Control Barrier Functions for Multi-Agent Systems under Conflicting Local Signal Temporal Logic Tasks

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***Abstract*—Motivated by the recent interest in cyber-physical and interconnected autonomous systems, we study the problem of dynamically coupled multi-agent systems under conflicting local signal temporal logic tasks. Each agent is assigned a local signal temporal logic task regardless of the tasks that the other agents are assigned to. Such a task may be dependent, i.e., the satisfaction of the task may depend on the behavior of more than one agent, so that the satisfaction of the conjunction of all local tasks may be conflicting. We propose a hybrid feedback control strategy using time-varying control barrier functions. Our control strategy finds least violating solutions in the aforementioned conflicting situations based on a suitable robustness notion and by initiating collaboration among agents.**

***Index Terms*—Multi-agent systems, signal temporal logic, autonomous systems, cooperative control, hybrid systems.**

I. Introduction

COLLABORATIVE control of multi-agent systems deals with achieving global tasks such as consensus, formation control, connectivity maintenance, and collision avoidance (see [1] for an overview). A recent trend has been to extend beyond these standard objectives and to consider more complex global or local task specifications by using temporal logics [2]–[5]. Most of these works use linear temporal logic (LTL) and require a discrete abstraction of the physical system to then employ computationally costly graph search methods. Signal temporal logic (STL) [6], on the other hand, allows to impose tasks with strict deadlines and offers a closer connection to the physical system by the introduction of robust semantics [7], [8], hence offering the benefit of not necessarily requiring an abstraction of the system. Control methods for STL then consider discrete-time systems and result, even for single-agent systems, in computationally costly mixed integer linear programs [9]–[11]. We recently proposed an alternative approach for continuous-time systems by using time-varying feedback control strategies [12]–[14], which are computationally efficient and inherently robust due its feedback nature. For multi-agent systems, [2] and [12] assume that each agent is subject to a local task regardless of the tasks that the other agents are assigned to. Since these tasks may be dependent, i.e., satisfaction of a task may depend on more than one agent, satisfiability of each local task does not imply satisfiability of the conjunction of all local tasks. In these conflicting cases, [2] finds least violating solutions for local LTL tasks, while [12] finds least violating solutions for the STL setup, but considering a limited class of STL (not allowing until operators) and assuming complete communication graphs for subgroups of agents.

Control barrier functions [15] guarantee the existence of a control law that renders a desired set forward invariant; [16] presents control barrier functions tailord for safe robot navigation, while [17] presents decentralized control barrier functions for safe multi-robot navigation. Nonsmooth and time-varying control barrier functions have appeared in [18] and [19], while robustness and input-to-state safety notions have been proposed in [20] and [21]. Barrier functions have also been used to control systems under temporal logic tasks; [13] establishes a connection between the semantics of an STL task and time-varying control barrier functions, while [22] considers finite time control barrier functions for LTL.

We consider coupled multi-agent systems under local STL task. First, we provide a barrier function-based control law that guarantees satisfaction of a local task despite dynamical couplings and when the task is not dependent. Therefore, the existence of a barrier function that accounts for the semantics of this STL task is assumed, as described in [13]. Based on this control law and motivated by a notion of input-to-state safety, we then propose a control law that finds a least violating solution for the case when the local task is dependent and when collaboration among agents is not possible or desired. In a second step, we introduce a local detection mechanism that detects critical events that may lead to a violation of the local task and that may be resolved or benefit from online collaboration with other agents. The proposed control strategy is robust and computationally efficient. In contrast to dependent local tasks, our previous work [14] considers global tasks and derives collaborative feedback control laws.

Sec. II presents the problem formulation, while our proposed problem solution is stated in Sec. III. Simulations are presented in Sec. IV followed by conclusions in Sec. V.

II. Preliminaries and Problem Formulation

Let $\mathbf{0}$ be a zero vector of appropriate size. Furthermore, an extended class $\mathcal{K}$ function $\alpha : \mathbb{R} \to \mathbb{R}$ is a locally Lipschitz continuous and strictly increasing function with $\alpha(0) = 0$.

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Lemma 1: The initial value problem $\dot{z} = -\alpha(z)$ with $z(0) \geq 0$ has the solution $z(t) = \beta(|z(0)|, t) \geq 0$ where $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a class $\mathcal{KL}$ function. For $\epsilon \in \mathbb{R}_{\geq 0}$ and if $\alpha(z)$ is a linear function, $\dot{z} = -\alpha(z) - \epsilon$ with $z(0) \geq 0$ has the solution $z(t)$ satisfying $z(t) \geq \beta(|z(0)|, t) + \alpha^{-1}(-\epsilon)$.

Proof: The first part follows by [23, Ch. 4]. The second part can easily be verified since $\alpha(z)$ is a linear function. ■

Consider M agents modeled by a directed graph $\mathcal{G} := (\mathcal{V}, \mathcal{E})$. The set of agents is $\mathcal{V} := \{1, \ldots, M\}$, while $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ indicates communication links, i.e., $(i, j) \in \mathcal{E}$ if agent j receives information from agent i. For each agent i, let $\boldsymbol{x}_i \in \mathbb{R}^{n_i}$ and $\boldsymbol{u}_i \in \mathbb{R}^{m_i}$ be the corresponding state and input, respectively. Also let $n := n_1 + \ldots + n_M$ and $\boldsymbol{x} := \begin{bmatrix} \boldsymbol{x}_1^T & \ldots & \boldsymbol{x}_M^T \end{bmatrix}^T \in \mathbb{R}^n$. The dynamics of agent i are

$$\dot{\boldsymbol{x}}_i = f_i(\boldsymbol{x}_i, t) + g_i(\boldsymbol{x}_i, t)\boldsymbol{u}_i + c_i(\boldsymbol{x}, t) \tag{1}$$

where $f_i : \mathbb{R}^{n_i} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_i}$, $g_i : \mathbb{R}^{n_i} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_i \times m_i}$, and $c_i : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_i}$ are locally Lipschitz continuous. The function $c_i(\boldsymbol{x}, t)$ may model *dynamical couplings* such as those induced by a mechanical connection between agents or such as those induced by a secondary controller; $c_i(\boldsymbol{x}, t)$ may also describe unmodelled dynamics or process noise. We assume that $c_i(\boldsymbol{x}, t)$ is bounded, but otherwise unknown so that the control design does not require knowledge of $\boldsymbol{x}$. In other words, there exists $C_i \geq 0$, known by agent i, such that $\|c_i(\boldsymbol{x}, t)\| \leq C_i$ for all $(\boldsymbol{x}, t) \in \mathbb{R}^n \times \mathbb{R}_{\geq 0}$.

Signal temporal logic (STL) [6] is based on predicates μ that are obtained after evaluation of a continuously differentiable predicate function $h : \mathbb{R}^d \to \mathbb{R}$ as $\mu := \top$ (True) if $h(\boldsymbol{\zeta}) \geq 0$ and $\mu := \bot$ (False) if $h(\boldsymbol{\zeta}) < 0$ for $\boldsymbol{\zeta} \in \mathbb{R}^d$. The STL syntax is then given by

$$\phi \ ::= \ \top \mid \mu \mid \neg\phi \mid \phi' \wedge \phi'' \mid \phi' \, U_{[a,b]} \, \phi''$$

where ϕ' and ϕ'' are STL formulas and where $U_{[a,b]}$ is the until operator with $a \leq b < \infty$. Also define $F_{[a,b]}\phi := \top \, U_{[a,b]} \, \phi$ (eventually operator) and $G_{[a,b]}\phi := \neg F_{[a,b]}\neg\phi$ (always operator). Let $\boldsymbol{\zeta}' \models \phi$ denote the satisfaction relation, i.e., if a signal $\boldsymbol{\zeta}' : \mathbb{R}_{\geq 0} \to \mathbb{R}^d$ satisfies ϕ (at time 0). These STL semantics are defined in [6]. A formula ϕ is satisfiable if $\exists \boldsymbol{\zeta}' : \mathbb{R}_{\geq 0} \to \mathbb{R}^d$ such that $\boldsymbol{\zeta}' \models \phi$. Robust semantics for STL [8] are denoted by $\rho^\phi(\boldsymbol{\zeta}')$ and determine how robustly $\boldsymbol{\zeta}'$ satisfies ϕ. The robust semantics for STL are defined in [8, Def. 3]. It holds that $\boldsymbol{\zeta}' \models \phi$ if $\rho^\phi(\boldsymbol{\zeta}') > 0$ [7, Prop. 16]. In this paper, we consider the STL fragment

$$\psi \ ::= \ \top \mid \mu \mid \neg\mu \mid \psi' \wedge \psi'' \tag{2a}$$

$$\phi \ ::= \ G_{[a,b]}\psi \mid F_{[a,b]}\psi \mid \psi' \, U_{[a,b]} \, \psi'' \mid \phi' \wedge \phi'' \tag{2b}$$

where ψ' and ψ'' are of the form (2a), whereas ϕ' and ϕ'' are of the form (2b). Each agent i is assigned a local task ϕ_i of the form (2b). Initially, each agent only knows its own formula, but it may obtain partial information of other agent's formulas. These tasks may be dependent, i.e., the satisfaction of ϕ_i may depend on the behavior of other agents $j \neq i$. By behavior of an agent i, we mean the state trajectory $\boldsymbol{x}_i(t)$ that evolves according to (1). Let the satisfaction of ϕ_i depend on a set of agents denoted by $\mathcal{V}_i \subseteq \mathcal{V}$ with $|\mathcal{V}_i| \geq 1$ where $|\mathcal{V}_i|$ denotes the cardinality of the set $\mathcal{V}_i$.

Assumption 1: It holds that $(j, i) \in \mathcal{E}$ for all $j \in \mathcal{V}_i \setminus \{i\}$.

For $j_1, \ldots, j_{|\mathcal{V}_i|} \in \mathcal{V}_i$, let $\bar{\boldsymbol{x}}_i := \begin{bmatrix} \boldsymbol{x}_{j_1}^T & \ldots & \boldsymbol{x}_{j_{|\mathcal{V}_i|}}^T \end{bmatrix}^T$ and $\bar{n}_i := n_{j_1} + \ldots + n_{j_{|\mathcal{V}_i|}}$, i.e., $\bar{\boldsymbol{x}}_i$ is the stacked state vector of all agents in $\mathcal{V}_i$. Since the elements of $\bar{\boldsymbol{x}}_i$ are contained in $\boldsymbol{x}$, let us also define the projection map $p_i : \mathbb{R}^n \to \mathbb{R}^{\bar{n}_i}$ as $p_i(\boldsymbol{x}) := \bar{\boldsymbol{x}}_i$ and let the projector from a set $\mathcal{S} \in \mathbb{R}^n$ onto the formula state-space $\mathbb{R}^{\bar{n}_i}$ be $P_i(\mathcal{S}) := \{\bar{\boldsymbol{x}}_i \in \mathbb{R}^{\bar{n}_i} | \exists \boldsymbol{x} \in \mathcal{S}, p_i(\boldsymbol{x}) = \bar{\boldsymbol{x}}_i\}$. Each agent is supposed to not collide with obstacles indicated by $\mathcal{O}_i \subset \mathbb{R}^{n_i}$. Note that satisfaction of all local tasks may not be possible. Therefore, this paper proposes a new notion of finding least violating solutions.

Problem 1: Consider M agents subject to the dynamics in (1) and where each agent i is subject to a task ϕ_i of the form (2b). Derive a local control law $\boldsymbol{u}_i$ so that $r_i \leq \rho^{\phi_i}(\bar{\boldsymbol{x}}_i)$ where r_i is maximized, while $\boldsymbol{x}_i(t) \notin \mathcal{O}_i$ for all $t \geq 0$.

III. Problem Solution

We use control barrier functions as in [13] where, for single-agent systems, conditions are imposed on a function $\mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)$ that account for the semantics of ϕ_i. If then

$$\mathfrak{C}_i(t) := \{\bar{\boldsymbol{x}}_i \in \mathbb{R}^{\bar{n}_i} | \mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t) \geq 0\}$$

is forward invariant, it holds that $\bar{\boldsymbol{x}}_i \models \phi_i$. These conditions also enforce that $\bar{\boldsymbol{x}}_i(t) \in \mathfrak{C}_i(t)$ implies $\bar{\boldsymbol{x}}_i(t) \in \mathfrak{B}_i$ for a compact set $\mathfrak{B}_i \subset \mathbb{R}^{\bar{n}_i}$; [14] presents a systematic procedure to construct $\mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)$ if all predicate functions in ϕ_i are concave (if the predicate function associated with $\neg\mu$ as in (2a) is convex, it can be rewritten as a concave predicate function) and if $g_i(\bar{\boldsymbol{x}}_i, t)$ has full row rank for all $(\bar{\boldsymbol{x}}_i, t) \in \mathbb{R}^{n_i} \times \mathbb{R}_{\geq 0}$. Due to this particular construction, invariance of $\mathfrak{C}_i(t)$ implies $r_i \leq \rho^{\phi_i}(\bar{\boldsymbol{x}}_i, 0)$ where $r_i \geq 0$ is maximized. In [13] and [14], the function $\mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)$ is piecewise continuous in the second argument with discontinuities at times $\{s_0 := 0, s_1, \ldots, s_q\}$ for some finite q. For each s_j with $j \in \{1, \ldots, q\}$, it holds that $\lim_{\tau \to s_j^-} \mathfrak{C}_i(\tau) \supseteq \mathfrak{C}_i(s_j)$ where $\lim_{\tau \to s_j^-} \mathfrak{C}_i(\tau)$ is the left-sided limit of $\mathfrak{C}_i(t)$ at $t = s_j$.

Theorem 1: For each ϕ_i, assume that $|\mathcal{V}_i| = 1$ and let $\mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)$ be a barrier function that satisfies the conditions in [13, Steps A, B, and C]. If, for some extended class $\mathcal{K}$ function α_i, for some open set $\mathfrak{D}_i$ with $\mathfrak{D}_i \supset \mathfrak{C}_i(t)$ for all $t \geq 0$, and for all $(\boldsymbol{x}_i, t) \in \mathfrak{D}_i \times (s_j, s_{j+1})$, there exists a continuous control law $\boldsymbol{u}_i(\boldsymbol{x}_i, t)$ such that

$$\begin{aligned} &\frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial \boldsymbol{x}_i}(f_i(\boldsymbol{x}_i, t) + g_i(\boldsymbol{x}_i, t)\boldsymbol{u}_i(\boldsymbol{x}_i, t)) \\ &+ \frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial t} \geq -\alpha_i(\mathfrak{b}_i(\boldsymbol{x}_i, t)) + \left\| \frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial \boldsymbol{x}_i} \right\| C_i, \end{aligned} \tag{3}$$

then $\boldsymbol{x}_i \models \phi_i$.

Proof: Note that there exist solutions $\boldsymbol{x} : [0, \tau_{\max}) \to \mathfrak{D}_1 \times \ldots \times \mathfrak{D}_M$ to (1) with $\tau_{\max} > 0$. Now, (3) implies

$$\begin{aligned} &\frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial \boldsymbol{x}_i}(f_i(\boldsymbol{x}_i, t) + g_i(\boldsymbol{x}_i, t)\boldsymbol{u}_i + c_i(\boldsymbol{x}, t)) \\ &\qquad + \frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial t} \geq -\alpha_i(\mathfrak{b}_i(\boldsymbol{x}_i, t)) \end{aligned} \tag{4}$$

so that, for all $t \in (0, \min(\tau_{\max}, s_1))$, $\dot{\mathfrak{b}}_i(\boldsymbol{x}_i(t), t) \geq -\alpha_i(\mathfrak{b}_i(\boldsymbol{x}_i, t))$. Due to Lemma 1, the Comparison Lemma [23, Ch. 3.4], and since $\mathfrak{b}_i(\boldsymbol{x}_i(0), 0) \geq 0$, it follows that $\mathfrak{b}_i(\boldsymbol{x}_i(t), t) \geq 0$, i.e., $\boldsymbol{x}_i(t) \in \mathfrak{C}_i(t)$, for all $t \in [0, \min(\tau_{\max}, s_1))$. Assuming $\tau_{\max} \geq s_1$, it holds $\boldsymbol{x}_i(t) \in \mathfrak{C}_i(t)$ for all $t \in [s_1, \min(\tau_{\max}, s_2))$. Note that $\boldsymbol{x}_i(s_1) \in \mathfrak{C}(s_1)$ since $\lim_{\tau \to s_j^-} \mathfrak{C}_i(\tau) \supseteq \mathfrak{C}_i(s_j)$. This argument can be repeated unless $\tau_{\max} < s_j$ for some j. Since $\mathfrak{b}_i(\boldsymbol{x}_i(t), t) \geq 0$ implies $\boldsymbol{x}_i(t) \in \mathfrak{B}_i$ for all $t \in [0, \tau_{\max})$, $\tau_{\max} = \infty$ due to [23, Thm. 3.3] so that $\boldsymbol{x}_i(t) \in \mathfrak{C}_i(t)$ for all $t \geq 0$. ■

Theorem 1 is hence established since $|\mathcal{V}_i| = 1$, while the dynamical couplings in $c_i(\boldsymbol{x}, t)$ are bounded.

Corollary 1: For each ϕ_i, assume that $|\mathcal{V}_i| = 1$. If ϕ_i contains only predicates associated with concave predicate functions, $g_i(\boldsymbol{x}_i, t)$ has full row rank for all $(\boldsymbol{x}_i, t) \in \mathbb{R}^{n_i} \times \mathbb{R}_{\geq 0}$, $\mathfrak{b}_i(\boldsymbol{x}_i, t)$ is constructed as in [14, Eq. (11)], and α_i satisfies [14, Lem. 4] which ensures $\frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial t} \geq -\alpha_i(\mathfrak{b}_i(\boldsymbol{x}_i, t)) + \chi$ for some $\chi > 0$ if $\frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial \boldsymbol{x}_i} g_i(\boldsymbol{x}_i, t) = \boldsymbol{0}$, then $\boldsymbol{u}_i(\boldsymbol{x}_i, t) = \hat{\boldsymbol{u}}_i$, where $\hat{\boldsymbol{u}}_i$ is given by

$$\operatorname*{argmin}_{\hat{\boldsymbol{u}}_i} \hat{\boldsymbol{u}}_i^T \hat{\boldsymbol{u}}_i \tag{5a}$$

$$\text{s.t. } \frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial \boldsymbol{x}_i}(f_i(\boldsymbol{x}_i, t) + g_i(\boldsymbol{x}_i, t)\hat{\boldsymbol{u}}_i) + \frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial t} \geq -\alpha_i(\mathfrak{b}_i(\boldsymbol{x}_i, t)) + \Big\|\frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial \boldsymbol{x}_i}\Big\| C_i, \tag{5b}$$

results in $r_i \leq \rho^{\phi_i}(\boldsymbol{x}_i)$ where r_i is maximized.

Proof: If $(\boldsymbol{x}_i, t) \in \mathbb{R}^{n_i} \times (s_j, s_{j+1})$ with $\frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial \boldsymbol{x}_i} g_i(\boldsymbol{x}_i, t) \neq \boldsymbol{0}$, (5) is feasible and $\boldsymbol{u}_i(\boldsymbol{x}_i, t)$ is locally Lipschitz continuous at $(\boldsymbol{x}_i, t)$ [20, Thm. 8]. Note that $\frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial \boldsymbol{x}_i} g_i(\boldsymbol{x}_i, t) = \boldsymbol{0}$ if and only if $\frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial \boldsymbol{x}_i} = \boldsymbol{0}$ since $g_i(\boldsymbol{x}_i, t)$ has full row rank. If $(\boldsymbol{x}_i, t) \in \mathbb{R}^{n_i} \times (s_j, s_{j+1})$ with $\frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial \boldsymbol{x}_i} = \boldsymbol{0}$, (5b) is satisfied since $\frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial t} \geq -\alpha_i(\mathfrak{b}_i(\boldsymbol{x}_i, t)) + \chi$ due the choice of α_i so that $\boldsymbol{u}_i(\boldsymbol{x}_i, t) := \boldsymbol{0}$. Due to continuity of $\frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i, t)}{\partial t}$ and $\alpha_i(\mathfrak{b}_i(\boldsymbol{x}_i, t))$, there exists a neighborhood $\mathcal{U}$ around $(\boldsymbol{x}_i, t)$ so that, for each $(\boldsymbol{x}_i', t') \in \mathcal{U}$, $\frac{\partial \mathfrak{b}_i(\boldsymbol{x}_i', t')}{\partial t} \geq -\alpha_i(\mathfrak{b}_i(\boldsymbol{x}_i', t'))$ and consequently $\boldsymbol{u}_i(\boldsymbol{x}_i', t') = \boldsymbol{0}$. Hence, $\boldsymbol{u}_i(\boldsymbol{x}_i, t)$ is continuous on $\mathbb{R}^{n_i} \times (s_j, s_{j+1})$. Theorem 1 guarantees invariance of $\mathfrak{C}_i(t)$ which implies $r_i \leq \rho^{\phi_i}(\boldsymbol{x}_i)$ where r_i is maximized. ■

If now $|\mathcal{V}_i| > 1$ for some i, satisfiability of each ϕ_i separately does not ensure satisfiability of $\phi_1 \wedge \ldots \wedge \phi_M$.

Example 1: Consider $M := 6$ agents. Agents $i \in \{1, 2, 3\}$ obey uncontrolled dynamics ($\boldsymbol{u}_i := \boldsymbol{0}$) with periodic solutions $\boldsymbol{x}_i(t) := \begin{bmatrix}\sin(t + \frac{2(i-1)}{3}\pi) & \cos(t + \frac{2(i-1)}{3}\pi)\end{bmatrix}^T$. Agent 4, 5, and 6 are supposed to track agent 1, 2, and 3, respectively, while being subject to connectivity constraints. In STL language, this may look as follows: $\phi_4 := G_{[10,\infty)} \bigwedge_{j=1,5,6}(\|\boldsymbol{x}_4 - \boldsymbol{x}_j\| \leq 0.3)$, $\phi_5 := G_{[10,\infty)} \bigwedge_{j=2,4,6}(\|\boldsymbol{x}_5 - \boldsymbol{x}_j\| \leq 0.3)$, and $\phi_6 := G_{[10,\infty)} \bigwedge_{j=3,4,5}(\|\boldsymbol{x}_6 - \boldsymbol{x}_j\| \leq 0.3)$. Each of ϕ_4, ϕ_5, or ϕ_6 is satisfiable on its own; however, $\phi_4 \wedge \phi_5 \wedge \phi_6$ is not satisfiable.

Denote $\bar{f}_i(\bar{\boldsymbol{x}}_i, t) := \begin{bmatrix} f_{j_1}(\boldsymbol{x}_{j_1}, t)^T & \ldots & f_{j_{|\mathcal{V}_i|}}(\boldsymbol{x}_{j_{|\mathcal{V}_i|}}, t)^T \end{bmatrix}^T$, $\bar{g}_i(\bar{\boldsymbol{x}}_i, t) := \operatorname{diag}(g_{j_1}(\boldsymbol{x}_{j_1}, t), \ldots, g_{j_{|\mathcal{V}_i|}}(\boldsymbol{x}_{j_{|\mathcal{V}_i|}}, t))$, $\bar{c}_i(\boldsymbol{x}, t) := \begin{bmatrix} c_{j_1}(\boldsymbol{x}, t)^T & \ldots & c_{j_{|\mathcal{V}_i|}}(\boldsymbol{x}, t)^T \end{bmatrix}^T$, and $\bar{\boldsymbol{u}}_i := \begin{bmatrix} \boldsymbol{u}_{j_1}{}^T & \ldots & \boldsymbol{u}_{j_{|\mathcal{V}_i|}}{}^T \end{bmatrix}^T$ for $j_1, \ldots, j_{|\mathcal{V}_i|} \in \mathcal{V}_i$, i.e., the stacked elements of all agents in $\mathcal{V}_i$. If now $|\mathcal{V}_i| > 1$, the barrier inequality (4) changes to

$$\frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)}{\partial \bar{\boldsymbol{x}}_i}(\bar{f}_i(\bar{\boldsymbol{x}}_i, t) + \bar{g}_i(\bar{\boldsymbol{x}}_i, t)\bar{\boldsymbol{u}}_i + \bar{c}_i(\boldsymbol{x}, t)) + \frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)}{\partial t} \geq -\alpha_i(\mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)) \tag{6}$$

where we, in the remainder, assume that $\mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)$ satisfies the conditions in [13, Steps A, B, and C] and is such that, for each $(\bar{\boldsymbol{x}}_i, t) \in \mathbb{R}^{\bar{n}_i} \times (s_j, s_{j+1})$, $\bar{\boldsymbol{u}}_i$ can be selected so that (6) holds, i.e., $\frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)}{\partial t} \geq -\alpha_i(\mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t))$ if $\frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)}{\partial \bar{\boldsymbol{x}}_i} \bar{g}_i(\bar{\boldsymbol{x}}_i, t) = \boldsymbol{0}$. This means that, if all agents in $\mathcal{V}_i$ collaborate, ϕ_i can be satisfied. If ϕ_i contains only predicates associated with concave predicate functions and $\bar{g}_i(\bar{\boldsymbol{x}}_i, t)$ has full row rank for all $(\bar{\boldsymbol{x}}_i, t) \in \mathbb{R}^{\bar{n}_i} \times \mathbb{R}_{\geq 0}$, then $\mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)$ can even be constructed as in [14, Eq. (11)] with α_i satisfying [14, Lem. 4] which again ensures that $\frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)}{\partial t} \geq -\alpha_i(\mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)) + \chi$ for some $\chi > 0$ if $\frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)}{\partial \bar{\boldsymbol{x}}_i} \bar{g}_i(\bar{\boldsymbol{x}}_i, t) = \boldsymbol{0}$. This ensures that all agents in $\mathcal{V}_i$ can use a collaborative control law as presented in [14, Thm. 1]. Thereby, we ensure that a possible violation of (6) in fact stems from conflicting local objectives.

A. Conflicting Local STL tasks without Online Collaboration

We first consider cases where *online collaboration*, i.e., agents can send and receive collaboration requests during runtime, is not desired (e.g., agents are not willing to collaborate) or possible (e.g., communication limitations in $\mathcal{E}$) and investigate the behavior of agent i while other agents $j \neq i$ are subject to the following assumption that we put in perspective later.

Assumption 2: Each agent $j \neq i$ applies a bounded and continuous control law $\boldsymbol{u}_j(\boldsymbol{x}, t)$ that achieves $\boldsymbol{x}_j(t) \in \mathfrak{B}_j$ for a compact set $\mathfrak{B}_j$ and for all $t \geq 0$.

For simplicity, let us re-write the dynamics of the set of agents $\mathcal{V}_i$ by re-indexing the agents as follows

$$\begin{aligned}\dot{\bar{\boldsymbol{x}}}_i &= \bar{f}_i(\bar{\boldsymbol{x}}_i, t) + \bar{g}_i(\bar{\boldsymbol{x}}_i, t)\bar{\boldsymbol{u}}_i + \bar{c}_i(\boldsymbol{x}, t) \\ &= \tilde{f}_i(\boldsymbol{x}_i, t) + \tilde{g}_i(\boldsymbol{x}_i, t)\boldsymbol{u}_i + \tilde{c}_i(\boldsymbol{x}, t)\end{aligned}$$

where $\tilde{f}_i(\boldsymbol{x}_i, t) := \begin{bmatrix} f_i(\boldsymbol{x}_i, t)^T & \boldsymbol{0}^T & \ldots & \boldsymbol{0}^T \end{bmatrix}^T$, $\tilde{g}_i(\boldsymbol{x}_i, t) := \begin{bmatrix} g_i(\boldsymbol{x}_i, t)^T & \boldsymbol{0} & \ldots & \boldsymbol{0} \end{bmatrix}^T$, $\tilde{c}_i(\boldsymbol{x}, t) := \bar{c}_i(\boldsymbol{x}, t) + \begin{bmatrix} \boldsymbol{0}^T & d_{j_1}(\boldsymbol{x}, t)^T & \ldots & d_{j_{|\mathcal{V}_i|}}(\boldsymbol{x}, t)^T \end{bmatrix}^T$ with $d_j(\boldsymbol{x}, t) := f_j(\boldsymbol{x}_j, t) + g_j(\boldsymbol{x}_j, t)\boldsymbol{u}_j(\boldsymbol{x}, t)$. Hence, (6) is equivalent to

$$\frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)}{\partial \boldsymbol{x}_i}(f_i(\boldsymbol{x}_i, t) + g_i(\boldsymbol{x}_i, t)\boldsymbol{u}_i) + \frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)}{\partial \bar{\boldsymbol{x}}_i}\tilde{c}_i(\boldsymbol{x}, t) + \frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)}{\partial t} \geq -\alpha_i(\mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)). \tag{7}$$

The above inequality may pose feasibility issues if $\frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)}{\partial \boldsymbol{x}_i} g_i(\boldsymbol{x}_i, t) = \boldsymbol{0}$ and $\frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)}{\partial \bar{\boldsymbol{x}}_i}\tilde{c}_i(\boldsymbol{x}, t) \neq \boldsymbol{0}$. Then, the satisfaction of (7) depends in particular on $\frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i, t)}{\partial \bar{\boldsymbol{x}}_i}\tilde{c}_i(\boldsymbol{x}, t)$ and hence on the behavior of the agents in $\mathcal{V}_i \backslash \{i\}$ according to $\tilde{c}_i(\boldsymbol{x}, t)$; $\tilde{c}_i(\boldsymbol{x}, t)$ is, however, unknown to agent i and may be favoring or acting against satisfying (7). It should be noted that these situations are inevitable in the given setup. In the sequel, $\tilde{c}_i(\boldsymbol{x}, t)$ is treated as an unknown disturbance. In particular, let $\tilde{C}_i$ be a positive constant such that $\|\tilde{c}_i(\boldsymbol{x},t)\| \leq \tilde{C}_i$ for all $(\boldsymbol{x},t) \in \mathfrak{D} \times \mathbb{R}_{\geq 0}$ where $\mathfrak{D} \in \mathbb{R}^n$ is an open and bounded set for which it holds that $P_i(\mathfrak{D}) \supset \mathfrak{C}_i(t)$ for all $t \geq 0$ as well as $P_j(\mathfrak{D}) \supset \mathfrak{B}_j$ for all $j \neq i$ (the relevance of $\mathfrak{D}$ becomes obvious in Theorem 2); $\tilde{C}_i$ exists since $c_j(\boldsymbol{x},t)$ and $\boldsymbol{u}_j(\boldsymbol{x},t)$ are bounded (Assumption 2) and $f_j(\boldsymbol{x}_j,t)$ and $g_j(\boldsymbol{x}_j,t)$ are continuous. Let, for a linear class $\mathcal{K}$ function α_i and each $(\bar{\boldsymbol{x}}_i,t) \in P_i(\mathfrak{D}) \times \mathbb{R}_{\geq 0}$, $\boldsymbol{u}_i(\bar{\boldsymbol{x}}_i,t) := \hat{\boldsymbol{u}}_i$ and $\epsilon_i(\bar{\boldsymbol{x}}_i,t) := \hat{\epsilon}_i$ where $\hat{\boldsymbol{u}}_i$ and $\hat{\epsilon}_i$ are given by

$$\operatorname*{argmin}_{\hat{\boldsymbol{u}}_i,\hat{\epsilon}_i} K_{i,1}\hat{\boldsymbol{u}}_i^T\hat{\boldsymbol{u}}_i + K_{i,2}\hat{\epsilon}_i^2 \tag{8a}$$

$$\begin{aligned} \text{s.t. } & \frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t)}{\partial \boldsymbol{x}_i}(f_i(\boldsymbol{x}_i,t) + g_i(\boldsymbol{x}_i,t)\hat{\boldsymbol{u}}_i) + \frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t)}{\partial t} \\ & \geq -\alpha_i(\mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t)) + \Big\|\frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t)}{\partial \bar{\boldsymbol{x}}_i}\Big\|\tilde{C}_i - \hat{\epsilon}_i \end{aligned} \tag{8b}$$

with $K_{i,1}, K_{i,2} \in [0,1]$ and $K_{i,1} + K_{i,2} = 1$; (8b) implies (7) when $\hat{\epsilon}_i = 0$ and $\hat{\epsilon}_i > 0$ relaxes (8b) when needed. Inspired by the notion of input-to-state safety [21], we describe the worst case level of infeasibility by considering $\epsilon_{i,\text{wc}} := \sup_{(\bar{\boldsymbol{x}}_i,t)\in P_i(\mathfrak{D})\times\mathbb{R}_{\geq 0}} \epsilon_i(\bar{\boldsymbol{x}}_i,t)$.

Theorem 2: Let Assumptions 1 and 2 hold and assume that $\tilde{C}_i$ is given. Then it holds that

$$\mathfrak{C}_{i,\text{wc}}(t) := \{\bar{\boldsymbol{x}}_i \in \mathbb{R}^{\bar{n}_i} | \mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t) \geq \alpha_i^{-1}(-\epsilon_{i,\text{wc}})\}$$

is forward invariant if $P_i(\mathfrak{D}) \supset \mathfrak{C}_{i,\text{wc}}(t)$ for all $t \geq 0$.

Proof: Note that $\boldsymbol{u}_i(\bar{\boldsymbol{x}}_i,t)$ is locally Lipschitz continuous due to [20, Thm. 8]. Consequently, there exists a solution $\boldsymbol{x} : [0,\tau_{\max}) \to \mathfrak{D}$ to (1) with $\tau_{\max} > 0$. Due to (8b) it holds that $\dot{\mathfrak{b}}_i(\bar{\boldsymbol{x}}_i(t),t) \geq -\alpha_i(\mathfrak{b}_i(\bar{\boldsymbol{x}}_i(t),t)) - \epsilon_{i,\text{wc}}$ for all $t \in [0,\min(\tau_{\max},s_1))$. By Lemma 1 and the Comparison Lemma [23, Ch. 3.4], we deduce $\mathfrak{b}_i(\bar{\boldsymbol{x}}_i(t),t) \geq \beta(|\mathfrak{b}_i(\bar{\boldsymbol{x}}_i(0),0)|,t) + \alpha_i^{-1}(-\epsilon_{i,\text{wc}}) \geq \alpha_i^{-1}(-\epsilon_{i,\text{wc}})$ for all $t \in [0,\min(\tau_{\max},s_1))$. The same iterative reasoning over $[s_1,\min(\tau_{\max},s_2))$ as in Theorem 1 applies if $\tau_{\max} \geq s_1$ until $\tau_{\max} < s_j$ for some j. It, however, holds that $\mathfrak{b}_i(\bar{\boldsymbol{x}}_i(t),t) \geq \alpha_i^{-1}(-\epsilon_{i,\text{wc}})$ for all $t \in [0,\tau_{\max})$, i.e., $\bar{\boldsymbol{x}}_i(t) \in \mathfrak{C}_{i,\text{wc}}(t)$ for all $t \in [0,\tau_{\max})$ where $\mathfrak{C}_{i,\text{wc}}(t)$ is compact since $P_i(\mathfrak{D}) \supset \mathfrak{C}_{i,\text{wc}}(t)$. By [23, Thm. 3.3], it follows that $\tau_{\max} = \infty$. ■

The above estimate may be conservative. For a given initial condition $\boldsymbol{x}(0)$ and the solution $\boldsymbol{x} : [0,\tau) \to \mathfrak{D}$ to (1) until time $\tau > 0$, it does not necessarily hold that $\epsilon_i(\bar{\boldsymbol{x}}_i(t),t) = \epsilon_{i,\text{wc}}$ for some $t \in [0,\tau)$. We can obtain a local estimate of the worst case at time τ by defining $\epsilon_{i,\max}(\tau) := \sup_{t\in[0,\tau)} \epsilon_i(\bar{\boldsymbol{x}}_i(t),t)$. Note that $\epsilon_{i,\max}(\tau) \leq \epsilon_{i,\text{wc}}$.

Corollary 2: Let Assumptions 1 and 2 hold and assume that $\tilde{C}_i$ is given. Given an initial condition $\boldsymbol{x}(0)$ and the solution $\boldsymbol{x} : [0,\tau) \to \mathfrak{D}$ to (1) until time $\tau > 0$. If $P_i(\mathfrak{D}) \supset \mathfrak{C}_{i,\max}(t)$ for all $t \in [0,\tau)$ where

$$\mathfrak{C}_{i,\max}(t) := \{\bar{\boldsymbol{x}}_i \in \mathbb{R}^{\bar{n}_i} | \mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t) \geq \alpha_i^{-1}(-\epsilon_{i,\max}(\tau))\},$$

then it holds that $\bar{\boldsymbol{x}}_i(t) \in \mathfrak{C}_{i,\max}(t)$ for all $t \in [0,\tau)$.

Proof: It holds that $\dot{\mathfrak{b}}_i(\bar{\boldsymbol{x}}_i(t),t) \geq -\alpha_i(\mathfrak{b}_i(\bar{\boldsymbol{x}}_i(t),t)) - \epsilon_{i,\max}(\tau)$ for all $t \in [0,\tau)$. By Lemma 1 and the Comparison Lemma [23, Ch. 3.4], $\bar{\boldsymbol{x}}_i(t) \in \mathfrak{C}_{i,\max}(t)$ for all $t \in [0,\tau)$. ■

Corollary 2 tells us that $\bar{\boldsymbol{x}}_i(t) \in \mathfrak{C}_{i,\max}(t)$ for all $t \in [0,\tau)$, but it does not tell us whether or not $\bar{\boldsymbol{x}}_i(t) \in \mathfrak{C}_{i,\max}(t)$ for all $t \geq \tau$. Corollary 2, however, motivates that minimizing $\hat{\epsilon}_i$ results in a *least violating solution*, i.e., achieving $\bar{\boldsymbol{x}}_i(t) \in \mathfrak{C}_{i,\max}(t)$ for all $t \geq 0$ depends on ensuring that $\epsilon_i(\bar{\boldsymbol{x}}_i(t),t) \leq \epsilon_{i,\max}(\tau)$ for $t \geq \tau$. This observation will be used in the online collaboration part presented in the next subsection. By least violating solution we hence mean a solution $\bar{\boldsymbol{x}}_i(t)$ such that $\bar{\boldsymbol{x}}_i(t) \in \mathfrak{C}_{i,\max}(t)$ where $\epsilon_{i,\max}(\infty)$ is minimized. The previous analysis relies on Assumption 2. If, however, each agent i solves (8), making Assumption 2 obsolete, the question is how an estimate of $\tilde{C}_i$ can be obtained. First, the set $\mathfrak{D}$ needs to be selected. A starting point is to select $\mathfrak{D}$ such that $P_i(\mathfrak{D}) \supset \mathfrak{B}_i$ for each i. Then, $\tilde{C}_i$ needs to be selected, for each agent i, such that $\|\tilde{c}_i(\boldsymbol{x},t)\| \leq \tilde{C}_i$ for all $(\boldsymbol{x},t) \in \mathfrak{D} \times \mathbb{R}_{\geq 0}$. If agents are subject to input limitations, i.e., $\boldsymbol{u}_i \in \mathcal{U}_i$ for some compact set $\mathcal{U}_i$, an estimate of $\tilde{C}_i$ can easily be obtained. This will be assumed in the next section.

B. Conflicting Local STL tasks with Online Collaboration

Online collaboration is initiated if a *critical event* (defined below) is detected by agent i and should account for $\mathcal{E}$. The structure of $\mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t)$ (see [13], [14] for details) is

$$\mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t) := -\frac{1}{\eta_i}\ln\Big(\sum_{l=1}^{p_i}\mathfrak{o}_i^l(t)\exp(-\eta_i\mathfrak{b}_i^l(\bar{\boldsymbol{x}}_i^l,t))\Big)$$

where $\eta_i > 0$, $\mathfrak{o}_i^l(t) : \mathbb{R}_{\geq 0} \to \{0,1\}$, and $p_i \in \mathbb{N}$; $\bar{\boldsymbol{x}}_i^l$ contains the stacked states of only a subset of agents $\mathcal{V}_i^l \subseteq \mathcal{V}_i$. This allows to collaborate only with a subset of agents. Let $\mathcal{A}_i(t)$ be such that $l \in \mathcal{A}_i(t)$ if and only if $\mathfrak{o}_i^l(t) = 1$. It holds that

$$\mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t) \leq \min_{l\in\mathcal{A}_i(t)} \mathfrak{b}_i^l(\bar{\boldsymbol{x}}_i^l,t) \leq \mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t) + \frac{\ln(|\mathcal{A}_i(t)|)}{\eta_i}. \tag{9}$$

Definition 1: A critical event happens at time $\tau > 0$ if $\mathfrak{b}_i(\bar{\boldsymbol{x}}_i(\tau),\tau) + \frac{\ln(|\mathcal{A}_i(\tau)|)}{\eta_i} < 0$ and $\epsilon_i(\bar{\boldsymbol{x}}_i(\tau),\tau) \geq \epsilon_{i,\text{th}}$ where $\epsilon_{i,\text{th}} > 0$ is a design parameter.

Collaboration requests are indicated by $\text{cr}_{i,j}^l : \mathbb{R}_{\geq 0} \to \{\top,\perp\}$ where $\text{cr}_{i,j}^l(t) := \perp$ by default. If a critical event is detected at $t = \tau$, there exists at least one $l \in \mathcal{A}_i(\tau)$ such that $\mathfrak{b}_i^l(\bar{\boldsymbol{x}}_i^l(\tau),\tau) < 0$ due to (9). For each $l \in \mathcal{A}_i(\tau)$ with $\mathfrak{b}_i^l(\bar{\boldsymbol{x}}_i^l(\tau),\tau) < 0$, agent i sends the function $\mathfrak{b}_i^l(\bar{\boldsymbol{x}}_i^l,t)$ to agent $j \in \mathcal{V}_i^l \setminus \{i\}$ and sets $\text{cr}_{i,j}^l(\tau) := \top$ if $(k,j) \in \mathcal{E}$ for each $k \in \mathcal{V}_i^l \setminus \{j\}$. Let $\mathcal{N}_i(t)$ and $\mathcal{L}_{i,j}(t)$ be such that $j \in \mathcal{N}_i(t)$ and $l \in \mathcal{L}_{i,j}(t)$ if and only if $\text{cr}_{j,i}^l(t') = \top$ for some $j \in \mathcal{V}$, $l \in \{1,\ldots,p_j\}$, and $t' \in [0,t]$; $\mathcal{N}_i(t) \subseteq \mathcal{V}$ is the set of agents from which a collaboration request has been received until time t, while $\mathcal{L}_{i,j}(t)$ is the set of corresponding indices l. Let also $\text{CR}_i(t) := \sum_{j\in\mathcal{N}_i(t)} \|\mathcal{L}_{i,j}(t)\|$ denote the number of received collaboration requests and let each pair $(j,l) \in \mathcal{N}_i(t) \times \mathcal{L}_{i,j}(t)$ be uniquely associated with $\nu_{j,l}(t) \in \{2,\ldots,\text{CR}_i(t)+1\}$. For $K_{i,k} \in [0,1]$ with $\sum_{k=1}^{\text{CR}_i(t)+2} K_{i,k} = 1$, agent i then solves

$$\operatorname*{argmin}_{\hat{\boldsymbol{u}}_i,\hat{\boldsymbol{\epsilon}}_i} K_{i,1}\hat{\boldsymbol{u}}_i^T\hat{\boldsymbol{u}}_i + \sum_{k=2}^{\text{CR}_i(t)+2} K_{i,k}\hat{\epsilon}_{i,k-1}^2 \tag{10a}$$

$$\begin{aligned} \text{s.t. } & \frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t)}{\partial \boldsymbol{x}_i}(f_i(\boldsymbol{x}_i,t) + g_i(\boldsymbol{x}_i,t)\hat{\boldsymbol{u}}_i) + \frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t)}{\partial t} \\ & \geq -\alpha_i(\mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t)) + \Big\|\frac{\partial \mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t)}{\partial \bar{\boldsymbol{x}}_i}\Big\|\tilde{C}_i - \hat{\epsilon}_{i,1} \end{aligned} \tag{10b}$$

$$\frac{\partial \mathfrak{b}_j^l(\bar{\boldsymbol{x}}_j^l,t)}{\partial \boldsymbol{x}_i}(f_i(\boldsymbol{x}_i,t)+g_i(\boldsymbol{x}_i,t)\hat{\boldsymbol{u}}_i)+\frac{\partial \mathfrak{b}_j^l(\bar{\boldsymbol{x}}_j^l,t)}{\partial t} \geq -\alpha_j(\mathfrak{b}_j^l(\bar{\boldsymbol{x}}_j^l,t))+\Big\|\frac{\partial \mathfrak{b}_j^l(\bar{\boldsymbol{x}}_j^l,t)}{\partial \bar{\boldsymbol{x}}_j^l}\Big\|\tilde{C}_j-\hat{\epsilon}_{i,\nu_{j,l}(t)} \quad \text{for each } j\in\mathcal{N}_i(t),\ l\in\mathcal{L}_{i,j}(t). \tag{10c}$$

Collaboration is indicated by (10c) and agent i hence not only aims to satisfy ϕ_i as in (10b), but also contributes to satisfying ϕ_j for each $j \in \mathcal{N}_i(t)$. Collaboration may come at the cost of not satisfying ϕ_i depending on the ratio of the parameters $K_{i,k}$. Note that (10) is a convex quadratic program with $m_i + 1 + \text{CR}_i(t)$ decision variables and $1 + \text{CR}_i(t)$ constraints.

For safety, barrier functions such as in [16] are used. Consider $\mathfrak{h}(\boldsymbol{x}_i) : \mathbb{R}^{n_i} \to \mathbb{R}$ with $\mathfrak{h}(\boldsymbol{x}_i) < 0$ for $\boldsymbol{x}_i \in \mathcal{O}_i$ and $\mathfrak{h}(\boldsymbol{x}_i) \geq 0$ for $\boldsymbol{x}_i \notin \mathcal{O}_i$. We also require that $\mathfrak{h}(\boldsymbol{x}_i) < 0$ for $\boldsymbol{x}_i \notin \mathfrak{B}_i$. For an extended class $\mathcal{K}$ function $\hat{\alpha}_i$, consider

$$\frac{\partial \mathfrak{h}_i(\boldsymbol{x}_i)}{\partial \boldsymbol{x}_i}(f_i(\boldsymbol{x}_i,t)+g_i(\boldsymbol{x}_i,t)\hat{\boldsymbol{u}}_i) \geq -\hat{\alpha}_i(\mathfrak{h}_i(\boldsymbol{x}_i))+\Big\|\frac{\partial \mathfrak{h}_i(\boldsymbol{x}_i)}{\partial \boldsymbol{x}_i}\Big\|C_i. \tag{11}$$

We assume that there exists a compact set $\hat{\mathcal{U}}_i$ such that, for each $\boldsymbol{x}_i \in P_i(\mathfrak{D})$, there exists $\hat{\boldsymbol{u}}_i \in \hat{\mathcal{U}}_i$ so that (11) holds. Given a function $\mathfrak{h}_i(\boldsymbol{x}_i)$ obtained, for instance, by a sum-of-squares procedure, this property can easily be verified; $\tilde{C}_i$ can then be obtained by considering $\boldsymbol{x}_i \in P_i(\mathfrak{D})$ and assuming $\boldsymbol{u}_i \in \mathcal{U}_i \supseteq \hat{\mathcal{U}}_i$. Let now $\boldsymbol{u}_i(\bar{\boldsymbol{x}}_{i,\text{c}},t) := \hat{\boldsymbol{u}}_i$ and $\epsilon_{i,k}(\bar{\boldsymbol{x}}_{i,\text{c}},t) := \hat{\epsilon}_{i,k}$ where $\hat{\boldsymbol{u}}_i$ and $\hat{\epsilon}_{i,k}$ are given by the quadratic program (10), which is additionally subject to (11) and $\hat{\boldsymbol{u}}_i \in \mathcal{U}_i$, and where $\bar{\boldsymbol{x}}_{i,\text{c}}$ is the stacked vector of the states of the agents in $\mathcal{V}_i \cup_{j\in\mathcal{N}_i(t)} \cup_{l\in\mathcal{L}_{i,j}(t)} \mathcal{V}_j^l$. Note that $\text{CR}_i(t)$ introduces discontinuities that, however, do not affect the existence of solutions since $\text{CR}_i(t)$ is piecewise continuous.

Theorem 3: Let Assumption 1 hold and assume that $\tilde{C}_i$ is given. If $\boldsymbol{u}_i(\bar{\boldsymbol{x}}_{i,\text{c}},t)$ is continuous, then $r_i \leq \rho^{\phi_i}(\bar{\boldsymbol{x}}_i)$ where $r_i \geq \kappa_i \geq \alpha_i^{-1}(-\epsilon_{i,\max}(\infty))$ with $\kappa_i := \inf_{t\geq 0} \mathfrak{b}_i(\bar{\boldsymbol{x}}_i(t),t)$ and where r_i is maximized, while $\boldsymbol{x}_i(t) \notin \mathcal{O}_i$ for all $t \geq 0$.

Proof: The quadratic program (10), additionally subject to (11) and $\hat{\boldsymbol{u}}_i \in \mathcal{U}_i$, is feasible for each $\boldsymbol{x}_i \in P_i(\mathfrak{D})$. If $\boldsymbol{u}_i(\bar{\boldsymbol{x}}_{i,\text{c}},t)$ is continuous, there exist solutions $\boldsymbol{x} : [0,\tau_{\max}) \to \mathfrak{D}$ to (1) with $\tau_{\max} > 0$. This implies that $\mathfrak{C}_{i,\text{safe}} := \{\boldsymbol{x}_i \in \mathbb{R}^{n_i} | \mathfrak{h}_i(\boldsymbol{x}_i) \geq 0\}$ is forward invariant and $\boldsymbol{x}_i(t) \notin \mathcal{O}_i$ for all $t \in [0,\infty)$. In particular, note that $\tau_{\max} = \infty$ due to [23, Thm. 3.3] so that $\mathfrak{b}_i(\bar{\boldsymbol{x}}_i(t),t) \geq \kappa_i \geq \alpha_i^{-1}(-\epsilon_{i,\max}(\infty))$ for all $t \geq 0$ so that $\rho^{\phi_i}(\bar{\boldsymbol{x}}_i) \geq r_i \geq \kappa_i$ by construction of $\mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t)$. ■

IV. Simulations

Consider $M := 6$ with $n_i := m_i := 2$. Agents 1, 2, and 3 are as in Example 1. Agents 4, 5, and 6 are subject to $\dot{\boldsymbol{x}}_i = c_i(\boldsymbol{x},t) + \boldsymbol{u}_i$ where $c_i(\boldsymbol{x},t) := \sum_{j\in\{4,5,6\}\setminus\{i\}} \text{sat}_1(\boldsymbol{x}_j - \boldsymbol{x}_i)$ and where, for $\boldsymbol{\zeta} := \begin{bmatrix}\zeta_1 & \zeta_2\end{bmatrix}^T \in \mathbb{R}^2$, $\text{sat}_1(\boldsymbol{\zeta}) := \begin{bmatrix}\bar{\zeta}_1 & \bar{\zeta}_2\end{bmatrix}^T$ with $\bar{\zeta}_c = \zeta_c$ if $|\zeta_c| \leq 1$, $\bar{\zeta}_c = 1$ if $\zeta_c > 1$, and $\bar{\zeta}_c = -1$ if $\zeta_c < -1$ for $c \in \{1,2\}$. We impose $\boldsymbol{u}_i \in \mathcal{U}_i := [-2,2]^2$ so that $\tilde{C}_i = 4$. Scenario 1 illustrates the approach in Section III-A, while Scenario 2 illustrates the online collaboration as in Section III-B; $\mathfrak{b}_i(\bar{\boldsymbol{x}}_i,t)$ for $i \in \{4,5,6\}$ are constructed as in [14, Eq. (11)] and we set $\alpha_i(r) := 10r$ and $\hat{\alpha}_i(r) := 500r$.

Scenario 1: Agents 4, 5, and 6 are subject to ϕ_4, ϕ_5, and ϕ_6 as in Example 1. It holds that $\mathcal{E} := \{(1,4),(5,4),(6,4),(2,5),(4,5),(6,5),(3,6),(4,6),(5,6)\}$. Note that ϕ_4, ϕ_5, and ϕ_6 already have a formula dependency in a favouring direction. The simulation results are shown in Fig. 1 where $K_{i,1} := 0.1$ and $K_{i,2} := 0.9$. Fig. 1a shows that $\kappa_4 = -0.395$, $\kappa_5 = -0.382$, and $\kappa_6 = -0.39$, while Fig. 1b shows that $\epsilon_{4,\max}(\infty) = 7.006$, $\epsilon_{5,\max}(\infty) = 6.808$, and $\epsilon_{6,\max}(\infty) = 7.271$ with $\epsilon_{i,\max}(\infty) = \sup_{t\geq 0} \epsilon_i(\hat{\boldsymbol{x}}_i(t),t)$. Theorem 3 hence predicts that $\mathfrak{b}_4(\hat{\boldsymbol{x}}_4(t),t) \geq -0.7006$, $\mathfrak{b}_5(\hat{\boldsymbol{x}}_5(t),t) \geq -0.6808$, and $\mathfrak{b}_6(\hat{\boldsymbol{x}}_6(t),t) \geq -0.7271$ and gives a more conservative estimate than what is actually obtained. The trajectories from $0-10$ s and from $10-35$ s are shown in Fig. 1c and 1d, respectively. Consider further one static obstacle $\mathcal{O}_i := \{\boldsymbol{o}\}$ for each agent $i \in \{4,5,6\}$ with $\boldsymbol{o} := \{\boldsymbol{x}_i \in \mathbb{R}^{n_i} | \|\boldsymbol{x}_i - \begin{bmatrix}0 & 0.3\end{bmatrix}^T\| \leq 0.2\}$, i.e., placed such that it intersects the agents trajectories in Fig. 1d. The simulation results are shown in Fig. 2.

Scenario 2: To illustrate the use of collaboration requests, consider now a slightly altered scenario with the formulas $\phi_4' := \phi_4$, $\phi_5' := G_{[5,\infty)}(\|\boldsymbol{x}_5 - \boldsymbol{x}_2\| \leq 0.3)$, and $\phi_6' := G_{[5,\infty)}(\|\boldsymbol{x}_6 - \boldsymbol{x}_3\| \leq 0.3)$ together with the edge set $\mathcal{E} := \{(1,4),(5,4),(6,4),(2,5),(4,5),(3,6)\}$ so that only agent 5 can colaborate with agent 4 in case of a critical event. The simulation results are shown in Fig. 3. A critical event happens at $\tau = 4.927$ s and then collaboration is established with agent 5 and the parameters $K_{5,1} := 0.1$, $K_{5,2} := 0.7$, and $K_{5,3} := 0.2$. Agent 5 deviates from its optimal trajectory, which would be similar to agent 6's trajectory (agent 6 can not collaborate due to $\mathcal{E}$), to collaborate with agent 4.

The computation times are, on average for each agent and on an Intel Core i7-6600U with 16 GB of RAM, 2 ms without collaboration and 2.5 ms when collaboration is initiated.

V. Conclusion

Based on control barrier functions, we presented a feedback control strategy to find least violating solutions for multi-agent systems under conflicting local signal temporal logic tasks. In particular, the barrier function inequality was relaxed whenever needed and a characterization of the violation was formalized. Furthermore, collaboration among agents was initiated when possible. For future work, not only collaboration, but also task re-assignment may be considered, i.e., defining a notion of least violating solutions for the discrete level.

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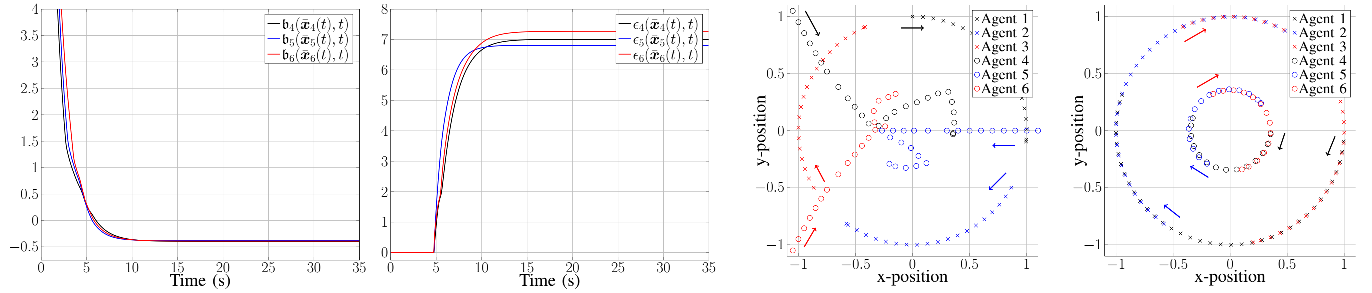
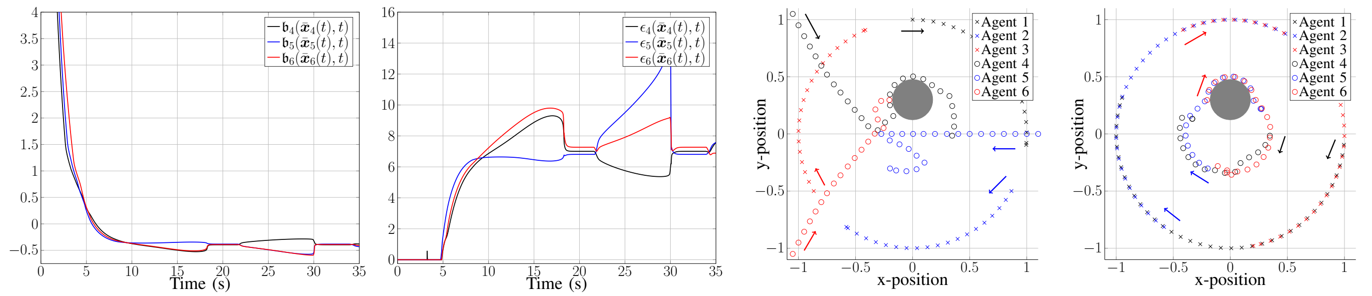


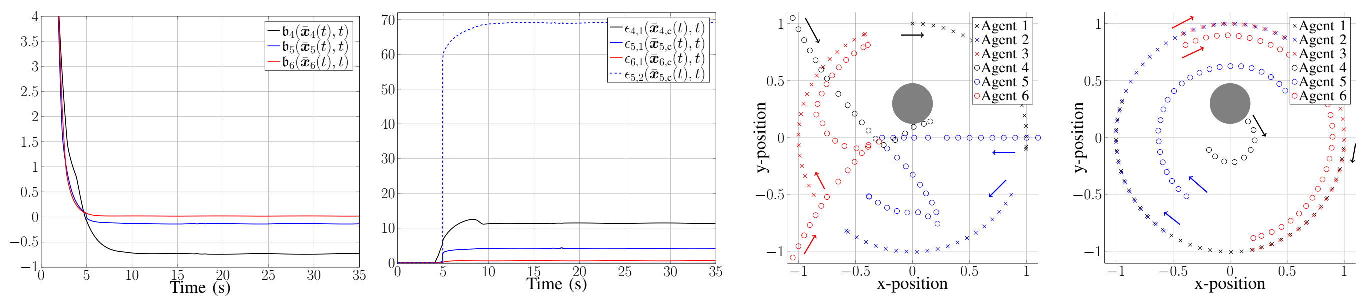
(a) Barrier Function Evolution (b) Slack Variable Evolution (c) Agent trajectories: 0-10 s (d) Agent trajectories: 10-35 s

Fig. 1: Barrier function evolution and agent trajectories for Scenario 1 without obstacles.



(a) Barrier Function Evolution (b) Slack Variable Evolution (c) Agent trajectories: 0-10 s (d) Agent trajectories: 10-35 s

Fig. 2: Barrier function evolution and agent trajectories for Scenario 1 with an obstacle indicated in grey.



(a) Barrier Function Evolution (b) Slack Variable Evolution (c) Agent trajectories: 0-10 s (d) Agent trajectories: 10-35 s

Fig. 3: Barrier function evolution and agent trajectories for Scenario 2 with an obstacle indicated in grey.

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