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Convergence in European GDP series:
a multivariate common converging trend-cycle decomposition

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Abstract

Convergence in gross domestic product series of five European countries is empirically identified using multivariate time series models that are based on unobserved components with dynamic converging properties. We define convergence in terms of a decrease in dispersion over time and model this decrease via mechanisms that allow for gradual reductions in the ranks of covariance matrices associated with the disturbance vectors driving the unobserved components of the model. The inclusion of such convergence mechanisms within the formulation of unobserved components makes the identification of various types of convergence possible. The common converging component model is estimated for the per capita gross domestic product of five European countries: Germany, France, Italy, Spain and the Netherlands. It is found that convergence features in trends and cycles are present and are associated with some key events in the history of European integration.

Keywords: Common trends and cycles; Dynamic factor model; Economic convergence; Kalman filter; Multivariate unobserved components time series models.

JEL classification: C13, C32, E32.

1 Introduction

The European economy has become more integrated in the last twenty-five years due to economic, political and institutional factors. Some key events in the recent history of Europe are the establishment of the Exchange Rate Mechanism in 1979, the entry of Spain to the European Union in 1986, the opening of the Common Market in 1993 and the introduction of the Euro in 2001. These events, together with a worldwide increase in trade and financial flows, have led to a closer synchronization of economic fluctuations across European countries. It is therefore of particular interest to investigate whether the cyclical components of gross domestic product series, which are closely related to the

business cycle, are evolving more closely over time as a result. One of the challenges we have set ourselves is to empirically identify country-specific cycles that are converging to a smaller number of common factors. This important task is of particular relevance to economic and monetary policy makers.

To investigate the existence of converging properties in economic time series we adopt unobserved components time series (UC) models that typically consist of interpretable components such as trend, cycle, seasonal and irregular components. Each component is separately modelled by an appropriate dynamic stochastic process which usually depends on normally distributed disturbances. The UC model with trend and cycle components enables a time series decomposition that is appropriate for many macroeconomic time series such as consumption, investment and national income. In a univariate time series analysis this model leads to a trend-cycle decomposition and can be regarded as a model-based alternative to analyses based on the Hodrick-Prescott filter of Hodrick and Prescott (1980) and the Beveridge-Nelson decomposition of Beveridge and Nelson (1981). Harvey and Jaeger (1993) have argued that a model-based trend-cycle decomposition for economic time series is to be preferred and can avoid the detection of spurious cycles in the time series. Working with UC models has the additional advantage that they can also be used for producing forecasts.

The multivariate extension of the UC model can be used for simultaneous decompositions of a group of related time series. As a result the components become vectors defined by stochastic functions of vector disturbances generated by multivariate distributions. In the case of Gaussian models, disturbance densities rely on variance matrices rather than scalar variances. The application of non-diagonal variance matrices requires that the time series be modelled simultaneously. The variance matrices are of interest from an economic standpoint. They determine, for example, whether the trends are positively correlated with each other, and whether a subset of the time series shares a common cycle. From an econometric perspective, the multivariate extension of UC models is of interest because it enables the modeller to identify specific stable relationships between time series. It is therefore interesting to note that the phenomenon of cointegration can explicitly be modelled within an UC framework by including common trends.

Some important contributions can be found in the economic literature that study the phenomenon of convergence in the context of economic growth theory, see, for example, Williamson (1996), Galor (1996) and Quah (1996). Various definitions of convergence have been suggested and several inference procedures have been developed for the detection of convergence based on cross-section and time series data, see, for example, Bernard and Durlauf (1996). Many contributions in the literature use straightforward techniques for the comparision of two specific time series. In such analyses the focus is on testing the hypothesis whether the production series of, say, Europe and Japan have converged, see Cook (2002). In other contributions, distinction is drawn between specific forms of convergence such as the convergence of growth rates and the convergence of overall variation, see, for example, Temple (1999) and the references therein.

We will adopt the definitions of convergence introduced by Barro and Sala-Martin (1992). Our starting point will be the definition of convergence as a reduction of the variation in a cross-section of

time series. Whether the reduction of variability is due to harmonization in the underlying dynamics of the growth, the cyclical behaviour, the volatility or a combination of these is to be deducted from our modelling strategy. This paper does not focus on testing hypotheses of convergence such as is done in Bernard and Durlauf (1995) and Harvey and Carvalho (2002). In the latter paper the authors analyse convergence using unobserved components with balanced growth trends and concentrate on the formulation of convergence tests.

In this paper we introduce multivariate time series models based on unobserved components with explicit time-varying rank-reduction mechanisms in order to identify convergence features present in the economic time series. We propose to model convergence via the gradual reduction in the number of eigenvalues of the covariance matrix associated with the disturbance vector driving the appropriate unobserved component. This convergence mechanism that we are introducing has not been used within the multivariate UC framework before. The stochastic process governing a particular unobserved component can be made subject to the proposed convergence mechanism. This makes the identification of various types of convergence possible. For example, the short-term business cycle dynamics of a cyclical process may convergence, while the long-term dynamics of a trend may not. In general this approach permits the investigation of convergence to be directed towards the identification of which types of convergence are present. Therefore, while it is possible that both the trend and cycle components may converge, it may also be the case that only a feature of the trend component, say, the growth of the trend, may be subject to convergence. In this paper we will present an illustration with a panel of time series of real gross domestic product (per capita, in logs) from five different European countries that appear to be subject to convergence in the rate of growth, cyclical behaviour and the overall variance. In our analysis we are able to identify these different types of convergence by defining a multivariate UC model with convergence mechanisms and by estimating the parameters using maximum likelihood methods based on the Kalman filter.

The multivariate unobserved converging component model is considered for the per capita real gross domestic product of five European countries: Germany, France, Italy, Spain and the Netherlands. Various UC models both with and without convergence have been fitted and a full account of the modeling process is given in this paper. We show that the main convergence features in these series are present in the cyclical pattern and in the overall volatility. For example, it is found that convergence of the cycle component began following the introduction of the Exchange Rate Mechanism in 1979, with complete convergence having taken place by the inception of the Common Market in 1993.

The remaining part of the paper is organized as follows. The multivariate unobserved components time series model with common trend and cycle components is discussed and is estimated for the European GDP series in section 2. The resulting salient features of the GDP series are presented and discussed. The multivariate common converging trend-cycle decomposition model is introduced in section 3. Empirical results of various model specifications for the European GDP series are presented in section 4 and some final remarks are made in section 5. The details of the convergence mechanism for a simple multivariate UC model are reviewed in the Appendix.

2 Trend-cycle decompositions of European GDP series

Many economic time series typically feature a long term trend with cyclical variations around this trend. Further they are often characterised by trends with different growth rates for different periods and by cycles with time-varying characteristics. The identification of these unobservable features can be improved by considering a multiple set of similar time series. Therefore a multivariate unobserved components time series model is considered.

2.1 Trend and cycle components

A general unobserved components model for a multiple set of non-seasonal economic time series can be expressed as

$$y_t = \mu_t + \psi_t + \varepsilon_t, \qquad t = 1, \ldots, n, \tag{1}$$

where y_t represents the actual time series, and the unobserved components consist of the trend μ_t, cycle ψ_t and irregular ε_t, all of which are vectors. These unobserved vectors are modelled as stochastic processes. For example, a simple model for the trend μ_t is given as part of the local level model (13) in the Appendix. However, the long-term trends of the GDP series are subject to positive growths which can be incorporated in the trend specification as follows,

$$\mu_{t+1} = \mu_t + \beta_t + \eta_t, \qquad \beta_{t+1} = \beta_t + \zeta_t, \qquad \eta_t \sim \mathcal{N}(0, \sigma^2 \Sigma_\eta), \quad \zeta_t \sim \mathcal{N}(0, \sigma^2 \Sigma_\zeta), \tag{2}$$

for $t = 1, \ldots, n$ and $\sigma^2 > 0$ where β_t is the $p \times 1$ vector of growth terms. This trend model is known as the local linear trend model. If $\Sigma_\eta = \Sigma_\zeta = 0$, then $\beta_{t+1} = \beta_t = \beta$, and $\mu_{t+1} = \mu_t + \beta$, so that the trend model (2) reduces to a deterministic linear trend. If $\Sigma_\eta = 0$ and $\Sigma_\zeta \neq 0$, the growth vector β_t is stochastically time-varying and the trend vector μ_t has become a cumulator function of growth terms which will result in a smooth trend component. Deficient ranks of variance matrices Σ_η and Σ_ζ imply common trend and growth components; see the discussion in Harvey (1989) and Harvey and Koopman (1997). The presence of common trends implies what is known as cointegration in the econometric literature; see Stock and Watson (1988). In short, it assumes that a $(p - r_\eta) \times p$ matrix of cointegrating vectors G exists such that Gy_t is stationary. This requires that $GA = 0$.

Various specifications for a cycle component ψ_t may be considered. For example, we may wish to generate multiple cycles by vector autoregressive processes. Alternatively we can represent the cyclical processes by a set of trigonometric terms with time-varying coefficients. A stochastic cyclical process can be incorporated in a multivariate time series model. The multiple cycle component is given by

$$\begin{pmatrix} \psi_{t+1} \\ \psi_{t+1}^+ \end{pmatrix} = \rho \left(\begin{bmatrix} c & s \\ -s & c \end{bmatrix} \otimes I_N \right) \begin{pmatrix} \psi_t \\ \psi_t^+ \end{pmatrix} + \begin{pmatrix} \kappa_t \\ \kappa_t^+ \end{pmatrix}, \tag{3}$$

where $c = \cos \lambda_c$, $s = \sin \lambda_c$ and I_k is the $k \times k$ identity matrix. The $p \times 1$ vector ψ_t consists of similar cycles that have a common frequency λ_c and a common autoregressive coefficient $|\rho| < 1$. The disturbance vectors are serially and mutually uncorrelated, and are normally distributed with mean

zero and variance matrix

$$\text{Var}\begin{pmatrix} \kappa_t \\ \kappa_t^+ \end{pmatrix} = I_2 \otimes \sigma^2 \Sigma_\kappa,$$

such that κ_t and κ_t^+ have a common variance matrix Σ_κ. This specification generates a stationary multiple cyclical process with a period of $f_c = 2\pi/\lambda_c$. The individual cycles in ψ_t have similar properties due to the common damping factor ρ and the cycle period λ_c. More details on similar cycles can be found in Harvey and Koopman (1997).

The model is completed by taking the irregular component ε_t as a normally distributed random vector with mean zero and variance matrix $\sigma^2 \Sigma_\varepsilon$. The irregular and other disturbances associated with the various components are mutually uncorrelated, both contemporaneously and between different time periods. We finally note that the constant $\sigma^2 > 0$ is the variance scalar that is common to all components of the model. It can be estimated when one of the non-zero elements in the variance matrices Σ_η, Σ_ζ, Σ_κ and Σ_ε is set to one. The common variance is introduced because we will allow it to vary over time in the next section in order to model variance convergence.

The unknown parameters in the trend plus cycle model are the variance matrices Σ_ε, Σ_η, Σ_ζ and Σ_κ together with the autoregressive coefficient ρ and the cycle frequency λ_c. These parameters are estimated by maximum likelihood for which the Kalman filter is employed to compute the loglikelihood function for a given set of parameters. The fixed common variance can be concentrated out of the loglikelihood function and can be estimated implicitly by the Kalman filter output; see Harvey (1989). The resulting concentrated loglikelihood function can be maximised numerically with respect to the vector of parameters. The variance matrices need to be non-negative definite while the cycle coefficients are subject to the restriction $0 \le \rho < 1$ and $\lambda_c = 2\pi/f_c$ with a cycle period of $f_c > 2$. The restrictions are implemented by representing the variance matrices via the Cholesky decompositions

$$\Sigma_\eta = AD_\eta A', \qquad \Sigma_\zeta = BD_\zeta B', \qquad \Sigma_\kappa = CD_\kappa C', \qquad \Sigma_\varepsilon = ED_\varepsilon E', \tag{4}$$

where the matrices A, B, C and E have a unity lower triangular structure. Their number of nonzero columns depends on the number of nonzero diagonal elements in the diagonal matrices D_η, D_ζ, D_κ and D_ε. To enforce the restriction that the diagonal matrices have positive values, we take

$$D_a = \text{diag}\left\{\exp(2d_{a,1}^*), \dots \exp(2d_{a,r_a}^*)\right\}, \tag{5}$$

for $a = \eta, \zeta, \kappa, \varepsilon$ with r_a as the rank of the corresponding variance matrix (assuming that the elements $d_{a,1}^*, \dots, d_{a,r_a}^*$ do not tend to $-\infty$). Finally, we have $\rho = |\theta_\rho| (1 + \theta_\rho^2)^{-1/2}$ and $f_c = 2 + \exp \theta_\lambda$ where θ_ρ and θ_λ are the parameters that are actually estimated together with the matrices from the Cholesky decomposition of the variance matrices.

The Kalman filter is used for the computation of the loglikelihood function, for more details see, for example, Durbin and Koopman (2001). This requires the model to be represented as a state space model. The computations are implemented using the object-oriented matrix programming environment `Ox` of Doornik (1999) and using the `Ox` library of `C` functions for state space models `SsfPack` by Koopman, Shephard, and Doornik (1999). The computations for the basic multivariate

model of this section can also be carried out by the STAMP package of Koopman, Harvey, Doornik, and Shephard (2000). The bulk of the parameters are part of the variance matrices of the model. Initial estimates are obtained via the EM algorithm, see discussion in section 7.3 of Durbin and Koopman (2001). The convergence UC model introduces two to three additional parameters for each convergence mechanism which can be estimated simultaneously with the other parameters. Specific adjustments are required for the EM method and for the evaluation of analytical scores.

2.2 Stylized facts of European GDP series

We consider a multiple time series of real gross domestic production per capita (GDP) for five European countries: Germany, France, Italy, Spain and the Netherlands. The time series are measured in local currencies on a quarterly basis and cover the period of the first quarter of 1970 to the first quarter of 2001. We obtained the required population and GDP series from OECD sources[1] and calculated the series after which we standardized each series to the value of 100 in 1970. The rescaling of the data is justified due to the fact that the series are measured in different currencies, see Knowles (2001) for a related discussion. We model the logarithm of these time series multiplied by 100. The resulting time series is denoted by

$$y_t = \log \ \text{GDP} \begin{pmatrix} \text{Germany} \\ \text{France} \\ \text{Italy} \\ \text{Spain} \\ \text{Netherlands} \end{pmatrix} \text{ at time } t = 1, \ldots, n \text{ corresponding to } 1970Q1, \ldots 2001Q1,$$

with $n = 125$. The five time series are presented in Figure 1. The typical features of trends and cycles in the GDP series can be detected. The unobserved components model introduced in the previous section is estimated using the Kalman filter and numerical optimisation methods. The resulting estimated transformed parameters of the diagonal variance matrices D_ε, D_η, D_ζ and D_κ in (4) are reported in Table 1 in the third row of each panel (sample 1970–2001).

It follows from these results that the estimates of the trend variance matrices Σ_η and Σ_ζ have lower ranks. In particular, the estimates imply that $r(\Sigma_\eta) = 1$ and $r(\Sigma_\zeta) = 3$, indicating that the five trends are relatively smooth. This conclusion is confirmed by Figure 2 where the estimated trends are presented (first column) together with the associated slope components (second column). The growth rates of the trends vary but the overall growth is high in the beginning of the 1970s and drops to a slow growth in the beginning of the 1990s. In the course of the 1990s all growth rates increase. These features are also reflected in the trends themselves. The growth rates for Germany, France and Italy are similar while the slopes for these three countries in turn is different from the similar pair of slopes for Spain and the Netherlands. The patterns from 1986 onwards are similar for all five countries. Finally, the swings in the growth rate of Spain are more dramatic.

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[1] See http://www.sourceoecd.org/content/html/index.htm.

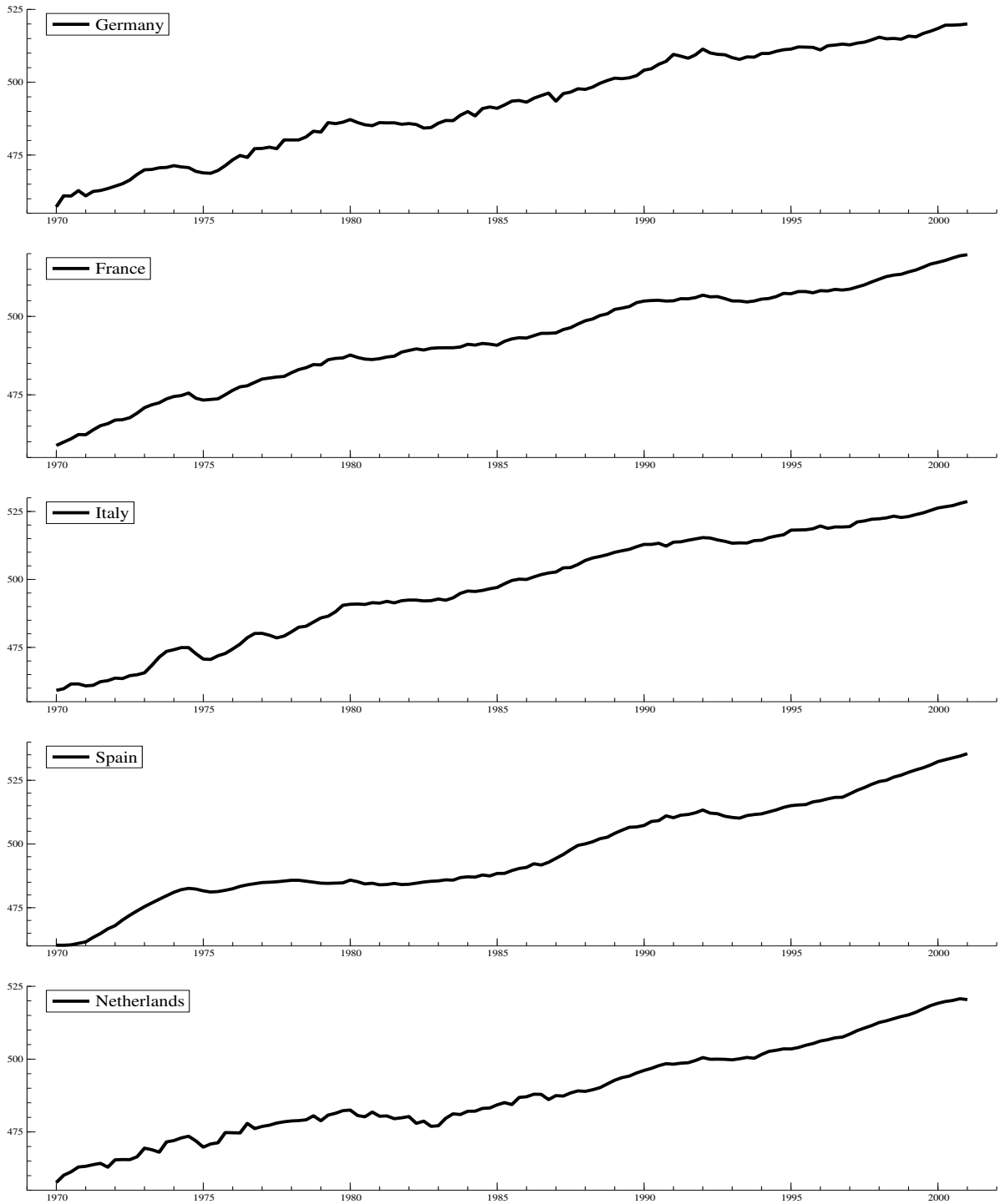Figure 1: Quarterly seasonal adjusted real gross domestic production (per capita) for Germany, France, Spain, Italy and the Netherlands for the period 1970Q1 – 2001Q1.
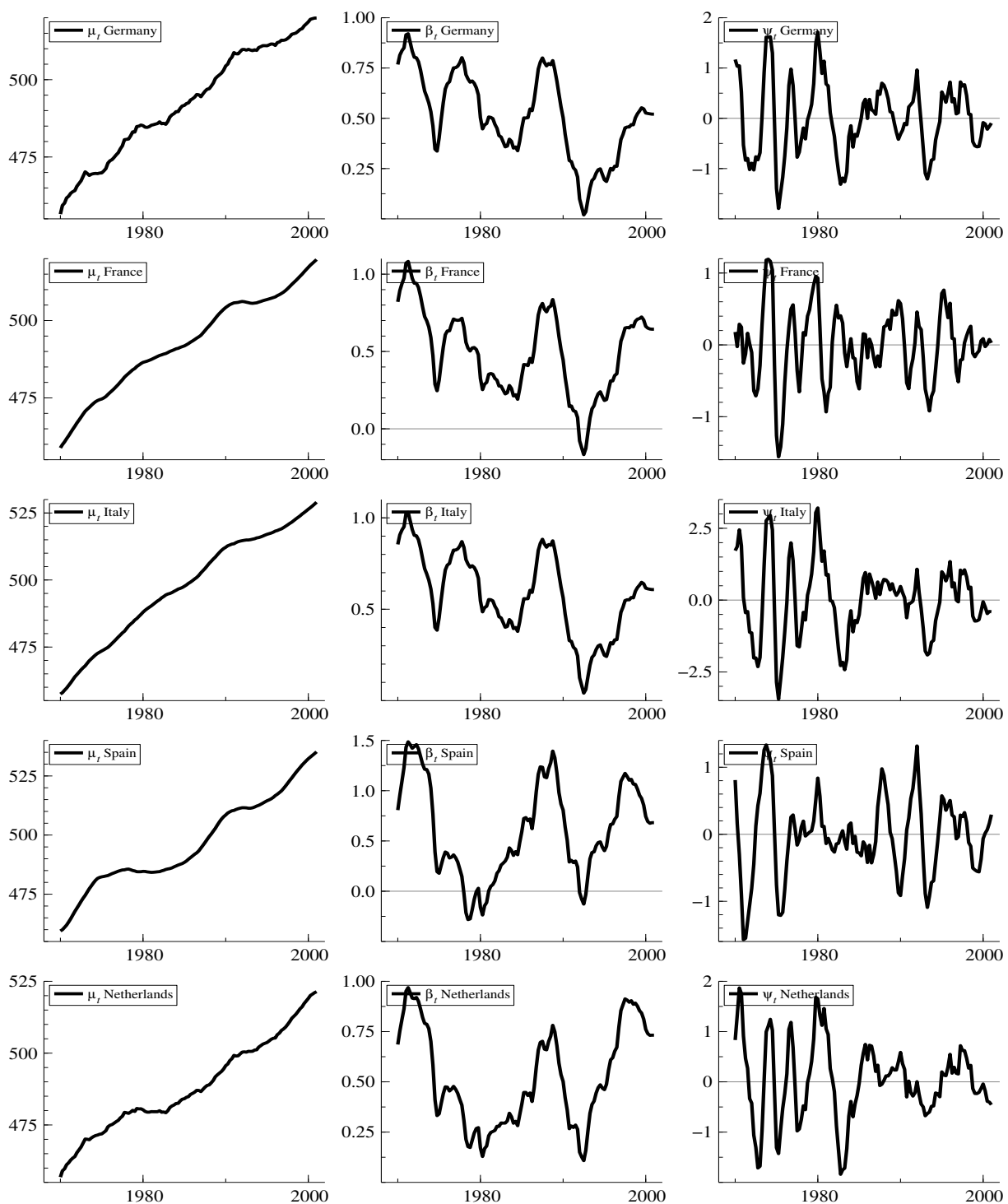
Figure 2: Estimated trend, slope (or growth rate) and cycle components for common trend-cycle model are presented column-wise for Germany, France, Spain, Italy and the Netherlands. Estimates are based on estimation sample 1970–2001.

Table 1: Estimated diagonal variance matrices of common trend-cycle model

	sample	Germany d_1^*	France d_2^*	Italy d_3^*	Spain d_4^*	Netherlands d_5^*
level D_η	$1970-1986$	-1.86	-	-	-	-
	$1987-2001$	-1.36	-	-1.51	-	-
	$1970-2001$	-0.47	-	-	-	-
slope D_ζ	$1970-1986$	-1.52	-2.41	-2.67	-2.11	-
	$1987-2001$	-2.42	-2.59	-	-	-
	$1970-2001$	-2.30	-3.49	-	-2.44	-
cycle D_κ	$1970-1986$	-1.17	-1.77	-2.34	-	-
	$1987-2001$	-1.47	-2.38	-	-	-
	$1970-2001$	-1.27	-1.51	-1.98	-	-
irregular D_ε	$1970-1986$	-0.47	-1.87	-2.01	-	-
	$1987-2001$	-0.77	-1.80	-1.48	-1.94	-
	$1970-2001$	-0.70	-1.60	-1.74	-	-1.06

The estimates are for the transformed values of the diagonal variance matrices in (4) for three different samples. The transformed value d_i^* is the logarithm of the square root of the ith diagonal element of D. Large negative values produce therefore variances which are close to zero.

The estimated parameters for the multiple cycle component are given by

$$\hat{\theta}_\rho = 2.48 \quad \text{(s.e } 0.38\text{)}, \qquad \hat{\theta}_\lambda = 2.72 \quad \text{(s.e } 0.09\text{)},$$

which imply a similar cycle with an estimated autoregressive coefficient ρ of 0.93 (with a 95% confidence interval between 0.87 and 0.96) and an estimated cycle period f_c of 17.2 quarters (with a 95% confidence interval between 14.8 and 20.0). The estimate of the variance matrix Σ_κ, which applies to both disturbance vectors κ_t and κ_t^+, has a rank of 3 according to the estimate of matrix D_κ reported in Table 1. The cycles of the five countries can therefore be described by three mutually uncorrelated common similar cycles. These empirical results alone can be of interest to economists. European policy decisions on integration can benefit from the empirical confirmation of the existence of common cycles, especially if the actual dependence of individual European countries on common cyclical movements can be empirically identified. To be able to adequately address this last point, we would need to report the estimate of the C matrix in (4). To limit the number of tables in this paper we will only present the estimates of the matrices A, B, C and E in the discussion of the empirical results for the converging models. The estimated cycle components presented in Figure 2 show that the oil crises in the 1970s have clearly induced two or three recessions in Europe. All countries but Spain show recessions in the beginning of the 1980s corresponding to the double-dip recession in the United States.

According to the diagnostic test statistics reported in Table 4 under the columns denoted by M, the estimated common components model is not entirely satisfactory. The table presents some standard

diagnostics based on the residuals of the estimated model. The standardized one-step ahead residuals are assumed to be standard normally distributed and serially uncorrelated. It follows from the reported statistics that France and the Netherlands may be subject to some moderate outliers, the dynamics are not well captured for Spain and for most series the residuals appear to be heteroskedastic. It is of interest to determine whether these model diagnostics improve when we consider converging models.

The common components model provides a satisfactory overall fit when we compare the sum of squared residuals with the sum of squared residuals from a model with only a constant and a fixed time trend. The percentage decrease of the common components model for all five series is at least 82% in relation to the naive model with a maximum decrease for Spain and a minimum decrease for Germany.

2.3   Preliminary evidence of convergence in European GDP series

We now focuses on the question whether it is reasonable to assume that the rank of a variance matrix changes over time to a lower rank. If this is the case we will consider this to be an indication of some converging behaviour by a particular set of components. The full sample is split into the two roughly equivalent subsamples of 1970–1986 and 1987–2001. The trend-cycle components model is re-estimated for both of these subsamples. The estimation results of the variance matrices D, which determine the rank of the variance matrices in (4), are produced in Table 1. It is perhaps not surprising that we do indeed find evidence that convergence mechanisms play a role within the multiple time series. The estimated variance matrix of the slope component has a rank of 4 for the first sample, whereas in the second sample its rank is 2. Similarly, in the case of the cycle variance matrix in the first sample, the estimate of D_κ indicates a rank of 3 while in the last sample it indicates a rank of 2. These results provide some evidence of slope and cyclical convergence although any conclusions made must be tentative ones, because the subsamples consist of five series with at most 68 observations, representing only a moderate sample size. Furthermore, the cycles have an estimated period of more than 4 years so that realistically no more than three, or at most four full cycles can be observed within each subsample.

3   Unobserved common converging components

In this section we introduce the converging mechanism for the growth and cycle components. Details of the implementation of convergence in an unobserved components time series model are given in the Appendix. The multivariate trend-cycle decomposition model is represented as a dynamic factor model for the $p \times 1$ vector of time series y_t and is given by

$$y_t = a + A\mu_t^* + C\psi_t^* + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma^2 \Sigma_\varepsilon), \tag{6}$$

where a is a $p \times 1$ vector with the first r_η elements equal to zero, A is a $p \times p$ unity lower triangular matrix and C is a $p \times r_\kappa$ unity lower triangular matrix. The matrix A differs somewhat from matrix A in (14) of the Appendix in that zero rows and zero columns, corresponding to the diagonal zero

elements in D_η, must be added. All of the diagonal elements of A, however, including those of the added zero rows and columns, are unity. The specification of common growth rates given below in subsection 3.1 necessitates the augmentation of the matrix A in this manner. The matrices A and C are sometimes referred to as factor loading matrices. The unobserved factor μ_t^* represents the $p \times 1$ vector of underlying trends and factor ψ_t^* represents the $r_\kappa \times 1$ vector of cyclical components.

An unobserved component with an activated convergence mechanism that models the gradual rank reduction over time such as (15) of the Appendix is defined as an *unobserved converging component*. From an economic viewpoint, convergence in growth and cycle components can be of major interest as we will show in the analysis of European GDP series in section 4 where models with unobserved converging components are estimated. In subsections 3.1 and 3.2 we introduce the convergence mechanisms for the trend and cycle component. In subsection 3.3 we turn our attention to the specification of variance convergence.

3.1 Common converging slope component

The trend component is specified as

$$
\begin{array}{llll}
\mu_{t+1}^* &=& \mu_t^* + b + B^*\beta_t^* + \eta_t^*, & \eta_t^* \sim \mathcal{N}(0, \sigma^2 D_\eta), \\
\beta_{t+1}^* &=& \beta_t^* + \zeta_t^*, & \zeta_t^* \sim \mathcal{N}(0, \sigma^2 D_\zeta),
\end{array}
\tag{7}
$$

where the $r_\zeta \times 1$ vector β_t^* consists of the underlying growth terms of the trend μ_t^* for $t = 1, \ldots, n$. The initial values for μ_1^* and β_1^* are treated as being generated from diffuse density functions. The vector b is a $p \times 1$ vector of unknown constants of which the first r_ζ are fixed at zero. The $p \times r_\zeta$ factor loading matrix B^* is unity lower triangular. The disturbance vectors η_t^* and ζ_t^* are mutually and serially uncorrelated and have diagonal variance matrices D_η and D_ζ, respectively. Note that the matrix B in 4 is given by $B = AB^*$, and therefore the matrix Σ_ζ can be expressed as $\Sigma_\zeta = AB^* D_\zeta B^{*\prime} A'$.

When $r_\eta = r_\zeta = p$ and $\psi_t = 0$ in (6), the multivariate local linear trend model reduces to a SUTSE specification in which the variance matrices Σ_η and Σ_ζ are of full rank. Common trend specifications are obtained when $r_\eta < p$ or $r_\zeta < p$ or both. Note that common trends imply cointegration.

In the application of this paper, the convergence of the trend takes place in the growth rate. We will therefore give the details of convergence for the slope component. A gradual change of a non-zero value for a particular element of D_ζ to a zero value can be accomplished by the logit function

$$
d_{\zeta,t,i} = \exp(2d_{\zeta,i}^*) \exp(s_{\zeta,t,i}) / \{1 + \exp(s_{\zeta,t,i})\}, \qquad i = 1, \ldots, r_\zeta,
\tag{8}
$$

where $d_{\zeta,t,i}$ is the ith element of the time-varying variance matrix $D_{\zeta,t}$ that replaces D_ζ in (7). The size of the logarithm of the standard deviation of the ith element of $D_{\zeta,t}$ is $d_{\zeta,i}^*$ for $i = 1, \ldots, r_\zeta$. The variable $s_{\zeta,t,i}$ is specified as

$$
s_{\zeta,t,i} = s_{\zeta,i}^* \times (t - \tau_{\zeta,i}), \qquad i = 1, \ldots, r_\zeta,
$$

where $s_{\zeta,i}^*$ is typically negative and determines how quickly the function $s_{\zeta,t,i}$ approaches zero and $\tau_{\zeta,i}$ determines the mid-point of this change over time. Further details of the converging mechanism are

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discussed in the Appendix. A similar logit-converging mechanism can be introduced for the variance matrix of the trend component itself, $D_{\eta,t}$, which is then also time-varying and replaces D_η in (7). The details of this convergence mechanism are the same as for the slope component and are discussed in the Appendix.

3.2 Common converging cycle component

The cycle component ψ_t^* is defined in a similar way as in equation (3) with $\psi_t = C\psi_t^*$ and $\psi_t^+ = C\psi_t^{*+}$ where

$$\begin{pmatrix} \psi_{t+1}^* \\ \psi_{t+1}^{*+} \end{pmatrix} = \rho \left(\begin{bmatrix} c & s \\ -s & c \end{bmatrix} \otimes I_{r_\kappa} \right) \begin{pmatrix} \psi_t^* \\ \psi_t^{*+} \end{pmatrix} + \kappa_t^\times, \qquad \kappa_t^\times \sim \mathcal{N}(0, I_2 \otimes \sigma^2 D_\kappa), \tag{9}$$

for $t = 1, \ldots, n$. The cycle vectors ψ_t^* and ψ_t^{*+} have dimension $r_\kappa \times 1$ and the vector of disturbances $\kappa_t^\times$ has dimension $2r_\kappa \times 1$. The fact that $\psi_t = C\psi_t^*$ such that $\Sigma_\kappa = CD_\kappa C'$ is only valid for similar cycles; see Harvey and Koopman (1997) for the technical details.

The convergence mechanism introduced for the trend component in the previous subsection can also be incorporated in the diagonal variance matrix of the cycle disturbances D_κ. It implies that matrix D_κ becomes time-varying and its i-th element will be modelled in the same manner defined in (8) for $i = 1, \ldots, r_\kappa$.

3.3 Common converging variance component

The specification of a common converging variance component as a formalization of convergence in the overall variance requires three parameters. Unlike the other forms of convergence for the trend, slope, and cycle components, the common variance σ^2 cannot be allowed to converge to zero unless we are willing to entertain the idea of convergence to a deterministic system of non-stochastic equations. To ensure that we retain some volatility after the convergence of the common variance, we introduce a third constant parameter into the convergence specification. This leads to the following specification for variance convergence,

$$\sigma_t^2 = \exp(2d_\sigma) + \exp(2d_\sigma^*) \exp(s_{\sigma,t})/\{1 + \exp(s_{\sigma,t})\}. \tag{10}$$

According to (10), the total volatility before convergence sets in is given by $\exp(2d_\sigma) + \exp(2d_\sigma^*)$, while only the constant term $\exp(2d_\sigma)$ remains after convergence has occurred. Both d_σ and d_σ^* are the logarithm of a standard deviation. The size of the constant component of the total volatility is determined by d_σ, while d_σ^* determines by how much volatility declines due to variance convergence. In an analogous fashion to the definition of $s_{\eta,t,i}$ in (16), the variable $s_{\sigma,t}$ is specified as

$$s_{\sigma,t} = s_\sigma^* \times (t - \tau_\sigma)$$

where $s_\sigma^* < 0$ determines the rate at which the function $s_{\sigma,t}$ approaches zero and τ_σ determines the mid-point of the change over time.

4 Convergence in European GDP series

We now turn our attention to the question of how the various types of covergence presented in the previous section can be applied in practice to develop an appropriate statistical model. We demostrate this by generalizing the trend-cycle model presented in section 2 to include the three different types of convergence.

4.1 Specification and estimation of converging trend-cycle model

To apply the convergence mechanisms, we first determine which diagonal elements $d_{a,i}$ from the matrices D_a for $i = 1, \ldots, r_a$ and $a = \eta, \zeta, \kappa$, are candidates to converge to zero. Given the structure of the Cholesky decompositions of the variance matrices in (4), it is possible to infer a number of simple rules that will hold in most situations. We illustrate these rules via a discussion of D_ζ.

Generally, it will not be the case that the first diagonal element $d_{\zeta,1}$ will converge to zero. Although technically possible, the result of such convergence is that the variance of the disturbance driving the slope component for the first series degenerates to zero. In other words, the element $d_{\zeta,1}$ represents the sole source of variability for the slope component of the first series. This can be verified directly via (4). In the case of our model, $d_{\zeta,1}$ corresponds to the variance of the slope component for Germany. Examination of the estimated values given in Table 1 for $d_{\zeta,1}$ suggests that this parameter is not subject to the convergence mechanism, given that its value is the largest one in the D_ζ matrices for each sample. The same can be said of the values for $d_{\eta,1}$ and $d_{\kappa,1}$.

The second diagonal element $d_{\zeta,2}$ represents the marginal contribution to the variance of the slope disturbance for the second series, France, after accounting for the contribution made by $d_{\zeta,1}$. The contribution of $d_{\zeta,1}$ is the result of the correlation between the slope disturbances for the first, Germany, and second series, France. In general, the structure of the Cholesky decomposition implies that the variance of the slope disturbance for series j is only affected by $d_{\zeta,1}, \ldots, d_{\zeta,j}$. The elements $d_{\zeta,i}$ for $i > j$ make no contribution to the variances of the slope disturbances for the series j. Therefore, in the case of the second series, France, it is also less likely that $d_{\zeta,2}$ is subject to the convergence mechanism, because this would imply that the slope disturbances of Germany and France become perfectly negatively or positively correlated after convergence has taken place. We wish to point out that it is a relatively strong statement about slope convergence for these two series. This is, of course, not to suggest that this could not have happened, but it is unlikely. In any case, the results in Table 1 indicate that the estimated value of $d_{\zeta,2}$ remains roughly constant over the entire sample period. This also applies to the estimates shown for $d_{\kappa,2}$, while in the case of $d_{\eta,2}$ all estimates indicate that France does not have its own variance for the trend disturbance in any of the sample periods.

For the series i for $i > 2$, the convergence of $d_{\zeta,i}$ to zero implies that the variance of the slope disturbance for the series i becomes a linear combination of slope disturbances of the first $i - 1$ series. For this reason the correlations between the slope disturbance for the series i with those for the first $i - 1$ series will still typically be less than one in absolute value after convergence. We therefore regard the last element d_{ζ,r_ζ} as the most likely candidate for convergence. This is a consequence of the fact

that the variance of the slope disturbance for the series r_ζ becomes a linear combination of all other values of $d_{\zeta,i}$ in the case that this element converges. Generally it is the case that the larger the number of elements that are involved in a linear combination, the greater the chance one element will converge. We therefore conclude for $a = \zeta, \kappa$ that the most likely candidate for convergence is $d_{a,i}$ for the largest available value of i, that is $i = r_a$. Inspection of the estimates in Table 1 supports this conclusion as well. Where there is evidence of $d_{a,i}$ dropping out of the model in the latter part of the sample period for $a = \eta, \zeta, \kappa$, it is for values of i of either 3 or 4. In the case of $d_{\eta,i}$, the estimates show no evidence of convergence. In fact the estimates obtained for the entire sample period indicate that only one value $d_{\eta,1}$ is responsible for the trend volatility for all five countries in Σ_η. For this reason we do not apply the convergence mechanism to D_η and restrict our attention to model specifications in which $r_\eta = 1$. This has the advantage of resulting in more parsimonious models, which, given the number of model parameters, is an important advantage.

Returning to the example of the slope component, we note that a further consequence of the Cholesky decomposition of Σ_ζ is that the variances and correlations among the slope disturbances for the first $i - 1$ series are not influenced by the existence of a convergence mechanism on $d_{\zeta,i}$. The variances of the slope disturbances for the series j, where $j \geq i$, can be altered by this convergence. The correlations between the slope disturbances for the series j, where $j \geq i$, with the slope disturbances from all other series can also be changed by convergence. For this reason we also consider it to be important, in the case of our example of GDP, to place the largest, most dominating and stable economies first in the observation vector, with the smaller, more dependent, and less stable economies last, because the latter economies are more likely to have been effected by convergence towards the economies of the more dominating countries of the European Union (EU). Although this rule does not lead to a unique ordering of the countries in the observation vector y_t, it does in any case seems reasonable to have Germany and France, two of the orginal member of the EU, as the first two elements of y_t, given the economic dominance of these countries. Italy too is one of the original members of the EU and also has a larger economy than either Spain or the Netherlands, although smaller than that of either Germany or France. For these reasons we opted to place Italy third in the observation vector. We chose to place Spain fourth in the vector, reflecting the fact that Spain only joined the EU in 1986. Although the Netherlands has been a member of the EU since its inception, it is a smaller country with little power to influence the larger economies in the EU, and as a result, the Netherlands is the last series.

In summary, based on the estimates for the common trend-cycle model without convergence mechanisms reported in Table 1, as well as on the model structure implied by the combination of the Cholesky decomposition parameterization of the variance matrices together with the convergence mechanism, we have opted to produce estimates for the trend-cycle model of the previous section with the incorporation of converging mechanisms for the slope β_t, the cycle ψ_t and the common variance σ_t components. At first the convergence mechanism is introduced to element 4 of the diagonal variance matrix of the common slope component β_t and the resulting model is indicated by M_β. In the same way, the convergence mechanism is placed on the third element of the diagonal matrix D_κ to investigate cycle

convergence only. This model is indicated by M_ψ. The trend-cycle model without any convergence mechanism is indicated by M and the model with only the convergence mechanism for σ^2 is indicated by M_σ. Although we have also investigated employing the convergence mechanism on other elements of D_ζ and D_κ, no other specification produced significant results based on the loglikelihood values and standard information criteria such as the Akaike information criterion (AIC). In the discussion of the results below we only consider the models with convergence mechanisms that are estimated significantly. Finally, the model with convergence considered for all components will be indicated by $M_{\beta\psi\sigma}$.

The full modelling and estimation process is as follows. First the non-converging model M is estimated for which some of the results for the European GDP series are reported and discussed in section 2. The estimated parameters of the M model act as the starting values for the parameters of the models M_β, M_ψ and M_σ. In particular, we maintain the restrictions of $r_\eta = 1$ and $r_\varepsilon = 4$ in all model specifications in the interests of parsimony. Initial values for τ and s^* of the convergence mechanisms are chosen so that the original values of D_ζ, D_κ or σ remain almost fixed throughout. We then explore the possibility that convergence can take place at various points in time. This amounts to setting s^* close to zero. We typically started with a value of $\tau = n/2$, but we also thoroughly explored values corresponding to the start date for the European Exchange Rate Mechanism in 1979Q1 and Spanish entry to the EU in 1986Q1. The initial values used in the maximum likelihood optimization routine for the converging models are similar to the values for M, some of which are reported in Table 1. The estimates for the models M_β, M_ψ and M_σ are then used to initialize the maximum likelihood routines for the models employing two type of convergence: $M_{\beta\psi}$, $M_{\beta\sigma}$ and $M_{\psi\sigma}$. Finally, estimates obtained for the the latter more complex converging models serve in turn as starting values for the final and most complex model of interest $M_{\beta\psi\sigma}$.

All models, including the converging trend-cycle models, are estimated by maximum likelihood for which the Kalman filter is used for the evaluation of the loglikelihood function. The Kalman filter can handle time-varying state space models which we require for representing the converging model in state space form since the variance matrices are time-varying due to the convergence mechanisms defined in (8) and (10).

4.2 Estimation results for European GDP series

The estimation results for model M are discussed in section 2 and the estimated elements of the diagonal variance matrices of the common components are reported in Table 1. The parameters in Table 1 correspond to the d^* parameters of the converging mechanism defined in (8) and (10) and their estimates for the three single convergence models are reported in Table 2 together with the convergence rate parameter s^* and the timing parameter τ. Estimates of the same parameters for the final model $M_{\beta\psi\sigma}$ are also reported. The models M_σ and $M_{\beta\psi\sigma}$ incorporate variance convergence which requires a third convergence parameter d_σ. This parameter is also reported in Table 2.

Since the variance matrix Σ_η for the level disturbance η_t is estimated to have rank one in model M, we have used this restriction for all converging models. The estimated value of $d^*_{\eta,1}$ is around

-0.5 for most models except for M_σ. In fact, the estimated d^* values in D_ζ and D_ε for the various convergence models do not vary a great deal. This indicates that a large number of parameters in the convergence models are estimated in a numerically stable manner. Of particular interest are the estimates of the parameters s^* and τ of the converging mechanisms. The actual values of s^* and τ are not straightforward to interpret and therefore we graphically present in Figure 3 the three different convergence mechanisms for the slope, cycle and common variance components which are estimated simultaneously for model $M_{\beta\psi\sigma}$. The graphs clearly show two different converging patterns. By the end of the sample period the slope and cycle convergence processes have taken place, while the common variance is still in the process of converging. It is estimated that the standard deviation of the variance will eventually converge to the value of 0.037. We note, however, that the final converged value of this variance is difficult to estimate accurately given that both the starting and converged values lie well outside the sample period. The midpoints of convergence for the slope is aproximately $1981Q2$ and for the cycle it is approximately $1983Q3$, or about two years later. This fact will be discussed further below.

The log-likelihood values of the various estimated models, denoted by $\log(L)$, are reported in Table 3 together with the number of parameters, n_p, that are present in the model and the Akaike information criterion (AIC) that we have computed as

$$\text{AIC} = -2\log(L) + 2n_p. \tag{11}$$

We use the minimum of the AIC to determine the "best" model within the class of converging trend-cycle models in terms of fit relative to the number of parameters required for the estimation. The minimum value of the AIC is found for the model $M_{\psi\sigma}$. This may indicate that cycle and common variance convergence is most relevant for the European GDP series. Although the slope convergence is strong, it is only relevant for the GDP of Spain and it has a lesser impact on the multiple GDP time series as a whole. However, for the illustrative purposes of this paper we will mainly discuss the estimation results of the full converging model $M_{\beta\psi\sigma}$ because it considers all aspects of the converging trend-cycle model.

The factor loading matrices A, B, C and E of the model $M_{\beta\psi\sigma}$ are estimated as

$$
\widehat{A} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0.08 & 1 & 0 & 0 & 0 \\
-0.42^* & 0 & 1 & 0 & 0 \\
-0.02 & 0 & 0 & 1 & 0 \\
0.51^* & 0 & 0 & 0 & 1
\end{pmatrix}, \qquad
\widehat{B} = \begin{pmatrix}
1 & 0 & 0 \\
1.48^{**} & 1 & 0 \\
1.03^* & 0.14 & 0 \\
1.91^{**} & 1.39^{***} & 1 \\
0.94^* & 1.33^* & 0.27^*
\end{pmatrix}
$$

$$
\widehat{C} = \begin{pmatrix}
1 & 0 & 0 \\
0.48^* & 1 & 0 \\
1.20^{**} & 0.49 & 1 \\
0.77^{**} & -0.17 & -0.28^* \\
0.32^* & -0.31 & 1.52
\end{pmatrix}, \qquad
\widehat{E} = \begin{pmatrix}
1 & 0 & 0 \\
0.18^* & 1 & 0 \\
0.05 & -0.27 & 1 \\
-0.06 & -0.16 & -1.04^{***} \\
-0.24 & 1.37^{**} & -0.46^*
\end{pmatrix}
$$

where *, ** and *** indicate that the t-statistic is estimated as being greater than 1, 2 and 3, respectively. Zero values in the loading matrice A are the result of the fact that the corresponding values of d_i are 0. This causes these parameters to drop out of the model, hence the reported values of 0. There are some restrictions which could be applied by assigning various parameters a value of 0, particularly in the matrix E. This will not be explored further.

It can be inferred from the factor loadings of the converging element of the slope component (that is the third column of B) that the slope convergence is strongest for Spain. The mid-point of the convergence is in the year 1982. Around this year the import and export activity of Spain with Europe increases exponentially while imports and exports with the countries of South America, for which tradionally strong trade links existed, hardly increases in this period. For example, European Union (EU) imports were five times higher in 1992 than it was in 1980 (for exports it was four times). This has heavily influenced the development of GDP in Spain and its dependence on the EU member countries' economies. Therefore it may not be surprising that Spanish growth converges during this period toward the growth pattern followed by the EU member countries. This mainly explains the fact that the slope component converges from three factors to two factors. There is also some weak evidence of the Dutch growth rate converging. This may partly be explained by the fact that Dutch GDP heavily depends on trade figures and more trade within Europe therefore likely leads to a stronger dependence on EU GDP growth. Finally it is noted that the fact that the factor loading matrices are restricted to be lower triangular and the fact that the fourth common slope component is converging, rather than the third, is evidence that the GDP growth of Germany, France and Italy converge before the beginning of the sample, that is before 1970. The GDP of these three countries can be described by two common slope factors for the whole sample. The GDP growth of Spain and the Netherlands roughly converges to these two factors by 1986, the year in which Spain officially joined the EU.

The characteristics of the cyclical convergence are more complex, because more countries are involved. In fact we can distinguish two main GDP cycles in mainland Europe: one for Germany and one for France. The GDP cycles of the other three countries considered in our study converge towards these two cycles with full convergence taking place by the beginning of the 1990s. Based on the estimated C matrix we can conclude that the five countries have the first common cycle component in common. The second common cycle is only significant for France since the factor loadings for the second cycle are not significant for Italy, Spain, or the Netherlands. Further, the converging cycle (the third column of C) is significant for Italy and the Netherlands, and to a lesser extent, for Spain. The empirical findings of cyclical convergence for Italy may well correspond to the inception of the Exchange Rate Mechanism. Arguably, stable economic conditions were required for entering the Exchange Rate Mechanism in 1979, as well for the later participation in the Euro, and as a result Italy was forced to follow the two dominating EU business cycles more closely. Given the discipline required by the goal of monetary union, it is even surprising that France deviates from the economic European cycle. However, the variation of the specific cycle for France (after correction for the European cycle) is moderate given its logged standard deviation estimated as -1.39 compared to the one of "Europe", that is -0.97, for model $M_{\beta\psi\sigma}$.
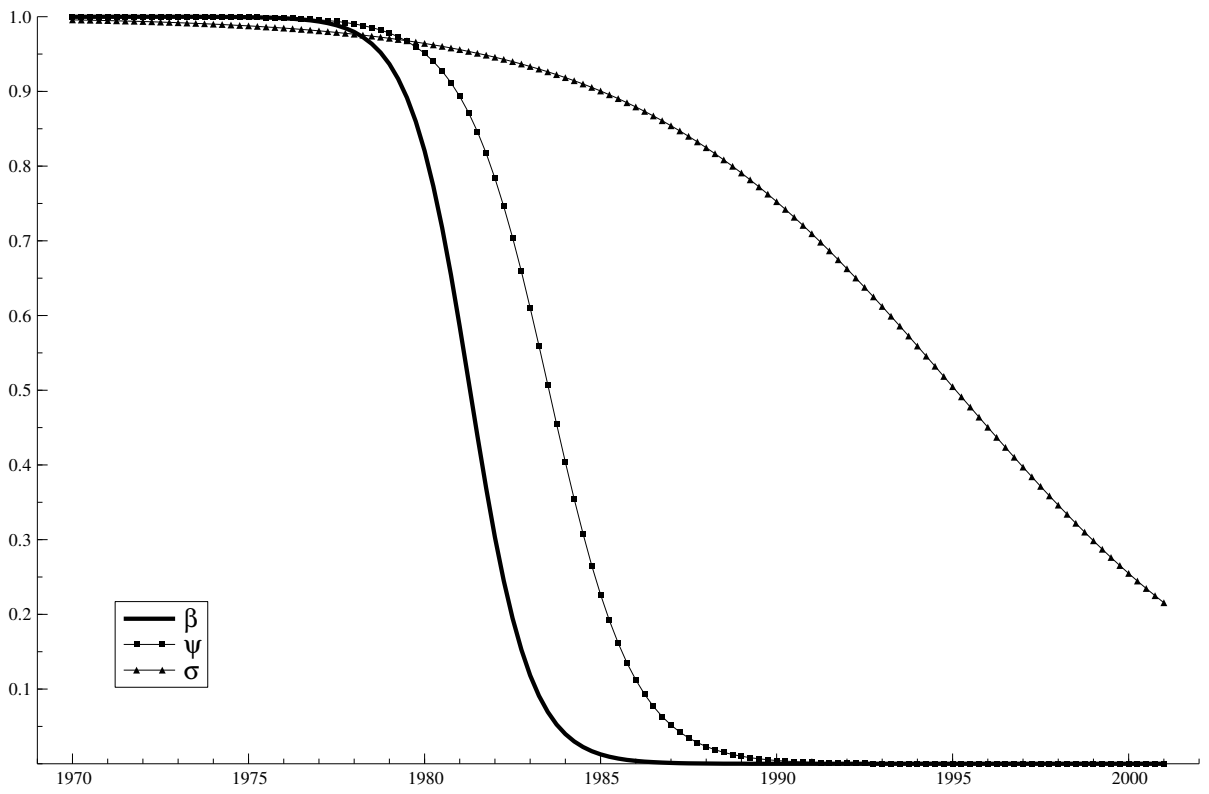
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Figure 3: The converging mechanisms for slope (β), cycle (ψ) and variance (σ) convergence in model $M_{\beta\psi\sigma}$.

Finally, the estimated Cholesky matrix E of the irregular variance matrix in (4) is not of any significant interest. However, it is worth mentioning that the irregular series of Italy and Spain are strongly negatively correlated implied by the highly significant value of -1.04 in E, element $(3, 4)$. This may well be explained by the competitive nature of the trade of both countries within Europe. When Spain is doing well in terms of GDP, Italy may have done less well as a result, and vice-versa. Casual observation would certainly seem to suggest that various products and services traded in Europe come from both Italy and Spain, for example, wine and tourism.

Table 2: Estimated diagonal variance matrices of converging trend-cycle model

| | | Ger | Fr | It | Sp | Neth | | | | |
UC	model	d_1^*	d_2^*	d_3^*	d_4^*	d_5^*	i	s_i^*	τ_i	time
Lvl D_η	M_β	-0.53	-	-	-	-	-	-	-	-
	M_ψ	-0.50	-	-	-	-	-	-	-	-
	M_σ	-0.33	-	-	-	-	-	-	-	-
	$M_{\beta\psi\sigma}$	-0.45	-	-	-	-	-	-	-	-
Slp D_ζ	M_β	-2.30	-3.66	-	-1.47	-	4	-0.43	48.2	1982Q1
	M_ψ	-2.18	-2.80	-	-2.34	-	-	-	-	-
	M_σ	-2.18	-2.92	-	-2.33	-	-	-	-	-
	$M_{\beta\psi\sigma}$	-2.30	-2.93	-	-1.45	-	4	-0.29	45.2	1981Q2
Cyc D_κ	M_β	-1.17	-1.48	-1.87	-	-	-	-	-	-
	M_ψ	-1.19	-1.46	-3.22	-	-	3	-0.50	66.2	1986Q3
	M_σ	-0.99	-1.42	-3.23	-	-	-	-	-	-
	$M_{\beta\psi\sigma}$	-0.97	-1.39	-0.86	-	-	3	-0.21	54.1	1983Q3
Irr D_ε	M_β	-0.69	-1.61	-1.74	-	-1.01	-	-	-	-
	M_ψ	-0.65	-1.63	-1.95	-	-	-	-	-	-
	M_σ	-0.60	-1.41	-1.52	-	-0.99	-	-	-	-
	$M_{\beta\psi\sigma}$	-0.54	-1.49	-1.34	-	-0.87	-	-	-	-
σ_t^2	M_σ	$d_\sigma = -3.16$		$d_{\sigma,t}^* = -0.60$			-	-0.055	93.3	1993Q2
	$M_{\beta\psi\sigma}$	$d_\sigma = -3.29$		$d_{\sigma,t}^* = -0.54$			-	-0.055	100.4	1995Q1

The estimates are for the transformed values of the diagonal variance matrices in (4) for four different models of convergence. See Table 3 for the definition of the four models. The transformed value d_i is the logarithm of the square root of the ith diagonal element of D. Large negative values produce therefore variances which are close to zero. To facilitate the comparison of the results for the various models, the values for d_i obtained for models with variance convergence are from the first quarter of 1970 and are reported after re-scaling the values to reflect the contribution due to σ^2. This amounts to the rescaling of the variance so that $\sigma_1^2 = 1$.

Table 3: Log-likelihood values of estimated models

model	description	$\log(L)$	n_p	AIC
M :	UC, $1970 - 2001$	-634.69	43	1355.37
M_{70-86} :	UC, $1970 - 1986$	-371.22	45	-
M_{87-01} :	UC, $1987 - 2001$	-184.80	42	-
M_{β} :	UCC, slope convergence	-628.42	45	1346.85
M_{ψ} :	UCC, cycle convergence	-616.72	45	1323.45
M_{σ} :	UCC, variance convergence	-604.28	46	1300.57
$M_{\beta\psi}$:	UCC, slope and cycle convergence	-609.62	47	1315.24
$M_{\beta\sigma}$:	UCC, slope and variance convergence	-602.08	48	1300.15
$M_{\psi\sigma}$:	UCC, cycle and variance convergence	-595.78	48	1287.57
$M_{\beta\psi\sigma}$:	UCC, slope, cycle, and variance convergence	-595.31	50	1290.63

The log-likelihood values are computed via the Kalman filter that is adapted for the diffuse initialisations of the trend and slope (nonstationary) components.

4.3 Diagnostic checking

The diagnostics of the standardised residuals (obtained for the estimated converging trend-cycle model $M_{\beta\psi\sigma}$ for normality, heteroskedasticity and serial correlation are presented in Table 4. They are for the most part satisfactory for the five residual series. In fact when they are compared with the diagnostics of the estimated non-converging model M they have generally improved. In particular, the heteroskedasticity tests and the normality tests are all satisfactory except for Spain. The residual series of Spain indicate that a more modest decline in the variance may be required for the observations after 1986.

Figure 4 presents the cumulative sum of squared residuals for all standardised prediction residuals, that is

$$C_j = c_j/c_n, \qquad c_j = \sum_{t=1}^{j} v_t' F_t^{-1} v_t,$$

for $j = 1, \ldots, n$, where v_t is the vector of one-step ahead prediction errors and F_t is its variance matrix at time t. Both v_t and F_t are computed by the Kalman filter. Relative large deviations of C_j from j, whether they are positive or negative, indicate some structural break in the variance. It is clearly seen from Figure 4 that for the non-converging model M such breaks occur right from the beginning of our sample. This is the main reason for including the common variance converging mechanism of (10) in the final model $M_{\beta\psi\sigma}$. The graph of C_j for the estimated model $M_{\beta\psi\sigma}$ in Figure 4 shows that the inclusion of variance convergence in the model has been effective in dealing with the heteroscedasticity in the time series due to breaks and other irregularities.
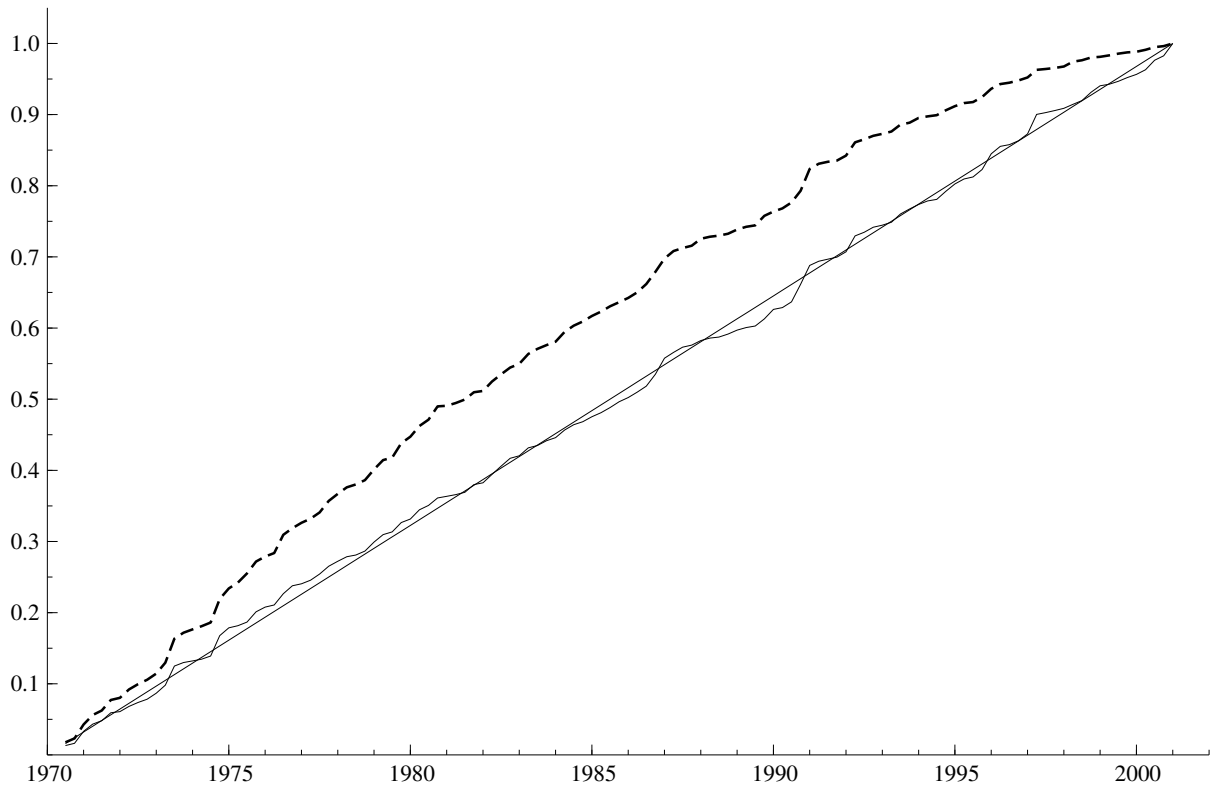
Figure 4: Cumulative sum of squared residuals C_j for common trend-cycle (M, broken line) and converging trend-cycle model ($M_{\beta\psi\sigma}$, solid line) together with the diagonal reference line (dotted line).

Table 4: Diagnostic checking for residuals of common converging trend-cycle model

	N_{DH}		$Q(10,7)$		$H(40)$		DW	
	M	$M_{\beta\psi\sigma}$	M	$M_{\beta\psi\sigma}$	M	$M_{\beta\psi\sigma}$	M	$M_{\beta\psi\sigma}$
Germany	1.24	0.34	7.58	8.72	0.28**	0.56*	1.86	1.89
France	7.85*	2.79	11.7	14.1	0.33**	0.72	1.79	2.00
Italy	0.37	1.63	11.2	5.09	0.32**	0.96	1.48**	1.62*
Spain	3.03	5.66	23.9**	14.3*	0.88	3.03**	1.91	1.86
Netherlands	6.25*	3.97	8.22	9.82	0.24**	0.89	1.93	1.75

M is the common trend-cycle model without convergence and $M_{\beta\psi\sigma}$ is the common converging trend-cycle model for the slope, cycle and variance components. The statistics are computed for the five standardized one-step ahead residuals obtained from the Kalman filter. N_{DH} is the asymptotic χ_2^2 normality test of Doornik and Hansen (1994). $Q(p,q)$ is the Box-Ljung test for the first p autocorrelations and is asymptotically χ_q^2 distributed. $H(k)$ is the standard heteroskedasticity test computed as the ratio of the sum of the first k and the sum of the last k squared residuals and is asymptotically $F(k,k)$ distributed. DW is the Durbin-Waston test which we use here as a diagnostic. The notation * indicates significance at the 5% level and ** indicates significance at the 1% level.

4.4 Multivariate decomposition into trends and cycles

Given that the model diagnostics are satisfactory for the $M_{\beta\psi\sigma}$ model, we now present the estimated trend, slope and cycle components based on all observations (smoothed estimates of elements of the state vector). In Figure 2 we present the estimated vector components for the common trend-cycle model M and in Figure 5 the same smoothed estimates of the components are presented for the converging model $M_{\beta\psi\sigma}$. Comparison of the two figures indicates that they are similar as may be expected. In the case of the similar cycle component, the estimated parameters for the $M_{\beta\psi\sigma}$ model are also close to those reported for the M model in section 2. For the $M_{\beta\psi\sigma}$ model, we obtained the following maximum likelihood estimates,

$$\hat{\theta}_\rho = 2.09 \quad \text{(s.e 0.29)}, \qquad \hat{\theta}_\lambda = 2.75 \quad \text{(s.e 0.10)}.$$

These values imply an estimated autoregressive coefficient ρ of 0.90 (with a 95% confidence interval between 0.83 and 0.94) and an estimated cycle period f_c of 17.7 quarters (with a 95% confidence interval between 14.9 and 21.1).

There are, however, some subtle differences in the estimated components obtained with the two models. The estimates of the slope and cycle components in the last ten years of the sample look more similar for the model $M_{\beta\psi\sigma}$ than those from the model M. This is to be expected because the slope and cycle components are both linear functions of only two underlying factors for the model $M_{\beta\psi\sigma}$ whereas for the model M they remain functions of three factors.
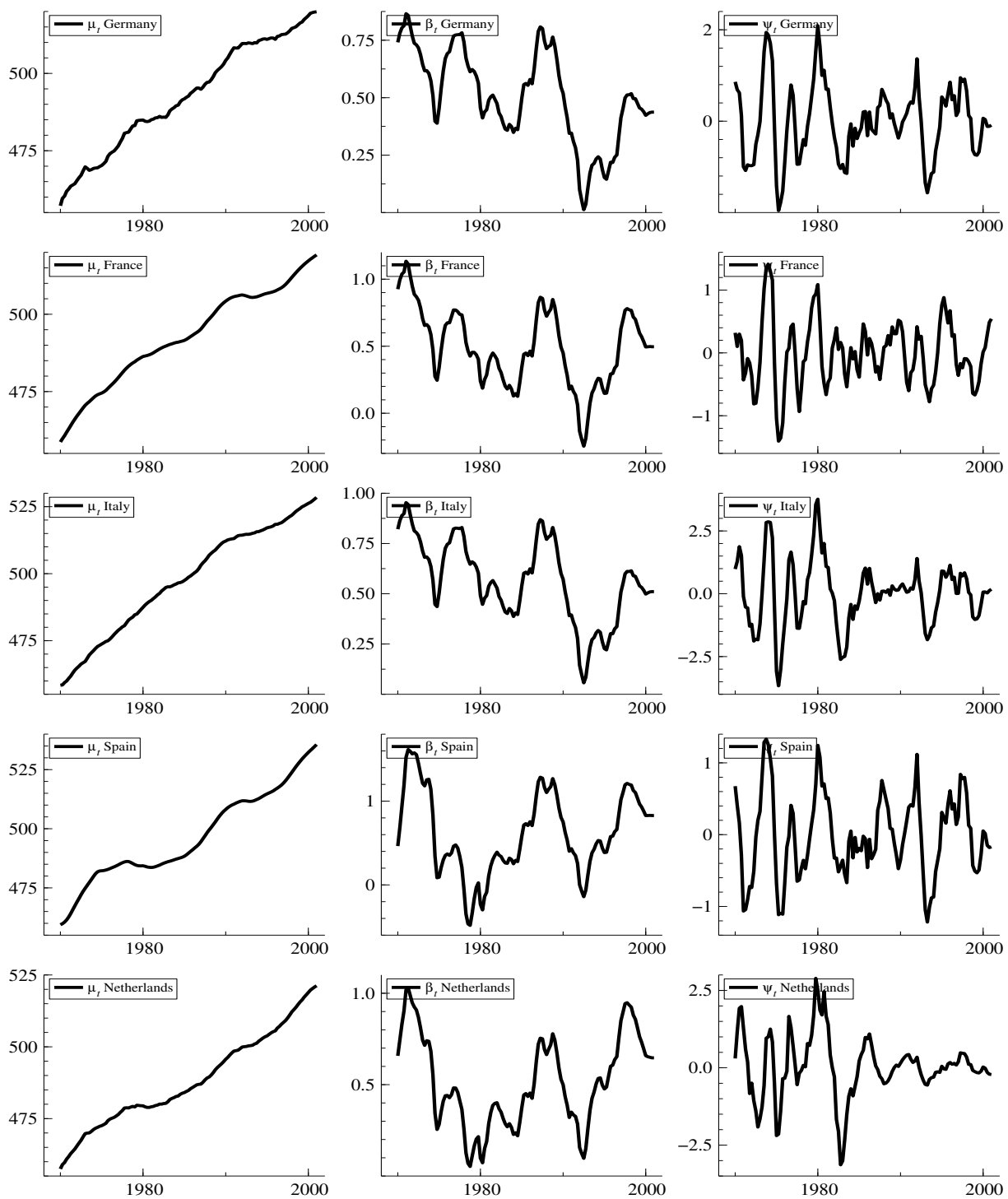
Figure 5: Trend, slope and cycle components for converging trend-cycle model.

4.5 Time-varying correlations for converging common components

A quantity of particular interest for converging trend-cycle models is the correlation between the individual elements of the converging component. Let us define the time-varying level correlations by

$$\varrho_{\eta,t,i,j} = \sigma_t \Sigma_{\eta,t}(i,j) / \sqrt{\{\sigma_t \Sigma_{\eta,t}(i,i)\sigma_t \Sigma_{\eta,t}(j,j)\}}, \tag{12}$$

where $\Sigma_{\eta,t} = AD_{\eta,t}A'$, the diagonal variance matrix $D_{\eta,t}$ being time-varying due to the convergence mechanism introduced in (15) of the Appendix. Further, $\Sigma_{\eta,t}(i,j)$ refers to the (i,j) element of variance matrix $\Sigma_{\eta,t}$. These time-varying correlations can be presented as graphs and they provide information about which two countries are converging to each other in terms of level dynamics. Similar quantities to those defined in (12) can be introduced for the slope ($\varrho_{\zeta,t,i,j}$) and cycle ($\varrho_{\kappa,t,i,j}$) components. Note that the correlations are not affected by variance convergence since the common variance σ_t cancels out in the computation of $\varrho_{\eta,t,i,j}$ and in the other correlations.

In Figure 6 we present a selection of the time-varying correlations for the slope and cycle components that in our view are of most interest. The slope correlations are presented for Germany, France, Italy, and Spain against Spain and the Netherlands. As pointed out earlier, the slope convergence is mainly due to Spain as the graphs make clear. The correlation in the slope disturbance between Spain and Germany is 0.62 at the beginning of the 1970s and it becomes 0.94 at the end of the 1980s which is a remarkable increase in about 15 years. Similar increases can be observed for the correlations of the slope disturbances with France and Italy. Slope correlation increases are also found for the Netherlands but on a smaller scale.

The correlation increases in the cyclical component are due to various countries as we have mentioned earlier. It is interesting to see that the cyclical correlations between Germany and the Netherlands increases in the 1980s from 0.2 to 0.8, between France and Italy from 0.57 to 0.79 and between Italy and Spain from 0.4 to 0.9. The most dramatic increase, however, is due to the change in the correlation between Spain and the Netherlands: from -0.18 to 0.91. A closer economic investigation may be required to provide some explanation for these particular changes in the cycle correlations. However, the figures clearly show that a closer integration of the dynamics of the GDP cycle can be identified beginning around the time of the start of the European Exchange Rate Mechanism in March of 1979, and in the case of the slope convergence, ending around January 1986 when Spain became a member of the EU. The cycle component converges more slowly, but still before the opening of the EU's Common Market in January of 1993. That the cycle component converges more slowly is also evidenced by the fact that the parameter governing the rate of convergence for the cycle, $s_{\kappa,3}^* = -0.21$ is closer to 0 than the rate for the slope convergence, $s_{\zeta,4}^* = -0.29$, as can be seen in Table 2. Finally, however, we wish to sound a note of caution about the exact interpretation of these rates of convergence. Our experience with estimating differing model specifications and differing sample periods has demonstrated that the estimates of these rates can vary as can been seen in Table 2 where the reported rate $s_{\kappa,3}^*$ for the model M_κ is further from 0 than the one for $s_{\zeta,4}^* = -0.29$ estimated for the model M_ζ. Nonetheless, both the slope and cycle components are consistently estimated as converging in the period around the early to mid 1980's.
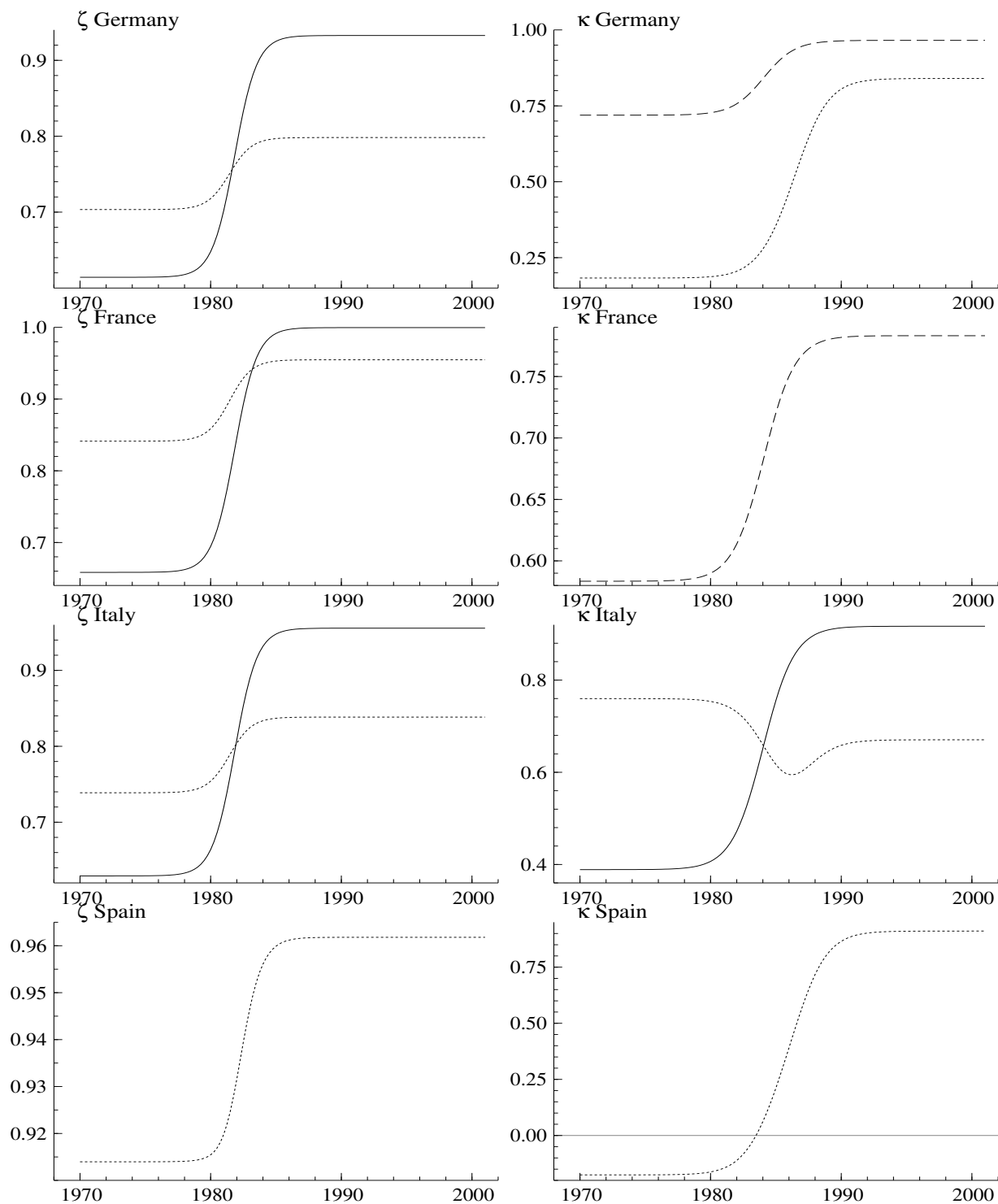
Figure 6: Some time-varying correlations $\varrho_{\zeta,t,i,j}$ and $\varrho_{\kappa,t,i,j}$ implied by the slope variance matrix $\Sigma_{\zeta,t}$ and the cycle variance matrix $\Sigma_{\kappa,t}$, respectively, for Spain (solid line), Netherlands (dotted line) and Italy (dashed line).

5 Conclusions

In this paper we have considered a multivariate unobserved components model for quarterly GDP series of five European countries from 1970 to 2001. The model decomposes the $p \times 1$ observation vector, for $p = 5$, into unobserved trend, cycle and irregular components. We refer to this model as the trend-cycle decomposition model. Each component depends linearly on $r \leq p$ independent common factors. Multiple time series with $r < p$ common trends imply cointegrating relationships within the set of time series. Common cycles can be introduced into the model in a similar fashion. This leads to a model framework that can incorporate stable relationships between economic variables for the long term (trends) and the middle term (business cycles or other stationary components).

The main contribution of this paper is the introduction of convergence mechanisms into the common trend-cycle model. At the beginning of the time series, for example, the vector cycle component is a linear function of three factors, and subsequently converges to being dependent on only two factors. The process of rank-reduction is modelled as a smooth logistic function of time that depends on a shape parameter and a parameter that determines the mid-point of the convergence process. Such a convergence mechanism is introduced not only for the slope and cycle components, but also for the overall variance of the model in an adjusted form. The multivariate converging trend-cycle model is estimated by maximum likelihood via the Kalman filter and the results are discussed in detail.

On the basis of the empirical results, we draw the following three main conclusions.

1. Both the slope and cycle components begin converging following the introduction of the Exchange Rate Mechanism for EU countries in 1979.

2. Spanish GDP growth converges to the common growth components of the other European countries by 1986 when Spain joined the EU.

3. The cyclical variations of the GDP series for Italy, Netherlands and Spain converge to the cycle processes of Germany and France by the beginning of the 1990's, before the inception of the Common Market in 1993.

The convergence of the Spanish growth rate coincides with the huge increase in Spanish imports from and exports to other EU countries during the 1980s as Spain prepared for entry to the EU, which occurred on 1 January 1986. The cycle convergence is consistent with the introduction of the Exchange Rate Mechanism in 1979 which demanded strong monetary coordination between participating EU countries, as well as with the necessary macroeconomic adjustments required by the EU member countries for the sucessful implementation of the Common Market in 1993, and the introduction of the Euro which followed. The convergence of the cycle by the beginning of the 1990's can therefore be taken to be a consequence of the economic discipline required for the creation of the European Common Market. Although the estimated convergence of the common variance is significant, it had not fully converged by the end of the sample period in 2001, and according to our estimates is still in the process of converging.

Appendix: The converging local level model

The concept of unobserved converging components is illustrated for the multivariate local level (LL) model which is the simplest multivariate unobserved components time series model. The LL model is given by

$$y_t = \mu_t + \varepsilon_t, \qquad \mu_{t+1} = \mu_t + \eta_t, \qquad t = 1, \ldots, n, \tag{13}$$

where y_t is a $p \times 1$ vector of observed time series that is decomposed into the unobserved level component μ_t and the irregular vector ε_t. The level is modelled as a multivariate random walk process and the irregular is a vector white noise process. Further we assume Gaussian densities for the disturbance vectors

$$\varepsilon_t \sim \mathcal{N}(0, \Sigma_\varepsilon), \qquad \eta_t \sim \mathcal{N}(0, \Sigma_\eta), \qquad t = 1, \ldots, n,$$

and both disturbance vectors are serially and mutually uncorrelated. The time series characteristics and properties of model (13) are discussed in detail in Harvey (1989). The level variance matrix Σ_η plays an important role in the interpretation of the LL model. When Σ_η is positive definite the model represents a set of seemingly unrelated time series equations (SUTSE). In the case of Σ_η with rank $r < p$, the model can be represented by the common level model

$$y_t = a + A\mu_t^* + \varepsilon_t, \qquad \mu_{t+1}^* = \mu_t^* + \eta_t^*, \qquad t = 1, \ldots, n, \tag{14}$$

where a is a $p \times 1$ vector with the first r elements equal to zero, A is a $p \times r$ unity lower triangular matrix and level μ_t^* is a $r \times 1$ vector that represents a multivariate random walk process as in (13) with innovation vector $\eta_t^* \sim \mathcal{N}(0, D_\eta)$ and $r \times r$ diagonal matrix D_η. It can be shown that $\mu_t = a + A\mu_t^*$ and $\Sigma_\eta = AD_\eta A'$ with $\text{rank}(\Sigma_\eta) = r$. The variance matrices Σ_η and Σ_ε are unknown and need to be estimated by maximum likelihood. The evaluation of the likelihood function can be based on the output of the Kalman filter; see Durbin and Koopman (2001) for a recent overview of such state space methods. The likelihood function is numerically maximised with respect to the variance matrices. The matrices are typically decomposed so that $\Sigma_\eta = AD_\eta A'$. The elements in A and D_η are then estimated, thereby ensuring that the restriction that Σ_η is non-negative definite holds. The elements of D_η are estimated in logs to make sure that they are always non-negative.

System convergence in this paper is modelled by a gradual reduction in the rank of the variance matrices associated with the disturbances driving the unobserved components. For example, when the rank of Σ_η is r_1 at the beginning of the sample and $r_2 < r_1$ at the end of the sample, the local level model is said to be subject to system covergence in levels. This feature of convergence can be modelled in various ways. It can be introduced as a "structural" break within the rank of the variance matrix. A more realistic alternative, however, is to allow for a smooth transition towards a lower rank regime. In this paper we introduce a convergence mechanism that is parsimonious and elementary. Similar mechanisms are used in the context of so-called smooth transition autoregressive (STAR) models which are reviewed in van Dijk, Terasvirta, and Franses (2002). Other convergence specifications can also be considered within our framework.

The convergence mechanism for the ith element of D_η is specified by the deterministically time-varying logit function

$$d_{\eta,t,i} = \exp(2d^*_{\eta,i}) \exp(s_{\eta,t,i})/\{1 + \exp(s_{\eta,t,i})\}, \tag{15}$$

where $d^*_{\eta,i}$ represents the logarithm of a standard deviation and therefore determines the size of the variance. The variable $s_{\eta,t,i}$ is given by

$$s_{\eta,t,i} = s^*_{\eta,i} \times (t - \tau_{\eta,i}), \tag{16}$$

where $s^*_{\eta,i}$ determines the rate of the variance change and $\tau_{\eta,i}$ determines the mid-timepoint of the change. According to the specification in (15), the variance matrix D_η, now $D_{\eta,t}$, is time-varying, as is the variance matrix Σ_η, which is replaced by

$$\Sigma_{\eta,t} = AD_{\eta,t}A', \qquad t = 1, \dots, n.$$

In this specification the variance $d_{\eta,t,i}$ will converge to zero as $t \to \infty$ for any finite $d^*_{\eta,i}$, $s^*_{\eta,i} < 0$ and $1 \le \tau_{\eta,i} \le n$. For the case $s^*_{\zeta,i} < 0$ and $t \ll \tau_{\zeta,i}$, we have $d_{\zeta,t,i}$ close to $\exp(2d^*_{\zeta,i})$. For the case $s^*_{\zeta,i} < 0$ and $t \gg \tau_{\zeta,i}$, the value of $d_{\zeta,t,i}$ approaches zero. Nonlinear smooth transition autoregressive models adopt a similar type of logit function but with the argument t in $s_{\zeta,t,i}$ replaced by the difference $y_t - y_{t-1}$, for more details see, for example, van Dijk, Terasvirta, and Franses (2002). Whether the convergence of $d_{\eta,t,i}$ to zero takes place within the sample range of $t = 1, \dots, n$ depends on the values of $s^*_{\eta,i}$ and $\tau_{\eta,i}$. If a particular element of the time-varying variance matrix $D_{\eta,t}$ converges to (virtually) zero via the convergence mechanism (15) with $t < n$, the rank of $\Sigma_{\eta,n}$ will be one less than the rank of $\Sigma_{\eta,1}$.

The convergence mechanism can be introduced for every diagonal element of $D_{\eta,t}$. The estimation of the variance parameters, including the additional coefficients $s^*_{\eta,i}$ and $\tau_{\eta,i}$ for $i = 1, \dots, r$, can take place simultaneously. However, the introduction of convergence mechanisms should take place after some prior analysis of the data in empirical work. The evaluation of the likelihood function can still be based on the Kalman filter although the converging model requires a state space representation with time-varying variances.

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