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Cooling of Pre-Galactic Gas Clouds by Hydrogen Molecule

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According to the expanding hot universe model, neutralization of the cosmic plasma ceases at the stage of radiation temperature $T_r \simeq 4000°K$. After then, a density contrast in the uniform medium grows into a contracting gas cloud with mass greater than $10^{5\sim6}M_{\odot}$. Hydrogen molecule is formed in the cloud after the stage of $T_r \simeq 300°K$, and the thermal evolution is largely affected by the cooling through H_2, whose processes are studied quantitatively in this paper. The present paper is a preliminary one for a more interesting problem of galaxy formation.

§ 1. Introduction

Since the discovery of the cosmic black-body radiation,[1] the hot universe model proposed by Gamow[2] has attracted much attention; many authors studied various evolutional stages of the universe: a stage of elementary particle interaction before 1 sec since the beginning of the universe,[3] a stage of nuclear interaction during $10\sim10^3$ sec,[4] a stage of atomic process after 10^5 years[5)~8)] and a stage of galaxy formation after $10^8\sim10^9$ years.[9)~11)] An aim of this paper is to discuss a close connection between the stage of atomic process and that of galaxy formation. As is well known in the study of star formation, a powerful cooling mechanism which converts kinetic energy into radiation is indispensable for a formation of gravitational bound objects.[12] In the pre-galactic matter, heavy elements such as carbon are not contained, and the sole possible cryogen is H_2 molecule.[13)~15)]

An evolutional stage in which H_2 molecules are able to be formed follows the recombination of hydrogen at the radiation temperature $T_r \simeq 4000°K$. By the recombination, almost all protons change into neutral hydrogens, but such a small fraction as $H^+/H = e/H = 10^{-5}\sim10^{-3}$ remains unrecombined;[6)~8)] the electrons, afterward, work as a catalyzer to form H_2 molecules through H^- ions. Since the degree of dissociation of H^- by the black-body radiation is very high in the early stage, the molecular formation is postponed until T_r becomes below 300°K.[15] Therefore, the formation of bound gas cloud is also possible only after this stage.

After the recombination, the matter and the radiation decouple each other, and the non-uniform density contrast grows into gravitationally contracting cloud with mass larger than about $10^5 M_{\odot}$.[9] Without cooling, hydrogen atoms in it would soon be ionized again and the cloud would finally be dispersed by radiation pressure. In the contracting dense cloud, however, sufficient amount of H_2 to cool the matter is formed until H_2 molecules are dissociated by the collision with hydrogen atoms.

As to the subsequent evolution of these gas clouds, Peebles and Dicke[11] have assumed them to be proto-globular clusters, and Dorshkevich et al.[10] have assumed them to be super-massive stars named Urstar. In this paper, we give no quantitative discussion about this problem but we make an assumption akin to Dorshkevich et al.'s assumption.

In § 2 the recombination of He and H, and the formation of H_2 in the expanding uniform medium are summarized.

In § 3 the evolutional track of thermal history in the contracting gas cloud is calculated, and the results are shown in § 4.

In § 5 some conjectures about the subsequent evolution of the contracting gas cloud are given.

§ 2. Recombination of atom and formation of hydrogen molecule

After the element formation at the early stage, the cosmic matter consisting of hydrogen and helium is in a fully ionized state. Recombination of hydrogen atom has been investigated by Peebles,[6] Zeldovich et al.[7] and us.[8] In this section, we describe the recombination processes of hydrogen and helium to see a subsequent formation of H_2 molecule. Evolutional sequence of these processes is shown in Fig. 1. Calculated evolution of the abundance is shown in Fig. 3 in the case of the flat model.*)

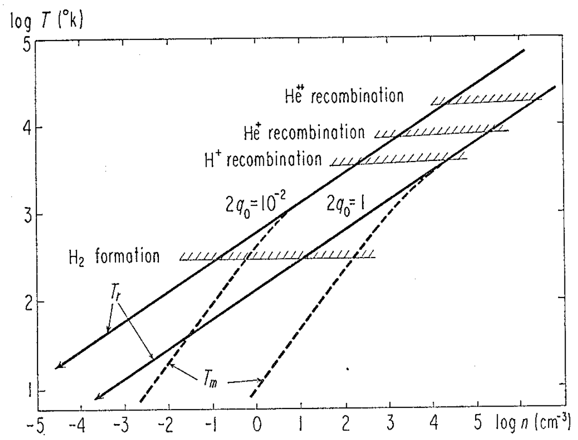

Fig. 1. Evolutional sequence in the expanding hot universe. Evolutional pathes of radiation temperature T_r and matter temperature T_m in the uniform medium are shown for the two models with the deceleration parameter, $2q_0=10^{-2}$ and $2q_0=1$.

(a) *Recombination of He^{++} into He^{+}*

When the radiation temperature decreases below $1.8\cdot10^{4}$°K, ionization equilibrium shifts suddenly towards He^{+} phase. However, detailed mechanism of electron capture is not so simple because recombination photons inhibit remarkably further recombination. The same mech-

*) Throughout this paper, we assume the universe model to be of the Friedman type characterized by the deceleration parameter $q_0=\rho_{m0}/(1.86\cdot10^{-29}\mathrm{g/cm}^3)$, where ρ_{m0} is the present matter density. The flat model means $q_0=1/2$.

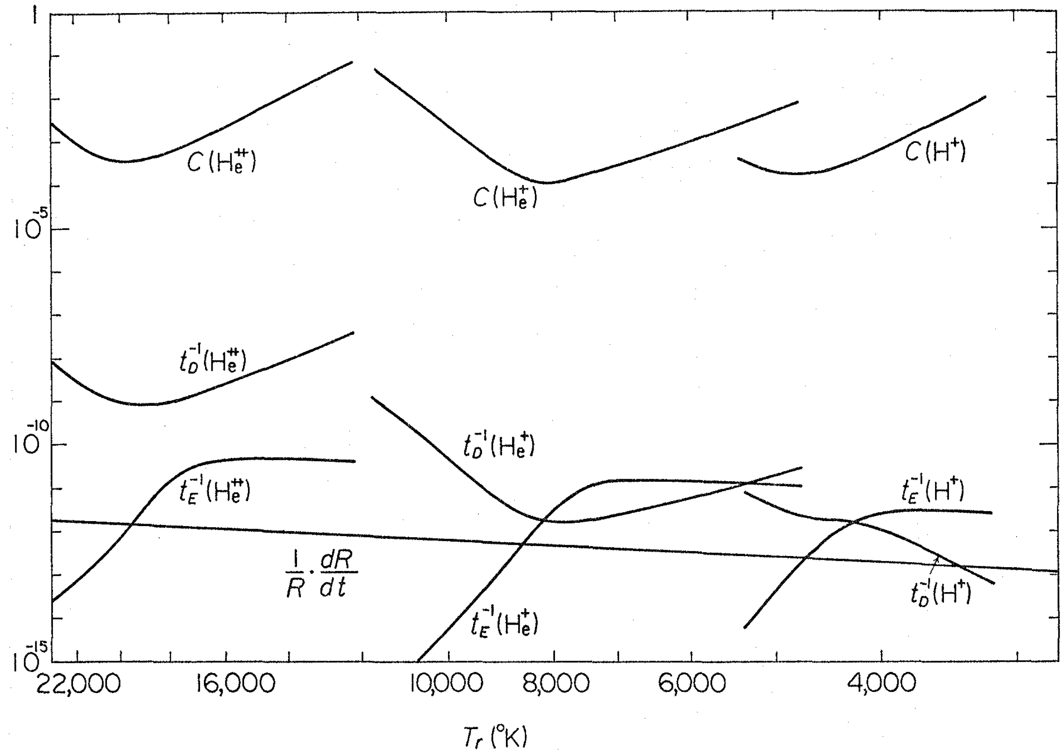

Fig. 2. Variation of the characteristic quantities of the recombination processes. Reduction factor of the direct capture C refers to a non-dimensional unit of the ordinate. The inverses of the time scale t_D^{-1}, t_E^{-1} and $(dR/dt)/R$ refer to a unit of sec^{-1} of the ordinate.

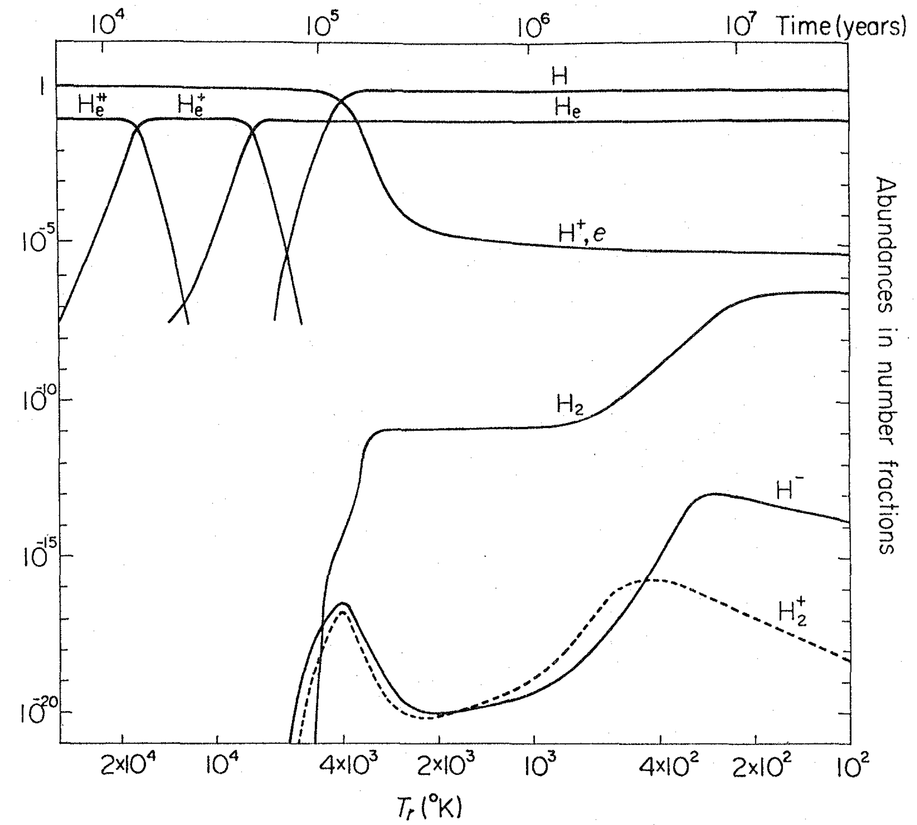

Fig. 3. Evolution of the abundances in the uniform medium for the flat universe model $2q_0=1$.

Table I. The characteristic quantities of the recombination processes. n_a, n_I, n_{LY} and $n_I(2s)$ are the number density of the each element, that of the photon above the ionization energy from the ground state, that of the photon above the excitation energy of $2p$ state and that of the photon above the ionization energy from the 1st excited state, respectively. The lowest row represents the ratio of the transition into the ground state through the allowed transition to that through the two photon emission at the temperature of recombination, where the degree of recombination is 1/2.

Atom	He^+	He	H
Ionization Energy B_1(ev)	54.40	24.59	13.6
Temperature of Recombination (°K)	18,000	8,000	4,000
n_a/n_I	$4.9\cdot10^3$	$8.6\cdot10^3$	$3.1\cdot10^6$
$(H/c)/(\sigma_I n_a)$	$1.6\cdot10^{-10}$	$1.3\cdot10^{-10}$	$4.4\cdot10^{-11}$
n_a/n_{LY}	$1.8\cdot10^3$	$1.5\cdot10^5$	$3.6\cdot10^5$
$H/c/(\sigma_{LY}n_{LY})$	$9.9\cdot10^{-15}$	$1.8\cdot10^{-14}$	$2.2\cdot10^{-5}$
$n_I(2s)/n_a$	$3.4\cdot10^6$	$2.9\cdot10^7$	$7.3\cdot10^4$
two photon emission rate A (sec^{-1})	526.5[a)]	46[b)]	8.227[c)]
Electron Capture rate α (cm^{-3} sec^{-1})	$1.14\cdot10^{-10}T^{-1/2}$	$6.3\cdot10^{-12}T^{-1/2}$	$2.84\cdot10^{-11}T^{-1/2}$
Lyman-α rate/two quantum rate	$4.0\cdot10^{-2}$	$1.5\cdot10^{-1}$	$1.8\cdot10^{-2}$

a) M. Lipeles, R. Novick and N. Tolk, Phys. Rev. Letters **15** (1965), 690.
b) A. Dalgarno, Month. Notices Roy. Astron. Soc. **131** (1966), 311.
c) L. Spitzer, JR. and J. L. Greenstein, Appl. J. **114** (1951), 407.

anism acts in the case of the recombinations of He^+ and H^+, and we explain this mechanism taking the recombination of He^{++} as an example.

As shown in Table I, the number density of the ionizing photons n_I of the black-body radiation at $T_r=1.8\cdot10^4$°K is much less than that of helium element n_α. Furthermore, a mean free path l_I of the ionizing photon at Lymann limit is very small compared with the radius of the horizon c/H. Therefore, a direct capture of electrons to the ground state of He^+ is followed immediately by the direct ionization of He^+ by the recombination photons. For the photon of Lyman series, the above situation is the same as that seen from Table I. Thus, the ionizing photons and Lyman series photons repeat their emission and absorption, and gradually split into Lyman α photons and lower energy photons. Since the accumulated Lyman α photons inhibit seriously the transition from $2p$ state to $1s$ state, the transition through the two-photon emission from $2s$ state to $1s$ state becomes the most frequent way to $1s$ state.

Following the notation of Peebles,[6] we write the equation of the recombination as follows,

$$-\frac{d}{dt}\left(\frac{n_{\mathrm{He}^{++}}}{n_\alpha}\right)=\left(\frac{\alpha n_e n_{\mathrm{He}^{++}}}{n_\alpha}-\frac{\beta n_{\mathrm{He}^+}}{n_\alpha}e^{-(B_1-B_2)/kT}\right)C\,, \tag{1}$$

where B_n is a binding energy in the n-th principal quantum number, α is recombination rate to all the excited states, β is photodissociation rate from these states and C is a factor which represents degree of reduction of direct capture to the ground state. The factor C is given by

$$C=\frac{1+K\Lambda n_{\mathrm{He}^+}}{1+K(\Lambda+\beta)\,n_{\mathrm{He}^+}} \tag{2}$$

and

$$K=\lambda^3 a/(8\pi da/dt)\,, \tag{3}$$

where a, λ and Λ are the scale factor of the universe, the wave length of the resonance line photon and the two photon emission rate given in Table I respectively. The value of C is given in Fig. 2. Using this value, we also give an inverse of the delated recombination rate $t_D{}^{-1}=\alpha n_e C$. Further, we give an inverse of the time scale of ionization equilibrium $t_E{}^{-1}=(dn_{\mathrm{eq}}/dt)/n_{\mathrm{eq}}$, n_{eq} being an equilibrium value given by the Saha formula, and an inverse of the time scale of cosmic expansion H. As seen from Fig. 2, $t_D{}^{-1}\gg t_E{}^{-1}$, and we can conclude that the recombination proceeds maintaining thermal equilibrium.

(b) *Recombination of He$^+$ into He*

In this case, we must pay attention to the existence of the two-spin states of He, because some of radiative transitions between them are highly forbidden; the decay time from the lowest triplet state 2^3s to the ground state 1^1s is larger than 10^{10} sec.[16a)] Therefore, we may be tempted to assume the triplet state to be a meta-stable state during the recombination process. As is known in the study of solar corona, however, such an assumption is wrong, since the thermal equilibrium between the two-spin states is maintained by the spin exchange collision of electrons. In the hot universe at $T_r\simeq10^4$°K, the transition times between neighbouring excited levels such as $2^1s\rightleftarrows1^3s$ and $2^1s\rightleftarrows2^3p$ are of the order of $10^2\sim10^3$ sec[16b)] which is shorter than the electron capture time. Then, we may assume that the excited states are in a thermal equilibrium in contact with the free electrons. Under these situations, the triplet state may be neglected when we consider the recombination to the ground state.

The recombination mechanism is similar to the case of He^{++}, and the characteristic quantities are given in Table I and Fig. 2. The reduction factor in this case is given by

$$C=\frac{1+K\Lambda n_{\mathrm{He}}e^{\Delta E/kT}}{1+K(\Lambda+\beta)\,n_{\mathrm{He}}e^{\Delta E/kT}}\,. \tag{4}$$

The difference of this expression from Eq. (2) arises from the non-degeneracy of 2^1s and 2^1p; the energy difference is given by $\Delta E=0.61$ eV.

Figure 2 shows that the ionization equilibrium is not maintained in the early stage of recombination, but the equilibrium is recovered before the time of H^+ recombination. In this way, almost all the α-particles become neutral.

(c) *Recombination of H^+ into H*[6)~8)]

The characteristic quantities are also given in Table I and Fig. 2. In comparison with the above two cases, the free electrons decrease appreciably during the recombination, and there remains a small fraction of ionized hydrogen. The fraction of the remaining ionized atoms is of an order of $H^+/H=10^{-3}\sim10^{-5}$, corresponding to the model with $2q_0=10^{-2}\sim1$.[8)]

(d) *Formation of H_2*[15)]

The remaining electrons and protons work as a catalyzer to form H_2 by the reactions given in Appendix B. In the uniform medium in the universe, the formation of H_2 is obstructed because H^- is photodissociated by the black-body radiation up to the stage of $T_r\simeq300°$K. The amount of produced H_2 is not sensitive to the universe model, and is limited to $H_2/H\simeq10^{-6.6}$ for $2q_0=10^{-2}\sim10^1$.

§ 3. Formation of local bound systems and their subsequent contraction

(a) *Mass spectrum of clouds*

After the recombination of hydrogen, the interaction between matter and radiation ceases because of disappearance of electrons. Uniform medium, which has been prevented from contracting by the radiation drag, begins to fragment into local condensations. There exists a lower limit of mass of condensation, M_c, above which matter can contract. In the fully ionized stage, M_c is as large as $10^{18}M_\odot$.[10)] However, it decreases rapidly with decoupling of plasma, and the limit M_c agrees with Jeans mass limit $M_J=10^{1.7}(T_m{}^3/n)^{1/2}M_\odot$ in the static medium, T_m and n being temperature and number density of matter in the universe, respectively. In the expanding medium, however, M_c becomes about $10M_J$ because the expansion prevents the contraction of matter.[17)] Though the magnitude of M_c depends both on model of the universe and on the epoch when the matter begins to contract, it is nearly constant, being of the order of $10^{5\sim6}M_\odot$.[10)] Suppose that the density fluctuation is random, the mass spectrum of the bound systems is given as follows,[11)]

$$N(M)\,dM\propto\begin{cases}dM/M^2 & M\geq M_c\,,\\ 0 & M<M_c\,,\end{cases}\tag{5}$$

where N is the number of bound systems whose masses lie between M and $M+dM$. Therefore, it seems that the bound systems with masses near M_c are formed most frequently. In the following, we consider the evolution of a contracting gas cloud which is left behind in the expanding universe.

(b) *Formation of H_2 in a contracting gas cloud*

If there are no cooling mechanisms, the temperature of a gas cloud rises adiabatically and collisional ionization occurs. Then, the gas cloud becomes opaque to radiation because of electron scattering. In Fig. 4, the opaque region is shown with hatched parts for a gas cloud with mass of $10^6 M_\odot$. When the gas cloud becomes optically thick, it confines the background radiation in it. After the epoch when T_r is lower than 100°K, the effect of the confined radiation is negligible, and the gas cloud can contract furthermore. In this way, the epoch after which a bound system is formed would not be the epoch of recombination of plasma but the epoch when $T_r = 100$°K, if there were no cooling mechanisms.

Now, we consider about cooling mechanisms. In the interstellar space, there are many kinds of cooling matter, such as molecules, ions of heavy elements, neutral atoms of heavy elements and grains;[12] they absorb kinetic energy of gas and set it free in a form of infrared radiation. In the early stage of the universe, however, there are no heavy elements nor grains, and the sole possibility is in H_2 molecules.[13)~15)] The H_2 molecules excited rotationally by the collision with H atoms can emit the infrared radiation. In the contracting stage of gas cloud, both the density and the temperature of gas rise, and the abundance of H_2 increases to $H_2/H \simeq 10^{-3}$.[15]

As atomic and molecular processes which take place in the gas cloud, we adopt the reactions given in Appendix B. In the following calculations, we neglect helium atoms and consider the pure hydrogen gas, because helium atom is inactive.

If $T_r > 300$°K, the H_2 molecules can hardly be formed because of the reaction equations (B·6) and (B·9). Therefore, a bound system begins to be formed after the epoch of $T_r = 300$°K.

(c) *Evolutional path of massive gas clouds*

The cooling rate per unit volume is denoted by Γ. Then, the cooling time scale of the gas is given by[12]

$$t_c = \frac{3nkT}{2\Gamma}, \tag{6}$$

where n and T are the mean number density and the mean temperature of matter in the cloud, respectively. The cooling rate Γ is taken for the range $n \leq 10^2$ and $T \leq 5 \cdot 10^{3}$°K from Takayanagi and Nishimura,[18] and for the range $n \leq 10^4$ and $T \leq 10^{4}$°K from Nishimura.[19] In the limit of high density, the population among the rotational levels approach the Boltzmann distribution, and Γ can be expressed in a form of analytic function. For the sake of simplicity, we adopt an approximation formula for Γ as follows,[11),13)]

$$\Gamma = \mathrm{Min}\{2.5\cdot10^{-31}T^{2.4}n^{2/3}n_{\mathrm{H}_2},\ 2.7\cdot10^{-30}T^{3}n_{\mathrm{H}_2}\}\ \mathrm{erg\ cm^{-3}\ sec^{-1}}. \tag{7}$$

Next, we consider the time scale of adiabatic heating. For the mass greater than M_c, pressure gradient is negligible and we assume that the contraction is nearly free fall; the heating time scale is the free fall time scale given by

$$t_f = \left(\frac{32\pi}{3}Gm_{\mathrm{H}}n\right)^{-1/2}, \tag{8}$$

where G and m_{H} are gravitational constant and the mass of hydrogen atom respectively.

The evolutional path of a gas cloud in the log n-log T plane can be given approximately by the line of $t_c = t_f$. The cooling rate Γ depends on the abundance of H_2 which increases with density, so we draw the line for $H_2/H = 10^{-4}$ at lower densities, and for $H_2/H = 10^{-3}$ at higher densities.

The temperature of a gas cloud contracting from the initial point A rises adiabatically as $T \propto n^{2/3}$ while the abundance of H_2 is small. As the abundance

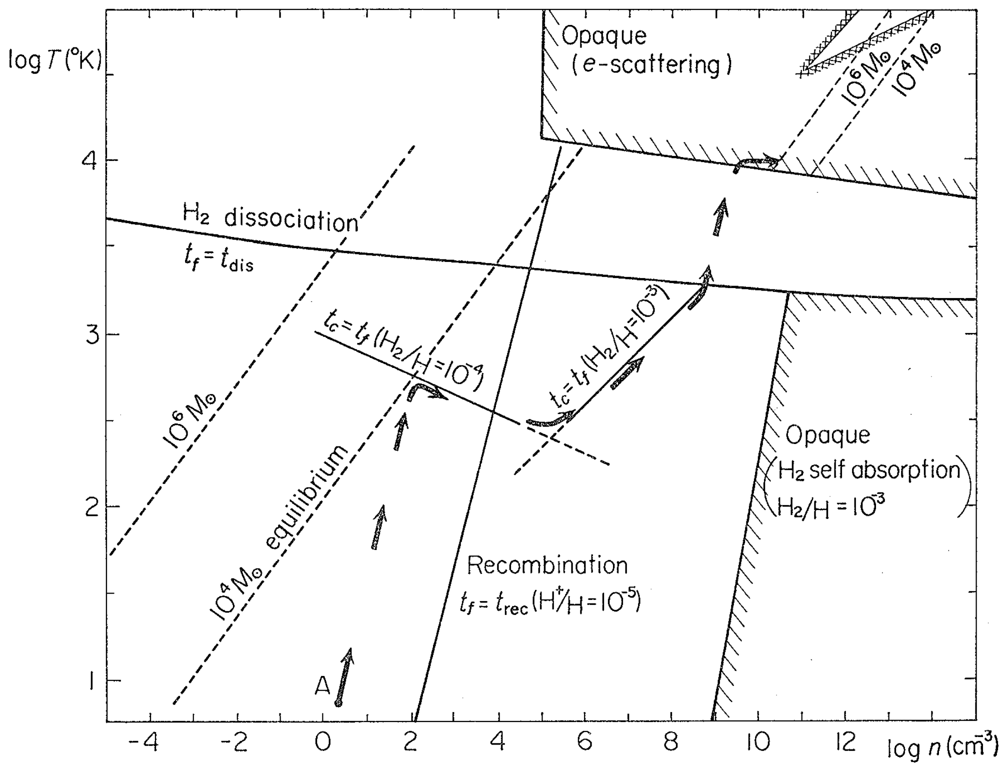

Fig. 4. The schematic features of evolution of the gas cloud with mass $10^6 M_\odot$ in the density temperature diagram. The time scales of cooling, free fall, recombination and dissociation are denoted by t_c, t_f, t_{rec} and t_{dis}, respectively. The thick arrow is an evolutional path in the gas cloud; this path approximately follows the line of $t_c = t_f$. The gas cloud is opaque in the hatched region. The dashed lines represent equilibrium lines of the respective masses, on which a mechanical equilibrium is attained but a thermal one is not necessarily attained. In the triangular region on the upper right side, a stable massive star can exist.[22)]

of H_2 increases, the temperature stops rising and rather decreases. With increasing density, the electrons and protons which act as a catalyzer for formation of H_2 molecules recombine with each other. If we denote the reaction rate of Eq. (B·1) as α, then the recombination time scale t_{rec} is written as

$$t_{rec}=1/\alpha n_e\,, \tag{9}$$

where n_e is the number density of electron. In Fig. 4, the line of $t_{rec}=t_f$ is drawn assuming $H^+/H=10^{-5}$; after the gas cloud pass through this line, the number of e and H^+ decreases appreciably and the formation of H_2 is suppressed.

The dissociation of H_2 molecules by the process Eq. (B·11) occurs at rather low temperature; if we denote the reaction rate of Eq. (B·11) as γ, we can define the dissociation time scale t_{dis} as

$$t_{dis}=1/\gamma n_H\,. \tag{10}$$

In Fig. 4 we draw the line of H_2 dissociation, $t_{dis}=t_f$; above this line, dissociation occurs, and the temperature of the contracting gas cloud rises adiabatically. When the temperature becomes above about 10^4°K, H atoms are ionized by electrons and the cloud becomes optically thick.

As to the source of opacity, we take two processes: one is electron scattering and the other is self-absorption of H_2 molecules. We define the opaque region as a region in which optical depth τ becomes larger than unity. Taking the electron scattering as a source of opacity, the opaque region is defined by

$$\tau_e=\sigma nxR\geq 1\,, \tag{11}$$

where σ, x and R are Thomson cross section, ionization degree and the radius of the gas cloud. In Fig. 4, such a line is given for the case $M=10^6M_\odot$. Similarly, the line of $\tau_{H_2}=1$ is given by[20]

$$n(\tau_{H_2}=1)=10^{11.32}T^{3/4}(M/M_\odot)^{-1/2}\{(n_{H_2}/n_H)/10^{-3}\}^{-3/2}. \tag{12}$$

In Fig. 4, this relation is drawn taking $H_2/H=10^{-3}$. When the gas cloud comes into the opaque region, the background radiation is trapped, but in this case those radiation does not play an important role because of the large matter density. However, a new thermal radiation emerges through the free-bound and free-free processes, and finally thermal equilibrium between the matter and the radiation is attained. As the radiation pressure is dominant in the opaque region, the temperature rises along $T\propto n^{1/3}$, and, sooner or later, the bounce of the core occurs by the pressure gradient force, i.e. a supermassive star is born.[21] The region where supermassive star can exist stably is shown in Fig. 4 on the upper right corner.[22]

§ 4. Numerical computation

In order to solve our cooling problem, we must connect the hydrodynamic calculation with the variation of chemical composition. For this purpose we add the term of heat sink to the hydrodynamic code, HYDAC, which was developed to calculate the collapse of massive gas spheres in a previous paper.[21] We calculate the variation of chemical composition as follows. As the time scales of destruction of H^- and H_2^+ are very short compared with free fall time, we may assume that those elements are in equilibrium with the electrons and the

Table II. The initial conditions of central temperature, central number density, electron number fraction and H_2 number fraction.

$2q_0$	$\log(T_c)_0$	$\log(n_c)_0$	$(e/\mathrm{H})_0$	$(\mathrm{H}_2/\mathrm{H})_0$
1	0.75	−0.40	$6.0\cdot10^{-5}$	$2.7\cdot10^{-7}$
10^{-2}	1.08	−2.30	$1.2\cdot10^{-4}$	$3.7\cdot10^{-7}$

protons. Furthermore, we set $n_e/n=n_{\mathrm{H}^+}/n=1-n_{\mathrm{H}}/n$, and we may calculate only the abundances of e and H_2 from differential equations.

As to the model of the universe, we adopt two cases: the flat model $2q_0=1$ and an open model of $2q_0=10^{-2}$. The initial evolutional track of the central temperature and central density does not depend on the initial point, if T_r falls below 300°K. We assume that the density distribution is expressed as the Emden function of polytrope and the temperature distribution is $T/T_c=(n/n_c)^{3/2}$. The initial conditions are tabulated in Table II.

In the contracting gas cloud, turbulence may be generated. In order to include the effect of turbulent stress, we use the artificial viscosity Q.[21] Figure 5 shows evolutional paths of contracting gas cloud with mass $10^6M_\odot$ in the case $2q_0=1$ for the two cases without and with turbulence. The two cases show no

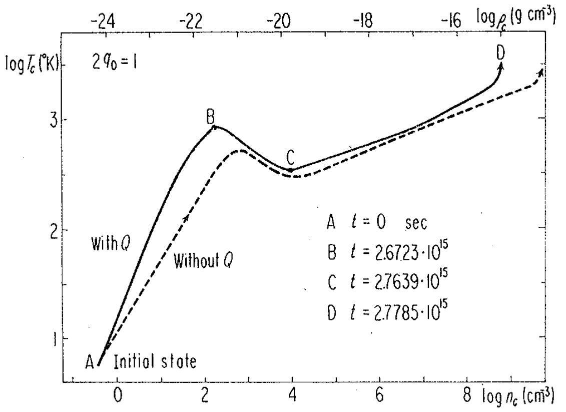

Fig. 5. Calculated evolutional path of the gas cloud with mass $10^6M_\odot$ in the density temperature diagram. The universe is supposed to be flat $2q_0=1$. Two cases are shown; the one with Q is the case that the artificial viscosity Q is added to the pressure term and the other is without.

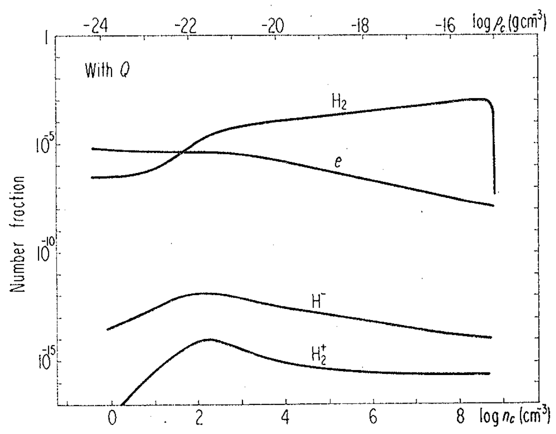

Fig. 6. Calculated time variation of chemical abundance at the center versus central density at that time.

serious difference, so a more accurate treatment of turbulence is unnecessary.

Figure 6 shows the variation of chemical elements in the innermost shell. Hydrogen molecules are easily destructed above 2000°K against the conjecture of Saslaw and Zipoy,[13] who supposed that H_2 is destructed above 6000°K. The distributions of the temperature T and the number density n are shown in Fig. 7, which shows that T and n in the innermost shell $(M_r/M \simeq 10^{-4})$ increase very much at the final stage of the calculation. The evolutional path of the different initial condition $(2q_0 = 10^{-2})$ is shown in Fig. 8. The final state D is almost the same for the both cases as can be seen from Figs. 5 and 8. The evolutional path does not sharply depend on the mass of gas cloud.

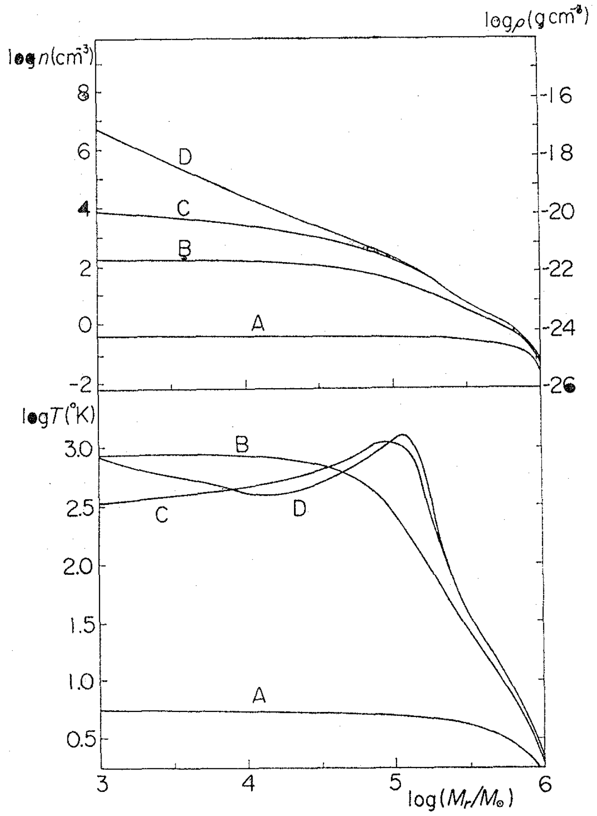

Fig. 7. Distribution of the density and the temperature at various times. The notations A, B, C and D represent the same time as in Fig. 5.

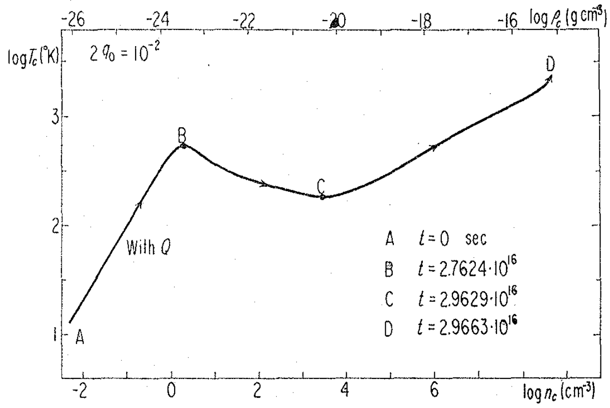

Fig. 8. Calculated evolutional path as in Fig. 5. The universe is supposed to be open $2q_0 = 10^{-2}$

§ 5. Subsequent evolution of the cloud into galaxies

Further evolution of the primordial gas cloud into galaxies has not been investigated quantitatively in this paper. Therefore, we only describe some assumptions about the subsequent evolution into galaxies.

(a) *Explosion of supermassive star and heavy element formation*

Structure of the falling cloud may, in general, have a condensed core and a diffuse envelope. As the contraction of core rushes into the opaque region, the contraction of the core may be stopped by a dominant radiation pressure and wait for the fall of the envelope. This growing core is assumed to become a single star with mass $M>10^4 M_\odot$ in contradiction with the assumption of fragmentation into ordinary stars, which we call a urstar.*),[10] The urstar will continue a slow gravitational contraction and the central temperature in it becomes high enough to start nuclear burning. A question as to whether the nuclear burning occurs in a quasistatic configuration or in a non-equilibrium state is not evident. However, the dynamical case is probable if we recall the general relativistic instability[22] and the pulsational instability[24] in nuclear burning concerning supermassive stars with dominant radiation. Nuclear burning may start from three α reactions, and rapid CNO cycle may follow using the products of 3α reaction.**) By the nuclear energy generation, the urstar may explode in some stage.***) Such a possibility is very probable, considering the calculation by Bisnovatyi-Kogan.[25]

Explosion of the urstar may disperse some fraction of the heavy elements produced in it.[26] Therefore, pre-galactic matter may already be polluted with the heavy elements. The origin of heavy elements on the surface of population II star may be these elements. These pregalactic heavy elements seem to be necessary to reconcile the age of old globular cluster with the age of the universe.

(b) *Heating of gas and generation of turbulence*

If the energy produced in the explosion is dispersed in a form of shock wave or cosmic ray, the surrounding medium may be heated[27] or may set in a turbulent motion. Energy flux of photon is ineffective to affect the gaseous medium, except that the ultraviolet photons ionize the hydrogen atoms and heat them up to $T_m \simeq 10^{4}$°K.****) If the hydrogen atoms are ionized again, the blackbody radiation suppresses the formation of newly-born clouds because of radiation drag, which, however, becomes ineffective again according to the decrease of the radiation temperature.

(c) *Formation of galaxies*

Galaxies are assumed to be formed after the explosion of the urstars. The surrounding medium after the explosion is changed in the following properties: (1) Because of heating of gas, Jeans' critical mass becomes larger. (2) Because of turbulent motion, a newly-born cloud may have angular momentum. (3) Because of powerful cryogen made from heavy elements, the cloud fragments into stars with ordinary masses.

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*) Though fragmentation occurs, masses of stars may be larger than $10^{2\sim3} M_\odot$.[23]

**) S. Ikeuchi, T. Matsuda, K. Nakazawa, H. Sato and H. Takeda, work in progress.

***) There is an upper mass limit of the exploding urstar, which is given as $10^{5.1} M_\odot$ in reference 25). Above this mass limit, the urstar may collapse infinitely and exist now as invisible mass.

****) If the urstar at the hydrogen burning were in an equilibrium state, the effective temperature of the surface is $6.4\cdot 10^{4}$°K and the Strömgren sphere around it has a dimension of $10^{1.55} \times (M/M_\odot)^{1/2}/n^{2/3}$ p.c. For $M>10^{-5} M_\odot/n^2$, this dimension is larger than the average distance of stars and the ambient medium may be in a state of HII region.

Appendix A

Method of calculation

In this Appendix, we show how to calculate the evolution of the abundances of e, H^+, H, H^-, H_2^+ and H_2. We denote the number fraction of these elements by $x_i (x_1 = n_e/n,\ x_2 = n_{H^+}/n$, etc., where n_e and n_{H^+} are the number density of electron and proton respectively and n is the total nucleon number density). We neglect the existence of helium, so $\sum_{i=1}^{6} x_i = 1$. Photon is numbered 7 and we put $x_7 = 1$. The rate of change of $x_i (i = 1, 2, \cdots 6)$ is then determined by the following equation

$$\frac{dx_i}{dt} = -\sum_j [i, j] x_i x_j + \sum_{k,l} [k, l] x_k x_l \,, \tag{A·1}$$

where the summation includes all reactions involving elements i. For reactions between photon and particle i, $[i, 7]$ is defined as

$$[i, 7] = c\langle n_\nu \sigma \rangle \,, \tag{A·2}$$

where n_ν and σ are the number density of photon and the cross section, respectively, and the average is taken over the Planck distribution. For reactions between two particles i and j, $[i, j]$ is defined as

$$[i, j] = n\langle \sigma v \rangle \,, \tag{A·3}$$

where v is the relative velocity and average is taken over the Maxwell distribution.

In actual calculations, there arise the cases that each term on right-hand side of Eq. (A·1) becomes very large compared with the term dx_i/dt, and the explicit calculation near the equilibrium state becomes difficult. In the case where the abundances of e, H^+, H and H_2 shift from the equilibrium abundances as in § 2, we solve Eq. (A·1) by the implicit method adopted by Wagoner.[28] In this method, we solve the six dimensional linear equations. However, the implicit method failed when T_r became lower than 400°K because the variation of H^- was too large. At lower temperature, therefore, we solve Eq. (A·1) as follows: x_4 and x_5 (or H^- and H_2^+) are determined by algebraic relation,

$$x_i = \sum_{k,l} [k, l] x_k x_l \Big/ \sum_j [i, j] x_j, \tag{A·4}$$

and the other elements are determined by the explicit method. In the calculation of § 4, the latter method is used.

Appendix B

Reaction rates

In this paper, we consider the following reactions; reaction rates are shown simultaneously.

$$H^+ + e \to H + \gamma^{29)} \quad [1,2] = 6.6 \cdot 10^{-11} T_m^{-1/2} n\,, \tag{B·1}$$

$$H_2^+ + e \to 2H + \gamma^{30)} \quad [1,5] = 10^{-7} n\,, \tag{B·2}$$

$$H^- + H^+ \to 2H + \gamma^{31} \quad [2,4] = 1.6 \cdot 10^{-6} T_m^{-0.45} n\,, \tag{B·3}$$

$$H + \gamma \to H^+ + e^{29)} \quad [3,7] = 1.6 \cdot 10^5 T_r \exp(-1.57 \cdot 10^5 / T_r)\,, \tag{B·4}$$

$$H + e \to H^- + \gamma^{11)} \quad [1,3] = 6.06 \cdot 10^{-19} T_m n\,, \tag{B·5}$$

$$H^- + \gamma \to H + e^{32)} \quad [4,7] = 1.53 \cdot 10^{-2} T_r^{2.4} \exp(-8.75 \cdot 10^3 / T_r)\,, \tag{B·6}$$

$$H^- + H \to H_2 + e^{33)} \quad [3,4] = 1.3 \cdot 10^{-9} n\,, \tag{B·7}$$

$$H^+ + H \to H_2^+ + \gamma^{34)} \quad [2,3] = 5.10^{-24} T_m^2 n\,, \tag{B·8}$$

$$H_2^+ + \gamma \to H^+ + H^{35)} \quad [5,7] = 1.08 \cdot 10^{-13} T_r^{5.34} \exp(-1.0 \cdot 10^4 / T_r)\,, \tag{B·9}$$

$$H_2^+ + H \to H_2 + H^{+14)} \quad [3,5] = 1.3 \cdot 10^{-9} n\,, \tag{B·10}$$

$$H_2 + H \to 3H^{36)} \quad [3,6] = 2.0 \cdot 10^{-6} T_m^{-1/2} \exp(-51944 / T_m) n\,, \tag{B·11}$$

$$H_2 + \gamma \to 2H^{37)} \quad [6,7] = 5.12 \cdot 10^7 \exp(-1.44 \cdot 10^5 / T_r)\,. \tag{B·12}$$

Here T_r and T_m are the temperature of radiation and of matter, respectively. [5, 7] is calculated from the assumption that $\sigma = 3.10^{-19}\text{cm}^2$ for $\lambda \leq 10^4$Å.[35)]

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