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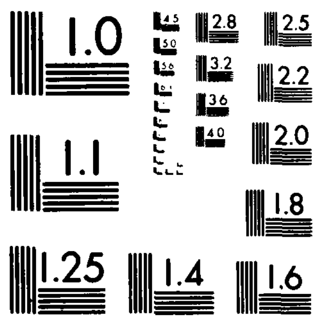

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Prof Banks and his co-workers continued their efforts on approximation techniques to be employed in parameter identification and optimal control problems. A general theoretical framework for such approximation schemes for partial differential equations was developed and tested numerically for the specific case of modal approximations. Significant advances were made in the difficult problems of parameter estimation for delay systems. Results for both semi-
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ITEM #20, CONTINUED: discrete and fully discrete methods for linear equations were obtained. ~~Profs Chow and Hale finished their~~ a book on Methods of Bifurcation Theory ~~to appear April 1982, Springer Verlag.~~ Much of their earlier work on nonlinear oscillations is included here. Hale prepared an extensive paper on dynamical systems in infinite dimensional spaces with the basic model being functional differential equations and parabolic partial differential equations. Hale and Rybakowski have completed a paper on gradient-like delay equations discussing in detail the maximal compact invariant set. Hale and Magalhaes have given a rather complete description of the flows defined by singularly perturbed delay differential equations. Abstracts of the papers ~~which have evolved~~ resulting from the support of the grant are included in this annual report, as well as a list of papers produced during this period ~~of the grant.~~

by Chow and Hale, * Publications include:

AFOSR-TR- 82-0456

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on

CONTROL OF DYNAMICAL SYSTEMS

Lefschetz Center for Dynamical Systems
Division of Applied Mathematics
Brown University
Providence, R. I. 02912

Principal Investigators:
H. T. Banks
J. K. Hale
E. F. Infante

Report prepared by:
H. T. Banks

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Banks, in collaboration with students and colleagues, continued his efforts on approximation techniques to be employed in parameter identification and optimal control problems. A general theoretical framework for such approximation schemes for partial differential equations was developed in [1] and tested numerically for the specific case of modal approximations. Alternative methods based on cubic spline and quintic spline approximations were treated in [2] and [3]. A survey of these methods, along with a treatment of variable coefficient estimation problems was given in [4].

Significant advances were made in the difficult problems of parameter (including multiple unknown delays) estimation for delay systems. Results for both semi-discrete (approximating ordinary differential equations) and fully discrete (approximating difference equations) methods for linear equations are summarized in [5]. Results for general nonlinear nonautonomous delay systems are given in [6],[7].

Chow and Hale finished their book on Methods of Bifurcation Theory to appear April, 1982, Springer-Verlag. Much of their earlier work on non-linear oscillations is included here.

Hale [9] prepared an extensive paper on dynamical systems in infinite dimensional spaces with the basic model being functional differential equations and parabolic partial differential equations.

Hale and Rybakowski [10] have completed a paper on gradient-like delay equations discussing in detail the maximal compact invariant set.

Hale and Magalhães [11] and Magalhães [12] have given a rather complete description of the flows defined by singularly perturbed delay differential equations.

Hale and Massatt [13] have discussed the ω-limit set of orbits of gradient-like systems proving, in particular, that it must be a point if the degeneracy is of order one.

Hale and Vegas [14] completed their work on bifurcation of equilibrium solutions of a parabolic equation when the parameter is the basic domain.

Infante pursued investigations centered on problems of stability and asymptotic behavior of infinite dimensional systems, particularly functional differential and integrodifferential equations.

In joint work with D. Abrahamson [15] the problem of the construction of a quadratic functional for the linear Volterra integro-differential equation

$$\dot{x}(t) = Ax(t) + \int_0^t B(t-\tau)x(\tau)d\tau, \quad t \geq t_0,$$

with initial condition $x(t) = f(t)$, $0 \leq t \leq t_0$, is attacked; the functional constructed yields very sharp asymptotic estimates, and represents a considerable generalization of previously used Liapunov functionals for such problems. Functionals of this type are also effectively applied to problems of a nonlinear nature and with nonconvolution kernels.

The approach to the construction of the Liapunov functionals used in [15], as well as previous experience in the construction of Liapunov functionals for neutral and retarded difference-differential equations, suggested that there is a general procedure that underlies these methods of construction. The procedure is centered on the use of Laplace transform techniques and in imitating the, by now, familiar method of constructing a quadratic form through the solution of the algebraic Liapunov equation $A^TB + BA = -C$. This methodology is presented and illustrated in [16].

Together with J. K. Hale and F. P. Tsen a number of results on the stability of retarded and neutral difference-differential equations of the neutral type have been obtained. In [17] some of these results are presented and illustrated by examples.

REFERENCES

[1] H.T. Banks and K. Kunisch, An approximation theory for nonlinear partial differential equations with applications to identification and control, LCDS Report #81-7.

[2] H.T. Banks, J.M. Crowley, K. Kunisch, Cubic spline approximation techniques for parameter estimation in distributed systems, LCDS Report #81-25.

[3] H.T. Banks and J.M. Crowley, Parameter estimation for distributed systems arising in elasticity, LCDS Report #81-24.

[4] H.T. Banks, A survey of some problems and recent results for parameter estimation and optimal control in delay and distributed parameter systems, LCDS Report #81-19.

[5] H.T. Banks and I.G. Rosen, Apprximation techniques for parameter estimation in hereditary control systems, Proc. 19th IEEE Conf. on Decision and Control, Dec. 10-12, 1980, Albuquerque, N.M., pp.741-743.

[6] P.L. Daniel, Spline based approximation methods for the identification and control of nonlinear functional differential equations, Ph.D. Thesis, Brown University, June 1981.

[7] H.T. Banks and P.L. Daniel, Estimation of delays and other parameters in nonlinear functional differential equations, LCDS Report #82-2.

[8] S.N. Chow and J.K. Hale, Methods of Bifurcation Theory, Springer-Verlag, 1982.

[9] J.K. Hale, Topics in Dynamic Bifurcation Theory, CBMS Reg. Conf. Math., No. 47, Am. Math. Soc., Prov., R.I., 1981.

[10] J.K. Hale and K. Rybakowskii, On a gradient-like integro differential equation. Proc. Royal Soc. Edinburgh. To appear.

[11] J.K. Hale and L. Magalhães, An example of boundary layer in a delay equation. Proc. Meeting on Functional Differential Equations, VPI, 1981. To appear.

[12] L. Magalhães, Singular perturbations of linear retarded functional differential equations, Ph.D. Thesis, Brown Univ., June, 1982.

[13] J.K. Hale and P. Massatt, Asymptotic behavior of gradient-like systems. Proc. Int. Meeting Dyn. Systems, Univ. Fla., Feb., 1981.

[14] J.K. Hale and J. Vegas, A nonlinear parabolic equation with varying domain. Arch. Rat. Mech. Ana. To appear.

[15] D.L. Abrahamson and E.F. Infante, A Liapunov functional for linear Volterra integrodifferential equations, to appear.

[16] E.F. Infante, Some results on the Liapunov stability of functional equations, Proc. of the Conf. on Volterra and Functional Equations, Marcel Dekker. To appear.

[17] E.F. Infante, A note on the stability in retarded delay equations for all delays, Proc. of Conf. on Dynamical Systems, Academic Press. To appear.

An Approximation Theory for Nonlinear Partial Differential Equations with Applications to Identification and Control

by

H.T. Banks and K. Kunisch

Abstract

Approximation results from linear semigroup theory are used to develop a general framework for convergence of approximation schemes in parameter estimation and optimal control problems for nonlinear partial differential equations. These ideas are used to establish theoretical convergence results for parameter identification using modal (eigenfunction) approximation techniques. Results from numerical investigations of these schemes for both hyperbolic and parabolic systems are given.

Cubic Spline Approximation Techniques for Parameter Estimation in Distributed Systems

H. T. Banks, J. M. Crowley, and K. Kunisch

ABSTRACT

Approximation schemes employing cubic splines in the context of a linear semigroup framework are developed for both parabolic and and hyperbolic second order partial differential equation parameter estimation problems. Convergence results are established for problems with linear and nonlinear systems and a summary of numerical experiments with the techniques proposed is given.

ESTIMATION OF DELAYS AND OTHER PARAMETERS IN NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS

H. T. Banks and P. L. Daniel

ABSTRACT

We discuss a spline-based approximation scheme for nonlinear nonautonomous delay differential equations. Convergence results (using dissipative type estimates on the underlying nonlinear operators) are given in the context of parameter estimation problems which include estimation of multiple delays and initial data as well as the usual coefficient-type parameters. A brief summary of some of our related numerical findings is also given.

APPROXIMATION TECHNIQUES FOR PARAMETER ESTIMATION IN HEREDITARY CONTROL SYSTEMS

by

H. T. Banks and I. G. Rosen

Abstract

We consider two approximation techniques for parameter identification problems for delay systems, one involving discretization in the state only, the other involving simultaneous discretization in state and time. Numerical comparisons are presented and discussed.

A SURVEY OF SOME PROBLEMS AND RECENT RESULTS FOR PARAMETER ESTIMATION AND OPTIMAL CONTROL IN DELAY AND DISTRIBUTED PARAMETER SYSTEMS

by

H. T. Banks

ABSTRACT

We survey a number of applications and problems motivating our current efforts on numerical techniques for parameter estimation in and optimal control of delay and partial differential equations. We then outline two different approaches for establishing theoretical convergence results for estimation algorithms. An application of modal techniques to the investigation of transport in brain tissue is briefly explained. A sketch of a convergence theory for spline techniques for function space parameter estimation problems is given.

PARAMETER ESTIMATION FOR DISTRIBUTED SYSTEMS ARISING IN ELASTICITY

H. T. Banks[+]

and

J. M. Crowley[+]

We discuss parameter estimation techniques for distributed systems such as the Euler-Bernoulli and Timoshenko equations of elasticity. The methods are based on cubic and quintic spline approximation schemes formulated in the context of a general functional analytic framework for abstract equations in Hilbert spaces. A number of examples with numerical results are presented to demonstrate efficacy of the techniques.

On a gradient-like integro-differential equation

by

Jack K. Hale and Krzysztof P. Rybakowski

Abstract

Let $b: [-1,0] \to \mathbb{R}$ be a C^2-function, $b(\theta) > 0$, $\theta \in (-1,0]$, $b(1) = 0$, $b'(\theta) \geq 0$, $b''(\theta) \geq 0$, $\theta \in [-1,0]$ and there is a θ_0 such that $b''(\theta_0) > 0$. Suppose $g: \mathbb{R} \to \mathbb{R}$ is a C^1-function such that $\int_0^x g(s)ds \to \infty$ as $|x| \to \infty$ and consider the equation

$$\dot{x}(t) = -\int_{-1}^{0} b(\theta)g(x(t+\theta))d\theta$$

Every solution of this equation approaches a zero of g. If the zeros of g are bounded, there is a maximal compact invariant set $A_{b,g}$ of this equation in $C([-1,0],\mathbb{R})$ which is one dimensional and consists only of the zeros of g and the unstable manifolds of these zeros. If g has only one zero, then $A_{b,g}$ is a point. If g has no more than three simple zeros, then the set $A_{b,g}$ is simply an arc with the unstable zero connected to the stable ones. In the class of g which have five simple zeros, we show that there are five distinct ways that the zeros of g can be connected by orbits in $A_{b,g}$. Only one of these preserves the order of the zeros on the reals. This shows clearly the importance of considering the set $A_{b,g}$ and the structure of the flow on this set rather than just asserting that every solution approaches a zero of g.

AN EXAMPLE OF BOUNDARY LAYER IN DELAY EQUATIONS

by

Jack K. Hale* and Luis Magalhães**

Abstract

Singular perturbation problems for functional differential equations have been studied by a number of authors [1-5]. These investigations were primarily concerned with the nature of convergence of the solutions to the degenerate problem for positive time. Very little is known about the boundary layer for the general case. The results of Halanay [4] and Klimushev[5] lead to a partial discussion of the boundary layer for a very special class of equations. In this note, we give a result for a special equation concerning necessary and sufficient conditions for the existence of an invariant subspace of finite codimension.

A NONLINEAR PARABOLIC EQUATION WITH VARYING DOMAIN

by

Jack K. Hale and José Vegas

ABSTRACT

For Ω a bounded convex domain, the only stable equilibrium solutions of the equation

$$u_t = \Delta u + f(u) \quad \text{in } \Omega$$

$$\frac{\partial u}{\partial n} = 0 \qquad \text{on } \partial\Omega$$

are spatially homogeneous. If $f(u) = \lambda u - u^3$, λ small, it is shown there are stable spatially nonhomogeneous solutions if $\Omega = \Omega_\varepsilon$ depends on a small parameter ε, $\Omega_\varepsilon \to \Omega_0$ as $\varepsilon \to 0$, Ω_0 is the union of two disjoint convex domains and some other technical conditions are satisfied.

ASYMPTOTIC BEHAVIOR OF GRADIENT-LIKE SYSTEMS

by

Jack K. Hale and Paul Massatt

ABSTRACT

For a class of gradient evolutionary equations, we prove that the ω-limit set of a bounded orbit is an equilibrium point if the dimension of the null space of the linear variational operator is no more than one. This implies the result of Matano [10] concerning a parabolic equation in one space dimension with separated boundary conditions. The statement about gradient systems is a consequence of a more general property which has applications, for example, to the stability of traveling waves.

A LIAPUNOV FUNCTIONAL FOR

LINEAR VOLTERRA INTEGRODIFFERENTIAL EQUATIONS

D.L.Abrahamson and E.F. Infante

ABSTRACT

Liapunov functionals of quadratic form have been used extensively for the study of the stability properties of linear ordinary, functional and partial differential equations. In this paper, a quadratic functional V is constructed for the linear Volterra integrodifferential equation

$$\dot{x}(t) = Ax(t) + \int_0^t B(t-\tau)x(\tau)d\tau \ , \ t \geq t_0 \ ,$$

$$x(t) = f(t), \ 0 \leq t \leq t_0 \ .$$

This functional, and its derivative $\dot{V}$, is more general than previously constructed ones and still retains desirable computational qualities; moreover, it represents a natural generalization of the Liapunov function for ordinary differential equations. The method of construction used suggests functionals which are useful for more general equations.

SOME RESULTS ON THE LIAPUNOV STABILITY
of FUNCTIONAL EQUATIONS

by

E.F. Infante

ABSTRACT

Liapunov functionals of quadratic form have been used extensively for the study of the stability properties of linear ordinary differential equations. In this brief paper, a simple method for the construction of desirable quadratic functionals for linear functional differential equations is outlined. These functionals are at the basis of the construction of Liapunov functionals for functional differential equations.

A NOTE ON THE STABILITY IN RETARDED DELAY

EQUATIONS FOR ALL DELAYS

by

E.F. Infante

ABSTRACT

In this note some results on the characterization of difference-differential equations, of retarded type, that retain the property of asymptotic stability or hyperbolicity irrespective of the value of the delays are presented. These results are illustrated through some simple examples.

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