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Converting Mathematics Tasks to Learning Opportunities: An Important Aspect of Knowledge for Mathematics Teaching

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As part of a research and professional development project that focused on the opportunities and constraints provided by different kinds of mathematical tasks, a group of 67 primary and 40 secondary practising teachers of mathematics were asked to complete a survey focusing on their use of tasks. In this article, we discuss responses to one particular item which sought teachers' ideas on taking a fraction comparison task (which is larger: 2/3 or 201/301?) and converting it into a mathematics lesson in the middle years of schooling. Drawing upon a number of components of 'mathematical knowledge for teaching' as a framework, we attempt to examine those aspects of mathematical knowledge which are involved in making such a conversion. Our recommendation following this analysis is that greater emphasis is necessary in professional development settings on taking a potentially useful task and converting it into a worthwhile mathematics learning experience for students. Knowing the relevant mathematics also seems necessary even if not sufficient to make this conversion.

Plenty of Interesting Ideas but Where are the Interesting Lessons?

One of the paradoxes facing those whose task it is to support mathematics teachers is that, while the availability and accessibility of interesting teaching ideas is steadily increasing, the challenge of converting those ideas to successful student learning is as substantial as ever. We have each worked for some time on developing tasks that can be used as the basis of mathematics lessons, and have conducted numerous sessions on interesting tasks for both prospective and practising teachers. There has been an implicit assumption in some of our work that teachers can convert tasks to lessons easily. We are currently examining that assumption.

We also note that the availability of interesting ideas and tasks seems to have had limited impact on Australian classroom teaching, as evidenced by results of the Third International Mathematics and Science Study (TIMSS) Video Study which aimed to investigate and describe Year 8 mathematics and science teaching practices in a variety of countries (Hollingsworth, Lokan, & McCrae, 2003). The researchers videotaped and analysed 638 Year 8 lessons from seven participating countries, including Australia. Altogether, 87 Australian schools with one teacher in each school were randomly selected in such a manner that the selection was representative of all states, territories, school sectors, and metropolitan and country areas. Each teacher was filmed for one complete lesson.

Although the report detailed the international findings, the following specifically related to Australia:

- More than three quarters of the problems used by teachers were rated as low in procedural complexity (requiring four or fewer steps to solve). There were more problems low in complexity than in any other country, and significantly more than for teachers from Japan (17%).
- Seventy-six percent of all problems were repetitions of one or more problems students had done earlier in the lesson.
- The majority of problems involved emphasis on correct use of procedures.
- One third of problems per lesson, on average, were solved publicly by giving the answer only.
- Just over one quarter of problems were set up with use of real-life connections (42% in The Netherlands).
- More than 90% of problems were presented to students as having only one solution.

(Hollingsworth, Lokan, & McCrae, 2003, pp. xviii – xxi)

The authors noted that "Australian students would benefit from more exposure to less repetitive, higher-level problems, more discussion of alternative solutions, and more opportunity to explain their thinking" (p. xxi). They noted that "there is an over-emphasis on 'correct' use of the 'correct' procedure to obtain 'the' correct answer. Opportunities for students to appreciate connections between mathematical ideas and to understand the mathematics behind the problems they are working on are rare" (p. xxi). The authors commented on "a syndrome of shallow teaching, where students are asked to follow procedures without reasons" (p. xxi).

We are seeking to explore this apparent anomaly in which it seems to us that there are many interesting tasks available to teachers, yet it does not appear that teachers are taking advantage of these tasks in an effective way.

In this article, we examine the connection between tasks and teaching and the challenge of converting tasks to lessons, and then describe an aspect of a larger study. We report responses to a particular question which sought teachers' views on what they interpreted an illustrative task to involve mathematically, and how the task might be used as the basis of a lesson. The results seem to confirm that many teachers might need support in learning to make this conversion effectively.

Tasks and Learning

Mathematical tasks are important for teaching, and the nature of student learning is determined by the type of task and the way it is used. The task set and the associated activity have been argued to form the basis of the interaction between teaching and learning (Christiansen & Walther, 1986), and researchers have suggested that "instructional tasks and classroom discourse moderate the relationship between teaching and learning" (Hiebert & Wearne, 1997, p. 420). When teachers pose higher order tasks, students have been found to give longer responses and demonstrate higher levels of performance on mathematical assessments (Hiebert & Wearne, 1997), and the greatest gains on performance assessments including questions that required high levels of mathematical thinking and reasoning were found to relate to the use of instructional tasks that engaged students in "doing mathematics or using procedures with connection to meaning" (Stein & Lane, 1996, p. 50).

There are some general characteristics of effective tasks. Christiansen and Walther (1986), from a mathematics education perspective, argued that non-routine tasks, because of the interplay between different aspects of learning, provide optimal conditions for cognitive development in which new knowledge is constructed relationally and items of earlier knowledge are recognised and evaluated. Ames (1992), from a motivational perspective, argued that students should see a meaningful reason for engaging in a task, that there needs to be enough but not too much challenge, and that variety is important. Fredericks, Blumfield, and Paris (2004), in a comprehensive review of studies on student engagement, argued that engagement is enhanced by tasks that are authentic, that provide opportunities for students' sense of ownership and personal meaning, that foster collaboration, that draw on diverse talent, and that are fun.

Our sense is that there is a variety of sources from which teachers can choose tasks that have at least some of these characteristics.

The data presented below are from the Task Type and Mathematics Learning[1] (TTML) project, which is investigating the best ways to use different types of mathematics tasks, particularly in Grades 5 to 8. Essentially the project focuses on four types of mathematical tasks that we describe as follows:

Type 1: Teacher uses a model, example, or explanation that elaborates or exemplifies the mathematics.

Type 2: Teacher situates mathematics within a contextualised practical problem to engage the students, but the motive is explicitly mathematics.

Type 3: Teacher poses open-ended tasks that allow students to investigate specific mathematical content.

Type 4: Teacher poses interdisciplinary investigations in which the assessment of learning in both mathematical and non mathematical domains is possible.

The four types of tasks are designed to represent potentially successful task types. The focus of our research is to describe in detail how the tasks respectively contribute to mathematics learning, the features of successful exemplars of each type, constraints which might be experienced by teachers, and teacher actions which can best support students' learning.

The Challenge of Converting Tasks to Lessons

A range of teacher actions is necessary to plan and deliver lessons utilising potentially interesting tasks. In other words, there are specific actions that teachers must take to transform tasks into effective lessons. One perspective on the process of converting tasks to lessons has been described as the *intended* curriculum, the *implemented* curriculum, and the *attained* curriculum (e.g., Robitaille et al., 1993). Similarly, Gehrke, Knapp, and Sirotnik (1992) described the *planned*, *enacted,* and *experienced* curriculum, while Burkhardt, Fraser, and Ridgway (1990) referred to the *ideal*, *adopted*, *implemented*, *achieved*, and *tested* curriculum. Our interest within the project is mainly in the implemented, or enacted, curriculum—the ways in which a teacher takes a syllabus or curriculum guidelines or standards or particular mathematics tasks and enacts them in the classroom—although this article focuses on the intended, or planned, or adapted, curriculum.

This article examines the "factors influencing set up" that Stein, Grover, and Henningsen (1996) presented in a model of task use. One aspect of their model described how the features of the mathematical task as set up in the classroom, and the cognitive demands the task makes of students, are informed by the mathematical task as represented in curriculum materials, and influenced by the teacher's goals, subject-matter knowledge, and knowledge of students. We draw on teachers' responses to some prompts about a particular task to make inferences about teacher knowledge and orientation, and to inform future emphases in our project.

We note that conventional wisdom would suggest that primary-level teachers have well developed language about pedagogy, and also that the mathematical knowledge of many was perhaps restricted. Likewise it is conventionally assumed that secondary teachers are strong mathematically but have less sophisticated language to describe pedagogy. Even though the project participants were from active clusters with a professional development orientation, there was no reason for assuming they were any different from teachers generally.

Categories of Teacher Knowledge

There are two major categories of knowledge needed to convert tasks to lessons, and for the teaching of mathematics generally: subject matter knowledge; and pedagogical content knowledge. Hill, Ball, and Schilling (2008) described diagrammatically components of these two types of knowledge. When converted to text, these categories and subcategories were:

Subject Matter Knowledge
common content knowledge
specialised content knowledge
knowledge at the mathematical horizon
Pedagogical Content Knowledge
knowledge of content and teaching
knowledge of content and students
knowledge of curriculum

Under the category of subject matter knowledge, the authors described three subcategories. The first, *common content knowledge*, is the mathematics needed to solve a task. This is the type of knowledge that someone who is not a teacher but who is good at mathematics might have.

The second is *specialised content knowledge*, or "the knowledge that allows teachers to engage in particularly *teaching* tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems" (Hill et al., 2008, p. 378). This category would also include the recognition that a task can be solved in different ways. *Specialised content knowledge* is also described as the mathematical knowledge and skill uniquely needed by teachers in the conduct of their work (Ball, Thames, & Phelps, in press). In looking for patterns in errors made by students or in considering whether a student-generated solution strategy could be generalised to other tasks, a teacher is drawing upon this knowledge.

The third subcategory, *knowledge at the mathematical horizon,* is described by Ball et al. (in press) as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (p. 42).

The second main category, pedagogical content knowledge (PCK), was first described by Shulman (1986) as:

> an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. If those preconceptions are misconceptions, which they so often are, teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners. (pp. 9–10)

Ball et al. (in press) also delineated three subcategories of PCK. The first, *knowledge of content of teaching,* includes an understanding of how to sequence particular content for instruction, of how to evaluate instructional advantages and disadvantages of particular representations, and of the knowledge required to make "instructional decisions about which student contributions to pursue and which to ignore or save for a later time" (p. 38).

The second subcategory, *knowledge of content and students,* is "knowledge that combines knowing about students and knowing about mathematics" (Ball et al., in press, p. 36). This subcategory includes, for example, anticipating students' cognitive and affective responses to particular tasks, and knowing what they will find easy and hard. "They [teachers] must also be able to hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language" (p. 37).

Ball et al. argue that these two subcategories coincide with the two central dimensions of Shulman's PCK: "the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of the most frequently taught topics and lessons", and "the ways of representing and formulating the subject that make it comprehensible for others" (Shulman, 1986, p. 9).

The third subcategory, *knowledge of curriculum,* is slightly different in that it relates closely to Shulman's (1986) 'curricular knowledge', in which curriculum "is represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances" (p. 10).

We use these subcategories to summarise and interpret the responses of the teachers who responded to our questionnaire items.

The particular research questions we explore in this article are:

- What do teachers see as the content focus of an illustrative mathematics task?
- What sorts of hypothetical lessons do teachers create to include the illustrative task?
- Are there differences between responses of secondary and primary level teachers?

We are also interested in whether the Ball et al. (in press) categories are helpful in describing different aspects of the knowledge that teachers might need to convert tasks to lessons.

The Illustrative Task

As part of a survey given to teachers, one prompt sought insights into the extent to which teachers could describe the content of a particular task, and the ways in which they might convert the task to a lesson. The survey was given initially to 41 primary teachers and 6 secondary teachers from the TTML project. To allow better primary/secondary comparisons, the survey was also completed by 26 willing experienced primary teachers who were participating in a mathematics teacher learning session, and 34 further secondary mathematics teachers who were in the same session.

The task was what we term *type 1*, in that it is focused on the mathematics (and was without a practical context and not particularly open-ended according to our definition). The task was presented to teachers as follows:

The following is a description of an idea that might be used as the basis of a lesson:

Which is bigger $\frac{2}{3}$ *or* $\frac{201}{301}$ *?*

Readers are encouraged to take a moment to consider how they might decide which of these fractions is larger. Although many people state the correct answer quickly, they often have difficulty articulating their reasons for the conclusion they draw. Strategies that emerged when we have previously posed this task to prospective and practising teachers include the following:

- Finding common denominators and comparing 602/903 to 603/903 (or comparing 602 to 603 after cross multiplying)
- Using a calculator to divide numerator by denominator to convert to decimals and then comparing 0.6666… to 0.66777…
- Realising that 2/3 is the same as 200/300, and then considering the shift from 200/300 to 201/301. The reasoning is often along two lines, one correct, one incorrect. The correct reasoning takes the form of "I'm adding the same to the numerator as the denominator, but proportionally more to the numerator, so the ratio is increased". Some try this with simple cases to convince themselves that doing so will increase a fraction. That is, by going from 4/5 to 5/6 to 7/8, you are increasing the fraction, as it is getting closer to 1 each time, as the small part to be added to take the fraction to 1 is decreasing—this has been termed *residual thinking* (see Clarke, Sukenik, Roche, & Mitchell, 2006). This will work for all fractions between 0 and 1. The incorrect reasoning is that we are adding 1 to the numerator and 1 to the

denominator, so that there is no real change, and the two fractions are the same.

- Adding 199 to the numerator and 298 to the denominator. Because the ratio of these two numbers is slightly more than 2/3 (0.66778…), the effect must be to increase the fraction overall.
- Some calculate the residual needed to build 200/300 and 201/301 to the whole, respectively. They determine that the first fraction requires 100/300 to do so, and the second 100/301, and because the 301ths are smaller parts, the residual needed is smaller.
- A teacher in the United States used a basketball analogy to give a convincing alternative strategy. He argued that we could think of 200/300 as a basketball player's free throw success rate, as 200 successful throws out of 300. In moving to 201/301, the basketball player has had one more throw, which was successful. His average must therefore have improved, and so 201/301 must be larger.

While any one of these strategies might be described as *common content knowledge*, awareness of the range of possible tasks is better considered as *specialised content knowledg*e.

What do Teachers Identify as the Content of the Task?

The first prompt to which teachers were invited to respond was:

If you developed a lesson based on this idea, what mathematics would you hope that the students would learn?

In our view, the fundamental mathematical concept is comparing fractions, with the task offering opportunities for students to seek alternate or intuitive strategies as well as considering formal approaches to comparisons, such as finding a common denominator or converting to decimals.

The teachers' responses to the survey question were inspected, common responses were identified, initial categories were proposed, responses were categorised, and then some categories were combined if doing so did not detract from the integrity of the categories. In what follows, the categories are presented, along with examples of the responses of teachers that were so categorised, and then some comparative data on the responses are considered.

Responses were categorised as *Students learn various ways of comparing* if we considered that teachers appreciated various ways of solving the task, and that they sought for students to learn more than a single approach. Some illustrative responses from the teachers were:

- Understanding size of fraction, some ideas of equivalence, ordering fractions, changing fractions into decimals (possibly).
- Fractional understandings – What is the size of the pieces? How many pieces are we talking about? What proportion of the whole is each fraction?
- Knowledge of equivalent fractions, ability to "cancel down", place value of decimal numbers.

In each case, teachers have identified multiple aspects of the task, and the aspects they described are indeed important for solving the task.

Responses were categorised as *A single specific concept (not comparing)* if they referred to only one concept. A number of responses used the words of only one concept, such as:

- Equivalent fractions.
- Place Value.
- Fractions.

There were also more descriptive responses, although they still focused on just one concept:

- Looking at concepts related to fractions equivalence.
- Concept of equivalent fractions.
- Students would learn to compare two fractions the method I would use would be changing fraction to a decimal.

The "place value" response is puzzling, but perhaps this implies an assumption of converting to decimals, although even in this case seeing the key concept as place value is difficult. It is also difficult to see the essence of the task as equivalence. Even though converting to denominators of 300 or 903 does indeed involve equivalent fractions, solving the task requires more. We suspect that these teachers either used language to describe concepts loosely, or they did not identify the relevant concepts in the task.

Responses were coded as *General and only vaguely related to multiple concepts* if they mentioned a number of concepts, but which were only vaguely related to the task. Examples of responses so categorised were:

- What is a fraction? What does the notation mean? What do we mean by bigger?
- Meaning of numerator/denominator/ equivalent fractions.
- What a fraction is. Fractions are just another way to write a number where they are used in everyday life.
- Concept of larger smaller.

These seem to be vague and general and sufficiently removed from the task to suggest that those teachers would have difficulty focusing a discussion with students on the key elements of the task.

Table 1 presents the number of teachers whose responses were coded in those three categories; the responses are also categorised according to whether the teacher was or was not involved in TTML (the project) and whether the teacher taught in primary or secondary school.

Table 1
Categorisation of Responses Describing the Focus of the Task

	TTML Primary	TTML Secondary	Other Primary	Other Secondary
Students learn various ways of comparing	8	4	7	19
A single specific concept (not comparing)	22	1	7	9
General and only vaguely related multiple concepts	11	1	12	6

Overall 58% of the secondary teacher respondents and 22% of the primary teachers not only identified the essence of the task as comparing fractions but also that students might well be exposed to different ways of comparing. This means that about two-fifths of the secondary teachers and over three quarters of the primary teachers choose a single content area (that was not "comparing"), or a range of vaguely related ones. This finding raises the possibility that some of these teachers were not able to identify readily the focus or potential of this mathematical task. It should be noted that the prompt for the teachers was not limiting and that considerable space was provided for the answer.

We recognise that these are teachers' responses to a particular item, completed in the midst of a larger survey, and so may tend to underestimate the possible responses of the teachers. Nevertheless, as Ball and Bass (2001) argued, in order to help students reason mathematically, the teacher must do the following: uncover the students' current base of common knowledge; establish and extend the students' base of common knowledge; and model and guide the construction of acceptable mathematical arguments. Taking these steps clearly requires the teachers' own *common content knowledge* to be well established. Also, a reasonable number of these teachers appeared to evidence limited *specialised content knowledge*, and one could suspect that these teachers would have difficulty matching tasks to curriculum statements, or matching resources to particular tasks. This identification or deciphering of the mathematics content that a task is likely to elicit goes beyond *common content knowledge*. This process requires *specialised content knowledge* in that the teachers are required to analyse and articulate the content in more detail than is required in simply solving the problem.

It is also noted that, in order to communicate a response, teachers need the language and concepts that would be associated with *knowledge of content and curriculum*. This includes knowing what the terms "equivalent fractions" and "fraction comparisons" mean and do not mean. It is possible that some limitations in the teachers' responses may be connected to inadequate understandings of the meaning of such key terms.

Designing a Lesson

The second related prompt on the teacher survey was as follows:

Describe, briefly, a lesson you might teach based on this idea.

We had hoped to see descriptions of lessons that would allow students to see that there were various ways to solve the task, and ideally that teachers would plan specifically to allow students to explore the task in their own way at some stage. As before, the responses overall were inspected, categories developed, the responses allocated to categories, and then categories and allocations adjusted. The resultant categories are presented, along with examples of responses of teachers whose responses were so categorised. Some quantification of the responses follows.

The first category was described as *student centred, perhaps using a meaningful example, but emphasising student generation of strategies and discussion.* This category would include teachers who seek to engage students in their learning and who remain open to the possibility of students generating their own approaches, including fostering discussion. The following are some examples of responses categorised in this group:

- Take in counters or used student nos in class say you want 2 out of 3 in one pile and 201 out of 301 in the other. Which is the larger proportion? Get students to answer qn demonstrate answer to class encourage diversity of answers praise all attempts even those that are wrong as long as they are thinking and prepared to write down their thinking
- 2/3 or 201/301 if you were going to explain the answer to someone choose either multi media, poster concrete material to do so. In your explanation include written language as well as pictorial
- Pose the problem brainstorm processes we could use to determine the solution have groups attempt to solve the problem using their choice of methods Have the group answer the question and explain their process going over the solution

discuss any errors in comprehension practice some other problems

These can be taken as a reasonable representation of the “reform” approach, and would certainly allow the possibility of the type of intended solutions that were described above.

The second category was described as *a teacher-centred lesson, incorporating a specific strategy for teaching the task*. This category would include teachers who adopt a direct teaching approach, and who see mathematics as a set of procedures to be learned effectively. Such teachers might focus on a specific approach and teach it explicitly. The following are some examples of responses categorised in this group:

- Change both factions to decimal numbers and find which one is bigger
- Firstly fractions what and how they are used, start with simpler fractions and build up you could do other examples of close but not quite the same fractions 6/7, 7/8 extend

While the latter response suggests an intent to build understanding, the phrasing implies teacher direction with specific teacher intent for the students’ learning. Perhaps this approach could be described as good traditional teaching. Given that the teachers identified an appropriate focus for the task, the lessons have a chance of producing useful learning for their students. Nevertheless, the teachers are still missing key opportunities that this question offers.

A third category was *real-life exemplars but only vaguely related to the concepts.* There is considerable emphasis in both curriculum documents and teacher support advice (e.g., National Council of Teachers of Mathematics, 2000) on the importance of using relevant examples. The responses in the third category seem to prioritise this aspect over the mathematical concepts:

- If I wanted to focus on fractions I would begin having the kids identify life situation where fractions are used. From this I would physically have them compare their ideas to see if they could rate them from smallest to largest.
- A man who orders a pizza asked the chef to cut the pizza into 6 slices because he won’t think he can eat the 8.
- I use Dove soap. Since I am allergic to moisturising cream, I use ¾ of the soap and throw the other ¼ away

These seem vague, and it is difficult to know how the contexts would clarify the mathematical concept that was the intended focus.

The fourth category was *teacher-centred but with a general strategy not specific to the task.* The following are some examples of the response categorised in this group:

- Converting between fraction decimals and percentage ratio another ways of writing ratio when doing drug calculations
- A lesson taught would involve place value and fractions dealing with whole numbers, this would require the use of concrete material to enable students a visual prompt
- Give students flashcards with fractions written on them. Ask them to place themselves in order from smallest to largest, followed by whole group discussion on what they need to know to complete the task

Such descriptions seem to address only vaguely the general topic of fractions, but have not communicated that they are in any way addressing the hypothetical focus.

A fifth and catch all category is described as *nothing or don't know*. There was a range of responses categorised as "don't know." Some of these teachers wrote the words, and others gave no response at all. Note that it is not inferred that this is due to fatigue in completing the survey, since all teachers completed subsequent questions including some that required open responses. There were also responses such as the following in this category:

- I feel it would be a very dry lesson unless I could find a hands on investigative approach. By looking at this problem I lack confidence so would probably shy away from it

It is noted that this teacher rated his/her knowledge of mathematics as 5/10, and his/her confidence at teaching mathematics at 4/10. A key development need for this teacher is clearly improved *common content knowledge*.

Table 2 presents the number of teachers whose responses were coded in these five categories of hypothetical lessons; the data are categorised according to whether the teacher was or was not involved in TTML, and whether the teacher taught at primary or secondary school.

Table 2
Categorisation of Teacher-Created Lessons

	TTML Primary	TTML Secondary	Other Primary	Other Secondary
Student centred, perhaps using a meaningful example, emphasising student generation of strategies and discussion (or the process)	5	1	4	4
A teacher-centred lesson, incorporating a specific strategy for teaching the task	7	3	4	6
Real-life or concrete examples but only vaguely related to the concepts	16	1	7	14
Teacher centred but with a general strategy not specific to the task	6	0	0	3
Nothing or don't know	6	1	13	3
Totals	40	6	28	30

About 13% of the lessons (that is, 14 out of the overall total of 104) were of the "reform" style; percentages were similar for primary and secondary teachers. Assuming that the teachers implement their hypothetical lesson effectively these lessons would be likely to foster learning, utilising the potential of the task.

Around 19% of the lessons (that is, 20 out of the overall total of 104) could be described as good traditional teaching, a higher proportion of secondary teachers (25%) than primary teachers (16%) described teaching of this type. Given that the teachers identified an appropriate focus for the task, such lessons have a good chance of producing useful learning for the students. On the other hand, it is less likely that important opportunities would be taken up in these lessons than in student-centred lessons.

The other 70 lessons described do not create confidence that these teachers can transform a basic idea into an effective mathematics lesson. For example, 35% of the hypothetical lessons could be described as meaningless use of relevant or real-life examples. This is not what the emphasis on relevance is intended to achieve. When these lessons are taken on face value, one could suspect that most students would not learn effectively in such lessons. While there is variation in the distribution of responses, the responses of the primary and secondary teachers are similar, and the TTML teachers were similar to the others.

The knowledge necessary for converting tasks to lessons is clearly part of PCK, and would contain elements of *knowledge of content and teaching*, and *knowledge of content and students.* Only some teachers appeared able to do this the way we expected. It is noted that if the teachers do not have the *common content knowledge*, then aspects of the relevant PCK may not be able to be enacted (Thames, 2006). It is important to note that this is a challenging problem for primary students, and that we did not encourage the teachers specifically to indicate how they would adapt the task for the grade levels they teach.

After One Year Working in the Project

As part of a subsequent survey of TTML teachers, we posed the same prompts after the teachers had been working on our project for one school year. Because we were asking for other sensitive information as well, we did not ask for names or level of teaching so it was not possible to exactly match responses. Nevertheless the overall responses were analysed in the same way.

Table 3 compares the responses of the TTML teachers at the start and the end of the first year of our project.

Table 3
Categorisation of Responses Describing the Focus of the Task

	TTML start N = 47	TTML end N = 34
Students learn various ways of comparing	26%	29%
A single specific concept (not comparing)	49%	15%
General and only vaguely related multiple concepts	26%	56%

There were slightly fewer teachers in the second administration of the survey. The main change seems to have been that those who had earlier given a single specific response, other than comparing, now are more likely to give a list of general and only vaguely related multiple concepts. There was no specific focus on developing a language of teaching and so it was not anticipated that there would be much change in the teachers' responses to this prompt.

In general it seems that the teachers gave longer answers than previously. For example, a response categorised as "Teacher centred but with general strategy not specific to the task" was:

> Objective would be to illustrate equivalence and also consolidate fraction sense in terms of collections. Begin with a fraction wall and illustrate how ¼ is the same as 2/8. Look at other examples, starting with fractions from a "whole". Then show simple equivalent fractions using collections e.g. (4 out of 8) and (2 out of 4) challenging student to come up other examples. Ask many questions like "how did you do that?" and provide opportunities for share time.

Table 4 compares the responses the teachers gave to the prompt about creating a lesson.

Table 4
Categorisation of Teacher-Created Lessons

	TTML start N = 45	TTML end N = 34
Student centred, perhaps using a meaningful example, emphasizing student generation of strategies and discussion (or the process)	11%	24%
A teacher-centred lesson, incorporating a specific strategy for teaching the task	22%	21%
Real-life or concrete examples but only vaguely related to the concepts	38%	3%
Teacher centred but with a general strategy not specific to the task	13%	53%
Nothing or don't know	16%	0

The main change seems to have been that the teachers moved away from inappropriate use of contexts or materials, but seemed instead to have created somewhat vague lessons. Since the project was about creating lessons from tasks, and the teachers may have even written up descriptions of lessons they had taught, they might have been expected to provide more focus. Their difficulties might suggest that teachers need to identify the purpose of the task before being able to design an appropriate lesson.

Indeed, it seems that to do this teachers need reasonable familiarity with all aspects of both subject matter knowledge and pedagogical content knowledge.

It could be noted in this context that six out of the eight teachers who described a student-centred lesson had identified the task as being about comparing, and their lessons consider different aspects of this. On the other hand, 16 out of the 18 teachers who did not identify the main concept as comparing described a general but nonspecific lesson. This tends to suggest that both *common content knowledge* and *specialised content knowledge* are prerequisites—as distinct from being corequisites (National Council of Supervisors of Mathematics, 2007)—for pedagogical content knowledge.

What Might Make this Difficult?

We also asked teachers in the second survey to indicate what might make creating a lesson from this task difficult. Although the teachers were asked to respond to a hypothetical lesson, it appears that they have responded instead as if asked what might make teaching difficult generally. Many, for example, commented on the constraints that aspects of the students' readiness might present. The following are some of their responses.

Nine teachers suggested that inadequate prior knowledge may be a barrier, with comments such as:

- Children's lack of understanding of simple fractions &/or place value concepts.

There were five teachers who commented on the difficulty of the specific concept, for example:

- Conceptually hard - start with easier 1/2 of 2/3

There were five others who commented on specific misconceptions:

- Student misconceptions around the meaning of fractions, e.g. 'larger is bigger' could create the need for focused teaching for some students.

There were three teachers who referred to student attitudes, for example:

- Children may feel concept is too challenging - especially as it involves fractions

One commented on class size:

- Too many students.

Two commented on equipment limitations:

- Not sufficient equipment e.g. models of fractions

These comments all seem to relate to student difficulties, when the item was about teachers' difficulties in creating a lesson from this task. As teachers' expectation of student abilities and level is clearly one of the "factors influencing set up", it is not surprising that teachers thought about creating a lesson for "someone" as distinct from hypothetically. In doing so, teachers may well seek to reduce the demand of the tasks.

Revisiting Subject Matter Knowledge and Pedagogical Content Knowledge

One of the purposes of writing this report was to use the data as a way of clarifying different aspects of knowledge for teaching mathematics.

Unless a teacher is planning to just 'throw' the fraction comparison task to the class without having solved it personally, we believe that clearly a teacher needs relevant *common content knowledge* to determine for himself or herself which of 2/3 and 201/301 is larger. Our observations during the completion of the questionnaire indicated that this task was problematic for a number of teachers. Given our belief that an understanding of relevant *common content knowledge* is a necessary condition for successful translation of such tasks into classroom use, teachers' difficulty with the task may explain the inadequacies of a number of the responses.

Conversations with teachers around this task indicated that many are not aware of the wide variety of strategies that students might bring to it, aside from attempting to find common denominators or using a calculator to divide the respective numerators by their denominators. Such awareness draws upon *specialised content knowledge*.

Although it could be argued that the content is not appropriate for the students of those Grade 5 and 6 teachers who completed the survey, one would nevertheless hope that the teachers possessed sufficient *knowledge at the mathematical horizon* to discuss how the problem might be used in a junior secondary classroom.

We are also somewhat concerned that many teachers seemed unable to describe the mathematical content in terms that indicated that they realised this task was actually about fraction comparison. This understanding connects directly to *knowledge of content and curriculum*. The responses call into question the sense teachers make of curriculum documents including syllabuses (i.e., the intended curriculum), when knowledge of content and curriculum is limited.

It is possible that *knowledge of content and students* had the effect of limiting the vision that teachers had for the use of the task.

Implications for Professional Development for Teachers of Mathematics

It is clear that many teachers found translating the fraction comparison task into a worthwhile learning experience for middle school students difficult, or at least had great difficulty in articulating how they might do so. It also seemed that primary and secondary teachers were equally likely to create student-focused investigative type lessons.

It became clear to the project team, particularly when considering topics that both students and teachers find difficult (such as rational number), that professional development leaders need to take the time to focus on all six components of knowledge for teaching mathematics. It is also clear that we need to give a greater time commitment to discussing how such content might be addressed across the middle years. Importantly, we should not take for granted that all or even most teachers can necessarily translate a good idea or task into a worthwhile learning experience for students, without considerable professional-development support.

Note

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