Controlling transient chaos in deterministic flows with applications to electrical power systems and ecology

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Transient chaos is a common phenomenon in nonlinear dynamics of many physical, biological, and engineering systems. In applications it is often desirable to maintain sustained chaos even in parameter regimes of transient chaos. We address how to sustain transient chaos in deterministic flows. We utilize a simple and practical method, based on extracting the fundamental dynamics from time series, to maintain chaos. The method can result in control of trajectories from almost all initial conditions in the original basin of the chaotic attractor from which transient chaos is created. We apply our method to three problems: (1) voltage collapse in electrical power systems, (2) species preservation in ecology, and (3) elimination of undesirable bursting behavior in a chemical reaction system. [S1063-651X(99)04902-8]

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I. INTRODUCTION

Transient chaos is a ubiquitous phenomenon in nonlinear dynamical systems. In such a case, a trajectory typically behaves chaotically for a finite amount of time before settling into a final (usually nonchaotic) state. The dynamical origin of transient chaos is known: it is due to nonattracting chaotic saddles in phase space [1–4]. A chaotic saddle is a bounded set, and it has fractal structures in both stable and unstable directions, in contrast to a chaotic attractor which exhibits a fractal structure only in the stable direction. Due to the fractal structure in the unstable direction, an infinite number of gaps of all sizes exists along the unstable manifold of the chaotic saddle. An initial condition is typically attracted toward the chaotic saddle along the stable direction, stays in its vicinity for a finite amount of time, and then leaves the chaotic saddle through one of the gaps in the unstable direction. It is known that chaotic saddles and transient chaos are responsible for important physical phenomena such as chaotic scattering [5] and particle transport in open hydrodynamical flows [6]. They are also speculated to be the culprit for catastrophic phenomena such as voltage collapse in electric power systems [7,8] and species extinction in ecology [9]. The aim of this paper is to address how to control transient chaos in flows, that is, systems described by autonomous ordinary differential equations. Our goal is to apply small and infrequent perturbations to the system so that transient chaos can be forced to become sustained chaos or periodic motion. Our motivation comes from the fact that transient chaos can be quite disastrous, as in situations of voltage collapse and species extinction. By converting transient chaos into sustained chaos using only small control, the natural dynamics of the system can be preserved, but with no catastrophes. This can be of significant interest in the areas of electrical engineering and environmental studies.

While there has been a tremendous amount of work in controlling chaos following the seminal work of Ott, Grebogi, and Yorke [10], there have been only a few papers on controlling transient chaos [11–14]. A problem in the existing methods is that only a fraction of initial conditions can be controlled. To address this problem, consider the typical route by which transient chaos is created: crisis [1]. At a crisis, a chaotic attractor collides with its own basin boundary and becomes a chaotic saddle. After the crisis, almost all trajectories in the original basin of the chaotic attractor exit the basin. With the existing control method, (e.g., Refs. [11] and [12]), trajectories from a fraction of these initial conditions can be controlled. There are trajectories which cannot be controlled.

In this paper, we propose a simple and practical method to control transient chaos in general deterministic flows. Following the general ideas of Schwartz and Triandaf [13] and Kapitaniak and Brindley [14], we identify small regions near a chaotic saddle through which trajectories escape and we search for a set of ''target'' points in these regions which yield trajectories that can stay near the chaotic saddle for relatively long time (long lifetime). By perturbing the trajectory, when it falls into one of the escaping regions, to one of the nearest target points, the trajectory can live near the chaotic saddle for an additional long period of time. Keeping doing this, a transient chaotic trajectory can be sustained. The difference between our method and previous ones [13,14] is that we obtain all the control information including the set of target points through the return map constructed from local maxima or minima of a measured time series. We find that we can control almost all initial conditions using small and very rarely applied perturbations. We apply our method to three practical problems: (1) the voltage-collapse problem [7,8], (2) the species extinction problem [9], and (3) elimination of undesirable bursts in a chemical reaction system [15,16]. Since all information required for the control can be obtained from time series, we expect our method to be accessible for experimental implementation.

The rest of the paper is organized as follows. In Sec. II, we describe our general method. In Sec. III, we apply the method to the voltage-collapse problem. In Sec. IV, the problem of controlling transient chaos to prevent species extinction is investigated. In Sec. V, we study a chemical reaction system to eliminate undesirable bursts in time series. In Sec. VI, we present a discussion.

II. METHOD OF CONTROL

We consider the following N-dimensional autonomous flow:

$$\frac{d\mathbf{x}}{dt}=\mathbf{F}(\mathbf{x},p), \tag{1}$$

where $x\in\mathcal{R}^N$ and p is a system parameter. Let p_c be the crisis value at which a chaotic attractor for $p<p_c$ is converted into a chaotic saddle for $p>p_c$. Let $\mathcal{B}$ denote, in the phase space, the basin of attraction for the chaotic attractor before the crisis ($p<p_c$). As p passes through p_c, we expect the volume of the original basin $\mathcal{B}$ to change smoothly. For simplicity we still use $\mathcal{B}$ to denote the phase-space region after the crisis that evolves from the basin of the original chaotic attractor before the crisis. For $p>p_c$, there is transient chaos. That is, a trajectory $\mathbf{x}(t)$ starting from a random initial condition $\mathbf{x}_0$ in $\mathcal{B}$ typically stays in $\mathcal{B}$ in a chaotic manner but only for a finite amount of time. The average time for a typical trajectory to behave chaotically is the average lifetime of the chaotic saddle, or the inverse of the escape rate of the chaotic saddle [4].

In an experimental situation involving transient chaos, usually one or several time series are measured. In contrast to situations of chaotic attractors, these time series consist of short segments of chaotic oscillations exhibiting a number of local maxima and minima. Let x_n ($n=1,...,L$) be the set of maxima (or minima) from one segment of measurement of one dynamical variable $x(t)$ which exhibits L maxima (or minima). A plot of x_{n+1} versus x_n thus yields only a few points. In order to detect the underlying dynamics, an ensemble of transient chaotic trajectories from a large number of random initial conditions are used, each yielding a number of points in the x_{n+1} versus x_n plot. As a result, we obtain a crude representation of the discrete map:

$$x_{n+1}=M(x_n). \tag{2}$$

If the underlying dynamics is approximately one dimensional, then the map $M(x)$ is roughly a one-dimensional smooth curve. For higher-dimensional dynamics, the plot $M(x)$ typically exhibits complicated structure such as fractal or even random patterns. In all cases, it is possible to identify regions of $\mathbf{x}_n=(x_n,x_{n+1})$ in which the chaotic saddle lies and regions where escaping from the chaotic saddle occurs. The key observation is that a typical trajectory must enter one of the escaping regions to escape from $\mathcal{B}$ and then, possibly, becomes nonchaotic. Thus, by applying small perturbations to an accessible set of dynamical variables at a time when the trajectory is in an escaping region to ''kick'' it back into $\mathcal{B}$, chaotic motion can be maintained for a finite period of time.

To compute the perturbations, we follow the ideas in Refs. [13] and [14] to preselect a set of target points in the vicinity of the escaping regions in $\mathcal{B}$. Due to the fractal structures in the stable and unstable foliations of the chaotic saddle, lifetimes of trajectories from points in $\mathcal{B}$ are highly nonuniform. It is thus possible to choose target points near the escaping regions such that trajectories starting from them can stay in $\mathcal{B}$ for relatively long times, comparing with the average lifetime of the chaotic saddle. The difference between our method and those in Refs. [13] and [14] is that, in our case, the escaping regions are identified only in the two-dimensional plane of $M(x)$, whereas in Refs. [13], and [14] these are identified in the full phase space. In this sense, information about target points in our method is not complete. Control may be lost if one simply perturbs the variable $x(t)$. The situation can be improved if more dynamical variables are experimentally accessible. In particular, let $\mathbf{x}_m$ be a subset of dynamical variables that can be observed in an experiment, where $x\in\mathbf{x}_m$. For each target point selected from the plot of $M(x)$ [which makes use of $x(t)$ only], we also store the corresponding values of all the remaining dynamical variables in the subset $\mathbf{x}_m$. These are the only information required for control, which are all experimentally accessible. To realize the control, one waits until the trajectory falls into an escaping region, at which time a target point is selected under the criterion that the difference between the real-time subset of variables $\mathbf{x}_m$ and those of the target point be made minimum. As such, only small perturbations are needed most of the time.

III. EXAMPLE 1: PREVENTION OF VOLTAGE COLLAPSE IN ELECTRICAL POWER SYSTEMS

Electrical power systems are essentially nonlinear dynamical systems. Most of major power-system failures in the past years have been reported to be caused by the dynamic response of the system to disturbances [7]. One type of instability is voltage collapses which occur when the system is heavily loaded. In such a case, dynamical variables of the system, such as various voltages, fluctuate in a random manner for a period of time before collapsing to zero suddenly, leading to a complete blackout of the system [17]. Due to an ever-increasing demand for electrical power nowadays, there is a tremendous interest in operating the power system very near the edge of its stability boundary. As a consequence, the system becomes highly nonlinear and can exhibit chaotic behaviors. One possible mechanism for voltage collapse is then as follows: the system is operating in a parameter region where there is a chaotic attractor. A disturbance or a temporal overload causes a shift in a system parameter so that a boundary crisis occurs, after which the system exhibits transient chaos, leading to a voltage collapse.

To understand the phenomenon of voltage collapse, Dobson and Chiang [7] introduced a model power system consisting of a generator, an infinite bus, a nonlinear load, and a capacitor in parallel with the nonlinear load. Subsequently, Wang and Abed pointed out that the presence of the capacitor could cause an increase of the voltage magnitude and the reactive power demand of the load to almost practically unreachable value even in normally encountered parameter regimes [8]. A modified model was then proposed [8], as shown in Fig. 1. The model consists of two generators with voltages E_0 and E_m, and a load consisting of an induction motor and a PQ load in parallel. The system dynamics are governed by four differential equations in terms of dynamical variables: (1) δ_m, the generator phase angle which is closely related to the mechanical angle of the rotor; (2) ω, the angular speed of the rotor; (3) δ, the load voltage phase angle; and (4) V, the magnitude of the voltage provided to the load. The dynamics between the two generators are gov-

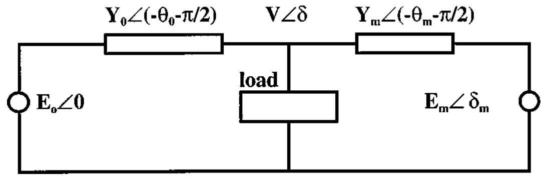

FIG. 1. The modified electric power-system model of Wang and Abed. The model consists of two generators with voltages E_0 and E_m, and a load consisting of an induction motor and a PQ load in parallel.

erned by the following swing equation:

$$M\ddot{\delta}_m + d_m\omega = P_m + E_m V Y_m \sin(\delta - \delta_m - \theta_m) + E_m^2 Y_m \sin\theta_m, \tag{3}$$

where M is generator inertia, d_m is the damping factor, and P_m is the mechanical power. The load model includes a dynamic induction motor and a constant PQ load in parallel. The induction motor model specifies the real and reactive power demands P and Q of the motor in terms of load voltage and frequency $\dot{\delta}$. The combined model for the motor and the PQ load is:

$$P = P_0 + P_1 + K_{pw}\dot{\delta} + K_{pv}(V + T\dot{V}), \tag{4}$$

$$Q = Q_0 + Q_1 + K_{qw}\dot{\delta} + K_{qv}V + K_{qv2}V^2, \tag{5}$$

where P_0 and Q_0 (real) are reactive powers of the motor, and P_1 and Q_1 are the powers for the PQ load. We choose Q_1 to be the bifurcation parameter [8] for a practical reason: increasing Q_1 corresponds to increasing the load reactive power demand. Rearranging those equations with $\theta_m = 0$ and $\theta_0 = 0$, we obtain four first-order, autonomous differential equations for the model:

$$\begin{aligned} \dot{\delta}_m &= \omega, \\ M\dot{\omega} &= -d_m\omega + P_m - E_m V Y_m \sin(\delta_m - \delta), \\ K_{qw}\dot{\delta} &= -K_{qv2}V^2 - K_{qv}V + Q(\delta_m, \delta, V) - Q_0 - Q_1, \\ T K_{qw} K_{pv}\dot{V} &= K_{pw}K_{qv2}V^2 + (K_{pw}K_{qv} - K_{qw}K_{pv})V \\ &\quad + K_{qw}[P(\delta_m, \delta, V) - P_0 - P_1] \\ &\quad - K_{pw}[Q(\delta_m, \delta, V) - Q_0 - Q_1]. \end{aligned} \tag{6}$$

The real and reactive powers supplied to the load by the network are

$$\begin{aligned} P(\delta_m, \delta, V) &= -E_0 V Y_0 \sin\delta + E_m V Y_m \sin(\delta_m - \delta), \\ Q(\delta_m, \delta, V) &= E_0 V Y_0 \cos\delta + E_m V Y_m \cos(\delta_m - \delta) \\ &\quad - (Y_0 + Y_m)V^2. \end{aligned} \tag{7}$$

In our numerical simulation, the load, the network, and the generator parameter values are chosen to be [8] $K_{pw} = 0.4$, $K_{pv} = 0.3$, $K_{qw} = -0.03$, $K_{qv} = -2.8$, $K_{qv2} = 2.1$, $T = 8.5$, $P_0 = 0.6$, $Q_0 = 0.3$, $P1 = 0.0$, $Y_0 = 3.33$, $Y_m = 5.0$, $P_m = 1.0$, $d_m = 0.05$, $M = 0.014\,64$, $E_m = 1.05$, $\theta_0 = 0$, and $\theta_m = 0$.

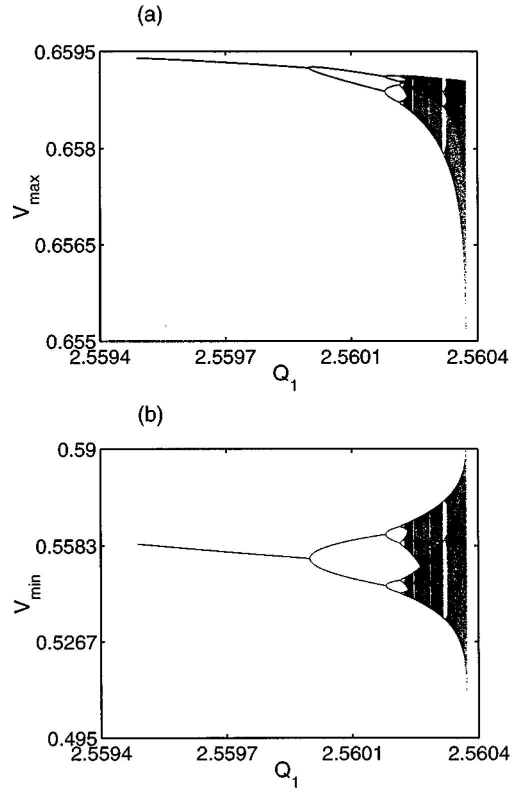

FIG. 2. Bifurcation diagrams of the power-system model Eq. (6): asymptotic values of (a) local maxima and (b) local minima of the voltage.

We first study the dynamical properties of Eq. (6). Figures 2(a) and 2(b) show bifurcation diagrams of Eq. (6), where 100 successive local maxima $V_{\max}$(a) and minima $V_{\min}$(b) of $V(t)$ are plotted for each of 1000 values of Q_1 (after disregarding an initial transient). There is a period-doubling cascade to chaos and a crisis occurs at $Q_{1c} \approx 2.560\,378\,33$ after which the chaotic attractor is converted into a chaotic saddle. Notice the range for the attractor is relatively small. Say now the system operates at some value of the load Q_1 before the crisis. A small change in Q_1 can then shift the system into the parameter regime after the crisis where there is only transient chaos. A voltage collapse can then occur. Figure 3 shows a time series $V(t)$ for $Q_1 = 2.560\,378\,4 > Q_c$, where $V(t)$ goes to zero suddenly after about 80 time units.

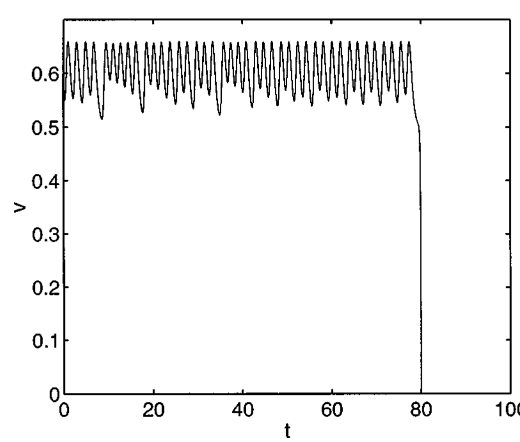

FIG. 3. For $Q_1 = 2.560\,378\,4 > Q_c$, an example of voltage collapse in the power system [Eq. (6)].

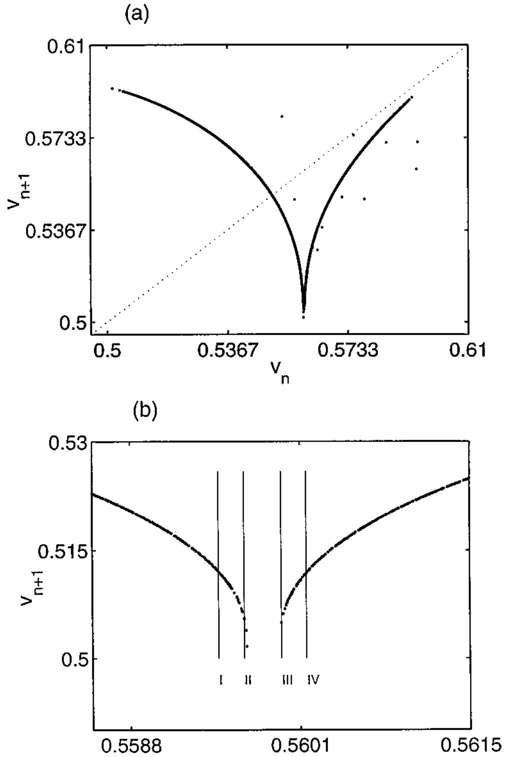

FIG. 4. The return maps constructed from the local minima of $V(t)$: (a) after the crisis at $Q_1 = 2.560\,378\,4$; and (b) a blowup part of (a) near the cusp. There is an escaping gap, enclosed between lines II and III, in the middle through which a trajectory asymptotes to the state with $V=0$ (voltage collapse). Two regions to the left (I–II) and the right (III–IV) of the gap are the regions from which a set of target points are chosen for control.

How to prevent voltage collapse? From Fig. 2, we see that a possible approach is to reduce the load Q_1 to bring the system back into the parameter regime where there is an attractor. It is, however, not practical to change the load in relatively short time. Our strategy is thus to construct a return map based on previously measured time series and to apply our method of control. Figure 4(a) shows the return map obtained from the local minima of $V(t)$ for $Q_1 = 2.560\,378\,4 > Q_{1c}$. There is an apparently escaping window below which $V(t)$ goes to zero quickly, as shown in Fig. 4(b), a blowup of part of Fig. 4(a). The vertical lines denote the regions from which target points are chosen. In particular, these regions are the one between line I and line II, and the one between line III and line IV. The escaping window is the primary gap on the chaotic saddle, which is a Cantor-like set. In contrast, before the crisis, there is no such a gap in the return map, as shown in Fig. 5 for $Q_1 = 2.560\,37$. In this case, there is a chaotic attractor and there is no escaping window. To achieve control in the regime of transient chaos, we select a set of 3000 target points in the vicinity of the escaping window with long lifetime. Figure 6 shows the lifetime versus the value of local minima. The plot is apparently not smooth, and in fact it contains an infinite number of singularities corresponding to points on the chaotic saddle. This singular structure renders possible selection of desired target points. Each target point contains the values of four dynamical variables in Eq. (6), although $V(t)$ is always at a local minimum. The set of target points is then stored for computing the control perturbation. When an actual trajectory falls into the escaping gap, the computer selects a target point such that the required perturbation to kick the trajectory onto the target point is minimum. Perturbations can be applied to the dynamical variables $\mathbf{x}$ directly. Or, if there is an accessible system parameter p that can be easily adjusted, perturbations can be applied to the parameter based on the difference between the trajectory point in the escaping window and the target point: $\Delta p = (\partial \mathbf{x}/\partial p)|_{\text{target}} \Delta \mathbf{x}$. In this case, more information is needed: the partial derivatives $(\partial \mathbf{x}/\partial p)|_{\text{target}}$. In the power system model [Eq. (6)], since all four dynamical variables can be perturbed, it may be convenient to apply control directly to these variables. As an example of control, Fig. 7(a) shows a controlled time series $V(t)$. The required control perturbations are shown in Fig. 7(b). In the time interval shown, only four small perturbations are required to sustain transient chaos. In general, the average time interval for applying perturbations is approximately the average lifetime of the chaotic saddle. We stress that, for Eq. (6), perturbations are required only when the system drifts into regime of transient chaos, because transient chaos is the culprit for voltage collapse in this model. No control is required when disturbances occur so that the system drifts back into the region with an attractor (either chaotic or periodic).

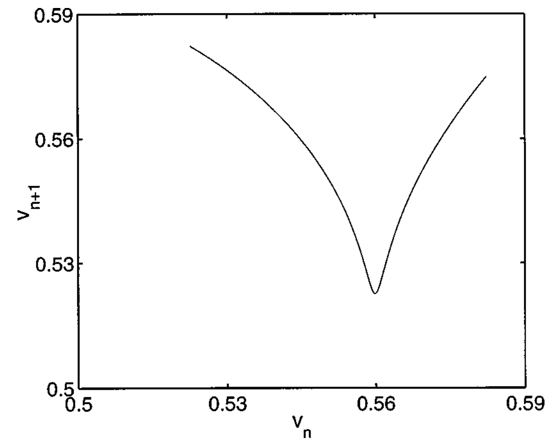

FIG. 5. The return map for $Q_1 = 2.560\,37 < Q_{1C}$ (before the crisis).

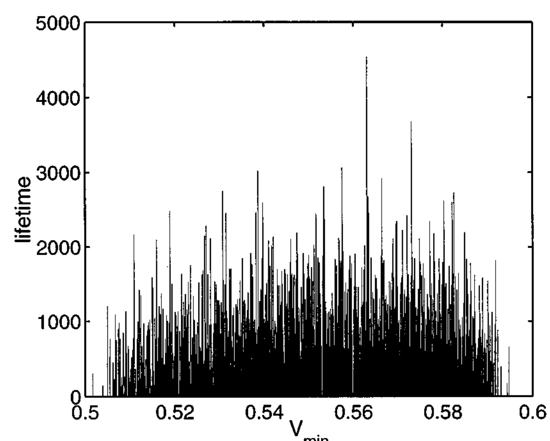

FIG. 6. Lifetime vs the local minima of $V(t)$ in the return map. The plot contains an infinite number of singularities corresponding to points on the chaotic saddle.

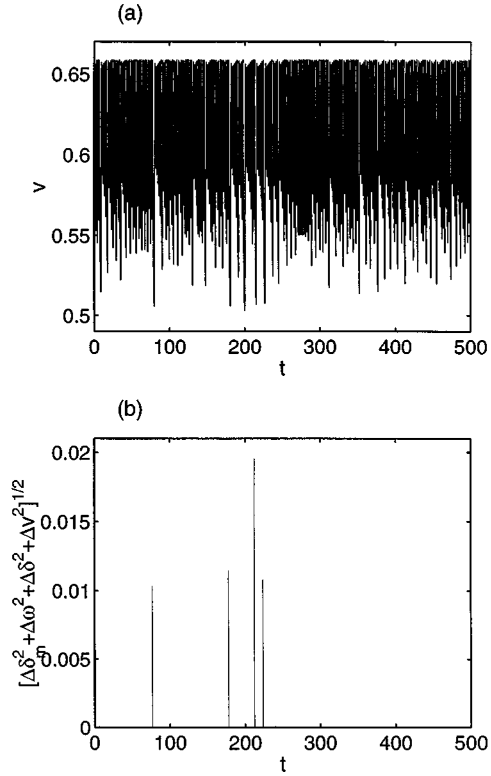

FIG. 7. For $Q_1 = 2.5603784$: (a) a controlled time series $V(t)$; and (b) required control perturbations. Apparently, only infrequent perturbations are needed to prevent voltage collapse.

A key question in any scheme of controlling transient chaos concerns the probability for an initial condition in the original basin $\mathcal{B}$ to be controlled [18]. Since the system performs normally before collapse and since control is activated only when $V(t)$ falls into the escaping gap, we expect almost all trajectories to be controlled. To test this numerically, we choose a two-dimensional region (V, δ_m) in the four-dimensional phase space and locate the cross section of the original basin with the V-δ_m plane, as shown in Fig. 8. We then uniformly distribute 25 000 initial conditions in the cross section and examine how many of them can be controlled for $Q_1 > Q_{1c}$. We find that all 25 000 initial conditions can be controlled. In practice, this implies that voltage collapse can be effectively prevented by using our control method.

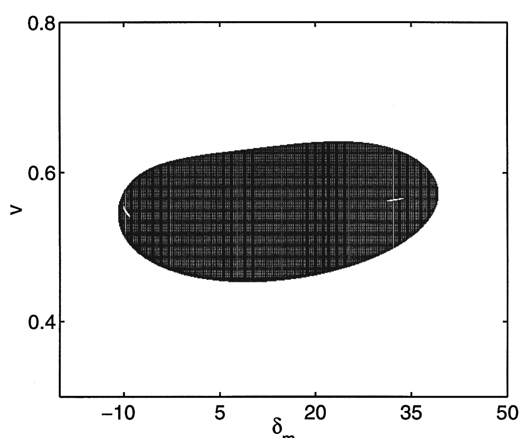

FIG. 8. For $Q_1 = 2.5603784$ after the crisis, a two-dimensional cross section of the basin of the original attractor $\mathcal{B}$. We find numerically that almost all initial conditions chosen from this region can be controlled.

IV. EXAMPLE 2: SPECIES PRESERVATION IN ECOLOGY

Extinction of species has been one of the biggest mysteries in nature [19]. A common belief about local extinction is that it is typically caused by external environmental factors such as sudden changes in climate. For a species of very small population size, small random changes in the population (known as ''demographic stochasticity'') can also lead to its extinction. Clearly, the question of how species extinction occurs is extremely complex, as each species typically lives in an environment that involves interaction with many other species (e.g., through competition for common food sources, predator-prey interactions, etc.) as well as physical factors such as the weather and disturbances (e.g., landslides). From a mathematical point of view, a dynamical model for the population size of a species is complex, involving temporal and spatial variations, external driving, and random perturbations. Such a system should, in general, be modeled by nonlinear partial differential equations with random and/or regular external driving forces. A difficulty associated with this approach is that the analysis and numerical solution of stochastic and/or driven nonlinear partial differential equations present an extremely challenging problem in mathematics.

Nonetheless, in certain situations the problem of species extinction may become simpler. Specifically, it was recently suggested by McCann and Yodzis [9] that deterministic chaos in very simple but plausible ecosystem models, mathematically described by a set of coupled ordinary differential equations, can provide a hint to how local species extinction can occur without the necessity to consider temporal or spatial variations and external factors. The key observation is that population dynamics of a large class of ecosystems can be effectively modeled by deterministic systems [20–23], and that the behavior of transient chaos is often typical in such systems [1,4]. For an ecosystem that exhibits transient chaos, the implication is that the population size of some species may behave chaotically for a (long) period of time and then decrease to zero in a relatively short period of time. It was shown by McCann and Yodzis [9] that such a transient chaotic behavior, which is responsible for species extinction, can indeed occur in a simple three-species food chain model which incorporates biologically reasonable assumptions about species interactions. We mention that the phenomenon of extremely long chaotic transients can also occur in other ecosystems, typically systems that involve both temporal and spatial variations [24]. Our idea is that if species extinction is caused by transient chaos, then it is possible for human being to intervene externally by applying perturbations so as to effectively prevent species from becoming extinct. The magnitude of the applied perturbation can be made arbitrarily small, and the perturbations need to be applied only occasionally. As such, the natural dynamics of the species population is hardly influenced, and yet, the population, though still exhibiting chaotic behavior, will never become zero. The implication is that in a realistic ecological environment, a very small amount of artificially imposed change to population sizes or some small disturbance to the environment, only very rarely applied, can prevent species extinction over long time scales. Potentially, our idea can be of paramount interest to the significant and growing environmental problem of species preservation.

We consider the following model of a simple three-species food chain by McCann and Yodzis [9]: a resource species, a prey (consumer), and a predator. The population densities of these three species, denoted by R, C, and P for resource, consumer, and predator, respectively, are governed by the following equations:

$$\frac{dR}{dt}=R\left(1-\frac{R}{K}\right)-\frac{x_C y_C C R}{R+R_0},$$
$$\frac{dC}{dt}=x_C C\left(\frac{y_C R}{R+R_0}-1\right)-\frac{x_P y_P P C}{C+C_0}, \quad (8)$$
$$\frac{dP}{dt}=x_P P\left(-1+\frac{y_P C}{C+C_0}\right),$$

where K is the resource carrying capacity, and x_C, y_C, x_P, y_P, R_0, and C_0 are parameters that are positive. The model carries the following biological assumptions: (1) The life histories of each species involve continuous growth and overlapping generations, with no age structure (this permits the use of differential equations). (2) The resource population (R) grows logistically. (3) Each consumer species (immediate consumer C, top consumer P) without food dies off exponentially. (4) Each consumer's feeding rate [e.g., $x_C y_C R/(R+R_0)$], saturates at high food levels.

The resource population, growing alone, equilibrates its carrying capacity K. The resource population and intermediate consumer, without the top consumer, either settle to a stable equilibrium, or to a stable limit cycle, a kind of ''biological oscillator.'' The oscillations are generated by the saturating feeding response, which permits the resource to periodically ''escape'' control by the consumer. With the top consumer, there are in a sense two coupled oscillators in the food chain. It is well known that coupled oscillators can lead to complex dynamics (see, for example, Ref. [25]). This provides an intuitive insight into why the model can give rise to chaotic dynamics.

Realistic values for parameters can be derived from bioenergetics. Following McCann and Yodzis [9], in our study we fix $x_C=0.4$, $y_C=2.009$, $x_P=0.08$, and $y_P=2.876$ so that both the consumer and the predator can be either invertebrate or vertebrate ectotherms (e.g., fish), with a reasonable predator to prey (consumer to resource) body mass ratio. We also fix $R_0=0.16129$ and $C_0=0.5$. Although the above parameter choices are rather arbitrary, they are ecologically meaningful [9]. The resource carrying capacity K, however, can be different in different environments. Thus we vary K over some reasonable range to assess different dynamical behaviors of the system.

To understand how species extinction can occur in the model Eq. (8), it is insightful to look into the dynamics of the predator population from the perspective of chaos. Figures 9(a) and 9(b) show bifurcation diagrams of local maxima $P_{\max}$ and local minima $P_{\min}$ versus K. There is a period-doubling cascade to chaos and a crisis at $K=K_c\approx 0.999\,76$. Figures 10(a) and 10(b) show, for $K=0.99<K_c$, the projections of the chaotic attractor onto the (R,P) and (C,P) planes, respectively. It is apparent from these plots that the trajectory $[R(t),C(t),P(t)]$ exhibits irregular and chaotic motion, but none of the populations will become extinct because the chaotic attractor is located in a phase-space region away from the origin $[(R,C,P)=(0,0,0)]$. In this parameter range, not all initial conditions yield motions on this chaotic attractor; there is in fact a second attractor, coexisting with the first one. This attractor is a limit cycle located in the plane of $P=0$. Trajectories on this attractor thus correspond to the situation where the top predator population is extinct. Therefore, for a fixed $K\lesssim K_c$, depending on the choice of the initial condition, the system either asymptotes to the chaotic attractor or to the limit cycle with $P=0$. As the carrying capacity K increases passing through the critical value K_c, the predator eventually becomes extinct for almost all initial conditions. This is quite counterintuitive, but it can be easily understood from the dynamics. At $K=K_c$, a crisis occurs where the tip of the chaotic attractor touches the basin boundary [1], after which there is transient chaos. Figure 11 shows a time series $P(t)$ for $K=1.02\gtrsim P_c$. It can be seen that $P(t)$ remains finite initially but decreases rapidly to zero. Thus we see that a species extinction can indeed occur as a result of transient chaos.

One way to prevent extinction of the predator population is to decrease the resource carrying capacity K so that sustained chaotic motion on the attractor is restored. But ecologically, it may not be easy to adjust the carrying capacity of an environment, and, if this can be done, it may take some

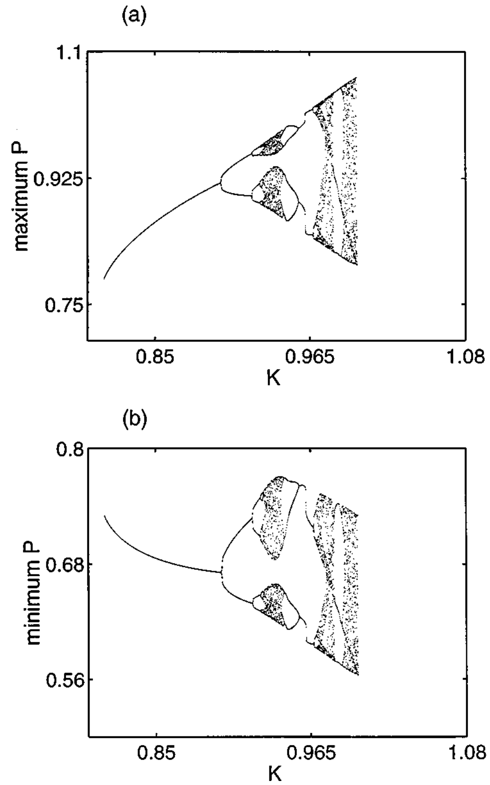

FIG. 9. Bifurcation diagrams of the ecological model from (a) the local maxima and (b) the local minima of $P(t)$.

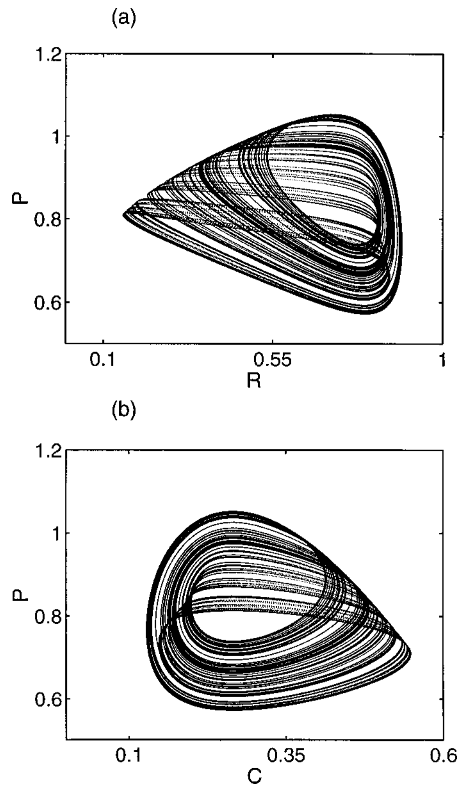

FIG. 10. For the ecological model [Eq. (8)] at $K=0.99$ (before the crisis), the projections of the chaotic attractor onto the (R,P) plane (a) and the (C,P) plane (b).

time to do so after detecting that the predator population is in danger. It may occur that the predator will already have become extinct before the carrying capacity is changed. Thus we suggest the use of small but occasional adjustments to the population at appropriate times to prevent species extinction. From an ecological point of view, it may be more feasible to make tiny adjustments to the local populations than to change the carrying capacity of the environment.

To apply control, we first construct a return map from the local minima $P_{\max}$ and identify the escape region, as shown in Fig. 12. The box enclosed by the dotted lines is the phase-space region in which the chaotic saddle lies. The vertical

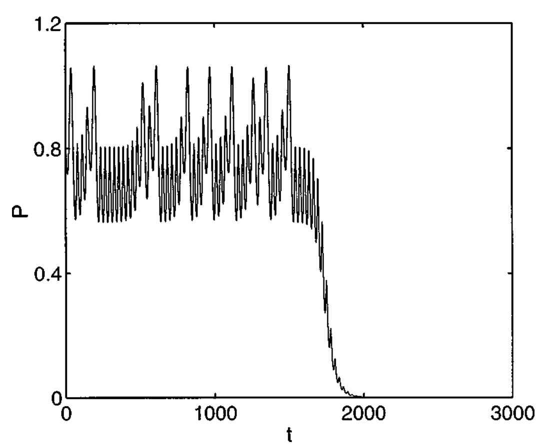

FIG. 11. A typical time series $P(t)$ after the crisis. It can be seen that $P(t)$ finite remains initially but decreases rapidly to zero, signifying species extinction.

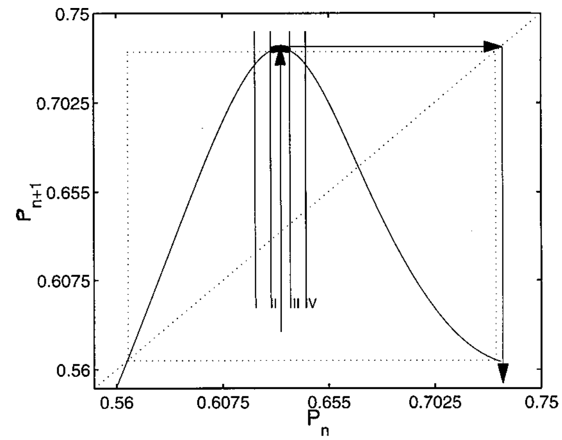

FIG. 12. For $K=1.02$ (after the crisis), the return map constructed from the local minima of $P(t)$. The existence of an escaping region (II–III) through which species extinction occurs is apparent. The arrow illustrates how a trajectory escapes from the chaotic saddle when falling into the escaping gap. The regions between lines I and II and between lines III and IV are the these from which target points are chosen.

lines denote the regions from which target points are chosen (the regions between lines I and II, and between lines III and IV). The escaping gap lies in between lines II and III. We then select a set of target points in both the right and left vicinities of the primary gap for computing the control perturbations. Figures 13(a) and 13(b) show a controlled time series $P(t)$ and the required magnitude of the perturbation $\Delta X(t) \equiv \sqrt{[\delta R(t)]^2+[\delta C(t)]^2+[\delta P(t)]^2}$. It can be seen

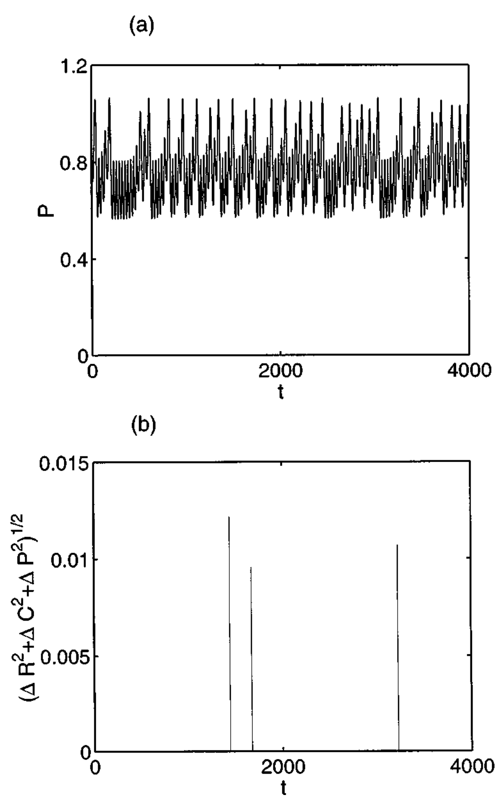

FIG. 13. For $K=1.02$ (after the crisis): (a) a controlled time series $P(t)$ for which extinction is prevented; (b) the required infrequent small control perturbations.

that the required perturbations are indeed small [$\Delta X(t) < 0.015$, compared with the size of the population which is about one] and rare [only three perturbations are applied in a time interval of (0, 4000)]. Numerical computations reveal that the chaotic population $P(t)$ can be maintained practically *indefinitely* through the use of occasional and small adjustments to all the populations, for almost all initial conditions chosen in the original basin of the chaotic attractor. Our approach can thus prevent species extinction effectively.

V. EXAMPLE 3: ELIMINATING UNDESIRABLE BURSTS IN A CHEMICAL REACTION

The preceding two examples are for transient chaos caused by boundary crises. There is another important class of crisis, the interior crisis, in which a chaotic attractor suddenly enlarges itself after a system parameter passes through a critical value. Dynamically, an interior crisis is triggered by the collision of a small chaotic attractor with a large chaotic saddle near the attractor [1,26,27]. Interior crisis occurs extremely commonly in chaotic systems because there is at least one event of interior crisis in every periodic window (at the end of the window), which is believed to be dense in parameter space. Physically, after an interior crisis, the dynamical variables exhibit intermittency in that a typical trajectory switches between distinct chaotic states in an intermittent fashion. In applications it may be desirable to keep the trajectory in one chaotic state. The aim of this section is to present an example to demonstrate that our control method can be employed to eliminate undesirable chaotic state from intermittent chaotic time series.

Our strategy is as follows. First, we construct a return map by using local maxima (or minima) from a measured time series. Second, we identify, on the return map, a critical region through which a switch from one chaotic state to another occurs. We then run the system to determine a set of target points near the critical region on the side of the desirable chaotic state. These target points are chosen such that trajectories originated from them can follow the desirable chaotic state for relatively long time, and they are the only information needed to achieve control if perturbations are to be applied directly to the dynamical variables. In real time, when a trajectory falls into the critical region, control perturbations are applied to force the trajectory onto one of the nearest target points. Desirable chaotic or periodic motion can then be maintained for a long period of time.

To demonstrate our strategy for controlling interior crisis, we consider the following model of a chemical reaction [15,16]:

$$\dot{x} = k_1 xz - k_2 x - \frac{k_3 xy}{x+K} + k_4 d,$$

$$\dot{y} = k_2 x - k_5 y + k_6, \tag{9}$$

$$\dot{z} = k_7 - k_1 xz - k_8 z,$$

where *x*, *y* and *z* are dynamical variables representing the concentrations of chemicals in the reaction, $k_1 - k_8$ and K are parameters, and we choose d to be the bifurcation parameter. Figure 14 shows a bifurcation diagram for $0.12 < d < 0.145$,

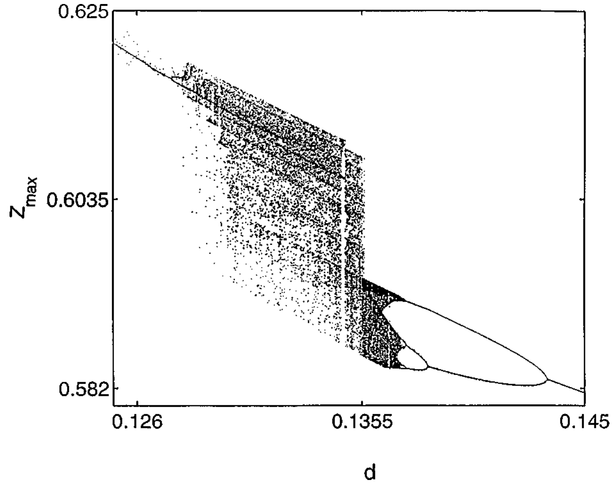

FIG. 14. For the chemical reaction model Eq. (9), a bifurcation diagram for $0.12 < d < 0.145$. An interior crisis occurs at $d = d_c \approx 0.1356$ as d is decreased.

where other parameters are set to be [15,16] $(k_1,k_2,k_3,k_4,k_5,k_6,k_7,k_8,K) = (2,0.4,1.0,0.0001,0.5,0.0002,0.005,0.0068,0.002)$. As d is decreased, an interior crisis occurs at $d = d_c \approx 0.1356$. For $d > d_c$, there is a small chaotic attractor. At the interior crisis, this attractor collides with a chaotic saddle which already exists for $d > d_c$. For $d < d_c$, there is a larger attractor consisting of essentially the former small chaotic attractor and the chaotic saddle [26,27]. A trajectory after the crisis typically visits both parts in an intermittent fashion, leading to an intermittent time series, as shown in Fig. 15(a) for $d = 0.135$. Figure 15(b) shows the return map constructed from the local maxima of the time series in Fig. 15(a), where the critical regions through which switching of the trajectory between the two chaotic states occurs are denoted by thick solid lines, and the desirable chaotic state is confined between lines I and II. Assuming that the small amplitude chaotic state is the desirable one, we determine from the return map a set of 3000 target points from which trajectories can stay in the small amplitude chaotic state for at least 200 time units. Figure 16(a) shows a controlled trajectory that only stays in the desirable chaotic state, and the required small perturbations are shown in Fig. 16(b). The controlled trajectory is in fact a periodic one embedded in the desirable chaotic state, since we apply the small control periodically. These results thus demonstrate that interior crisis in deterministic flow can also be controlled to yield a sustained desirable *periodic* motion [28,29].

VI. DISCUSSION

In this work, we have studied a scheme to control transient chaos in general deterministic flows. In principle, the control strategy does not require detailed knowledge of the underlying dynamical equations: a time series and accessibility of dynamical variables are enough to achieve control. Thus we expect our method, or some variants of it, to be applicable to practical situations where sustained periodic or chaotic motion is desired.

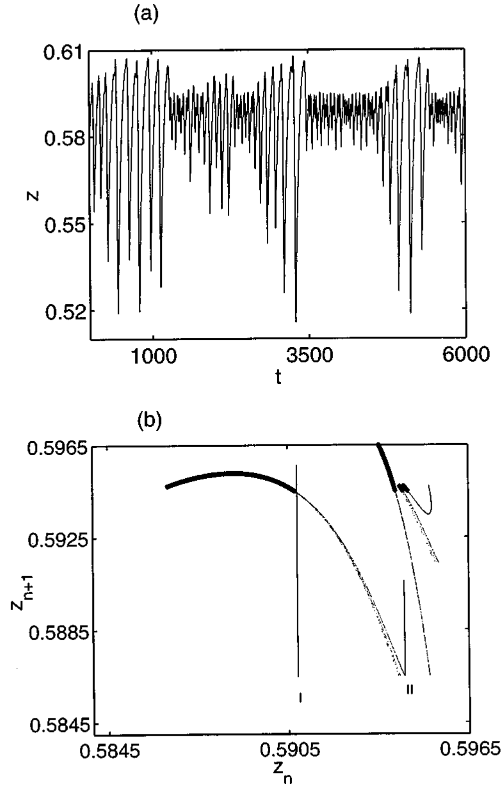

FIG. 15. For the chemical reaction model Eq. (9) at $d=0.135<d_c$ (after the crisis): (a) an intermittent chaotic time series; and (b) the return map constructed from the local maxima of the time series in (a). The desirable small-amplitude chaotic state lives in the region bounded by lines I and II.

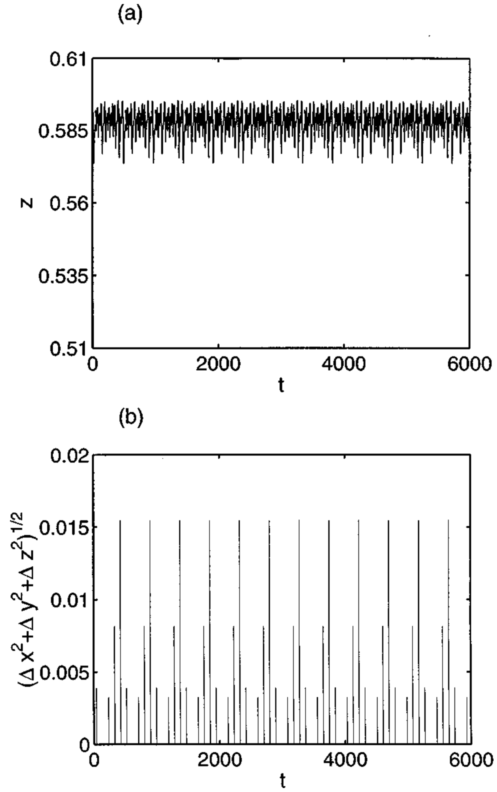

FIG. 16. For $d=0.135$ (after the crisis): (a) a controlled time series where the corresponding trajectory is periodic and restricted to the desirable small chaotic state; and (b) the required small perturbations. This example shows that our method can be readily adapted to stablizing periodic motion from transient chaos.

An important issue in any scheme of chaos control is noise. Typically noise can destabilize an already controlled trajectory [10]. In the case of transient chaos, noise can be devastating because a trajectory, when kicked out of the region of desirable chaotic motion by noise, can go to the undesirable state, e.g., voltage collapse or extinction of a species, in an irreversible manner. Our control strategy is, however, robust against noise in so far as the magnitude of control perturbations can exceed the noise level. This is because under the influence of noise, the return map becomes fuzzy, but the fuzziness hardly affect the control as we activate the control whenever a dynamical variable falls into an escaping region, although a precise control of a target point becomes difficult.

The issue of noise thus motivates us to consider an alternative way to sustain transient chaos. Specifically, we ask, can control still be achieved if we randomly perturb the trajectory to kick it back into the region of desirable chaotic motion after a dynamical variable falls below a critical value? Take Eq. (8), for example: what if we simply apply some small kick so that the trajectory falls back into the region where $P>P_{\rm crit}=0.56$. To address this question, we have undertaken the following numerical experiment. Say the population $P(t)$ falls slightly below the critical level at time *t*. Let (R_-,C_-,P_-) be the values of the state variables at this time, where $P_-\lesssim P_{\rm crit}$, and let (R_+,C_+,P_+) be the values of the state variables a little before *t*, where $P_+\gtrsim P_{\rm crit}$. The criteria for choosing $P_{\rm crit}$ are (1) ecologically it is chosen with respect to a population that can become extinct, and (2) dynamically it should be sufficiently close to the original basin $\mathcal{B}$. The plane $P=P_{\rm crit}$ thus represents a critical level of the endangered population at which human intervention must be introduced to prevent the extinction of the species *P* [30]. At time *t*, arbitrarily small *random* adjustments $[\delta R(t),\delta C(t),\delta P(t)]$ are made to *all* the populations so that the trajectory falls into a point, in the phase space, within a small ball centered at (R_+,C_+,P_+). With a nonzero probability, the trajectory will be close to one of the points in the small ball with long lifetime so that sustained chaotic motion can be resumed. Insofar as the trajectory executes a recurrent chaotic motion for $P>P_{\rm crit}$ no external perturbations are necessary. The control is successful and, further, we find that only small perturbations to the populations are needed.

The procedure we have presented in this paper applies generally to controlling transient chaos in deterministic flows. The key feature of our method (or a variant of it, discussed in the preceding paragraph) is that we set a control region based on the most relevant dynamical variable that is experimentally accessible. Control is activated only when the variable falls into the region. As such, only infrequent small control perturbations are required, and we also overcome the difficulty caused by the more standard use of the discrete-map-type of controlling procedure so that almost all transient chaotic trajectories can be controlled. To our knowledge, in previously purposed methods for controlling transient chaos, only a fraction of trajectories can be controlled. Controlling transient chaos and maintaining sustained chaotic motion [31,32] have become an interesting area of recent investigation due to their potential relevance to problems such as biological health [33], and our work may thus help to provide broadly useful insights into this rapidly growing area of research.

ACKNOWLEDGMENTS

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[1] C. Grebogi, E. Ott, and J. A. Yorke, Phys. Rev. Lett. **48**, 1507 (1982); Physica D **7**, 181 (1983).

[2] H. Kantz and P. Grassberger, Physica D **17**, 75 (1985).

[3] G. H. Hsu, E. Ott, and C. Grebogi, Phys. Lett. A **127**, 199 (1988).

[4] T. Tél, in *Directions in Chaos*, edited by Bai-lin Hao (World Scientific, Singapore, 1990), Vol. 3; T. Tél, in *STATPHYS 19*, edited by Bai-lin Hao (World Scientific, Singapore, 1996).

[5] E. Ott and T. Tél, Chaos **3**, 417 (1993), focus issue on chaotic scattering, edited by E. Ott and T. Tél.

[6] C. Jung and E. Ziemniak, J. Phys. A **25**, 3929 (1992); C. Jung, T. Tél, and E. Ziemniak, Chaos **3**, 555 (1993); E. Ziemniak, C. Jung, and T. Tél, Physica D **76**, 123 (1994); Á. Péntek, T. Tél, and Z. Toroczkai, J. Phys. A **28**, 2191 (1995); Fractals **3**, 33 (1995); Á. Péntek, Z. Toroczkai, T. Tél, C. Grebogi, and J. Yorke, Phys. Rev. E **51**, 4076 (1995); Z. Neufeld and T. Tél, *ibid.* **57**, 2832 (1998); Z. Toroczkai, G. Károlyi, Á. Péntek, T. Tél, and C. Grebogi, Phys. Rev. Lett. **80**, 500 (1998).

[7] I. Dobson and H.-D. Chiang, Syst. Control Lett. **13**, 253 (1989).

[8] H. Wang, E. H. Abed, and A. M. A. Hamdan, in *Proceedings of the 1992 American Control Conference (Chicago)* (American Autom. Control Council, Evanston, IL, 1992), pp. 2084–2088; H. Wang and E. H. Abed, in *Proceedings of NOLCOS'92: Nonlinear Control System Design Symposium, Bordeaux, France*, edited by M. Fliess (Pergamon, Oxford, UK, 1992), pp. 283–288; H. Wang, Ph.D. thesis, University of Maryland, 1993.

[9] K. McCann and P. Yodzis, Am. Nat. **144**, 873 (1994).

[10] E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990).

[11] T. Tél, J. Phys. A **24**, L1359 (1991), Int. J. Bifurcation Chaos Appl. Sci. Eng. **3**, 757 (1993).

[12] Y.-C. Lai and C. Grebogi, Phys. Rev. E **49**, 1094 (1994); Y.-C. Lai, T. Tél, and C. Grebogi, *ibid.* **48**, 709 (1993); Y.-C. Lai, C. Grebogi, and T. Tél, in *Towards the Harnessing of Chaos*, edited by M. Yamaguti (Elsevier, Amsterdam, 1994).

[13] I. Schwartz and I. Triandaf, Phys. Rev. Lett. **77**, 4740 (1996).

[14] T. Kapitaniak and J. Brindley, Phys. Lett. A **241**, 41 (1998).

[15] J. L. Hudson, O. E. Rössler, and H. Killory, Chem. Eng. Commun. **46**, 158 (1986).

[16] P. Parmananda and M. Eiswirth, Phys. Rev. E **54**, R1036 (1996).

[17] Instances of voltage collapses in the past were December 1978 in France, August 1981 in Belgium, etc. [7].

[18] We address initial conditions only in the original basin of the attractor because, before the collapse, the system performs normally and operates in the parameter regime before the crisis. We are thus not concerned with initial conditions outside the basin although they usually yield trajectories that asympotote to $V=0$. A voltage collapse can thus be regarded as a catastrophic event. Our control method is applicable to this type of catastrophes.

[19] S. L. Pimm, *The Balance of Nature* (University of Chicago Press, Chicago, 1991).

[20] R. May, Nature (London) **261**, 459 (1976).

[21] A. Hastings *et al.*, Annu. Rev. Ecol. Syst. **24**, 1 (1993).

[22] R. May, Am. Math. Soc. Trans. **32**, 291 (1995).

[23] R. D. Holt and M. A. McPeek, Am. Nat. **148**, 709 (1997).

[24] A. Hastings and K. Higgins, Science **263**, 1133 (1994).

[25] K. Alligood, T. Sauer, and J. A. Yorke, *Chaos: An Introduction to Dynamical Systems* (Springer, New York, 1997).

[26] Y.-C. Lai, C. Grebogi, and J. A. Yorke, in *Applied Chaos*, edited by J. H. Kim and J. Stringer (Wiley, New York, 1992).

[27] K. G. Szabó and T. Tél, Phys. Lett. A **196**, 173 (1994); K. G. Szabó, Y.-C. Lai, T. Tél, and C. Grebogi, Phys. Rev. Lett. **77**, 3102 (1996).

[28] The issue of sustaining a desirable chaotic state from an intermittent time series as a result of interior crisis was addressed by Y. Nagai and Y.-C. Lai, Phys. Rev. E **51**, 3842 (1995). A method based on stabilizing a long target chaotic trajectory was also proposed in that work. The applicability of this method is, however, limited, as it may be difficult to find a long target chaotic trajectory and small control perturbations needed to be applied at each time step.

[29] A somewhat similar method for selecting a desirable chaotic state was proposed in Ref. [16].

[30] The concept of a ''threshold population size'' may provide a useful rule of thumb for manipulating the dynamics. Similar ideas have been used elsewhere in conservation theory [R. Gomulkiewicz and R. D. Holt, Evolution (Lawrence, Kans.) **49**, 201 (1995)].

[31] M. Ding, E. Ott, and C. Gebogi, Phys. Rev. E **50**, 4228 (1994).

[32] V. In, S. Mahan, W. L. Ditto, and M. L. Spano, Phys. Rev. Lett. **74**, 4420 (1995); V. In, M. Spano, J. Neff, W. L. Ditto, C. Daw, K. D. Edwards, and K. Nguyen, Chaos **7**, 605 (1997).

[33] S. J. Schiff, K. Jerger, D. H. Duong, T. Chang, M. L. Spano, and W. L. Ditto, Nature (London) **370**, 615 (1994).