

## CONTROLLABILITY IMPROVEMENT FOR LINEAR TIME-INVARIANT DYNAMICAL MULTI-AGENT SYSTEMS

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*ABSTRACT.* For two types of linear time-invariant dynamical multi-agent systems under leader-follower framework, the problem of graph topology adjustment is addressed to improve system controllability. As important concepts and theoretical foundations, the maximum controllability index of square matrices is defined and analyzed, and a generalized controllability canonical form is introduced for single-input systems. Based on these concepts, approaches for adjusting the leader-follower and follower-follower communication architectures are presented respectively.

**Keywords:** Controllability, Graph, Multi-agent systems

**1. Introduction.** The study of controllability of composite systems started long ago. Gilbert [1], Davison and Wang [2], and Porter [3] were some of the early researchers endeavoring to develop criteria for checking the controllability of composite systems. Later, based on graph theory, Davison [4] defined the concept “connectivity”, which is crucial for the controllability of composite systems. Also based on graph theory, Lin [5] introduced the concept “structural controllability”. Zazworsky and Knudsen [6] presented conditions for the controllability of compartmental models, focusing on the configuration of interconnections. Kobayashi et al. [7,8] discussed the criteria for controllability of decentralized configurations concerning fixing-mode and graph topology respectively.

During recent years, dynamical multi-agent systems have been extensively studied by scholars in the field of control theory. The consensus problem [9-11,25,26] attracts most attention, which is essentially a stability problem. Nonetheless, only a few researchers have started to notice the controllability problems. Mesbahi [12] proposed the concept “state-dependent graph” and defined “controllability” for state-dependent graphs. Tanner [13] postulated that more information exchange might be detrimental to controllability. Ji et al. [14] gave sufficient conditions for controllability, based on the algebraic characteristics of certain matrices about the graphs. Rahmani et al. [15] extended the work of Ji et al. in [14], concentrating on the relationship between graph symmetry and controllability. Cai et al. [16-19] proposed the concept “formation controllability” and studied the condition for controllability of high-order systems. Liu et al. [20] discussed the controllability of discrete time systems with switching graph topologies. Ji et al. [21] presented conditions for graph controllability, which are analogous to the results in [6]. Liu et al. [22] endeavored to