Contributions of Individual Generators to Loads and Flows

Daniel Kirschen Ron Allan Goran Strbac
UMIST
Manchester, United Kingdom

Abstract - Because of the introduction of competition in the electricity supply industry, it has become much more important to be able to determine which generators are supplying a particular load, how much use each generator is making of a transmission line and what is each generator's contribution to the system losses. This paper describes a technique for answering these questions which is not limited to incremental changes and which is applicable to both active and reactive power. Starting from a power flow solution, the technique first identifies the busses which are reached by power produced by each generator. Then it determines the sets of busses supplied by the same generators. Using a proportionality assumption, it is then possible to calculate the contribution of each generator to the loads and flows. The applicability of the proposed technique is demonstrated using a 30-bus example.

Keywords: power system operations, transmission access, power flow, spot pricing, location-dependent pricing, power system economics.

Introduction

In many parts of the world, the electricity supply industry is undergoing unprecedented changes. While these changes take many forms (separation of traditional vertically integrated utilities into generation, transmission and distribution companies, introduction of retail wheeling, creation of markets for electric energy) the goal is always the introduction of competition and a lowering of the average consumer price.

While competition is introduced in generation and retail (or supply), it is widely agreed that transmission is a natural monopoly and should remain centrally controlled. It is also widely recognized that the operation of the transmission system can have an enormous impact on a competitive market. Competition will flourish only if all actual and potential market participants are convinced that the market is operating fairly.

Transparency in the operation of the transmission system is an essential ingredient in establishing this confidence. In this respect, generators, suppliers and network operating companies are likely to want accurate and indisputable answers to questions such as "how far is the power generated by this unit really going?" or "which generators are supplying this load?" or even "which generator is making the biggest use of this transmission line?". Before the introduction of competition, these questions were of limited and mostly academic interest because all of the power was generated by the same utility company or bought under fairly straightforward contracts. Furthermore, conventional wisdom suggested that, except for radial networks and other special configurations, they did not have any answer.

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Since the introduction of competition in various countries around the world and the introduction of wheeling in North America, these questions have had to be addressed in various forms. While the approaches which have been implemented are reasonable and reflect sound engineering judgement, it is probably fair to say that their scope is limited and that their application is not entirely satisfactory. In the United Kingdom these issues were deemed too complex and were deliberately set aside. Consequently, a single non-geographically differentiated electricity market was created and generators are compensated if they are not allowed to produce due to transmission constraints[1]. Connection charges depend on the location but are based on capacity and not energy. On the other hand, the longitudinal nature of the Chilean power system makes possible the introduction of the concept of influence area based on sensitivity analysis [2]. These areas of influence are used to allocate the cost of the transmission system among the competing generators. In North America, the introduction of wholesale wheeling has led to the development of concepts such as "contract paths" and the pricing of transmission services based on "MW-miles"[3]. The problem of "loop flows" or "parallel paths" in the Eastern Great Lakes region required the implementation of a complex agreement involving many utilities[4].

This paper describes a technique for determining which generators are supplying a particular load, how much use each generator is making of a transmission line and what is each generator's contribution to the system losses. The proposed technique is not limited to incremental changes and is applicable to both active and reactive power. Starting from a power flow solution, the technique first identifies the busses which are reached by power produced by each generator. Then it determines the sets of busses supplied by the same generators. Using a proportionality assumption, it is then possible to calculate the contribution of each generator to the loads and flows.

The concepts which form the basis of the proposed method and the algorithms which are required to put it into practice are described in the following sections with the help of simple examples. Possible applications are then briefly discussed. Finally the applicability of the method is demonstrated using the standard 30-bus test system.

Concepts and Algorithms

Overview

Based on the active or reactive branch flows from a solved power flow or state estimation computation, the proposed method organizes the busses and branches of the network into homogeneous groups according to a few concepts which are introduced below. Once this organization is complete, it is possible to answer questions such as "how far does the power produced by this generating unit go?" or "which generators are supplying this load?". It is also possible to represent the state of the system by a directed, acyclic graph. Further processing of this graph provides the answer to questions such as "how much use is this generator making of this line?" or "what proportion of the system losses is produced by that generator?".

This method is applicable independently to both active and reactive power flows. In the following description, the term "power" can be replaced by either "active power" or "reactive power" depending on the desired application.

Domain of a Generator

The domain of a generator is defined as the set of busses which are reached by power produced by this generator. Power from a generator reaches a particular bus if it is possible to find a path through the network from the generator to the bus for which the direction of travel is always consistent with the direction of the flow as computed by a power flow program or a state estimator.

For example, it can easily be seen that, for the small system shown in Fig. 1, the domain of generator A encompasses all the busses while the domain of generator B includes only busses 3, 4, 5 and 6 and the domain of generator C is limited to bus 6. As could be expected, there is a significant overlap between the domains of the various generators.

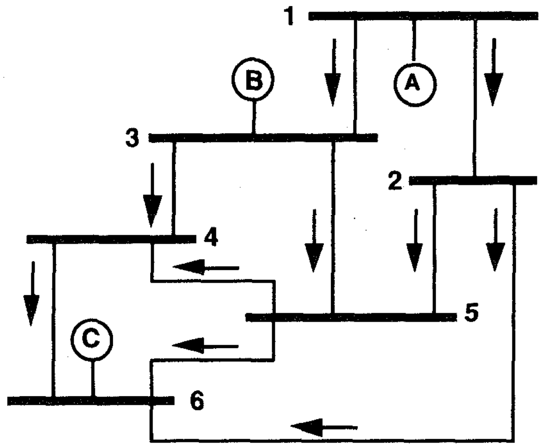

Fig. 1: 6-bus example used to illustrate the concepts

For larger systems, the domain of a generator can be determined using the following algorithm:

Place the bus where the generator is connected on the open list

While there are busses on the open list do:
Remove the first bus from the open list
Add this bus to the domain of the generator
Loop over all the branches connected to this bus:
If the power on this branch flows away from this bus and if the bus at opposite end of the branch is not yet part of the domain, then:
Add the opposite bus to the list of open busses.
End if
End loop
End while

Note that the "active domain" of a generator does not usually cover the same set of busses as its "reactive domain".

The concept dual to the domain of a generator could be dubbed the catchment area of a load and is defined as the set of busses which are reached by power consumed by this load. Its extent can be computed using the same algorithm as above but starting from the load and considering only the branches which carry power flowing towards the load. In the example of Fig. 1, the catchment area of a load connected to bus 5 includes busses 5, 3, 2 and 1 and hence generators A and B.

Commons

By itself the domain of a generator is an interesting concept but its applicability is limited due to the heavy overlap between the domains of the various generators. The concept of commons is more useful, albeit somewhat less intuitive. A common is defined as a set of contiguous busses supplied by the same generators. Unconnected sets of busses supplied by the same generators are treated as separate commons. A bus therefore belongs to one and only one common. The rank of a common is defined as the number of generators supplying power to the busses comprising this common. It can never be lower than one or higher than the number of generators in the system.

The example of Fig. 1 contains three commons:

- Busses 1 and 2 which are supplied by generator 1 only (common 1, rank 1)
- Busses 3, 4 and 5 which are supplied by both generators 1 and 2 (common 2, rank 2)
- Bus 6 which is supplied by all three generators (common 3, rank 3)

For networks of a more realistic size, the following algorithm determines the commons efficiently:

Determine the domain of each generator
Record with each bus the generators which supply this bus
Loop over all the busses
If this bus is not yet part of a common, then:
Create a new common based on the generators supplying this bus
Recursively propagate this common to all the busses connected to this bus
End if
End loop

Links

Having divided the busses into commons, each branch is either internal to a common (i.e. it connects two busses which are part of the same common) or external (i.e. it connects two busses which are part of different commons). One or more external branches connecting the same commons form what will be called a link. It is very important to note that the actual flows in all the branches of a link are all in the same direction. Furthermore, this flow in a link is always from a common of rank N to a common of rank M where M is always strictly greater than N.

In the example of Fig.1, there are three links:

- Link 1 which connects commons 1 and 2 and consists of branches 1-3 and 2-5
- Link 2 which connects 2 and 3 and consists of branches 4-6 and 5-6
- Link 3 which connects commons 1 and 3 and consists of branch 2-6

Branches 3-4, 3-5 and 4-5 are internal to common 2. Branch 1-2 is internal to common 1. There are no internal branches in common 3.

State Graph

Given the direction of the flows in all the branches of the network, the algorithms described above produce unique sets of commons and links. If the commons are represented as nodes and the links as branches, the state of the system can be represented by a directed, acyclic graph. This graph is directed because the direction of the flow in a link is specified. It is acyclic because links can only go from a common supplied by fewer generators to a common supplied by more generators. Typically, the root nodes of such a graph correspond to a common of rank one while the leaves consists of the highest ranked commons.

The state graph of the system of Fig.1 is shown in Fig.2. Such a small system obviously leads to an almost trivial graph. A much more interesting example is given in the section presenting the results obtained with the 30 bus test system.

It should be emphasized that a reversal in the direction of the flow of power in a single transmission line or transformer can radically alter the size and shape of this state graph representation of the system. Such a reversal can considerably increase or decrease the domain of a generator and hence cause the creation or the disappearance of several commons and links.

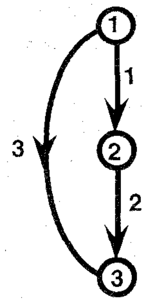

Fig. 2: State graph for the 6-bus example of Fig. 1.

Contribution to the Load of a Common

The results obtained so far provide a qualitative view of the system. To obtain quantitative information, a few more definitions and a fundamental assumption are required.

The inflow of a common is defined as the sum of the power injected by sources connected to busses located in this common and of the power imported in this common from other commons by links. This inflow is always strictly positive. For root nodes of the state graph it includes only the power injected within the common as there are no imports. The outflow of a common is equal to the sum of the power exported through links from this common to commons of higher rank. The inflow of a common is equal to the sum of its outflow and of all the loads connected to the busses comprising the common.

Further results are dependent on the following proportionality assumption:

For a given common, if the proportion of the inflow which can be traced to generator i is x_i, then the proportion of the outflow which can be traced to generator i is also x_i.

Like all postulates, this assumption can neither be proven or disproven and its only justification is that it appears more reasonable than any other possible assumption. These other assumptions would imply that the power traceable to some generators is disproportionately consumed in the common while the power traceable to other generators is disproportionately transmitted to other commons. Considering that the definition of a common states that all busses within the common are reached by power traceable to the same set of generators, these competing assumptions do not seem to have any reasonable physical basis.

It can easily be shown that the following statement is a corollary or an alternate formulation of the proportionality assumption:

For a given common, if the proportion of the inflow which can be traced to generator i is x_i, then the proportion of the load which can be traced to generator i is also x_i.

This assumption provides the basis of a recursive method for determining the contribution of each generator to the load in each common. Using the following notations:

C_{ij}: Contribution of generator i to the load and the outflow of common j

C_{ik}: Contribution of generator i to the load and the outflow of common k

F_{jk}: Flow on the link between commons j and k

F_{ijk}: Flow on the link between commons j and k due to generator i.

I_k : Inflow of common k

then:

$$F_{ijk} = C_{ij} * F_{jk} \quad (1)$$

$$I_k = \sum_j F_{jk} \quad (2)$$

$$C_{ik} = \frac{\sum_j Fijk}{I_k} \quad (3)$$

These recursive equations can be used to compute the contribution of each generator to each common if they can be initialized. Fortunately, the inflow of the root nodes of the state graph is produced entirely by the generators embedded in these commons. The proportion of the outflow traceable to each of these generators can therefore be readily computed and propagated to commons of higher rank.

An example based on the system shown in Fig. 1 is used to clarify this procedure. Figure 3 provides additional data about generations, loads in commons and flows on links.

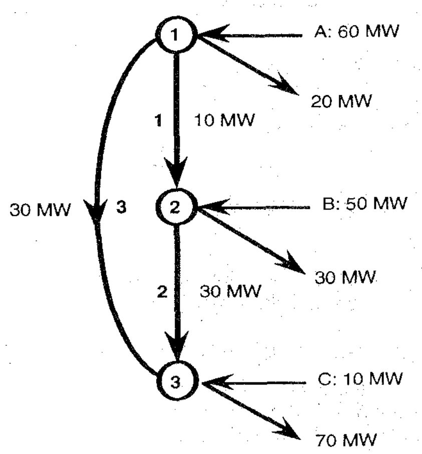

Fig. 3: Additional load, generation and flow data for the 6-bus example. Losses are neglected.

First compute the inflows of each common:

common 1: 60 MW
common 2: 50 + 10 = 60 MW
common 3: 10 + 30 + 30 = 70 MW

Then, compute the contributions starting from the root node of the state graph:

Relative contributions to the load and outflow of common 1:

Generator A: 60 / 60 = 1.0 p.u.
Absolute contributions to the inflow of common 2:

Generator A: 10 x 1.0 = 10 MW
Generator B: 50 MW

Relative contributions to the load and outflow of common 2:

Generator A: 10 / 60 = 0.167 p.u.
Generator B: 50 / 60 = 0.833 p.u.

Absolute contributions to the inflow of common 3:

Generator A: 30 x 1.0 + 30 x 0.167 = 35 MW

Generator B: 30 x 0.833 = 25 MW
Generator C: 10 MW
Relative contributions to the load of common 3 (and to its outflow if there was any):

Generator A: 35 / 70 = 0.500 p.u.
Generator B: 25 / 70 = 0.357 p.u.
Generator C: 10 / 70 = 0.143 p.u.

In other words, it is now possible to conclude that generator A produces 50% of the load consumed in common 3 but only 16.7% of the load consumed in common 2.

Contributions to Individual Loads and Branch Flows

Considering that all busses within a common are indistinguishable from each other as far as power tracing is concerned, it is reasonable to apply the proportionality assumption not only to the common taken as a whole but also to each bus load and to each branch flow taken independently within a common. In other words, if x_{ij} is the contribution of generator i to common j, it is also the contribution of generator i to every bus load and to every branch flow within common j and to every branch flow in the outward links of common j.

Knowing the common to which a bus belongs and the contributions of each generator to each common therefore gives the ability to compute how much power each generator contributes to each load. It also makes it possible to compute what proportion of the use of each branch can be apportioned to each generator. For branches linking busses in separate commons, the proportion of usage should be based on the contribution of the generator to the lower ranked common.

Since it is reasonable to assume that generators contribute to the losses in a branch in proportion to their use of this branch, it is possible to compute what proportion of the output of generator is dissipated in losses in the system.

Applications

Identification of the commons and the calculation of the contributions does not require much computer time and could therefore be carried out on-line (based on the output of a state estimator) as well as off-line (based on the results of a power flow program). On-line computations would have to be performed every few minutes to track the evolution of the system as the load and generation patterns change during the day. The concepts described in this paper could have the following applications:

- Geographically-differentiated spot pricing: the price charged to consumers could be computed on the basis of the relative contribution of each generator to their load and the price of each of these generators.
- Pricing of transmission services: generators could be charged for transmission services based on their actual use of each transmission line.
- Apportionment of the losses: the proposed method makes it possible to compute the fraction of a generator's output which is actually delivered to consumers. In a fair market for electricity, generators should be compensated on this basis, not on the basis of their output.
- By comparing the contributions to the active and reactive power flows in a branch, it may be possible to determine whether each generator is producing its "fair share" of the reactive power needed to keep the system operating.
- Visualization: the concepts of domains and commons could be used to help operators get a better understanding of the state of the power system.

It is clear that the application of the proposed method to pricing problems raises important and complex issues of fairness. Short of relocating, consumers would have no control on the price they would be charged. Similarly, generators might complain that they would be charged for transmission services without having any control on how the power they produce reaches its destination, or even what that destination is.

Numerical Results

A computer program has been written to show how the concepts described above can be put into practice and to test the correctness of the proposed algorithms. The results obtained with this program are illustrated using the standard 30-bus test system shown on Fig. 4. This network is sufficiently large to demonstrate the various situations which can be encountered and small enough to allow an intuitive understanding of the results.

Tables A-1 and A-2 in the appendix give the active power generation, load and flow data required to reproduce the results discussed in this section. Table 1 shows the extent of the domain of the six generators of this system. Note that the size of these domains varies between 1 bus for the generator at FIELDALE and 26 busses (i.e. most of the system) for the generator at GLEN-LYN. Having established the domains, it is now possible to determine the commons. These are shown as closed contours in Fig. 4. Note that in this example, no common is supplied by all six of the generators in the system. Note also that commons 1, 3 and 6 are supplied by the same generators but are treated as separate commons because they are unconnected.

Generator	Domain
B11	CLOVERDL, FIELDALE, BLAINE, ROANOKE, B21, B17, B26, B30, B29, B27, B25, B24, B22, B19, B20, B10, B9, B11
B13	B17, B16, B14, B19, B18, B26, B30, B29, B27, B25, B24, B23, B15, B12, B13
FIELDALE	FIELDALE
REUSENS	FIELDALE, BLAINE, B21, B17, B26, B25, B24, B22, B19, B20, B10, ROANOKE, B30, B29, B27, CLOVERDL, REUSENS
CLAYTOR	FIELDALE, CLOVERDL, BLAINE, B21, B22, B20, B10, ROANOKE, B17, B16, B14, B19, B18, B26, B30, B29, B27, B25, B24, B23, B15, B12, HANCOCK, CLAYTOR
GLEN-LYN	CLAYTOR, CLOVERDL, FIELDALE, BLAINE, B21, B22, B20, B10, ROANOKE, B17, B16, B14, B19, B18, B26, B30, B29, B27, B25, B24, B23, B15, B12, HANCOCK, KUMIS, GLEN-LYN

Table 1: Domain of the generators for the 30 bus example

Figure 5 shows how these commons and the 14 links which join them form the directed, acyclic state graph. Using the information contained in Tables A-1 and A-2, it is then possible to compute the load and the inflow of each common as well as the flows on the links. Starting from the root nodes of the state graph (commons 5, 7, 10 and 11) and moving towards the leave nodes (commons 1, 3, 6 and 9) it is finally possible to compute the contributions of the generators to each of the commons. These contributions are summarized in the matrix shown in Table 2. The sparsity of this matrix is an indication of how much "power mixing" takes place in the system at a particular time. It is also interesting that for the commons where power mixing does take place, the contributions vary from almost 100% to almost nothing.

On the basis of this matrix of contributions, it is possible to claim that 16.4% of the 9 MW load at bus B17 is supplied by the generator at bus B11, representing 3.0% of the output of this generator. Similarly, generator REUSENS is responsible for 20.6% of the flow on the line between busses B10 and B21.

By allocating the losses in each line on the basis of each generator's contribution to the flow in that line, it is possible to show that generator GLEN-LYN is responsible for 60.2% of the system losses but provides only 23.6% of its total generation. On the other hand, generator REUSENS provides 17.3% of the system generation but causes only 5.6% of the losses.

Conclusions

A simple method for computing the contribution of each generator to a given load or to the flow in a line has been described and demonstrated. It is applicable independently to active and reactive power flows and is not limited to incremental changes. The numerical example demonstrates that this approach can objectively assess the contributions made by each generator in the system and can answer the rhetorical questions posed in the Introduction. This method could be used to resolve some of the difficult pricing and costing issues which arise from the introduction of competition in the electricity supply industry and to ensure fairness and transparency in the operation of the transmission system.

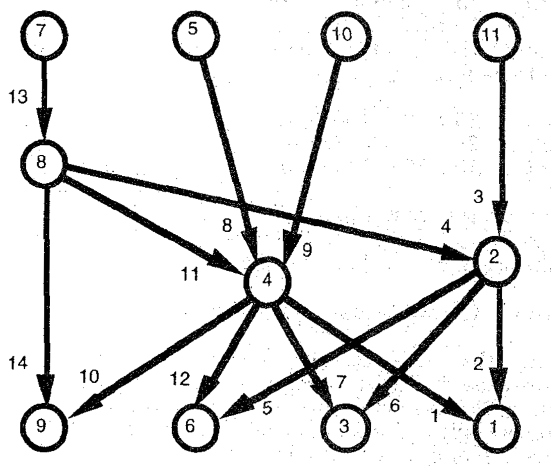

Fig. 5: State graph of the 30-bus example.

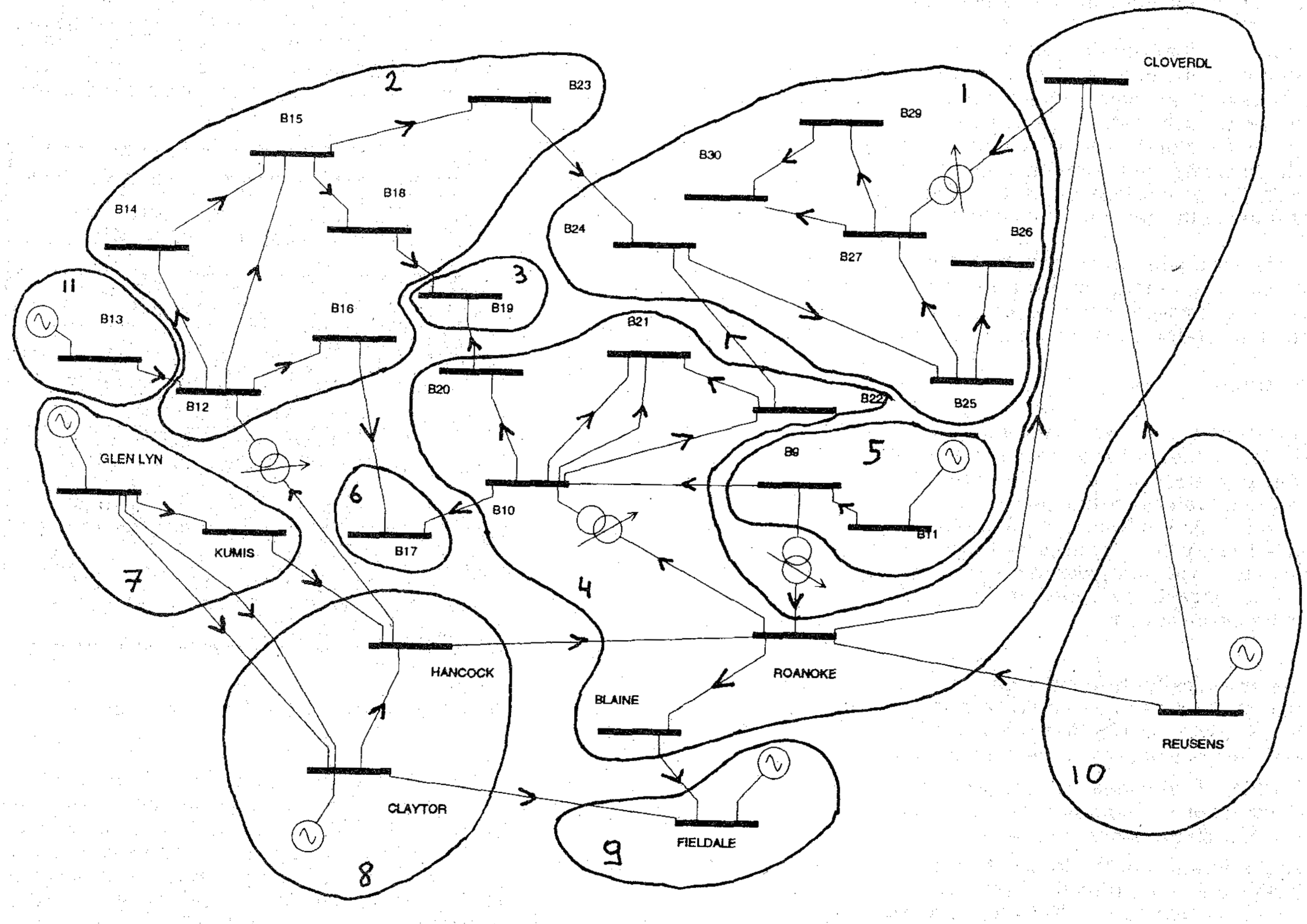

Fig. 4: One-line diagram of the 30-bus test system. The arrows on the branches represent the direction of the active power for the case under consideration. The commons are indicated by closed contours.

	1	2	3	4	5	6	7	8	9	10	11
B13	0.173	0.987	0.402			0.674					1.00
REUSENS	0.170		0.122	0.206		0.065			0.039	1.00	
B11	0.427		0.307	0.517	1.00	0.164			0.098		
GLEN-LYN	0.141	0.008	0.104	0.170		0.059	1.00	0.613	0.328		
CLAYTOR	0.089	0.005	0.065	0.107		0.038		0.387	0.207		
FIELDALE									0.328		

Table 2: Contribution of the generators to the commons of the 30-bus example.

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Biographies

Daniel S. Kirschen received his Electrical and Mechanical Engineer's degree from the University of Brussels, Belgium and his Master's and PhD degrees in electrical engineering from the University of Wisconsin-Madison. From 1985 to 1994 he worked for Siemens-Empros where he did research on network, scheduling and artificial intelligence applications. He is currently a Lecturer at UMIST. His e-mail address is daniel.kirschen@umist.ac.uk.

Ron Allan is Professor of Electrical Energy Systems in the Manchester Centre for Electrical Energy at UMIST, Manchester, England. He obtained a DipTech from Portsmouth Polytechnic in 1961, and his MSc and PhD from UMIST in 1963 and 1966 respectively. In 1987, he was awarded a DSc from the University of Manchester in recognition of his research contributions. He joined the UMIST as an academic member of staff in 1966. He is a Fellow of the IEEE, of the IEE (UK), and the Safety and Reliability Society (UK). He is a registered Chartered Engineer in the UK.

Goran Strbac is a Lecturer in the Manchester Centre for Electrical Energy at UMIST, Manchester, England. He obtained his BSc in Electrical Engineering in 1983 from the University of Novi Sad, and his MSc and PhD from the University of Belgrade in 1983 and 1989 respectively. He was a System Engineer at Elektro-Vojvodina, an electricity company in Novi Sad between 1984 and 1987, a Research and Teaching Associate at the University of Novi Sad between 1987 and 1993, and a Research Assistant at Imperial College, London from 1993 until he joined UMIST in November 1994.

Appendix: Case data for the 30-bus example

From Bus	To Bus	From Flow	To Flow
B15	B23	7.76	7.70
B30	B29	-3.67	-3.70
B29	B27	-6.10	-6.19
CLOVERDL	B27	12.96	12.96
B27	B25	-0.33	-0.33
B25	B26	3.54	3.50
B25	B24	-3.88	-3.91
B24	B23	-4.48	-4.50
B14	B15	2.29	2.28
B15	B12	-20.81	-21.09
B12	B14	8.58	8.49
B16	B12	-9.67	-9.75
B15	B18	7.12	7.07
B18	B19	3.87	3.86
B19	B20	-5.63	-5.65
B20	B10	-7.85	-7.92
B22	B24	8.22	8.13
B21	B22	-0.33	-0.32
B22	B10	-8.54	-8.61
B10	B9	-39.00	-38.99
B9	B11	-50.00	-50.00
B10	B17	2.87	2.85
B17	B16	-6.14	-6.17
HANCOCK	B12	0.64	0.64
GLEN-LYN	KUMIS	24.06	23.61
KUMIS	HANCOCK	21.21	21.10
CLAYTOR	HANCOCK	14.09	13.98
ROANOKE	B10	3.51	3.51
HANCOCK	ROANOKE	26.84	26.74
ROANOKE	BLAINE	41.20	40.76
BLAINE	FIELDALE	17.96	17.81
FIELDALE	CLAYTOR	-45.53	-46.50
REUSENS	CLOVERDL	5.38	5.36
ROANOKE	REUSENS	-14.58	-14.61
B12	B13	-50.00	-50.00
B21	B10	-8.58	-8.65
B10	B21	8.65	8.58
ROANOKE	B9	-11.00	-11.00
B30	B27	-6.92	-7.10
GLEN-LYN	CLAYTOR	22.13	21.14
ROANOKE	CLOVERDL	7.61	7.60
CLAYTOR	GLEN-LYN	-21.14	-22.13

Table A-2: Active power flows in the branches

Bus	Generation (MW)	Load (MW)
B15	0.00	8.20
B23	0.00	3.20
B24	0.00	8.70
B25	0.00	0.00
B27	0.00	0.00
B29	0.00	2.40
B30	0.00	10.60
B14	0.00	6.20
B12	0.00	11.20
B18	0.00	3.20
B19	0.00	9.50
B20	0.00	2.20
B10	0.00	5.80
B22	0.00	0.00
B26	0.00	3.50
B16	0.00	3.50
B17	0.00	9.00
B13	50.00	0.00
B9	0.00	0.00
B11	50.00	0.00
B21	0.00	17.50
HANCOCK	0.00	7.60
KUMIS	0.00	2.40
GLEN-LYN	68.34	0.00
CLAYTOR	40.00	21.70
BLAINE	0.00	22.80
CLOVERDL	0.00	0.00
REUSENS	50.00	30.00
ROANOKE	0.00	0.00
FIELDALE	30.85	94.20

Table A-1: Active power generations and loads

Discussion

Rodrigo Palma and Hugh Rudnick (Universidad Católica de Chile, Santiago, Chile): We congratulate the authors for an original methodology that is very attractive for its ease of application. The critical aspect of the proposal is that of the proportionality assumption which states that: "For a given common, if the proportion of the inflow which can be traced to generator i is xi, then the proportion of the outflow which can be traced to generator i is also xi.". This assumption identifies the method and makes it different to other proposed in the literature.

We are interested in studying in more depth the applications foreseen by the authors and would appreciate their comments on the following questions related to those applications.

1.- Geographically-differentiated spot prices: we agree that based on generation costs, a weighted average of the contributions of each generator to a particular load could be determined. However, it is not clear how line saturation or cost of unserved load would be reflected. Have the authors made consistency studies on these aspects?

2.- Pricing of transmission services: The transmission pricing method can be classified as based on system usage [A,B]. We suggest the authors to qualitative and numerically compare their proposal with others being applied worldwide, such as postage stamp and contract path methods or marginally based schemes. Have they done such comparison at this stage?

Can the authors comment on the impact of such pricing method on the expansion of the transmission system? How would sunk costs affect the application of the method in a competitive generation environment?

We would like to discuss the application of the method to two simple systems, where the method provides interesting results:

2.1.- Let us assume a radial south-north system (common in countries in South America), where two generators inject at the extreme south, where resultant flows are always south-north

```
G1-|-----|3       4        5          6
   1     |------|--------|---------|... etc.
         |  -->    -->       -->
G2-|-----|
   2
```

Based on the authors' nomenclature, this system has 3 commons:
common 1: generators: 1, buses: 1
common 2: generators: 2, buses: 2
common 3: generators: 1 and 2, buses: 3, 4, 5, 6, etc.

The method indicates that lines l1-3 y l2-3 have to be fully paid by generators G1 and G2 respectively. Lines l3-4, l4-5, l5-6, etc., have to be paid in a constant proportion among generators, given their generation. If we assume that generation is defined by each plant's firm power, the method takes the character of a postage stamp one, no directly linked to use. Even if the network meshes, starting from bus 4, the domains of both generators remain the same. This implies that a method that pretends to allocate transmission payments based on system usage, may not remain so depending on how assignations are made.

2.2.- Let us look at another simple 3 bus radial system

```
   -->|       G2-->|             |
G1    |----------|-----------|
      |    -->     |->10  -->    |->20
          f12             f23
      1            2             3
```

where **bus 1** has no load and a low cost generation plant (the marginal plant), with no restrictions on minimum generation; **bus 2** has a load of 10 MW, a high cost generation plant with minimum generation 10 MW; **bus 3** has a load of 20 MW, no generation.

The resultant economic dispatch, no losses considered, is: G1=20 MW and G2=10 MW, with resultant flows F1-2=20 MW and F2-3=20 MW.

A sensitivity analysis would free generator 2 from transmission payments, given it only supplies local load. This generator would no agree on paying for line l2-3, at least for this dispatch condition.

The application of the method proposed by the authors indicates two commons:
common 1: generators: 1, buses: 1
common 2: generators: 1 y 2, buses: 2 y 3

Therefore, generator 2 has to pay for line l2-3 in proportion to its generation. How would this relate to the previous analysis that assigned no responsibility on that line?.

3.- Apportionment of the losses: the method is most interesting and could be of use in applications related to retail wheeling in subtransmission and/or distribution systems.

[A] Pérez-Arriaga, I., Rudnick, H., Stadlin, W. "International power system transmission open access experience". IEEE Transactions on Power Systems, Vol. 10, Nº1, February 1995, pp. 554-561.
[B] J.W.M.Lima, "Allocation of transmission fixed charges: an overview", Paper 95SM574-4 PWRS
Manuscript received February 26, 1996.

William W. Hogan (Harvard University, Cambridge, MA): The paper summarizes an interesting approach to reducing network flows to a representation in a directed, acyclical graph that might be used for purposes of cost allocation amongst the participants in a market. Although I have not verified the proofs of the method, it appears sound and would provide one way of visualizing the interactions in a network.

The analysis starts from an assumption that there must exist a well-defined and acceptable method for allocating total flows on a network with the intent of using these allocations for purposes of costing. This is closely akin to many other average cost pricing mechanisms which have been the dominant approach in regulated electric industries throughout the world. However, with the introduction of competitive markets, the primacy of place of average cost pricing gives way to the competitive principles of marginal cost pricing. According to these principles, one feature of a competitive market would be consistency with least cost dispatch and the use of locationally differentiated prices that represent the marginal cost of load at each location in the network. This, of course, is a completely different approach, again from first principles, which would produce quite different results in cost recovery.

For example, as the authors point out, correctly, marginal cost pricing on a spot market basis would be insufficient for covering the total cost of the system. Hence, some other method would be required to recover the total cost. However, it does not follow that the other method must replace locational marginal cost pricing. It is entirely possible, and, in fact, widely recognized, that pricing mechanisms could be extended to "two-part" tariffs with access charges to recover the remaining revenue requirement and locational marginal cost pricing to provide the correct economic signals. The difference in the impact on prices, compared with the allocation method described, can be seen clearly by the simple Figure 1 example. Even though this is an acyclic network, it contains parallel flows. These parallel flows, sometimes described less precisely as "loop flows", are the source of the difficulty in electric networks where prices at one location can be impacted by constraints on the system at distant locations. Reduction of the network to acyclical directed network, through the creation of "commons" and "links", does not eliminate this loop flow problem, which is the central complicating feature in pricing in a competitive market within an electric network. However, the least-cost dispatch framework, with locational pricing, does deal with the parallel flow problem and seems to have much to recommend it as a way of achieving economic efficiency.

The place where the average allocation methods, of the type the authors describe, probably have their greatest application, would be in sharing of fixed costs. Whether in allocating the existing costs of the system or in dividing the cost for investments among the various participants in a joint venture, there needs to be some method for finding an acceptable allocation. The flow-based methods, such as the one described, may have the advantage of being intuitively plausible as a way of allocating joint fixed costs and achieving an allocation which would not provide sufficient incentive for any of the participants to defect from the coalition needed to support the network.

A few years ago, Trans Power of New Zealand, when they were trying to allocate transmission and generation access costs amongst the various distribution suppliers in the system, adopted an algorithm for cost-sharing which is similar to the authors in the use of a proportionality assumption to distribute load back through the network to identify the fraction of each line or transformer "used" by each load. As I recall, they found this to produce a reasonably fair allocation of these fixed costs. However, when they returned to the issue of spot pricing, they used the locational-based marginal cost approach as the theoretical foundation of their pricing methodology.

This use of locational pricing is important both because of its economic theory and because of its practical consequences as it is being applied elsewhere in the world. It is already in place in one form or another in Argentina and Chile, Norway, New Zealand and Australia, and is being proposed seriously in the various power pools in the United States. It is also part of the recent decision by the California Public Utilities Commission, "A Structure of the Market for California". The challenge is to marry these economically-efficient locational pricing packages with access charges that will cover the remaining costs of the transmission grid. The right approach would seem to be some merger of these locational pricing methods with flow based allocation methods such as the one proposed by authors.

Raymond Johnson (Pacific Gas and Electric, San Francisco, CA): This discusser would like to congratulate the authors on their innovative proposed solution to the problem of determining the contribution of specific generators to loads, flows and losses.

Although the proportionality assumption used in determining the contribution of each generator in a common seems reasonable on a physical basis, it will have to be reconciled with bilateral contracting and wheeling practices. So for example, in Figure 3 of the paper, Generator B may have contracted to supply all of the 30 MW of the load in Common 2 as opposed to the 25 MW deemed to be its contribution. Similarly, Generator A may be wheeling power through Common 2 to supply loads in Common 3. This difference between 'physical' flows and 'contract' flows is the root cause of the loop flow problem. How can the allocations resulting from the proposed technique be reconciled with bilateral contracts and wheeling transactions?

Another and even more contentious issue arises in the allocation of losses. In the 30-bus example, over 60% of system losses are allocated to a single generator. This discusser suspects that GLEN-LYN is the slack generator in the load flow solution and that an optimal power flow solution may distribute losses more evenly. Nevertheless, the proposed technique will produce loss allocations significantly different from those resulting from the more commonly used marginal techniques such as penalty factors. How can the allocation of total losses be reconciled with marginal loss allocation methods?

Finally, for reactive power applications, how will authors extend the technique to handle those cases when reactive power flows into a line from both ends?

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D. S. Kirschen, R.N. Allan, G. Strbac (UMIST, Manchester, UK): Before replying to the specific issues raised by the discussers, we would like to address a fundamental point which underlies these discussions and informal conversations which we have had with colleagues since the presentation of the paper.

It must be stressed that the proposed approach is not an incremental method, i.e. it does not say anything about what would change if a small change was introduced in one of the variables. Instead, it provides a rigorous and accurate characterization of the flows and injections for a specific system condition. There is therefore no contradiction when our method shows that a particular injection does not contribute to the flow in some lines while sensitivities indicate that a *change* in this injection would have an effect on *all* line flows. Besides its simplicity and transparency, the proposed method has therefore the added advantage that its results are independent of the arbitrary choice of a slack generator.

Application to Transmission Pricing:

A large part of the discussions revolve around the applicability of the proposed method to transmission pricing. While our method can be used to compute the contribution of each network user to physical flow, we do not believe that it provides by itself a basis for computing transmission prices and charges because it does not take into account the cost of providing network security. Therefore, we do not believe that a detailed comparison with other pricing methods is warranted at this point.

We agree with Professor Hogan that methods based on marginal cost provide economic signals which are more likely to lead to an efficient market than methods based on average costs. Locational marginal cost pricing has the further advantage that it provides a consistent framework for handling congestion charges. However, we would like to point out that the proposed method could be applied to historical records to determine the transmission rights which underlie the "contract networks" method which Prof. Hogan has proposed.

Developing a pricing system which will provide the long term incentives necessary to foster an efficient expansion of the transmission system is an urgent, important and difficult problem. Short run marginal cost pricing creates perverse incentives not to expand the network and does not usually generate enough revenues to operate and expand the system. A method which reflects actual system usage (such as the method which we propose) could be used to provide these revenues.

Net Injections vs. Individual Generations and Loads

Example 2.2 of Mr Palma and Prof Rudnick illustrates that the contributions method can be applied either to the net injections at each bus or to the loads and generations taken individually and separately.

In this example, the net injection at bus 2 is indeed zero. On this basis, the inflow and outflow of common 2 (busses 2 and 3) is equal to 20 MW and generator 1 contributes 100% of this inflow and 100% of the flow on line 2-3.

On the other hand, if we consider the load and the generation at bus 2 separately, the inflow and outflow of common 2 are equal to 30 MW, 2/3 being provided by generator 1 and 1/3 by generator 2. The flow on line 2-3 is divided in the same proportions.

We believe that treating loads and generations separately reflects more accurately the physical world and is therefore "fairer." If the owners of the generator and the load at bus 2 object to paying system charges on the basis that "they are not using the transmission network," they should isolate themselves completely from the system and pay the price in terms of lost reliability.

Contracts and Transactions

In response to Dr Johnson's question, we don't believe that the proposed method can be "reconciled" with contracts and transactions which do not reflect the physical world. In fact, we believe that one of the benefits of the proposed method is to demonstrate the absurdity of some of the assumptions upon which these transactions are based.

LossAllocation

Contrary to what Dr Johnson suggests, the fact that GLEN-LYN might be the slack generator has no effect on the allocation of the losses. The proposed method allocates a large fraction of the losses to this generator because a significant part of these losses takes place in a line whose flow is contributed mostly this generator. As we stressed earlier, one of the benefits of this method for allocating the losses is that it is *independent* of the arbitrary choice of a slack bus.

Extension to Reactive Power Flows

Dr Johnson points out one of several difficulties which arise when the method is applied to reactive power:

- reactive power flows into some lines from both ends
- reactive power flows out of some lines from both ends
- reactive losses depend heavily on active flows
- even loads with unity power factor cause reactive power to flow in the network

It does not make much sense to apportion reactive flows on the basis of reactive loads because a significant part of the reactive flows are due to reactive losses which are caused by active flows. While it is justifiable to treat active power independently from reactive power, the converse is harder to justify. We are currently trying to develop a comprehensive framework to handle these issues and we hope to report on our results in the near future.

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