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Research Article

H_∞ Control Theory Using in the Air Pollution Control System

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In recent years, air pollution control has caused great concern. This paper focuses on the primary pollutant SO_2 in the atmosphere for analysis and control. Two indicators are introduced, which are the concentration of SO_2 in the emissions (PSO_2) and the concentration of SO_2 in the atmosphere (ASO_2). If the ASO_2 is higher than the certain threshold, then this shows that the air is polluted. According to the uncertainty of the air pollution control systems model, H_∞ control theory for the air pollution control systems is used in this paper, which can change the PSO_2 with the method of improving the level of pollution processing or decreasing the emissions, so that air pollution system can maintain robust stability and the indicators ASO_2 are always operated within the desired target.

1. Introduction

The main feature of the H_∞ control theory is based on the frequency design method of using the state-space model, and this theory presents an effective method to solve the uncertainty problem of external disturbance to the system. In order to overcome the drawbacks of the classical control theory and the modern control theory, H_∞ control theory established technology and method of the loop shaping in the frequency domain, which combines the classic frequency-domain and the modern state-space method. The design problem of the control system is converted to the H_∞ control problem, which made the system closer to the actual situation and meet the actual needs. So it gives the robust control system design method, which obtains H_∞ controller by solving two Riccati equations. This method fully considered the impact of system uncertainty, which not only can ensure the robust stability of the control system, but also can optimize some performance indices. It is the optimal control theory in frequency domain, and the parameters design of H_∞ controller is more effective than optimal regulator [1].

So the H_∞ control theory for the air pollution control systems can solve the uncertainty of the air pollution control systems model, which can get the control strategy to change the PSO_2, so that air pollution system can maintain robust stability and the indicators ASO_2 are always operated within the desired target.

2. Standard H_∞ Control Problem

2.1. Problem Description. The standard H_∞ control problem is shown in Figure 1, which consists of G and K. G is a generalized control target which is a given part of the system. K is H_∞ controller, and it needs to be designed.

It is supposed that G and K are described as the transfer function matrix of the linear time invariant system [2]. Then, $G(s)$ and $K(s)$ are proper rational matrices, Decomposing $G(s)$ as

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}. \tag{1}$$

Its state-space realization is

$$\begin{aligned} \dot{x} &= Ax + B_1\omega + B_2u, \\ z &= C_1x + D_{11}\omega + D_{12}u, \\ y &= C_2x + D_{21}\omega + D_{22}u. \end{aligned} \tag{2}$$

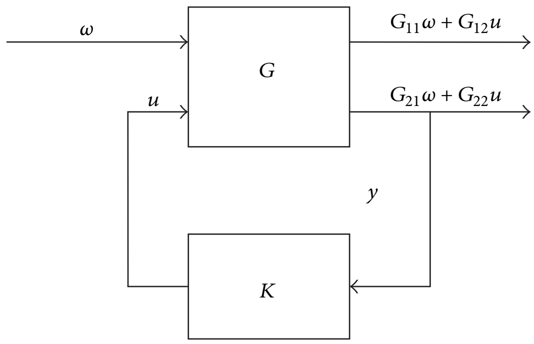


FIGURE 1: The diagram of the standard H_∞ control problem.

It is denoted as:

$$G = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}, \quad (3)$$

where $x \in R^n$ is the state vector, ω is the external input, u is the control input, z is the controlled output, and y is the measured output. They are all vector signals. To compare (1) and (3), we can obtain

$$G_{ij} = C_i(sI - A)^{-1}B_j + D_{ij}, \quad i, j = 1, 2. \quad (4)$$

Obviously, the closed-loop transfer function matrix from ω to z can be expressed as

$$T_{z\omega}(s) = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21} = F_l(G, K). \quad (5)$$

The problem of H_∞ optimal control theory is to find a proper rational controller K for closed-loop control system, and make the closed-loop control system internally stabile, and then minimize the H_∞ norm of closed-loop transfer function matrix $T_{z\omega}(s)$ [3]:

$$\min_{K \text{ stability } G} \left\|F_l(G, K)\right\|_\infty < 1. \quad (6)$$

2.2. State-Space H_∞ Output-Feedback Control. We assume that the state-space representation of the generalized controlled object G state-space representation is (2), where $x \in R^n, \omega \in R^{m_1}, u \in R^{m_2}, z \in R^{p_1}, y \in R^{P_2}, A \in R^{n\times n}, B_1 \in R^{n\times m_1}, B_2 \in R^{n\times m_2}, C_1 \in R^{p_1\times n}, C_2 \in R^{p_2\times n}$, D_{21} and D_{22} are the corresponding dimension of the real matrix, and controller K is dynamic output feedback compensator [4].

Consider that G have a special form:

$$G = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ \begin{bmatrix} I \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ I \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{bmatrix}, \quad (7)$$

which satisfies the following conditions:

(1) (A, B_1) is stabilizable, and (C_1, A) is detectable;

(2) (A, B_2) is stabilizable;

(3) $D_{12}^T\left[C_1 \;\; D_{12}\right] = \left[0 \;\; I\right]$.

Obviously, $y = \left[\begin{smallmatrix} x \\ \omega \end{smallmatrix}\right]$; that is, the state x of generalized control object and the external input signal ω could be measured; then, it can be directly used to constitute control law.

As the result, we can obtain Theorem 1.

Theorem 1. *If and only if $H_\infty \in \text{dom(Ric)}$, $X_\infty = \text{Ric}(H_\infty) \geq 0$, there exists H_∞ controller*

$$u = -B_2^T X_\infty x + Q(s)\left(\omega - \gamma^{-2}B_1^T X_\infty x\right). \quad (8)$$

That is,

$$K(s) = \left[-B_2^T X_\infty - Q(s)\gamma^{-2}B_1^T X_\infty Q(s)\right], \quad (9)$$

where $Q(s) \in RH_\infty$, $\|Q(s)\|_\infty < \gamma$, which is made to stabilize the closed-loop control system, and the closed-loop transfer function matrix $T_{z\omega}(s)$ from ω to z is satisfied as $\|T_{z\omega}(s)\|_\infty < \gamma$.

Proof. If Hamilton matrix H_∞ was considered, there exist $X_\infty = \text{Ric}(H_\infty)$, and then the derivative of $x^T(t)X_\infty x(t)$ can be achieved, such that

$$\frac{d}{dt}x^T X_\infty x = \dot{x}^T X_\infty x + x^T X_\infty \dot{x}. \quad (10)$$

Substituting (2) into (10) and considering Riccati equation (11) and orthogonality condition

$$\begin{aligned} &A^T X_\infty + X_\infty A + \gamma^{-2}X_\infty B_1 B_1^T X_\infty \\ &\quad - X_\infty B_2 B_2^T X_\infty + C_1^T C_1 = 0. \end{aligned} \quad (11)$$

So the following formula can be achieved:

$$\begin{aligned} \frac{d}{dt}x^T X_\infty x &= x^T\left(A^T X_\infty + X_\infty A\right)x \\ &\quad + \omega^T\left(B_1^T X_\infty x\right) + \left(B_1^T X_\infty x\right)^T \omega \\ &\quad + u^T\left(B_2^T X_\infty x\right) + \left(B_2^T X_\infty x\right)^T u \\ &= -\left\|C_1 x\right\|^2 - \gamma^{-2}\left\|B_1^T X_\infty x\right\|^2 \\ &\quad + \left\|B_2^T X_\infty x\right\|^2 + \omega^T\left(B_1^T X_\infty x\right) + \left(B_1^T X_\infty x\right)^T \omega \\ &\quad + u^T\left(B_2^T X_\infty x\right) + \left(B_2^T X_\infty x\right)^T u \\ &= -\|z\|^2 + \gamma^2\|\omega\|^2 - \gamma^2\left\|\omega - \gamma^{-2}B_1^T X_\infty x\right\|^2 \\ &\quad + \left\|u + B_2^T X_\infty x\right\|^2. \end{aligned} \quad (12)$$

If $x(0) = x(\infty) = 0$, $\omega \in L_2[0, +\infty)$, to integral above equation from $t = 0$ to $t = \infty$, then

$$\begin{aligned} &\|z\|_2^2 - \gamma^2\|\omega\|_2^2 \\ &\quad = \left\|u + B_2^T X_\infty x\right\|_2^2 - \gamma^2\left\|\omega - \gamma^{-2}B_1^T X_\infty x\right\|_2^2. \end{aligned} \quad (13)$$

According to the equivalence relation,

$$\|T_{z\omega}(s)\|_\infty < \gamma \Longleftrightarrow \|z\|_2^2 < \gamma^2 \|\omega\|_2^2. \tag{14}$$

Form (12) and (13), we can get that

$$\begin{aligned}\|T_{z\omega}(s)\|_\infty < \gamma &\Longleftrightarrow \left\|u + B_2^T X_\infty x\right\|_2^2 \\ &< \gamma^2 \left\|\omega - \gamma^{-2} B_1^T X_\infty x\right\|_2^2 \\ &\Longleftrightarrow u + B_2^T X_\infty x \\ &= Q(s)\left(\omega - \gamma^{-2} B_1^T X_\infty x\right),\end{aligned} \tag{15}$$

where ‖ • ‖ represent Euclidean norm.

Hence, $u = -B_2^T X_\infty x + Q(s)(\omega - \gamma^{-2} B_1^T X_\infty x)$; the proof is completed. □

u is the input to ensure $\|T_{z\omega}(s)\|_\infty < \gamma$, where $Q(s)(\omega - \gamma^{-2} B_1^T X_\infty x)$ is the output of the transfer system $Q(s)$ by the input $(\omega - \gamma^{-2} B_1^T X_\infty x)$. From (12), the decay of $x^T(t) X_\infty x(t) \ge 0$ is the slowest when $\omega = \gamma^{-2} B_1^T X_\infty x$. Then for this kind of disturbance, we can stabilize (12) by using control strategy u.

3. Analysis and Synthesis of Air Pollution Control System

Atmospheric quality management is essentially the process of the analysis and synthesis to the air pollution control system. So-called atmospheric system analysis is qualitative and quantitative research for an atmospheric area system or facilities system, which include that to evaluate the current situation, to find out the main environment problems, to put a series of optional target projects, and to establish the quantitative relation between emission and the quality of the permissive atmosphere [5]. The systems synthesis means the plan and design of an air pollution control system on the basis of system analysis to determine the target and to determine a management method of a system, in other words, in order to achieve certain environmental goal to select the optimal planning scheme, to optimal design, to find the optimal management method, and so forth. So the process of the synthesis should include three main steps: determine the target, form better feasibility scheme of system, and optimization decision [6].

Usually, the atmosphere has certain self-purification ability; namely, the atmospheric environment has a certain capacity. It refers to the permissible pollutant emissions within the natural purification capacity, which reach the limiting quantity in order not to destruct the nature material circulation [7]. We can make the quantity of pollutant discharged that meets a certain environmental goal to be permissible total emission. Only when the pollutant emissions beyond atmospheric self-purification ability, namely, exceeds the environmental capacity, there may be air pollution [8].

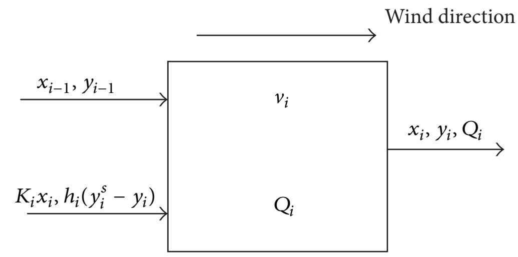


Figure 2: The stirred reactor model of an ideal atmosphere section.

In this paper, the objective of applying the H_∞ control theory in air pollution control system is to find out regularity, to make full use of atmospheric environmental self-purification ability; we cannot only develop production but also protect the environment.

4. H_∞ Control of Air Pollution

In recent years, air pollution control has caused great concern. This paper focuses on primary pollutant SO_2 in the atmosphere for analysis and control, which is mainly pollutant of the PM2.5. We introduced two indicators, which are the concentration of SO_2 in the emissions (PSO_2) and the concentration of SO_2 in the atmosphere (ASO_2). Meanwhile, we can change the content of SO_2 in the emissions (with the method of improving the methods of processing), with the aim of returning atmospheric quality back to the desired value.

4.1. The Mathematical Model of Air Pollution. A certain volume of air mainly accepted the controlled pollutants which discharged from certain purification equipments. In addition, considering that the atmospheric self-purification ability is mainly affected by wind and other meteorological factors, a certain volume of air can be defined as atmospheric section. Thus, we can put forward a second order state-space equation; it describes the relationship between PSO_2 and ASO_2 on an average point of the atmospheric section [9]. The basic idea of modeling is to consider each section as ideal stirred reactor, as shown in Figure 2. So, the parameters and variables of the whole section are consistent, and the output concentration of PSO_2 and ASO_2 is equal to the counterpart concentration in this section. Hence, from the point of view of the mass balance, we can get the following equation.

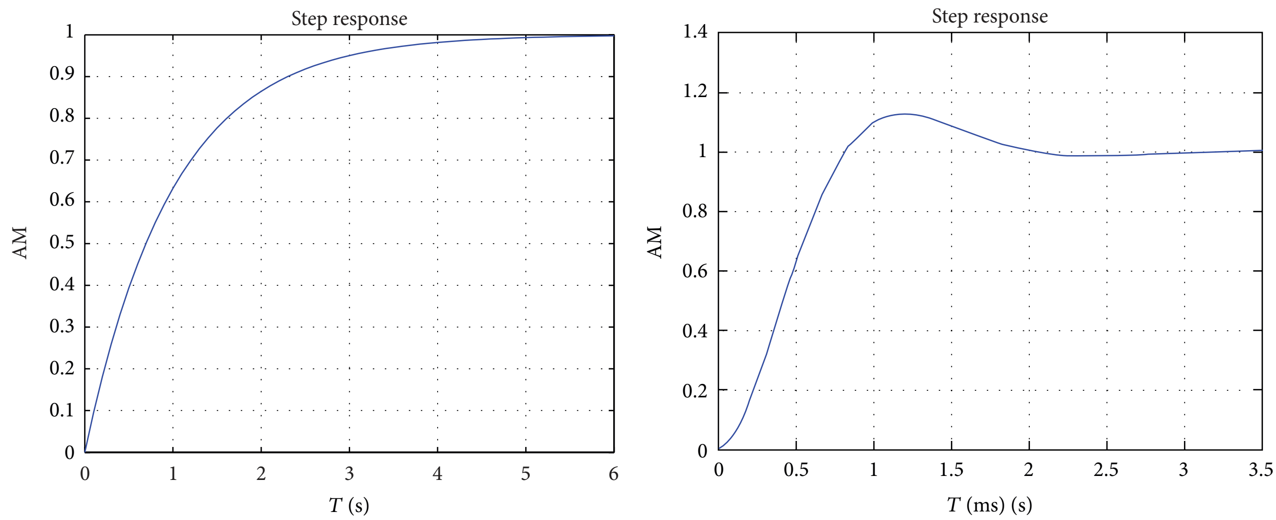
PSO_2 balance equation:

$$\dot{x}_i = -k_i x_i + \frac{Q_{i-1}}{v_i} x_{i-1} - \frac{Q_i + Q_E}{v_i} x_i. \tag{16}$$

ASO_2 balance equation:

$$\dot{y}_i = h_i\left(y_i^s - y_i\right) + \frac{Q_{i-1}}{v_i} y_{i-1} - \frac{Q_i + Q_E}{v_i} y_i - k_i x_i, \tag{17}$$

where x_i, x_{i-1} are the PSO_2 of Section i and Section $i - 1$ (mg/m^3), v_i is the atmospheric capacity of Section i (m^3);



(a) The original closed-loop system (b) The closed-loop system with H_∞ control strategy

Figure 3: The system step response.

Q_E is PSO_2 gas flow rate of Section i (m^3/d), y_i, y_{i-1} are ASO_2 of Section i and Section $i-1$ (mg/m^3), k_i, h_i are daily decay rate of PSO_2 of Section i and supply rate of ASO_2 of Section i, Q_i, Q_{i-1} are atmosphere gas flow rates of Section i and Section $i-1$ (m^3/d), and y_i^s is saturating capacity of SO_2 of Section i (mg/m^3).

From [5], we can conclude that it is advisable to take the following values as the coefficients in above equations:

$$k_i = 0.32/\text{day}, \qquad h_i = 0.2/\text{day}, \qquad y_i^s = 0.36\,\text{mg/m}^3,$$
$$\frac{Q_E}{v_i} = 0.1, \qquad \frac{Q_i}{v_i}, \frac{Q_{i-1}}{v_i} = 0.9. \tag{18}$$

Thus, the mathematical model of Section i air pollution is

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = \begin{bmatrix} -1.32 & 0 \\ -0.32 & -1.2 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0.9x_{i-1} \\ 0.9y_{i-1} + 1.9 \end{bmatrix}, \tag{19}$$

where u is control strategy.

4.2. Simulation of Air Pollution H_∞ Control. H_∞ control problem of air pollution is basically how to control emissions of pollutants in the best way, in order to properly handle the cost of cleaning and the price we pay for too much atmospheric pollution. Using H_∞ control theory can better achieve this goal, which not only saves investment but also is easy to be realized [10].

In this paper, the simulation object is two sections of the H_∞ control of atmospheric pollution, the state-space equation is described as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} -1.32 & 0 & 0 & 0 \\ -0.32 & -1.2 & 0 & 0 \\ 0.9 & 0 & -1.32 & 0 \\ 0 & 0.9 & -0.32 & -1.2 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 5.35 \\ 1.09 \\ 4.19 \\ 1.9 \end{bmatrix}. \tag{20}$$

Using the MATLAB to simulate [11], we can conclude the step response curve of the air pollution control system. We have

$$u(s) = \frac{570613.9602\,(s+7.702)\,(s+1.32)^2(s+1.2)^2}{(s+4012)\,(s+1.77)\,(s^2+2.5s+1.563)\,(s^2+2.4s+1.44)}, \tag{21}$$

$$K(s) = \begin{bmatrix} -0.67 & -1.77 & -2.32 & -1.52 & -0.4 & 0.1 \\ 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 \\ 8.63 & 4.36 & 8.24 & 6.92 & 2.18 & 0.14 \end{bmatrix} Q(s), \tag{22}$$

which is the transfer function and the state-space expression of robust H_∞ control strategy.

It could be seen from Figure 3, that the response time of the original system is 3.6 s, while the response time of closed loop system is 0.026 s by using H_∞ control strategy. It means that H_∞ control strategy makes the step response of the atmosphere system with an improvement; the response time is greatly reduced. Therefore, H_∞ control strategy is a practical control strategy, which can ensure that air pollution control system is operating steadily within the desired target [12].

5. Conclusion

In this paper, the H_∞ control theory and methods have a great application value on air pollution control system. It can help the environmental protection departments at various levels to analyze the air pollution system, which can ensure the atmosphere quality steady work within the desired target value. Of course, the analysis and control of a large-scale air pollution system that introduced other influencing factors will be very complicated, which will be the focus of the study in this field.

Acknowledgments

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