

Convexification and Global Optimization in Continuous and Mixed-Integer Nonlinear Programming

Theory, Algorithms, Software, and Applications

by

Mohit Tawarmalani

*Purdue University,
West Lafayette, IN, U.S.A.*

and

Nikolaos V. Sahinidis

*University of Illinois,
Urbana, IL, U.S.A.*



KLUWER ACADEMIC PUBLISHERS

DORDRECHT / BOSTON / LONDON

Contents

Preface	xiii
Acknowledgments	xvii
List of Figures	xix
List of Tables	xxiii
1 Introduction	1
1.1 The Mixed-Integer Nonlinear Program	5
1.2 Branch-and-Bound	6
1.3 Illustrative Example	8
1.3.1 A Separable Relaxation	9
1.3.2 Tighter Relaxation	12
1.3.3 Optimality-Based Range Reduction	12
1.3.4 Drawing Inferences from Constraints	16
1.3.5 Branching on the Incumbent	17
1.4 Outline of this Book	18
2 Convex Extensions	25
2.1 Introduction	26
2.2 Convex Extensions of l.s.c. Functions	29
2.3 Multilinear Functions	40
2.4 Analysis of Convex Underestimators of x/y	43
2.4.1 Convex Envelope of x/y	44
2.4.2 Closed-Form Expression of Convex Envelope	45
2.4.3 Theoretical Comparison of Underestimators	47
2.4.4 Numerical Example	52

2.4.5	Concave Envelope of x/y	56
2.4.6	Relaxing the Positivity Requirement	57
2.4.7	Semidefinite Relaxation of x/y	60
2.5	Generalizations and Applications	62
2.5.1	Envelopes of $(ax + by)/(cx + dy)$	64
2.5.2	Convex Envelope of $f(x)y^2$	65
2.5.3	Convex Envelope of $f(x)/y$	69
2.5.4	Summation of Functions	69
3	Product Disaggregation	71
3.1	Introduction	72
3.2	Preliminaries	75
3.3	Reformulations of a Rational Function	77
3.4	Tightness of the Reformulation Scheme	81
3.5	Special Instances of the Reformulation	91
3.6	Examples of the Reformulation Scheme	93
3.6.1	Example 1: Hock & Schittkowski (1981)	94
3.6.2	Example 2: Nuclear Reactor Reload Pattern Design	95
3.6.3	Example 3: Catalyst Mixing for Packed Bed Reactor	97
3.7	Reformulations of Hyperbolic Programs	100
3.8	Upper Bounding of 0–1 Hyperbolic Programs	105
3.9	A Branch-and-Bound Algorithm	108
3.10	Cardinality Constrained Hyperbolic Programs	110
3.11	Computational Results for CCH Programs	111
3.11.1	Comparison of Bounds	112
3.11.2	Performance of the Proposed Algorithm	112
3.11.3	p -Choice Facility Location	115
4	Relaxations of Factorable Programs	125
4.1	Nonlinear Relaxation Construction	125
4.1.1	Concavoconvex Functions	130
4.2	Polyhedral Outer-Approximation	132
5	Domain Reduction	147
5.1	Preliminaries	147
5.1.1	Legendre-Fenchel Transform	148
5.1.2	Lagrangian Relaxation	152
5.2	An Iterative Algorithm for Domain Reduction	153

5.3	Theoretical Framework: Abstract Minimization	154
5.4	Application to Traditional Models	160
5.5	Geometric Intuition	163
5.6	Domain Reduction Problem: Motivation	163
5.7	Relation to Earlier Works	164
5.7.1	Bounds Via Monōtone Complementarity	177
5.7.2	Tightening using Reduced Costs	178
5.7.3	Linearity-based Tightening	179
5.8	Probing	181
5.9	Learning Reduction Procedure	183
6	Node Partitioning	189
6.1	Introduction	189
6.2	Partitioning Factorable Programs	190
6.2.1	Branching Variable Selection	190
6.2.2	Branching Point Selection	194
6.3	Finiteness Issues	196
6.3.1	Stochastic Integer Programs	197
6.3.2	The Question of Finiteness	198
6.3.3	Key to Finiteness	199
6.3.4	Lower Bounding Problem	200
6.3.5	Upper Bounding	202
6.3.6	Branching Scheme	203
6.3.7	Finiteness Proof	205
6.3.8	Enhancements	205
6.3.9	Extension to Mixed-Integer Recourse	207
6.3.10	Computational Results for Stochastic Programs	207
7	Implementation	213
7.1	Design Philosophy	213
7.2	Programming Languages and Portability	215
7.3	Supported Optimization Solvers	216
7.4	Data Storage and Associated Algorithms	216
7.4.1	Management of Work-Array	216
7.4.2	List of Open Nodes	217
7.4.3	Module Storage: Factorable Programming	218
7.5	Evaluating Derivatives	219
7.6	Algorithmic Enhancements	221

7.6.1	Multiple Solutions	221
7.6.2	Local Upper Bounds	222
7.6.3	Postponement	222
7.6.4	Finite Branching Schemes	223
7.7	Debugging Facilities	224
7.8	BARON Interface	224
8	Refrigerant Design Problem	229
8.1	Introduction	229
8.2	Problem Statement	230
8.3	Previous Work	231
8.4	Optimization Formulation	232
8.4.1	Modeling Physical Properties	235
8.4.2	Modeling Structural Constraints	239
8.5	Multiple Solutions	249
8.6	Computational Results	249
9	The Pooling Problem	253
9.1	Introduction	254
9.2	The p- and q-Formulations	256
9.2.1	The p-Formulation	256
9.2.2	The q-Formulation	261
9.3	The pq-Formulation	264
9.3.1	Properties of the pq-Formulation	266
9.3.2	Lagrangian Relaxations	273
9.4	Global Optimization of the Pooling Problem	276
9.4.1	Branching Strategy	278
9.4.2	Computational Experience	279
10	Miscellaneous Problems	285
10.1	Separable Concave Quadratic Programs	285
10.2	Indefinite Quadratic Programs	289
10.3	Linear Multiplicative Programs	293
10.4	Generalized Linear Multiplicative Programs	297
10.5	Univariate Polynomial Programs	298
10.6	Miscellaneous Benchmark Problems	298
10.7	Selected Mixed-Integer Nonlinear Programs	305
10.7.1	Design of Just-in-Time Flowshops	305

10.7.2	The Gupta-Ravindran Benchmarks	311
11	GAMS/BARON: A Tutorial	313
11.1	Introduction	314
11.2	Types of Problems GAMS/BARON Can Solve	315
11.2.1	Factorable Nonlinear Programming: MIP, NLP, and MINLP	315
11.2.2	Special Cases of BARON's Factorable Nonlinear Programming Solver	316
11.3	Software and Hardware Requirements	320
11.4	Model Requirements	320
11.4.1	Variable and Expression Bounds	320
11.4.2	Allowable Nonlinear Functions	321
11.5	How to Run GAMS/BARON	321
11.6	System Output	322
11.6.1	System Log	322
11.6.2	Termination Messages, Model and Solver Status	324
11.7	Algorithmic and System Options	325
11.8	Application to Multiplicative Programs	325
11.8.1	LMPs of Type 1	326
11.8.2	Controlling Local Search Requirements	329
11.8.3	Reducing Memory Requirements via Branching Op- tions	331
11.8.4	Controlling Memory Requirements via Probing	333
11.8.5	Effects of Reformulation	334
11.8.6	LMPs of Type 2	335
11.8.7	Controlling Time Spent on Preprocessing LPs	339
11.8.8	LMPs of Type 3	342
11.8.9	Comparison with Local Search	347
11.9	Application to Pooling Problems	356
11.9.1	Controlling Time Spent in Preprocessing	364
11.9.2	Reducing Memory Requirements	368
11.9.3	Controlling the Size of the Search Tree	368
11.9.4	Controlling Local Search Time During Navigation	371
11.9.5	Reduced Branching Space	371
11.9.6	Pooling Problem Computations	372
11.10	Problems from <code>globallib</code> and <code>minplib</code>	376
11.11	Local Landscape Analyzer	380

11.12 Finding the K Best or All Feasible Solutions	383
11.12.1 Motivation and Alternative Approaches	383
11.12.2 Finding All Solutions to Combinatorial Optimization Problems	385
11.12.3 Refrigerant Design Problem	391
11.12.4 Finding All Solutions to Systems of Nonlinear Equa- tions	394
A GAMS Models for Pooling Problems	403
A.1 Problems Adhya 1, 2, 3, and 4	403
A.2 Problems Bental 4 and 5	411
A.3 Problems Foulds 2, 3, 4, and 5	416
A.4 Problems Haverly 1, 2, and 3	428
A.5 Problem RT 2	431
Bibliography	435
Index	463
Author Index	469