COOLING OF NON-NEUTRAL PLASMA BY ENERGY EXCHANGE

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Abstract

Crystallization of beams and non neutral plasmas is a fundamental process. A factor limiting the cooling of beams and non neutral plasmas is the energy exchange between the different degrees of freedom. To understand this problem, we study the crystallization of a strongly magnetized, non neutral plasma, where the exchange between perpendicular and parallel energy is small (in the limit of gyroradius small compared to Debye length). We introduce a microwave bath to transfer energy from the poorly cooled parallel degree of freedom to the well cooled (by synchrotron radiation) perpendicular degree of freedom. Results for the cooling of the parallel degree of freedom will be presented.

1 INTRODUCTION

The concept of crystalline non-neutral plasma, regarded as a new state of matter, has been studied for a variety of fundamental and applied physics areas, including the study of space-charge-dominated beams, the study of Coulomb crystals, the realization of high luminosity ion colliders, the application to ultra-high resolution nuclear experiments and to the atomic physics research, etc. Crystallization occurs as non-neutral plasmas and beams are cooled below the transition temperature. In fact, as seen in many Penning trap experiments, the non-neutral plasmas has three different phases: fluid, fcc, and bcc[1]. Crystallization in one dimension has been observed in the beams at the Aarhus accelerator[2], in agreement with calculations[3], and crystallization in three dimension has been observed in the ion Penning trap at NIST[4] and in dusty plasmas[5].

In high energy physics, Penning traps and antiparticle storage rings have been used for experimental tests of the CPT theorem, which predicts equivalence of various physical parameters such as masses, charge-to-mass ratio, magnetic moments, and gyromagnetic ratio for particles and antiparticles. Charged particles can be confined perfectly in an ideal cylindrically symmetric trap with a uniform axial magnetic field, which is the basic setup of the Penning trap. This approach, the use of Penning trap, has been favored and widely used because the particle can be cooled down to a temperature of the order of tens of mK. Penning traps at CERN have been used to capture antiparticles for high-resolution measurements for proton mass and for mass spectrometry of nuclei[6].

Recently, laser cooling has been the primary approach towards obtaining such ultra cool beams and plasmas. A laser that is tuned to a frequency below the resonant frequency of the ion is directed at the ionic plasma. Ions moving towards the laser beams see an upshifted laser beam, and thus can absorb the light. Subsequently they spontaneously emit a photon isotropically. Thus, in the full process, they lose momentum by recoil. This leads to cooling. Such a method naturally works only for ions, not electrons or protons, as they have the internal resonances needed for narrow absorption. For non-ionic beams, electron cooling has been used, but such cooling has not produced ultra cool beams[7].

For this reason we already investigated phase transition of strongly magnetized electron plasmas in Penning traps and we concluded that the phase transition can occur on the condition that longitudinal temperature is below a certain value irrespective of transverse temperature. Now the question is how to decrease the longitudinal temperature to the critical value. As one of the possible ways we suggest a microwave cooling method. Applying a tuned microwave into the longitudinal direction, the longitudinal energy can be reduced and then the temperature can be dropped down below the critical value. This means that the electron plasma crystallization can be achieved. In the following section we explain the thermodynamics by molecular dynamics simulation of the strongly magnetized plasmas and show how their phase responds to the longitudinal temperature. In the next section we suggest how to reduce the longitudinal temperature and the result is shown.

2 STRONGLY MAGNETIZED PLASMA

2.1 MD SIMULATIONS

The strongly magnetized non-neutral plasma is an ideal system of mobile particles of charge Q, number density n, and temperature k_BT, immersed in a heat bath. With the Wigner-Seitz radius $a \equiv (3/4\pi n)^{1/3}$ and $\sqrt{3}\omega_p^{-1}$ ($\omega_p \equiv \sqrt{4\pi n Q^2/m}$) as the units of length and time, the thermodynamics of the non-neutral plasma can be described in terms of some dimensionless parameters. One of the important parameters, namely the Coulomb coupling parameter, is

$$\Gamma = \frac{Q^2}{ak_BT} \tag{1}$$

which is roughly the ratio of the Coulomb potential energy to the thermal energy per particle.

The plasma immersed in a heat bath finally equilibriates with the bath, so that the temperature of the system is identical to the bath. Therefore, the system can be described

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by a dissipative system with a fluctuation. The dissipative force continues to take the energy out of the system before the system equilibriates to a stable state due to the equalization between the dissipation and the fluctuation. Also, a strong magnetic field forces gyromotions of the particles much faster than drift motions, so that the adiabatic invariance of the magnetic moment effectively reduces the system from the actual motion to a very good approximation, guiding center approximation. In this case the guiding center position, X and Y are conjugate to each other, so that the system can be described by two pairs of conjugate variables. The dimensionless equations of motion in MD simulations are

$$\frac{d\xi_z^{(k)}}{d\tau} = V_z^{(k)} \tag{2}$$

$$\frac{dV_z^{(k)}}{d\tau} = -\sum_{j\neq k}^{N} \frac{\partial\hat{\Phi}}{\partial\xi_z}(\vec{\xi}^{(k)} - \vec{\xi}^{(j)}) + \nu V_z^{(k)} + \delta_z^{(k)} \tag{3}$$

$$\frac{d\vec{\xi}_\perp^{(k)}}{d\tau} = -\frac{\omega_p}{\sqrt{3}\Omega}\sum_{j\neq k}^{N} \nabla_\perp\hat{\Phi}(\vec{\xi}^{(k)} - \vec{\xi}^{(j)}) \times \hat{z} - \frac{\nu}{3}\frac{\omega_p^2}{\Omega^2}\sum_{j\neq k}^{N} \nabla_\perp\hat{\Phi}(\vec{\xi}^{(k)} - \vec{\xi}^{(j)}) + \vec{\delta}_\perp^{(k)} \times \hat{z} \tag{4}$$

where τ is the dimensionless time, $\vec{\xi}^{(k)}$ is the dimensionless location of k-th particle, Ω is a gyrofrequency of the plasma ($\Omega = QB/mc$), and ν is the dissipative coefficient and $\delta^{(k)}$ is the fluctuation. The dimensionless potential $\hat{\Phi} = a\Phi/Q^2$ for the one component plasma

$$\hat{\Phi}(\vec{\xi}) = \sum_{\vec{n}} \frac{erfc(\sqrt{\pi}|\vec{\xi} + \vec{n}\Lambda|/\Lambda)}{|\vec{\xi} + \vec{n}\Lambda|} + \sum_{\vec{n}\neq 0} \frac{exp(-\pi|\vec{n}|^2)cos(2\pi\vec{n}\cdot\vec{\xi}/\Lambda)}{\pi|\vec{n}|^2\Lambda} \tag{5}$$

which is the well-known Ewald potential for the OCP[8], is the effective pair potential describing the interaction of any two particles ($\vec{\xi} = \vec{\xi}_k - \vec{\xi}_j$) with all periodic images of the latter ($\Lambda \equiv (4\pi N/3)^{1/3}$). From the equations we get the coupling parameter of the equilibrium state,

$$\Gamma_{||} \equiv \frac{Q^2}{ak_BT_{||}} \approx \frac{2\nu}{\langle\sum_k|\vec{\delta}_k|^2/N\rangle_\tau} \tag{6}$$

with a small ν and a constraint, $\langle\sum_k|\vec{\delta}_k|^2/N\rangle_\tau = const.$

In MD simulations with the equations of motion of the system and a given initial state (bcc or fluid), all the thermodynamic quantities are determined as time averages of the physical quantities including internal energy for a canonical ensemble after a typical relaxation time. Taking possible initial states, such as bcc or fluid with their initial velocities of particles, the energies can be calculated, and then the phase transition can be found at the intersection of the two Helmholtz free energies calculated from the internal energies for bcc and fluid.

2.2 PHASE TRANSITION

By taking various temperatures with bcc states as their initial states, we calculated the internal energies for the various temperatures. In some cases, phase transition was found during the simulation. The transition depends on the initial state and the heat bath temperature. Fig. 1 shows the phase transition during the simulation. The initial state is a fluid state and the $\Gamma_{||}$ is almost 200. A sudden change in the potential of a particle is seen at $\tau \approx 1350$. At this time, the transition from fluid to bcc can be found.

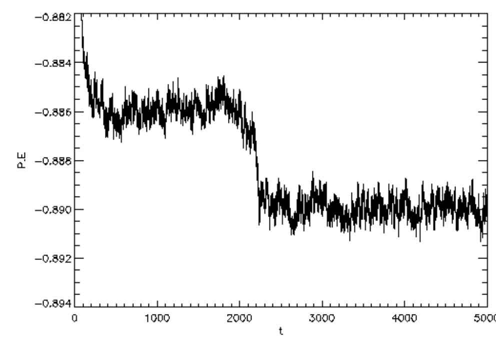

Figure 1: The potential is suddenly changed at $\tau \approx 1350$.

With the internal energies and the Helmholtz free energies for various values of Γ, we finally conclude that the phase transition temperature is almost the same to that of unmagnetized plasma. In Fig. 2 and Fig. 3, the energies are shown as functions of the Coulomb coupling parameter $\Gamma_{||}$ and the phase transition is found at $\Gamma = 170$.

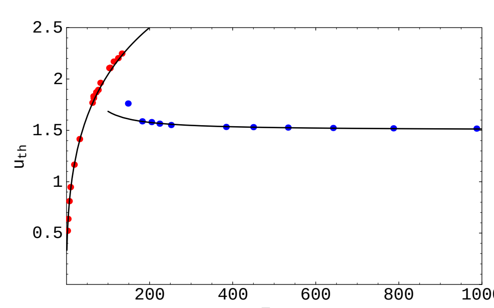

Figure 2: The internal energies as functions of $\Gamma_{||}$. Red - $fluid$ and $Blue$ - bcc.

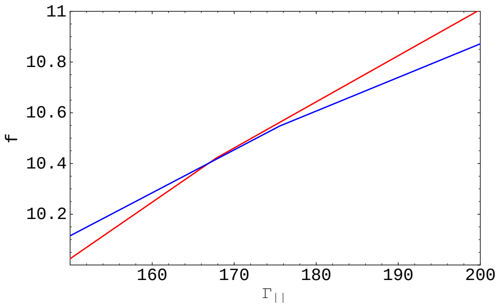

Figure 3: The free energies as functions of $\Gamma_{||}$. Red - $fluid$ and $Blue$ - bcc.

3 MICROWAVE COOLING

In the low transverse temperature limit, the longitudinal energy can be reduced by the microwave radiation. Absorption of a microwave photon in Penning trap by an electron, thus moving it up one in Landau state, can reduce the parallel energy, just as laser cooling works for ionic plasma and equilibria. Then the spontaneous radiation reduces the transverse energy, so that the transverse state move back to the original Landau state and finally the transverse temperature is the same to the heat bath temperature. From the entire process only the longitudinal temperature can be decreased.

The longitudinal velocity distributions during the process can be described by Master equations. With an assumption that most of particles are in the ground or first excited state, the equations are

$$\frac{\partial f_0(u_z)}{\partial t} = \int dv_z f_1(v_z)D(v_z,u_z) - f_0(u_z)\int dv_z W(v_z,u_z) \quad (7)$$

$$\frac{\partial f_1(u_z)}{\partial t} = \int dv_z f_0(v_z)W(v_z,u_z) - f_1(u_z)\int dv_z D(v_z,u_z) \quad (8)$$

where $f_n(p_z)$ is the distribution of transverse quantum number n. The spontaneous and the stimulated transition probabilities are defined as

$$P_{||} = \frac{2e^2\omega_R^4}{3c^3}\frac{1}{2}\frac{\hbar\Omega}{m\Omega^2} \quad (9)$$

$$W_{||} = \frac{I\sigma_0}{\hbar\omega_R}\frac{(\gamma/2)^2}{(\gamma/2)^2+\Delta^2(\omega,k_{||},u_z,v_z)} \quad (10)$$

$$D_{||} = \frac{P_{||}}{\hbar\omega_R} \quad (11)$$

with resonance conditions,

$$\Delta \equiv \omega - \omega_R \quad (12)$$

$$\hbar k_{||} = m(v_z - u_z) \quad (13)$$

$$\hbar\omega_R = \hbar\Omega + \frac{m}{2}(v_z^2 - u_z^2). \quad (14)$$

Taking appropriate conditions about the initial state, the longitudinal temperature can be estimated after the state reaches an equilibrium. We apply 5T as its magnetic field, 4.2K as its initial temperatures, $0.7\times10^9/cm^3$ as its number density to the plasma in the trap, and 10^5 as the Q factor of microwave cavity. With the values the longitudinal temperature decreases to 1.8mK which is below the critical temperature (T_c = 14mK). Fig 4 shows the longitudinal velocity distribution of the ground state after the state reaches to an equilibrium.

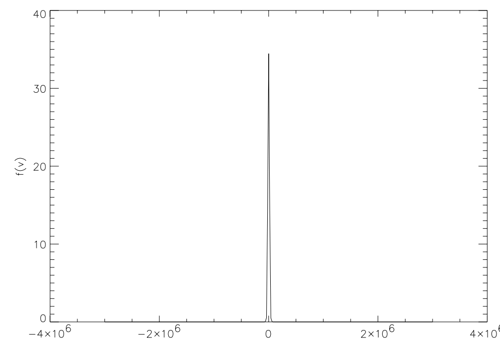

Figure 4: The velocity distribution for the ground state.

4 DISCUSSION

We applied the molecular dynamics simulation to a strongly magnetized plasma and concluded that the crystallization can be achieved below a longitudinal critical temperature irrespective of transverse temperature. Applying microwaves into the longitudinal direction, the temperature can be decreased below the critical temperature by exchange of energy between two degrees of freedoms. With the condition that an appropriate low temperature heat bath reduces the transverse energy of the plasma continuously, the electron plasma crystallization can be achieved in Penning traps.

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