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## **Control of focusing forces and emittances in plasma-based accelerators using near-hollow plasma channels**

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A near-hollow plasma channel, where the plasma density in the channel is much less than the plasma density in the walls, is proposed to provide independent control over the focusing and accelerating forces in a plasma accelerator. In this geometry the low density in the channel contributes to the focusing forces, while the accelerating fields are determined by the high density in the channel walls. The channel also provides guiding for intense laser pulses used for wakefield excitation. Both electron and positron beams can be accelerated in a nearly symmetric fashion. Near-hollow plasma channels can effectively mitigate emittance growth due to Coulomb scattering for high-energy physics applications.

## I. INTRODUCTION

Plasma-based accelerators are of interest because of their ability to sustain large acceleration gradients, enabling compact accelerating structures. The electric field of the electron plasma wave is on the order of  $E_0 = cm_e\omega_p/e$ , or  $E_0[\text{V/m}] \simeq 96\sqrt{n_0[\text{cm}^{-3}]}$ , where  $\omega_p = k_p c = (4\pi n_0 e^2/m_e)^{1/2}$  is the electron plasma frequency,  $n_0$  is the ambient electron number density,  $m_e$  and  $e$  are the electron rest mass and charge, respectively, and  $c$  is the speed of light in vacuum. This field can be several orders of magnitude greater than conventional accelerators, which are limited by material breakdown. Electron plasma waves with relativistic phase velocities may be excited by the ponderomotive force of an intense laser<sup>1</sup> or the space-charge force of a charged particle beam.<sup>2,3</sup> High-quality 1 GeV electron beams have been produced using 40 TW laser pulses in cm-scale plasmas.<sup>4</sup> Beam-driven plasma waves have also been used to double the energy of a fraction of electrons in the beam tail by the plasma wave excited by the beam head.<sup>5</sup> These experimental successes have resulted in further interest in the development of plasma-based acceleration as a basis for future linear colliders.<sup>6-8</sup>

The focusing forces acting on beams originate from the transverse wakefields in the plasma. For beam or laser drivers in the blow-out regime the focusing forces are determined by the background ion density.<sup>1</sup> For laser-driven plasma waves in the quasi-linear regime, the focusing forces can be controlled by controlling the transverse wakefields using shaped transverse laser intensity profiles.<sup>9</sup> Matching the beam to the focusing force is required to prevent emittance growth. Independent control of the transverse focusing and longitudinal accelerating forces is desired for control of the beam radius, enabling matched propagation.<sup>10</sup>

For high-energy physics applications, beams require ultra-low emittance to achieve the required luminosity. For fixed beam size, matching the beam in a plasma accelerator requires adjusting the focusing force of the wakefield as the beam accelerates such that  $k_\beta = \epsilon_n/(\gamma\sigma_x^2)$ , where  $k_\beta$  is the betatron wavenumber of the focusing force,  $\epsilon_n$  is the normalized transverse emittance,  $\sigma_x$  the beam transverse size, and  $\gamma m_e c^2$  is the beam particle energy. For a beam density greater than the plasma density  $n_b \gg n_0$ , the beam will blowout the surrounding electrons such that  $k_\beta = k_p/\sqrt{2\gamma}$ , and for sufficiently high beam density in a uniform plasma  $n_b/n_0 \gg M_i/m_e$ , where  $M_i$  is the ion mass, the background plasma ions will move on the

plasma period time-scale, leading to emittance growth.<sup>11</sup> Maintaining  $n_b \lesssim n_0$  typically requires weak focusing  $k_\beta = \epsilon_n/(\gamma\sigma_x^2) \ll k_p/\sqrt{\gamma}$ . This weak focusing can lead to emittance growth via Coulomb scattering with background ions in the plasma.

Hollow plasma channels, with zero density out to the channel radius and constant density for larger radii, have been studied<sup>12–14</sup> due to the beneficial properties of the accelerating structure. In a hollow plasma channel, the transverse profile of the driver is largely decoupled from the transverse profile of the accelerating mode. Therefore, for a relativistic driver, the accelerating gradient is transversely uniform and the focusing fields are linear. In addition, the accelerating mode of the hollow plasma channel is primarily electromagnetic, unlike the electrostatic fields excited in a homogeneous plasma. Methods for hollow plasma channel creation are actively being explored.<sup>15</sup>

In this work, we propose to use a partially-filled (near-hollow) plasma channel, with plasma density in the channel much less than the plasma density in the wall, to provide independent control of the focusing forces. In this geometry, the plasma density in the channel contributes to the focusing force, and the accelerating force is determined by the plasma density in the walls. For a sufficiently relativistic driver, the focusing wakefield is transversely linear and axially uniform. Hence any projected transverse emittance growth due to beam head-to-tail mismatch is eliminated.<sup>16</sup> It is also shown that the accelerating and focusing forces in this geometry can mitigate emittance growth induced by Coulomb collisions with background ions, preserving ultra-low emittance for high-energy physics applications. Ion motion in the channel is also negligible for resonant beams. A near-hollow channel, as described below, allows matched beam propagation with a constant beam density without significant emittance growth via scattering. Both electrons and positrons beams can be accelerated in a nearly symmetrical fashion.

## II. WAKEFIELDS IN NEAR-HOLLOW PLASMA CHANNELS

Consider a plasma channel with an initial electron plasma density of the form  $n(r) = n_c$  for  $r < r_c$  and  $n(r) = n_w$  for  $r \geq r_c$ , where  $n_w$  is the density in the wall,  $n_c$  is the density in the channel ( $n_c \ll n_w$ ), and  $r_c$  is the channel radius. We will consider  $k_w r_c \sim 1$  where  $k_w^2 = 4\pi n_w e^2/m_e c^2$  is the plasma wavenumber corresponding to the wall density. To provide weak focusing of an electron beam we will consider  $k_c^2 \ll k_w^2$ , where  $k_c^2 = 4\pi n_c e^2/m_e c^2$  is the

plasma wavenumber corresponding to the channel density.

Excitation of large amplitude plasma waves requires high-intensity lasers,  $a_0 \sim 1$ , where  $a_0^2 \simeq 7.32 \times 10^{-19} \lambda_0^2 [\mu\text{m}] I_0 [\text{W}/\text{cm}^2]$  with  $\lambda_0$  the laser wavelength and  $I_0$  the peak laser intensity. Note that such a plasma channel can effectively guide a laser pulse,<sup>12</sup> as the channel depth (not the on-axis density) provides for laser guiding.<sup>1</sup> The laser spot size  $w_0$  for quasi-matched propagation can be computed following Ref. 17, and  $w_0 \sim r_c > k_w^{-1}$  is typically required for guiding. For example, guiding for a transversely Gaussian laser pulse is provided in the low intensity, low power limit for a laser spot size  $w_0 = r_c / [\ln(k_w r_c)]^{1/2}$ . In this regime  $a_0^2 / (1 + a_0^2/2)^{1/2} > k_c^2 w_0^2 / 2$ , and the transverse ponderomotive force of the laser will expel the channel electrons, leaving an ion column.<sup>1</sup> This ion column can provide linear, phase-independent focusing for an ultra-relativistic witness bunch. In the following we will consider wakefields excited by a laser driver, although a particle beam driver can also excite wakefields with similar properties in a near-hollow plasma channel.

The wake excited in such a channel will consist of an electromagnetic wake owing to surface currents driven in the channel walls and a wake owing to the background ions in the channel. In the limit  $k_c^2 \ll k_w^2$ , the accelerating field in the channel is dominated by the currents in the wall and has the form<sup>12</sup>

$$E_z \simeq -E_w \Omega \int_{-\infty}^{\zeta} k d\zeta' \cos[k(\zeta - \zeta')] a^2(r = r_c, \zeta') / 4, \quad (1)$$

where  $a(r, \zeta) = eA/m_e c^2$  is the normalized transverse vector potential profile of the laser,  $\zeta = z - \beta_p c t$ ,  $\beta_p c$  is the driver velocity and  $\gamma_p = (1 - \beta_p^2)^{-1/2} \gg 1$ . For a laser driver,  $\gamma_p \sim k_0/k_w = 2\pi/(k_w \lambda_0)$ . The excited mode wavenumber<sup>12,14</sup> is  $k = k_w \Omega$  with

$$\Omega = \left[ 1 + \frac{k_w r_c K_0(k_w r_c)}{2K_1(k_w r_c)} \right]^{-1/2}, \quad (2)$$

and, for typical parameters,  $\Omega \sim 1$ . The focusing field excited in the channel is given by

$$E_r - \beta B_\theta \simeq E_c \frac{k_c r}{2} - E_w \frac{k_w r}{4} (\gamma^{-2} + \gamma_p^{-2}) \Omega^2 \int_{-\infty}^{\zeta} k d\zeta' \sin[k(\zeta - \zeta')] a^2(r = r_c, \zeta') / 4, \quad (3)$$

where  $\gamma^2 = 1/(1 - \beta^2) \gg 1$  and  $c\beta$  is the witness beam velocity. Here  $E_w = m_e c^2 k_w / e$  and  $E_c = m_e c^2 k_c / e$ . The first term on the right-hand side of Eq. (3) is due to the ion column and the second term is due to the currents driven in the channel walls. The focusing force is linear, to order  $\mathcal{O}(\gamma_p^{-2})$ , with respect to the radial position  $E_r - \beta B_\theta \propto r$  and hence the rms normalized transverse (slice) emittance is conserved to that order. In the regime

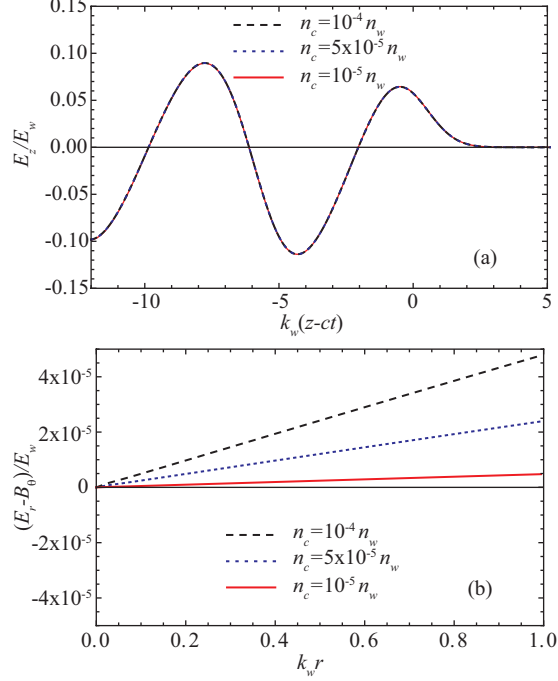


FIG. 1. Plasma wakefield excited by a quasi-matched, resonant Gaussian laser pulse with  $a_0 = 1$  and  $k_w w_0 = 2.3$  in a near-hollow plasma channel with  $k_w r_c = 1.5$  and  $n_c/n_w = 10^{-5}$ ,  $5 \times 10^{-5}$  and  $10^{-4}$ . (a) Accelerating wakefield  $E_z/E_w$  in the channel versus  $k_w(z - ct)$ . (b) Focusing wakefield  $(E_r - B_\theta)/E_w$  in the channel (at the peak  $E_z$ ) versus  $k_w r$ .

$n_c/n_w > a_0^2(r_c)/(8\gamma_p^2)$  and  $\gamma^2 \gg \gamma_p^2$ , the focusing from the channel ion density dominates  $E_r - \beta B_\theta \simeq E_c k_c r/2$  and  $k_\beta = k_c/\sqrt{2\gamma}$ . In this case the focusing force is uniform (in phase) over the entire bunch, eliminating any betatron mismatch between the head and tail of the beam.<sup>16</sup> Matched propagation is achieved for

$$\frac{n_c}{n_w} = \frac{2(k_w \epsilon_n)^2}{\gamma(k_w \sigma_x)^4}. \quad (4)$$

For the case of an effectively hollow plasma channel, the focusing forces can be controlled by using an external (permanent magnet) focusing system. Acceleration of positron beams would operate in this regime.

Figure 1 shows the normalized longitudinal  $E_z/E_w$  and transverse  $(E_r - B_\theta)/E_w$  wakefields excited by a quasi-matched,<sup>17</sup> resonant Gaussian laser with  $a_0 = 1$ ,  $k_0/k_w = 100$ , and  $k_w w_0 = 2.3$ , in a channel with  $k_w r_c = 1.5$  and  $n_c/n_w = 10^{-5}$ ,  $5 \times 10^{-5}$  and  $10^{-4}$ . The wakefields shown in Fig. 1 were computed using the particle-in-cell code INF&RNO.<sup>18</sup> Figure 1(a) shows

the accelerating field (determined by the wall density) is independent of the channel density for  $n_c \ll n_w$ . In addition, the accelerating field is uniform with respect to radial position inside the channel. Figure 1(b) shows independent control over the focusing field on a witness bunch (for fixed accelerating field) by varying the channel density. These transverse wakefields have excellent properties for beam emittance preservation, namely linear  $\propto r$  and axially constant throughout a witness beam.

### III. EMITTANCE GROWTH BY SCATTERING

It is well-known that emittance growth can occur by elastic scattering with the plasma ions.<sup>7,19,20</sup> Coulomb collisions results in a change of the rms divergence of the beam particle.<sup>21</sup> For the case of a near-hollow plasma channel,

$$\frac{d\langle\Delta\theta_x^2\rangle}{dz} = \frac{4\pi r_e^2}{\gamma^2} \int_{r_{\min}}^{r_{\max}} \frac{dr}{r} Z^2 n_i = \frac{r_e}{\gamma^2} \left[ Z_c k_c^2 \ln\left(\frac{r_c}{r_{\min}}\right) + Z_w k_w^2 \ln\left(1 + \frac{\lambda_D}{r_c}\right) \right], \quad (5)$$

where  $\Delta\theta_x = \Delta p_x/p_z$  is the ratio of perturbed transverse particle momentum to longitudinal particle momentum (the brackets indicate an rms average over many scattering events),  $Z_{c,w}$  is the charge state of the ions, and  $r_e = e^2/m_e c^2$ . In the quasi-linear wakefield regime  $a^2(r = r_c) < 1$ , background electrons are present and provide screening in the channel walls, such that the maximum impact parameter is  $r_{\max} \sim r_c + \lambda_D$  where  $\lambda_D = (k_B T/4\pi n_w e^2)^{1/2}$  is the Debye length. For typical laser-ionized plasmas, the electron plasma temperature  $k_B T$  is on the order of eV. For  $k_w r_c \sim 1$ ,  $\lambda_D/r_c \sim (k_B T/m_e c^2)^{1/2} = \beta_{\text{th}} \ll 1$ . The minimum impact parameter is the effective nuclear radius,  $r_{\min} = 1.4A^{1/3}$  fm, with  $A$  the mass number.<sup>22</sup> For typical parameters,  $\ln(r_c/r_{\min}) \sim 10$  and  $\ln(1 + \lambda_D/r_c) \sim \beta_{\text{th}} \sim 10^{-3}$ . For a relativistic particle undergoing linear focusing ( $F_x/\gamma m_e c^2 = -k_\beta^2 x$ ) with acceleration and an approximately matched beam, the resulting rms normalized emittance  $\epsilon_n = \gamma \epsilon_x$  growth is

$$\frac{d\epsilon_n}{dz} = \frac{\gamma}{2k_\beta} \frac{d\langle\theta_x^2\rangle}{dz}. \quad (6)$$

Strong focusing (large  $k_\beta$ ) suppresses the emittance growth from scattering.

#### A. Constant focusing force in homogeneous plasma

To compare to the case of a near-hollow plasma channel, we first review the case of scattering in a homogeneous plasma.<sup>7,19,20</sup> Consider a constant focusing force, with  $k_\beta =$

$\kappa/\gamma^{1/2}$  and  $\kappa = \text{constant}$ . For the quasi-linear laser-driven wake regime,  $\kappa^2 = (\phi_0/r_\perp^2) \sin \psi$ , where  $\psi$  is phase location of the beam in the plasma wave,  $r_\perp$  is the transverse scale length of the wakefield, and  $\phi_0$  is the wakefield amplitude. The accelerating field is  $d\gamma/dz = -k_p E_z/E_0$ . For the quasi-linear laser-driven wake regime,  $d\gamma/dz = k_p \phi_0 \cos \psi$ . In the highly-nonlinear bubble regime,<sup>20</sup>  $\kappa = k_p/\sqrt{2}$  and  $E_z/E_0 = k_p r_B/2$ , where  $r_B$  is the bubble radius (approximately the nonlinear plasma wavelength).

For constant wakefield focusing, the emittance growth is

$$\frac{d\epsilon_n}{d\gamma} = \frac{k_p r_e Z}{2\kappa(E_z/E_0)\gamma^{1/2}} \ln\left(\frac{r_{\max}}{r_{\min}}\right), \quad (7)$$

which may be solved to yield

$$\Delta\epsilon_n = \left[ \frac{k_p}{\kappa(E_z/E_0)} \right] r_e Z \ln\left(\frac{r_{\max}}{r_{\min}}\right) (\gamma_f^{1/2} - \gamma_i^{1/2}), \quad (8)$$

where  $\Delta\epsilon_n = \epsilon_{nf} - \epsilon_{ni}$  with  $\epsilon_{nf}$  and  $\epsilon_{ni}$  the final and initial normalized emittances, respectively, and  $\gamma_f$  and  $\gamma_i$  are the final and initial beam energies, respectively. Note that  $r_{\max} = \lambda_D$  in the quasi-linear regime and  $r_{\max} = r_B$  in the nonlinear bubble regime. Typically  $k_p r_\perp \sim 1$  and  $k_p r_B \sim 1$ , such that  $k_p/[\kappa(E_z/E_0)] \sim 1$  and there is a weak dependence on the plasma density. Scattering is suppressed by the strong focusing provided by the plasma wave. As the beam accelerates, the beam radius decreases and the peak beam density increases. For sufficiently high beam density  $n_b/n_0 \gtrsim M_i/m_e$ , ion motion will occur.<sup>11</sup>

## B. Constant beam density in homogeneous plasma

If fixed beam density is desired, i.e.,  $\sigma_x = \text{constant}$ , one may consider varying the focusing force with energy such that the beam remains matched  $k_\beta = \epsilon_n/(\gamma\sigma_x^2)$  or  $\kappa \propto 1/\gamma^{1/2}$ . In this case, the emittance growth is

$$\frac{d\epsilon_n}{d\gamma} = \frac{\sigma_x^2 k_p r_e Z}{2\epsilon_n(E_z/E_0)} \ln\left(\frac{\lambda_D}{r_{\min}}\right), \quad (9)$$

which may be solved to yield

$$\epsilon_{nf}^2 - \epsilon_{ni}^2 = \frac{k_p r_e Z \sigma_x^2}{(E_z/E_0)} \ln\left(\frac{\lambda_D}{r_{\min}}\right) (\gamma_f - \gamma_i). \quad (10)$$

This emittance growth can be prohibitively large for high-energy physics applications. Note that a similar expression was obtained in Ref. 23.



### C. Constant beam density in near-hollow plasma channel

Consider the case of scattering in a near-hollow plasma channel. We assume  $n_c/n_w \ll 1$ , and  $n_c \ln(r_c/r_{\min}) < n_w \ln(1 + \lambda_D/r_c) \approx n_w \beta_{\text{th}}/(k_w r_c)$ , where the scattering is dominated by interaction of the beam with the ions in the wall of the plasma channel. In this case the scattering rate is

$$\frac{d\epsilon_n}{d\gamma} = \frac{r_e Z_w \beta_{\text{th}}}{2(E_z/E_w) r_c k_\beta \gamma}, \quad (11)$$

where  $E_z$  is given by Eq. (1). With a focusing force that maintains constant beam density,  $\sigma_x^2 = \epsilon_n/(\gamma k_\beta) = \text{constant}$ , the emittance growth is

$$\epsilon_{nf} = \left[ \epsilon_{ni}^2 + \frac{\sigma_x^2 r_e Z_w \beta_{\text{th}}}{(E_z/E_w) r_c} (\gamma_f - \gamma_i) \right]^{1/2}. \quad (12)$$

For typical parameters,  $E_z/E_w \sim a^2 \lesssim 1$ ,  $k_w \sigma_x \sim 1$ ,  $k_w r_c \sim 1$ ,  $Z_w \sim 1$ , and  $\beta_{\text{th}} \sim 10^{-3}$ . Hence, for  $\epsilon_{nf} \gg \epsilon_{ni}$  and  $\gamma_f \gg \gamma_i$ ,  $\epsilon_{nf} \sim (\gamma_f r_e \beta_{\text{th}}/k_w)^{1/2}$ . For high-energy physics applications  $\gamma_f = 10^6$ , and if the laser-plasma accelerator is operating at a density of  $n_w = 10^{17} \text{ cm}^{-3}$ , then  $\epsilon_{nf} \sim 10^{-8} \text{ m}$ . This emittance growth is acceptable for high-energy physics applications. By contrast, Eq. (10) implies  $\epsilon_{nf} \sim 10^{-6} \text{ m}$  for similar parameters.

### D. Constant focusing force in near-hollow plasma channel

For a near-hollow plasma channel, the beam density need not be constant and a constant focusing force can be considered. In this case, the beam pinches as it accelerates, however, beam-induced plasma blow-out is not an issue since the beam is propagating in an ion channel and on-axis electrons are not required to provide focusing. Furthermore, the beam induced wake (beam loading) excited in the channel walls will be determined by the total beam charge, and not the peak beam density. The on-axis peak density can increase until limits imposed by ion motion.

For constant focusing, the rate of scattering is

$$\frac{d\epsilon_n}{d\gamma} = \frac{r_e Z_w \beta_{\text{th}}}{2(E_z/E_w) r_c \kappa \gamma^{1/2}}, \quad (13)$$

and the emittance growth is

$$\Delta\epsilon_n = \frac{r_e Z_w \beta_{\text{th}}}{(E_z/E_w) r_c \kappa} \left( \gamma_f^{1/2} - \gamma_i^{1/2} \right). \quad (14)$$

For typical laser-plasma parameters,  $E_z/E_w \sim a^2 \lesssim 1$ ,  $k_w r_c \sim 1$ ,  $Z_w \sim 1$ , and  $\beta_{\text{th}} \sim 10^{-3}$ . Hence, for  $\gamma_f \gg \gamma_i$ ,  $\Delta\epsilon_n \sim r_e \beta_{\text{th}} \gamma_f^{1/2} k_w / \kappa$ . For high-energy physics applications  $\gamma_f = 10^6$ , and if  $\kappa/k_w = 10^{-3}$  (to prevent ion motion), then  $\Delta\epsilon_n \sim 10^{-12}$  m, which is orders of magnitude smaller than that given by Eqs. (10) or (12).

#### IV. SUMMARY AND CONCLUSIONS

In this work, we have described a method to independently control the focusing and accelerating forces provided by a plasma accelerator by using a near-hollow ( $n_c/n_w \ll 1$ ) plasma channel. The accelerating wakefield is determined by the wall density  $n_w$ . In the limit  $n_c/n_w > a_0^2(r_c)/(8\gamma_p^2)$  the channel density  $n_c$  determines the focusing wakefield, and  $k_\beta = k_c/\sqrt{2\gamma}$ . This control of the focusing field allows matched propagation of a witness beam accelerated by the wakefield driven by an intense laser pulse or relativistic beam. In principle, this channel geometry can provide perfectly matched propagation of a high-energy beam, reducing emittance growth. Emittance growth via Coulomb scattering is mitigated using this transverse plasma profile, enabling high-energy physics applications. Use of a near-hollow plasma channel removes the need for low density bunches  $n_b < n_0$  for control of the focusing forces, thereby allowing strong focusing to be applied (in the quasi-linear regime of laser-plasma acceleration), further reducing the emittance growth.

The near-hollow plasma channel in the limit described above, in which the focusing is provided by the channel ions  $n_c$ , is applicable to accelerating and focusing electron beams. For positron beams, required for high-energy physics applications, one can consider operating in the limit of a hollow plasma channel and the focusing is provided externally (e.g., permanent magnets). In this regime, electrons and positrons can be accelerated in a nearly symmetrical fashion.

In this analysis we have neglected ion motion. Ion motion in the channel due to the presence of a beam can be estimated as  $\Delta r/r_0 \sim Z_c(m_e/M_i)(n_b/n_w)(k_w c \Delta t)^2$ , where  $r_0$  is the channel ion position,  $\Delta r$  is the displacement of the ion during interaction with the witness beam, and  $\Delta t$  is the interaction time between the witness beam and a background ion. For resonant beams  $k_w c \Delta t \lesssim 1$ . Sufficiently weak focusing should be provided such that the beam density satisfies  $Z_c(m_e/M_i)(n_b/n_w) \ll 1$ , and the motion of ions in the channel is negligible.

Other non-ideal effects may influence the wakefield structure. For example, finite wall thickness in the channel (i.e., a finite gradient in plasma density at the wall) will result in a slow decay of the wakefield, due to mode coupling at  $k_w(r) = k$ , with characteristic length scale  $L_d \sim k_w/(\partial_r k_w)$ ,<sup>24</sup> typically orders of magnitude larger than  $k_w^{-1}$ . This analysis assumed a linear plasma response, valid for  $|E_z| < E_w$ . For larger laser intensities at the channel wall such that  $|E_z| \gtrsim E_w$ , wall motion and other nonlinear effects will contribute to the wakefields. Driver mis-alignment will also result in excitation of additional (non-axis-symmetric) mode structure.<sup>14</sup> These modes could introduce transverse instabilities (hosing and beam break-up), however, since the drivers are short compared to the plasma wavelength, such instabilities will be suppressed.

Although the example presented in Fig. 1 considered a laser driver, a beam driver will produce a similar wakefield structure in a near-hollow plasma channel with the wakefield phase velocity approximately the drive beam velocity.<sup>14</sup> Use of relativistic beams would enable the focusing to be dominated by the channel ions for lower channel densities.

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