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Cooperation vs. Competition in R&D: the Role of Stability of Equilibrium

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We consider a model in which firms first choose process R&D expenditures and then compete in an output market. We show the symmetric equilibrium under R&D competition is sometimes unstable, in which case two asymmetric equilibria must also exist. For the latter, we find, in contrast to the literature that total profits are sometimes higher with R&D competition than with research joint venture cartelization (due to the cost asymmetry of the resulting duopoly in the noncooperative case). Furthermore, these equilibria provide another instance of R&D-induced firm heterogeneity.

Keywords: strategic R&D, research joint venture, unstable equilibrium.

JEL classification: C72, L13, O31.

1 Introduction

Recent studies by d'Aspremont and Jacquemin (1988, 1990) and Kamien et al. (1992) provide a performance comparison between various R&D cooperation scenarios, ranging from full cooperation, as in a cartelized research joint venture (RJV), to pure (strategic) R&D competition (see also Katz, 1986). One of the main results is that a cartelized RJV, which may be viewed as a situation where firms run one joint R&D laboratory at equal cost to each and fully share R&D results, yields the best performance among all the scenarios considered, in terms of R&D propensity, consumer surplus, and producer surplus. This result is based on a comparison of each scenario's symmetric equilibrium.

Our study shows that under R&D competition the firms' reaction functions may cross the "wrong" way. In this case the R&D game has three equilibria – one equilibrium is symmetric, interior, and *unstable* under Cournot best-reply dynamics. The other two equilibria are asymmetric, on the boundary, and locally stable. (According to Cournot best-reply dynamics, each firm best responds to his rival's action in the previous period.) The reaction functions in R&D decisions, given the unique second-stage equilibrium in outputs, are depicted in Fig. 1. If reaction functions cross as in Fig. 1a, then under Cournot best reply the firms' actions converge to the symmetric equilibrium. If they cross as in Fig. 1b then, depending on the starting point, actions either converge to one of the asymmetric equilibria or they eventually cycle between both firms choosing the maximal action and both choosing the best response to the maximal action.

When the R&D game has three equilibria, there are a number of reasons to take the asymmetric equilibria as a benchmark for the outcome under R&D competition. First, if Cournot best-reply dynamics converges, then it converges to one of the asymmetric equilibria and never to the unstable symmetric equilibrium. Another justification for selecting the asymmetric equilibria is suggested by results for Cournot oligopolies. Seade (1980) and Amir and Lambson (1997) show the (unique) unstable symmetric equilibrium in a Cournot oligopoly has economically unintuitive properties; in particular, the price is increasing in the number of firms. Furthermore, experimental evidence obtained for Cournot output games suggests that the interior Nash equilibrium predicts play well if the equilibrium is stable, but predicts play poorly if it is unstable (see Holt, 1995; Cox and Walker, 1997). Finally, we note that while the classical refinements of the Nash-equilibrium solution concept for one-shot games (such as normal-form perfection or stability in the sense of Kohlberg and Mertens, 1986) do not rule out the unstable equilibrium (in the sense of best-reply dynamics), some of the selection criteria from evolutionary game theory (Kandori et al., 1993) or adaptive learning (Milgrom and Roberts, 1991) do.

Using the asymmetric Nash equilibria as a benchmark for the outcome under R&D competition, we find in contrast to Kamien et al. (1992) that total profits are sometimes higher with R&D competition than with RJV cartelization. This result is obtained for a particular specification of the R&D production function, and for simplicity we assume there are no spillovers. For our example, like Kamien et al. we find that R&D propensity and consumer surplus are both higher under RJV cartelization than under the asymmetric equilibrium with R&D competition. Overall, therefore, our results only partially confirm the central conclusion of Kamien et al. We also show that the superiority of RJV cartelization over R&D competition can be re-established by imposing an additional condition insuring that demand is sufficiently high, relative to initial unit costs.

The analysis of Kamien et al. (1992) builds on d'Aspremont and Jacquemin (1988, 1990), the primary difference between the two, beyond the greater generality of the former, is that in Kamien et al. spillovers are in R&D expenditures while in d'Aspremont and Jacquemin spillovers are in R&D results (see Amir, 1997, for a discussion of the significance of this difference). In a perceptive note, Henriques (1990) points out that in d'Aspremont and Jacquemin the symmetric equilibrium under R&D competition is sometimes unstable. We go further by identifying the conditions under which the symmetric equilibrium is unstable, and by providing a re-examination of the conclusions of Kamien et al. in this case.

The existence of asymmetric Nash equilibria when firms engage in R&D competition provides an endogenous explanation of firm heterogeneity. Amir and Wooders (1997a, b) show that firm heterogeneity also arises endogenously when R&D spillovers flow only from the more R&D-intensive firm to its rival. Salant and Shaffer (1992) make the important point that an R&D cartel (maximizing total profit while keeping the spillover parameter at its original value) may well find it globally optimal to choose different levels of R&D for the two participating firms: This is another instance of R&D-induced firm heterogeneity.

2 The Models

Consider an industry in which two firms, each with unit cost $c > 0$, engage in a two-stage game. At the first stage, firms 1 and 2 choose R&D expenditures x_1 and x_2, respectively. An expenditure of x_i by firm i reduces its unit cost of production by $\lambda\sqrt{x_i}$, where $\lambda > 0$. Since firms cannot reduce their costs below zero, we have $x_1, x_2 \in [0, c^2/\lambda^2]$. At the second stage each firm observes its rival's R&D expenditure, and then chooses its output. In the output market the firms face a linear inverse demand with quantity units normalized so that $P(q_1, q_2) = a - (q_1 + q_2)$, where q_i is the output of firm i. Attention is restricted to subgame-perfect equilibria. Thus, our setup follows Kamien et al. (1992) with the spillover parameter set to zero, in which case the model is clearly equivalent to that of d'Aspremont and Jacquemin (1988).[1]

Assumption 1: Demand is sufficiently high relative to costs so that $a > 2c$.

1 Therefore, one can alternatively use cost reductions as the decision variables. Furthermore, given the absence of spillover effects, the model at hand is also equivalent to the model of Amir and Wooders (1997a), and both are special cases of the setup of Brander and Spencer (1983).

Assumption 1 insures that every subgame at the second stage has a unique Nash equilibrium with both firms in the market. In the subgame where firm i's R&D expenditure is x_i and its rival's expenditure is x_j, firm i's Nash output and profit are, respectively, $(a - c + 2\lambda\sqrt{x_i} - \lambda\sqrt{x_j})/3$ and $[(a - c + 2\lambda\sqrt{x_i} - \lambda\sqrt{x_j})/3]^2$. Firm i's profit in the overall game is its profit at the second stage less its R&D costs at the first stage. Specifically, when firm i's R&D expenditure is x_i and its rival's expenditure is x_j, then its profit in the overall game is

$$\Pi(x_i, x_j) = \left(\frac{a - c + 2\lambda\sqrt{x_i} - \lambda\sqrt{x_j}}{3}\right)^2 - x_i \tag{1}$$

Clearly firm i's payoff is concave in its R&D expenditure.[2]

Since the second-stage game has a unique Nash equilibrium, every Nash equilibrium of the game with payoffs given by (1) induces a subgame-perfect equilibrium of the two-stage game, and vice versa. For simplicity, we refer to the Nash equilibria of the game with payoffs given by (1) rather than the subgame-perfect equilibrium of the two-stage game.

Our next assumption insures that firm i's best response to an R&D expenditure of c^2/λ^2 by its rival is less than c^2/λ^2.

Assumption 2: The R&D production function is not too productive, i.e., $2\lambda^2 a/c < 9$.

If Assumption 2 did not hold, then it would be a dominant strategy for each firm to choose its maximal R&D expenditure, which is an uninteresting case.

Define the best-response functions in the usual way. For firm i, say, and $x_j \in [0, c^2/\lambda^2]$ let $r_i(x_j) = \arg\max\{\Pi(x_i, x_j) \mid x_i \in [0, c^2/\lambda^2]\}$. Since the game is symmetric, the firms' reaction functions are the same, i.e., $r_i(x) = r_j(x)$, and thus to reduce notation we write $r(x)$ for a firm's best response to an autonomous cost reduction of x by its rival.

Proposition 1: Suppose Assumptions 1 and 2 hold. Then each firm's

2 We have that $\partial^2\Pi(x_i, x_j)/\partial x_i^2 = -(\lambda/9)(a - c - \lambda\sqrt{x_j})x_i^{-3/2} < 0$ for $x_i > 0$.

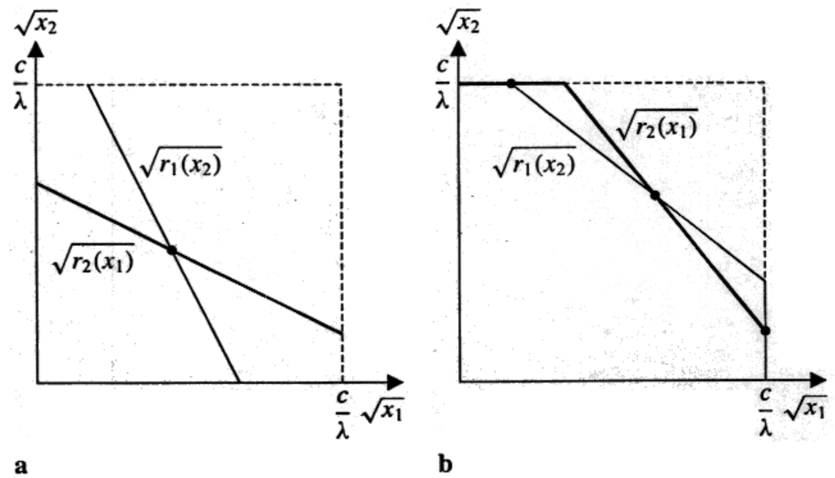

Fig. 1: a, $2\lambda^2 < 3$; b, $2\lambda^2 > 3$

reaction to an R&D expenditure of x by its rival, is given by

$$r(x) = 4\lambda^2\Big(\frac{a - c - \lambda\sqrt{x}}{9 - 4\lambda^2}\Big)^2 \wedge \Big(\frac{c}{\lambda}\Big)^2$$

Furthermore, (x^*, x^*) is a Nash equilibrium of the R&D game, with $x^* = 4\lambda^2((a-c)/(9-2\lambda^2))^2$. If $2\lambda^2 < 3$, then (x^*, x^*) is unique and stable. If $2\lambda^2 > 3$, then this equilibrium is unstable and the game has, in addition, two (locally) stable equilibria of the form $(\underline{x}, \bar{x})$ and $(\bar{x}, \underline{x})$ with $\bar{x} = (c/\lambda)^2 > 4\lambda^2((a-2c)/(9-4\lambda^2))^2 = \underline{x}$.

The two possible configurations for the reaction functions appear in Figs. 1a and b. Reaction functions have been "linearized" by taking the decision variables to be the square root of R&D expenditures.

We now consider R&D cooperation via a joint laboratory. With this form of cooperation, the R&D expenditure of a jointly owned laboratory is chosen in order to maximize the sum of the firms' profits when R&D results are fully shared. The R&D expenditure of the joint laboratory is given by the solution to

$$\max_x\Big\{\tfrac{2}{9}(a - c + \lambda\sqrt{x})^2 - x\Big\}. \qquad (2)$$

The solution to (2), denoted by x_J, is $x_J = 4\lambda^2((a-c)/(9-2\lambda^2))^2$.[3] Denote by Π_J each individual firm's profit under the joint laboratory when costs are equally shared. We note that the joint laboratory is equivalent to the case CJ, or cartelized RJV, in Kamien et al. (1992) in which firms conduct R&D in separately owned laboratories, and coordinate their R&D efforts to maximize total profits upon setting the spillover parameter to 1.[4]

Comparing the symmetric Nash equilibrium under R&D competition to the equilibrium under the joint laboratory yields: (i) total cost reduction is the same in each case, i.e., $2\lambda\sqrt{x^*} = 2\lambda\sqrt{x_J}$, and therefore total output and the output's price are also the same in each case, and (ii) total profits are higher under the joint laboratory. The preceding analysis is clearly a special case of the result of Kamien et al. establishing the superiority of the joint laboratory over R&D competition when the outcome under R&D competition is given by the symmetric Nash equilibrium.

3 R&D Competition (Asymmetric Equilibrium) versus the Joint Laboratory

When $r'(x^*) < -1$ (i.e., $2\lambda^2 > 3$), the symmetric Nash equilibrium is unstable and, as argued in the Introduction, the relevant benchmark for the outcome under R&D competition is the asymmetric Nash equilibrium. Hereafter, we focus on the comparison of the joint laboratory and the asymmetric Nash equilibrium under R&D competition. We begin with a comparison of R&D propensities, showing that $2\lambda\sqrt{x_J} > \lambda\sqrt{\underline{x}} + \lambda\sqrt{\bar{x}}$, i.e.,

$$4\lambda^2\frac{a-c}{9-2\lambda^2} > 2\lambda^2\frac{a-2c}{9-4\lambda^2} + c$$

3 The second derivative of the objective function is $-(1/9)\lambda(a-c)x^{-3/2} < 0$, and therefore the first-order condition is sufficient. Assumption 2 implies that $x_J < c^2/\lambda^2$.

4 The cartelized RJV solves $\max_{x_1,x_2} 2\Pi(x_1+x_2, x_1+x_2) + x_1 + x_2$, where Π is given by (1) and x_i is firm i's contribution to the RJV's R&D expenditure. If $(\hat{x}_1, \hat{x}_2)$ solves this problem, then $\hat{x}_1 + \hat{x}_2 = x_J$. Thus the joint laboratory and the cartelized RJV call for the same total R&D expenditure and cost reductions. In the symmetric solution to this problem, i.e., $\hat{x}_1 = \hat{x}_2$, each firm obtains profit Π_J.

This inequality can be rewritten as

$$\frac{3(2\lambda^2-3)(9c-2a\lambda^2)}{(9-2\lambda^2)(9-4\lambda^2)} > 0$$

which holds since $2\lambda^2 > 3$, and since $9c > 2a\lambda^2$ by Assumption 2. Therefore we have the following result.

Proposition 2: If $2\lambda^2 > 3$, it follows that $r'(x^*) < -1$, and the symmetric Nash equilibrium under R&D competition is unstable. Then total cost reduction under the joint laboratory is greater than total cost reduction in the asymmetric Nash equilibrium under R&D competition, i.e., $2\lambda\sqrt{x_J} > \lambda\sqrt{\underline{x}} + \lambda\sqrt{\bar{x}}$.

By Proposition 2, the superiority of the joint laboratory in terms of R&D propensity still holds. In addition, with linear demand, consumer surplus is increasing in total cost reductions, and therefore is also higher under the joint laboratory.

The following example shows, however, that total profit is sometimes higher in the asymmetric Nash equilibrium under R&D competition than under the joint laboratory.

Example 1: Let $a = 2.01$, $c = 1$, and $\lambda = 1.3115$. With R&D competition the R&D expenditures are $\underline{x} = 1.5308 \times 10^{-4}$ and $\bar{x} = 0.5814$, the cost reductions are 1.6227×10^{-2} and 1.0, respectively, and profits are $\Pi(\underline{x}, \bar{x}) = 4.717 \times 10^{-5}$ and $\Pi(\bar{x}, \underline{x}) = 0.41446$. The R&D expenditure of the joint laboratory is $x_J = 0.22703$, each firm's cost reduction is 0.6249, and each firm's profit is 0.18347. Total profit is greater under R&D competition.

The intuition underlying this example is that R&D competition conveys the (potential) advantage that firms compete asymmetrically at the second stage, while in the joint laboratory, symmetry at the second stage is built in. This advantage can more than offset the disadvantageous aspects of R&D competition that firms do not coordinate R&D decisions to maximize total profits and that R&D effort is duplicated since results are not shared.

To see that asymmetry can be advantageous, consider the hypothetical problem in which a joint laboratory has the know-how to lower costs by $k \leq c$, and this know-how is to be distributed to the firms in order to maximize total profits. Denoting by k_i the know-how distributed to firm i, the joint laboratory's problem is

$$\max_{0 \leq k_1, k_2 \leq k} \tfrac{1}{9}(a - c + 2k_1 \quad k_2)^2 + \tfrac{1}{9}(a - c + 2k_2 - k_1)^2$$

The objective function is jointly (strictly) convex in k_1 and k_2. Therefore, the solution has maximal differentiation (i.e., $k_i = 0$ and $k_j = k$, $i \neq j$) if $3k - 2(a - c) \geq 0$ and minimal differentiation (i.e., $k_1 = k_2 = k$) otherwise. In the former case, differentiation is advantageous.

Our next proposition shows that the superiority of the joint laboratory in terms of total profits can be re-established with a strengthening of Assumption 1.

Proposition 3: If demand is high relative to initial unit costs, then total profit is higher under the joint laboratory than under the asymmetric Nash equilibrium with R&D competition. Specifically, $a/c \geq 5/(2 + \lambda^2 2/9)$ implies $2\Pi_J > \Pi(\underline{x}, \bar{x}) + \Pi(\bar{x}, \underline{x})$.

Since $r'(x^*) < -1$ only if $\lambda^2 > 3/2$, it is clear that $a/c \geq 5/[2 + (2/9)(3/2)] \approx 2.14$ is sufficient to insure that the sum of profits is always higher under the joint laboratory than under R&D competition. Defining social welfare to be the sum of total profits and consumer surplus, this condition also guarantees that social welfare is higher under the joint laboratory than under R&D competition.

4 Conclusion

If the symmetric equilibrium under R&D competition is unstable, then the relevant benchmark when comparing this scenario to others is its asymmetric equilibrium. Using this benchmark, we find that total firm profits are sometimes higher under R&D competition than under RJV cartelization.

Appendix

Proof of Proposition 1

A straightforward calculation shows that

$$\frac{\partial \Pi(x_i, x_j)}{\partial x_i} = \frac{2\lambda}{9\sqrt{x_i}} \frac{a - c + 2\lambda\sqrt{x_i} - \lambda\sqrt{x_j}}{3}$$

Since $\lim_{x_i \to 0} \partial \Pi(x_i, x_j)/\partial x_i = \infty$, we have that $r_i(x_j) \in (0, c^2/\lambda^2]$ $\forall x_j \in [0, c^2/\lambda^2]$. For $r_i(x_j) \in (0, c^2/\lambda^2)$, we obtain $r_i(x_j) = 4\lambda^2((a - c - \lambda\sqrt{x_j})/(9 - 4\lambda^2))^2$ from the first-order condition

$$\frac{2\lambda}{\;} \frac{a - c + 2\lambda\sqrt{r_i(x_j)} - \lambda\sqrt{x_j}}{3} \qquad 0$$

By the concavity of $\Pi(x_i, x_j)$ in x_i, the first-order condition is sufficient. The concavity of $\Pi(x_i, x_j)$ also implies that $r_i(x_j)$ is single-valued and continuous and, hence, $r_i(x_j)$ is as given in Proposition 1.

To find the Nash equilibria of the R&D game it is useful to "linearize" reaction functions. We have that $\sqrt{r(x)} = 2\lambda(a - c - \lambda\sqrt{x})/(9 - 4\lambda^2) \wedge c/\lambda$ and, therefore, the square root of a firm's reaction (on the interior) is a linear function of the square root of its rival's R&D expenditure. If $-2\lambda^2/(9-4\lambda^2) > -1$ (i.e., $2\lambda^2 < 3$), then the linearized reaction functions cross as illustrated in Fig. 1a. In this case there is a unique Nash equilibrium (x^*, x^*) where x^* satisfies $\sqrt{x^*} = 2\lambda(a - c - \lambda\sqrt{x^*})/(9-4\lambda^2)$, and which is stable. Solving this equation yields the expression for x^* provided in the proposition. If $-2\lambda^2/(9-4\lambda^2) < -1$ (i.e., $2\lambda^2 > 3$), then the linearized reaction functions cross as illustrated in Fig. 1b. In this case, in addition to the interior Nash equilibrium (x^*, x^*), there are two boundary equilibria satisfying, for $i, j \in \{1, 2\}$, $i \neq j$, that $\sqrt{x_j} = c/\lambda$ and $\sqrt{x_i} = 2\lambda(a - 2c)/(9 - 4\lambda^2)$. In this case the interior Nash equilibrium is unstable, while each of the boundary equilibria is (locally) stable. □

Proof of Proposition 3

We first show that $a/c \geq 5/(2 + \lambda^2 2/9)$ implies that total profits in the second stage (ignoring R&D costs) when both firms reduce their cost by c exceeds total profits when costs are reduced by the asymmetric Nash equilibria amounts, i.e., we establish

$$\frac{2}{9}a^2 \geq \left(\frac{a + c - 2\lambda^2\left(\frac{a - 2c}{9 - 4\lambda^2}\right)}{3}\right)^2 + \left(\frac{a - 2c + 4\lambda^2\left(\frac{a - 2c}{9 - 4\lambda^2}\right)}{3}\right)^2$$

This inequality can be rewritten as

$$c^2 \frac{\left(9 - 2\lambda^2 \frac{a}{c}\right)\left(\frac{a}{c}\left(2 + \frac{2}{9}\lambda^2\right) - 5\right)}{(9 - 4\lambda^2)^2} \geq 0 ,$$

which follows from Assumption 2 and $a/c \geq 5/(2 + \lambda^2 2/9)$.

We have, therefore, that

$$\frac{2}{9}a^2 - \left(\frac{c}{\lambda}\right)^2 > \left(\frac{a + c - 2\lambda^2\left(\frac{a - 2c}{9 - 4\lambda^2}\right)}{3}\right)^2 - \left(\frac{c}{\lambda}\right)^2 + \left(\frac{a - 2c + 4\lambda^2\left(\frac{a - 2c}{9 - 4\lambda^2}\right)}{3}\right)^2 - 4\lambda^2\left(\frac{a - 2c}{9 - 4\lambda^2}\right)^2$$

The left-hand side is total profits under the joint laboratory were it to choose the maximal R&D expenditure, and the right-hand side is total profits under R&D competition, i.e., $\Pi(\underline{x}, \bar{x}) + \Pi(\bar{x}, \underline{x})$. Since x_J maximizes total profits under the joint laboratory, we have $2\Pi_J \geq a^2 \cdot 2/9 - (c/\lambda)^2$ and, therefore $2\Pi_J > \Pi(\underline{x}, \bar{x}) + \Pi(\bar{x}, \underline{x})$. □

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References

Amir, R. (1997): "Modelling Imperfectly Appropriable R&D via Spillovers." Mimeo, Odense University, Odense, Denmark.

Amir, R., and Lambson, V. E. (1997): "On the Effects of Entry in Cournot Markets." Mimeo, Odense University, Odense, Denmark.

Amir, R., and Wooders, J. (1997a): "Effects of One-Way Spillovers on Market Shares, Industry Price, Welfare and R&D Cooperation." University of Arizona Discussion Paper 97-1, Tucson, AZ.

—— (1997b): "One-Way Spillovers, Endogenous Innovator/Imitator Roles and Research Joint Ventures." University of Arizona Discussion Paper 97-2, Tucson, AZ.

d'Aspremont, C., and Jacquemin, A. (1988): "Cooperative and Noncooperative R&D in Duopoly with Spillovers." *American Economic Review* 78: 1133–1137.
—— (1990): "Erratum." *American Economic Review* 80: 641–642.
Brander, J., and Spencer, B. (1983): "Strategic Commitment with R&D: the Symmetric Case." *Bell Journal of Economics* 14: 225–235.
Cox, J., and Walker, M. (1997): "Learning to Play Cournot Duopoly Strategies." *Journal of Economic Behavior and Organization* (forthcoming).
Henriques, I. (1990): "Cooperative and Noncooperative R&D in Duopoly with Spillovers: Comment." *American Economic Review* 80: 638–640.
Holt, C. (1995): "Industrial Organization: a Survey of the Results of Laboratory Experiments." In *Handbook of Experimental Economics*, edited by A. Roth and J. Kagel. Princeton: Princeton University Press.
Kamien, M., Muller, E., and Zang, I. (1992): "Research Joint Ventures and R&D Cartels." *American Economic Review* 82: 1293–1306.
Kandori, M., Mailath, G., and Rob, R. (1993): "Learning, Mutation and Long-Run Equilibria in Games." *Econometrica* 61: 21–56.
Katz, M. (1986): "An Analysis of Cooperative Research and Development." *Rand Journal of Economics* 17: 527–543.
Kohlberg, E., and Mertens, J. (1986): "On the Strategic Stability of Equilibria." *Econometrica* 54: 1003–1037.
Milgrom, P., and Roberts, J. (1991): "Adaptive and Sophisticated Learning in Repeated Normal-Form Games." *Games and Economic Behavior* 3: 82–100.
Salant, S., and Shaffer, G. (1992): "Optimal Asymmetric Strategies in Research Joint Ventures: a Comment on the Literature." Mimeo, University of Michigan, Ann Arbor, MI.
Seade, J. (1980): "On the Effects of Entry." *Econometrica* 48: 479–489.

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