

## CONVERSES OF COPSON'S INEQUALITIES ON TIME SCALES

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*Abstract.* In this paper, we will prove some new dynamic inequalities on a time scale  $\mathbb{T}$ . These inequalities when  $\mathbb{T} = \mathbb{N}$  contain the discrete inequalities due to Bennett and Leindler which are converses of Copson's inequalities. The main results will be proved using the Hölder inequality and Keller's chain rule on time scales.

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