

CONTROLLED QUEUEING SYSTEMS¹

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The purpose of this book is to collect together the newest results on the theory of Markov decision processes needed for queueing models and to demonstrate their applications to main types of control in queueing systems. These types are:

- control of arrivals;
- control of service mechanisms;
- control of service discipline.

This book, to the best of our knowledge, is the first one which is completely devoted to the subject pointed out in its title. We realize that an attempt to be encyclopedic would defeat the purpose of the book because of the overwhelming amount of material. Therefore, within each of these classes we have chosen representatives where the general ideas were expressed in the most clear form.

The emphasis is placed on conditions providing "good" structural properties of optimal strategies such as monotonicity, threshold and hysteretic character, and priority. These properties allow us to restrict the range of search of the optimal strategy and sometimes to explicitly construct it.

We hope that this book will turn out to be useful to applied mathematicians interested in queueing systems, specialists in system analysis and performance evaluation, and for graduate and postgraduate students of corresponding orientations.

Each chapter is followed by exercises which form an essential logical part of the book. In most exercises, the reader is offered to complete technical fragments of proofs. Other exercises, grouped in series, are intended to lead the reader step by step to justification of some facts given in the text as auxiliary. This strategy allowed us to reduce proofs and emphasize guiding ideas.

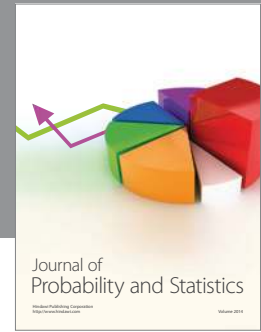
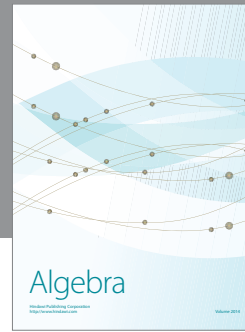
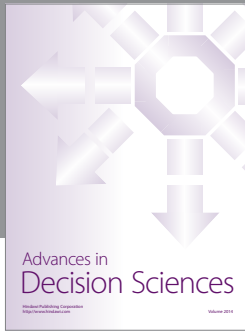
In bibliographical remarks as a rule, we only refer to those sources which were directly used in the text. It was not our intention to give an overall picture of the literature on one topic or another. The reader, interested in the history of the subject or in topics not treated in the book, may consult recent surveys which are cited in the book.

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