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Control, Estimation and Optimization of Energy Efficient Buildings

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Abstract— **Commercial buildings are responsible for a significant fraction of the energy consumption and greenhouse gas emissions in the U.S. and worldwide. Consequently, the design, optimization and control of energy efficient buildings can have a tremendous impact on energy cost and greenhouse gas emission. Buildings are complex, multi-scale in time and space, multi-physics and highly uncertain dynamic systems with wide varieties of disturbances. Recent results have shown that by considering the whole building as an integrated system and applying modern estimation and control techniques to this system, one can achieve greater efficiencies than obtained by optimizing individual building components such as lighting and HVAC. We consider estimation and control for a distributed parameter model of a multi-room building. In particular, we show that distributed parameter control theory, coupled with high performance computing, can provide insight and computational algorithms for the optimal placement of sensors and actuators to maximize observability and controllability. Numerical examples are provided to illustrate the approach. We also discuss the problems of design and optimization (for energy and CO2 reduction) and control (both local and supervisory) of whole buildings and demonstrate how sensitivities can be used to address these problems.**

I. Introduction

Whole buildings are complex, multi-scale, multi-physics, highly uncertain dynamic systems with wide varieties of disturbances. By itself, whole building simulation is a significant computational challenge. However, when addressing the additional requirements that center on design, optimization (for energy and CO_2) and control (both local and supervisory) of whole buildings, it becomes an immense challenge to develop practical computational tools that are scalable and widely applicable to current and future building stock.

At a fundamental level, there are several potential solutions to the design and control of high performance buildings. Roughly speaking, these approaches include: (1) Simulation Based Design, (2) Holistic Fully Integrated Design and (3) Hybrid Design Methods. Regardless of the approach, it is clear that computing resources and the development of computational methods will be an enabling science because at some point in the design and control process, numerical methods must be employed. A major question is, "When does one introduce the approximations"? In the best case one keeps the physics of the problem as long as possible and then introduce approximations at the last stage of the design. The current state is the opposite; the physics is approximated by a numerical (lumped) model and then used as a design model. This is what is known as simulation based design. At the other end of the spectrum is the holistic approach where the design problem is abstracted and then computational methods and tools are developed to solve the fully integrated design, optimization or control problem. Here the numerical approximations are introduced at the last stage. Hybrid methods attempt to take advantages of both approaches. In this paper we show that distributed parameter control, combined with high performance computing can be used to provide practical insight into important issues such as sensor/actuator placement and state estimation for control.

The Model and Problem Formulation

Before focusing on a specific problem it is important to note that whole buildings are very complex multi-scale (in time and space) systems as Fig. 1 below illustrates. Optimal design and control of these systems are very challenging problems and are often done by first developing a reduced order model and then basing the design on the simplified model. In this short paper we show that distributed parameter control theory can provide useful information about building design and control and then we suggest future areas of research.

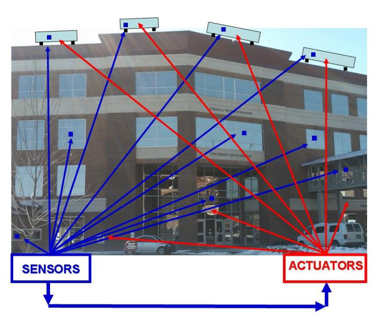

Fig. 1. A Whole Building is a Complex System

In order to illustrate some of the ideas, we consider the problem illustrated by a single room shown in Fig. 2 below.

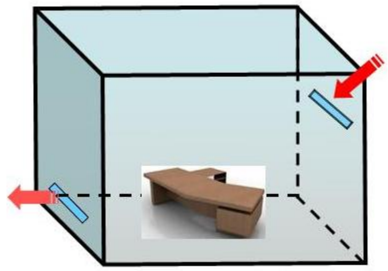

Fig. 2. Room Control Problem

Here, the goal is to design the room (locate vents, place sensors, etc.) in order to control the room temperature near the workspace and minimize energy. The problems of design and control should be considered simultaneously because the type and effectiveness of the controller depends on the type and quality of the sensed information and conversely. In this problem the system is governed by the Navier-Stokes equations in the room denoted by Ω and given by

$$\frac{\partial \mathbf{v}(t,\mathbf{x})}{\partial t} + \mathbf{v}(t,\mathbf{x})\cdot\nabla\mathbf{v}(t,\mathbf{x}) = -\nabla\mathbf{p}(t,\mathbf{x}) + \frac{1}{\mathrm{Re}}\Delta\mathbf{v}(t,\mathbf{x}) \quad \text{(I.1)}$$

$$\nabla\cdot\mathbf{v}(t,\mathbf{x}) = 0, \quad \text{(I.2)}$$

$$\frac{\partial T(t,\mathbf{x})}{\partial t} + \mathbf{v}(t,\mathbf{x})\cdot\nabla T(t,\mathbf{x}) = \frac{1}{\mathrm{RePr}}\Delta T(t,\mathbf{x}) + B(t,\mathbf{x}), \quad \text{(I.3)}$$

where $\mathbf{x}\in\Omega\subset\mathbb{R}^3$, $\mathbf{v}(t,\mathbf{x})$ is the velocity vector, $p(t,\mathbf{x})$ is the pressure and $T(t,\mathbf{x})$ is the temperature. Nondimensionalization has been carried out such that Re is the Reynolds number and Pr is the Prandtl number. Note that for this study, the energy equation (I.3) does not influence the momentum equation (I.1).

The ideas presented in this work should ultimately be studied on the Boussinesq equations, where temperature introduces a bouyancy force in (I.1). For ease of presentation, we assume inflow is fixed and the control term is given by $B(t,\mathbf{x}) = b(\mathbf{x})u(t)$ where $b(\mathbf{x})$ is a given distribution and $u(t)$ is a thermal control input. The case where the control is applied at the boundary is slightly more complex and requires a different technical framework. However, for the discussion here it is sufficient to think of $b(\mathbf{x})$ as a function with support near the wall vent defined on the domain Ω. The controlled output, w, of the system will be defined by a weighted average over the sub-domain in the room occupied by the workspace. In particular, let

$$w(t) = \int_{\Omega_W} d(\mathbf{x})T(t,\mathbf{x})d\mathbf{x}, \quad \text{(I.4)}$$

where $\Omega_W\subset\Omega$ is specified to be a region around the desk. Consider the problem of finding the control that minimizes

$$J(u) = \int_0^\infty \left\{[w(t)-r(t)]^2 + R[u(t)]^2\right\}dt, \quad \text{(I.5)}$$

where $R>0$ and $r(t)$ is a desired average temperature to be tracked. For the discussion here, we set $r(t)=0$.

Also, assume that one has p sensors with supports in the regions Ω_i. In particular, for $i=1,2,...,p$, let

$$y_i(t) = \int_{\Omega_i} c_i(\mathbf{x})T(t,\mathbf{x})d\mathbf{x}, \quad \text{(I.6)}$$

and hence the sensed output is given by

$$\mathbf{y}(t) = [y_1(t),\ y_2(t),\ \cdots,\ y_p(t)]^T.$$

II. Abstract Formulation of the Control Problem

Under suitable assumptions and applying the appropriate boundary conditions, we formulate (I.1)-(I.3) as a differential equation on an infinite dimensional (Hilbert) space, Z, of the form

$$\dot z(t) = \mathcal{A}z(t) + \mathcal{N}(z(t)) + \mathcal{B}u(t), \quad t>0, \quad \text{(II.1)}$$
$$z(0) = z_0\in Z,$$

where $\mathcal{A}: D(\mathcal{A})\subseteq Z\to Z$ generates a C_0-semigroup $S(t)$ on Z, $\mathcal{N}: D(\mathcal{N})\subseteq Z\to Z$ is a nonlinear operator and $\mathcal{B}: U\to Z$ is a linear input operator (perhaps unbounded) from the control space U to the state space Z. If the operator $\mathcal{E}: Z\to\mathbb{R}$ is defined by (I.4), then the cost function (I.5) has the form

$$J(u) = \int_0^\infty \left[\langle \mathcal{Q}z(t), z(t)\rangle_Z + \langle Ru(t), u(t)\rangle_U\right]dt, \quad \text{(II.2)}$$

where $\mathcal{Q} = \mathcal{E}^*\mathcal{E}$.

Also, if the operator $\mathcal{C}: Z\to\mathbb{R}^p$ is defined by (I.6), then the measured output is defined by

$$\mathbf{y}(t) = \mathcal{C}z(t). \quad \text{(II.3)}$$

Here, $Z = V\times\mathbb{R}$ where V is the space of divergent free vector fields on Ω, and $\mathcal{A}$ has the form

$$\mathcal{A} = \begin{bmatrix} \mathcal{A}_O & 0 \\ -(\nabla T)^T & \mathcal{A}_{AD} \end{bmatrix}$$

where $\mathcal{A}_O$ is the (linear) Oseen operator and $\mathcal{A}_{AD}$ is the (linear) advection-diffusion operator. The state $z(t)\in Z$ is identified with $[z(t)](\mathbf{x}) = [\tilde{\mathbf{v}}(t,\mathbf{x})\ T(t,\mathbf{x})]^T$, where $\tilde{\mathbf{v}}$ is a perturbation to the mean flow $\bar{\mathbf{v}}(\mathbf{x}) = \lim_{\tau\to\infty}\frac{1}{\tau}\int_0^\tau \mathbf{v}(s,\mathbf{x})ds$, i.e. $\mathbf{v}(t, \mathbf{x}) = \bar{\mathbf{v}}(\mathbf{x}) + \tilde{\mathbf{v}}(t, \mathbf{x})$ (see [18], [27] and [29] for details). When one linearizes the system about the steady flow, the resulting linearized equation has the form

$$\dot{z}(t) = \mathcal{A}z(t) + \mathcal{B}u(t). \tag{II.4}$$

The corresponding linear quadratic regulator (LQR) control problem is defined by minimizing the cost (II.2) subject to the linear dynamics (II.4). This approach to the control of Navier-Stokes may be found in [12], [17] and [27].

One can show that under reasonable conditions (see [25]) the LQR problem has an optimal control, in feedback form,

$$u^{opt}(t) = -\mathcal{K}z(t), \tag{II.5}$$

where $\mathcal{K} : Z \to R^m$ is a bounded linear "gain" operator. In addition, $\mathcal{K} = R^{-1}\mathcal{B}^*\Pi$ where $\Pi : Z \to Z$ is a bounded linear operator, $\Pi = \Pi^*$ and Π satisfies the Riccati equation

$$\mathcal{A}^*\Pi + \Pi\mathcal{A} - \Pi\mathcal{B}\mathbf{R}^{-1}\mathcal{B}^*\Pi + \mathcal{Q} = 0. \tag{II.6}$$

The Riesz Representation Theorem implies that there exist a divergence free vector field $\mathbf{k}_v(\mathbf{x})$ and a function $k_T(\mathbf{x})$ such that

$$\mathcal{K}z(t) = \int_{\Omega} \langle \mathbf{k}_v(\mathbf{x}), \mathbf{v}(t, \mathbf{x}) \rangle \, d\mathbf{x} + \int_{\Omega} k_T(\mathbf{x}) T(t, \mathbf{x}) d\mathbf{x} \tag{II.7}$$

where the kernels $\mathbf{k}_v(\mathbf{x})$, and $k_T(\mathbf{x})$ are called functional gains. The functional gains define the optimal LQR controller and can be used to place sensors and design low order controllers (see [1], [2], [11], [12], [13] and [14]).

Note that we have made two simplifying assumptions. First of all, we have assumed that the control input only appears in the energy equation (I.3). Secondly, we have neglected the Boussinesq term in (I.1). This results in the lower triangular structure of the operator $\mathcal{A}$. For further simplicity, we assume that the flow is fixed. Thus, the "fan" is always "on" and only the temperature of the air is controlled. Then (II.7) can be written as

$$\mathcal{K}z(t) = \int_{\Omega} k_T(\mathbf{x}) T(t, \mathbf{x}) d\mathbf{x}. \tag{II.8}$$

We now have developed practical methods for computing 3D functional gains $k_T(\mathbf{x})$ (see [8]), and we can use this gain to guide the choice and placement of the sensors. As we show below, the functional gains often have localized support and one can use this fact to determine what regions in space are most important to the controller. For example, if

$$k_T(\mathbf{x}) = \begin{cases} k_T(\mathbf{x}) > 0, & \mathbf{x} \in \Omega_S \\ 0, & \mathbf{x} \notin \Omega_S \end{cases},$$

then there is no reason to locate sensors outside Ω_S. Also, it is important (if possible) to place sensors in the regions where $k_T(\mathbf{x}) \gg 0$.

Consider the linearized system

$$\dot{z}_T(t) = \mathcal{A}_{AD} z_T(t) + \mathcal{B}_T u, \tag{II.9}$$

with a single sensed output

$$y(t) = \mathcal{C}_T z_T(t),$$

as defined above in (I.6). Standard linear state estimators (observers) have the form

$$\dot{z}_e(t) = \mathcal{A}_e z_e(t) + \mathcal{F}y(t), \tag{II.10}$$

where $\mathcal{F} : \mathbb{R} \to Z$ has the representation

$$[\mathcal{F}y](\mathbf{x}) = [f_v(\mathbf{x}), f_T(\mathbf{x})]^T y,$$

in general, and

$$[\mathcal{F}y](\mathbf{x}) = f_T(\mathbf{x}) y$$

in this particular case, given the simplifications above. The functions $f_v(\mathbf{x})$ and $f_T(\mathbf{x})$ are observer functional gains. It is often not possible to locate sensors in the "optimal" places and/or to have full state information even with optimal sensor location. Hence, one must construct a state estimator (observer) and use sensed information to construct (partial) state information needed for the control. In a practical setting, one can only measure a localized state such as the temperature on a wall and this data is often used as the "room" temperature measurement. Considerable improvements can be made by employing state estimation and reduced order modeling for estimation. In [31] the authors use reduced order model reduction techniques based on Lagrangian Coherent Structures (LCS) to locate sensors for estimating contaminant transport in a room. Although reduced order modeling was employed in [31], understanding the flow physics provides insight into the number of sensors needed to estimate the flow. If one combines such ideas with the spatial insight gained from the functional gains, then the problem of optimal sensor/actuator location becomes feasible. This requires that functional (feedback and observer) gains be computed. In the next section we illustrate this computation for the 3D room above.

III. Numerical Examples and Conclusions

Here we use distributed parameter LQR theory combined with finite elements to compute both feedback functional gains and observer functional gains for thermal control of the 3D room above (Fig. 2) with a fixed flow.

The flow is computed in a cubical room with an inflow duct at $(x = 0,\ 0.375 < y < 0.625,\ 0.75 < z < 0.825)$ with a bi-quadratic flow profile. The Reynolds number, Re, is 50 based on the height of the room and the maximum inlet flow velocity. The Prandtl number is $\mathrm{Pr} = 0.7$, which is appropriate for air. A finite element method using (35,000) tetrahedral Taylor-Hood elements was used for the flow simulation. Streamlines as well as velocity vectors and velocity magnitude contours are shown in Fig. 3. The advection diffusion equation (II.9) for the control problem uses a uniform inlet temperature of $u(t)$ and insulated boundary conditions elsewhere.

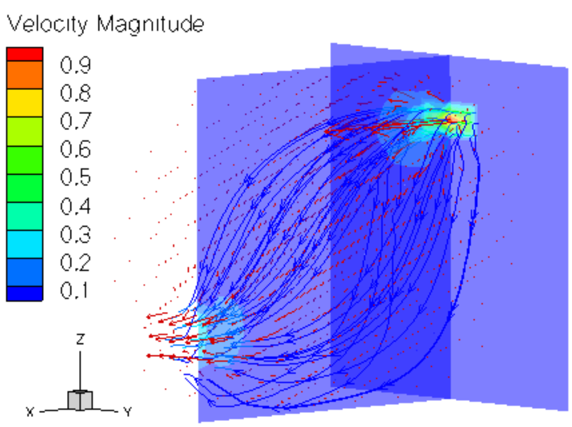

Fig. 3. Flow Through the Room

Fig. 4 shows the optimal LQR feedback functional gain $k_T(\mathbf{x})$. Note that a large portion of the support of $k_T(\mathbf{x})$ is concentrated in the center of the room and is zero near the inflow vent. The maximum value in the center of the room corresponds with our choice of $\Omega_W = (0.25, 0.75)^3$. Thus, the functional gain illustrates the point above, suggesting that "optimal" sensor placements should be focused near the workspace. However, in practice one must use "wall" sensors and use state estimation methods to re-construct the state.

Figs. 5–7 below contain the observer functional gains, F_T, corresponding to three different single sensor locations: $\Omega_1 = (.25, .50) \times (.875, 1) \times (.25, .375)$, $\Omega_2 = (.50, .75) \times (.875, 1) \times (.25, .375)$, and $\Omega_3 = (.25, .50) \times (.75, .875) \times (.875, 1)$, see (I.6). Note that when the sensor is placed near the outflow vent the support of the observer gain is the largest, while the support of the observer gain when the sensor is placed on the top wall the smallest. Thus, as indicated quantitatively in Table I below, placing a sensor near the exit vent will tend to enhance "observability."

TABLE I

L_2 Norm of Observer, $\mathcal{F}$

Sensor Location	$\|\mathcal{F}\|_2$	$\max(f_T)$
centered on side wall	1.37×10^{-6}	1.83×10^{-6}
on side wall near exit	1.43×10^{-6}	1.94×10^{-6}
on ceiling near side wall	1.22×10^{-6}	1.61×10^{-6}

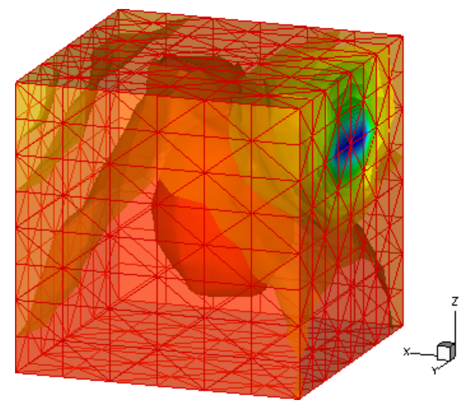

Fig. 4. Feedback Functional Gain

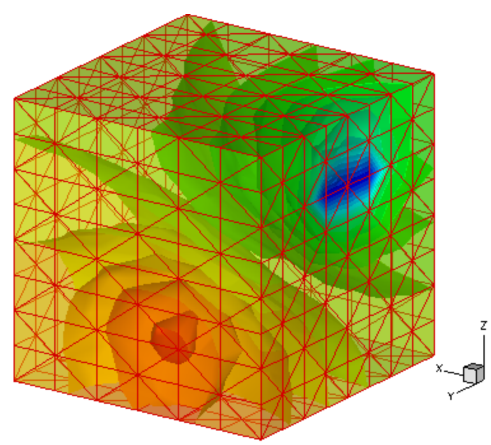

Fig. 5. Observer Gain: Sensor Centered on Side Wall

IV. Conclusions and Future Work

Although these results are preliminary, they illustrate how one can compute and use infinite dimensional theory to develop insight into control problems that arise in the design and operation of high performance buildings. For example, optimization could be used in the wall sensor placement problem to maximize $\|\mathcal{F}\|_2$. Likewise, similar optimization algorithms can be envisioned to optimally place the inflow vent. However, considerable work needs to be done before these ideas become useful tools for whole building design and control.

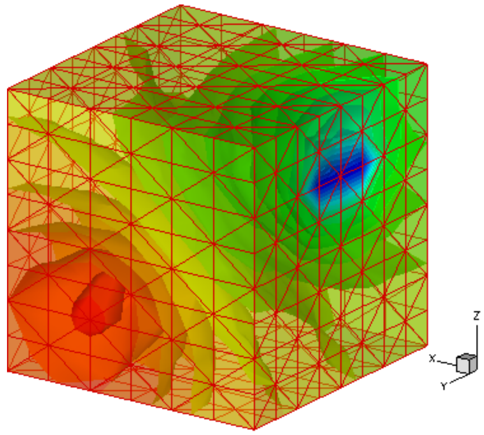

Fig. 6. Observer Gain: Sensor on Side Wall Near Vent

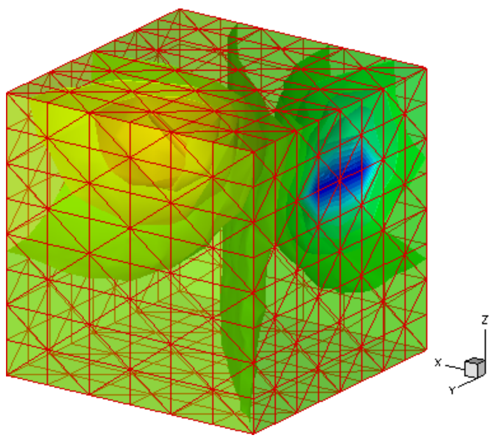

Fig. 7. Observer Gain: Sensor on Top Wall

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