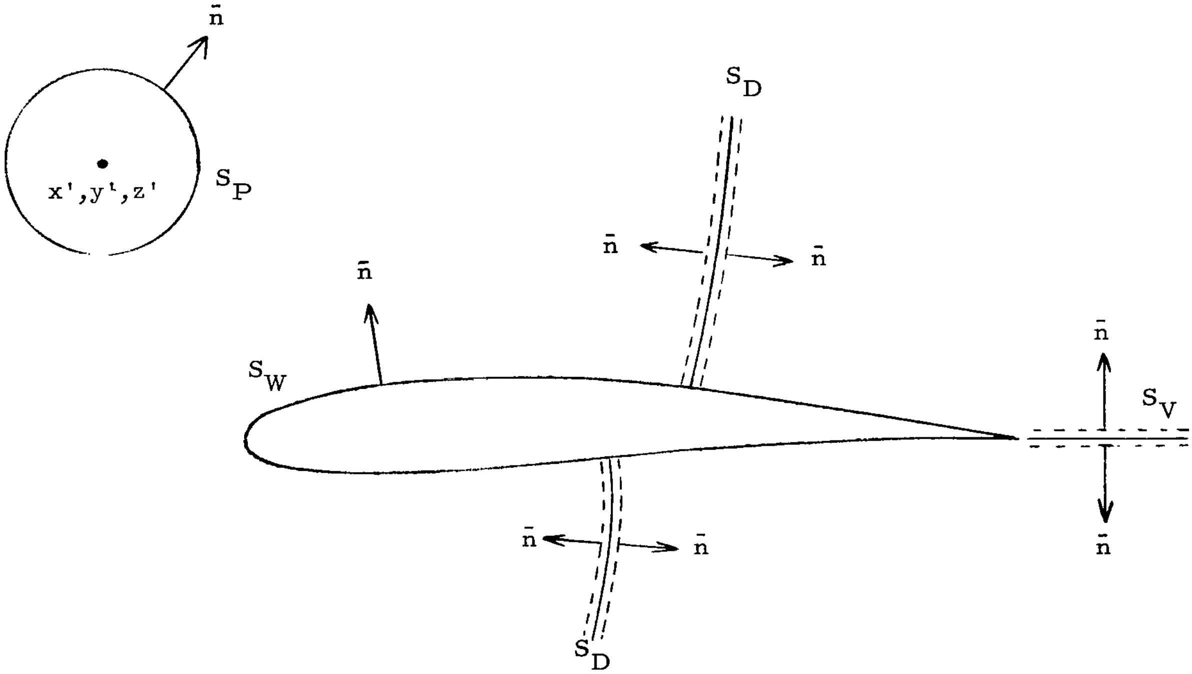

Figure 1.- Geometric surface comprising boundary surface S.

surface S_P surrounding the singular point (x',y',z'), (3) the surface S_D around any shock discontinuity, (4) the wing surface S_W, and (5) the trailing vortex sheet S_V. The integral over the surface at infinity, with ψ given by equation (5), vanishes under the condition $\phi \sim R^{-\epsilon}$, for $\epsilon > 0$, everywhere at infinity except in a region surrounding the trailing vortex sheet where ϕ is required to be bounded and antisymmetric only in z. The far-field expression for ϕ will be shown to have these properties and hence the integration over the surface at infinity vanishes and need not be considered further.

In the following development the potential at the field point (x',y',z'), which is obtained by integration over the surface S_P, is related to the remaining surface and volume integrals with the use of equation (3). The integrals over any shock discontinuity combine, so the integrand vanishes and thus the combined integral makes no contribution to ϕ. The integration over the trailing vortex sheet is transformed through an integration by parts so that the potential at the field point is given in terms of only surface integrals over the wing, which represent source and vortex distributions, and a volume integration throughout the field.

Integral over volume V.- Since $L(\psi) \equiv 0$, the only contribution to the left-hand side of equation (3) comes from the term $\psi L(\phi)$. The reduction is made with the use of the right-hand side of equation (2) for $L(\phi)$ together with an integration by parts in the x'-direction. The integration by parts leads to integrals over the bounding surface of the volume V, all of which vanish except those over any shock discontinuity. Thus,

$$\int_V \psi L(\phi) dV' = \frac{\gamma + 1}{2\beta^2} \left\{ \int_{S_D} \psi \left[u^2\right] \cos \vartheta_1' \, dS' - \int_V \psi_{\xi'} u^2 dV' \right\} \tag{7}$$

where the jump across a shock surface is

$$\left[u^2\right] = u_2^2 - u_1^2$$

with the subscripts 1 and 2 referring to the upstream and downstream values, respectively, at the shock-discontinuity surface S_D. The upstream unit vector normal to the shock has been taken as

$$\bar{n} = \bar{i} \cos \vartheta_1' + \bar{j} \cos \vartheta_2' + \bar{k} \cos \vartheta_3'$$

and in equation (7)

$$\cos \vartheta_1' \, dS' = \mathrm{sgn}\left(\cos \vartheta_1'\right) d\eta' \, d\zeta' = -d\eta' \, d\zeta'$$

Integral over surface surrounding singular point S_P.- The field point (x',y',z') is a singularity of the equations for $R = 0$ and must be excluded from the region of integration. The surface surrounding the field point is taken as the sphere $R = R_o$. Then

$$\frac{\partial\psi}{\partial n} = -\frac{1}{4\pi R_o^2}$$

and the element of area is $dS' = R_o^2 d\omega$, where ω is the solid angle. Integration over the surface S_P gives

$$\int_{S_P}\left(\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n}\right)dS' = \lim_{R_o\to 0}\frac{1}{4\pi}\int_0^{4\pi}\left(-\frac{\phi}{R_o^2} - \frac{1}{R_o}\frac{\partial\phi}{\partial R_o}\right)R_o^2 d\omega$$

$$= -\phi(x',y',z') \tag{8}$$

Integral over surface of discontinuity S_D.- The quantities ψ, $\frac{\partial\psi}{\partial n}$, and ϕ are continuous across any shock surface and the normal velocity $\frac{\partial\phi}{\partial n}$ is discontinuous there. Thus, these surfaces must be taken as boundaries of the region of integration in order to satisfy the requirements of Green's theorem The integral over S_D becomes

$$\int_{S_D}\left(\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n}\right)dS' = \int_{S_D}\left[\frac{\partial\phi}{\partial n}\right]\psi\, dS' \tag{9}$$

where $\left[\frac{\partial\phi}{\partial n}\right]$ denotes the jump in $\frac{\partial\phi}{\partial n}$ across the shock and $\frac{\partial}{\partial n}$ is in the direction of the upstream normal to S_D. Equation (9) will be used with $\left[\frac{\partial\phi}{\partial n}\right]$ expressed in the form

$$\left[\frac{\partial\phi}{\partial n}\right] = -\left[\phi_{\xi'}\right]\cos\vartheta_1' - \left[\phi_{\eta'}\right]\cos\vartheta_2' - \left[\phi_{\zeta'}\right]\cos\vartheta_3' \tag{10}$$

Integral over wing surface S_W.- For a thin wing $\frac{\partial}{\partial n} = \pm\frac{\partial}{\partial\zeta'}$ on the wing surface. It is convenient to employ the notation $\Delta\phi$ and $\Delta\frac{\partial\phi}{\partial\zeta'}$ to designate the value on the upper surface minus the value on the lower surface. Thus, $\Delta\phi$ is the jump in potential across the wing surface and $\Delta\frac{\partial\phi}{\partial\zeta'}$ is related to the wing thickness distribution. The integral over the wing surface then becomes

$$\int_{S_W}\left(\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n}\right)dS' = \int_{S_W}\left(\frac{\partial\psi}{\partial\zeta'}\Delta\phi - \psi\Delta\frac{\partial\phi}{\partial\zeta'}\right)dS' \tag{11}$$

Integral over trailing vortex sheet S_V.- For lifting wings there is a trailing vortex sheet S_V which extends downstream from the wing trailing edge and lies in the X'Y'-plane in the small-disturbance approximation. The quantities ψ, $\frac{\partial\psi}{\partial n}$, and $\frac{\partial\phi}{\partial n}$ are continuous on S_V whereas ϕ is discontinuous there. Thus,

$$\int_{S_V}\left(\phi \frac{\partial\psi}{\partial n} - \psi \frac{\partial\phi}{\partial n}\right)dS' = \int_{S_V} \frac{\partial\psi}{\partial\zeta'} \Delta\phi \, dS' \tag{12}$$

Integral equation for velocity potential.- Equations (7) to (12) provide the relations required to relate the potential at the field point to the surface integrals over the wing, the trailing vortex sheet, and any shock discontinuities as well as to a volume integral throughout the space. Substitution into equation (3), together with a transformation to the unprimed variables, gives

$$\phi = \int_{S_W}\left(\frac{\partial\psi}{\partial\zeta} \Delta\phi - \psi \, \Delta w\right) dS + \int_{S_V} \frac{\partial\psi}{\partial\zeta} \Delta\phi \, dS + \frac{\gamma + 1}{2} \int_V \psi_\xi u^2 dV$$

$$+ \int_{S_D} \psi \left(\left\{\beta^2[u] - \frac{\gamma + 1}{2}[u^2]\right\} \cos \vartheta_1 + [v] \cos \vartheta_2 + [w] \cos \vartheta_3\right) dS \tag{13}$$

where

$$\psi = \frac{1}{4\pi R}$$

with R now defined as

$$R = \left\{(x - \xi)^2 + \beta^2(y - \eta)^2 + \beta^2(z - \zeta)^2\right\}^{1/2} \tag{14}$$

and where the velocity components are $u = \phi_\xi$, $v = \phi_\eta$, and $w = \phi_\zeta$. The relations

$$\cos \vartheta_1' \, dS' = \beta^2 \cos \vartheta_1 \, dS$$

$$\cos \vartheta_2' \, dS' = \beta \cos \vartheta_2 \, dS$$

$$\cos \vartheta_3' \, dS' = \beta \cos \vartheta_3 \, dS$$

have been used in the transformation of the surface integrals from the primed to unprimed coordinates.

The direct expansion of the shock polar equation for weak jumps gives the equation

$$\left\{\beta^2[u] - \frac{\gamma + 1}{2}[u^2]\right\} [u] + [v]^2 + [w]^2 = 0 \tag{15}$$

which may be regarded as a small-disturbance approximation to the shock jump conditions. Applying the divergence theorem to the irrotationality conditions gives

$$[v]\cos\vartheta_1 - [u]\cos\vartheta_2 = 0$$

$$[w]\cos\vartheta_1 - [u]\cos\vartheta_3 = 0$$

$$[w]\cos\vartheta_2 - [v]\cos\vartheta_3 = 0$$

and these relations combined with equation (15) give

$$\left\{\beta^2[u] - \frac{\gamma + 1}{2}[u^2]\right\}\cos\vartheta_1 + [v]\cos\vartheta_2 + [w]\cos\vartheta_3 = 0 \tag{16}$$

as a second form of the shock jump conditions for weak jumps. The left-hand side of equation (16) is a factor in the integrand of the integral over S_D in equation (13) and, consequently, that integral does not contribute to the potential ϕ. The two forms of the shock jump conditions are equivalent to those given in reference 4.

The terms in equation (13) with integrands $\frac{\partial\psi}{\partial\xi}\Delta\phi$ correspond to doublets, and the region of integration is over both the wing and the trailing vortex sheet. These integrals can be transformed to one extending only over the wing surface through integration by parts in the ξ-direction and by making use of the fact that $\Delta\phi$ is a function only of η on S_V. The potential becomes

$$\phi = \frac{z}{4\pi}\int_{S_W}\frac{\Delta u}{(y - \eta)^2 + z^2}\left(1 + \frac{x - \xi}{R}\right)dS - \int_{S_W}\psi\,\Delta w\,dS + \frac{\gamma + 1}{2}\int_V u^2\psi_\xi\,dV \tag{17}$$

The first term on the right-hand side of equation (17) corresponds to a vortex distribution and represents the lifting effects; the second term corresponds to a source distribution whose strength is related to the wing thickness distribution; and the third term, which arises from the nonlinearity of the flow equations, has the form of a doublet with its axis in the stream direction and with a strength given by the local value of u^2 and influences the results for both lifting and nonlifting wings.

The corresponding relation for two-dimensional flow is found from equation (17) by regarding Δu, u, and Δw to be functions only of the argument ξ and integrating on η from $-\infty$ to $+\infty$. Thus, for a two-dimensional flow,

$$\phi = \frac{1}{2\pi} \int_C \Delta u \left\{ \frac{\pi}{2} \operatorname{sgn}(z) + \tan^{-1}\left(\frac{x - \xi}{\beta z}\right) \right\} d\xi + \frac{1}{2\pi\beta} \int_C \Delta w \log \left\{ (x - \xi)^2 \right.$$

$$\left. + \beta^2 z^2 \right\}^{1/2} d\xi + \frac{\gamma + 1}{4\pi\beta} \int_S u^2 \frac{(x - \xi)dS}{(x - \xi)^2 + \beta^2(z - \zeta)^2} \tag{18}$$

where C designates the airfoil section and S is the two-dimensional region exterior to the airfoil. The term sgn(z) in the first integral is required to account for the jump in potential downstream of the trailing edge on $z = 0$ with the principal value taken for the arc tangent. Equations (17) and (18) provide the basis for obtaining the asymptotic form of the potential in the far field for the three-dimensional and two-dimensional wings, respectively.

Far-Field Representation

In the far field the distance from the wing is large compared with the dimensions of the wing. Each integral in the expression for the potential given by equation (17) or (18) can be approximated and simplified at large distances from the wing on this basis.

Lift integral.- There are two regions of the far field to consider for the approximation of the integral associated with the lift: (1) the region where the distance from the wing $\left(x^2 + y^2 + z^2\right)^{1/2}$ is large with both y and z exterior to a region O(b) surrounding the trailing vortex sheet and (2) the region where only x is large and y and z lie in a neighborhood of the trailing vortex sheet.

For the first region, where $\left(x^2 + y^2 + z^2\right)^{1/2}$ is large compared with the wing dimension,

$$x - \xi \rightarrow x$$

$$y - \eta \rightarrow y$$

$$R \rightarrow \left\{ x^2 + \beta^2\left(y^2 + z^2\right) \right\}^{1/2}$$

and the contribution from the integral associated with the lift in equation (17) becomes

$$\phi_{lift} \sim \frac{1}{4\pi} \frac{z}{y^2 + z^2} \left(1 + \frac{x}{R}\right) \int_{S_W} \Delta u \, dS \tag{19}$$

For the second region (in the far field in the neighborhood of the trailing vortex sheet), $x \gg b$ and both y and z are O(b). The term in parentheses in the first integral in equation (17) approaches a value of 2 as $x \to \infty$, and the contribution of the integral associated with the lift is

$$\phi_{lift} \sim \frac{z}{2\pi} \int_{-b/2}^{b/2} \frac{\Gamma(\eta)}{(y - \eta)^2 + z^2} d\eta \tag{20}$$

where the circulation Γ is

$$\Gamma(\eta) = \int_C \Delta u(\xi,\eta) d\xi$$

and the integration is over the chord. On the trailing vortex sheet, the only contribution from equation (20) comes when the denominator approaches zero. This limiting form is readily evaluated as

$$\left.\begin{aligned} \phi_{lift} &\sim \frac{\Gamma(y)}{2} & \left(z \to +0;\ -\frac{b}{2} \leqq y \leqq \frac{b}{2}\right) \\ \phi_{lift} &\sim -\frac{\Gamma(y)}{2} & \left(z \to -0;\ -\frac{b}{2} \leqq y \leqq \frac{b}{2}\right) \end{aligned}\right\} \tag{21}$$

The two approximations given by equations (19) and (20) are in agreement outside a region where y is $O(b)$ and z is $O(b)$.

Thickness integral.- In equation (17) the integral with the integrand $\psi\ \Delta w$ is related to the wing thickness distribution through the surface boundary condition

$$\Delta w = 2F_\xi(\xi,\eta)$$

where $F(\xi,\eta)$ is the wing thickness distribution. Integrating by parts and letting ξ and η approach zero gives the leading term of the far-field representation for the non-lifting part of the potential as

$$\phi_{thickness} \sim -\frac{x}{2\pi R^3} \int_{S_W} F(\xi,\eta) dS \tag{22}$$

which is of higher order than equation (19).

Volume integral.- The contribution to the far-field potential from the volume integral is shown to be of higher order than that associated with the lift. The volume integral in equation (17) extends over an infinite region; however, this integral can be limited to a finite interior region V_I with little error since u decays rapidly far from the wing. Differentiation of equation (17) shows that $u \sim \frac{1}{R^2}$ for large R. Then the contribution to the potential from the region exterior to V_I is

$$\phi \sim O\left(\frac{1}{R^3}\right)$$

and the contribution from the neighborhood of the field point is of even higher order.

The order of magnitude of the far-field potential due to the integral over the interior region V_I can be estimated with the use of the Schwarz inequality

$$\left(\int_{V_I} u^2\psi_\xi \, dV\right)^2 \leqq \int_{V_I} u^4 dV \int_{V_I} \psi_\xi{}^2 dV$$

The integral of $\psi_\xi{}^2$ can be obtained in closed form in cylindrical coordinates and the result can then be expanded in powers of $\frac{\bar{r}}{R}$ where $\bar{r}$ is related to the region of integration as shown in figure 2. The region of integration is the annular sector defined by

$$r - \bar{r} \leqq \tilde{r} \leqq r + \bar{r}$$

$$-\frac{\bar{r}}{r} \leqq \mu \leqq \frac{\bar{r}}{r}$$

$$-\bar{r} \leqq \xi \leqq \bar{r}$$

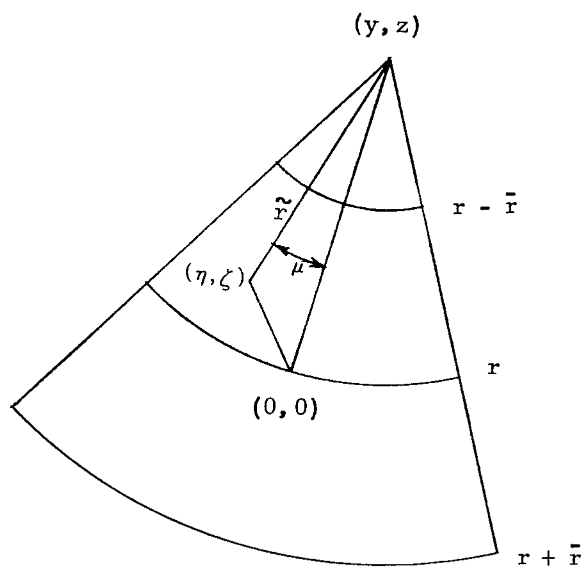

Figure 2.- Polar coordinates in YZ-plane.

where $r = \beta\sqrt{y^2 + z^2}$ and μ is the polar angle (fig. 2). The Schwarz inequality gives

$$\left(\int_{V_I} u^2 \psi_\xi \, dV\right)^2 \leqq \overline{u^4} V_I^2 \left\{\frac{x^2}{R^6} + \frac{\bar{r}^2}{R^6}\left(\frac{r^2}{R^6} - \frac{7}{6}\right) + O\left(\frac{\bar{r}^3}{R^7}\right)\right\} \tag{23}$$

where

$$\overline{u^4} = \frac{1}{V_I} \int_{V_I} u^4 dV$$

and the volume of the interior region is

$$V_I = (2\bar{r})^3$$

Equation (23) yields

$$\int_{V_I} u^2 \psi_\xi \, dV \sim O\left(\frac{x}{R^3}\right)$$

Thus the dominant contribution to the far-field potential from the volume integral is

$$\phi \sim \frac{\gamma + 1}{2} \int_{V_I} u^2 \psi_\xi \, dV$$

which is $O\left(\frac{x}{R^3}\right)$.

Velocity potential for the far field.- The leading terms of the potential for the far field are

$$\phi \sim \phi_{lift} - \frac{x}{2\pi R^3} \int_{S_W} F(\xi,\eta) dS + \frac{\gamma + 1}{2} \int_{V_I} u^2 \psi_\xi \, dV$$

where the various forms of the contribution due to lift ϕ_{lift} are given by equations (19) to (21). The terms associated with the lift largely dominate the far-field potential. These terms provide the proper jump across the trailing vortex sheet and they decay like the reciprocal of the distance in the regions above and below the wing as well as far downstream, whereas the contributions from the thickness and the volume integrals are $O\left(\frac{x}{R^3}\right)$. It should be noted that ϕ, as required by Green's theorem, has the proper form far from the wing $\left(\phi \sim R^{-\epsilon}\right.$, for $\epsilon > 0$, and ϕ is bounded and antisymmetric in z near the trailing vortex sheet$\left.\right)$ to ensure that the integral over the surface at infinity vanishes.

The far-field representation for the two-dimensional airfoil is found from equation (18) to be

$$\phi \sim \frac{\Gamma}{2\pi}\left\{\frac{\pi}{2}\,\mathrm{sgn}(z) + \tan^{-1}\left(\frac{x}{\beta z}\right)\right\} + \frac{1}{\pi\beta}\,\frac{x}{x^2 + \beta^2 z^2}\int_C F(\xi)d\xi$$

$$+ \frac{\gamma + 1}{4\pi\beta}\int_{S_I} u^2 \frac{(x - \xi)dS}{(x - \xi)^2 + \beta^2(z - \zeta)^2}$$

where C designates the airfoil section and S_I is the two-dimensional equivalent of region V_I. The two-dimensional potential is singular for $M_\infty = 1$; the free-stream Mach number can approach 1 only as the thickness ratio τ simultaneously approaches 0 so that the transonic similarity parameter $\beta^2/\tau^{2/3}$ remains of order 1.

Application of the far-field equation.- The far-field representation developed herein would be used for a three-dimensional-wing computation in the same manner as the corresponding two-dimensional far-field representation was used in the numerical calculations of Murman and Cole (ref. 4) and Krupp and Murman (ref. 7). A numerical solution of equation (1) would be made within a limited region surrounding the wing and the values of the potential would be matched to the far-field values on the outer boundary of the computational grid to satisfy, in effect, the boundary conditions at infinity. Only that part of the potential which is associated with the lift would be used for the far-field representation for lifting wings since the other terms are of higher order and may be neglected. The far-field value of the potential associated with the lift as given by equation (19) would be used everywhere except in some neighborhood of the trailing vortex sheet far downstream of the wing where equation (20) would be used. The region where the different forms of the equation for ϕ_{lift} (see eqs. (19) to (21)) merge would have to be established by trial. The integrands in the far-field representation contain the unknown velocity component u and consequently have to be evaluated during the finite-difference computation. Thus an iterative procedure is required in the numerical process.

CONCLUDING REMARKS

A far-field representation for the transonic flow about lifting wings is developed for the case where the free-stream Mach number is subsonic. The term associated with the lift is shown to dominate the far-field solution. This representation provides a means for both limiting the computational network for a finite-difference computation (thereby limiting the computer storage requirements) and satisfying the requirement of vanishing disturbances at infinity. A finite-difference computation would be made within a limited region surrounding the wing and the values of the potential would be matched on the outer boundary of the computational grid to satisfy, in effect, the boundary conditions at infinity.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., September 24, 1971.

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