GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION
SPONSORED PROJECT INITIATION

Date: December 1, 1976

Project Title: Convection and Thermoelastic Effects in Narrow Verticle al Fracture Spaces

Project No: G-35-623

Project Director: Robert P. Lowell

Sponsor: U. S. Department of Interior; Geological Survey

Agreement Period: From 9/1/76 Until 8/31/77

Type Agreement: Grant No. 14-08-0001-G-365

Amount:
$19,100 U.S. Geological Survey
5,485 GIT (G-35-322)
$24,585 Total
=======

Reports Required:
Quarterly Program Management Reports; Semi-Annual Technical; Final Scientific Report

Sponsor Contact Person (s):

Technical Matters

Geologic Division
Office of Geochemistry & Geophysics

Contractual Matters
(thru OCA)

U.S. Geological Survey
Procurement & Contracts
Eastern Region, MS 291
12201 Sunrise Valley Drive
Reston, Virginia 22092

Defense Priority Rating:

Assigned to: Geophysical Science (School/Laboratory)

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GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION

Date: June 6, 1978

Project Title: Convection and Thermoelastic Effects in Narrow Vertical Fracture Spaces

Project No: G-35-623

Project Director: Dr. Robert P. Lowell

Sponsor: U.S. Department of Interior; Geological Survey

Effective Termination Date: 3/1/78 (Grant Expiration)

Clearance of Accounting Charges: by 3/1/78

Grant/Contract Closeout Actions Remaining:

- _ Final Invoice and Closing Documents
- X Final Fiscal Report due by 6/1/78.
- _ Final Report of Inventions
- _ Govt. Property Inventory & Related Certificate
- _ Classified Material Certificate
- _ Other ______

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CA-4 (3/76)

G-35-625

1976-78

Program Management Report /

Grant #: 14-08-001-G-365 (This research is sponsored by the U.S. Geological Survey as part of the Geothermal Research Program)

Grantee: Georgia Institute of Technology, Atlanta, Georgia 30332

Principal Investigator: Dr. Robert P. Lowell

Title: Convection and thermoelastic effects in narrow, vertical fracture spaces.

Effective date: 9/1/76 Termination date: 8/30/77

Funded amount: $19,100

Period of time covered by report: 9/1/76 - 11/30/76

Date of submission: 12/15/76

The purpose of this research is to examine convection and thermoelastic effects in long, deep, but very narrow vertical fracture spaces. The initiation and time development of the physical processes are to be studied on the basis of mathematical-physical approximation methods and finite element and/or difference methods. The results are to be interpreted with regard to known geothermal systems in the Basin and Range province and other regions where major faults are the dominant controlling structures.

This work represents a joint effort with Dr. G. Bodvarsson at Oregon State University. Analytical methods are the main emphasis of the Oregon State group, whereas numerical analysis is the main emphasis of the Georgia Tech group.

Major Accomplishments: none.

Problems: Due to administrative difficulties, the grant was not initiated at Georgia Tech until December 1. Therefore, only preliminary work was done during the quarter covered by this report. Moreover, a graduate student qualified to carry out the numerical computations is not currently available to the Georgia Tech group.

Actions Required by the Government: If a suitable graduate student is not found in the near future, delay in carrying out the proposed research program may result. We will then formally request that the termination date be extended so that the research program may be satisfactorily completed. Such an extension would be at no additional cost. At this time, however, it is hoped that research will progress at a reasonable rate during the next quarter and thereafter.

Respectfully submitted,

Robert P. Lowell
Principal Investigator

G-35-623

Program Management Report No. 2

Grant #: 14-08-001-G-365 (This research is sponsored by the U.S. Geological Survey as part of the Geothermal Research Program)

Grantee: Georgia Institute of Technology, Atlanta, Georgia 30332

Principal Investigator: Dr. Robert P. Lowell

Title: Convection and thermoelastic effects in narrow, vertical fracture spaces.

Effective date: 9/1/76 Termination date: 8/31/77

Funded amount: $19,100

Period of time covered by report: 12/01/76 - 2/28/77

Date of submission: 3/10/77

The purpose of this research is to examine convection and thermoelastic effects in long, deep, but very narrow vertical fracture spaces. The initiation and time development of the physical processes are to be studied on the basis of mathematical-physical approximation methods and finite element and/or difference methods. The results are to be interpreted with regard to known geothermal systems in the Basin and Range province and other regions where major faults are the dominant controlling structures.

This work represents a joint effort with Dr. G. Bodvarsson at Oregon State University. Analytical methods are the main emphasis of the Oregon State group, whereas numerical analysis is the main emphasis of the Georgia Tech group.

Major Accomplishments:

(1) Pertinent material on convection in porous media has been collected and reviewed. The collection and review of pertinent thermal and elastic data is underway.

(2) An approximate solution to the problem of the onset of convection in a long deep, narrow vertical fault zone has been attained.

Problems:

A graduate student qualified to carry out the required numerical computations has not been available to the Georgia Tech group. Therefore, the numerical work is not yet underway.

Changes in plans for conducting research:

I visited with Bodvarsson the 1st week of February, 1977 and we decided to request funds to be transferred from the Georgia Tech grant to the Oregon State grant to support a graduate student, Mr. Chuen-Tien Shyu, who is qualified to carry out the numerical modeling. The request for transfer of funds was initiated March 1 through the Office of Research Administration at Georgia Tech. The numerical program which is to be carried out by Mr. Shyu will be communicated to him shortly.

Actions required by the Government:

Quick action on the proposed transfer of funds will help the research proceed more smoothly. There have already been considerable delays, however, and it is quiet likely that the research work will not be completed on schedule. If this proves to be the case, a formal request will be made to extend the termination date of the grant. Such an extension will be made at no additional cost.

Respectfully submitted,

Robert P. Lowell
Principal Investigator

RPL:cma

G-35-623

Program Management Report No. 3

Grant #: 14-08-001-G-365 (This research is sponsored by the U.S. Geological Survey as part of the Geothermal Research Program)

Grantee: Georgia Institute of Technology, Atlanta, Georgia 30332

Principal Investigator: Dr. Robert P. Lowell

Title: Convection and thermoelastic effects in narrow, vertical fracture spaces.

Effective date: 9/1/76 Termination date: 8/31/77

Funded amount: $19,100

Period of time covered by report: 3/1/77 - 5/31/77

Date of submission: 6/14/77

The purpose of this research is to examine convection and thermoelastic effects in long, deep, but very narrow vertical fracture spaces. The initiation and time development of the physical processes are to be studied on the basis of mathematical-physical approximation methods and finite element and/or difference methods. The results are to be interpreted with regard to known geothermal systems in the Basin and Range province and other regions where major faults are the dominant controlling structures.

This work represents a joint effort with Dr. G. Bodvarsson at Oregon State University. Analytical methods are the main emphasis of the Oregon State group, whereas numerical analysis is the main emphasis of the Georgia Tech group.

Major Accomplishments:

(1) Collection and review of pertinent thermal and elastic data is continuing.

(2) The condition for the onset of convection in a long, deep, narrow

vertical fault zone filled with saturated permeable material of anisotropic permeability has been attained. The results of this work were recently presented at the Industrial Associates Program on Geothermal Problems held at Lamont-Doherty Geological Observatory and at the Spring Annual meeting of the American Geophysical Union.

(3) The sub-contract arrangement with Oregon State University for numerical modeling has been accomplished and the work is currently underway. I expect a report from the OSU group shortly.

Problems:

There has been considerable delay in getting the numerical modeling underway; and although the work is now expected to progress relatively smoothly, it is unlikely that the proposed work will be completed on schedule.

Action Required by the Government:

No actions are required at this time. Because of the aforementioned delays; however, a formal request for a no cost extension of the grant termination date will be forthcoming. This request should arrive at your office by July 15, 1977.

Respectfully submitted,

Robert P. Lowell
Principal Investigator

RPL:cma

G-35-623

Program Management Report No. 4

Grant #: 14-08-001-G-365 (This research is sponsored by the U.S. Geological Survey as part of the Geothermal Research Program)

Grantee: Georgia Institute of Technology, Atlanta, Georgia 30332

Principal Investigator: Dr. Robert P. Lowell

Title: Convection and thermoelastic effects in narrow, vertical fracture spaces.

Effective date: 9/1/76 Termination date: 8/31/77

Funded amount: $19,100

Period of time covered by report: 6/1/77 - 8/31/77

Date of submission: 9/15/77

The purpose of this research is to examine convection and thermoelastic effects in long, deep, but very narrow vertical fracture spaces. The initiation and time development of the physical processes are to be studied on the basis of mathematical-physical approximation methods and finite element and/or difference methods. The results are to be interpreted with regard to known geothermal systems in the Basin and Range province and other regions where major faults are the dominant controlling structures.

This work represents a joint effort with Dr. G. Bodvarsson at Oregon State University. Analytical methods are the main emphasis of the Oregon State group, whereas numerical analysis is the main emphasis of the Georgia Tech group.

Major Accomplishments:

(1) Collection and review of pertinent thermal and elastic data is continuing

(2) Mr. Shyu has begun performing the numerical analysis in accordance with the subcontract agreement between Georgia Tech and Oregon State. He has used a Galerkin method to obtain the critical condition for the onset of convection in a box filled with water-saturated permeable material. His results apply to various types of box geometries, including the special case of a long, deep, and narrow box which represents a fault zone. His results presently hold for insulated and perfectly conducting side walls.

I recently visited Mr. Shyu at Oregon State to discuss his results and to plan future work. The next steps in the numerical program are to include the effects of imperfectly conducting boundaries and to examine finite-amplitude convection in a fault zone shaped box with conducting side walls.

Problems:

Although the numerical work now appears to be progressing smoothly, there has been considerable delay and the proposed work had not been completed as the grant expired on August 31, 1977. A formal request for an extension of the grant termination date was submitted in late July.

Actions Required by the Government:

Prompt action on the request for an extension of the grant termination date would be most appreciated. This would allow the numerical work to come to a fruitful completion without further delay.

Respectfully submitted,

Robert P. Lowell
Principal Investigator

RPL/lt

G-35-625

Program Management Report No. 5

Grant #: 14-08-001-G-365 (This research is sponsored by the U.S. Geological Survey as part of the Geothermal Research Program)

Grantee: Georgia Institute of Technology, Atlanta, Ga. 30332

Principal Investigator: Dr. Robert P. Lowell

Title: Convection and thermoelastic effects in narrow, vertical fracture spaces with emphasis on numerical techniques

Effective date: 9/1/76 Termination date: 3/1/78

Funded amount: $19,100

Period of time covered by report: 9/1/77 - 11/30/77

Date of submission: 12/15/77

The purpose of this research is to examine convection and thermoelastic effects in long, deep, but very narrow vertical fracture spaces. The initiation and time development of the physical processes are to be studied on the basis of mathematical-physical approximation methods and finite element and/or difference methods. The results are to be interpreted with regard to known geothermal systems in the Basin and Range province and other regions where major faults are the dominant controlling structures.

This work represents a joint effort with Dr. G. Bodvarsson at Oregon State University. Analytical methods are the main emphasis of the Oregon State group, whereas numerical analysis is the main emphasis of the Georgia Tech group.

Major Accomplishments:

(1) Collection and review of pertinent thermal and elastic data is continuing.

(2) Mr. Shyu is continuing the numerical calculations on the onset of convection in a fault zone filled with saturated porous material. This work is in accordance with the subcontract agreement between Georgia Tech and Oregon State. Initial results have been favorable; however, there has been some difficulty in obtaining useful results for boundary

conditions other than insulated side walls. The subcontract agreement has been extended until December 31, 1977 and it is anticipated that results will be obtained by that time. Dr. Bodvarsson (Mr. Shyu's advisor at O.S.U.) and I have been in periodic contact with regard to these matters.

(3) The results of the critical condition determined by the Principal Investigator for the onset of convection in a fault zone with anisotropic permeability are being used to analyze the nature of lineated hydrothermal systems as are formed in the Galapagos Spreading Center, the TAG area, and the Basin and Range.

Problems:

None-- other than the slowness with which some of the numerical work is progressing.

Actions Required by the Government:

None at this time.

Respectfully submitted,

Robert P. Lowell
Principal Investigator

RPL:nlm

SEMI-ANNUAL TECHNICAL LETTER REPORT NO. 1

CONVECTION AND THERMOELASTIC EFFECTS IN NARROW, VERTICAL FRACTURE SPACES.

Effective date: 9/1/76 Termination date: 8/31/77

Funded Amount: $19,100

Principal Investigator: Robert P. Lowell
School of Geophysical Sciences

Grantee: Georgia Institute of Technology
Atlanta, Georgia 30332

Reporting Period: September 1, 1976 - February 28, 1977

Date Submitted: March, 1977

This research is sponsored by the U.S. Geological Survey as part of the Geothermal Research Program under grant no.: 14-08-001-G-365.

Introduction

Geothermal systems consist principally of a heat source and a groundwater circulation system within a reservoir rock. The heat source is usually a deeply situated magmatic intrusive. The heat transferred to the overlying reservoir initiates thermal convection of the groundwater which fills the interconnected pore and/or fracture spaces within the reservoir rock. An understanding of the initiation and time development of such convection systems is essential to the understanding of the energetics of a geothermal system.

Several known geothermal systems are controlled by deep, nearly vertical fault and fracture zones of tectonic origin. Examples are found in the Basin and Range province (Hose and Taylor, 1974) and in Iceland; and hydrothermal circulation in the oceanic crust (e.g., Lister, 1972) may be controlled by vertical fractures and faults (Bodvarsson and Lowell, 1972; Lowell, 1975). These zones of high permeability may have a linear extent of tens of kilometers and a depth of several kilometers. The fault and fracture zones are often quite narrow, however, having a width of the order of several tens of meters.

The purpose of this research is to examine convection and thermoelastic effects in long, deep, but very narrow vertical fractures spaces. The initiation and time development of the physical processes are to be studied on the basis of mathematical-physical approximation methods and finite element and/or difference methods. The results are to be interpreted with regard to known geothermal systems in the Basin and Range province and other regions where major faults are the dominant controlling structures.

This work represents a joint effort with Dr. G. Bodvarsson at Oregon State University. Analytical methods are the main emphasis of the Oregon State group, whereas numerical analysis is the main emphasis of the Georgia Tech group.

Convection in Permeable Media

The basic model for convection of a fluid in a porous medium is due to Lapwood (1948). Lapwood determined the critical condition for the onset of convection in an infinite slab of homogeneous, isotropic porous material of thickness h, bounded by horizontal planes. Within the slab, a vertical conduction temperature gradient, β, was maintained. The bottom of the layer was held at temperature T_o and the upper layer at T_1, where $T_1 < T_o$. Using stability analysis, Lapwood showed that $R = \rho_f s\alpha \ \beta h^2 K/\nu\lambda \geq 4\pi^2$ for convection to occur between impermeable boundaries, where ρ_f, s, α, ν represent the density, specific heat, thermal expansivity and kinematic viscosity of the fluid, respectively; K is the permeability, λ is the thermal conductivity of the rock, and g is the acceleration of gravity. The critical Rayleigh number for the other boundary conditions are given in Nield (1968); and the critical number for case of temperature dependent viscosity (Kassoy and Zebib, 1975) temperature dependent expansivity, viscosity, and density (Straus and Schubert, 1977) and anisotropic permeability (Wooding, 1976) have recently been determined.

Finite amplitude calculations on convective flows in a porous medium have been carried out by Wooding (1963), Straus (1974), Ribando et al. (1976), and Lowell and Patterson (1977). Convection in narrow, open fractures imbedded in an impermeable rock has been treated by Bodvarsson and Lowell (1972) and Lowell (1975).

Of particular interest for the situation of convection in a fault zone, however, are the various models for convection in a closed rectangular container. In this case the presence of vertical walls affects the critical number and the form of convection cells at $R = R_c$. The onset of convection of a viscous fluid in rectangular enclosures has been treated by Davis (1967), and convection in a box of water-saturated porous material has been treated by Beck (1972) and Holst and Aziz (1972). The onset of convection of a variable viscosity fluid in a porous box has been considered by Zebib and Kassoy (1977). All of the above authors have considered the special box geometry in which one horizontal dimension is much much less than the height and the other horizontal dimension (Figure 1). With the exception of Davis (1967), however, they all consider the case of insulated vertical walls. This in not realistic for a fault zone within the earth's crust. A more realistic boundary condition would be an applied geothermal gradient along the walls $y < 0$, $y > L_y$ (see Figure 1). It is also of great interest to examine finite amplitude convection models in such a geometry and to consider the effect of conductive cooling of the impermeable rock in the regions $y < 0$, $y > L_y$. Kassoy (1976) and Kassoy and Zebib (Kassoy, personal communication) are considering some related problems with regard to flow of fluid in a fault zone.

The Onset of Convection in a Fault Zone

Consider a model fault zone (Figure 1) as a rectangular slab of water-saturated , homogenous, isotropic porous material imbedded in impermeable rock. Let the dimensions of the porous zone be L_x, L_y, L_z with L_x, $L_z >> L_y$. Let a uniform geothermal gradient, β, be applied to the material such that the temperature at the upper surface is $T = 0$ and at the lower surface $T = T_o$ ($T_o > 0$). Assuming an incompressible, constant property fluid, the linearized perturbation equations become:

$$(\nu/K)\nabla^2 W + \alpha g \nabla_1^2 \theta = 0 \qquad (1)$$

$$\rho_f s \beta W = \lambda \nabla^2 \theta \qquad (2)$$

where W, θ are the vertical velocity and temperature perturbations, respectively

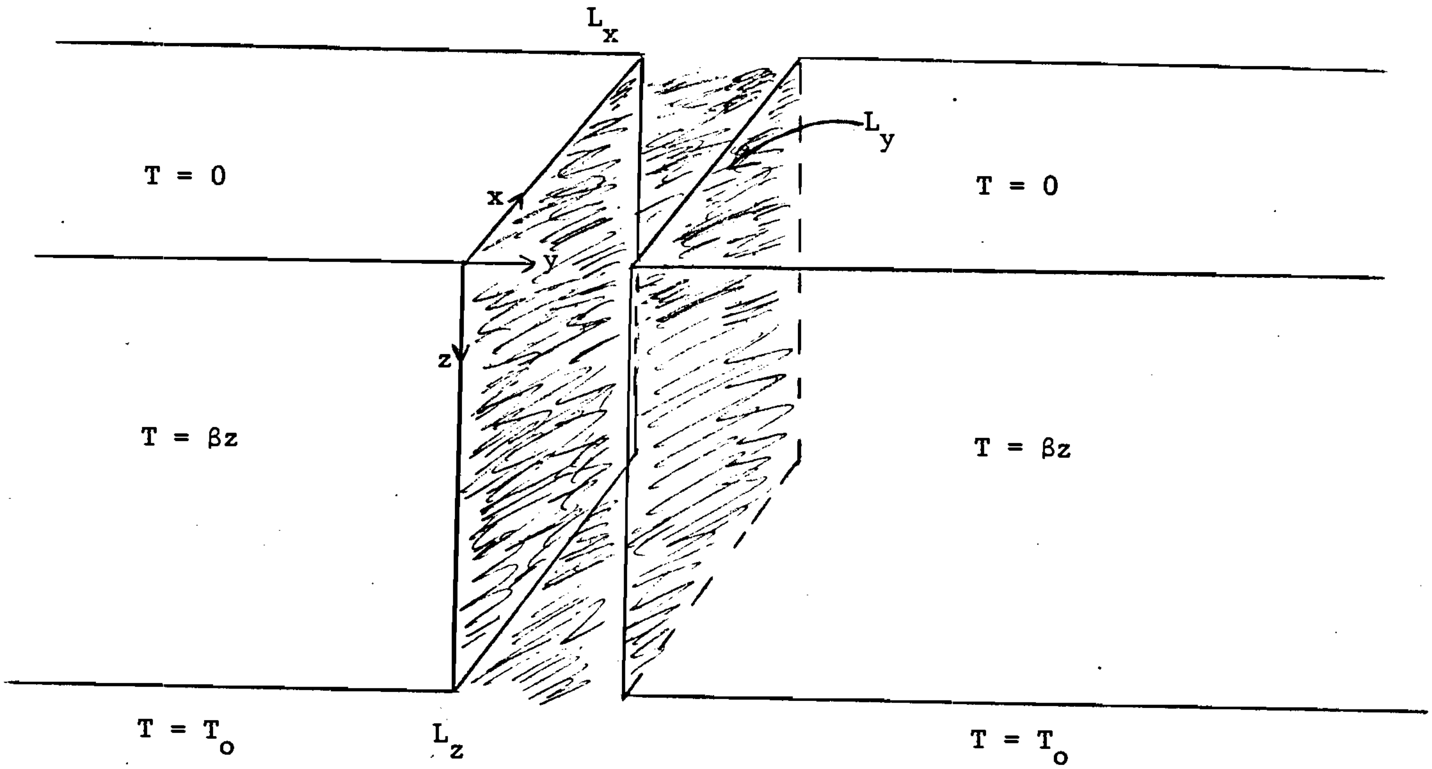

Figure 1. Model of narrow vertical fault zone.

and ∇_1^2 is the horizontal Laplacian. Eliminating W between the equations gives

$$\nabla^4\theta = \frac{-\rho_f s\beta\alpha g K}{\nu\lambda} \nabla_1^2\theta \tag{3}$$

Non-dimensionalizing the spatial coordinates using L_z as a length scale gives

$$\nabla'^4\theta = -R\nabla_1'^2\theta \tag{4}$$

where R is the usual Rayleigh parameter. The boundary conditions are taken as

$$\begin{aligned} \theta &= \theta'' = 0 \quad \text{at} \quad z' = 0,\ 1 \\ \theta &= \theta'' = 0 \quad \text{at} \quad y' = 0,\ a, \text{ where } a = \frac{L_y}{L_z} \\ \theta' &= \theta''' = 0 \quad \text{at} \quad x' = 0,\ b, \text{ where } b = \frac{L_x}{L_z} \end{aligned} \tag{5}$$

The conditions on the temperature derivatives correspond to the conditions $W = 0$ at $z' = 0, 1$; $\frac{\partial W}{\partial z} = 0$ at $y' = 0, a$; $x' = 0, b$. The solution to (4) which satisfies theses conditions is

$$\theta = \cos\frac{m\pi x'}{b} \sin\frac{n\pi y'}{a} \sin p\pi z' \tag{6}$$

leading to the condition that

$$R = \frac{\left(\frac{m^2\pi^2}{b^2} + \frac{n^2\pi^2}{a^2} + p^2\pi^2\right)}{\frac{m^2\pi^2}{b^2} + \frac{n^2\pi^2}{a^2}} \tag{7}$$

By inspection it is clear that the minimum value of R occurs when $m = 0$, $n = p = 1$. However, $n = 1$ corresponds to a vertical flow W which rises everywhere. Hence, one must take $n = 2$, corresponding to a flow which rises along one wall ($y' = 0$) and descends along the other ($y' = 1$). The flow is therefore a roll with its axis along the x direction. Finally, since $a << 1$, the critical number is approximately

$$R_c \simeq \left(\frac{2\pi}{a}\right)^2 \tag{8}$$

which is much larger than the value $R_c = 4\pi^2$ which Lapwood (1948) has derived for an infinite slab. The result (8) shows that as $L_y \to 0$, $R_c \to \infty$. Thus for a slab of infinitesimal thickness, the convection will not occur for an infinitesimal perturbation. This result agrees with Bodvarsson's (1977, personal communication) result for the onset of convection in a thin open fracture filled with a viscous fluid.

Assuming a ratio $L_y/L_z = 0.01$, $R_c = 4 \times 10^5$. For reasonable values for the physical parameters, $\rho_f s = 1$ cal/cm^3-^{o}C, $\alpha = 1.5 \times 10^{-3}$ ^{o}C, $\nu = 2 \times 10^{-3}$ cm^2/sec, $\lambda = 5 \times 10^{-3}$ cal/cm-^{o}C-sec, $L = 3 \times 10^5$ cm, $\beta = 10^{-3}$ ^{o}C/cm, where α, ν, λ are taken from Straus and Schubert (1977) as average values over the depth range, it is found that $K \simeq 1.0$ Darcy for convection. This is a fairly large value, though not unreasonable in typical geothermal systems where the permeability is often due to interconnected fractures within the rock volume. Assuming the permeability is due to a series of flat parallel fractures of width d and separation h, Bear (1972, p. 164) has shown that

$$K = \frac{d^3}{12h}$$

Thus if $h = 1$ m, $d \simeq 0.2$ mm which does not appear to be physically unreasonable.

It must now be confessed that these results are only approximate since the boundary condition $\frac{\partial W}{\partial z} = 0$ on the vertical boundaries is not the approximate condition. The condition $\frac{\partial W}{\partial z} = 0$ is in fact a redundant condition which would be satisfied automatically if the true condition $\vec{v} \cdot \hat{n} = 0$ on the boundaries had been applied, where $\vec{v}$ is the velocity vector and $\hat{n}$ is the unit vector normal to the wall. The equations, however, are not separable using the correct boundary conditions,and some other technique such as the Galerkin method must be used to find R_c (Davis, 1967). It is probable that the condition given by (8) is an underestimate, since the true boundary condition would act to more severely restrict the eigenfunctions. Ostrach and Pneuli (1963) have used the condition $\frac{\partial W}{\partial z} = 0$ to determine R_c in a box filled with viscous fluid. Their results give a R_c less than that given by Davis (1967), who applied the full boundary conditions.

The main result of the calculations given here is that the onset of convection in a narrow, vertical fault zone is much more difficult when heat is lost by conduction through the vertical boundaries. It would appear, however, that convection is possible, provided the fault zone is filled with fractured rock or debris. One might expect that in such geothermal systems, the actual Rayleigh number lies fairly close to the critical Rayleigh number.

Future Work

Due to delays in grant initiation at Georgia Tech and the inability to obtain a qualified graduate student, the numerical computations which are to be part of the work on this grant have not yet begun. We have proposed to transfer funds from this grant to the companion grant at Oregon State to support Mr. Chuen-Tien Shyu, a graduate student, who will perform the numerical modeling. The research plan which has recently been transmitted to Mr. Shyu via Dr. Gunnar Bodvarsson involves several cases of convection in a fault zone to which finite difference or finite element modeling will be applied. These cases are outlined briefly below. This will serve as an initial guide as to the direction of the proposed research. As the calculations are made and the results communicated to me,modifications will be made in the programs as deemed necessary.

Case 1 Steady convection: The temperature boundary conditions are $T = 0$ at $z = 0$, $T = T_o$ at $z = L_z$, $T = \beta z = (T_o/L_z)z$ at $y = 0$, $y = L_y$. The velocity boundary conditios are $\vec{v} \cdot \hat{n} = 0$ on all boundaries, where $\hat{n}$ is the normal unit vector. With the boundary conditions, the critical Rayleigh number and the cell form at the critical Rayleigh number will be determined. Flow and temperature fields will also be studied for finite amplitude conditions, $R \geq R_c$.

Case 2: Same as Case 1 except employ a radiation boundary condition $dT/dz+HT = 0$ at $z = 0$, where H is the heat transfer coefficient. This is the more realistic case in natural systems which are bounded above by a thin, impermeable cap rock. Several values of H will be considered.

Case 3: Same as Case 1 except that variable rock and water properties will be considered. That is, the permeability is $K = K(P,T)$, and the density, viscosity and thermal expansivity of water are taken to be temperature dependent. The function forms will be taken from Straus and Schubert (1977).

Case 4: Allow conductive cooling of the impermeable region surrounding the fault zone. That is, at time $t = 0$, $T = \beta z$ along the walls, but as convection occurs, heat is extracted from the surrounding rock. The time development of the system will be investigated for $R \geq R_c$.

This numerical modeling program involves a great amount of time and effort. Problems of the type proposed in Case 4 have not been treated in any fashion and it is not yet clear what new numerical techniques will be required. (Lowell (1975) has considered the simpler case of uni-directional flow in an open fracture.) It is not likely, therefore, that the entire program presented will be achieved within the grant period.

The results obtained will be interpreted with regard to the energetics of known geothermal systems of the Basin and Range province and other systems where fault and fracture zones are the dominant controlling structures.

It is expected that some of the preliminary results of this work will be presented at the Spring Annual Meeting of the American Geophysical Union, May 30-June 3, 1977.

Respectfully submitted,

Robert P. Lowell
Principal Investigator

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SEMI-ANNUAL TECHNICAL LETTER REPORT NO. 2

CONVECTION AND THERMOELASTIC EFFECTS IN NARROW, VERTICAL FRACTURE SPACES.

Effective date: 9/1/76 Termination date: 3/1/78

Funded Amount: $19,100

Principal Investigator: Robert P. Lowell
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Reporting Period: March 1, 1977 - August 31, 1977

Date Submitted: October, 1977

This research is sponsored by the U.S. Geological Survey as part of the Geothermal Research Program under grant no.: 14-08-001-G-365.

Introduction

Geothermal systems consist principally of a sub-surface heat source and a groundwater circulation system within a reservoir rock. The heat source is usually a deeply situated intrusive, and in most cases little is known with regard to the magnitude and precise location of the heat source. Heat is conducted to the overlying reservoir, and thermal convection is initiated in the ground water which fills the inter-connected pore and/or fracture spaces within the reservoir rock. In order to extract energy from the underground reservoir, one must drill bore holes which penetrate the reservoir rock and extract the heated fluid. The amount of energy which can be extracted depends upon the total heat content of the reservoir and the rate at which fluid is extracted. It is clear, therefore, that the energetics of a natural geothermal reservoir depend in a fundamental way on the circulation pattern of the ground water and the heat transfer between the ground water and the reservoir rock. An understanding of these processes, in a practical sense is in fact more important than an understanding of the conditions pertaining to the ultimate heat source. It is therefore extremely important to investigate the various types of convection systems which may exist in the permeable rocks of the earth's crust.

In many cases, the permeability of the crustal rocks is tectonically controlled. Several known geothermal systems are controlled by deep, long, nearly vertical fault or fracture zones. Examples are found in the Basin and Range Province (Hose and Taylor, 1974) and in Iceland; and hydrothermal circulation in the ocean crust (e.g. Lister, 1972; Williams, et al., 1974) may be controlled by vertical fractures and faults (Wolery and Sleep, 1976; Lowell, 1975). These zones of high permeability may have a linear extent of tens of kilometers and a depth of several kilometers. The zones may be quite narrow, however, having widths of the order of tens of meters.

The purpose of this research is to examine convection and thermoelastic effects in long, deep, but very narrow vertical fractures spaces. The initiation and time development of the physical processes are to be studied on the basis of mathematical-physical approximation methods and finite element and/or difference methods. The results are to be interpreted with regard to known geothermal systems in the Basin and Range province and other regions where major faults are the dominant controlling structures.

This work represents a joint effort with Dr. G. Bodvarsson and T. C. Shyu of Oregon State University. Bodvarsson is mainly working on problems of convection in open fracture spaces, whereas Lowell and Shyu are investigating convection in fracture/fault zones filled with permeable material.

Thermal and Elastic Properties of Rocks

In order to develop meaningful models of thermal convection and thermoelastic phenomena in the earth's crust it is important to have reliable data on the physical properties of rocks which may be pertinent to geothermal reservoirs. In particular, it is important to know how the parameters vary with temperature and pressure as well as with porosity, permeability, and saturation with thermal fluids. A cursory review of the literature indicates that data on pertinent parameters such as thermal conductivity, thermal diffusivity, thermal expansion coefficient, and elastic parameters of rocks in a geothermal environment are quite sketchy. Theoretical understanding is also quite rudimentary.

For example, consider the thermal conductivity data. Birch and Clark (1940) have determined the thermal conductivity, K, of several rocks as a function of temperature. Their work, however, did not include the effects of porosity, fracturing or alteration. Their data and other early data on thermal conductivity of rock materials of geologic interest is reviewed by Clark (1966). On the other hand, Horai (1972) has determined K for 166 minerals at room temperature by means of the powder method. There have also been measurements on unconsolidated porous materials and sandstones saturated with water, oil, or gas (Woodside and Messmer, 1961 a,b). These measurements were made at room temperature. Manghnani _et al._ (1976) has measured the thermal conductivity of basalt as a function of temperature and porosity, but without taking into account the effect of fluid in the pore spaces. Huang (1971), Woodside and Messmer (1961a), and Brailsford and Major (1964) have given some elementary theories of the thermal conductivity of porous rock-fluid systems. There does not appear to be any laboratory data on the thermal conductivity of common igneous rocks in the neighborhood 200°C as functions of porosity, permeability and fluid content.

Even if such laboratory data were available, it may be difficult to apply to some particular geothermal systems. Our knowledge of _in situ_ porosity, permeability, and fluid content is often not well known; consequently, it would be helpful to devise means to measure the thermal conductivity _in situ_. Recently, Goss and Combs (1975) have derived an empirical relationship between the thermal conductivity and compressional seismic wave velocity. They suggest that conventional well-log data may be used to predict K better than laboratory divided-bar apparatus measurements, particularly in the case of unconsolidated rocks. Conaway (1977) also indicates that temperature gradient logs (T-logs) may be useful for determining formation thermal conductivity. Clearly, much more work is needed on the problem of determining thermal conductivity in geothermal reservoirs.

A review of other pertinent thermal and elastic parameters will be given in later reports.

Convection in Permeable Media

The basic model for convection of a fluid in a porous medium is due to Lapwood (1948). Lapwood determined the critical condition for the onset of convection in an infinite slab of homogeneous, isotropic porous material of thickness h, bounded by horizontal planes. Within the slab, a vertical conduction temperature gradient, β, was maintained. The bottom of the layer was held at temperature T_o and the upper layer at T_1, where $T_1 < T_o$. Using stability analysis, Lapwood showed that $R = \rho_f s\alpha\ \beta h^2 K/\nu\lambda \geq 4\pi^2$ for convection to occur between impermeable boundaries, where ρ_f, s, α, ν represent the density, specific heat, thermal expansivity and kinematic viscosity of the fluid, respectively; K is the permeability, λ is the thermal conductivity of the rock, and g is the acceleration of gravity. The critical Rayleigh number for other boundary conditions are given in Nield (1968); and the critical number for case of temperature dependent viscosity (Kassoy and Zebib, 1975), temperature dependent expansivity, viscosity, and density (Straus and Schubert, 1977), and anisotropic permeability (Wooding, 1976) have recently been determined.

Finite amplitude calculations of convective flows in a porous medium have been carried out by Wooding (1963), Straus (1974), Ribando et al. (1976), and Lowell and Patterson (1977). Convection in narrow, open fractures imbedded in an impermeable rock has been treated by Bodvarsson and Lowell (1972) and Lowell (1975).

Of particular interest for the situation of convection in a fault zone, however, are the various models for convection in a closed rectangular container. In this case the presence of vertical walls affects the critical number and the form of convection cells at $R = R_c$. The onset of convection of a viscous fluid in rectangular enclosures has been treated by Davis (1967), and convection in a box of water-saturated porous material has been treated by Beck (1972) and Holst and Aziz (1972). The onset of convection of a variable viscosity fluid in a porous box has been considered by Zebib and Kassoy (1977). All of the above authors have considered the special box geometry in which one horizontal dimension is much much less than the height and the other horizontal dimension (Figure 1). With the exception of Davis (1967), however, they all consider the case of insulated vertical walls. This is not realistic for a fault zone within the earth's crust. A more realistic boundary condition would be an applied geothermal gradient along the walls $y < 0$, $y > L_y$ (see Figure 1). It is also of great interest to examine finite amplitude convection models in such a geometry and to consider the effect of conductive cooling of the impermeable rock in the regions $y < 0$, $y > L_y$. It is also of some interest to examine the effect of anisotropic permeability on the onset of convection in a fault zone. Kassoy (1976) and Kassoy and Zebib (Kassoy, personal communication) are considering some related problems with regard to flow of fluid in a fault zone.

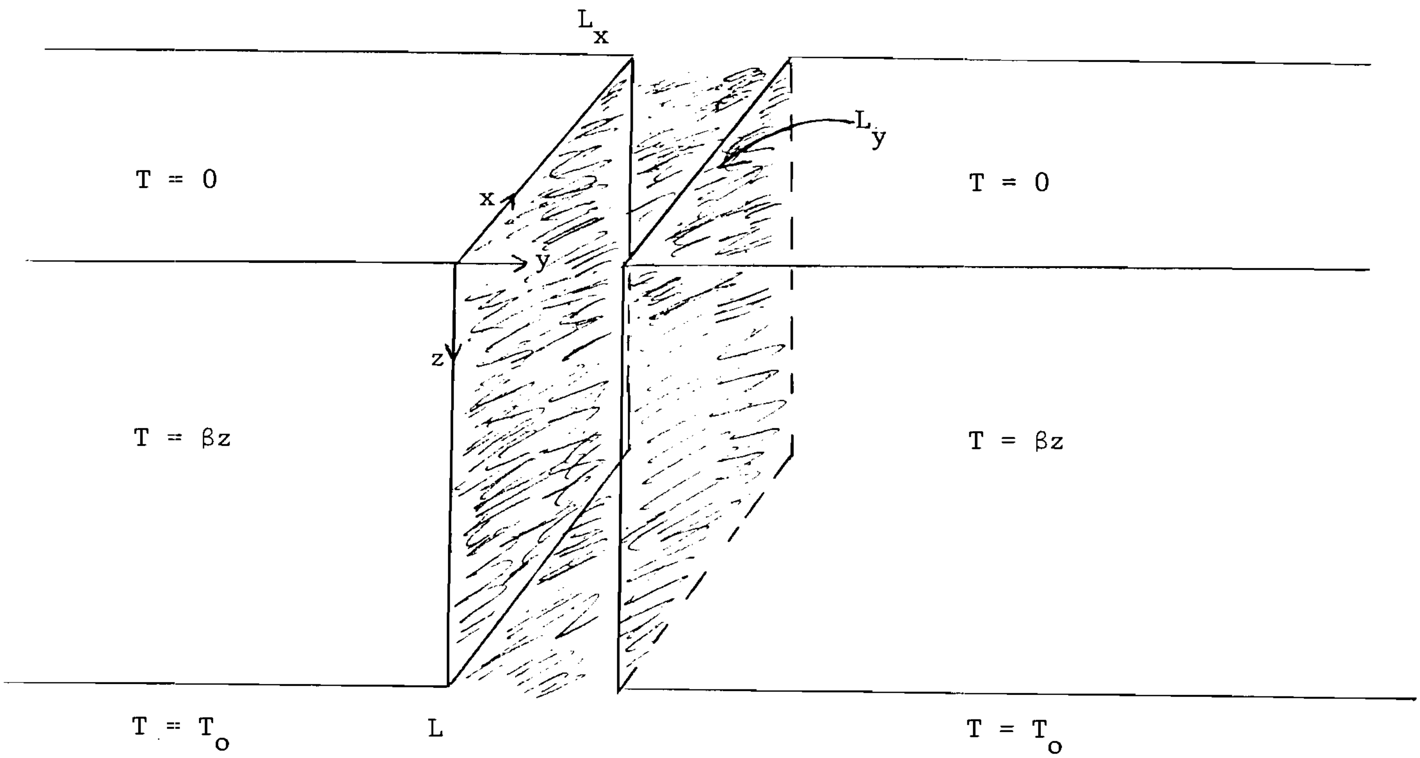

Figure 1. Model of narrow vertical fault zone.

The Onset of Convection in a Fault Zone

a. The effect of anisotropic permeability

Consider a model fault zone (Figure 1) as a rectangular slab of water-saturated, porous material imbedded in impermeable rock. Let the dimensions of the porous zone by L_x, L_y, L_z. Let a uniform geothermal gradient, β, be applied to the material such that the temperature at the upper surface is $T = 0$ and at the lower surface $T = T_o$ $(T_o > 0)$.

With z defined positively downward, the linearized, steady state perturbation equations are:

$$\lambda \nabla^2 T^* = \rho_f s w^* \beta \tag{1}$$

$$-\nabla P^* - \rho_f \alpha T^* g \hat{z} - \rho_f \nu \underline{K}^{-1} . \vec{u}^* = 0 \tag{2}$$

$$\nabla . \vec{u}^* = 0 \tag{3}$$

where the * refers to dimensional perturbation quantities and

Where all the parameters are defined as previously. w^* is the vertical velocity; P^* the pressure perturbation; $\vec{u}^*$ the velocity vector. Note that $\underline{K}$ is now treated as a tensor. Non-dimensionalizing the above equations by using L_z as a length scale, T_o as a temperature scale, and a reference mean flow $q_r = gK_o \alpha T_o / \nu_o$, where K_o, ν_o refer to values at the top of the layer; and eliminating P^* from the equations gives

$$Rw = \nabla^2 T \tag{4}$$

$$(\partial\sigma/\partial z)(\partial w/\partial z) + \{(1/\gamma_1)\partial^2/\partial x^2 + (1/\gamma_2)\partial^2/\partial z^2\}(T + \sigma w) = 0 \tag{5}$$

where $\sigma = (K_o/K_3)(\nu/\nu_o)$; $\gamma_1 = K_3/K_1$; $\gamma_2 = K_3/K_2$

and

$$R = \rho_f s\alpha\beta L_z^2 \, K_o/\nu_o\lambda$$

Equations (4) and (5) have been derived by Wooding (1976) for the case of an infinite horizontal porous layer. To determine the onset of convection in a box-like region equations (4) and (5) must be solved subject to the boundary conditions:

$$\vec{u} \cdot \hat{n} = 0 \quad \text{at all walls} \tag{6}$$

$$T = 0 \text{ at } z = 0, 1 \tag{7}$$

$$\begin{aligned} \partial T/\partial x &= 0 \text{ at } x = 0, a \qquad a = L_x/L_z \\ \partial T/\partial y &= 0 \text{ at } y = 0, b \qquad b = L_y/L_z \end{aligned} \tag{8}$$

where for simplicity insulated vertical walls are assumed. For convenience let $\sigma = 1$. This corresponds to the case in which the decrease in permeability with depth is matched by the decrease in viscosity with depth, which is not an unrealistic assumption. Substituting solutions of the form.

$$w = \hat{w}(z) \cos \alpha x \cos \beta y$$

$$T = \hat{T}(z) \cos \alpha x \cos \beta y$$

where $\alpha = m\pi x/a$, $\beta = n\pi y/b$

into equations (4) and (5) with $\sigma = 1$, and combing the resulting equations to obtain a single equation for $\hat{T}$ gives:

$$\{D^4 - D^2(\alpha^2(1 + \frac{1}{\gamma_1}) + \beta^2(1 + \frac{1}{\gamma_2})) + (\frac{\alpha^2}{\gamma_1} + \frac{\beta^2}{\gamma_2})(\alpha^2 + \beta^2 - R)\} \hat{T} = 0 \tag{9}$$

where D represents d/dz. letting $\hat{T} = \sin p\pi z$ gives an equation for R:

$$R = \frac{p^4\pi^4 + p^2\pi^2(\alpha^2(1 + 1/\gamma_1) + \beta^2(1 + 1/\gamma_1)) + (\alpha^2/\gamma_1 + \beta^2/\gamma_2)(\alpha^2 + \beta^2)}{(\alpha^2/\gamma_1 + \beta^2/\gamma_2)} \quad (10)$$

The problem is then to determine the minimum value of R for given values of γ_1, γ_2, a, b and to find the cell pattern corresponding to the minimum R. It is clear that the minimum of R occurs for p=1 and that a spectrum of critical numbers can be obtained for different values of γ_1, γ_2 and different box geometries. It should be pointed our that equation (10) reduces to the results of Beck (1967) for isotropic media by letting $\gamma_1 = \gamma_2 = 1$; and that if the box dimensions are allowed to approach infinity, (10) reduces to Wooding's (1976) result.

Rather than treat the full range of box geometries and permeability ratios, only examples representive of fault zone geometries will be considered. Furthermore, for simplicity let $\gamma_1 = \gamma_2 = \gamma$ (i.e. the horizontal permeabilities are equal). Then (10) simplifies to:

$$R = \frac{\pi^2\{\gamma^{\frac{1}{2}} + m^2/a^2 + n^2/b^2)^2 + (\gamma^{\frac{1}{2}} - 1)^2(m^2/a^2 + n^2/b^2)\}}{m^2/a^2 + n^2/b^2} \quad (11)$$

Note that (11) is symmetric with respect to a and b. For the special case of a fault zone let a > 1, b << 1. In this case a cursory examination of (11) shows that the R_{min} will occur for n = 0. That is the cell will take form of rolls with their axis parallel to the shorter side. This is in agreement with the result of Beck (1972) for an isotropic medium. Setting n = 0 in (11) gives:

$$R = \pi^2\{\gamma(1 + \frac{a^2}{m^2}) + \frac{m^2}{a^2} + 1\} \quad (12)$$

Table 1. gives the critical Rayleigh number R_c and the number of rolls contained in the fault zone for several values of anisotropy γ and for fault lengths up to 4 times the depth. The results in Table 1 show that if the horizontal permeability is greater than the vertical ($\gamma < 1$), the critical Rayleigh number is reduced substantially from the isotropic values of $4\pi^2$; and that if $\gamma > 1$, the critical Rayleigh number is increased substantially. Moreover, the results indicate a remarkable change in aspect ratio as the anisotropy changes from $\gamma < 1$ to $\gamma > 1$. For example if a = 2, there is only one roll present at $R = R_c$ for $\gamma = 0.1$; whereas there are two rolls for isotropic materials and four rolls for $\gamma = 10$.

Although the calculations have been carried out for a rigid upper boundary, the results should be similar for a free upper surface. The effect of anisotropy, in the case of a free upper surface, would be to change the discharge locations along the axis of the fault zone. If $\gamma < 1$ the hot springs would be situated much further apart than if $\gamma > 1$.

It is useful to compare these results with observations on natural fault controlled geothermal systems. The observations of Hose and Taylor (1974) on hot springs in the Basin and Range province in Northern Nevada indicate that in some cases the springs embarge at rather regular spacings along a fault zone. One such example is found in the Double Hot Springs area just west of the Black Rock Range. Here seven springs (or seeps) are spared at roughly 1.5 Km along a fault. Geochemical temperatures of some of the springs indicate that the waters circulate to depths of the order of 3 Km. Thus if this spring system is a

TABLE 1. CRITICAL RAYLEIGH NUMBERS AND MODE

FOR SEVERAL VALUES OF ANISTROPY AND FAULT ZONE LENGTH

a↓ γ→	0.01	0.1	1	10
1	$R_c = 2.02\pi^2$ $m = 1$	$R_c = 2.2\pi^2$ $m = 1$	$R_c = 4\pi^2$ $m = 1$	$R_c = 17.5\pi^2$ $m = 2$
2	$R_c = 1.3\pi^2$ $m = 1$	$R_c = 1.75\pi^2$ $m = 1$	$R_c = 4\pi^2$ $m = 2$	$R_c = 17.5\pi^2$ $m = 4$
3	$R_c = 1.21\pi^2$ $m = 1$	$R_c = 1.77\pi^2$ $m = 2$	$R_c = 4\pi^2$ $m = 3$	$R_c = 17.4\pi^2$ $m = 5$
4	$R_c = 1.23\pi^2$ $m = 1$	$R_c = 1.75\pi^2$ $m = 2$	$R_c = 4\pi^2$ $m = 4$	$R_c = 17.3\pi^2$ $m = 7$

manifestation of roll-like convection within the fault, the small aspect ratio (cell wavelength) suggests that the permeability of the fault zone is anisotropic. The vertical permeability would be greater than the horizontal.

b. Convection with conducting vertical walls

One difficulty with the work done to date on convection in a box-like porous region, including the above work on anisotropic permeability, has been the assumption of insulated vertical walls. Part of the work on this grant has been devoted to determining the onset of convection in a fault zone assuming conducting vertical walls. This is a difficult problem in that the eigenfunctions are not separable (Davis, 1967). Lowell (1977) has presented an approximate solution for this case. The results are in error, however, since the eigenfunctions obtained do not satisfy the continuity equation (i.e., the velocity conditions on the vertical wells of the fault zone). Presently, Mr. T. C. Shyu, as part of a subcontract arrangement between Georgia Tech and Oregon State, is trying to solve the problem correctly by using a Galerkin technique for determing the minimum eigenvalue (Rayleigh number) and the form of the cell pattern at $R = R_c$. At this writing the results have not yet been obtained; however, they are expected within the next few weeks.

Work Plan for Coming Report Period

The work plan for the next six month period is concerned with the accomplishment of two principal tasks. First, the review of pertinent thermal and elastic data will be completed. Second, more numerical work on the problem of convection in a fault zone filled with water-saturated porous material will be undertaken. Once the basic problem of the onset of convection in a fault zone with conducting vertical boundaries is obtained, attention will be focused on a) finite amplitude convection and b) time dependent convection in a fault zone with imperfectly conducting walls. It is hoped that useful estimate of the heat content of fault controlled geothermal systems can be obtained and that the results of the numerical calculations can be applied to known fault-controlled systems in the Basin and Range Province and elsewhere.

Respectfully submitted,

Robert P. Lowell
Principal Investigator

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G-35-623

FINAL TECHNICAL REPORT

SUBMITTED BY: Robert P. Lowell

DATE: April, 1978

TITLE: CONVECTIVE AND THERMOELASTIC EFFECTS IN NARROW VERTICAL FRACTURE SPACES WITH EMPHASIS ON NUMERICAL TECHNIQUES

GRANT PERIOD: 9/1/76 - 2/28/78

FUNDED AMOUNT: $19,100

PRINCIPAL INVESTIGATOR: Robert P. Lowell
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GRANTEE: Georgia Institute of Technology
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REPORT PERIOD: 9/1/76 to 2/28/78

This research has been sponsored by the U.S. Geological Survey as part of the Extramural Geothermal Research Program under grant no.: 14-08-0001-G-365

ABSTRACT

The energetics of geothermal systems depend in a fundamental way upon the nature of the subsurface circulation pattern of the thermal fluid. The circulation pattern in turn is largely controlled by the nature and distribution of permeability in the crustal rocks. In many cases, the permeability is not of a primary nature, that is, due to interconnected pore spaces within the rock volume. Instead, the permeability is due to fractures and faults and is tectonically controlled. For example, in the Basin and Range Province and in Iceland the geothermal systems often appear to be controlled by deep, long, nearly vertical fault or fracture zones. Hydrothermal circulation in the oceanic crust may also be mainly controlled by such tectonic features. Such zones of high permeability may have a linear extent and a depth of several kilometers, but with a width of the order of tens of meters or less.

The principal purpose of this research was to examine convection and thermoelastic effects in long, deep, but very narrow vertical fracture spaces. The initiation and time development of the physical processes were to be studied on the basis of mathematical-physical approximation methods as well as finite difference and/or finite element methods. The results were to be interpreted with regard to known geothermal systems where faults and fractures are the dominant controlling structures.

Several useful results on convection in fault/fracture zones have been obtained; however, the research must be considered to have been only partially successful. The emphasis has been mainly on determining conditions for the onset of thermal convection under various types of physical conditions rather than on the evolutionary and finite amplitude aspects of the convective systems. For example, the condition for the onset of convection in a box filled with water-saturated anisotropic porous material has been determined. The results

show that the critical Rayleigh number and the cell wavelength depend strongly on the box dimensions and the degree of anisotropicity. Furthermore, the effect of conducting vertical walls is to greatly increase the critical Rayleigh number for boxes with fault zone geometry.

The research has also entailed a review of pertinent thermal properties of rocks as well as a study of a thermal problem related to transient temperature inversions in geothermal boreholes. The principal results of this work are that a) much more data is required on the thermal properties of rocks, particularly with regard to their setting in a geothermal environment and b) transient temperature inversions may provide useful information on the fluid conductivity of geothermal reservoirs.

I. Introduction

This document is the final technical report on our research on convection in fault and fracture zones and related problems. This work represents a joint effort with Dr. Gunnar Bodvarsson of Oregon State University. Bodvarsson has worked principally on problems related to convection and thermoelastic effects in hydrothermal systems controlled by narrow, vertical, fluid filled open fractures. Bodvarsson has stressed the evolutionary aspects of such systems and has applied his results to the systems in the Basin and Range. The Georgia Tech group has been more concerned with the onset of convection in confined zones of water-saturated porous material, also as might be applicable to the Basin and Range.

There has also been a subcontract agreement between Georgia Tech and Oregon State under which Mr. T.C. Shyu, a graduate student at OSU,has been supported. He has performed several of the numerical calculations on the conditions for the onset of convection. In addition to Mr. T.C. Shyu, Mr. D. Synnes and Ms. P.L. Patterson have been supported under this research grant.

All of the pertinent results of this research have been detailed in preceding technical reports or are presented below. Work which has been described fully in preceding reports is summarized here.

II. Thermal Properties of Rocks

In order to develop meaningful models of thermal convection and thermoelastic phenomena in the earth's crust, it is necessary to have reliable data on the physical properties of rocks which may be pertinent to geothermal systems. In particular, it is important to know how the parameters vary with temperature, pressure, porosity, amount of fluid content, etc. A cursory review of the literature, however, indicates that data on parameters such as thermal conductivity and thermal expansion coefficient of rocks in a geothermal environment are quite sketchy. Theoretical understanding is also quite rudimentary.

A. Thermal Conductivity

The data on thermal conductivity were reviewed in an earlier technical report (Lowell, 1977 b). The principal conclusions to be drawn from the survey of current literature are a) there is little or no laboratory data on the thermal conductivity of common igneous rocks in the neighborhood of 200°C as functions of permeability, porosity and fluid content, b) empirical relationships between thermal conductivity and P-wave velocity may be used to determine *in situ* thermal conductivity.

B. Thermal Expansion Coefficient

One of the fundamental thermal properties to be considered in the modeling of geothermal systems is the fractional volume increase which a rock undergoes as its temperature is raised. This parameter is termed the coefficient of thermal expansion. Thermal expansion is of particular importance with regard to various physical phenomena such as a) subsidence related to injection of cold water into heated rock, b) closing of fractures by flow of heated fluids, c) migration of and channeling in fractures as a result of cool circulating fluids, d) triggering of earthquakes as a result of thermoelastic stresses. Some simple examples of such thermoelastic effects have been given by Bodvarsson (1976) and in his companion report on this joint work (Bodvarsson 1978).

Laboratory data on the thermal expansion coefficient of typical igneous rocks has only recently become available. This data is primarily from G. Simmons and his colleagues at MIT, for his results indicate the inaccuracy and misinterpretation which were inherent in earlier measurements (e.g., Richter and Simmons, 1974; Cooper and Simmons, 1977).

Their measurements show that high heating rates (greater than 2°C/min.) cause thermal fractures and hence increased volume expansion; and that even at low heating rates, rocks that undergo several heating and cooling cycles show a hysteresis effect. That is, the rocks expand more on the first heating cycle than on subsequent ones. The hysteresis effect is also related to microfractures (Cooper and Simmons, 1977). Their results also indicate that the thermal expansion coefficient decreases linearly with crack porosity in rocks which are cracked in their "virgin" state The reason for this correlation is apparently that the respective mineral grains expand into the fracture space, thus lowering the overall expansion of the rock compared to the amount of expansion of the individual component minerals. Lastly, the thermal expansion coefficient of some igneous rocks is quite strongly temperature dependent-- the coefficient may increase by a factor of 3 or 4 in heating from 25°C to 400°C.

Crack formation and the hysteresis effect may be of fundamental importance with regard to thermoelastic processes in geothermal systems. These thermal expansion effects are not taken into account in the simple models of geothermal systems which have been developed so far, but it must be admitted that, because of confining pressure, the importance of crack formation in a natural field setting may be somewhat reduced. On the other hand, the percent of water saturation may have a significant effect on the thermal expansivity of cracked rocks (Hockman and Kessler 1950).

Clearly much more data is needed on the thermal expansion properties of rocks, particularly with regard to their setting in a geothermal environment.

III. The Onset of Convection in a Fault Zone

The basic model for convection of a fluid in a porous medium is due to Lapwood (1948). Lapwood determined the critical condition for the onset of convection in an infinite slab of homogeneous, isotropic porous material of thickness h, bounded by horizontal planes. Within the slab, a vertical conduction temperature gradient, β, was maintained. The bottom of the layer was held at temperature T_o and the upper layer at T_1, where $T_1 < T_o$. Using stability analysis, Lapwood showed that $R = \rho_f s\alpha\ \beta h^2 K/\nu\lambda \geq 4\pi^2$ for convection to occur between impermeable boundaries, where ρ_f, s, α, ν represent the density, specific heat, thermal expansivity and kinematic viscosity of the fluid, respectively; K is the permeability, λ is the thermal conductivity of the rock, and g is the acceleration of gravity. The critical Rayleigh number for other boundary conditions are given in Nield (1968); and the critical number for case of temperature dependent viscosity (Kassoy and Zebib, 1975), temperature dependent expansivity, viscosity, and density (Straus and Schubert, 1977), and anisotropic permeability (Wooding, 1976) have recently been determined.

Finite amplitude calculations of convective flows in a porous medium have been carried out by Wooding (1963), Straus (1974), Ribando *et al*. (1976), and Lowell and Patterson (1977). Convection in narrow, open fractures imbedded in an impermeable rock has been treated by Bodvarsson and Lowell (1972) Lowell (1975), and Murphy, (1978); and some additional work on convection in fractures has been carried out by Bodvarsson (1978) as part of this joint research project.

The models of convection in infinite horizontal slabs do not carry over to the geothermal setting in which the fluid conductivity is controlled by fault and/or fracture zones. Examples of such systems are found in the Basin and Range (Hose and Taylor, 1974) in Iceland, and, perhaps, in the oceanic crust

(Lowell, 1975; Wolery and Sleep, 1976); and it is these types of systems which have been emphasized in the present study of convection in a porous medium.

Of particular interest for the situation of convection in a fault or fracture zone, however, are the various models for convection in a closed rectangular container. In this case the presence of vertical walls affects the critical number and the form of convection cells at $R = R_c$. The onset of convection of a viscous fluid in rectangular enclosures has been treated by Davis (1967), and convection in a box of water-saturated porous material has been treated by Beck (1972) and Holst and Aziz (1972). The onset of convection of a variable viscosity fluid in a porous box has been considered by Zebib and Kassoy (1977). All of the above authors have considered the special box geometry in which one horizontal dimension is much less than the height and the other horizontal dimension (Figure 1). With the exception of Davis (1967), however, they all consider the case of insulated vertical walls. This is not realistic for a fault zone within the earth's crust. A more realistic boundary condition would be an applied geothermal gradient along the walls $y< 0$, $y> L_y$ (see Figure 1). It is also of great interest to examine finite amplitude convection models in such a geometry and to consider the effect of conductive cooling of the impermeable rock in the regions $y< 0$, $y> L_y$. It is also of some interest to examine the effect of anisotropic permeability on the onset of convection in a fault zone. Kassoy (1976) and Zebib and Kassoy (Kassoy, personal communication) are considering some related problems with regard to flow of fluid in a fault zone.

A. The Effect of Anisotropic Permeability

In order to introduce the pertinent equations, consider the fault zone (Figure 1) as a rectangular slab of water-saturated porous material imbedded in impermeable rock. Let the dimensions of the porous zone by L_x, L_y, L_z. Let a uniform geothermal gradient, β, be applied to the material such that the temperature at the upper surface is T = 0 and at the lower surface $T = T_o$ ($T_o > 0$).

With z defined positively downward, the linearized, steady state perturbation equations are:

$$\lambda \nabla^2 T^* = \rho_f s w^* \beta \tag{1}$$

$$-\nabla P^* - \rho_f \alpha T^* g \hat{z} - \rho_f \nu \underline{K}^{-1} . \vec{u}^* = 0 \tag{2}$$

$$\nabla . \vec{u}^* = 0 \tag{3}$$

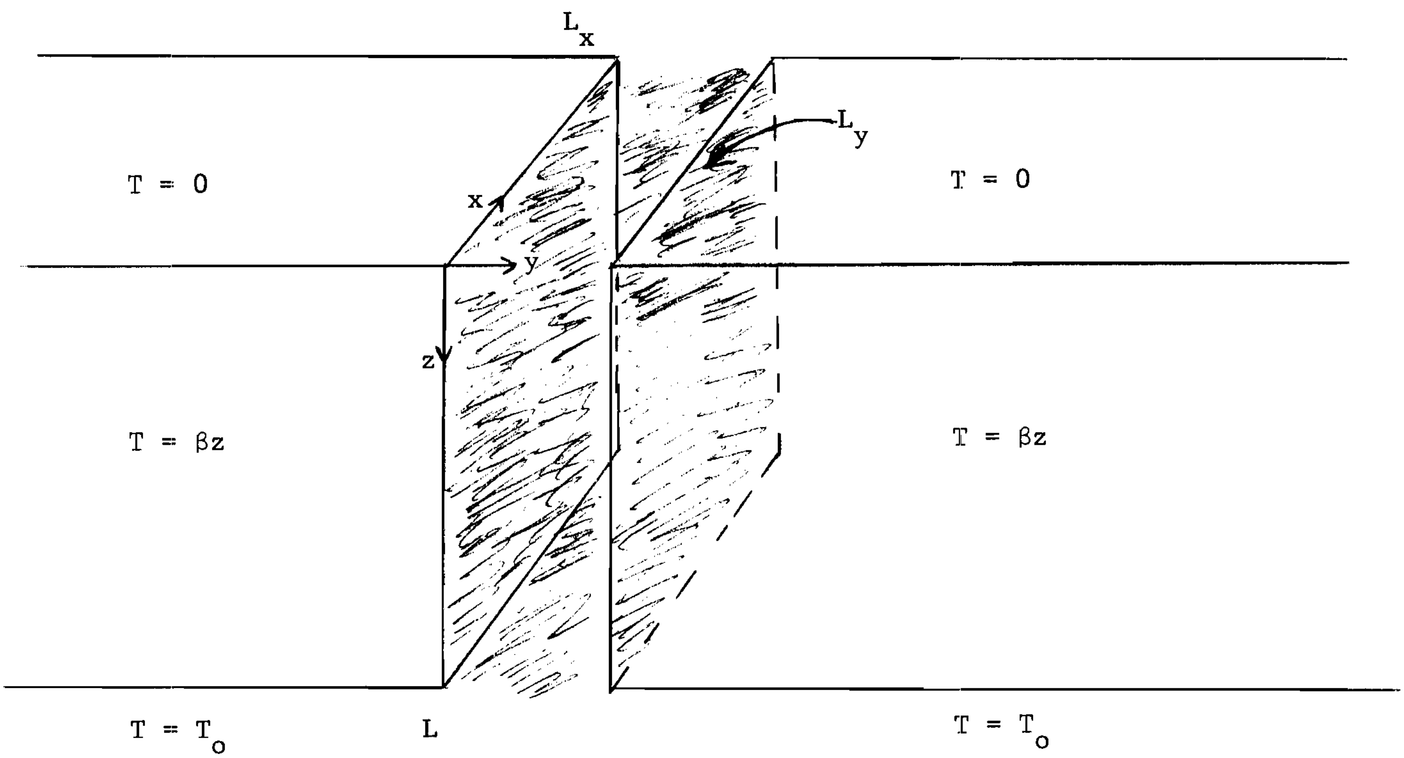

Figure 1. Model of Narrow Vertical Fault Zone

where the * refers to dimensional perturbation quantities and

Where all the parameters are defined as previously. w^* is the vertical velocity; P^* the pressure perturbation; $\vec{u}^*$ the velocity vector. Note that $\underline{K}$ is now treated as a tensor. Non-dimensionalizing the above equations by using L_z as a length scale, T_o as a temperature scale, and a reference mean flow $q_r = gK_o\alpha T_o/\nu_o$, where K_o, ν_o refer to values at the top of the layer; and eliminating P^* from the equations gives

$$Rw = \nabla^2 T \tag{4}$$

$$(\partial\sigma/\partial z)(\partial w/\partial z) + \{(1/\gamma_1)\partial^2/\partial x^2 + (1/\gamma_2)\partial^2/\partial z^2\}(T + \sigma w) = 0 \tag{5}$$

where $\sigma = (K_o/K_3)(\nu/\nu_o)$; $\gamma_1 = K_3/K_1$; $\gamma_2 = K_3/K_2$

and $R = \rho_f s\alpha\beta L_z^2\, K_o/\nu_o\lambda$

Equations (4) and (5) have been derived by Wooding (1976) for the case of an infinite horizontal porous layer. To determine the onset of convection in a box-like region equations (4) and (5) must be solved subject to the boundary conditions.

$$\vec{u} \cdot \hat{n} = 0 \quad \text{at all walls} \tag{6}$$

$$T = 0 \text{ at } z = 0,\ 1 \tag{7}$$

$$\begin{aligned} \partial T/\partial x &= 0 \text{ at } x = 0,\ a \qquad a = L_x/L_z \\ \partial T/\partial y &= 0 \text{ at } y = 0,\ b \qquad b = L_y/L_z \end{aligned} \tag{8}$$

where for simplicity insulated vertical walls are assumed. For convenience let $\sigma = 1$. This corresponds to the case in which the decrease in permeability with depth is matched by the decrease in viscosity with depth, which is not an unrealistic assumption.

The solution to equations (4) and (5) are subject to conditions (6)-(8) were solved and discussed for some special cases (i.e., $\gamma_1 = \gamma_2 = \gamma$; $a > 1$; $b << a$) in Lowell (1977b). This work will not be reproduced here; however, Table 1 gives the main results for a fault zone geometry. In Table 1, m corresponds to the mode or number of convective rolls within the fault zone. The rolls have their axis parallel to the short side of the box.

B. Convection with Conducting Vertical Walls

A principal difficulty in applying the results on convection in a confined porous region, including the above results on anisotropic permeability, to realistic geophysical situations has been the assumption of perfectly insulated vertical walls. In real-earth situations, one would expect there to be lateral heat transfer between the porous, convecting region and the impermeable rock adjacent to it. In general, one would expect this heat transfer to be imperfect; however, in the interests of simplicity, the case of conducting walls has been treated. For the onset of convection in the case of imperfectly conducting walls, R_c should lie somewhere between that for insulating and that for conducting walls.

For the case of conducting walls, the second boundary condition (8) is replaced by

$$T = 0; \; y = 0, b \qquad (9)$$

The determination of R_c in the case of conducting walls is a difficult problem because the eigenfunctions are not separable (Davis, 1967).

1. Approximate Results

Approximate solutions can be obtained, however, if one relaxes the velocity

TABLE 1. CRITICAL RAYLEIGH NUMBERS AND MODE

FOR SEVERAL VALUES OF ANISTROPY AND FAULT ZONE LENGTH

a↓ γ→	0.01	0.1	1	10
1	$R_c = 2.02\pi^2$ $m = 1$	$R_c = 2.2\pi^2$ $m = 1$	$R_c = 4\pi^2$ $m = 1$	$R_c = 17.5\pi^2$ $m = 2$
2	$R_c = 1.3\pi^2$ $m = 1$	$R_c = 1.75\pi^2$ $m = 1$	$R_c = 4\pi^2$ $m = 2$	$R_c = 17.5\pi^2$ $m = 4$
3	$R_c = 1.21\pi^2$ $m = 1$	$R_c = 1.77\pi^2$ $m = 2$	$R_c = 4\pi^2$ $m = 3$	$R_c = 17.4\pi^2$ $m = 5$
4	$R_c = 1.23\pi^2$ $m = 1$	$R_c = 1.75\pi^2$ $m = 2$	$R_c = 4\pi^2$ $m = 4$	$R_c = 17.3\pi^2$ $m = 7$

boundary conditions on the vertical walls and assumes a fault zone of infinite length. Lowell (1977a) has found separable solutions to equations (4) and (5) (with $\gamma_1 = \gamma_2 = \sigma = 1$) subject to conditions (7) and (9) and

$$w = 0 \quad \text{at} \quad z = 0,1$$

$$\frac{dw}{dz} = 0 \quad \text{at} \quad y = 0,b$$

Lowell (1977a) has shown that

$$R_c \simeq 4\pi^2/b^2 \qquad (b<<1)$$

which is much greater than the value $R_c = 4\pi^2$ determined by Lapwood (1948) for an infinite horizontal slab. For example, if $b = 0.01$, $R_c \simeq 4 \times 10^5$. The approximate results also show that the flow at R_c takes place as a roll with its axis along the strike of the fault. This is opposite to the roll orientation in the case of insulated walls (Beck, 1972; Lowell, 1977b).

2. Galerkin Results

In order to find accurate results for the onset of convection, using the full boundary conditions, the Galerkin method was used. The work was carried out by Mr. T.C. Shyu, a graduate student under Dr. G. Bodvarsson at Oregon State University. The work presented below was performed under a sub-contract agreement between Georgia Tech and Oregon State.

To apply the Galerkin method one combines equations (4) and (5) to form a single equation for the perturbation temperature T. With $\gamma_1 = \gamma_2 = \sigma = 1$ this gives

$$\nabla^4 T = -R\nabla_h^{\ 2} T \tag{10}$$

where $\nabla_h^{\ 2}$ is the horizontal Laplacian. One then chooses a set of trial functions T_i which satisfy the boundary conditions and solves equation (10) using variational techniques. That is, one solves

$$\int(\nabla^2(\nabla^2 T) + R\nabla_h^2 T)\cdot T_k \, dV = 0 \tag{11}$$

where the integral is taken over the volume of the box. In (11) T is replaced by trial functions:

$$T = \sum_{i=1}^{N} C_i T_i \tag{12}$$

and T_K is one component of the set of trial functions (12). Substitutions of the trial functions leads to a matrix equation of the form

$$M_A \vec{C} = R M_B \vec{C} \tag{13}$$

and the condition for non-trivial solution requires that the determinant of the coefficients be zero. That is:

$$|M_A^{-1} M_B - R^{-1} I| = 0 \tag{14}$$

Equation (14) represents an eigenvalue problem for the Rayleigh number R and the minimum eigenvalue found, $R = R_c$, represents the critical condition for the onset of convection. We choose trial functions of the form

$$\begin{aligned} T_{ijk} = \big[& (b/k(j-1))((i/a)^2 + (j-1/b)^2 + k^2)\cos(i\pi x/a)\sin((j-1)\pi y/b)\cos k\pi z \\ & -(b/k(j+1))((i/a)^2 + (j+1/b)^2 + k^2)\cos(i\pi x/a)\sin((j+1)\pi y/b)\sin k\pi z \big] \end{aligned} \tag{15}$$

and the results for the critical Rayleigh number, for different box dimensions, are given in Table 2.

Table 2 shows several interesting features of convection in a box with two conducting walls. First, it is clear that for large box dimensions, the wall conditions become immaterial and the critical number approaches the value $4\pi^2$ given by Lapwood

TABLE 2

Critical Rayleigh Numbers for 2-D and 3-D Convection in a Porous Box With Two Vertical Conducting Walls

b↓	2-D (i=0)	3-D, a → 1	5	10	20	50
0.01	878644.0	878652.0	878645.0	878645.0	878645.0	878645.0
0.05	35163.1	35170.4	35163.4	35163.2	35163.1	35163.1
0.1	8804.31	8811.60	8804.61	8804.39	8804.34	8804.32
0.5	369.68	375.91	369.95	369.75	369.70	369.69
1.0	106.95	110.48	107.15	107.00	106.97	106.96
1.5	60.50	61.73	60.58	60.52	60.50	60.50
3	43.57	40.85	40.86	40.61	40.66	40.66
5	41.01	39.61	39.62	39.62	39.60	39.60
10	39.87	39.48	39.48	39.49	39.49	39.49
50	39.48	39.48	39.48	39.48	39.48	39.48

(1948) for an infinite slab. Secondly, if the conducting walls are far apart ($b \gtrsim 2$), three dimensional motion is preferred over two dimensional at the onset of convection; and as the separation between the insulated walls increases ($a \rightarrow \infty$), the three-dimensional values of R_c approach the two-dimensional values. This suggests that in natural systems the motions may tend to be fully three-dimensional rather than roll-like in character. Moreover, those results suggest that at finite amplitude (i.e., $R > R_c$) the two-dimensional solutions may be unstable and the motion at finite amplitude may become three-dimensional. Lastly, in the case of a fault/fracture zone geometry ($b<<1$, $a \geq 1$). Table 2 shows that R_c is several orders of magnitude greater than the value $4\pi^2$ for for an infinite slab, and the motion at $R = R_c$ takes the form of a roll with its axis parallel to the long horizontal dimension of the box. This results in agreement with the approximate analytical results of Lowell (1977 a). The Galerkin method give R_c about a factor of 2 greater than the analytical method. This is as expected since the boundary conditions satisfied in the analytical methods were somewhat less restrictive than the actual conditions.

These results are being submitted for publication in the Journal of Heat Transfer.

IV. Transient Temperature Inversions in Geothermal Boreholes

A problem of a somewhat different nature than the convection problems treated above, but a problem of great practical importance, concerns the problem of temperature inversions in geothermal systems. Such temperature inversions, that is, regions where the temperature decreases with depth, may be of useful diagnostic value with regard to the structural aspects and fluid conductivity of a geothermal reservoir. Bodvarsson (1973) has discussed several causes of inversions; however, he neglected to examine transient temperature inversions which may result from fluid losses into permeable formations during the drilling process. Some data from the Reykir geothermal area in Iceland have been examined in the course of this research, and some simple models for interpretation of the data have been developed. The principal results of this study are: 1) if drilling fluid losses occur into thin horizontal fractures, small but detectable temperature inversions are set up in the neighborhood of the fracture; 2) the decay of such inversions after drilling ceases may take place by thermal conduction and disappear on a time scale comparable to the relaxation of other drilling disturbance effects; 3) rapid decay of temperature inversions in thick, permeable formations is indicative of transverse flow in the formation penetrated by the borehole; 4) the rate of decay of such broad temperature inversions may be used to indicate flow rates and, possibly, permeabilities in the formations; 5) repeated temperature logging of geothermal boreholes as they return to equilibrium can be of great assistance in accurate analysis of transient temperature inversions resulting from drilling fluid losses.

The details of this work will be presented at the Geothermal Resources Council Annual Meeting in Hilo, Hawaii, July 25-26, 1978. A copy of the extended summary of this work is attached as an Appendix to this report.

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V. Publications and Papers

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Lowell, R.P. and T.C. Shyu, 1978, The onset of convection in a box fluid with water-saturated porous material: the effect of conducting walls (to be submitted to J. of Heat Transfer).

VI. Thesis

The research performed on this grant will form a part of Mr. T.C. Shyu's Ph.D. dissertation at Oregon State University.

VII. Reports

Program Management Reports:

Submitted quarterly on: 12/15/76; 3/15/77; 6/15/77; 9/15/77; 12/15/77

Technical Reports:

Lowell, R.P., 1977, Convection and thermoelastic effects in narrow, vertical fracture spaces with emphasis on numerical techniques, Semi-annual technical letter report no. 1, USGS grant #14-08-001-G-365, 10 p.

Lowell, R.P., 1977, Convection and thermoelastic effects in narrow, vertical fracture spaces with emphasis on numerical techniques, Semi-annual technical report no. 2, USGS grant #14-08-001-G-365, 13 p.

APPENDIX

ON TRANSIENT TEMPERATURE INVERSIONS IN GEOTHERMAL BOREHOLES

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Introduction

Normally, the temperature increases with depth in the earth's crust. In geothermal areas, however, temperature profiles often show a rapid increase with depth over the first few hundred meters followed by a nearly isothermal region; and in many cases, there exist temperature inversions, regions in which the temperature decreases with depth. Healy (1971) has observed temperature inversions from the El Tatio geothermal area in Northern Chile, and Bodvarsson and Palmason (1964) have shown some examples of temperature inversions in Iceland. Bodvarsson (1973) has discussed several possible causes for such temperature inversions and has derived some simple models to show the diagnostic value of such inversions with regard to the structural and flow characteristics of geothermal systems.

There is one class of temperature inversion which has not been discussed by Bodvarsson (1973). This involves transient temperature inversions due to drilling disturbances. Very often in drilling a geothermal well, drilling fluid is lost from the wellbore into the adjacent rock formation. Since the drilling fluid is typically much cooler than the rock, injection of the drilling fluid will cool the rock. Temperature logs taken shortly after the injection of drilling fluid will show a region containing a temperature decrease with depth. After drilling ceases, the borehole will gradually return to equilibrium, and the artificially induced temperature inversion will disappear. An example of such a case is given in Figure 1.

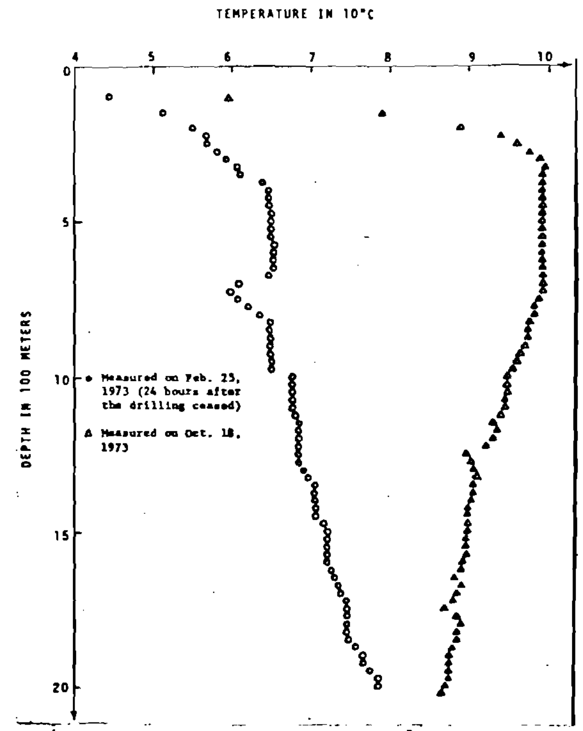

Figure 1 Temperature data from borehole MG-16 in the Reykir geothermal area, Iceland

The purpose of this paper is to give some simple models for the generation of temperature inversions which may result from the injection of drilling fluid into a formation and the subsequent relaxation of such inversions after drilling ceases.

Injection of Fluid Into Thin, Horizontal Fractures

Often in geothermal areas, the bulk permeability of the rock formations is quite low, and the permeability is due principally to narrow fractures. A good example of this is the Hengill geothermal area in Iceland where there appears to be just a few narrow, widely-spaced, nearly horizontal contacts which act as fluid conductors (Bodvarsson, 1970).

It is useful to model the temperature inversion which would result from the loss of drilling fluid into a thin fracture. To consider this problem in an approximate way, we will neglect the usual drilling disturbance associated with energy dissipation due to drilling and fluid circulation through the drill stem and annulus. We will treat the disturbance simply as a result of drilling fluid being injected into the formation at a given flow rate and temperature. The mathematical formulation of the problem is then similar to that involved in thermal reinjection wells (Bodvarsson, 1972). Let r be the radial distance from the borehole, and z be the coordinate perpendicular to the fracture, which is located at $z = 0$. Neglecting heat conduction in the rock in radial direction, the pertinent heat transfer equation for the rock is

$$a\partial^2 T/\partial z^2 = \partial T/\partial t \qquad (1)$$

where a is the thermal diffusivity and T the temperature of the rock. The heat transfer by the fluid in the fracture is considered to be a boundary condition at $z = 0$. This condition takes the form

$$sq\partial T/\partial r = 4\pi r\lambda \partial T/\partial z|_{z=0} \qquad (2)$$

where s is the specific heat, q the mass flow rate of the fluid, and λ the thermal conductivity of the rock. Let the fluid enter the formation at temperature T = 0 and let the formation at time t = 0 be at $T = T_o$. The solution to these equations has been given by Bodvarsson (1972) as

$$T = T_o \mathrm{erf}[(2\pi\lambda r^2/sq + z)/2\sqrt{at}] \qquad (3)$$

Equation (3) can be used to estimate as a function of time the vertical distance along the borehole wall and the radial distance into the rock over which the temperature field is appreciably disturbed. To estimate the temperature disturbance along the borehole wall, we note that R, the borehole radius is small such that $2\pi\lambda R^2/sq \doteq 10^{-5}$m for reasonable parameters. Consequently, we can take the limit as $r \to o$ and note that the disturbance along the borehole wall varies as

$$T = T_o \mathrm{erf}(z/2\sqrt{at}) \qquad (4)$$

Since we may fix the ratio T/T_o, at which the disturbance is assumed to be negligible, (4) shows that the maximum vertical extent of the disturbance varies as $t^{\frac{1}{2}}$. For example, a drilling time of the order of 1 month gives a temperature inversion of less than 10 meters width. The total zone of temperature disturbance at the borehole wall is twice this amount. A continuous temperature log made a short time after drilling ceases would show a temperature pertubation indicating the loss of drilling fluid into the formation. Since the perturbed zone is thin (~20 m), however, a temperature log taken at discrete intervals of 10 m or more may miss the temperature inversion.

Relaxation of the Drilling Disturbance

After drilling ceases, the borehole tends to return to equilibrium. For the situation in which the temperature perturbation has been due to the circulation of drilling fluid through the drill stem annulus and the heat generated by the drill stem and drill bit, there have been approximate calculations of the time required for the borehole to equilibrate. Bullard (1947) has shown that the time required for a return to within 1% of equilibrium is a factor of 10 or more longer than the time after which the drill bit passed a given depth. There have been no calculations of the relaxation of a drilling disturbance in the situation where drilling fluid has been lost to the formation.

To estimate the time required for the dissipation of the temperature disturbance due to drilling fluid loss into a thin fracture, we consider the cooling of an infinite region in which the initial condition is given by equation (3) with $t = t_o$, where t_o is the drilling time. Analytical solution of the conduction equation with this initial condition is somewhat tedious (Chen (1975) gives an approximate analytical treatment); consequently, we approximated the initial condition on a discrete finite difference grid and solved the heat conduction equation using finite differences. As an example, we let $t_o = 10^6$ sec. The time for the temperature to return to within 1% of equilibrium at the point of maximum temperature disturbance was approximately 2.2×10^7 sec., more than a factor of 20 larger than the drilling time. This is of roughly the same order as the time estimated by Bullard (1947) for the relaxation of a drilling disturbance in the absence of fluid losses.

Large Scale Fluid Losses

Fluid losses during drilling do not always take place by flow in thin fractures. In some cases, fluid is lost into thick, permeable formations. For example, the zone of anomalous temperatures in the borehole in Figure 1 is approximately 200 m thick. This region of anomalous temperature is known from drilling records to be a zone in which more than 40 kg/sec was lost into the formation. The large width of the temperature inversion precludes the possibility that the fluid losses took place in a thin fracture; but what is more striking is the rapid relaxation of the temperature disturbance. Figure 1 shows that the temperature inversion has disappeared within 8 months after drilling ceased. Such rapid relaxation, over such a thick zone, could not have been due to thermal conduction. The temperature disturbance must have been removed by convective processes.

In order the consider this problem quantitatively, we consider first the problem of fluid loss into the permeable formation. Since the flow rate is so great, we consider heat transport in the permeable formation as a purely convective phenomenon. Following Bodvarsson (1972) we write heat transport equation as:

$$\partial T/\partial t + (s/\rho c)\vec{q}\cdot\nabla T = 0 \qquad (5)$$

where ρc is the heat capacity of the rock and the other terms are as previously defined. The solution to (5) in cylindrical coordinates gives a temperature field which is translated radially into the solid. The radius r of the advance of the front is (Bodvarsson, 1972)

$$r = \sqrt{sM/\pi\rho ch} \qquad (6)$$

where M = qt, q being the mass rate of drilling fluid loss and t the time. h is the thickness of the formation. For the present example, $t \approx 3 \times 10^6$ sec, $q \approx 40$ kg/sec, $h \approx 200$ m. This gives $r \approx 15$ m. This is probably an underestimate of r, since it is not likely that the fluid is lost uniformly over the 200 m thickness of the formation.

We will assume that the borehole returns to equilibrium as a result of horizontal sheet flow in the permeable layer. Once the cold temperature front resulting from the drilling fluid loss passes the borehole wall, equilibrium is restored. The borehole temperature data show that less than 8 months or approximately 2.4×10^7 sec was required for equilibrium to be attained. The minimum velocity required in the formation is therefore given by

$$v \leq (\rho c/\rho_f s) x/t \qquad (7)$$

where x is the horizontal distance between the temperature front and the borehole wall. Inserting x = 15 m and t = 2.4 X 10^7 sec, and $\rho c/\rho_f s$ = 0.9 gives v = 5.6 X 10^{-7} m/sec. which is a rather reasonable velocity. For example, assume that the flow is driven by a pressure gradient mainly of convective origin of $-\partial P/\partial x = 10^3$ Nt/m^3. Applying Darcy's law

$$\rho v = -K/\nu \, \partial P/\partial x \qquad (8)$$

where K is the permeability and ν the kinematic viscosity; ν = 0.3 X 10^{-6} m^2/s gives K = 0.18 Darcy. This is a rather high but not unrealistic permeability for a single permeable layer. If this layer gives rise to the main horizontal permeability over the upper 2000 m of the thermal reservoir, the bulk horizontal permeability is roughly (0.18) X (200/2000) = 0.02 Darcy, which is in line with gross scale horizontal permeability estimated for the Reykjavik temperature area by Palmason (1967).

Conclusions

In most cases borehole temperature data is taken in order to estimate equilibrium subsurface temperatures. Either equilibrium data is used or techniques are applied to non-equilibrium data in order to estimate equilibrium temperatures. Anomalous data are often discarded. In some cases, however, anomalous temperature data may be of significant diagnostic value. The above example of a transient temperature inversion due to drilling fluid penetrating a permeable rock formation is a case in point. It may be possible to determine the permeability of formations or to isolate thin zones of high permeability on the basis of observed temperature inversions in boreholes. It is clear that in the example discussed here, that the data is much too sketchy to permit more than a very crude estimate of the flow rate and formation permeability. The data discussed above serve primarily as an example of how transient temperature inversions may be utilized to obtain information about the permeability structure of geothermal reservoirs. It would be useful to take frequent logs as the borehole returns to equilibrium. This would enable one to make a much more accurate treatment of the problem.

Acknowledgments

The senior author thanks Dr. Jens Tomasson of the National Energy Authority, Reykjavik, Iceland for making the borehole data available and for useful discussions regarding borehole data. The senior author also thanks the Georgia Tech Foundation for travel support for the trip to Iceland which made these discussions possible. This research has been supported in part by the U.S. Geological Survey, Department of the Interior, under USGS Grant No. 14-08-001-G-365 and in part by the National Science Foundation, Division of Ocean Sciences under NSF Grant No. OCE-76-81876.

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