Control of shimmy vibration in aircraft landing gears based on tensor product model transformation and twisting sliding mode algorithm

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Abstract. Shimmy vibration is a common phenomenon in landing gear systems during either the take-off or landing of aircrafts. The shimmy vibration is undesirable since it can damage the landing gear and discomforts the pilots and passengers. In this work, tensor product model transformation (TPMT) and twisting sliding mode algorithm (TSMA) are utilized to design a robust controller for suppression of the shimmy vibration. The design has two steps. First, the TPMT is applied to determine the first part of the controller to suppress the vibration of the undisturbed system. After that, the TSMA is adopted to obtain another part of the controller to eliminate the remaining vibration caused by disturbances. By integrating these two parts, the proposed controller is obtained. Simulation studies are provided to demonstrate the effectiveness of the controller.

1 Introduction

Shimmy vibration is a common phenomenon in landing gear systems during either the take-off or landing of aircrafts. It is the state of self-excite oscillations in lateral and torsional directions caused by the interaction between the tires and the runway. The energy input is provided by the kinetic energy of the forward motion of the aircraft [1, 2].

The shimmy vibration is undesirable since it can damage the landing gear and discomforts the pilots and passengers. The damage of the landing gear can also lead to accidents. Thus, suppression of the shimmy vibration is needed. Passive dampers are normally used to suppress the shimmy vibration due to their simplicity [3]. However, damping requirements often conflict with good high-speed direction control [4]. Thus, active shimmy vibration control strategies have drawn attention recently [5-7]. In [5], an active control strategy based on model predictive control and tensor product model transformation is proposed. It was shown that the proposed active control method can effectively suppress shimmy vibration. In [6], robust and nonlinear optimal control of shimmy vibration is developed. The controller is designed by integrating sliding mode control together with State-Dependent Riccati Equation. In [7], an active shimmy vibration controller based on a filtered PID scheme is proposed. The parameters of the controller are tuned by means of a population decline particle swarm optimizer. Results under different scenarios were investigated and stochastic robustness verification was addressed to verify the controller effectiveness.

Tensor product model transformation (TPMT) is an effective numerical technique based on the recently developed high order singularity value decomposition (HOSVD) [8–10]. It transforms a linear parameter varying (LPV) system into a tensor product (TP) model form, which is described by a convex combination of linear time invariant (LTI) systems. Various types of convex hulls also can be derived. The TPMT has been successfully applied in many challenging problems [5, 11–14]. In [11], the TPMT was adopted for designing a controller to stabilize shared impedance/admittance-based bilateral telemanipulation under varying time delay. The controller fulfilled the stability requirement within the time delay domain considered in the design. A TPMT-based control and synchronization scheme for fractional-order chaotic systems was proposed in [12]. Numerical results of the fractional-order Lorenz and Liu chaotic systems illustrated the effectiveness of the scheme. In [13], a TPMT-based modelling and control design approach for morphing aircraft in transition process was investigated. Its effectiveness was shown by simulations of a variable-sweep morphing aircraft. In [14], a TPMT approach for level control of a three tanks system was presented. Simulation results were obtained to validate the controller design. A generalization of the TPMT for control design can be found in [15].

High order sliding mode (HOSM) control was first introduced by Levant [16, 17]. Twisting sliding model algorithm (TSMA) is one of the best choices among other HOSMs for the stabilization of nonlinear systems under external disturbances or uncertainties. Examples demonstrating applications of the TSMA can be found in [18–22].

This paper presents a method based the TPMT and the TSMA to design a robust controller to supress shimmy vibration in aircraft landing gears. The rest of the paper is

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organized as follows. In the next section, some preliminaries are provided, including the mathematical model of the shimmy vibration, the TPMT and the TSMA. The proposed robust control design is presented in Section 3, followed by simulation results in Section 4. The conclusion is drawn in Section 5.

2 Preliminaries

2.1 Mathematical model

Consider an aircraft landing gear system as shown in Figure 1. The mathematical model describing shimmy vibration in the landing gear can be written as [5]:

$$\dot{\mathbf{x}} = \mathbf{A}(V, e)\mathbf{x} + \mathbf{B}u + \mathbf{D}, \qquad (1)$$

where $\mathbf{x} = \begin{bmatrix} \psi \\ \dot{\psi} \\ y \end{bmatrix}$ is the system state,

ψ is the yaw angle (rad),
y is the lateral deflection (m),
u is the control command (volt),

$$\mathbf{A}(V, e) = \begin{bmatrix} 0 & 1 & 0 \\ -\dfrac{K}{I_z} & -\dfrac{c}{I_z} + \dfrac{\kappa}{VI_z} & \dfrac{F_z}{I_z\sigma}(C_{M\alpha} - eC_{F\alpha}) \\ V & e-a & -\dfrac{V}{\sigma} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ k_e \\ 0 \end{bmatrix},$$

V is the taxiing velocity (m/s),
e is the wheel caster length (m),
$T = k_e u$ is the input torque (N·m) and
$\mathbf{D} = [0, d, 0]^T$ is the bounded disturbance torque (N·m).

Here, V and e are considered as variable parameters. Thus, the system (1) is a linear parameter varying (LPV) system in which V and e are the time-varying parameters.

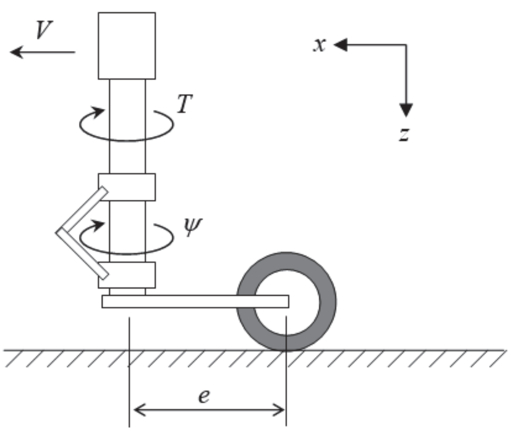

Fig. 1. Aircraft landing gear system.

The values of the system parameters used in this paper are summarized in Table 1. Examples of uncontrolled system (i.e., u = 0) are shown in Figures 2 and 3. Note that the system is stable in velocity 25 m/sec while it is unstable in velocity 75 m/sec.

Table 1. System parameters [5].

Symbol	Parameter	Value
V	Taxiing velocity	20–80 m/s
e	Wheel caster length	0.1–0.5 m
a	Haft contact length	0.1 m
I_z	Moment of inertia	1 kg·m^2
F_z	Vertical force	9 kN
K	Torsional spring constant	20 N·m/(rad·s)
c	Torsional damping constant	20 N·m/(rad·s)
$C_{F\alpha}$	Side force derivative	20 rad^{-1}
$C_{M\alpha}$	Moment derivative	-2 m/rad
κ	Tread width moment constant	-270 N·m^2/rad
σ	Relaxation length	$3a$ = 0.3 m
k_e	Input torque constant	10 kN·m/volt

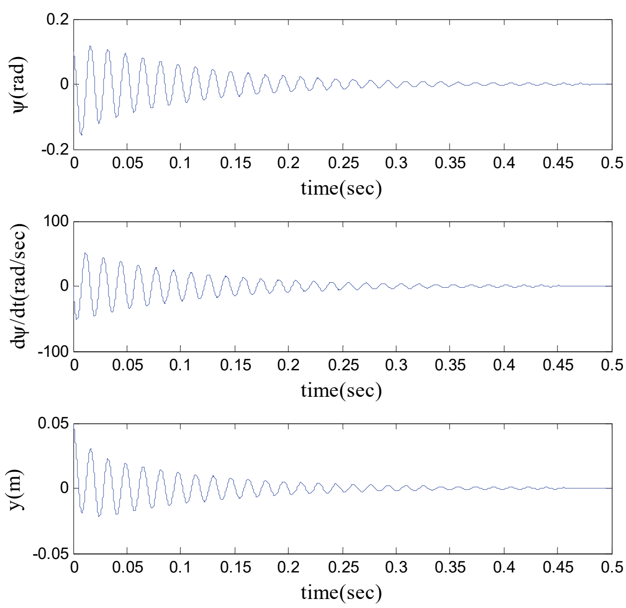

Fig. 2. Shimmy vibration at V = 25 m/sec and e = 0.3 m.

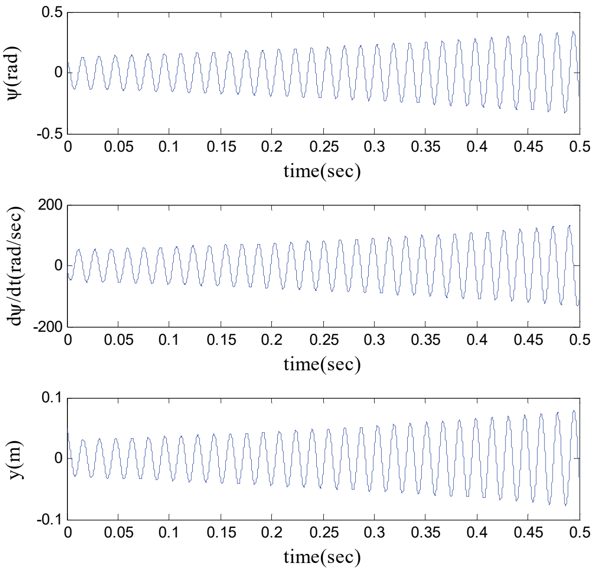

Fig. 3. Shimmy vibration at $V = 75$ m/sec and $e = 0.3$ m.

2.2 Tensor product model transformation

Tensor product model transformation (TPMT) is an effective numerical methodology to transform linear parameter varying (LPV) systems into convex tensor product (TP) model representations [8, 9]. In this paper, the LPV system is written as:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}(\mathbf{p}(t))\mathbf{x} + \mathbf{B}(\mathbf{p}(t))\mathbf{u} \\ &= \mathbf{S}(\mathbf{p}(t))\begin{bmatrix}\mathbf{x}\\ \mathbf{u}\end{bmatrix},\end{aligned} \tag{2}$$

where

$$\mathbf{S}(\mathbf{p}(t)) = [\mathbf{A}(\mathbf{p}(t)) \quad \mathbf{B}(\mathbf{p}(t))] \tag{3}$$

is the system matrix, $\mathbf{x}$ is the state vector, $\mathbf{u}$ is the input vector and $\mathbf{p}(t)$ is a time varying vector in a bounded space. The system is called quasi LPV (qLPV) if $\mathbf{p}$(t) includes some elements of $\mathbf{x}$.

The TPMT converts the system matrix (3) into a convex combination of R constant linear time invariant (LTI) system matrices

$$\mathbf{S}_r = [\mathbf{A}_r \quad \mathbf{B}_r], \quad r = 1, 2, ..., R \tag{4}$$

as

$$\mathbf{S}(\mathbf{p}(t)) = \sum_{r=1}^{R} \omega_r(\mathbf{p}(t))\mathbf{S}_r, \tag{5}$$

where $\omega_r(\mathbf{p}(t))$ is the weighting function. Thus, the system (2) can be written as

$$\dot{\mathbf{x}} = \sum_{r=1}^{R} \omega_r(\mathbf{p}(t))(\mathbf{A}_r\mathbf{x} + \mathbf{B}_r\mathbf{u}). \tag{6}$$

Based on a parallel distributed compensation (PDC) controller design framework, the control law can be expressed as

$$\mathbf{u} = -\left(\sum_{r=1}^{R} \omega_r(\mathbf{p}(t))\mathbf{K}_r\right)\mathbf{x}, \tag{7}$$

where the matrix gains $\mathbf{K}_r$ can be obtained by solving the following LMIs [8, 9]

$$-\mathbf{X}\mathbf{A}_r^T - \mathbf{A}_r\mathbf{X} + \mathbf{M}_r^T\mathbf{B}_r^T + \mathbf{B}_r\mathbf{M}_r > 0 \tag{8}$$

for all r and

$$\begin{aligned}&-\mathbf{X}\mathbf{A}_r^T - \mathbf{A}_r\mathbf{X} - \mathbf{X}\mathbf{A}_s^T - \mathbf{A}_s\mathbf{X} \\ &\quad + \mathbf{M}_s^T\mathbf{B}_r^T + \mathbf{B}_r\mathbf{M}_s + \mathbf{M}_r^T\mathbf{B}_s^T + \mathbf{B}_s\mathbf{M}_r \geq 0\end{aligned} \tag{9}$$

for all $r < s \leq R$, where $\mathbf{K}_r = \mathbf{M}_r\mathbf{X}^{-1}$. The control law (7) renders the closed loop system of the system (2) asymptotically stable. The reader refers to [8, 9] for more details. Note that the TPMT and the control law (7) can be determined by using the TP tool [10].

2.3 Twisting sliding model algorithm

Twisting sliding model algorithm (TSMA) is one of the popular high order sliding mode (HOSM) control. The TSMA was first introduced in [16, 17].

Consider the second order system described by

$$\begin{aligned}&\dot{x} = y, \quad \dot{y} = u + \delta(t, x, y) \\ &|\delta(t, x, y)| \leq D, \quad D > 0,\end{aligned} \tag{10}$$

where x, y are the state variables, u is the control input and $\delta(t,x,y)$ is the bounded disturbance. The TSMA control law can be written as [16]

$$\begin{aligned}u &= -M_1\mathrm{sign}(x) - M_2\mathrm{sign}(y) \\ &= -M(\mathrm{sign}(x) + 0.5\mathrm{sign}(y)),\end{aligned} \tag{11}$$

where $M_1 = M$ and $M_2 = 0.5M$. The control law renders the closed loop system a finite time stable equilibrium, provided that $M > 2D$.

Note that if the bound D is unknown, the fixed gain M can be replaced by an adaptive gain M(t) [23].

3 Control design

The control command u is expressed as

$$u = u_{TPMT} + u_{TSMA}. \tag{12}$$

The control design has two steps. In the first step, u_{TPMT} is determined by assuming that the system (1) is undisturbed (i.e., $d = 0$). Thus, the system (1) can be expressed as

$$\dot{\mathbf{x}} = \mathbf{A}(V, e)\mathbf{x} + \mathbf{B}u_{TPMT}, \tag{13}$$

which is the same form as the system (2). Then, by applying the TPMT and solving the LMIs (8) and (9), it yields

$$u_{TPMT} = -\left(\sum_{r=1}^{R} \omega_r(\mathbf{p}(t))\mathbf{K}_r\right)\mathbf{x}. \tag{14}$$

Thus, the closed loop system containing the system (13) and the control law (14) is asymptotically stable. By substituting (12) into (1), it results in

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}(V,e)\mathbf{x} + \mathbf{B}(u_{TPMT} + u_{TSMA}) + \mathbf{D} \\ &= \mathbf{A}(V,e)\mathbf{x} + \mathbf{B}u_{TPMT} + \mathbf{B}u_{TSMA} + \mathbf{D}.\end{aligned} \tag{15}$$

In the second step, since $\dot{\mathbf{x}} = \mathbf{A}(V,e)\mathbf{x} + \mathbf{B}u_{TPMT}$ is asymptotically stable, x_1, x_2 and x_3 are bounded since d is bounded. Then, we simplify the system (15) as

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= k_e u_{TSMA} + \delta(t),\end{aligned} \tag{16}$$

where $\delta(t)$ includes d and any unmodelled parts of the system. Note that $\delta(t)$ is bounded since x_1, x_2, x_3 and d are bounded. From (11), we obtain

$$u_{TSMA} = -M\,(\mathrm{sign}\,(x_1) + 0.5\mathrm{sign}\,(x_2))\,. \tag{17}$$

Therefore, the control command (12) can be written as

$$u = -\left(\sum_{r=1}^{R} \omega_r(\mathbf{p}(t))\mathbf{K}_r\right)\mathbf{x} - M\,(\mathrm{sign}\,(x_1) + 0.5\mathrm{sign}\,(x_2)). \tag{18}$$

4 Simulation results

The Runge–Kutta method with the time step of 0.0001 sec was used in all simulations. The initial condition was set as $\psi(0) = 0.1$ rad, $\mathrm{d}\psi(0)/\mathrm{d}t = 0$ rad/sec and $y(0) = 0.05$ m.

The time varying vector $\mathbf{p}(t)$ contains V and e. From Table 1, the space of $\mathbf{p}(t)$ is selected as [20, 80] × [0.1, 0.5]. By executing the TPMT of the system (13), using the TP Tool [10] with 100 × 100 sampling grid points, the ranks of the sampled tensors were found to be 3 and 2 on V and e, respectively. Thus, six vertex systems can exactly represent the system. The weighting functions are shown in Figure 4.

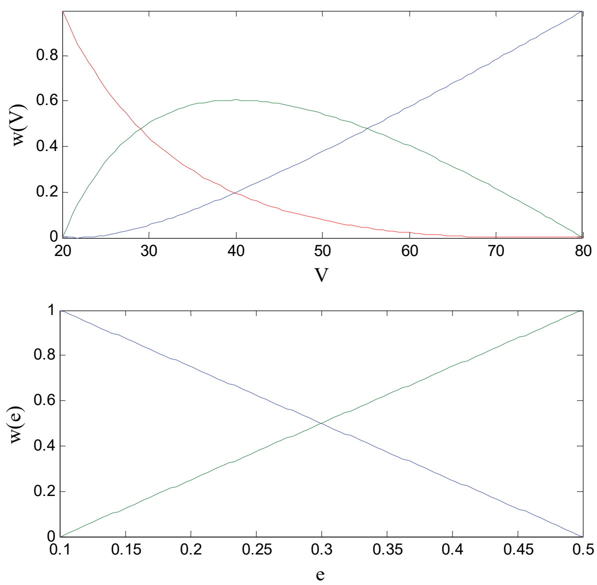

Fig. 4. Weighting functions on V and e.

By solving the LMIs (8) and (9), the following six linear feedback gains were obtained:

$$\begin{aligned}\mathbf{K}_1 &= [6.5065 \quad 0.0222 \quad -15.7799], \\ \mathbf{K}_2 &= [6.7727 \quad 0.0223 \quad -11.2479], \\ \mathbf{K}_3 &= [7.4875 \quad 0.0226 \quad -10.5572], \\ \mathbf{K}_4 &= [8.1582 \quad 0.0239 \quad -13.6185], \\ \mathbf{K}_5 &= \mathbf{K}_6 = [0.0 \quad 0.0 \quad 0.0].\end{aligned} \tag{19}$$

The closed loop control result of the system (1) using only u_{TPMT} for $V = 75$ m/sec, $e = 0.3$ m and $d = 0$ kN·m is shown in Figure 5. It shows that the shimmy vibration was suppressed within 0.07 sec.

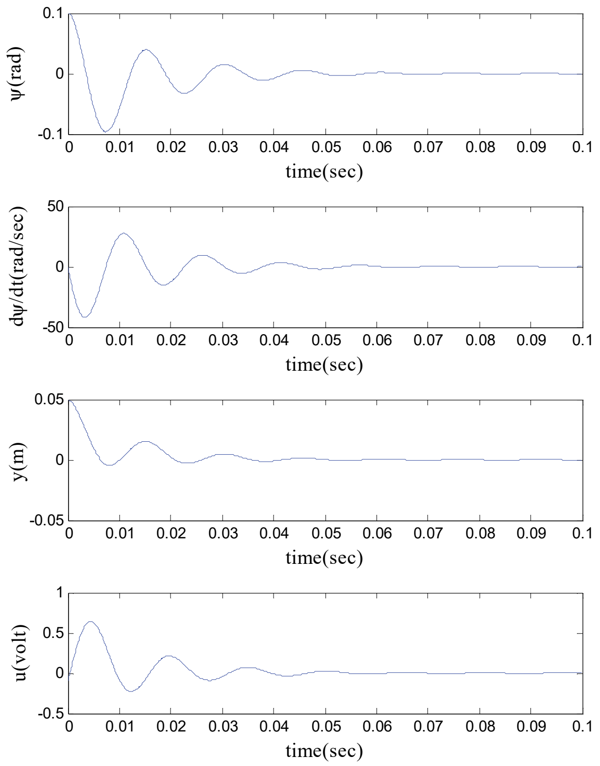

Fig. 5. Control responses of the undisturbed system using only the TPMT controller.

Next, the following disturbance torque was imposed into the system:

$$d(\mathrm{t}) = 5\sin(1000\mathrm{t})\ \mathrm{kN{\cdot}m}. \tag{19}$$

The result is shown in Figure 6. It is observable that additional vibration due to the disturbance was occurred although the controller had continuously responded to suppress the vibration.

To eliminate the effect of the disturbance, u_{TSMA} was integrated into the control law. Here, $M = 1$ was selected. In summary, the control law becomes:

$$u = -\left(\sum_{r=1}^{R} \omega_r(\mathbf{p}(t))\mathbf{K}_r\right)\mathbf{x} - (\mathrm{sign}\,(x_1) + 0.5\mathrm{sign}\,(x_2)). \tag{20}$$

where the weighting functions are given in Figure 4 and the linear feedback gains in Eq.(19).

The closed loop control result is shown in Figure 7. The controller was able to suppress the vibration as desired. However, due to the discontinuity nature of the TSMA, chattering was observed.

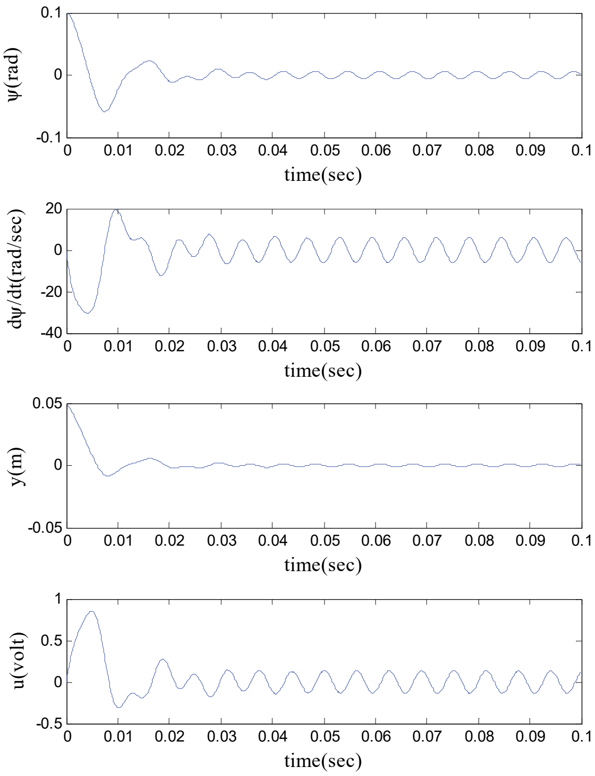

Fig. 6. Control responses of the disturbed system using only the TPMT controller.

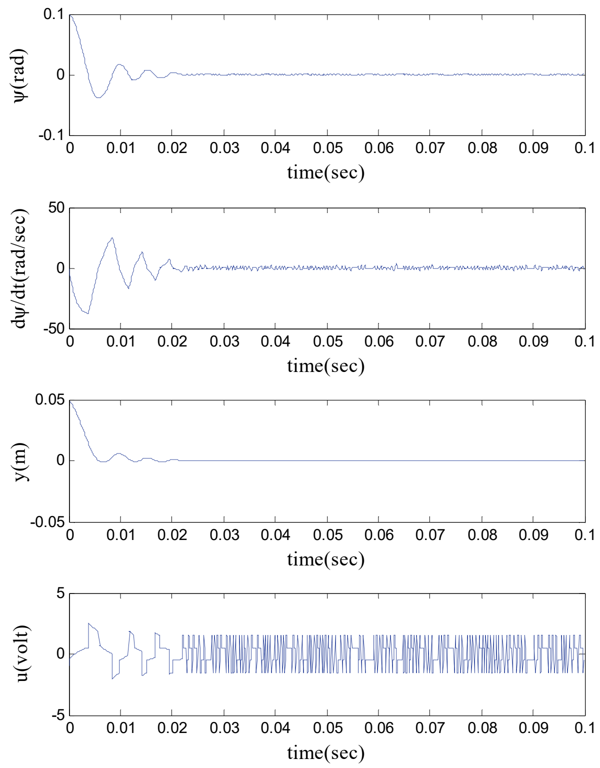

Fig. 7. Control responses of the disturbed system using both TPMT and TSMA controllers.

Finally, by replacing the discontinuous function sign(x) with the function tanh(5x) to solve the chattering problem, smoother control responses were achieved, as shown in Figure 8.

Similar results were also obtained for different values of V and e.

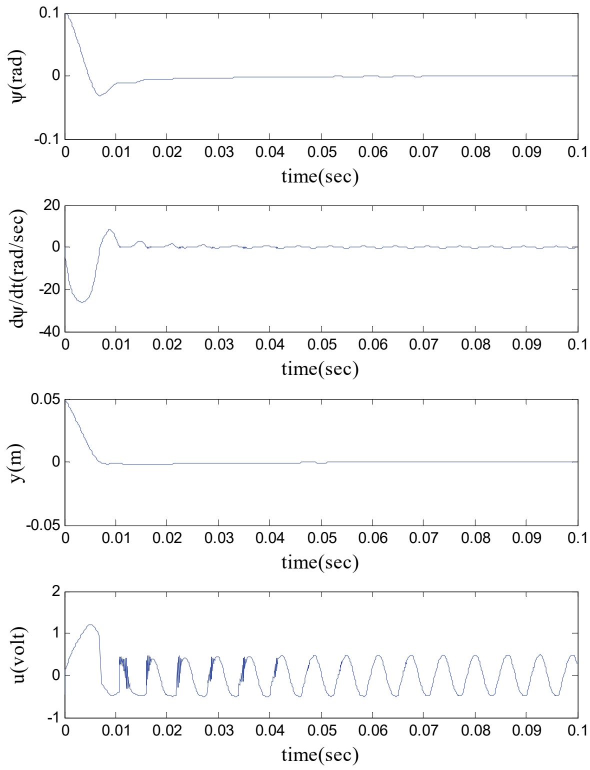

Fig. 8. Control responses of the disturbed system using both TPMT and TSMA controllers (replacing sign(x) with tanh(5x)).

5 Conclusions

The tensor product model transformation (TPMT) and twisting sliding mode algorithm (TSMA) are utilized to design a robust controller for suppression of the shimmy vibration in aircraft landing gears. The shimmy vibration is undesirable and it can lead to accidents. Simulation results illustrated that the proposed controller effectively suppressed the vibration even though the system was subjected to an external disturbance torque.

References

1. G. Somieski, Aerospace Science and Technology, **8**, 545–555 (1997)
2. E. Esmailzadeh, K.A. Farzaneh, Journal of Vibration and Control, **5**, 45–56 (1999)
3. Y. Li, C. Howcroft, S.A. Neild, J.Z. Jiang, Journal of Vibration and Control, **408**, 234–251 (2017)
4. J. Pritchard, Journal of Aircraft, **38**, 130–137 (2001)
5. A. Hajiloo, W.F. Xie, Asian Journal of Control, **17**, 476–485 (2015)
6. H. Tourajizadeh, S. Zare, Aerospace Science and Technology, **50**, 1–14 (2016)

7. C. Orlando, A. Alaimo, Mechanical Systems and Signal Processing, **84**, 551–569 (2017)
8. P. Baranyi, IEEE Transactions on Industrial Electronics, **51**, 387–400 (2004)
9. P. Baranyi, Y. Yam, P. Várlaki, *Tensor Product Model Transformation in Polytopic Model-Based Control*, (Florida: CRC Press, 2013)
10. S. Nagy, Z. Petres, P. Baranyi, 8th International Symposium of Hungarian Researchers on Computational Intelligence and Informatics, 483–495 (2007)
11. P. Galambos, P. Baranyi, G. Arz, Proc. IMechE Part C: J Mechanical Engineering Science, **228**, 765–777 (2014)
12. S. Kuntanapreeda, Asian Journal of Control, **17**, 371–380 (2015)
13. Z. He, M. Yin, Y. Lu, Proc. IMechE Part G: J Aerospace Engineering, **230**, 378–391 (2016)
14. E.-L. Hedrea, C.-A. Bojan-Dragos, R.-E. Precup, T.-A. Teban, 21st International Conference on Intelligent Engineering Systems, 113–118 (2017)
15. J. Kuti, P. Galambos, P. Baranyi, IFAC PaperOnline, **50**, 5604–5609 (2017)
16. A. Levant, International Journal of Control, **58**, 1247–1263 (1993)
17. A. Levant, Automatica, **43**, 576–586 (2007)
18. W. Xu, Y. Xu, X. Zheng, 5th Annual IEEE International Conference on Cyber Technology in Automation, Control and Intelligent Systems, 1379–1384 (2015)
19. H. Chen, Z. Sun, W. Sun, T. W. Yeow, 34th Chinese Control Conference, 8331–8336 (2015)
20. T. Han, Q. Hu, Aerospace Science and Technology, **70**, 428–434 (2017)
21. P. Khamsuwan, S. Kuntanapreeda, T. Sangpet, 12th IEEE Conference on Industrial Electronics and Applications, 798–803 (2017)
22. R. Luo, H. Su, Chinese Journal of Physics, **55**, 2199–2207 (2017)
23. Y.B. Shtessel, J.A. Moreno, L.M. Fridman, Automatica, **75**, 229–235 (2017)