Cooperative Beamforming in Cognitive Radio Networks

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Abstract*—In this paper, the problem of cooperative beamforming is investigated in a heterogeneous underlay cognitive network. A major difference exists between a conventional cooperative beamforming problem and cooperative beamforming in a CR (Cognitive Radio)network due to the interference constraints imposed by primary network. These constraints can greatly icrease the complexity of the related optimization problem and make it much more challenging. Here, we have assumed a typical CR network where a set of cognitive relay nodes collaboratively assist the secondary transmitter to relay its message signal to the destination in the presence of a primary transmitter-receiver pair. Each relay employs the Amplify-and-Forward (AF) protocol to retransmit the signal. In fact, the relays comprise a distributed beamformer to beamform the signal towards its destination while maintaining the QoS in the primary user. The objective is designing the beamformer in order to maximize the target SINR in the cognitive network subject to the network power limitations and interference constraints on the primary network. First, the cooperative beamforming problem has been solved and the improved performance in terms of target SINR in the secondary network has been investigated by simulations. Secondly, a parameter has been derived to demonstrate the distance between the optimal beamweights in this problem and the beamweights in a conventional beamforming problem. Simulations show that the target SINR can be dramatically increased by using cooperative beamforming. Moreover, they show that there is a great distinction between the optimal solutions in conventional cooperative beamforming and cooperative beamforming in cognitive radios.

I. INTRODUCTION

Cognitive Radio is a new approach which is introduced to cope with the spectral limitations and improve spectral efficiency. Cognitive users actively detect the given channel environment at specific times and change their transmission or reception parameters to communicate efficiently avoiding interference with licensed or unlicensed users [1], [2]. Cooperation is increasingly regarded as a key technology for tackling the challenges of a practical implementation of CR network [3]. As a form of cooperative communication, distributed beamforming [4] can be used in the CR network to improve the network performance. By means of a virtual array of N antennas in distributed beamforming, an N-fold power gain can be ideally yielded in comparison to single antenna transmission. Furthermore, interference reduction which results in increasing the network capacity is another important benefit. In addition to these benefits, distributed beamforming offers dramatic increase in range and energy efficiency [4].

Cooperative beamforming would bring substantial enhancements to CR networks. A CR network can reap the benefits of cooperative beamforming to mitigate the interference power on the primary users and improve the secondary performance especially in wide range communications. The state of art in cooperative beamforming shows that a detailed design of distributed beamforming in the network scope is the current issue [4]. Conceding that every type of network has its own special objectives, limitations, and characteristics, cooperative beamforming in CR networks as another class of networks should be considered in a different scope. A cognitive network has additional imposing constraints corresponding to the interference effects on the primary users. This can significantly increase the complexity of the problem. In fact, cooperative beamforming is very new to the cognitive networks.

In this paper, we are studying the advantage of exploiting cooperative beamforming in CR networks.We consider an underlay CR network comprising a primary and secondary transmitter-receiver pair. Beamforming is performed by multiple ad-hoc cognitive relays. They beamform the secondary message to its destination in order to improve the target SINR while maintaining the QoS of the primary network. In fact, this is considered as a way to overcome the power limitation on the signal transmitter to achieve higher SINR or to simply communicate with a distant receiver. The relaying strategy is the AF protocol which seems to be the best choice for simplifying the system.

In relative current works on cognitive relay networks, this type of cooperation is performed by single relay selection among a number of relays and only one relay is selected to forward the secondary message [5]–[7].

In [8], multiple relays which employ Regenerative Decode-and-Forward (RDF) protocol are used to relay the signal in an overlay CR network. However, overlay network is not the case in this paper.

In [9] a distributed beamformer for the CR network has been considered where the synchronization among beamforming nodes is coordinated by transmitting a beacon as a reference signal periodically. Primary users are located in the side lobes of the beam pattern. The paper argues that distributed beamforming gives useful benefits in a CR system when the number of beamforming nodes increases. In the above work, a conventional beamforming is performed where the phase and power of beamweights are assigned based on path phase compensation and water

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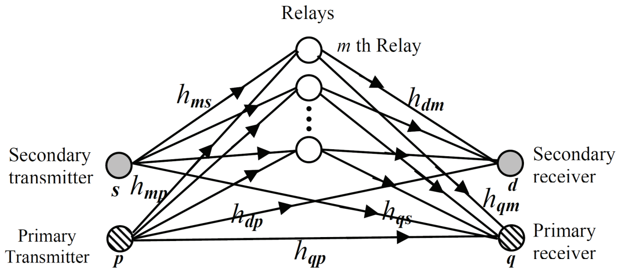

Fig. 1. System Model

filling method. Though, in our work, beamweights are obtained by solving an optimization problem with the target SINR as the objective function. The problem is solved under the individual power constraints for the secondary transmitter and relays. Besides, there is an interference constraint imposed by the primary user which does not exist in beamforming problems in any other types of network. This additional constraint makes the analytical solution much more difficult to find. However, the optimal solution for the beamwights completely differs from the conventional beamweights in [9] and this affirms the idea that cooperative beamforming should be separately investigated as a case study in CR networks. First steps towards this goal have been taken in this paper.

In the following section, the system model is described. The SINR maximization problem is formulated in Section III which is solved by using the Genetic Algorithm. Additionally, a parameter has been derived to measure the distance between the optimal beamweights in this problem and the beamweights in a conventional beamforming problem. The simulation results are presented in part IV and finally part V concludes the paper.

II. SYSTEM MODEL

Consider an underlay cognitive network which consists of a transmitter, a receiver, and a set of M cooperating relay nodes which receive and beamform the source signal from the transmitter to the receiver in a half duplex mode where only one side of the transmission link can transmit at a time. The network coexists with a primary transmitter receiver pair and we study the effect of co-operative beamforming in the cognitive network in the presence of primary nodes. The system is depicted in Fig. 1. We assume that the source and cooperative transmitters are synchronized and beamforming is performed via a two-step AF protocol. In the first step, the secondary transmitter broadcasts the message signal $\sqrt{P_s}x_s$, where P_s is its transmitting power and x_s is the message with unit energy which means $E\{|x_s|^2\} = 1$. $E\{.\}$ denotes the statistical expectation and $|.|$ represents the amplitude of a complex number. The primary transmitter is continuously transmitting with power P_p. For distinguishing the transmitted signal of the primary user in the two steps, we assume that the primary transmitter sends $\sqrt{P_p}x_p^{(1)}$ and $\sqrt{P_p}x_p^{(2)}$ respectively in the first and second steps where $E\{|x_p^{(1)}|^2\} = 1$ and $E\{|x_p^{(2)}|^2\} = 1$. There is a slowly flat fading channel where the instantaneous channel gains represent the path loss and fading effects. The channel gain between the secondary transmitter and mth relay is shown by $h_{ms} = d_{ms}^{-\alpha} g_{ms}$ and the one between the primary transmitter and mth relay is denoted by $h_{mp} = d_{mp}^{-\alpha} g_{mp}$, where d_{ms} and d_{mp} are respectively the distance from the cognitive transmitter and primary transmitter to the mth relay, α is the path loss component, and g_{ms} and g_{mp} are circular complex Gaussian variables with zero mean and unit variance. Consequently, the mth relay receives

$$y_m = h_{mp}\sqrt{P_p}x_p^{(1)} + h_{ms}\sqrt{P_s}x_s + n_m, \tag{1}$$

where n_m represents the additive circularly symmetric white Gaussian noise with variance σ_m^2 at mth relay. The next step is the simultaneous retransmission of the signal in the second half time slot by relays. The mth relay applies a beam weight w_m and transmits the signal

$$\begin{aligned} x_m &= w_m \frac{y_m}{\sqrt{P_p|h_{mp}|^2 + P_s|h_{ms}|^2 + \sigma_m^2}} \\ &= w_m \frac{h_{mp}\sqrt{P_p}x_p^{(1)} + h_{ms}\sqrt{P_s}x_s + n_m}{\sqrt{P_p|h_{mp}|^2 + P_s|h_{ms}|^2 + \sigma_m^2}} \end{aligned} \tag{2}$$

with the power limit of P_t. Denoting the channel gain between the mth relay and the secondary receiver with $h_{dm} = d_{dm}^{-\alpha} g_{dm}$, where d_{dm} represents their distance and g_{dm} is the corresponding zero mean circular complex Gaussian variable with unit variance, the total received signal at the secondary receiver in the second half time slot can be obtained as

$$y_d = \sum_{m=1}^{M} h_{dm}x_m + \sqrt{P_p}h_{dp}x_p^{(2)} + n_d, \tag{3}$$

where the first term in the summation is the received signal from the relays, the second term corresponds to the signal of the primary user which is received through a channel with gain h_{dp}. n_d is the additive white Gaussian complex noise with variance σ_d at the receiver. y_d can be rewritten as

$$\begin{aligned} y_d \quad = \quad & \sum_{m=1}^{M} w_m h_{dm} \frac{h_{mp}\sqrt{P_p}x_p^{(1)} + h_{ms}\sqrt{P_s}x_s + n_m}{\sqrt{P_p|h_{mp}|^2 + P_s|h_{ms}|^2 + \sigma_m^2}} + \\ & \sqrt{P_p}h_{dp}x_p^{(2)} + n_d \end{aligned} \tag{4}$$

which can be represented in a matrix form as

$$\begin{aligned} y_d \quad = \quad & \underline{w}^H H_d \left(\underline{h}_p \sqrt{P_p}x_p^{(1)} + \underline{h}_s \sqrt{P_s}x_s + \underline{n} \right) + \\ & h_{dp}\sqrt{P_p}x_p^{(2)} + n_d. \end{aligned} \tag{5}$$

The vector $\underline{w}$ contains the beamforming weights as $\underline{w} = [w_1^*, w_2^*, \ldots, w_M^*]^T$ where $(.)^*$ denotes complex conjugate, and $(.)^T$ and $(.)^H$ are respectively the transposition and Hermitian operator. H_d has the channel gains between relays and receiver on its diagonal;

$$H_d = diag(h_{d,m=1}, h_{d,2}, \ldots, h_{d,M}). \tag{6}$$

We have defined $M \times 1$ channel vectors $\underline{h}_p$, $\underline{h}_s$, and the relay noise vector $\underline{n}$ such that their mth elements, respectively denoted by $\underline{h}_p^m$, $\underline{h}_s^m$, and $\underline{n}^m$ for $m = 1, 2, \ldots, M$ are

$$\underline{h}_p^m = \frac{h_{mp}}{\sqrt{P_p|h_{mp}|^2 + P_s|h_{ms}|^2 + \sigma_m^2}}, \tag{7}$$

$$\underline{h}_s^m = \frac{h_{ms}}{\sqrt{P_p|h_{mp}|^2 + P_s|h_{ms}|^2 + \sigma_m^2}}, \tag{8}$$

and

$$\underline{n}^m = \frac{n_m}{\sqrt{P_p|h_{mp}|^2 + P_s|h_{ms}|^2 + \sigma_m^2}}. \tag{9}$$

The desired part of y_d for the receiver node is the sum of all the terms which contain x_s as the secondary transmitted signal. Thus, according to (5) the signal power can be written as

$$S = E|\underline{w}^H H_d \underline{h}_s \sqrt{P_s} x_s|^2. \tag{10}$$

Since x_s has unit energy and channel gains have been assumed to be constant during beamforming, the signal power can be obtained as

$$S = P_s E\{\underline{w}^H H_d \underline{h}_s \underline{h}_s^H H_d^H \underline{w}\}. \tag{11}$$

The noise power can be similarly derived as

$$\begin{aligned} N &= \underline{w}^H H_d E\{\underline{n}\underline{n}^H\} H_d^H \underline{w} + \sigma_d^2 \\ &= \underline{w}^H H_d \Sigma_n H_d^H \underline{w} + \sigma_d^2, \end{aligned} \tag{12}$$

where Σ_n is the covariance matrix of $\underline{n}$;

$$\Sigma_n = E\underline{n}\underline{n}^H, \tag{13}$$

and because the noise signals on relays are uncorrelated, Σ_n is a diagonal matrix whose mth element on its diagonal, presented by $\Sigma_n^{m,m}$ is equal to

$$\Sigma_n^{m,m} = \frac{\sigma_m^2}{P_p|h_{mp}|^2 + P_s|h_{ms}|^2 + \sigma_m^2}. \tag{14}$$

The interference power imposed by primary on the secondary receiver can be obtained as

$$\begin{aligned} I = P_p E\Big\{ &\left(\underline{w}^H H_d \underline{h}_p x_p^{(1)} + h_{dp} x_p^{(2)}\right) \times \\ &\left(\underline{w}^H H_d \underline{h}_p x_p^{(1)} + h_{dp} x_p^{(2)}\right)^H \Big\} \end{aligned} \tag{15}$$

If the messages $x_p^{(1)}$ and $x_p^{(2)}$ have a correlation coefficient equal to r, the Interference power can be calculated as

$$\begin{aligned} I &= P_p \underline{w}^H H_d \underline{h}_p \underline{h}_p^H H_d^H \underline{w} + P_p |h_{dp}|^2 + \\ &\quad 2P_p \ Re\left[r \underline{w}^H H_d \underline{h}_p h_{dp}^*\right]. \end{aligned} \tag{16}$$

where $Re(\cdot)$ takes the real part of its argument. SINR in the secondary receiver is therefore written as

$$\begin{aligned} SINR &= \frac{S}{I+N} \\ &= \frac{P_s \underline{w}^H Q_s \underline{w}}{\underline{w}^H \left(Q_I + Q_n\right) \underline{w} + 2 \ Re\left(\underline{w}^H V_I\right) + c}, \end{aligned} \tag{17}$$

where

$$\begin{aligned} Q_s &= H_d \underline{h}_s \underline{h}_s^H H_d^H, && (18) \\ Q_I &= P_p H_d \underline{h}_p \underline{h}_p^H H_d^H, && (19) \\ Q_n &= H_d \Sigma_n H_d^H, && (20) \\ V_I &= P_p r H_d \underline{h}_p h_{dp}^*, && (21) \\ c &= P_p |h_{dp}|^2 + \sigma_d^2. && (22) \end{aligned}$$

III. SINR MAXIMIZATION

Depending on the systems goals, different beamforming weights can be allocated to relays. One approach is setting the beamforming weights in order to improve the quality of service of the cognitive radio. In our scenario, this goal is interpreted as SINR maximization at the secondary receiver. Therefore the objective function in our optimization problem is the SINR function in (17). This optimization should be performed with respect to the interference constraints on the primary network. The Interference power received by the primary receiver during both steps of transmission should be less than the interference temperature Ip. In the first step, the interference constraint can be written as

$$P_s |h_{qs}|^2 \leq I_p, \tag{23}$$

where h_{qs} is the channel gain between the primary receiver and the secondary transmitter. During the second step of transmission, the interference on primary is received from relays. It contains the secondary signal x_s and the primary signal transmitted in the first time slot i.e. $x_p^{(1)}$. If there is a correlation between the primary messages $x_p^{(1)}$ and $x_p^{(2)}$, or in other words $r \neq 0$, a part of relays' transmit signals can be used to realize $x_p^{(2)}$ at the primary receiver. However, we consider the worst case and assume that every signal other than $x_p^{(2)}$ cause interference on the primary and thus the total relay signals received at the primary receiver form the interference power and the interference constraint on the primary in the second step can be written as

$$E|\sum_{m=1}^{M} h_{qm} x_m|^2 \leq I_p, \tag{24}$$

where h_{qm} is the channel gain between mth relay and the primary receiver which is represented as node q in Fig. 1. According to (2), It is straight forward to rewrite (24) as

$$\begin{aligned} E|\sum_{m=1}^{M} h_{qm} x_m|^2 &= P_p \underline{w}^H H_q \underline{h}_p \underline{h}_p^H H_q^H \underline{w} + \\ &\quad P_s \underline{w}^H H_q \underline{h}_s \underline{h}_s^H H_q^H \underline{w} + \\ &\quad \underline{w}^H H_q \Sigma_n H_q^H \underline{w} \leq I_p, \end{aligned} \tag{25}$$

where H_q is defined as a diagonal matrix which has the channel gains h_{qm} for $m = 1, 2, \ldots, M$ on its main diagonal.

Considering the interference constraints and individual transmit power constraint on all secondary transmitters, the optimization problem can be considered as

$$\begin{aligned} \max_{\underline{w}} \quad & SINR = \frac{P_s \underline{w}^H Q_s \underline{w}}{\underline{w}^H \left(Q_I + Q_n\right) \underline{w} + 2 \ Re\left(\underline{w}^H V_I\right) + c} \\ \text{s.t.} \quad & \\ & |w_m|^2 \leq P_t, \\ & P_s \leq P_t, \\ & P_s |h_{qs}|^2 \leq I_p, \\ & \underline{w}^H H_q \left(P_p \underline{h}_p \underline{h}_p^H + P_s \underline{h}_s \underline{h}_s^H + \Sigma_n\right) H_q^H \underline{w} \leq I_p \end{aligned} \tag{26}$$

The first set of M constraints are the individual transmit power constraints of the relays. The next one refers to the transmit power constraint for the secondary source. The objective function and the forth constraint are not convex. One of powerful tools for solving optimization prolems is the Genetic Algorithm. We have employed this tool to solve this non-convex optimization.

According to the SINR relation and the equivalent value of Q_s in (18), the SINR value will be nonzero if and only if the beamforming vector $\underline{w}$ have a nonzero projection on the vector $H_d\underline{h}_s$. This is because Q_s is a single ranked matrix whose eigen vector corresponding to its single nonzero eigen value is proportional to $H_d\underline{h}_s$. This part of solution pertains to the solution of ordinary distributed beamforming when each beamforming weight is selected such that it is proportional to the conjugate of corresponding channel gain to cancel out the phase difference and have the signals constructively received at the receiver. Consequently, the optimal beam forming vector $\underline{w}_o$ can be decomposed to two orthogonal vectors as

$$\begin{aligned} &\underline{w}_o = \beta\underline{b} + \underline{v}, \\ &\underline{b} = H_d\underline{h}_s, \ \underline{v}^H\underline{b} = 0, \end{aligned} \tag{27}$$

The first part of vector $\beta\underline{b}$ is proportional to an ordinary beamforming solution and has the responsibility to bring the desired signal power to the destination. The second vector $\underline{v}$ does not have any effect on the signal power S or the numerator of $SINR$ since it is orthogonal to $H_d\underline{h}_s$. This vector is the part that is utilized to combat the interference power I at secondary receiver and control the interference on the primary. The difference between power allocation to the first part and second part of $\underline{w}_o$, can show the distance between ordinary beamforming and the optimal beamforming solution in this scenario which is investigated in the simulation results.

IV. SIMULATION RESULTS

In our simulations, we have compared the maximum SINR obtained by cooperative beamforming with that of direct transmission. The secondary transmitter and receiver are respectively located at $(0, 0)$ and $(0, 100m)$ in Cartesian coordinates. The primary transmitter and receiver are located at $(40m, -20m)$ and $(80m, -20m)$ respectively. The primary transmitter power is $P_p = 50$ dB. The correlation coefficient r between consecutive messages of the primary transmitter is set to $r = 0.7$. The noise variance on relays and the secondary receiver are equal to $\sigma_m^2 = \sigma_d^2 = 1$ dB. The propagation loss α has been chosen as $\alpha = 2$. There are three relays at $(40m, 25m)$, $(20m, 20m)$, and $(30m, 10m)$. In this scenario, the obtained SINR in the secondary receiver is compared with the case where the secondary transmitter directly transmits without using relays. In direct transmission, the power limit of the secondary transmitter is set to the sum of power of relays and the secondary transmitter at the optimal point of cooperative beamforming. Firstly, the primary interference threshold is assumed as $I_p = 7$ dB and the resulted SINR values are plotted versus the value of P_t. The graph is illustrated in Fig. 2. The results have been averaged over 100 channel realizations. As it can be seen, SINR has been tremendously improved by cooperative beamforming and can be increased by raising the individual power level of cognitive source and cooperative relays. Fig. 3 shows the value of SINR versus I_p where the primary transmit power is set to $P_p = 50$ dB. As it is expected, the SINR value can be increased by decreasing the limitations imposed by the primary network.

The total power of relays at optimal point is denoted by P_r and can be obtained as

$$P_r = \underline{w}_o^H\underline{w}_o. \tag{28}$$

A part of this amount is utilized to generate signal power at secondary receiver which relates to the part of $\underline{w}_o$ that contains the ordinary beamforming beamforming weights. According to (27), this part of power can be written as

$$S_r = |\beta|^2\underline{b}\underline{b}^2. \tag{29}$$

We have calculated the relation of this power to the total power of relays at the optimal point for different values of I_p in the same scenario. The results are illustrated in Fig. 4. These results show that not only do the relays allocate the whole or even the most part of power to ordinary distributed beamforming, but also they utilize a considerable amount to combat primary interference while controling the primary's QoS.

V. CONCLUSIONS

We have applied cooperative beamforming to a CR network and have shown that by using multiple relays we can improve the target SINR in the secondary network while the intereference constraint for the primary network is maintained. In direct transmission, the power constraint imposed by the primary receiver restricts the SINR of the secondary receiver to a very small amount. The simulation results show that distributed beamforming in a CR network can highly improve the perfomance and the solution to this problem in CR netwrks is very different from an ordinary distributed beamforming approach.

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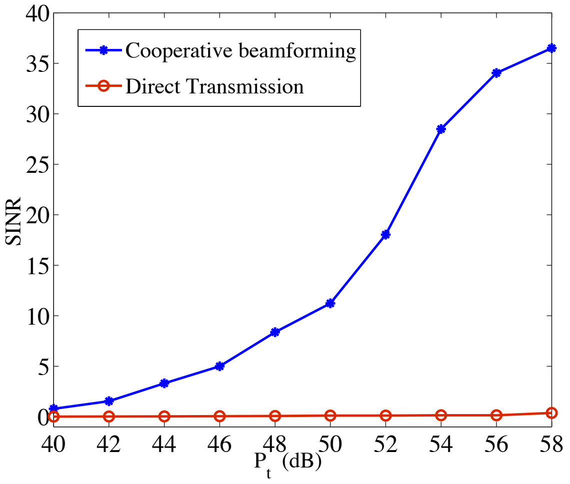

Fig. 2. SINR vs. maximum transmit power in the cognitive network

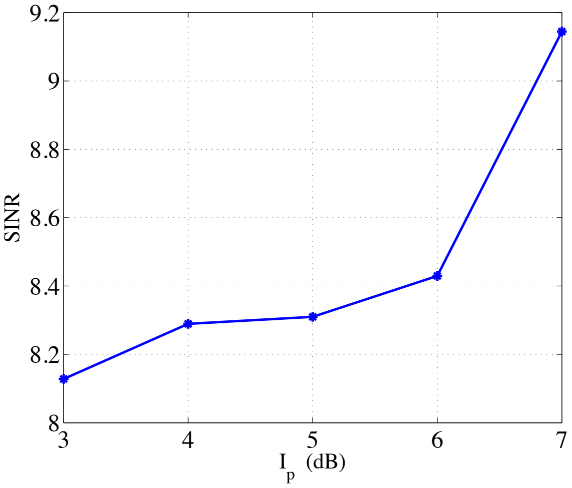

Fig. 3. SINR vs. primary interference constraint (I_p) in cooperative beamforming

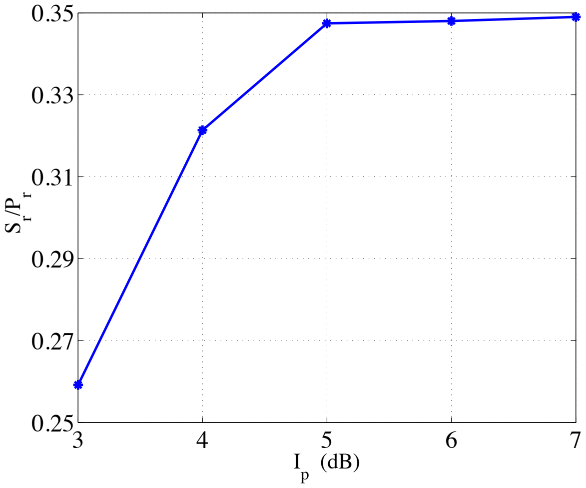

Fig. 4. The ratio of S_r to P_r. It can be seen that as the primary interference power becomes less restrictive, the relays allocate a higher amount of their power to align the phases of the received signals at the secondary destination.

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