Control of a Ground Vehicle Using Quadratic Programming Based Control Allocation Techniques

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***Abstract*— This paper examines the use of control allocation techniques for the control of multiple inputs to a ground vehicle to track a desired yaw rate trajectory while minimizing vehicle sideslip. The proposed controller uses quadratic programming accompanied by linear quadratic regulator gains designed around a linear vehicle model to arrive at a combination of vehicle commands. Several failure scenarios are examined and the results for two different quadratic programming approaches are presented along with a discussion of the advantages each method has to offer.**

I. Introduction

Control allocation (CA) of over-actuated vehicles involves generating an optimal set of effector commands that match actual body torque to the desired body torque as closely as possible while minimizing the control effort and obeying the position and rate constraints of the effectors. A control allocation approach is generally used when different combinations of effector commands can produce the same result and when the number of effectors available exceeds the number of states being controlled. A key feature of control allocation is that of reconfiguration. In the event an effector failure is detected, the control effort is redistributed among the remaining active effectors to minimize the tracking error. Different methods of control allocation have been developed for aerospace vehicles [3], [6], marine vessels [17], and other areas where this proves to be a valuable safety aspect. Increasing driver/passenger safety is a constant motivation for related research on ground vehicles. Research has been conducted on the control of ground vehicles via state feedback using steering angle, differential braking, and even aerodynamic actuators as control inputs [15], [19]. The addition of differential braking as a control parameter has proven to be successful in maintaining vehicle stability during drastic maneuvers [18] and in rollover prevention [4].

This paper examines the use of quadratic programming in determining an on-line solution to the control allocation problem to control a ground vehicle with redundant effector inputs (e.g. differential braking and steering) to track a desired yaw rate trajectory while minimizing vehicle sideslip. The proposed controller uses quadratic programming (QP) accompanied by linear quadratic regulator (LQR) gains designed around a linear vehicle model to arrive at a combination of vehicle commands.

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II. Control Allocation

The control allocation problem has become harder to solve as over-actuated vehicles become increasingly complex with the advancement of science. The general control allocation problem is well stated in [9] as the computation of an optimal set of effector commands u that will produce some desired overall control effect, $\bar{u}$. In other words, given a desired response $\bar{u}$, determine u such that $Bu = \bar{u}$ subject to $u^- \leq u \leq u^+$, where u^+ and u^- are upper and lower bounds placed on the effectors and B is a matrix defining the effectiveness of the effectors. If multiple solutions exist, choose one that will minimize the predetermined cost function. If there are no solutions, find u such that Bu approximates $\bar{u}$ as well as possible.

Traditional CA approaches are centered around a simple least squares approach. The least squares method uses a pseudo-inverse of a reference model and determines the effector commands as a function of the commanded, or desired, effects. Although this method is easily implemented and computationally efficient, it does not consider effector command limitations [2].

Model predictive control (MPC) has recently gained popularity in the vehicle control community due to advancements that significantly reduce the computational time required to solve this type of optimization. MPC has been developed significantly in the chemical industry where plant dynamics allow for sufficient computational time. Recent advancements in MPC however, allow for a faster on-line solution by shifting some of the computational burden off-line [1]. This has been proven to be an effective CA technique for rollover prevention of ground vehicles [4] but still possesses significant computational complexity along with a trade off between simplicity of on-line solution and memory to store off-line computed solutions.

III. Vehicle

A. Nonlinear Vehicle Model

The vehicle model used to study the proposed controller is a 4 wheeled ground vehicle with three actuators available to control the moment about the yaw axis: differential braking of the front and rear tires and steering angle of the front tires. The free body diagram is shown below:

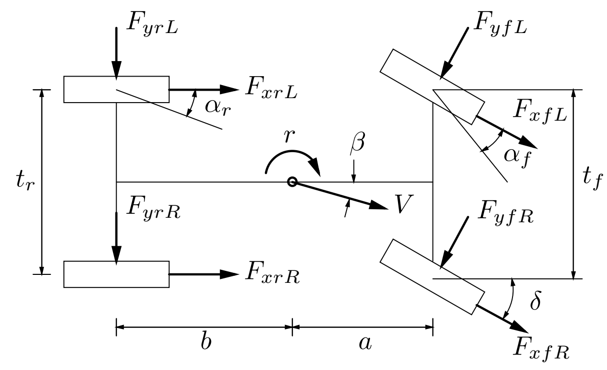


Fig. 1. Free body diagram of vehicle model

where r is the yaw rate, V is the velocity vector acting at the vehicle's center of gravity, β represents sideslip angle, and δ is the steering angle. Subscripts f, r, R, L denote front, rear, right, and left sides of the vehicle respectively. Note that all forces F and slip angles α are drawn in the positive direction such that lateral force $F_y = -C_\alpha \alpha$ where C_α represents tire cornering stiffness.

Applying Newton's laws of motion to the free body diagram (Fig. 1), the nonlinear equation of motion about the yaw axis at the center of gravity can be written as

$$\begin{aligned}\ddot{\Psi} = \dot{r} = \quad & [a(F_{yfL}\cos(\delta) + F_{yfR}\cos(\delta) \\ & -F_{xfL}\sin(\delta) - F_{xfR}\sin(\delta)) \\ & -b(F_{yrL} + F_{yrR}) + \tfrac{t_r}{2}(\Delta F_{xr}) \\ & +\tfrac{t_f}{2}(F_{yfR}\sin(\delta) - F_{yfL}\sin(\delta) \\ & +F_{xfR}\cos(\delta) - F_{xfL}\cos(\delta))]/I_z\end{aligned} \tag{1}$$

where I_z is the moment of inertia about the yaw axis. A good estimate of sideslip angle can be written $\beta = tan^{-1}(\frac{V_x}{V_y})$. Differentiating this w.r.t. time gives the equation of motion for sideslip:

$$\begin{aligned}\dot{\beta} = \quad & (F_{yrL} + F_{yrR} + F_{yfL}\cos(\delta) + F_{yfR}\cos(\delta) \\ & -F_{xfL}\sin(\delta) - F_{xfR}\sin(\delta))/mV\cos(\beta) \\ & -\dot{V}\tan(\beta)/V - r\end{aligned} \tag{2}$$

A Pacejka tire model [7] is used to model the behavior of the tires. This nonlinear tire model uses tire slip angle and vertical force to approximate the lateral force acting on the tire. Similar to vehicle sideslip, the equations for tire slip angle can be written:

$$\begin{aligned}\alpha_{rL} &= \tan^{-1}\left[\frac{V\sin(\beta) - rb}{V\cos(\beta) + r\frac{t_r}{2}}\right] \\ \alpha_{rR} &= \tan^{-1}\left[\frac{V\sin(\beta) - rb}{V\cos(\beta) - r\frac{t_r}{2}}\right] \\ \alpha_{fL} &= \tan^{-1}\left[\frac{V\sin(\beta) + ra}{V\cos(\beta) + r\frac{t_f}{2}}\right] - \delta_L \\ \alpha_{fR} &= \tan^{-1}\left[\frac{V\sin(\beta) + ra}{V\cos(\beta) - r\frac{t_f}{2}}\right] - \delta_R\end{aligned} \tag{3}$$

Vertical forces are a function of roll angle and yaw rate. For the maneuvers considered in this paper, the roll dynamics are assumed to be slower than the input dynamics so that the roll angle is strictly proportional to lateral acceleration. Therefore, roll angle is approximated as in [8] by:

$$\phi = \frac{WhV\frac{r}{g}}{K_{\phi f} + K_{\phi r} - Wh} \tag{4}$$

where W is the vehicle weight, h is the distance between the center of gravity and the roll axis, g is acceleration due to gravity, and K_ϕ is the total roll stiffnesses of the axle. A more accurate roll model is used in [4] where transient roll dynamics are not ignored. By further assuming that the vehicle travels on flat terrain and neglecting longitudinal weight transfer, the vertical load difference is approximated for each axle by taking the moment about the roll axis and combining it with the above expression for ϕ.

$$\begin{aligned}\Delta F_{zf} &= \left[\frac{W_f V r h_f}{g} + \phi K_{\phi f}\right]/t_f \\ \Delta F_{zr} &= \left[\frac{W_r V r h_r}{g} + \phi K_{\phi r}\right]/t_r\end{aligned} \tag{5}$$

The vertical load at each tire is:

$$\begin{aligned}F_{zfR} &= F_{zf} - \Delta F_{zf} \\ F_{zfL} &= F_{zf} + \Delta F_{zf} \\ F_{zrR} &= F_{zr} - \Delta F_{zr} \\ F_{zrL} &= F_{zr} + \Delta F_{zr}\end{aligned} \tag{6}$$

B. Linear Vehicle Model

The proposed control law is designed around a linearized model of equations (1-2) given in state space form below.

$$\begin{bmatrix}\dot{\beta} \\ \dot{r}\end{bmatrix} = \begin{bmatrix} -\frac{C_0}{mV} & -\frac{C_1}{mV^2} - 1 \\ -\frac{C_1}{I_z} & -\frac{C_2}{VI_z}\end{bmatrix}\begin{bmatrix}\beta \\ r\end{bmatrix} + \begin{bmatrix}\frac{C_{\alpha f}}{mV} & 0 & 0 & 1 \\ a\frac{C_{\alpha f}}{I_z} & \frac{t_f}{2I_z} & \frac{t_r}{2I_z} & 0\end{bmatrix}\begin{bmatrix}\delta \\ \Delta F_{xf} \\ \Delta F_{xr} \\ \nu\end{bmatrix} \tag{7}$$

$$\begin{aligned}C_0 &= C_{\alpha f} + C_{\alpha r} \\ C_1 &= aC_{\alpha f} - bC_{\alpha r} \\ C_2 &= a^2 C_{\alpha f} + b^2 C_{\alpha r}\end{aligned}$$

where $C_{\alpha f}$ and $C_{\alpha r}$ are constants representing the front and rear linearized tire cornering stiffness values (per axle). The state space model also utilizes small angle approximations, the assumption of constant vehicle velocity, and neglects lateral and longitudinal weight transfer.

The nature of the input matrix B implies that the differential braking inputs can only affect vehicle sideslip indirectly through the coupling between the two states. Because of this, it is necessary to add a fourth virtual input ν. This relaxes the equality constraint on the sideslip and removes some of the control responsibility from the steering angle without affecting the yaw rate.

IV. Controller Design

Observe that, for a given desired derivative vector $\dot{x}_{des} = \begin{bmatrix} \dot{\beta} & \dot{r} \end{bmatrix}^T$, there is more than one possible corresponding input vector u. One may, for example, use a least squares weighted pseudo-inverse to compute an input vector u to match $\dot{x}_{des}$, but such a procedure does not necessarily obey effector constraints (e.g., position limits). The proposed controller uses QP-based CA to compute corresponding optimal input commands that, when possible, match the desired derivative vector while obeying effector constraints.

The control law assumes full state feedback of yaw rate and sideslip are available. These values can be obtained using traditional estimation [18], or measured by using Global Positioning System and inertial sensors [5], [11]. Considering the availability of such measurements along with how inaccuracies due to the linearization of the vehicle model affect the QP optimization, a more robust form of QP based control allocation presented in [10] may be more appropriate.

The proposed controller can be thought of as being split into two separate parts: a control law which defines a total control effect $\bar{u}$ that the vehicle must produce, and a control allocator that calculates an optimal combination of effector commands u that when applied to the vehicle will produce the desired control effect $\bar{u}$.

LQR gains designed for a modified linear vehicle model are used to produce a desired control effect, $\bar{u}$. The modified system assumes a perfect input matrix $B = I_{2x2}$, and also includes the addition of an integrator to place more emphasis on yaw rate tracking and less on minimization of sideslip. The resulting state space model takes the form:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{e}_r \end{bmatrix} = \begin{bmatrix} -\frac{C_0}{mV} & -\frac{C_1}{mV^2} - 1 & 0 \\ -\frac{C_1}{I_z} & -\frac{C_2}{VI_z} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ e_r \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{u}_\beta \\ \bar{u}_r \end{bmatrix} \tag{8}$$

Applying the Algebraic Riccati Equation yields a gain matrix K_{2x3} such that

$$\begin{bmatrix} \bar{u}_\beta \\ \bar{u}_r \end{bmatrix} = -K \begin{bmatrix} \beta \\ r_e \\ e_r \end{bmatrix} \tag{9}$$

where r_e is defined as the difference between actual and desired yaw rate, and $e_r = \dot{r}_e$. The controller gains K serve to drive the vector of signals $\begin{bmatrix} \beta & r_e & e_r \end{bmatrix}^T$ to zero, and $\begin{bmatrix} \bar{u}_\beta & \bar{u}_r \end{bmatrix}^T$ are the resulting sideslip and yaw rate control effects the control allocator tries to match.

A. Quadratic Programming

The control allocation is achieved by solving a quadratic programming problem which involves the minimization of a quadratic cost function subject to both equality and inequality constraints. The general form of the quadratic programming problem is:

$$\begin{aligned} \min_{x} \quad & \tfrac{1}{2}u^T Q u + c^T u \\ \text{subject to} \quad & Bu = \bar{u} \\ \text{and} \quad & u^- \leq u \leq u^+ \end{aligned} \tag{10}$$

where u is the set of effector commands and c and Q are weights placed respectively on the linear and quadratic parts of the cost function. These weights can be chosen to favor certain effectors and/or weight the frequency content of the effector commands over time.

The incorporation of inequality constraints ensures that the set of commands u will always be inside the attainable operating ranges of the effectors whether they be position limits, rate limits or any other limiting factor associated with the effectors. The input effectiveness matrix B from the linear vehicle model (7) is incorporated into the equality constraint which serves to ensure that the solution vector u matches the desired control effect vector $\bar{u}$.

Proper selection of the quadratic weight matrix Q significantly affects the optimization with the addition of the virtual sideslip effector ν. A large quadratic penalty is placed on ν to reduce its use and leave most of the control responsibility up to the real effectors.

B. Sign Preserving Quadratic Programming

In the event that $\bar{u}$ exists such that the solution u must lie outside the inequality constraints, the QP problem is deemed infeasible. The method of sign preserving quadratic programming (SPQP) proposed in [16] guarantees feasibility by allowing scaling of the control effort.

Sign preserving quadratic programming introduces slack variables σ for each control effort so that the problem is modified as follows:

$$\begin{aligned} \min_{u,\sigma} \quad & \tfrac{1}{2}u^T Q_u u + {c_u}^T u + \tfrac{1}{2}Q_\sigma(1-\sigma_\beta)^2 \\ & +\tfrac{1}{2}Q_\sigma(1-\sigma_r)^2 \\ s.t. \quad & Bu - \Sigma\bar{u} = 0 \\ & \begin{bmatrix} u_c^- \\ 0 \\ 0 \end{bmatrix} \leq \begin{bmatrix} u_c \\ \sigma_\beta \\ \sigma_r \end{bmatrix} \leq \begin{bmatrix} u_c^+ \\ 1 \\ 1 \end{bmatrix} \\ where \quad & \Sigma = \begin{bmatrix} \sigma_\beta & 0 \\ 0 & \sigma_r \end{bmatrix} \end{aligned} \tag{11}$$

The slack variables σ_β and σ_r allow for individual scaling of the components of the control effect vector with the least control authority (function of states u in input effectiveness matrix B). The separate scaling of $\bar{u}_\beta$ and $\bar{u}_r$ maintains problem feasibility while preserving the sign of the total desired control effect vector.

The SPQP problem incorporates the equality constraint of $Bu - \Sigma\bar{u} = 0$ just as the nominal QP problem (10) meets the equality constraint of $Bu - \bar{u} = 0$. This “allocation error” provides a measure of how well the controller is able to match the desired control effect. Due to SPQP’s ability to scale $\bar{u}$ by Σ, the exclusion of Σ when calculating allocation error for the SPQP method produces a larger allocation error for the part of the control effect being scaled. Therefore, allocation error for the individual states are included in the results section to give the reader a way to observe which parts of $\bar{u}$ are being scaled for the different scenarios presented.

V. Simulation

The full nonlinear vehicle model with the nonlinear tire model (equations 1-6) and the proposed controller were implemented in Matlab/Simulink. A single sine wave oscillation corresponding to a double lane change maneuver was computed off-line and used as the desired yaw rate trajectory.

Three different scenarios are presented to demonstrate reconfiguration ability: 1) nominal case, no failures experienced 2) front brake failure at 2.75 seconds, and 3) steering failure at 2.75 seconds. For each of these cases the vehicle was simulated at constant velocities of 45, 55, and 65mph for both regular QP and SPQP algorithms. This assumes a separate controller regulates fuel flow to the engine during differential braking commands to maintain a constant speed.

Failures were implemented by scaling the columns of the input effectiveness matrix B from equation (7) corresponding to the failed effector. Actual implementation would require the controller to be alerted of a failure through an on-board vehicle diagnostic system such as the one demonstrated in [13]. Therefore, on-line calculation of input effectiveness matrix B would be necessary due to its dependence on both failure mode and velocity.

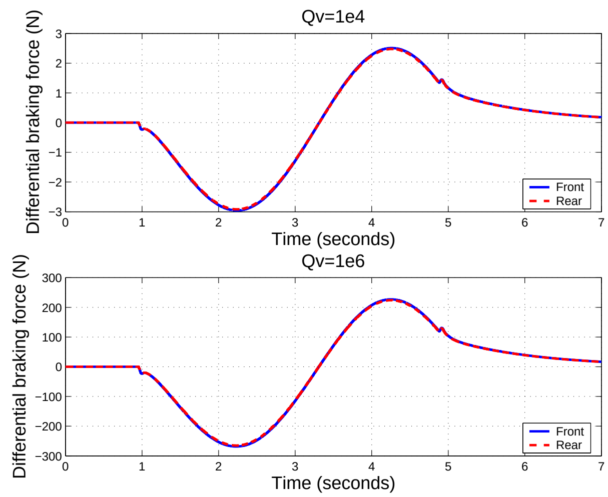
A constant position limit of $\pm 0.5\text{rad}(\approx\ 30\text{deg})$ was placed on the steering angle of the front tires. The limits placed on the differential braking commands were calculated on-line by taking 75% of the vertical force on the inside tires as the approximate maximum braking force that can be applied without producing slippage. Researchers have shown that on-line estimates of vertical force and road friction coefficient are possible through the use of extended Kalman-Bucy filtering and Bayesian hypothesis selection [14]. Alternatively, the underestimation of road friction coefficient has been shown to provide conservative approximations of maximum braking force available to the vehicle [12].

VI. Results

Since neither disturbances nor sensor noise were simulated in the experiment, the results presented here are for relative comparison only and are not a measure of true performance (qualitative not quantitative). The root mean squared errors of yaw rate and sideslip tracking are used to compare algorithms and assess the effectiveness of the controller in each scenario. Root mean squared allocation error, although subjective in nature, is also included for reasons stated in section IV-B.

As mentioned earlier, the quadratic penalty placed on the virtual effector ν significantly affects the control action. A quadratic penalty of $Q_\nu = 1e4$ resulted in minimal use of the differential braking commands as the controller relies heavily on the virtual effector to minimize sideslip. This value provided better yaw rate tracking for the straight QP approach by placing most of the control effort in the steering command and treating the differential braking as more of a secondary/backup input. A larger penalty of $Q_\nu = 1e6$ was also studied, in which case the differential braking commands were relied on significantly to minimize sideslip. The difference in magnitude of the braking commands is shown in Fig. 2 with plots of the commanded braking force for different values of Q_ν.

Fig. 2. nominal case braking commands at 45mph using QP



The results for each of the mentioned virtual effector weights $Q_\nu = 1e4$ and $Q_\nu = 1e6$ are included in Tables I and II, respectively. For the sake of consistency and to better demonstrate the control action taken on by the braking commands, all following figures are results from simulations run with a quadratic penalty of $Q_\nu = 1e6$ on the virtual effector.

The advantage of the SPQP algorithm's ability to scale the control effort is made clear by the comparatively lower yaw rate tracking error it provides for speeds of 45 and 55mph. The scaling occurs only in the sideslip portion of the control effect $\bar{u}_\beta$ and is a function of the weight placed on the virtual effector ν. A higher weight on ν requires a larger scaling of $\bar{u}_\beta$ resulting in greater sideslip allocation error and sideslip tracking error than the straight QP approach (Fig. 4). The effect of this scaling is apparent in the differential braking commands shown in figure 3 which appear to be significantly different than those resulting from the regular QP algorithm (Fig. 2).

Due to the coupling in equations (1) and (2), the QP optimization produces a differential braking command which opposes the steering angle command in an effort to keep the sideslip angle small. This leaves the yaw rate tracking

Fig. 3. nominal case: commands at 45mph using SPQP

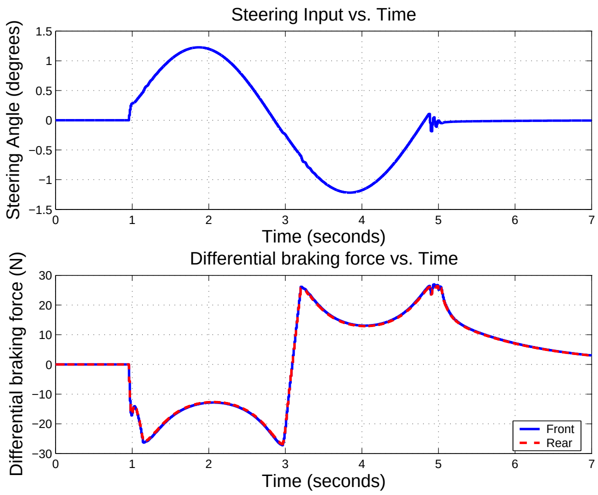


Fig. 4. nominal case: tracking at 45mph using SPQP

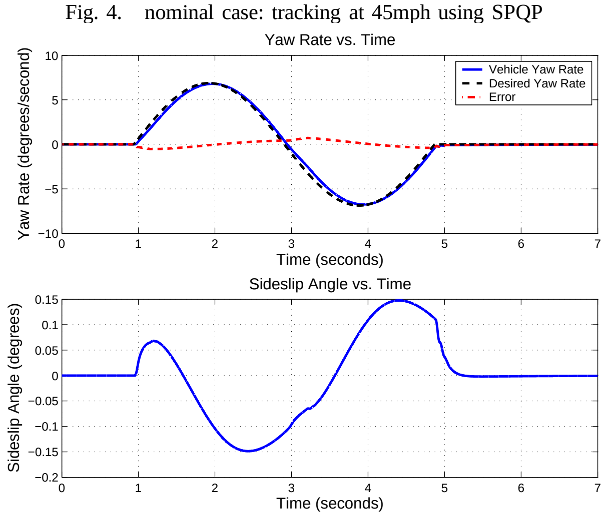


Fig. 5. Steering Failure: effector commands at 65mph using QP

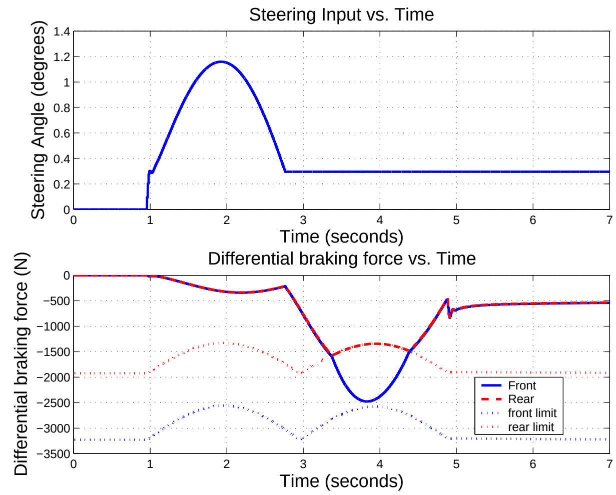
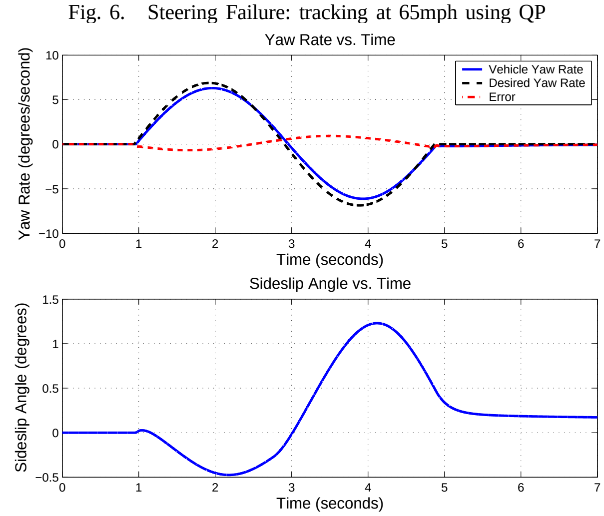


Fig. 6. Steering Failure: tracking at 65mph using QP



up to the steering command which has the most authority over both states. Less braking force opposing the steering angle requires less steering input resulting in improved yaw rate tracking. Better sideslip tracking is also achieved due to the direct impact of the steering input on β. For this reason, better performance of the straight QP algorithm is seen in the case of a brake failure just as it is when braking input is reduced by a small quadratic penalty on the virtual effector.

In the event of a steering failure however, yaw rate tracking is left solely up to the differential braking commands which are then used to minimize the yaw rate error after the failure occurs. Figures 5 and 6 display the controller's ability to maintain best possible yaw rate tracking through reconfiguration while respecting the limitations of the remaining effectors.

VII. Conclusion and Future Work

Control allocation and its reconfiguration abilities prove to be very useful in dealing with over-actuated systems such as the proposed ground vehicle model. QP based CA in particular, has been shown to provide intelligent distribution of control effort among the effectors in both nominal and failure cases. SPQP gives the best yaw rate tracking performance for the 45 and 55mph cases but regular QP consistently outperforms SPQP in the area of sideslip minimization. The two types of QP examined in this paper clearly offer different advantages and degrees of freedom which should be taken into account depending on the specific design goal. Additionally, for basic control allocation purposes a QP based method provides fast on-line commands yielding good results.

Better performance is achievable if the full potential of the braking commands can be exploited. Future work involves using a linear programming optimization in accordance with the tire friction circle to calculate a more accurate estimate of differential braking limits. Improvements in performance that may be obtained with additional inputs such as differential acceleration and 4 wheel steering will also be investigated.

TABLE I

RESULTS FOR QUADRATIC PENALTY OF $1e4$ ON VIRTUAL EFFECTOR ν

Vel (mph)	Alg	RMS r (deg/s)	RMS β (deg)	RMS r Allocation $\|Bu - \bar{u}\|$	RMS β Allocation $\|Bu - \bar{u}\|$
Nominal					
45	QP	0.4112	0.0179	2.755E-07	1.249E-06
45	SPQP	0.4111	0.0180	2.755E-07	1.249E-06
55	QP	0.3185	0.0899	3.656E-07	2.389E-06
55	SPQP	0.3044	0.0915	3.644E-07	9.702E-06
65	QP	0.2937	0.2047	5.008E-07	4.534E-06
65	SPQP	0.1853	0.2414	4.883E-07	3.081E-04
Front brake failure					
45	QP	0.4111	0.0179	2.755E-07	1.249E-06
45	SPQP	0.4111	0.0180	2.755E-07	1.250E-06
55	QP	0.3184	0.0900	3.656E-07	2.390E-06
55	SPQP	0.3043	0.0916	3.644E-07	9.703E-06
65	QP	0.2937	0.2049	5.008E-07	4.536E-06
65	SPQP	0.1852	0.2416	4.883E-07	3.083E-04
Steering failure					
45	QP	0.3124	0.2701	3.325E-07	2.539E-06
45	SPQP	0.3124	0.2699	3.325E-07	2.540E-06
55	QP	0.2618	0.3813	4.689E-07	4.617E-06
55	SPQP	0.2490	0.3815	4.669E-07	1.047E-05
65	QP	0.2605	0.5246	6.707E-07	7.870E-06
65	SPQP	0.1808	0.5424	6.585E-07	2.523E-04

TABLE II

RESULTS FOR QUADRATIC PENALTY OF $1e6$ ON VIRTUAL EFFECTOR ν

Vel (mph)	Alg	RMS r (deg/s)	RMS β (deg)	RMS r Allocation $\|Bu - \bar{u}\|$	RMS β Allocation $\|Bu - \bar{u}\|$
Nominal					
45	QP	0.4318	0.0053	2.698E-07	1.091E-06
45	SPQP	0.1103	0.0218	2.734E-07	4.674E-04
55	QP	0.3348	0.0522	3.505E-07	1.988E-06
55	SPQP	0.0843	0.1252	3.506E-07	1.119E-03
65	QP	0.3077	0.1360	4.630E-07	3.677E-06
65	SPQP	0.3223	0.3377	4.448E-07	2.709E-03
Front brake failure					
45	QP	0.4284	0.0083	2.700E-07	1.128E-06
45	SPQP	0.1097	0.0221	2.738E-07	4.686E-04
55	QP	0.3313	0.0603	3.518E-07	2.075E-06
55	SPQP	0.0843	0.1256	3.512E-07	1.121E-03
65	QP	0.3039	0.1506	4.679E-07	3.854E-06
65	SPQP	0.3230	0.3383	4.451E-07	2.712E-03
Steering failure					
45	QP	0.3157	0.3198	3.294E-07	2.511E-06
45	SPQP	0.1016	0.3418	2.486E-07	8.494E-04
55	QP	0.2645	0.4388	4.623E-07	4.542E-06
55	SPQP	0.3587	0.5726	3.188E-07	2.129E-03
65	QP	0.2628	0.5916	6.570E-07	7.695E-06
65	SPQP	1.0405	0.9854	4.616E-07	4.963E-03

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