Cooperation vs. hierarchy: an information-theoretic comparison — **Source link** ⧉

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Cooperation vs. Hierarchy: An Information-theoretic Comparison

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Abstract— **The performance of source-cooperation in a multi-access network is compared to that of using a wireless relay. The former network is modeled as a multi-access channel with generalized feedback and the latter as a multi-access relay channel. Using power as the cost metric, achievable rates and outage probabilities for the two networks are compared under a total transmit power constraint and a specified geometry. The use of a relay is shown to be advantageous for a variety of wireless fading channels.**

I. INTRODUCTION

Cooperation in communication networks results when terminals use their power, time, and bandwidth resources to mutually enhance their transmissions. Cooperation can be induced in several ways, and we consider two approaches. First, we allow source nodes to forward data for each other and second, we introduce wireless relay nodes when cooperation between source nodes is either undesirable or not possible. We refer to networks employing the former approach as *cooperative networks* and those employing the latter as *hierarchical networks*.

There are important differences between cooperative and hierarchical networks that are not easy to analyze from an information-theoretic point of view. For example, in cooperative networks one likely needs economic incentives to induce cooperation [1]. On the other hand, hierarchical networks incur infrastructure costs [2]. While incentives and infrastructure costs are important elements that need to be considered in comparing the two networks, we use only the total transmit power as the cost metric for our comparisons.

The paper is organized as follows. In Section II, we present the network models. The cooperative network is modeled as a multi-access channel with generalized feedback (MAC-GF) [3], [4] and the hierarchical network as a multi-access relay channel (MARC) [5]–[7]. We place *half-duplex* constraints on the transmit-receive capabilities of all nodes. In Section III, we develop *partial decode-and-forward* (PDF) strategies for both networks. Finally, in Section IV, we compare the rate and diversity gains for the two networks under the PDF strategy and a total average transmit power constraint.

II. CHANNEL AND NETWORK MODELS

A. Network Model

Our networks consist of M source nodes numbered $1, 2, \ldots, M$ and a destination node $M+1$. For the MARC there is one additional node (the relay node $M + 2$). The input and output alphabets of node m are the respective $\mathcal{X}_m$ and $\mathcal{Y}_m$. We write $\mathcal{S} = \{1, 2, \ldots, M\}$ and $x_{\mathcal{G}} = \{x_m : m \in \mathcal{G}\}$ for $\mathcal{G} \subseteq \mathcal{S}$. For the MARC, we write $\mathcal{T} = \{1, 2, \ldots, M - 1, M, M + 2\}$ for the set of transmitters.

Let $X_{m,i} \in \mathcal{X}_m$ be the input of node m at time i. For complex Gaussian channels, the output of node m at time i is

$$Y_{m,i} = \left(\sum_{k \neq m} h_{m,k,i} X_{k,i} \right) + Z_{m,i} \tag{1}$$

where the $Z_{m,i}$ are independent, proper, complex, zero-mean, unit variance Gaussian noise random variables, and $h_{m,k,i}$ is the fading gain between transmitter k and receiver m at time i. Note that $X_{M+1,i} = 0$ for both the MAC-GF and the MARC. The transmitted signals have average block power constraints

$$\sum_{i=1}^{n} E\left|X_{m,i}\right|^2 \leq nP_m \quad m \in \mathcal{T}. \tag{2}$$

The above model allows the network nodes to transmit and receive simultaneously, which may not possible in practice. We therefore impose a *half-duplex* constraint on every node, i.e., each node can be in one of two states, *listen* or *transmit* (*LoT*) [8]. We then have, for all m and i,

$$Y_{m,i} = 0 \quad \text{if } X_{m,i} \neq 0. \tag{3}$$

We define $\sigma_{m,i} \in \{L, T\}$ to be the *LoT* state at node m and time i. We assume that the $\sigma_{m,i}$ are known universally to all nodes at all times.

B. Cooperative Network

We model the cooperative network as a Gaussian MAC-GF. The average block power at the mth source node is $P_m^{(C)}$ and is subject to the constraint (2), i.e., $P_m^{(C)} \leq P_m$. We refer to the half-duplex MAC-GF as the HD-MAC-GF.

There are many ways to duplex M sources over two states. We here divide the total time into $M + 1$ slots. In slot t,

$t = 1, 2, \ldots, M$, only source t transmits. In slot $M + 1$, all sources transmit. Let α_t denote the fraction of time allocated to slot t. Note that the HD-MAC-GF can be viewed as a MAC-GF with $(M + 1)$ *LoT* states where the probability of the tth state is α_t.

C. Hierarchical Network

We model the hierarchical network as a Gaussian MARC with $M + 1$ inputs $X_{m,i}$, $m \in \mathcal{T}$, and two outputs $Y_{M+1,i}$ and $Y_{M+2,i}$ given by (1) [7]. We write $P_m^{(H)}$ for the average block power in (2), so that $P_m^{(H)} \leq P_m$.

A half-duplex MARC, or HD-MARC, results when the relay listens for a fraction α of the total time (or bandwidth) and transmits for the remaining fraction. We refer to the fraction α as the *listen* (*L*) slot and the fraction $(1 - \alpha)$ as the *talk* (*T*) slot. Note that the HD-MARC can be viewed as a MARC with two *LoT* states.

D. Network Power Constraint

We assume that the HD-MARC sources only transmit. Similarly, the destination node $M+1$ in both the HD-MAC-GF and the HD-MARC only listens. We will additionally impose a network-wide power constraint of

$$\sum_{k=1}^{M} P_k^{(C)} = \sum_{k \in \mathcal{T}} P_k^{(H)} \leq P_{tot}. \tag{4}$$

E. Fading Model

We model the fading gains as

$$h_{m,k,i} = \frac{A_{m,k,i}}{\sqrt{d_{m,k}^{\gamma}}} \tag{5}$$

where $d_{m,k}$ is the distance between the mth receiver and the kth source, γ is the path-loss exponent, and $A_{m,k,i}$ is a proper complex fading random variable. We consider three kinds of fading channels in this paper.

1) no fading $A_{m,k,i} = 1$ for all m, k, i.
2) ergodic phase-fading with $A_{m,k,i} = e^{j\theta_{m,k,i}}$ where $\theta_{m,k,i}$ are jointly independent and identically distributed (i.i.d) uniformly between $[-\pi, \pi)$ for all i.
3) Rayleigh quasi-static (slow) fading where $A_{m,k,i}$ are jointly i.i.d zero-mean, unit variance proper, complex Gaussian random variables.

III. COOPERATIVE STRATEGIES

Three commonly used cooperative communication strategies are *decode-and-forward* (DF), *compress-and-forward* (CF), and *amplify-and-forward* (AF) [5], [7], [9], [10]. The DF, CF, and AF strategies result when the cooperating node or relay decodes, compresses or amplifies, respectively, its received signal before forwarding to the destination. The DF strategy can be generalized to become a *partial decode-and-forward* (PDF) strategy where the relay decodes only one of two message streams from each source [11, theorem 7]. We here consider the PDF strategy and numerically compare

the two networks with $M = 2$ source nodes. However, for completeness we next describe strategies for $M \geq 2$ source nodes.

A. Cooperative Networks

Willems presents a coding strategy for the MAC-GF with $M = 2$ in [3, chap. 7]. For a network of $M > 2$ sources, there is a combinatorial explosion in the number of code constructions due to the potential to broadcast messages to any subset of the sources. For simplicity, we assume that all nodes cooperate to transmit the messages they decode from all other nodes. We denote the messages at node m by w_{m1} and w_{m2}, where w_{m1} and w_{m2} take on $2^{nR_{m1}}$ and $2^{nR_{m2}}$ values, respectively. We use codewords $\underline{x}_m(w_{m1}, w_{m2}, w_0)$, $\underline{v}_m(w_{m2}, w_0)$, and $\underline{u}(w_0)$ to transmit w_{m1}, w_{m2}, and a cooperative message w_0, where w_{m0} takes on $2^{nR_{m0}}$ values. The stream w_{m2} is decoded by all sources while the destination decodes all streams. Let $\mathcal{G}^c$ be the complement of $\mathcal{G}$ in $\mathcal{S}$, $R_{\mathcal{G},j} = \sum_{m \in \mathcal{G}} R_{m,j}$, $j = 1, 2$, and $R_{\mathcal{G}} = R_{\mathcal{G},1} + R_{\mathcal{G},2}$. We use backward decoding at the destination [3] to prove the following theorem.

Theorem 1: The rate tuple $(R_1, R_2, \ldots, R_M)$ is achievable for an M-source MAC-GF if, for all $\mathcal{G} \subseteq \mathcal{S}$, we have

$$R_{\mathcal{G},2} \leq \min_{m \in \mathcal{G}^c} \{I(V_{\mathcal{G}}; Y_m | X_m, U, V_{\mathcal{G}^c})\} \tag{6}$$

$$R_{\mathcal{G},1} \leq I(X_{\mathcal{G}}; Y_{M+1} | X_{\mathcal{G}^c}, V_{\mathcal{S}}, U) \tag{7}$$

$$R_{\mathcal{S}} \leq I(X_{\mathcal{S}}; Y_{M+1}) \tag{8}$$

where the joint probability distribution function of the random variables factors as

$$p(u) \cdot \left[\prod_{m=1}^{M} p(x_i, v_i | u) \right] \cdot p(y_{\mathcal{S}}, y_{M+1} | x_{\mathcal{S}}). \tag{9}$$

The code construction for the MAC-GF can be applied to the HD-MAC-GF model as the *partial decode-and-forward* (PDF) strategy. Recall that we are now restricting attention to strategies where only node m transmits during slot m for $m = 1, 2, \ldots, M$. We obtain the following rate bounds.

Theorem 2: The rate tuple $(R_1, R_2, \ldots, R_M)$ is achievable for an M-source HD-MAC-GF if, for all $\mathcal{G} \subseteq \mathcal{S}$, we have

$$R_{m,2} \leq \min_{k \in [1,M], k \neq m} \{\alpha_m I(V_m; Y_k)\} \tag{10}$$

$$R_{\mathcal{G},1} \leq \begin{pmatrix} \sum_{m \in \mathcal{G}} \alpha_m I(X_m; Y_{M+1} | V_m) + \\ \alpha_{M+1} I(X_{\mathcal{G}}; Y_{M+1} | X_{\mathcal{G}^c}, U) \end{pmatrix} \tag{11}$$

$$R_{\mathcal{S}} \leq \sum_{m=1}^{M} \alpha_m I(X_m; Y_{M+1}) + \alpha_{M+1} I(X_{\mathcal{S}}; Y_{M+1}) \tag{12}$$

where the joint probability distribution of the random variables factors as

$$p(u, \sigma_{\mathcal{S}}) \cdot \left[\prod_{m=1}^{M} p(x_i, v_i | u, \sigma_{\mathcal{S}}) \right] \cdot p(y_{\mathcal{S}}, y_{M+1} | x_{\mathcal{S}}, \sigma_{\mathcal{S}}). \tag{13}$$

Remark 3: The above bounds can also be obtained from $(6)-(8)$ by conditioning on the $(M + 1)$ *LoT* states.

For a Gaussian HD-MAC-GF, consider proper complex Gaussian inputs. The resource allocation problem then involves optimizing over the

- time fractions α_m,
- power tuples $(P_1^{(C)}, P_2^{(C)}, \ldots, P_M^{(C)})$,
- fractions $\gamma_{m,t}$ of power $P_m^{(C)}$ transmitted by the source m in time slot t, and
- fractions $\beta_{m,t}$ of power $\gamma_{m,t} P_m^{(C)}$ allocated by the source m to transmitting w_{m1}.

B. Hierarchical Networks

Consider the PDF strategy where each source sends two messages in any n-length block. Thus, the source m transmits w_{m1} and w_{m2} in each block using codewords $\underline{x}_m(w_{m1}, w_{m2}, w_{m0})$, $\underline{v}_m(w_{m2}, w_{m0})$, and $\underline{q}_m(w_{m0})$, where w_{m0} is the message transmitted cooperatively with the relay via the relay's codeword $\underline{x}_{M+2}(w_{10}, w_{20} \ldots, w_{M0})$. As before, w_{mj} takes on $2^{nR_{mj}}$ values. In each block, the relay decodes the current w_{m2}, $m = 1, 2, \ldots, M$, while the destination decodes (w_{m1}, w_{m0}). The mth source rate is $R_m = R_{m,1} + R_{m,2}$. We use backward decoding at the destination to prove the following theorem.

Theorem 4: The rate tuple $(R_1, R_2, \ldots, R_M)$ is achievable for an M-source MARC if, for all non-empty sets $\mathcal{A}, \mathcal{G}, \mathcal{A} \subseteq \mathcal{G} \subseteq \mathcal{S}$, we have

$$R_{\mathcal{G},2} \leq I(V_{\mathcal{G}}; Y_{M+2}|V_{\mathcal{G}^c}, Q_{\mathcal{S}}, X_{M+2}) \qquad (14)$$

$$R_{\mathcal{G},1} \leq I(X_{\mathcal{G}}; Y_{M+1}|X_{\mathcal{G}^c}, V_{\mathcal{S}}, Q_{\mathcal{S}}, X_{M+2}) \qquad (15)$$

$$R_{\mathcal{G},1} + R_{\mathcal{A},2} \leq I(X_{\mathcal{G}}, X_{M+2}; Y_{M+1}|X_{\mathcal{G}^c}, V_{\mathcal{A}^c}, Q_{\mathcal{A}^c}) \qquad (16)$$

where the joint probability distribution factors as

$$\left[\prod_{m=1}^{M} p(q_m, v_m, x_m)\right] \cdot p(x_{M+2}|q_{\mathcal{S}}) \cdot p(y_{M+1}, y_{M+2}|x_{\mathcal{T}}). \qquad (17)$$

Remark 5: The rate bounds for the HD-MARC are obtained by conditioning the bounds above on the relay's L and T states with probability α and $(1 - \alpha)$ respectively.

For the Gaussian HD-MARC, consider proper complex Gaussian inputs. The resource allocation problem is then to optimize over the

- time fraction α,
- power tuples $(P_1^{(H)}, P_2^{(H)}, \ldots, P_M^{(H)}, P_{M+2}^{(H)})$,
- fractions $\gamma_{m,t}$ of power $P_m^{(H)}$ transmitted by the source m in time slot t,
- fractions $\beta_{m,t}$ of power $\gamma_{m,t} P_m^{(H)}$ allocated by the source m to transmitting w_{m1}, and
- fractions $\alpha_{M+2,m}$ that the relay allocates to forwarding message w_{m0}.

IV. COOPERATION VS. HIERARCHY

We consider networks with $M = 2$ and with the planar geometry shown in Fig. 1. The sources are positioned symmetrically on the y-axis, and the destination is a unit distance
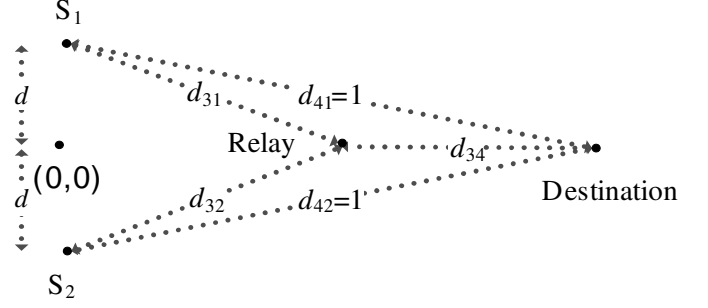

Fig. 1. Two-source geometry for HD-MAC-GF and HD-MARC

from both sources. For the HD-MARC, the relay is placed along the x-axis between the source and the destination. We choose the path loss exponent γ as 4 and plot the rates in bits per channel use. The individual and total power constraints are $P_1 = P_2 = 4$, $P_4 = 6$, and $P_{tot} = 8$. We present and analyze the results separately for the three fading models mentioned in Section II-E.

A. No Fading

In Fig. 2, we plot the maximum sum rate $R_1 + R_2$ achieved by the PDF strategy against the symmetric distance d of the two sources from the origin for the HD-MAC-GF and HD-MARC. We also plot the non-cooperative multi-access sum capacity (labeled as MAC) and the cooperative sum capacity achieved when all messages are known *a priori* at both sources (labeled as full cooperation). The HD-MAC-GF sum rate is maximized by $P_m^{(C)} = P_m = 4$, $m = 1, 2$. For the HD-MARC, the sum rate, maximized over all parameters and relay position, is plotted for several combinations of $P_1^{(H)} = P_2^{(H)} \leq P_1$ and $P_4^{(H)} \leq P_4$ subject to $2P_1^{(H)} + P_4^{(H)} = P_{tot}$ where each curve is characterized by a different $P_{ratio} \triangleq P_4^{(H)} \Big/ P_{tot}$.

Fig. 2 shows the effect of cooperation when the sources are closer to each other than to the destination. The sum rate decreases as the inter-source distance $2d$ increases until it coincides with the MAC sum capacity. The maximum HD-MARC sum rate is plotted for several choices of P_{ratio}. Fig. 2 also shows the rate gains that result from using a relay with non-zero, albeit small, power $P_4^{(H)}$ relative to HD-MAC-GF sum rate and MAC sum capacity. The gain is a direct result of the path-loss dependent channel model and optimal positioning of the relay. The curves for various relay powers also reveal the effect of sharing P_{tot} between source message transmissions and relay cooperation.

We remark that, because the total power is held constant, the two networks can also be compared by scaling the rates in Fig. 2 by P_{tot} to obtain the *network utility* in bits/Joule [12] that results from cooperation.
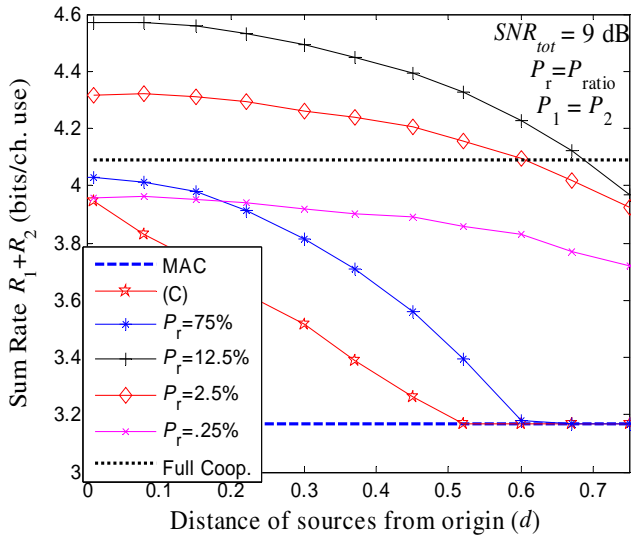
Fig. 2. Plot of $R_1 + R_2$ for HD-MAC-GF and HD-MARC

Fig. 3. Comparison of the outage probability for both networks at the symmetric rate point

B. Ergodic phase fading

For an ergodic fading channel with phase information unknown at the transmitter, the rates achieved for the DF and PDF strategies are maximized when the cooperating nodes transmit independently [5], [13]. Thus, since the source nodes cannot cooperate, the maximum sum rates for the HD-MAC-GF simplify to the MAC sum capacity. For the HD-MARC, rate gains relative to the MAC may be possible for certain choices of $(P_1^{(H)}, P_2^{(H)}, P_4^{(H)})$.

The optimal HD-MARC sum rate curves for the no-fading case in Fig. 2 are obtained when $\beta_{m,T} = 1$, i.e., when the relay and the sources transmit independently. Thus, these curves are the same as the curves for the ergodic phase-fading channel for this example. The maximum achievable HD-MAC-GF sum rate is given by the MAC sum rate in Fig. 2. This model further reveals the advantage of reducing source power while sharing the total power with a relay that is closer to the destination than the sources. Similar results can also be expected for ergodic Rayleigh fading channels.

C. Quasi-static Rayleigh Fading

We next consider the case where the channel parameters are chosen randomly but held fixed for all time. The best achievable rates are therefore random variables that are functions of the random channel parameters. We consider the above geometry with $2d = 0.5$, and set $R_1 = R_2 = R = .5$. We determine the probability P_{out} that the two sources experience an outage. In Fig. 3, we plot P_{out} for various models and resource allocations. We observe that half-duplexing the nodes to allow either source-source or source-relay cooperation can increase the spatial diversity at the destination, thereby decreasing P_{out} for the two networks relative to the MAC. The advantage of source cooperation is clearly highlighted by the near-optimal performance of the HD-MAC-GF as compared to full cooperation.
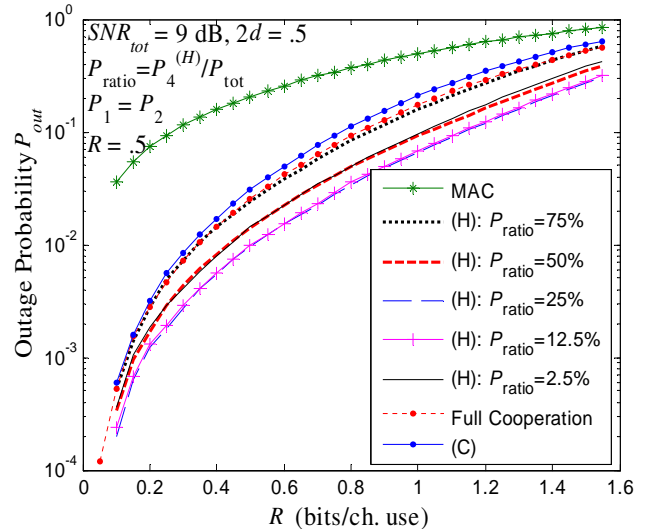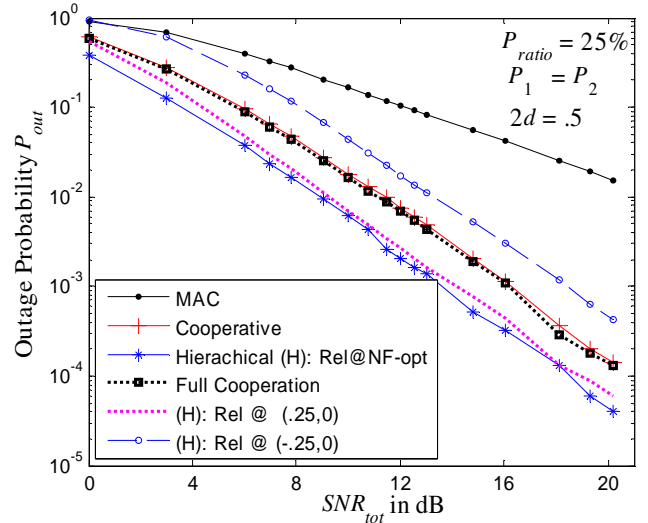

Fig. 4. Outage probability P_{out} vs. total SNR SNR_{tot}

Fig. 3 also reveals a performance advantage of the hierarchical over the cooperative network. Quantifying this further, we define SNR_{tot} as the ratio of P_{tot} to the noise power at the destination. In Fig. 4, we plot P_{out} against SNR_{tot} for $R = .5$, $2d = .5$, and power constraints $P_1 = P_2 = P_4 = P_{tot}/2$. For the two models, P_{out} is plotted for $P_m^{(C)} = P_m$, $m = 1, 2$, and $P_1^{(H)} = P_2^{(H)}$, $P_4^{(H)} = P_4/2$ subject to (4). We observe an increase in diversity from 1 to 2 for both models relative to the MAC. However, the SNR required for the HD-MARC is 2.5-3 dB lower than for the HD-MAC-GF (recall that we have positioned the relay to maximize the non-fading rate). Such gains in SNR are, in general, dependent on the relative position of the sources and relay, as evidenced by the outage curves in Fig. 4 for relay positions at $(.25, 0)$ and $(-.25, 0)$.

We remark that the above analysis can be applied to other geometries, where the maximum diversity remains at 2 for $M = 2$. For $M > 2$, however, cooperative networks can achieve diversity gains of M, while hierarchical networks can achieve diversity gains of at most 2 (recall that, by assumption, the MARC source nodes can cooperate only with the relay). Comparable diversity gains in the hierarchical network require either multiple antennas at the relay or multiple relays. Finally, the above networks can also be compared when using the AF and CF strategies, a topic for future work.

V. Acknowledgement

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