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Cooperative Communication Protocols in Wireless Networks: Performance Analysis and Optimum Power Allocation

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Abstract In this paper, symbol-error-rate (SER) performance analysis and optimum power allocation are provided for uncoded cooperative communications in wireless networks with either decode-and-forward (DF) or amplify-and-forward (AF) cooperation protocol, in which source and relay send information to destination through orthogonal channels. In case of the DF cooperation systems, closed-form SER formulation is provided for uncoded cooperation systems with PSK and QAM signals. Moreover, an SER upper bound as well as an approximation are established to show the asymptotic performance of the DF cooperation systems, where the SER approximation is asymptotically tight at high signal-to-noise ratio (SNR). Based on the asymptotically tight SER approximation, an optimum power allocation is determined for the DF cooperation systems. In case of the AF cooperation systems, we obtain at first a simple closed-form moment generating function (MGF) expression for the harmonic mean to avoid the hypergeometric functions as commonly used in the literature. By taking advantage of the simple MGF expression, we obtain a closed-form SER performance analysis for the AF cooperation systems with PSK and QAM signals. Moreover, an SER approximation is also established which is asymptotically tight at high SNR. Based on the asymptotically tight SER approximation, an optimum power allocation is determined for the AF cooperation systems. In both the DF and AF cooperation systems, it turns out that an equal power strategy is good, but in general not optimum in cooperative communications. The optimum power allocation depends on the channel link quality. An interesting result is that in case that all channel links are available, the optimum power allocation does not

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depend on the direct link between source and destination, it depends only on the channel links related to the relay. Finally, we compare the performance of the cooperation systems with either DF or AF protocol. It is shown that the performance of a systems with the DF cooperation protocol is better than that with the AF protocol. However, the performance gain varies with different modulation types and channel conditions, and the gain is limited. For example, in case of BPSK modulation, the performance gain cannot be larger than 2.4 dB; and for QPSK modulation, it cannot be larger than 1.2 dB. Extensive simulation results are provided to validate the theoretical analysis.

Keywords Cooperative communications · Amplify-and-forward protocol · Decode-and-forward protocol · Symbol error rate · Performance analysis · Optimum power allocation · Wireless networks

1 Introduction

In conventional point-to-point wireless communications, channel links can be highly uncertain due to multipath fading and therefore continuous communications between each pair of transmitter and receiver is not guaranteed [1]. Recently, the concept of cooperative communications, a new communication paradigm, was proposed for wireless networks such as cellular networks and wireless ad hoc networks [2–6]. The basic idea of the cooperative communications is that all mobile users or nodes in a wireless network can help each other to send signals to the destination cooperatively. Each user's data information is sent out not only by the user, but also by other users. Thus, it is inherently more reliable for the destination to detect the transmitted information since from a statistical point of view, the chance that all the channel links to the destination go down is rare. Multiple copies of the transmitted signals due to the cooperation among users result in a new kind of diversity, i.e., cooperative diversity, that can significantly improve the system performance and robustness. The discussion of cooperative communications can be traced back in 1970s [7, 8], in which a basic three-terminal communication model was first introduced and studied by van der Meulen in the context of mutual information. A more thorough capacity analysis of the relay channel was provided later in [9] by Cover and El Gamal, and there are more recent work that further addressed the information-theoretic aspect of the relay channel, for example [10, 11] on achievable capacity and coding strategies for wireless relay channels, [12] on capacity region of a degraded Gaussian relay channel with multiple relay stages, [13] on capacity of relay channels with orthogonal channels, and so on.

Recently, many efforts have also been focused on design of cooperative diversity protocols in order to combat the effects of severe fading in wireless channels. Specifically, in [2, 3], various cooperation protocols were proposed for wireless networks, in which when a user helps other users to forward information, it serves as a relay. The relay may first decode the received information and then forward the decoded symbol to the destination, which is termed as a *decode-and-forward* (DF) cooperation protocol, or the relay may simply amplify the received signal and forward it, which results in an *amplify-and-forward* (AF) cooperation protocol. In both DF and AF cooperation protocols, source and relay send information to destination through orthogonal channels. Extensive outage probability performance analysis has been provided in [3] for such cooperation systems. The concept of user cooperation diversity was also proposed in [4, 5], where a two-user cooperation scheme was investigated for CDMA systems and substantial performance gain was demonstrated with comparison to the non-cooperative approach.

In this paper, we analyze the symbol-error-rate (SER) performance of uncoded cooperation systems with either DF or AF cooperation protocol. For the DF cooperation systems, we derive closed-form SER formulation explicitly for the systems with PSK and QAM signals. Since the closed-form SER formulation is complicated, we establish an upper bound as well as an approximation to show the asymptotic performance of the DF cooperation systems, in which the approximation is asymptotically tight at high signal-to-noise ratio (SNR). Based on the SER performance analysis, we are able to determine an asymptotic optimum power allocation for the DF cooperation systems. It turns out that an equal power strategy [3] is in general not optimum and the optimum power allocation depends on the channel link quality. In case that all channel links are available, an interesting observation is that the optimum power allocation does not depend on the direct link between source and destination and it depends only on the channel links related to the relay.

For the AF cooperation systems, in order to analyze the SER performance, we have to find the statistics of the harmonic mean of two random variables, which are related to the instantaneous SNR at the destination [14]. The moment generating function (MGF) of the harmonic mean of two exponential random variables was derived in [14] by applying the Laplace transform and the hypergeometric functions [15]. However, the result involves an integration of the hypergeometric functions and it is hard to use for analyzing the AF cooperation systems. In the second part of this paper, we first obtain a simple MGF expression for the harmonic mean which avoids the hypergeometric functions. Then, by taking advantage of the simple MGF expression, we are able to obtain a closed-form SER performance analysis for the AF cooperation systems with PSK and QAM signals. Moreover, an asymptotically tight SER approximation is established to reveal the performance of the AF cooperation systems. Based on the asymptotically tight SER approximation, we then determine an optimum power allocation for the AF cooperation systems. Note that the optimum power allocation for the AF cooperation systems is not modulation-dependent, which is different from that for the DF cooperation systems in which the optimum power allocation depends on specific M-PSK or M-QAM modulation. This is due to the fact that in the AF cooperation systems, the relay amplifies the received signal and forwards it to the destination regardless what kind of the received signal is.

Finally, we compare the performance of the cooperation systems with either DF or AF cooperation protocol. It turns out that the performance of the cooperation systems with the DF cooperation protocol is better than that with the AF protocol. However, the performance gain varies with different modulation types and channel conditions, and the gain is limited. For example, in case of BPSK modulation, the performance gain cannot be larger than 2.4 dB; and for QPSK modulation, it cannot be larger than 1.2 dB. There are tradeoff between these two cooperation protocols. Extensive simulation results are also provided to validate the theoretical analysis.

The rest of the paper is organized as follows. In Sect. 2, we describe the cooperation systems with either DF or AF cooperation protocol. In Sect. 3, we analyze the SER performance and determine an asymptotic optimum power allocation for the DF cooperation systems. We investigate the SER performance for the AF cooperation systems in Sect. 4. First, we derive a simple closed-form MGF expression for the harmonic mean of two random variables. Then, based on the simple MGF expression, closed-form SER formulations are given for the AF cooperation systems. We also provide a tight SER approximation to show the asymptotic performance determine an optimum power allocation. In Sect. 5, we provide performance comparison between the cooperation systems with the DF and AF protocols. The simulation results are presented in Sect. 6, and some conclusions are drawn in Sect. 7.

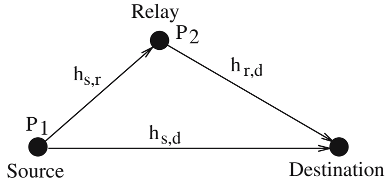


Fig. 1 A simplified cooperation model

2 System Model

We consider a cooperation strategy with two phases in wireless networks which can be mobile ad hoc networks or cellular networks [2–5]. In Phase 1, each mobile user (or node) in a wireless network sends information to its destination, and the information is also received by other users at the same time. In Phase 2, each user helps others by forwarding the information that it receives in Phase 1. Each user may decode the received information and forward it (corresponding to the DF protocol), or simply amplify and forward it (corresponding to the AF protocol). In both phases, all users transmit signals through orthogonal channels by using TDMA, FDMA or CDMA scheme [3, 5]. For better understanding the cooperation concept, we focus on a two-user cooperation scheme. Specifically, user 1 sends information to its destination in Phase 1, and user 2 also receives the information. User 2 helps user 1 to forward the information in Phase 2. Similarly, when user 2 sends its information to its destination in Phase 1, user 1 receives the information and forwards it to user 2s destination in Phase 2. Due to the symmetry of the two users, we will analyze only user 1s performance. Without loss of generality, we consider a concise model as shown in Fig. 1, in which source denotes user 1 and relay represents user 2.

In Phase 1, the source broadcasts its information to both the destination and the relay. The received signals $y_{s,d}$ and $y_{s,r}$ at the destination and the relay respectively can be written as

$$y_{s,d} = \sqrt{P_1}\, h_{s,d}\, x + \eta_{s,d}, \tag{1}$$

$$y_{s,r} = \sqrt{P_1}\, h_{s,r}\, x + \eta_{s,r}, \tag{2}$$

in which P_1 is the transmitted power at the source, x is the transmitted information symbol, and $\eta_{s,d}$ and $\eta_{s,r}$ are additive noise. In (1) and (2), $h_{s,d}$ and $h_{s,r}$ are the channel coefficients from the source to the destination and the relay respectively. They are modeled as zero-mean, complex Gaussian random variables with variances $\delta^2_{s,d}$ and $\delta^2_{s,r}$ respectively. The noise terms $\eta_{s,d}$ and $\eta_{s,r}$ are modeled as zero-mean complex Gaussian random variables with variance $\mathcal{N}_0$.

In Phase 2, for a DF cooperation protocol, if the relay is able to decode the transmitted symbol correctly, then the relay forwards the decoded symbol with power P_2 to the destination, otherwise the relay does not send or remains idle. The received signal at the destination in Phase 2 in this case can be modeled as

$$y_{r,d} = \sqrt{\tilde{P}_2}\, h_{r,d}\, x + \eta_{r,d}, \tag{3}$$

where $\tilde{P}_2 = P_2$ if the relay decodes the transmitted symbol correctly, otherwise $\tilde{P}_2 = 0$. In (3), $h_{r,d}$ is the channel coefficient from the relay to the destination, and it is modeled as a zero-mean, complex Gaussian random variable with variance $\delta^2_{r,d}$. The noise term $\eta_{r,d}$ is also modeled as a zero-mean complex Gaussian random variable with variance $\mathcal{N}_0$. Note that for analytical tractability, we assume in this paper an ideal DF cooperation protocol that the relay is able to detect whether the transmitted symbol is decoded correctly or not, which is also referred as a *selective-relaying protocol* in literature. In practice, we may apply an SNR threshold at the relay. If the received SNR at the relay is higher than the threshold, then the symbol has a high probability to be decoded correctly. More discussions on threshold optimization at the relay can be found in [16].

For an AF cooperation protocol, in Phase 2 the relay amplifies the received signal and forwards it to the destination with transmitted power P_2. The received signal at the destination in Phase 2 is specified as [3]

$$y_{r,d} = \frac{\sqrt{P_2}}{\sqrt{P_1 |h_{s,r}|^2 + \mathcal{N}_0}} h_{r,d}\, y_{s,r} + \eta_{r,d}, \tag{4}$$

where $h_{r,d}$ is the channel coefficient from the relay to the destination and $\eta_{r,d}$ is an additive noise, with the same statistics models as in (3), respectively. Specifically, the received signal $y_{r,d}$ in this case is

$$y_{r,d} = \frac{\sqrt{P_1 P_2}}{\sqrt{P_1 |h_{s,r}|^2 + \mathcal{N}_0}} h_{r,d}\, h_{s,r}\, x + \eta'_{r,d}, \tag{5}$$

where $\eta'_{r,d} = \frac{\sqrt{P_2}}{\sqrt{P_1 |h_{s,r}|^2 + \mathcal{N}_0}} h_{r,d}\, \eta_{s,r} + \eta_{r,d}$. Assume that $\eta_{s,r}$ and $\eta_{r,d}$ are independent, then the equivalent noise $\eta'_{r,d}$ is a zero-mean complex Gaussian random variable with variance $\left(\frac{P_2 |h_{r,d}|^2}{P_1 |h_{s,r}|^2 + \mathcal{N}_0} + 1\right) \mathcal{N}_0$.

In both the DF and AF cooperation protocols, the channel coefficients $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ are assumed to be independent to each other and the mobility and positioning of the nodes is incorporated into the channel statistic model. The channel coefficients are assumed to be known at the receiver, but not at the transmitter. The destination jointly combines the received signal from the source in Phase 1 and that from the relay in Phase 2, and detects the transmitted symbols by using the maximum-ratio combining (MRC) [17]. In both protocols, we assume the total transmitted power $P_1 + P_2 = P$.

3 SER Analysis for DF Cooperative Communications

In this section, we analyze the SER performance for the DF cooperative communication systems. First, we derive closed-form SER formulations explicitly for the systems with M-PSK and M-QAM[1] modulations. Then, we provide an SER upper bound as well as an approximation to reveal the asymptotic performance of the systems, in which the approximation is asymptotically tight at high SNR. Finally, based on the tight SER approximation, we are able to determine an asymptotic optimum power allocation for the DF cooperation systems.

3.1 Closed-Form SER Analysis

With knowledge of the channel coefficients $h_{s,d}$ and $h_{r,d}$, the destination detects the transmitted symbols by jointly combining the received signal $y_{s,d}$ (1) from the source and $y_{r,d}$ (3) from the relay. The combined signal at the MRC detector can be written as [17]

$$y = a_1 y_{s,d} + a_2 y_{r,d}, \tag{6}$$

[1] Throughout the paper, QAM stands for a square QAM constellation whose size is given by $M = 2^k$ with k even.

in which the factors a_1 and a_2 are determined such that the SNR of the MRC output is maximized, and they can be specified as $a_1 = \sqrt{P_1}h_{s,d}^*/\mathcal{N}_0$ and $a_2 = \sqrt{\tilde{P}_2}h_{r,d}^*/\mathcal{N}_0$. Assume that the transmitted symbol x in (1) and (3) has average energy 1, then the SNR of the MRC output is [17]

$$\gamma = \frac{P_1|h_{s,d}|^2 + \tilde{P}_2|h_{r,d}|^2}{\mathcal{N}_0}. \tag{7}$$

If M-PSK modulation is used in the system, with the instantaneous SNR γ in (7), the conditional SER of the system with the channel coefficients $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ can be written as [18]

$$P_{\text{PSK}}^{h_{s,d},h_{s,r},h_{r,d}} = \Psi_{\text{PSK}}(\gamma) \triangleq \frac{1}{\pi}\int_0^{(M-1)\pi/M} \exp\left(-\frac{b_{\text{PSK}}\gamma}{\sin^2\theta}\right) d\theta, \tag{8}$$

where $b_{\text{PSK}} = \sin^2(\pi/M)$. If M-QAM ($M = 2^k$ with k even) signals are used in the system, the conditional SER of the system can also be expressed as [18]

$$P_{\text{QAM}}^{h_{s,d},h_{s,r},h_{r,d}} = \Psi_{\text{QAM}}(\gamma), \tag{9}$$

where

$$\Psi_{\text{QAM}}(\gamma) \triangleq 4K\,Q(\sqrt{b_{\text{QAM}}\gamma}) - 4K^2Q^2(\sqrt{b_{\text{QAM}}\gamma}), \tag{10}$$

in which $K = 1 - \frac{1}{\sqrt{M}}$, $b_{\text{QAM}} = 3/(M-1)$, and $Q(u) = \frac{1}{\sqrt{2\pi}}\int_u^\infty \exp\left(-\frac{t^2}{2}\right) dt$ is the Gaussian Q-function [19]. It is easy to see that in case of QPSK or 4-QAM modulation, the conditional SER in (8) and (9) are the same.

Note that in Phase 2, we assume that if the relay decodes the transmitted symbol x from the source correctly, then the relay forwards the decoded symbol with power P_2 to the destination, i.e., $\tilde{P}_2 = P_2$; otherwise the relay does not send, i.e., $\tilde{P}_2 = 0$. If an M-PSK symbol is sent from the source, then at the relay, the chance of incorrect decoding is $\Psi_{\text{PSK}}(P_1|h_{s,r}|^2/\mathcal{N}_0)$, and the chance of correct decoding is $1 - \Psi_{\text{PSK}}(P_1|h_{s,r}|^2/\mathcal{N}_0)$. Similarly, if an M-QAM symbol is sent out at the source, then the chance of incorrect decoding at the relay is $\Psi_{\text{QAM}}(P_1|h_{s,r}|^2/\mathcal{N}_0)$, and the chance of correct decoding is $1 - \Psi_{\text{QAM}}(P_1|h_{s,r}|^2/\mathcal{N}_0)$.

Let us first focus on the SER analysis in case of M-PSK modulation. Taking into account the two scenarios of $\tilde{P}_2 = P_2$ and $\tilde{P}_2 = 0$, we can calculate the conditional SER in (8) as

$$\begin{aligned}
P_{\text{PSK}}^{h_{s,d},h_{s,r},h_{r,d}} &= \Psi_{\text{PSK}}(\gamma)\,|_{\tilde{P}_2=0}\Psi_{\text{PSK}}\left(\frac{P_1|h_{s,r}|^2}{\mathcal{N}_0}\right) \\
&\quad + \Psi_{\text{PSK}}(\gamma)\,|_{\tilde{P}_2=P_2}\left[1 - \Psi_{\text{PSK}}\left(\frac{P_1|h_{s,r}|^2}{\mathcal{N}_0}\right)\right] \\
&= \frac{1}{\pi^2}\int_0^{(M-1)\pi/M} \exp\left(-\frac{b_{\text{PSK}}P_1|h_{s,d}|^2}{\mathcal{N}_0\sin^2\theta}\right) d\theta \\
&\quad \times \int_0^{(M-1)\pi/M} \exp\left(-\frac{b_{\text{PSK}}P_1|h_{s,r}|^2}{\mathcal{N}_0\sin^2\theta}\right) d\theta \\
&\quad + \frac{1}{\pi}\int_0^{(M-1)\pi/M} \exp\left(-\frac{b_{\text{PSK}}\left(P_1|h_{s,d}|^2 + P_2|h_{r,d}|^2\right)}{\mathcal{N}_0\sin^2\theta}\right) d\theta \\
&\quad \times \left[1 - \frac{1}{\pi}\int_0^{(M-1)\pi/M} \exp\left(-\frac{b_{\text{PSK}}P_1|h_{s,r}|^2}{\mathcal{N}_0\sin^2\theta}\right) d\theta\right].
\end{aligned} \tag{11}$$

Averaging the conditional SER (11) over the Rayleigh fading channels $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$, we obtain the SER of the DF cooperation system with M-PSK modulation as follows:

$$\begin{aligned} P_{\text{PSK}} &= F_1\left(1+\frac{b_{\text{PSK}}P_1\delta_{s,d}^2}{\mathcal{N}_0\sin^2\theta}\right)F_1\left(1+\frac{b_{\text{PSK}}P_1\delta_{s,r}^2}{\mathcal{N}_0\sin^2\theta}\right)\\ &\quad + F_1\left(\left(1+\frac{b_{\text{PSK}}P_1\delta_{s,d}^2}{\mathcal{N}_0\sin^2\theta}\right)\left(1+\frac{b_{\text{PSK}}P_2\delta_{r,d}^2}{\mathcal{N}_0\sin^2\theta}\right)\right)\left[1-F_1\left(1+\frac{b_{\text{PSK}}P_1\delta_{s,r}^2}{\mathcal{N}_0\sin^2\theta}\right)\right], \end{aligned} \tag{12}$$

where $F_1(x(\theta)) = \frac{1}{\pi}\int_0^{(M-1)\pi/M}\frac{1}{x(\theta)}d\theta$, in which $x(\theta)$ denotes a function with variable θ.

For DF cooperation systems with M-QAM modulation, the conditional SER in (9) with the channel coefficients $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ can be similarly determined as

$$\begin{aligned} P_{\text{QAM}}^{h_{s,d},h_{s,r},h_{r,d}} &= \Psi_{\text{QAM}}(\gamma)|_{\tilde{P}_2=0}\Psi_{\text{QAM}}\left(\frac{P_1|h_{s,r}|^2}{\mathcal{N}_0}\right)\\ &\quad + \Psi_{\text{QAM}}(\gamma)|_{\tilde{P}_2=P_2}\left[1-\Psi_{\text{QAM}}\left(\frac{P_1|h_{s,r}|^2}{\mathcal{N}_0}\right)\right]. \end{aligned} \tag{13}$$

By substituting (10) into (13) and averaging it over the fading channels $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$, the SER of the DF cooperation system with M-QAM modulation can be given by

$$\begin{aligned} P_{\text{QAM}} &= F_2\left(1+\frac{b_{\text{QAM}}P_1\delta_{s,d}^2}{2\mathcal{N}_0\sin^2\theta}\right)F_2\left(1+\frac{b_{\text{QAM}}P_1\delta_{s,r}^2}{2\mathcal{N}_0\sin^2\theta}\right)\\ &\quad + F_2\left(\left(1+\frac{b_{\text{QAM}}P_1\delta_{s,d}^2}{2\mathcal{N}_0\sin^2\theta}\right)\left(1+\frac{b_{\text{QAM}}P_2\delta_{r,d}^2}{2\mathcal{N}_0\sin^2\theta}\right)\right)\\ &\quad \times\left[1-F_2\left(1+\frac{b_{\text{QAM}}P_1\delta_{s,r}^2}{2\mathcal{N}_0\sin^2\theta}\right)\right], \end{aligned} \tag{14}$$

where

$$F_2(x(\theta)) = \frac{4K}{\pi}\int_0^{\pi/2}\frac{1}{x(\theta)}d\theta - \frac{4K^2}{\pi}\int_0^{\pi/4}\frac{1}{x(\theta)}d\theta, \tag{15}$$

in which $x(\theta)$ denotes a function with variable θ. In order to get the SER formulation in (14), we used two special properties of the Gaussian Q-function as follows: $Q(u) = \frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{u^2}{2\sin^2\theta}\right)d\theta$ and $Q^2(u) = \frac{1}{\pi}\int_0^{\pi/4}\exp\left(-\frac{u^2}{2\sin^2\theta}\right)d\theta$ for any $u \geq 0$ [18, 20]. Note that for 4-QAM modulation,

$$\begin{aligned} F_2(x(\sin^2(\theta))) &= \frac{2}{\pi}\int_0^{\pi/2}\frac{1}{x(\sin^2(\theta))}d\theta - \frac{1}{\pi}\int_0^{\pi/4}\frac{1}{x(\sin^2(\theta))}d\theta\\ &= \frac{1}{\pi}\int_0^{\pi/2}\frac{1}{x(\sin^2(\theta))}d\theta + \frac{1}{\pi}\int_{\pi/4}^{\pi/2}\frac{1}{x(\sin^2(\theta))}d\theta\\ &= \frac{1}{\pi}\int_0^{3\pi/4}\frac{1}{x(\sin^2(\theta))}d\theta, \end{aligned}$$

which shows that the SER formulation in (14) for 4-QAM modulation is consistent with that in (12) for QPSK modulation.

3.2 SER Upper Bound and Asymptotically Tight Approximation

Even though the closed-form SER formulations in (12) and (14) can be efficiently calculated numerically, they are very complex and it is hard to get insight into the system performance from these. In the following theorem, we provide an upper bound as well as an approximation which are useful in demonstrating the asymptotic performance of the DF cooperation scheme. The SER approximation is asymptotically tight at high SNR.

Theorem 1 *The SER of the DF cooperation systems with M-PSK or M-QAM modulation can be upper-bounded as*

$$P_s \leq \frac{(M-1)\mathcal{N}_0^2}{M^2} \cdot \frac{MbP_1\delta_{s,r}^2 + (M-1)bP_2\delta_{r,d}^2 + (2M-1)\mathcal{N}_0}{(\mathcal{N}_0 + bP_1\delta_{s,d}^2)(\mathcal{N}_0 + bP_1\delta_{s,r}^2)(\mathcal{N}_0 + bP_2\delta_{r,d}^2)}, \tag{16}$$

where $b = b_{\mathrm{PSK}}$ for M-PSK signals and $b = b_{\mathrm{QAM}}/2$ for M-QAM signals. Furthermore, if all of the channel links $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ are available, i.e., $\delta_{s,d}^2 \neq 0$, $\delta_{s,r}^2 \neq 0$ and $\delta_{r,d}^2 \neq 0$, then for sufficiently high SNR, the SER of the systems with M-PSK or M-QAM modulation can be tightly approximated as

$$P_s \approx \frac{\mathcal{N}_0^2}{b^2} \cdot \frac{1}{P_1\delta_{s,d}^2}\left(\frac{A^2}{P_1\delta_{s,r}^2} + \frac{B}{P_2\delta_{r,d}^2}\right), \tag{17}$$

where in case of M-PSK signals, $b = b_{\mathrm{PSK}}$ and

$$A = \frac{M-1}{2M} + \frac{\sin\frac{2\pi}{M}}{4\pi}, \qquad B = \frac{3(M-1)}{8M} + \frac{\sin\frac{2\pi}{M}}{4\pi} - \frac{\sin\frac{4\pi}{M}}{32\pi}; \tag{18}$$

while in case of M-QAM signals, $b = b_{\mathrm{QAM}}/2$ and

$$A = \frac{M-1}{2M} + \frac{K^2}{\pi}, \qquad B = \frac{3(M-1)}{8M} + \frac{K^2}{\pi}. \tag{19}$$

Proof First, let us show the upper bound in (16). In case of M-PSK modulation, the closed-form SER expression was given in (12). By removing the negative term in (12), we have

$$\begin{aligned} P_{\mathrm{PSK}} \leq\ & F_1\left(1 + \frac{b_{\mathrm{PSK}}P_1\delta_{s,d}^2}{\mathcal{N}_0\sin^2\theta}\right)F_1\left(1 + \frac{b_{\mathrm{PSK}}P_1\delta_{s,r}^2}{\mathcal{N}_0\sin^2\theta}\right) \\ & + F_1\left(\left(1 + \frac{b_{\mathrm{PSK}}P_1\delta_{s,d}^2}{\mathcal{N}_0\sin^2\theta}\right)\left(1 + \frac{b_{\mathrm{PSK}}P_2\delta_{r,d}^2}{\mathcal{N}_0\sin^2\theta}\right)\right). \end{aligned} \tag{20}$$

We observe that in the right hand side of the above inequality, all integrands have their maximum value when $\sin^2\theta = 1$. Therefore, by substituting $\sin^2\theta = 1$ into (20), we have

$$\begin{aligned} P_{\mathrm{PSK}} \leq\ & \frac{(M-1)^2}{M^2} \cdot \frac{\mathcal{N}_0^2}{(\mathcal{N}_0 + b_{\mathrm{PSK}}P_1\delta_{s,d}^2)(\mathcal{N}_0 + b_{\mathrm{PSK}}P_1\delta_{s,r}^2)} \\ & + \frac{M-1}{M} \cdot \frac{\mathcal{N}_0^2}{(\mathcal{N}_0 + b_{\mathrm{PSK}}P_1\delta_{s,d}^2)(\mathcal{N}_0 + b_{\mathrm{PSK}}P_2\delta_{r,d}^2)} \\ =\ & \frac{(M-1)\mathcal{N}_0^2}{M^2} \cdot \frac{Mb_{\mathrm{PSK}}P_1\delta_{s,r}^2 + (M-1)b_{\mathrm{PSK}}P_2\delta_{r,d}^2 + (2M-1)\mathcal{N}_0}{(\mathcal{N}_0 + b_{\mathrm{PSK}}P_1\delta_{s,d}^2)(\mathcal{N}_0 + b_{\mathrm{PSK}}P_1\delta_{s,r}^2)(\mathcal{N}_0 + b_{\mathrm{PSK}}P_2\delta_{r,d}^2)}, \end{aligned}$$

which validates the upper bound in (16) for M-PSK modulation. Similarly, in case of M-QAM modulation, the SER in (14) can be upper bounded as

$$P_{\mathrm{QAM}} \le F_2\left(1+\frac{b_{\mathrm{QAM}}P_1\delta_{s,d}^2}{2\mathcal{N}_0\sin^2\theta}\right)F_2\left(1+\frac{b_{\mathrm{QAM}}P_1\delta_{s,r}^2}{2\mathcal{N}_0\sin^2\theta}\right) + F_2\left(\left(1+\frac{b_{\mathrm{QAM}}P_1\delta_{s,d}^2}{2\mathcal{N}_0\sin^2\theta}\right)\left(1+\frac{b_{\mathrm{QAM}}P_2\delta_{r,d}^2}{2\mathcal{N}_0\sin^2\theta}\right)\right). \tag{21}$$

Note that, the function $F_2(x(\theta))$ defined in (15) can be rewritten as

$$F_2(x(\theta)) = \frac{4K}{\pi\sqrt{M}}\int_0^{\pi/2}\frac{1}{x(\theta)}d\theta + \frac{4K^2}{\pi}\int_{\pi/4}^{\pi/2}\frac{1}{x(\theta)}d\theta, \tag{22}$$

which does not contain negative term. Moreover, the integrands in (21) have their maximum value when $\sin^2\theta = 1$. Thus, by substituting (22) and $\sin^2\theta = 1$ into (21), we have

$$\begin{aligned} P_{\mathrm{QAM}} &\le \left(\frac{2K}{\sqrt{M}}+K^2\right)^2 \frac{\mathcal{N}_0^2}{(\mathcal{N}_0+\frac{b_{\mathrm{QAM}}}{2}P_1\delta_{s,d}^2)(\mathcal{N}_0+\frac{b_{\mathrm{QAM}}}{2}P_1\delta_{s,r}^2)} \\ &\quad + \left(\frac{2K}{\sqrt{M}}+K^2\right)\frac{\mathcal{N}_0^2}{(\mathcal{N}_0+\frac{b_{\mathrm{QAM}}}{2}P_1\delta_{s,d}^2)(\mathcal{N}_0+\frac{b_{\mathrm{QAM}}}{2}P_2\delta_{r,d}^2)} \\ &= \frac{(M-1)\mathcal{N}_0^2}{M^2}\cdot\frac{M\frac{b_{\mathrm{QAM}}}{2}P_1\delta_{s,r}^2+(M-1)\frac{b_{\mathrm{QAM}}}{2}P_2\delta_{r,d}^2+(2M-1)\mathcal{N}_0}{(\mathcal{N}_0+\frac{b_{\mathrm{QAM}}}{2}P_1\delta_{s,d}^2)(\mathcal{N}_0+\frac{b_{\mathrm{QAM}}}{2}P_1\delta_{s,r}^2)(\mathcal{N}_0+\frac{b_{\mathrm{QAM}}}{2}P_2\delta_{r,d}^2)}, \end{aligned}$$

in which $K = 1-\frac{1}{\sqrt{M}}$. Therefore, the upper bound in (16) also holds for M-QAM modulation.

In the following, we show the asymptotically tight approximation (17) with the assumption that all of the channel links $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ are available, i.e., $\delta_{s,d}^2 \neq 0$, $\delta_{s,r}^2 \neq 0$ and $\delta_{r,d}^2 \neq 0$. First, let us consider the M-PSK modulation. In the SER formulation (12), we observe that for sufficiently large power P_1 and P_2, $1+\frac{b_{\mathrm{PSK}}P_1\delta_{s,d}^2}{\mathcal{N}_0\sin^2\theta} \approx \frac{b_{\mathrm{PSK}}P_1\delta_{s,d}^2}{\mathcal{N}_0\sin^2\theta}$, $1+\frac{b_{\mathrm{PSK}}P_1\delta_{s,r}^2}{\mathcal{N}_0\sin^2\theta} \approx \frac{b_{\mathrm{PSK}}P_1\delta_{s,r}^2}{\mathcal{N}_0\sin^2\theta}$ and $1+\frac{b_{\mathrm{PSK}}P_2\delta_{r,d}^2}{\mathcal{N}_0\sin^2\theta} \approx \frac{b_{\mathrm{PSK}}P_2\delta_{r,d}^2}{\mathcal{N}_0\sin^2\theta}$, i.e., the 1s are negligible with sufficiently large power. Thus, for sufficiently high SNR, the SER in (12) can be tightly approximated as

$$\begin{aligned} P_{\mathrm{PSK}} &\approx F_1\left(\frac{b_{\mathrm{PSK}}P_1\delta_{s,d}^2}{\mathcal{N}_0\sin^2\theta}\right)F_1\left(\frac{b_{\mathrm{PSK}}P_1\delta_{s,r}^2}{\mathcal{N}_0\sin^2\theta}\right) \\ &\quad + F_1\left(\frac{b_{\mathrm{PSK}}^2P_1P_2\delta_{s,d}^2\delta_{s,r}^2}{\mathcal{N}_0^2\sin^4\theta}\right)\left[1-F_1\left(\frac{b_{\mathrm{PSK}}P_1\delta_{s,r}^2}{\mathcal{N}_0\sin^2\theta}\right)\right] \\ &\approx F_1\left(\frac{b_{\mathrm{PSK}}P_1\delta_{s,d}^2}{\mathcal{N}_0\sin^2\theta}\right)F_1\left(\frac{b_{\mathrm{PSK}}P_1\delta_{s,r}^2}{\mathcal{N}_0\sin^2\theta}\right) + F_1\left(\frac{b_{\mathrm{PSK}}^2P_1P_2\delta_{s,d}^2\delta_{s,r}^2}{\mathcal{N}_0^2\sin^4\theta}\right), \\ &= \frac{A^2\mathcal{N}_0^2}{b_{\mathrm{PSK}}^2P_1^2\delta_{s,d}^2\delta_{s,r}^2} + \frac{B\mathcal{N}_0^2}{b_{\mathrm{PSK}}^2P_1P_2\delta_{s,d}^2\delta_{r,d}^2}, \end{aligned} \tag{23}$$

in which $A = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \sin^2\theta d\theta = \frac{M-1}{2M} + \frac{\sin\frac{2\pi}{M}}{4\pi}$, and $B = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \sin^4\theta d\theta = \frac{3(M-1)}{8M} + \frac{\sin\frac{2\pi}{M}}{4\pi} - \frac{\sin\frac{4\pi}{M}}{32\pi}$. Note that the second approximation is due to the fact that

$$1 - F_1\left(\frac{b_{\mathrm{PSK}} P_1 \delta_{s,r}^2}{\mathcal{N}_0 \sin^2\theta}\right) = 1 - \frac{\mathcal{N}_0}{\pi b_{\mathrm{PSK}} P_1 \delta_{s,r}^2}\int_0^{(M-1)\pi/M} \sin^2\theta d\theta \approx 1$$

for sufficiently large P_1. Therefore, the asymptotically tight approximation in (17) holds for the M-PSK modulation. In case of M-QAM signals, similarly the SER formulation in (14) can be tightly approximated at high SNR as follows

$$\begin{aligned} P_{\mathrm{QAM}} &\approx F_2\left(\frac{b_{\mathrm{QAM}} P_1 \delta_{s,d}^2}{2\mathcal{N}_0 \sin^2\theta}\right) F_2\left(\frac{b_{\mathrm{QAM}} P_1 \delta_{s,r}^2}{2\mathcal{N}_0 \sin^2\theta}\right) + F_2\left(\frac{b_{\mathrm{QAM}}^2 P_1 P_2 \delta_{s,d}^2 \delta_{r,d}^2}{4\mathcal{N}_0^2 \sin^4\theta}\right) \\ &= \frac{4A^2\mathcal{N}_0^2}{b_{\mathrm{QAM}}^2 P_1^2 \delta_{s,d}^2 \delta_{s,r}^2} + \frac{4B\mathcal{N}_0^2}{b_{\mathrm{QAM}}^2 P_1 P_2 \delta_{s,d}^2 \delta_{r,d}^2}, \end{aligned} \tag{24}$$

where

$$A = \frac{4K}{\pi\sqrt{M}}\int_0^{\pi/2} \sin^2\theta d\theta + \frac{4K^2}{\pi}\int_{\pi/4}^{\pi/2} \sin^2\theta d\theta = \frac{M-1}{2M} + \frac{K^2}{\pi},$$

$$B = \frac{4K}{\pi\sqrt{M}}\int_0^{\pi/2} \sin^4\theta d\theta + \frac{4K^2}{\pi}\int_{\pi/4}^{\pi/2} \sin^4\theta d\theta = \frac{3(M-1)}{8M} + \frac{K^2}{\pi}.$$

Thus, the asymptotically tight approximation in (17) also holds for the M-QAM signals. □

In Fig. 2, we compare the asymptotically tight approximation (17) and the SER upper bound (16) with the exact SER formulations (12) and (14) in case of QPSK (or 4-QAM) modulation. In this case, the parameters b, A and B in the upper bound (16) and the approximation (17) are specified as $b = 1$, $A = \frac{3}{8} + \frac{1}{4\pi}$ and $B = \frac{9}{32} + \frac{1}{4\pi}$. We can see that the upper bound (16) (dashed line with '·') is asymptotically parallel with the exact SER curve (solid line with '◇'), which means that they have the same diversity order. The approximation (17) (dashed line with '∘') is loose at low SNR, but it is tight at reasonable high SNR. It merges with the exact SER curve at an SER of 10^{-3}. Both the SER upper bound and the approximation show the asymptotic performance of the DF cooperation systems. Specifically, from the asymptotically tight approximation (17), we observe that the link between source and destination contributes diversity order one in the system performance. The term $\frac{A^2}{P_1\delta_{s,r}^2} + \frac{B}{P_2\delta_{r,d}^2}$ also contributes diversity order one in the performance, but it depends on the balance of the two channel links from source to relay and from relay to destination. Therefore, the DF cooperation systems show an overall performance of diversity order two.

3.3 Optimum Power Allocation

Note that the SER approximation (17) is asymptotically tight at high SNR. In this subsection, we determine an asymptotic optimum power allocation for the DF cooperation protocol based on the asymptotically tight SER approximation.

Specifically, we try to determine an optimum transmitted power P_1 that should be used at the source and P_2 at the relay for a fixed total transmission power $P_1 + P_2 = P$. According

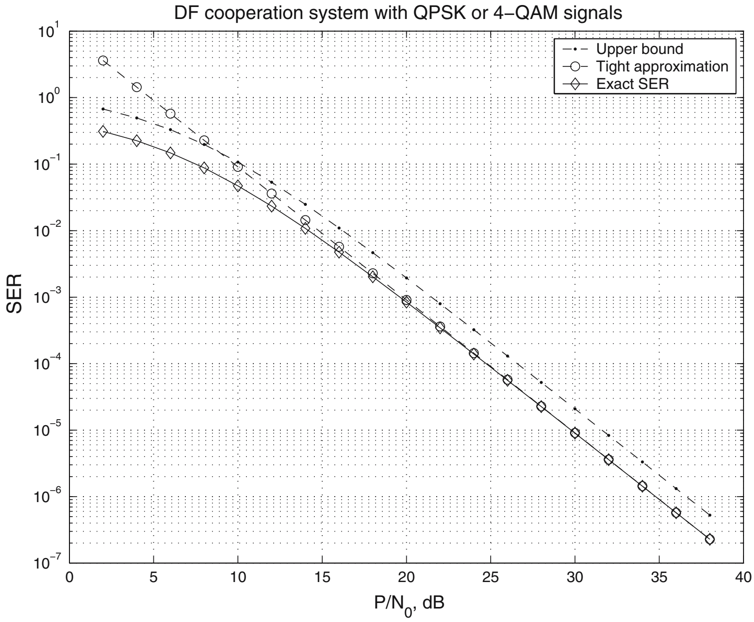


Fig. 2 Comparison of the exact SER formulation, the upper bound and the asymptotically tight approximation for the DF cooperation system with QPSK or 4-QAM signals. We assumed that $\delta^2_{s,d} = \delta^2_{s,r} = \delta^2_{r,d} = 1, \mathcal{N}_0 = 1$, and $P_1 = P_2 = P/2$

to the asymptotically tight SER approximation (17), it is sufficient to minimize

$$G(P_1, P_2) = \frac{1}{P_1\delta^2_{s,d}}\left(\frac{A^2}{P_1\delta^2_{s,r}} + \frac{B}{P_2\delta^2_{r,d}}\right).$$

By taking derivative in terms of P_1, we have

$$\frac{\partial G(P_1, P_2)}{\partial P_1} = \frac{1}{P_1\delta^2_{s,d}}\left(-\frac{A^2}{P_1^2\delta^2_{s,r}} + \frac{B}{P_2^2\delta^2_{r,d}}\right) - \frac{1}{P_1^2\delta^2_{s,d}}\left(\frac{A^2}{P_1\delta^2_{s,r}} + \frac{B}{P_2\delta^2_{r,d}}\right).$$

By setting the above derivation as 0, we come up with an equation as follows:

$$B\delta^2_{s,r}(P_1^2 - P_1P_2) - 2A^2\delta^2_{r,d}P_2^2 = 0.$$

With the power constraint, we can solve the above equation and arrive at the following result.

Theorem 2 *In the DF cooperation systems with M-PSK or M-QAM modulation, if all of the channel links $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ are available, i.e., $\delta^2_{s,d} \neq 0$, $\delta^2_{s,r} \neq 0$ and $\delta^2_{r,d} \neq 0$, then for sufficiently high SNR, the optimum power allocation is*

$$P_1 = \frac{\delta_{s,r} + \sqrt{\delta^2_{s,r} + 8(A^2/B)\delta^2_{r,d}}}{3\delta_{s,r} + \sqrt{\delta^2_{s,r} + 8(A^2/B)\delta^2_{r,d}}}P, \tag{25}$$

$$P_2 = \frac{2\delta_{s,r}}{3\delta_{s,r} + \sqrt{\delta^2_{s,r} + (8A^2/B)\delta^2_{r,d}}}P, \tag{26}$$

where A and B are specified in (18) and (19) *for M-PSK and M-QAM signals respectively.*

The result in Theorem 2 is somewhat surprising since the asymptotic optimum power allocation does not depend on the channel link between source and destination, it depends only on the channel link between source and relay and the channel link between relay and destination. Moreover, we can see that the optimum ratio of the transmitted power P_1 at the source over the total power P is less than 1 and larger than $1/2$, while the optimum ratio of the power P_2 used at the relay over the total power P is larger than 0 and less than $1/2$, i.e.,

$$\frac{1}{2} < \frac{P_1}{P} < 1 \quad \text{and} \quad 0 < \frac{P_2}{P} < \frac{1}{2}.$$

It means that we should always put more power at the source and less power at the relay. If the link quality between source and relay is much less than that between relay and destination, i.e., $\delta_{s,r}^2 << \delta_{r,d}^2$, then from (25) and (26), P_1 goes to P and P_2 goes to 0. It implies that we should use almost all of the power P at the source, and use few power at the relay. On the other hand, if the link quality between source and relay is much larger than that between relay and destination, i.e., $\delta_{s,r}^2 >> \delta_{r,d}^2$, then both P_1 and P_2 go to $P/2$. It means that we should put equal power at the source and the relay in this case.

We interpret the result in Theorem 2 as follows. Since we assume that all of the channel links $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ are available in the system, the cooperation strategy is expected to achieve a performance diversity of order two. The system is guaranteed to have a performance diversity of order one due to the channel link between source and destination. However, in order to achieve a diversity of order two, the channel link between source and relay and the channel link between relay and destination should be appropriately balanced. If the link quality between source and relay is bad, then it is difficult for the relay to correctly decode the transmitted symbol. Thus, the forwarding role of the relay is less important and it makes sense to put more power at the source. On the other hand, if the link quality between source and relay is very good, the relay can always decode the transmitted symbol correctly, so the decoded symbol at the relay is almost the same as that at the source. We may consider the relay as a copy of the source and put almost equal power on them. We want to emphasize that this interpretation is good only for sufficiently high SNR scenario and under the assumption that all of the channel links $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ are available. Actually, this interpretation is not accurate in general. For example, in case that the link quality between source and relay is the same as that between relay and destination, i.e., $\delta_{s,r}^2 = \delta_{r,d}^2$, the asymptotic optimum power allocation is given by

$$P_1 = \frac{1 + \sqrt{1 + 8A^2/B}}{3 + \sqrt{1 + 8A^2/B}} P, \tag{27}$$

$$P_2 = \frac{2}{3 + \sqrt{1 + 8A^2/B}} P, \tag{28}$$

where A and B depend on specific modulation signals. For example, if BPSK modulation is used, then $P_1 = 0.5931P$ and $P_2 = 0.4069P$; while if QPSK modulation is used, then $P_1 = 0.6270P$ and $P_2 = 0.3730P$. In case of 16-QAM, $P_1 = 0.6495P$ and $P_2 = 0.3505P$. We can see that the larger the constellation size, the more power should be put at the source.

It is worth pointing out that even though the asymptotic optimum power allocation in (25) and (26) are determined for high SNR, they also provide a good solution to a realistic moderate SNR scenario as in Fig. 3, in which we plotted exact SER as a function of the ratio P_1/P for a DF cooperation system with QPSK modulation. We considered the DF cooperation system with $\delta_{s,r}^2 = \delta_{r,d}^2 = 1$ and three different qualities of the channel link between source

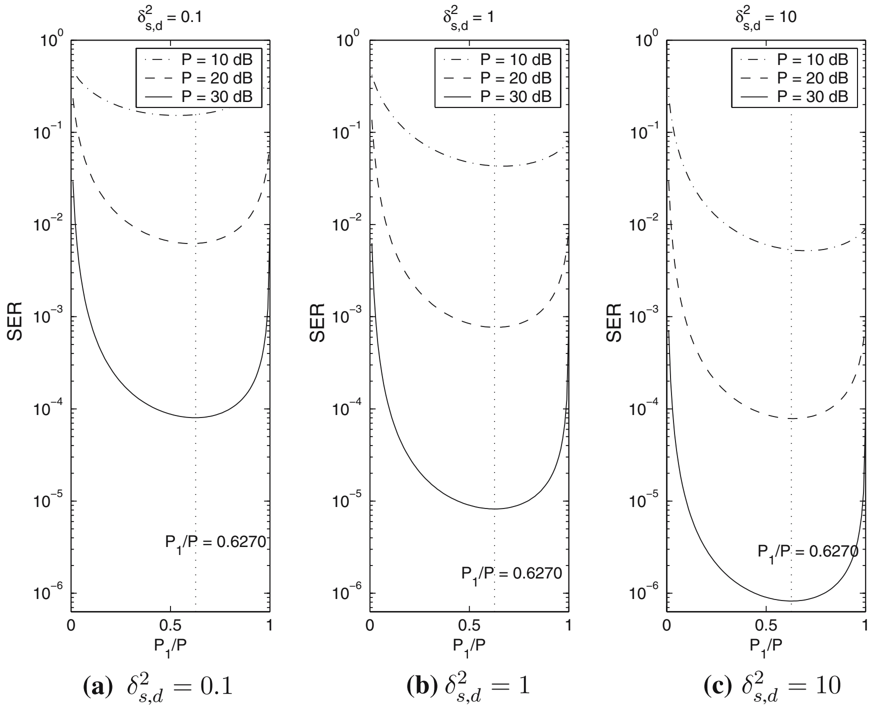


Fig. 3 SER of the DF cooperation systems with $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 1$: (**a**) $\delta^2_{s,d} = 0.1$; (**b**) $\delta^2_{s,d} = 1$; and (**c**) $\delta^2_{s,d} = 10$. The asymptotic optimum power allocation is $P_1/P = 0.6270$ and $P_2/P = 0.3730$.

and destination: (a) $\delta^2_{s,d} = 0.1$; (b) $\delta^2_{s,d} = 1$; and (c) $\delta^2_{s,d} = 10$. The asymptotic optimum power allocation in this case is $P_1/P = 0.6270$ and $P_2/P = 0.3730$. From the figures, we can see that the ratio $P_1/P = 0.6270$ almost provides the best performance for different total transmit power $P = 10, 20, 30\,\text{dB}$.

3.4 Some Special Scenarios

We have determined the optimum power allocation in (25) and (26) for the DF cooperation systems in case that all of the channel links $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ are available. In the following, we consider some special cases that some of the channel links are not available.

Case 1 If the channel link between relay and destination is not available, i.e., $\delta^2_{r,d} = 0$, according to (12), the SER of the DF system with M-PSK modulation can be given by

$$P_{\text{PSK}} = F_1\left(1 + \frac{b_{\text{PSK}} P_1 \delta^2_{s,d}}{\mathcal{N}_0 \sin^2\theta}\right) \le \frac{A\mathcal{N}_0}{b_{\text{PSK}} P_1 \delta^2_{s,d}}, \tag{29}$$

where A is specified in (18). Similarly, from (14), the SER of the system with M-QAM modulation is

$$P_{\text{QAM}} = F_2\left(1 + \frac{b_{\text{QAM}} P_1 \delta^2_{s,d}}{2\mathcal{N}_0 \sin^2\theta}\right) \le \frac{2A\mathcal{N}_0}{b_{\text{QAM}} P_1 \delta^2_{s,d}}, \tag{30}$$

where A is specified in (19). From (29) and (30), we can see that for both M-PSK and M-QAM signals, the optimum power allocation is $P_1 = P$ and $P_2 = 0$. It means that we should use the direct transmission from source to destination in this case.

Case 2 If the channel link between source and relay is not available, i.e., $\delta_{s,r}^2 = 0$, from (12) and (14), the SER of the DF system with M-PSK or M-QAM modulation can be upper bounded as $P_s \leq \frac{2A\mathcal{N}_0}{bP_1\delta_{s,d}^2}$, where in case of M-PSK modulation, $b = b_{\text{PSK}}$ and A is specified in (18), while in case of M-QAM modulation, $b = b_{\text{QAM}}/2$ and A is specified in (19). Therefore, the optimum power allocation in this case is $P_1 = P$ and $P_2 = 0$.

Case 3 If the channel link between source and destination is not available, i.e., $\delta_{s,d}^2 = 0$, according to (12) and (14), the SER of the DF system with M-PSK or M-QAM modulation can be given by

$$P_s = F_i\left(1 + \frac{bP_1\delta_{s,r}^2}{\mathcal{N}_0 \sin^2\theta}\right) + F_i\left(1 + \frac{bP_2\delta_{r,d}^2}{\mathcal{N}_0 \sin^2\theta}\right)\left[1 - F_i\left(1 + \frac{bP_1\delta_{s,r}^2}{\mathcal{N}_0 \sin^2\theta}\right)\right], \quad (31)$$

in which $i = 1$ and $b = b_{\text{PSK}}$ for M-PSK modulation, and $i = 2$ and $b = b_{\text{QAM}}/2$ for M-QAM modulation. If $\delta_{s,r}^2 \neq 0$ and $\delta_{r,d}^2 \neq 0$, then by the same procedure as we obtained the SER approximation in (17), the SER in (31) can be asymptotically approximated as

$$P_s \approx \frac{A\mathcal{N}_0^2}{b^2}\left(\frac{1}{P_1\delta_{s,r}^2} + \frac{1}{P_2\delta_{r,d}^2}\right), \quad (32)$$

where in case of M-PSK modulation, $b = b_{\text{PSK}}$ and A is specified in (18), while in case of M-PSK modulation, $b = b_{\text{QAM}}/2$ and A is specified in (19). From (32), we can see that with the total power $P_1 + P_2 = P$, the optimum power allocation in this case is

$$P_1 = \frac{\delta_{r,d}}{\delta_{s,r} + \delta_{r,d}} P \quad (33)$$

$$P_2 = \frac{\delta_{s,r}}{\delta_{s,r} + \delta_{r,d}} P \quad (34)$$

for both M-PSK and M-QAM modulations.

Note that when the channel link between source and destination is not available (i.e., $\delta_{s,d}^2 = 0$), the system reduces to a two-hop communication scenario [21]. It is worth noting that the optimum power allocation in (33) and (34), which is determined from minimizing the SER approximation (32), is consistent with the result in [21], in which the optimum power allocation was determined for multi-hop communication systems from a minimizing outage probability point of view.

4 SER Analysis for AF Cooperative communications

In this section, we investigate the SER performance for the AF cooperative communication systems. First, we derive a simple closed-form MGF expression for the harmonic mean of two independent exponential random variables. Second, based on the simple MGF expression, closed-form SER formulations are given for the AF cooperation systems with M-PSK and M-QAM modulations. Third, we provide an SER approximation, which is tight at high SNR, to show the asymptotic performance of the systems. Finally, based on the tight approximation, we are able to determine an optimum power allocation for the AF cooperation systems.

4.1 SER Analysis by MGF Approach

In the AF cooperation systems, the relay amplifies not only the received signal, but also the noise as shown in (4) and (5). The equivalent noise $\eta'_{r,d}$ at the destination in Phase 2 is a zero-mean complex Gaussian random variable with variance $\left(\frac{P_2|h_{r,d}|^2}{P_1|h_{s,r}|^2+\mathcal{N}_0}+1\right)\mathcal{N}_0$. Therefore, with knowledge of the channel coefficients $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$, the output of the MRC detector at the destination can be written as [17]

$$y = a_1 y_{s,d} + a_2 y_{r,d}, \tag{35}$$

where a_1 and a_2 are specified as

$$a_1 = \frac{\sqrt{P_1} h^*_{s,d}}{\mathcal{N}_0} \quad \text{and} \quad a_2 = \frac{\sqrt{\frac{P_1 P_2}{P_1|h_{s,r}|^2+\mathcal{N}_0}}\; h^*_{s,r} h^*_{r,d}}{\left(\frac{P_2|h_{r,d}|^2}{P_1|h_{s,r}|^2+\mathcal{N}_0}+1\right)\mathcal{N}_0}. \tag{36}$$

Note that to determine the factor a_2 in (36), we considered the equivalent received signal model in (5). By assuming that the transmitted symbol x in (1) has average energy 1, we know that the instantaneous SNR of the MRC output is [17]

$$\gamma = \gamma_1 + \gamma_2, \tag{37}$$

where $\gamma_1 = P_1|h_{s,d}|^2/\mathcal{N}_0$, and

$$\gamma_2 = \frac{1}{\mathcal{N}_0}\frac{P_1 P_2 |h_{s,r}|^2 |h_{r,d}|^2}{P_1|h_{s,r}|^2 + P_2|h_{r,d}|^2 + \mathcal{N}_0}. \tag{38}$$

It has been shown in [14] that the instantaneous SNR γ_2 in (38) can be tightly upper bounded as

$$\tilde{\gamma}_2 = \frac{1}{\mathcal{N}_0}\frac{P_1 P_2 |h_{s,r}|^2 |h_{r,d}|^2}{P_1|h_{s,r}|^2 + P_2|h_{r,d}|^2}, \tag{39}$$

which is the harmonic mean of two exponential random variables $P_1|h_{s,r}|^2/\mathcal{N}_0$ and $P_2|h_{r,d}|^2/\mathcal{N}_0$. According to (8) and (9), the conditional SER of the AF cooperation systems with M-PSK and M-QAM modulations can be given as follows:

$$P_{\text{PSK}}^{h_{s,d},h_{s,r},h_{r,d}} \approx \frac{1}{\pi}\int_0^{(M-1)\pi/M} \exp\left(-\frac{b_{\text{PSK}}(\gamma_1+\tilde{\gamma}_2)}{\sin^2\theta}\right) d\theta, \tag{40}$$

$$P_{\text{QAM}}^{h_{s,d},h_{s,r},h_{r,d}} \approx 4KQ\left(\sqrt{b_{\text{QAM}}(\gamma_1+\tilde{\gamma}_2)}\right) - 4K^2Q^2\left(\sqrt{b_{\text{QAM}}(\gamma_1+\tilde{\gamma}_2)}\right), \tag{41}$$

where $b_{\text{PSK}} = \sin^2(\pi/M)$, $b_{\text{QAM}} = 3/(M-1)$ and $K = 1 - \frac{1}{\sqrt{M}}$. Note that we used the SNR approximation $\gamma \approx \gamma_1 + \tilde{\gamma}_2$ in the above derivation.

Let us denote the MGF of a random variable Z as [18]

$$\mathcal{M}_Z(s) = \int_{-\infty}^{\infty} \exp(-sz) p_Z(z) dz, \tag{42}$$

for any real number s. By averaging over the Rayleigh fading channels $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ in (40) and (41), we obtain the SER of the AF cooperation systems in terms of MGF $\mathcal{M}_{\gamma_1}(s)$

and $\mathcal{M}_{\tilde{\gamma_2}}(s)$ as follows:

$$P_{\text{PSK}} \approx \frac{1}{\pi}\int_0^{(M-1)\pi/M} \mathcal{M}_{\gamma_1}\left(\frac{b_{\text{PSK}}}{\sin^2\theta}\right)\mathcal{M}_{\tilde{\gamma_2}}\left(\frac{b_{\text{PSK}}}{\sin^2\theta}\right)d\theta, \tag{43}$$

$$P_{\text{QAM}} \approx \left[\frac{4K}{\pi}\int_0^{\pi/2} - \frac{4K^2}{\pi}\int_0^{\pi/4}\right]\mathcal{M}_{\gamma_1}\left(\frac{b_{\text{QAM}}}{2\sin^2\theta}\right)\mathcal{M}_{\tilde{\gamma_2}}\left(\frac{b_{\text{QAM}}}{2\sin^2\theta}\right)d\theta, \tag{44}$$

in which, for simplicity, we use the following notation

$$\left[\frac{4K}{\pi}\int_0^{\pi/2} - \frac{4K^2}{\pi}\int_0^{\pi/4}\right]x(\theta)d\theta \triangleq \frac{4K}{\pi}\int_0^{\pi/2} x(\theta)d\theta - \frac{4K^2}{\pi}\int_0^{\pi/4} x(\theta)d\theta,$$

where $x(\theta)$ denotes a function with variable θ.

From (43) and (44), we can see that the remaining problem is to obtain the MGF $\mathcal{M}_{\gamma_1}(s)$ and $\mathcal{M}_{\tilde{\gamma_2}}(s)$. Since $\gamma_1 = P_1|h_{s,d}|^2/\mathcal{N}_0$ has an exponential distribution with parameter $\mathcal{N}_0/(P_1\delta_{s,d}^2)$, the MGF of γ_1 can be simply given by [18]

$$\mathcal{M}_{\gamma_1}(s) = \frac{1}{1+\frac{sP_1\delta_{s,d}^2}{\mathcal{N}_0}}. \tag{45}$$

However, it is not easy to get the MGF of $\tilde{\gamma_2}$ which is the harmonic mean of two exponential random variables $P_1|h_{s,r}|^2/\mathcal{N}_0$ and $P_2|h_{r,d}|^2/\mathcal{N}_0$. This has been investigated in [14] by applying Laplace transform and a solution was presented in terms of hypergeometric function as follows:

$$\mathcal{M}_{\tilde{\gamma_2}}(s) = \frac{16\beta_1\beta_2}{3(\beta_1+\beta_2+2\sqrt{\beta_1\beta_2}+s)^2}\left[\frac{4(\beta_1+\beta_2)}{\beta_1+\beta_2+2\sqrt{\beta_1\beta_2}+s}\,{}_2\right.$$
$$\left. +\ \mathrm{F}_1\left(3,\frac{3}{2};\frac{5}{2};\frac{\beta_1+\beta_2-2\sqrt{\beta_1\beta_2}+s}{\beta_1+\beta_2+2\sqrt{\beta_1\beta_2}+s}\right){}_2\mathrm{F}_1\left(2,\frac{1}{2};\frac{5}{2};\frac{\beta_1+\beta_2-2\sqrt{\beta_1\beta_2}+s}{\beta_1+\beta_2+2\sqrt{\beta_1\beta_2}+s}\right)\right], \tag{46}$$

in which $\beta_1 = \mathcal{N}_0/(P_1\delta_{s,r}^2)$, $\beta_2 = \mathcal{N}_0/(P_2\delta_{r,d}^2)$, and ${}_2\mathrm{F}_1(\cdot,\cdot;\cdot;\cdot)$ is the hypergeometric function[2] Because the hypergeometric function ${}_2\mathrm{F}_1(\cdot,\cdot;\cdot;\cdot)$ is defined as an integral, it is hard to use in an SER analysis aimed at revealing the asymptotic performance and optimizing the power allocation. Using an alternative approach, we found a simple closed-form solution for the MGF of $\tilde{\gamma_2}$ as shown in the next subsection.

4.2 Simple MGF Expression for the Harmonic Mean

In this subsection, we obtain at first a general result on the probability density function (pdf) for the harmonic mean of two independent random variables. Then, we are able to determine a simple closed-form MGF expression for the harmonic mean of two independent exponential random variables. The results presented are useful beyond this paper.

[2] A hypergeometric function with variables α, β, γ and z is defined as [15]

$${}_2\mathrm{F}_1(\alpha,\beta;\gamma;z) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-tz)^{-\alpha}dt,$$

where $\Gamma(\cdot)$ is the Gamma function.

Theorem 3 *Suppose that X_1 and X_2 are two independent random variables with pdf $p_{X_1}(x)$ and $p_{X_2}(x)$ defined for all $x \geq 0$, and $p_{X_1}(x) = 0$ and $p_{X_2}(x) = 0$ for $x < 0$. Then the pdf of $Z = \frac{X_1 X_2}{X_1 + X_2}$, the harmonic mean of X_1 and X_2, is*

$$p_Z(z) = z \int_0^1 \frac{1}{t^2(1-t)^2}\, p_{X_1}\left(\frac{z}{1-t}\right) p_{X_2}\left(\frac{z}{t}\right) dt \cdot U(z), \tag{47}$$

in which $U(z) = 1$ for $z \geq 0$ and $U(z) = 0$ for $z < 0$.

Note that we do not specify the distributions of the two independent random variables in Theorem 3. The proof of this theorem can be found in Appendix. Suppose that X_1 and X_2 are two independent exponential random variables with parameters β_1 and β_2 respectively, i.e., $p_{X_1}(x) = \beta_1 e^{-\beta_1 x} \cdot U(x)$ and $p_{X_2}(x) = \beta_2 e^{-\beta_2 x} \cdot U(x)$. Then, according to Theorem 3, the pdf of the harmonic mean $Z = \frac{X_1 X_2}{X_1 + X_2}$ can be simply given as

$$p_Z(z) = z \int_0^1 \frac{\beta_1 \beta_2}{t^2(1-t)^2}\, e^{-(\frac{\beta_1}{1-t} + \frac{\beta_2}{t})z} dt \cdot U(z). \tag{48}$$

The pdf of the harmonic mean Z has been presented in [14] in term of the zero-order and first-order modified Bessel functions [15]. The pdf expression in (48) is critical for us to obtain a simple closed-form MGF result for the harmonic mean Z.

Let us start calculating the MGF of the harmonic mean of two independent exponential random variables by substituting the pdf of Z (48) into the definition (42) as follows:

$$\begin{aligned} \mathcal{M}_Z(s) &= \int_0^\infty e^{-sz} z \int_0^1 \frac{\beta_1 \beta_2}{t^2(1-t)^2}\, e^{-(\frac{\beta_1}{1-t} + \frac{\beta_2}{t})z} dt dz \\ &= \int_0^1 \frac{\beta_1 \beta_2}{t^2(1-t)^2} \left(\int_0^\infty z\, e^{-(\frac{\beta_1}{1-t} + \frac{\beta_2}{t} + s)z} dz \right) dt, \end{aligned} \tag{49}$$

in which we switch the integration order. Since

$$\int_0^\infty z\, e^{-(\frac{\beta_1}{1-t} + \frac{\beta_2}{t} + s)z} dz = \left(\frac{\beta_1}{1-t} + \frac{\beta_2}{t} + s \right)^{-2},$$

the MGF in (49) can be determined as

$$\mathcal{M}_Z(s) = \int_0^1 \frac{\beta_1 \beta_2}{\left[\beta_2 + (\beta_1 - \beta_2 + s)t - st^2\right]^2} dt, \tag{50}$$

which is an integration of a quadratic trinomial and has a closed-form solution [15]. For notation simplicity, denote $\alpha = (\beta_1 - \beta_2 + s)/2$. According to the results on the integration over quadratic trinomial ([15], Eqs. 2.103.3 and 2.103.4), for any $s > 0$, we have

$$\begin{aligned} \int_0^1 \frac{1}{(\beta_2 + 2\alpha t - st^2)^2} dt &= \left. \frac{st - \alpha}{2(\beta_2 s + \alpha^2)(\beta_2 + 2\alpha t - st^2)} \right|_0^1 \\ &\quad + \frac{s}{4(\beta_2 s + \alpha^2)^{\frac{3}{2}}} \ln \left. \left| \frac{-st + \alpha - \sqrt{\beta_2 s + \alpha^2}}{-st + \alpha + \sqrt{\beta_2 s + \alpha^2}} \right| \right|_0^1 \\ &= \frac{\beta_2 s + \alpha(\beta_1 - \beta_2)}{2\beta_1\beta_2(\beta_2 s + \alpha^2)} + \frac{s}{4(\beta_2 s + \alpha^2)^{\frac{3}{2}}} \\ &\quad \times \ln \frac{\left(\beta_2 + \alpha + \sqrt{\beta_2 s + \alpha^2}\right)^2}{\beta_1 \beta_2}. \end{aligned} \tag{51}$$

By substituting $\alpha = (\beta_1 - \beta_2 + s)/2$ into (51) and denoting $\Delta = 2\sqrt{\beta_2 s + \alpha^2}$, we obtain a simple closed-form MGF for the harmonic mean Z as follows:

$$\mathcal{M}_Z(s) = \frac{(\beta_1 - \beta_2)^2 + (\beta_1 + \beta_2)s}{\Delta^2} + \frac{2\beta_1\beta_2 s}{\Delta^3} \ln \frac{(\beta_1 + \beta_2 + s + \Delta)^2}{4\beta_1\beta_2}, \quad s > 0, \quad (52)$$

where $\Delta = \sqrt{(\beta_1 - \beta_2)^2 + 2(\beta_1 + \beta_2)s + s^2}$. We can see that if β_1 and β_2 go to zero, then Δ can be approximated as s. In this case, the MGF in (52) can be simplified as

$$\mathcal{M}_Z(s) \approx \frac{\beta_1 + \beta_2}{s} + \frac{2\beta_1\beta_2}{s^2} \ln \frac{s^2}{\beta_1\beta_2}. \quad (53)$$

Note that in (53), the second term goes to zero faster than the first term. As a result, the MGF in (53) can be further simplified as

$$\mathcal{M}_Z(s) \approx \frac{\beta_1 + \beta_2}{s}. \quad (54)$$

We summarize the above discussion in the following theorem.

Theorem 4 *Let X_1 and X_2 be two independent exponential random variables with parameters β_1 and β_2 respectively. Then, the MGF of $Z = \frac{X_1X_2}{X_1+X_2}$ is*

$$\mathcal{M}_Z(s) = \frac{(\beta_1 - \beta_2)^2 + (\beta_1 + \beta_2)s}{\Delta^2} + \frac{2\beta_1\beta_2 s}{\Delta^3} \ln \frac{(\beta_1 + \beta_2 + s + \Delta)^2}{4\beta_1\beta_2} \quad (55)$$

for any $s > 0$, in which

$$\Delta = \sqrt{(\beta_1 - \beta_2)^2 + 2(\beta_1 + \beta_2)s + s^2}. \quad (56)$$

Furthermore, if β_1 and β_2 go to zero, then the MGF of Z can be approximated as

$$\mathcal{M}_Z(s) \approx \frac{\beta_1 + \beta_2}{s}. \quad (57)$$

We can see that the closed-form solution in (55) does not involve any integration. If X_1 and X_2 are i.i.d exponential random variables with parameter β, then according to the result in Theorem 4, the MGF of $Z = \frac{X_1X_2}{X_1+X_2}$ can be simply given as

$$\mathcal{M}_Z(s) = \frac{2\beta}{4\beta + s} + \frac{4\beta^2 s}{\Delta_0^3} \ln \frac{2\beta + s + \Delta_0}{2\beta}, \quad (58)$$

where $s > 0$ and $\Delta_0 = \sqrt{4\beta s + s^2}$. Note that we still do not see how the MGF expression in (46) in terms of hypergeometric function can be directly reduced to the simple closed-form solution (55) in Theorem 4. The approximation in (57) will provide a very simple solution for the SER calculations in (43) and (44) as shown in the next subsection.

4.3 Closed-Form SER Expressions and Asymptotically Tight Approximation

Now let us apply the result of Theorem 4 to the harmonic mean of two random variables $X_1 = P_1|h_{s,r}|^2/\mathcal{N}_0$ and $X_2 = P_2|h_{r,d}|^2/\mathcal{N}_0$ as we considered in Sect. 4.1. They are two independent exponential random variables with parameters $\beta_1 = \mathcal{N}_0/(P_1\delta_{s,r}^2)$ and $\beta_2 = \mathcal{N}_0/(P_2\delta_{r,d}^2)$, respectively.

With the closed-form MGF expression in Theorem 4, the SER formulations in (43) and (44) for AF systems with M-PSK and M-QAM modulations can be determined respectively as

$$P_{\text{PSK}} \approx \frac{1}{\pi}\int_0^{(M-1)\pi/M} \frac{1}{1+\frac{b_{\text{PSK}}}{\beta_0 \sin^2\theta}} \left\{ \frac{(\beta_1-\beta_2)^2+(\beta_1+\beta_2)\frac{b_{\text{PSK}}}{\sin^2\theta}}{\Delta^2} + \frac{2\beta_1\beta_2 b_{\text{PSK}}}{\Delta^3 \sin^2\theta} \ln \frac{(\beta_1+\beta_2+\frac{b_{\text{PSK}}}{\sin^2\theta}+\Delta)^2}{4\beta_1\beta_2} \right\} d\theta, \tag{59}$$

$$P_{\text{QAM}} \approx \left[\frac{4K}{\pi}\int_0^{\pi/2} - \frac{4K^2}{\pi}\int_0^{\pi/4}\right] \frac{1}{1+\frac{b_{\text{QAM}}}{2\beta_0 \sin^2\theta}} \left\{ \frac{(\beta_1-\beta_2)^2+(\beta_1+\beta_2)\frac{b_{\text{QAM}}}{2\sin^2\theta}}{\Delta^2} + \frac{\beta_1\beta_2 b_{\text{QAM}}}{\Delta^3 \sin^2\theta} \ln \frac{(\beta_1+\beta_2+\frac{b_{\text{QAM}}}{2\sin^2\theta}+\Delta)^2}{4\beta_1\beta_2} \right\} d\theta, \tag{60}$$

in which $\beta_0 = \mathcal{N}_0/(P_1\delta_{s,d}^2)$, $\beta_1 = \mathcal{N}_0/(P_1\delta_{s,r}^2)$, $\beta_2 = \mathcal{N}_0/(P_2\delta_{r,d}^2)$, and $\Delta^2 = (\beta_1-\beta_2)^2 + 2(\beta_1+\beta_2)s + s^2$ with $s = b_{\text{PSK}}/\sin^2\theta$ for M-PSK modulation and $s = b_{\text{QAM}}/(2\sin^2\theta)$ for M-QAM modulation. We observe that it is hard to understand the AF system performance based on the SER formulations in (59) and (60), even though they can be numerically calculated. In the following, we try to simplify the SER formulations by taking advantage of the MGF approximation in Theorem 4 to reveal the asymptotic performance of the AF cooperation systems.

We focus on the AF system with M-PSK modulation at first. Note that both $\beta_1 = \mathcal{N}_0/(P_1\delta_{s,r}^2)$ and $\beta_2 = \mathcal{N}_0/(P_2\delta_{r,d}^2)$ go to zero when the SNR goes to infinity. According to the MGF approximation (57) in Theorem 4, the SER formulation in (59) can be approximated as

$$\begin{aligned} P_{\text{PSK}} &\approx \frac{1}{\pi}\int_0^{(M-1)\pi/M} \frac{1}{1+\frac{b_{\text{PSK}}}{\beta_0\sin^2\theta}} \cdot \frac{\beta_1+\beta_2}{\frac{b_{\text{PSK}}}{\sin^2\theta}} d\theta \\ &= \frac{1}{\pi}\int_0^{(M-1)\pi/M} \frac{(\beta_1+\beta_2)\sin^4\theta}{b_{\text{PSK}}(\sin^2\theta + \frac{b_{\text{PSK}}}{\beta_0})} d\theta \qquad (61)\\ &\approx \frac{B}{b_{\text{PSK}}^2}\beta_0(\beta_1+\beta_2), \qquad (62) \end{aligned}$$

where $B = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \sin^4\theta d\theta = \frac{3(M-1)}{8M} + \frac{\sin\frac{2\pi}{M}}{4\pi} - \frac{\sin\frac{4\pi}{M}}{32\pi}$. To obtain the approximation in (62), we ignore the term $\sin^2\theta$ in the denominator in (61), which is negligible for sufficiently high SNR. Similarly, for the AF system with M-QAM modulation, the SER formulation in (60) can be approximated as

$$\begin{aligned} P_{\text{QAM}} &\approx \left[\frac{4K}{\pi}\int_0^{\pi/2} - \frac{4K^2}{\pi}\int_0^{\pi/4}\right] \frac{1}{1+\frac{b_{\text{QAM}}}{2\beta_0\sin^2\theta}} \cdot \frac{\beta_1+\beta_2}{\frac{b_{\text{QAM}}}{2\sin^2\theta}} d\theta \\ &= \left[\frac{4K}{\pi}\int_0^{\pi/2} - \frac{4K^2}{\pi}\int_0^{\pi/4}\right] \frac{4(\beta_1+\beta_2)\sin^4\theta}{b_{\text{QAM}}(2\sin^2\theta + \frac{b_{\text{QAM}}}{\beta_0})} d\theta \qquad (63)\\ &\approx \frac{4B}{b_{\text{QAM}}^2}\beta_0(\beta_1+\beta_2), \qquad (64) \end{aligned}$$

where $B = \left[\frac{4K}{\pi}\int_0^{\pi/2} - \frac{4K^2}{\pi}\int_0^{\pi/4}\right]\sin^4\theta\, d\theta = \frac{3(M-1)}{8M} + \frac{K^2}{\pi}$. Since for sufficiently high SNR, the term $2\sin^2\theta$ in the denominator in (63) is negligible, we ignore it to have the approximation in (64). We summarize the above discussion in the following theorem.

Theorem 5 *At sufficiently high SNR, the SER of the AF cooperation systems with M-PSK or M-QAM modulation can be approximated as*

$$P_s \approx \frac{B\mathcal{N}_0^2}{b^2} \cdot \frac{1}{P_1\delta_{s,d}^2}\left(\frac{1}{P_1\delta_{s,r}^2} + \frac{1}{P_2\delta_{r,d}^2}\right), \tag{65}$$

where in case of M-PSK signals, $b = b_{\mathrm{PSK}}$ and

$$B = \frac{3(M-1)}{8M} + \frac{\sin\frac{2\pi}{M}}{4\pi} - \frac{\sin\frac{4\pi}{M}}{32\pi}; \tag{66}$$

while in case of M-QAM signals, $b = b_{\mathrm{QAM}}/2$ and

$$B = \frac{3(M-1)}{8M} + \frac{K^2}{\pi}. \tag{67}$$

We compare the SER approximations (59), (60) and (65) with SER simulation result in Fig. 4 in case of AF cooperation system with QPSK (or 4-QAM) modulation. It is easy to check that for both QPSK and 4-QAM modulations, the parameters B in (66) and (67) are the same, in which $B = \frac{9}{32} + \frac{1}{4\pi}$. We can see that the theoretical calculation (59) or (60) matches with the simulation curve, except for a little bit difference between them at low SNR which is due to the approximation of the SNR $\tilde{\gamma}_2$ in (39). Furthermore, the simple SER approximation in (65) is tight at high SNR, which is good enough to show the asymptotic performance of the AF cooperation system. From Theorem 5, we can conclude that the AF cooperation systems also provide an overall performance of diversity order two, which is similar to that of DF cooperation systems.

It is interesting to note that the SER approximation in (65) is similar to a result in [22] where an SER approximation was obtained by investigating the behavior of the probability density function of γ around zero. Specifically, in case of BPSK modulation, the SER approximation in (65) with $B/b^2 = 3/16$ coincides with the result in [22]. However, for other modulation, the SER approximation in (65) is slightly different from the result in [22] with a constant factor. For example, in case of QPSK modulation, the factor B/b^2 in (65) is 1.4433 while an equivalent factor in [22] is 1.5; in case of 16-QAM, the factor B/b^2 in (65) is 53.06 while an equivalent factor in [22] is 56.25. Moreover, the approximation in [22] was obtained only for some types of modulation that the conditional SER can be expressed as a Gaussian Q-function like $Q(\sqrt{k\gamma})$ with a modulation dependent constant k and instantaneous SNR γ.

4.4 Optimum Power Allocation

We determine in this subsection an asymptotic optimum power allocation for the AF cooperation systems based on the tight SER approximation in (65) for sufficiently high SNR.

For a fixed total transmitted power $P_1 + P_2 = P$, we are going to optimize P_1 and P_2 such that the asymptotically tight SER approximation in (65) is minimized. Equivalently, we try to minimize

$$G(P_1, P_2) = \frac{1}{P_1\delta_{s,d}^2}\left(\frac{1}{P_1\delta_{s,r}^2} + \frac{1}{P_2\delta_{r,d}^2}\right).$$

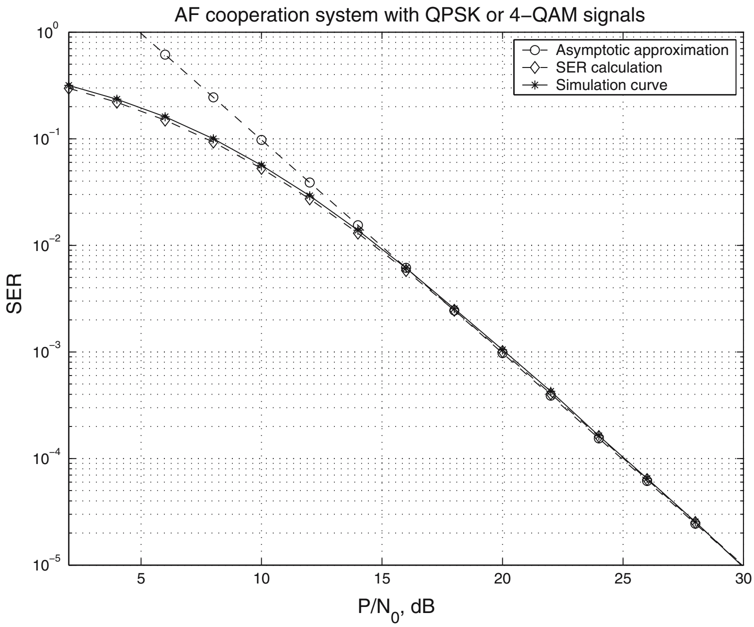


Fig. 4 Comparison of the SER approximations and the simulation result for the AF cooperation system with QPSK or 4-QAM signals. We assumed that $\delta_{s,d}^2 = \delta_{s,r}^2 = \delta_{r,d}^2 = 1$, $\mathcal{N}_0 = 1$, and $P_1/P = 2/3$ and $P_2/P = 1/3$

By taking derivative in terms of P_1, we have

$$\frac{\partial G(P_1, P_2)}{\partial P_1} = \frac{1}{P_1 \delta_{s,d}^2}\left(-\frac{1}{P_1^2 \delta_{s,r}^2} + \frac{1}{P_2^2 \delta_{r,d}^2}\right) - \frac{1}{P_1^2 \delta_{s,d}^2}\left(\frac{1}{P_1 \delta_{s,r}^2} + \frac{1}{P_2 \delta_{r,d}^2}\right).$$

By setting the above derivation as 0, we have $\delta_{s,r}^2(P_1^2 - P_1 P_2) - 2\delta_{r,d}^2 P_2^2 = 0$. Together with the power constraint $P_1 + P_2 = P$, we can solve the above equation and arrive at the following result.

Theorem 6 *For sufficiently high SNR, the optimum power allocation for the AF cooperation systems with either M-PSK or M-QAM modulation is*

$$P_1 = \frac{\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}}{3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}} P, \tag{68}$$

$$P_2 = \frac{2\delta_{s,r}}{3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}} P. \tag{69}$$

From Theorem 6, we observe that the optimum power allocation for the AF cooperation systems is not modulation-dependent, which is different from that for the DF cooperation systems in which the optimum power allocation depends on specific M-PSK or M-QAM modulation as stated in Theorem 2. This is due to the fact that in the AF cooperation systems, the relay amplifies the received signal and forwards it to the destination regardless what kind of received signal is. While in the DF cooperation systems, the relay forwards information to the destination only if the relay correctly decodes the received signal, and the decoding at the relay requires specific modulation information, which results in the modulation-dependent optimum power allocation scheme.

On the other hand, the asymptotic optimum power allocation scheme in Theorem 6 for the AF cooperation systems is similar to that in Theorem 2 for the DF cooperation systems, in the sense that both of them do not depend on the channel link between source and destination, and depend only on the channel link between source and relay and the channel link between relay and destination. Similarly, we can see from Theorem 6 that the optimum ratio of the transmitted power P_1 at the source over the total power P is less than 1 and larger than $1/2$, while the optimum ratio of the power P_2 used at the relay over the total power P is larger than 0 and less than $1/2$. In general, the equal power strategy is not optimum. For example, if $\delta_{s,r}^2 = \delta_{r,d}^2$, then the optimum power allocation is $P_1 = \frac{2}{3}P$ and $P_2 = \frac{1}{3}P$.

5 Comparison of DF and AF Cooperation Gains

Based on the asymptotically tight SER approximations and the optimum power allocation solutions we established in the previous two sections, we determine in this section the overall cooperation gain and diversity order for the DF and AF cooperation systems respectively. Then, we are able to compare the cooperation gain between the DF and AF cooperation protocols.

Let us first focus on the DF cooperation protocol. According to the asymptotically tight SER approximation (17) in Theorem 1, we know that for sufficiently high SNR, the SER performance of the DF cooperation systems can be approximated as

$$P_s \approx \frac{\mathcal{N}_0^2}{b^2} \cdot \frac{1}{P_1 \delta_{s,d}^2} \left(\frac{A^2}{P_1 \delta_{s,r}^2} + \frac{B}{P_2 \delta_{r,d}^2} \right), \tag{70}$$

where A and B are specified in (18) and (19) for M-PSK and M-QAM signals, respectively. By substituting the asymptotic optimum power allocation (25) and (26) into (70), we have

$$P_s \approx \Delta_{DF}^{-2} \left(\frac{P}{\mathcal{N}_0} \right)^{-2}, \tag{71}$$

where

$$\Delta_{DF} = \frac{2\sqrt{2}\, b \delta_{s,d} \delta_{s,r} \delta_{r,d}}{\sqrt{B}} \frac{\left(\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8(A^2/B)\delta_{r,d}^2}\right)^{1/2}}{\left(3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8(A^2/B)\delta_{r,d}^2}\right)^{3/2}}, \tag{72}$$

in which $b = b_{\text{PSK}}$ for M-PSK signals and $b = b_{\text{QAM}}/2$ for M-QAM signals. From (71), we can see that the DF cooperation systems can guarantee a performance diversity of order two. Note that the term Δ_{DF} in (72) depends only on the statistics of the channel links. We call it the *cooperation gain* of the DF cooperation systems, which indicates the best performance gain that we are able to achieve through the DF cooperation protocol with any kind of power allocation. If the link quality between source and relay is much less than that between relay and destination, i.e., $\delta_{s,r}^2 << \delta_{r,d}^2$, then the cooperation gain is approximated as $\Delta_{DF} = \frac{b\delta_{s,d}\delta_{s,r}}{A}$, in which $A = \frac{M-1}{2M} + \frac{\sin\frac{2\pi}{M}}{4\pi} \to \frac{1}{2}$ (M large) for M-PSK modulation, or $A = \frac{M-1}{2M} + \frac{K^2}{\pi} \to \frac{1}{2} + \frac{1}{\pi}$ (M large) for M-QAM modulation. For example, in case of QPSK modulation, $A = \frac{3}{8} + \frac{1}{4\pi} = 0.4546$. On the other hand, if the link quality between source and relay is much larger than that between relay and destination, i.e., $\delta_{s,r}^2 >> \delta_{r,d}^2$, then the cooperation gain can be approximated as $\Delta_{DF} = \frac{b\delta_{s,d}\delta_{r,d}}{2\sqrt{B}}$, in which $B = \frac{3(M-1)}{8M} + \frac{\sin\frac{2\pi}{M}}{4\pi} - \frac{\sin\frac{4\pi}{M}}{32\pi} \to \frac{3}{8}$ (M large) for M-PSK modulation, or $B = \frac{3(M-1)}{8M} + \frac{K^2}{\pi} \to \frac{3}{8} + \frac{1}{\pi}$ (M large) for M-QAM modulation. For example, in case of QPSK modulation, $B = \frac{9}{32} + \frac{1}{4\pi} = 0.3608$.

Similarly, for the AF cooperation protocol, from the asymptotically tight SER approximation (65) in Theorem 5, we can see that for sufficiently high SNR, the SER performance of the AF cooperation systems can be approximated as

$$P_s \approx \frac{B\mathcal{N}_0^2}{b^2} \cdot \frac{1}{P_1\delta_{s,d}^2}\left(\frac{1}{P_1\delta_{s,r}^2} + \frac{1}{P_2\delta_{r,d}^2}\right), \tag{73}$$

where $b = b_{\text{PSK}}$ for M-PSK signals and $b = b_{\text{QAM}}/2$ for M-QAM signals, and B is specified in (66) and (67) for M-PSK and M-QAM signals respectively. By substituting the asymptotic optimum power allocation (68) and (69) into (73), we have

$$P_s \approx \Delta AF^{-2}\left(\frac{P}{\mathcal{N}_0}\right)^{-2}, \tag{74}$$

$$\Delta_{AF} = \frac{2\sqrt{2}\, b\delta_{s,d}\delta_{s,r}\delta_{r,d}}{\sqrt{B}} \frac{\left(\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}\right)^{1/2}}{\left(3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}\right)^{3/2}}, \tag{75}$$

which is termed as the *cooperation gain* of the AF cooperation systems that indicates the best asymptotic performance gain of the AF cooperation protocol with the optimum power allocation scheme. From (74), we can see that the AF cooperation systems can also guarantee a performance diversity of order two, which is similar to that of the DF cooperation systems.

Since both the AF and DF cooperation systems are able to achieve a performance diversity of order two, it is interesting to compare their cooperation gain. Let us define a ratio $\lambda = \Delta_{DF}/\Delta AF$ to indicate the performance gain of the DF cooperation protocol compared with the AF protocol. According to (72) and (75), we have

$$\lambda = \left(\frac{\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8(A^2/B)\delta_{r,d}^2}}{\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}}\right)^{1/2}\left(\frac{3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}}{\delta_{s,r} + \sqrt{3\delta_{s,r}^2 + 8(A^2/B)\delta_{r,d}^2}}\right)^{3/2}, \tag{76}$$

A and B are specified in (18) and (19) for M-PSK and M-QAM signals respectively. We further discuss the ratio λ for the following three cases.

Case 1 If the channel link quality between source and relay is much less than that between relay and destination, i.e., $\delta_{s,r}^2 << \delta_{r,d}^2$, then

$$\lambda = \frac{\Delta_{DF}}{\Delta AF} \to \frac{\sqrt{B}}{A}. \tag{77}$$

In case of BPSK modulation, $A = \frac{1}{4}$ and $B = \frac{3}{16}$, so $\lambda = \sqrt{3} > 1$. In case of QPSK modulation, $A = \frac{3}{8} + \frac{1}{4\pi}$ and $B = \frac{9}{32} + \frac{1}{4\pi}$, so $\lambda = 1.3214 > 1$. In general, for M-PSK modulation (M large), $A = \frac{M-1}{2M} + \frac{\sin\frac{2\pi}{M}}{4\pi} \to \frac{1}{2}$ and $B = \frac{3(M-1)}{8M} + \frac{\sin\frac{2\pi}{M}}{4\pi} - \frac{\sin\frac{4\pi}{M}}{32\pi} \to \frac{3}{8}$, so

$$\lambda \to \frac{\sqrt{6}}{2} \approx 1.2247 > 1.$$

For M-QAM modulation (M large), $A = \frac{M-1}{2M} + \frac{K^2}{\pi} \to \frac{1}{2} + \frac{1}{\pi}$ and $B = \frac{3(M-1)}{8M} + \frac{K^2}{\pi} \to \frac{3}{8} + \frac{1}{\pi}$,

$$\lambda \to \frac{\sqrt{\frac{3}{8} + \frac{1}{\pi}}}{\frac{1}{2} + \frac{1}{\pi}} \approx 1.0175 > 1.$$

We can see that if $\delta_{s,r}^2 << \delta_{r,d}^2$, the cooperation gain of the DF systems is always larger than that of the AF systems for both M-PSK and M-QAM modulations. The advantage of the DF cooperation systems is more significant if M-PSK modulation is used.

Case 2 If the channel link quality between source and relay is much better than that between relay and destination, i.e., $\delta_{s,r}^2 >> \delta_{r,d}^2$, from (76) we have $\lambda = \frac{\Delta_{DF}}{\Delta_{AF}} \to 1$. This implies that if $\delta_{s,r}^2 >> \delta_{r,d}^2$, the performance of the DF cooperation systems is almost the same as that of the AF cooperation systems for both M-PSK and M-QAM modulations. Since the DF cooperation protocol requires decoding process at the relay, we may suggest the use of the AF cooperation protocol in this case to reduce the system complexity.

Case 3 If the channel link quality between source and relay is the same as that between relay and destination, i.e., $\delta_{s,r}^2 = \delta_{r,d}^2$, we have

$$\lambda = \left(\frac{1 + \sqrt{1 + 8(A^2/B)}}{4} \right)^{1/2} \left(\frac{6}{3 + \sqrt{1 + 8(A^2/B)}} \right)^{3/2}.$$

In case of BPSK modulation, $A = \frac{1}{4}$ and $B = \frac{3}{16}$, so $\lambda \approx 1.1514 > 1$. In case of QPSK modulation, $A = \frac{3}{8} + \frac{1}{4\pi}$ and $B = \frac{9}{32} + \frac{1}{4\pi}$, so $\lambda \approx 1.0851 > 1$. In general, for M-PSK modulation (M large), $A = \frac{M-1}{2M} + \frac{\sin \frac{2\pi}{M}}{4\pi} \to \frac{1}{2}$ and $B = \frac{3(M-1)}{8M} + \frac{\sin \frac{2\pi}{M}}{4\pi} - \frac{\sin \frac{4\pi}{M}}{32\pi} \to \frac{3}{8}$, so

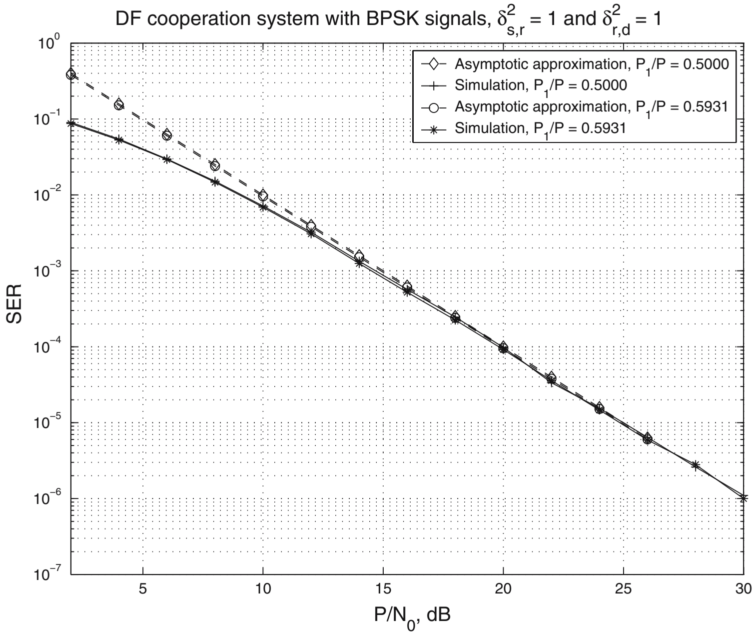
$$\lambda \to \left(\frac{1 + \sqrt{1 + 16/3}}{4} \right)^{1/2} \left(\frac{6}{3 + \sqrt{1 + 16/3}} \right)^{3/2} \approx 1.0635 > 1.$$

For M-QAM modulation (M large), $A = \frac{M-1}{2M} + \frac{K^2}{\pi} \to \frac{1}{2} + \frac{1}{\pi}$ and $B = \frac{3(M-1)}{8M} + \frac{K^2}{\pi} \to \frac{3}{8} + \frac{1}{\pi}$,

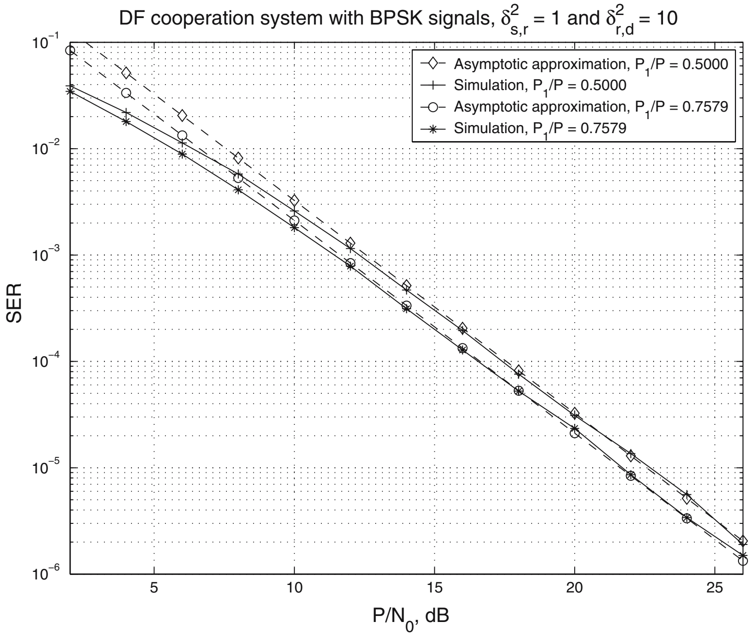
$$\lambda \to \left(\frac{1 + \sqrt{1 + 8(\frac{1}{2} + \frac{1}{\pi})^2/(\frac{3}{8} + \frac{1}{\pi})}}{4} \right)^{1/2} \left(\frac{6}{3 + \sqrt{1 + 8(\frac{1}{2} + \frac{1}{\pi})^2/(\frac{3}{8} + \frac{1}{\pi})}} \right)^{3/2} \approx 1.0058.$$

We can see that if the modulation size is large, the performance advantage of the DF cooperation protocol is negligible compared with the AF cooperation protocol. Actually, with QPSK modulation, the ratio of the cooperation gain is $\lambda \approx 1.0851$ which is already small.

From the above discussion, we can see that the performance of the DF cooperation protocol is always not less than that of the AF cooperation protocol. However, the performance advantage of the DF cooperation protocol is not significant unless (i) the channel link quality between the relay and the destination is much stronger than that between the source and the relay; and (ii) the constellation size of the signaling is small. There are tradeoff between these two cooperation protocols. The complexity of the AF cooperation protocol is less than that of the DF cooperation protocol in which decoding process at the relay is required. For high data-rate cooperative communications (with large modulation size), we may use the AF cooperation protocol to reduce the system complexity while the performance is comparable.



(a) $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 1$



(b) $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 10$

Fig. 5 Performance of the DF cooperation systems with BPSK signals: optimum power allocation versus equal power scheme

6 Simulation Results

To illustrate the above theoretical analysis, we perform some computer simulations. In all simulations, we assume that the variance of the noise is 1 (i.e., $\mathcal{N}_0 = 1$), and the variance of the channel link between source and destination is normalized as 1 (i.e., $\delta_{s,d}^2 = 1$). The performance of the DF and AF cooperation systems varies with different channel conditions. We simulate two kinds of channel conditions: (a) $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 1$; and (b) $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 10$. For fair comparison, we present average SER curves as functions of $P/\mathcal{N}_0$.

6.1 Performance of the DF Cooperation Systems

First, we simulate the DF cooperation systems with different modulation signals and different power allocation schemes. We compare the SER simulation curves with the asymptotically tight SER approximation in (17). We also compare the performance of the DF cooperation systems using the optimum power allocation scheme in Theorem 2 with that of the systems using the equal power scheme, in which the total transmitted power is equally allocated at the source and at the relay ($P_1/P = P_2/P = 1/2$).

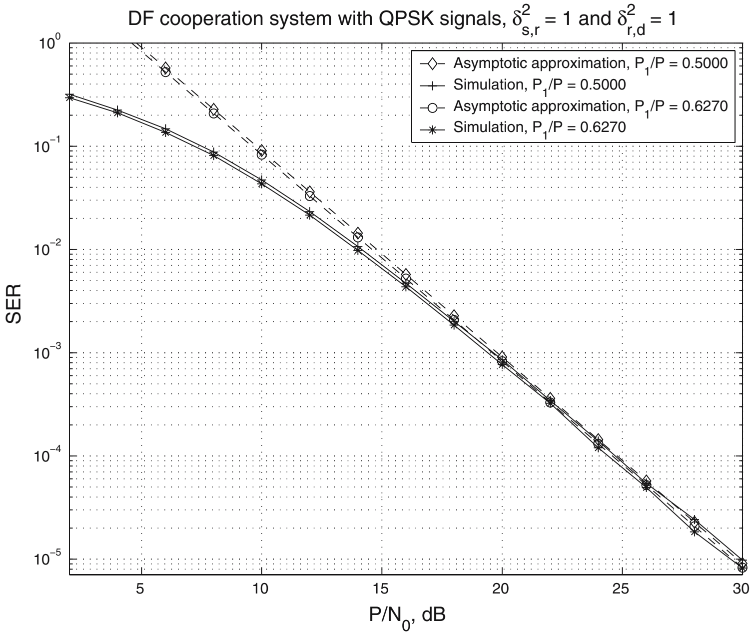
Figure 5 depicts the simulation results for the DF cooperation systems with BPSK modulation. We can see that the SER approximations from (17) are tight at high SNR in all scenarios. From the figure, we observe that in case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 1$, the performance of the optimum power allocation is almost the same as that of the equal power scheme, as shown in Fig. 5(a). In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 10$ in Fig. 5(b), the optimum power allocation scheme outperforms the equal power scheme with a performance improvement of about 1 dB. According to Theorem 2, the optimum power ratios are $P_1/P = 0.7579$ and $P_2/P = 0.2421$ in this case.

Figure 6 shows the simulation results for the DF cooperation systems with QPSK modulation. In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 1$ in Fig. 6(a), the optimum power ratios in this case are $P_1/P = 0.6270$ and $P_2/P = 0.3730$ by Theorem 2. From the figure, we observe that the performance of the optimum power allocation is a little bit better than that of the equal power case, and the two SER approximations are consistent with the simulation curves at high SNR respectively. In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 10$, the optimum power ratios are $P_1/P = 0.7968$ and $P_2/P = 0.2032$ according to Theorem 2. From Fig. 6(b), we can see that the optimum power allocation scheme outperforms the equal power scheme with a performance improvement of about 1 dB. Note that if the ratio of the link quality $\delta_{r,d}^2/\delta_{s,r}^2$ becomes larger, we will observe more performance improvement of the optimum power allocation over the equal power case. In all of the above simulations, we can see that the SER approximation in (17) is asymptotically tight at high SNR.

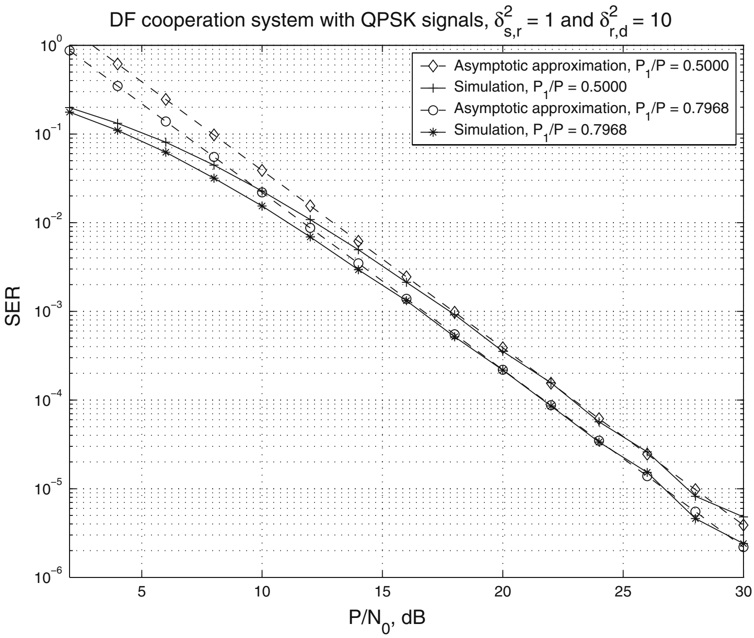
6.2 Performance of the AF Cooperation Systems

We also simulate the AF cooperation systems to compare the asymptotic tight SER approximation in (65) with the SER simulation curves. Moreover, we compare the performance of the AF cooperation systems using the optimum power allocation scheme in Theorem 6 with that of the systems using the equal power scheme.

Figure 7 provides the simulation results for the AF cooperation systems with BPSK modulation. In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 1$ in Fig. 7(a), we can see that the performance of the optimum power allocation is a little bit better than that of the equal power case, in which the optimum power ratios are $P_1/P = 2/3$ and $P_2/P = 1/3$ according to Theorem 6.



(a) $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 1$



(b) $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 10$

Fig. 6 Performance of the DF cooperation systems with QPSK signals: optimum power allocation versus equal power scheme

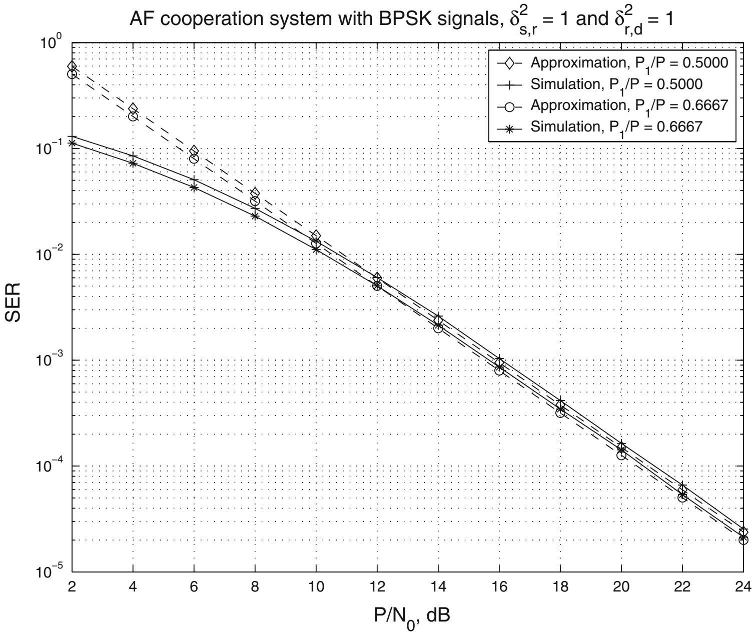
In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 10$, the optimum power ratios are $P_1/P = 0.8333$ and $P_2/P = 0.1667$ according to Theorem 6. We observe from Fig. 7(b) that the optimum power allocation scheme outperforms the equal power scheme with a performance improvement of more than 1.5 dB. Note that all SER approximations from (65) are respectively consistent with the simulation curves at reasonable high SNR.

We show the simulation results of the AF cooperation systems with QPSK modulation in Fig. 8. In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 1$ in Fig. 8(a), the optimum power ratios in this case are $P_1/P = 2/3$ and $P_2/P = 1/3$ which are the same as those for the case of BPSK modulation. From the figure, we can see that the performance of the optimum power allocation is better than that of the equal power case, and the two SER approximations are consistent with the simulation curves at high SNR respectively. In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 10$, the optimum power ratios are $P_1/P = 0.8333$ and $P_2/P = 0.1667$ according to Theorem 6. From Fig. 8(b), we observe that the optimum power allocation scheme outperforms the equal power scheme with a performance improvement of about 2 dB. If the ratio of the channel link quality $\delta_{r,d}^2/\delta_{s,r}^2$ becomes larger, we expect to see more performance improvement of the optimum power allocation over the equal power case. Moreover, from the figures we can see that in all of the above simulations, the SER approximations from (65) are tight enough at high SNR.

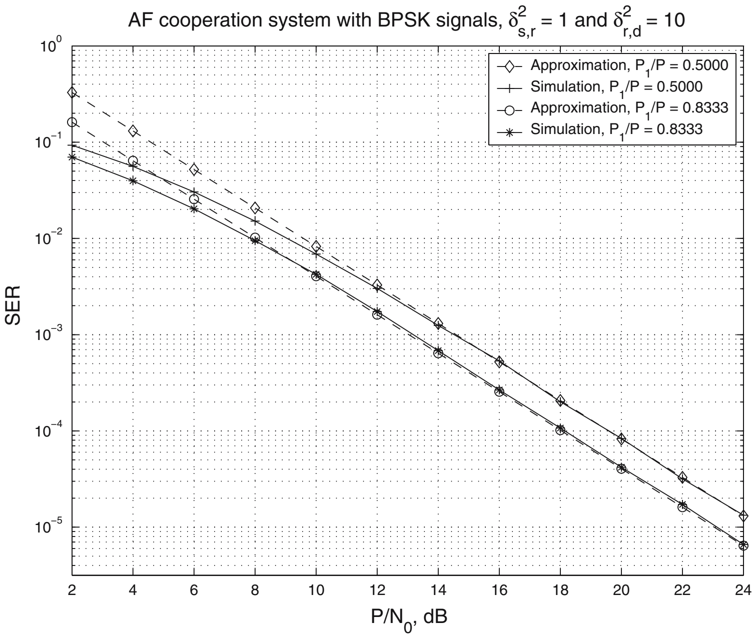
6.3 Performance Comparison between DF and AF Cooperation Protocols

Finally, we compare the performance of the cooperation systems with either DF or AF cooperation protocol. We demonstrate the performance comparison of the two cooperation protocols with BPSK modulation in Fig. 9. In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 1$, the performance of the DF cooperation protocol is better than that of the AF protocol about 1 dB, as shown in Fig. 9(a). In this case, the optimum power ratios for the DF cooperation protocol are $P_1/P = 0.5931$ and $P_2/P = 0.4069$ according to Theorem 2, while the optimum ratios for the AF protocol are $P_1/P = 2/3$ and $P_2/P = 1/3$ according to Theorem 6. In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 10$, from Fig. 9(b) we can see that the DF cooperation protocol outperforms the AF protocol with a SER performance about 2 dB. In this case, the optimum power ratios for the DF cooperation protocol are $P_1/P = 0.7579$ and $P_2/P = 0.2421$, while the optimum ratios for the AF protocol are $P_1/P = 0.8333$ and $P_2/P = 0.1667$. It seems that the larger the ratio of the channel link quality $\delta_{r,d}^2/\delta_{s,r}^2$, the more performance gain of the DF cooperation protocol compared with the AF protocol. However, the performance gain cannot be larger than $\lambda = \sqrt{3} \approx 2.4$ dB as shown in (77) in case of BPSK modulation.

Figure 10 shows the performance comparison of the two cooperation protocols with QPSK modulation. In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 1$, the performance of the DF cooperation protocol is better than that of the AF protocol, but not significant as shown in Fig. 10(a). In this case, the optimum power ratios for the DF cooperation protocol are $P_1/P = 0.6270$ and $P_2/P = 0.3730$ according to Theorem 2, while the optimum ratios for the AF protocol are $P_1/P = 2/3$ and $P_2/P = 1/3$ which are independent to the modulation types. In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 10$, from Fig. 10(b) we can see that the DF cooperation protocol outperforms the AF protocol with a SER performance about 1 dB, which is less than the performance gain of 2 dB in the case of BPSK modulation. The optimum power ratios for the DF cooperation protocol in this case are $P_1/P = 0.7968$ and $P_2/P = 0.20321$, while the optimum ratios for the AF protocol are $P_1/P = 0.8333$ and $P_2/P = 0.1667$. As shown in (77), in case of QPSK modulation, the performance gain of

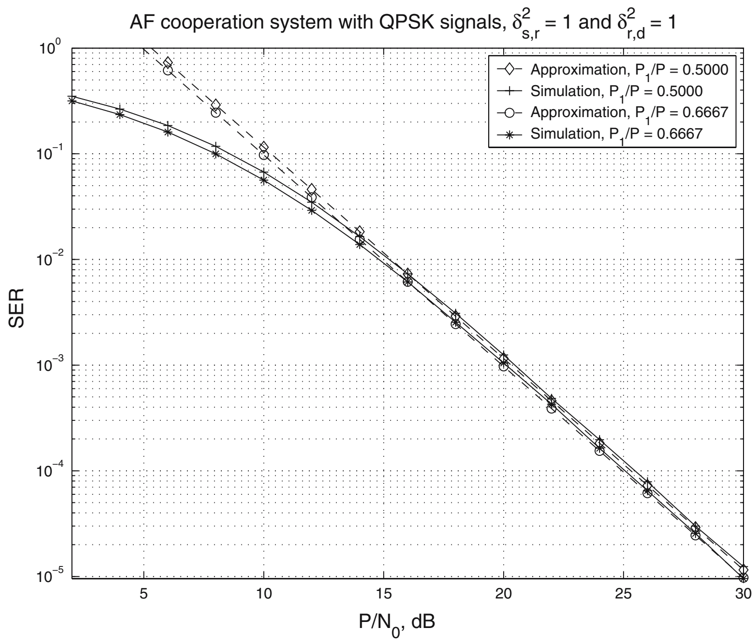


(a) $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 1$

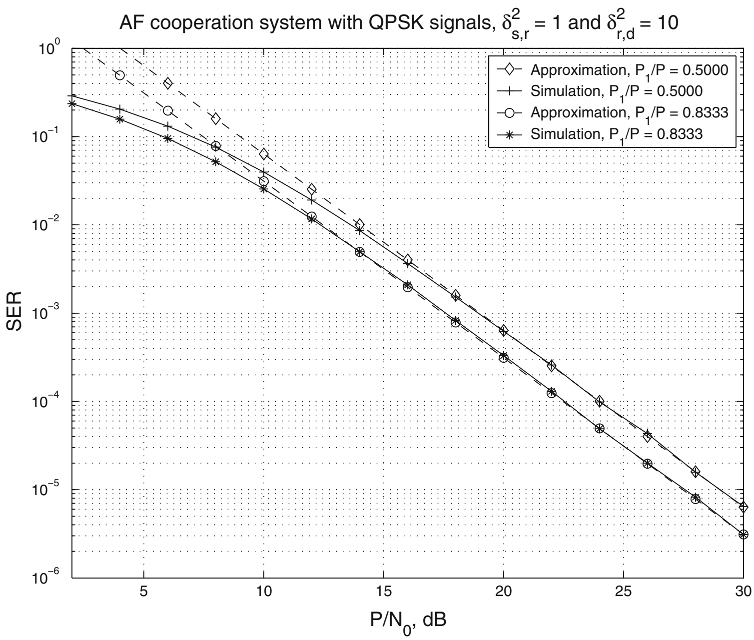


(b) $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 10$

Fig. 7 Performance of the AF cooperation systems with BPSK signals: optimum power allocation versus equal power scheme

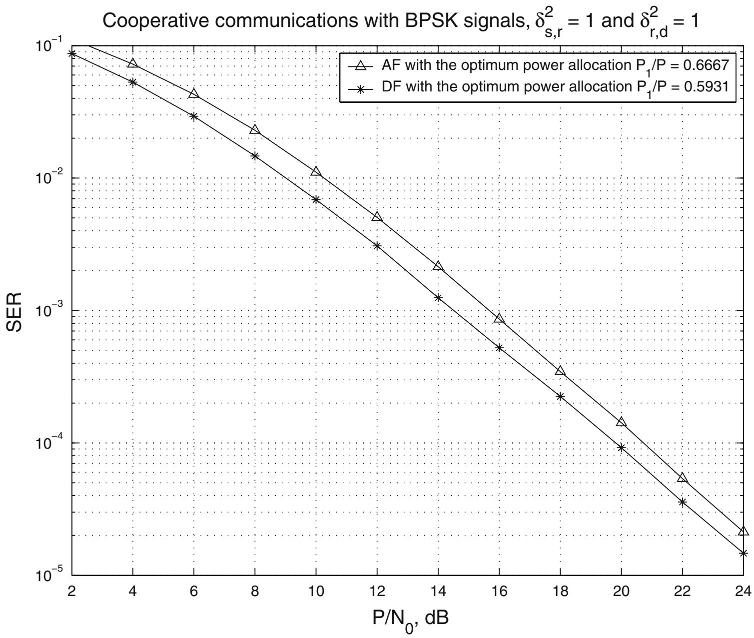


(a) $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 1$

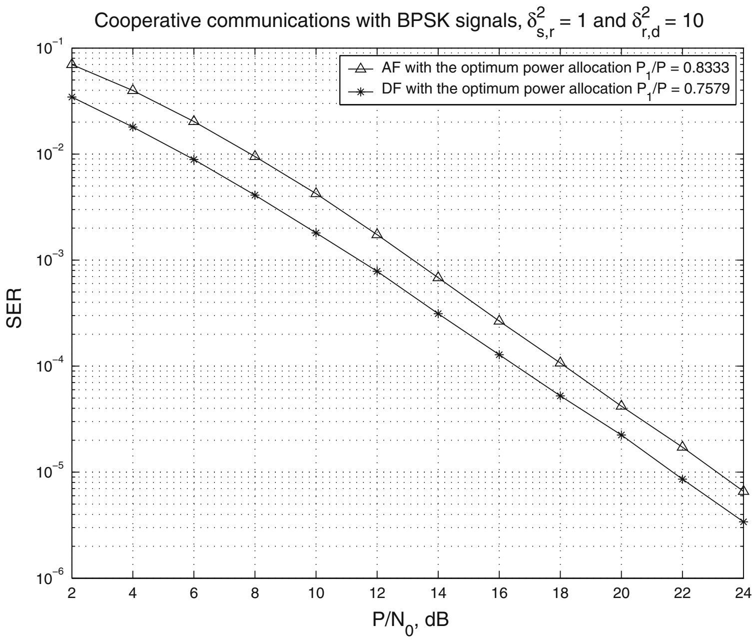


(b) $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 10$

Fig. 8 Performance of the AF cooperation systems with QPSK signals: optimum power allocation versus equal power scheme

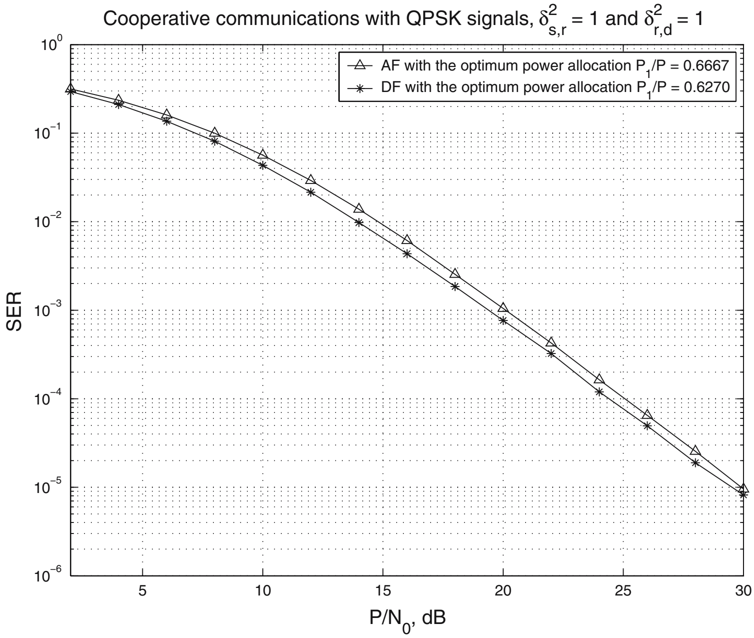


(a) $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 1$

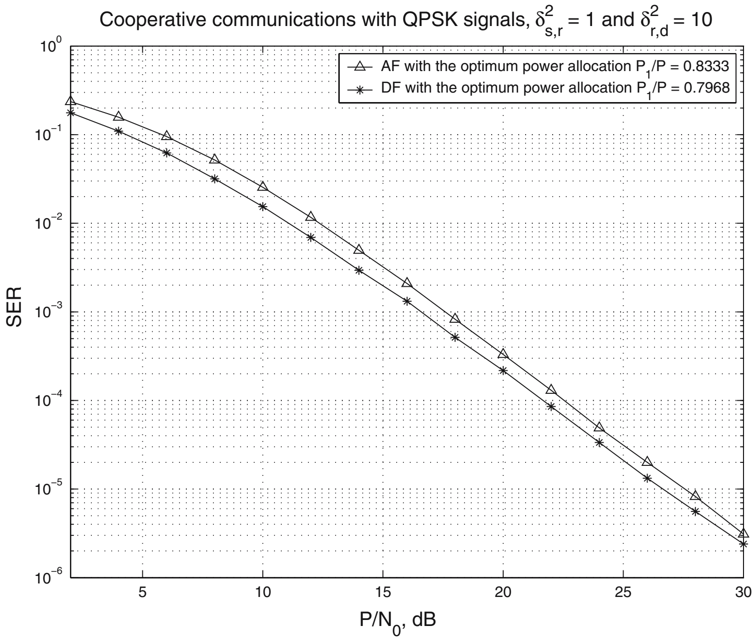


(b) $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 10$

Fig. 9 Performance comparison of the cooperation systems with either AF or DF cooperation protocol with BPSK signals



(a) $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 1$



(b) $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 10$

Fig. 10 Performance comparison of the cooperation systems with either AF or DF cooperation protocol with QPSK signals the DF cooperation protocol compared with the AF protocol is bounded by $\lambda = 1.3214 \approx 1.2\,\text{dB}$.

From the simulation results, we can see that the performance of the DF cooperation protocol is better than that of the AF protocol, but the performance gain varies in different channel situations and different modulation types. The larger the signal constellation size, the less the performance gain. So the DF cooperation protocol shows the best performance gain in case of BPSK modulation. Moreover, the larger the ratio of the channel link quality $\delta_{r,d}^2/\delta_{s,r}^2$, the more performance gain of the DF cooperation protocol compared with the AF protocol. But the performance gain is bounded by 2.4 dB in case of BPSK modulation, and 1.2 dB in case of QPSK modulation.

7 Conclusion

We have analyzed the SER performances of the uncoded cooperation systems with DF and AF cooperation protocols, respectively, and also compare their performances. From the theoretical and simulation results, we can draw the following conclusions. First, the equal power strategy is good, but in general not optimum in the cooperation systems with either DF or AF protocol, and the optimum power allocation depends on the channel link quality. Second, in case that all channel links are available in the DF or AF cooperation systems, the optimum power allocation does not depend on the direct link between source and destination, it depends only on the channel link between source and relay and that between relay and destination. Specifically, if the link quality between source and relay is much less than that between relay and destination, i.e., $\delta_{s,r}^2 << \delta_{r,d}^2$, then we should put the total power at the source and do not use the relay. On the other hand, if the link quality between source and relay is much larger than that between relay and destination, i.e., $\delta_{s,r}^2 >> \delta_{r,d}^2$, then the equal power strategy at the source and the relay tends to be optimum. Third, we observe that the performance of the cooperation systems with the DF protocol is better than that with the AF protocol. However, the performance gain varies with different modulation types. The larger the signal constellation size, the less the performance gain. In case of BPSK modulation, the performance gain cannot be larger than 2.4 dB; and for QPSK modulation, it cannot be larger than 1.2 dB. Therefore, for high data-rate cooperative communications (with large signal constellation size), we may use the AF cooperation protocol to reduce system complexity while maintains a comparable performance. Finally, we want to emphasize that the discussion of the optimum power allocation and the performance comparison in the paper is based on the asymptotically tight SER approximations that hold in sufficiently high SNR region, they may not be valid for low to moderate SNR regions. However, from the simulation results, we observe that the results from the high- SNR approximations also provide good match to the system performance in the moderate-SNR region.

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Appendix: Proof of Theorem 3

In the following, we list two Lemmas which will be used in the proof of Theorem 3.

Lemma 1 ([23]): *Let X be a random variable with pdf $p_X(x)$ for all $x \geq 0$ and $p_X(x) = 0$ for $x < 0$. Then, the pdf of $Y = 1/X$ is*

$$p_Y(y) = \frac{1}{y^2}\ p_X\left(\frac{1}{y}\right) \cdot U(y). \tag{78}$$

Lemma 2 ([23]): *Let X_1 and X_2 be two independent random variables with pdf $p_{X_1}(x)$ and $p_{X_2}(x)$ defined for all x. Then, the pdf of the sum $Y = X_1 + X_2$ is*

$$p_Y(y) = \int_{-\infty}^{\infty} p_{X_1}(y-x)\ p_{X_2}(x)dx, \tag{79}$$

which is the convolution of $p_{X_1}(x)$ and $p_{X_2}(x)$.

Proof of Theorem 3 Since X_1 and X_2 are two random variables with pdf $p_{X_1}(x)$ and $p_{X_2}(x)$ defined for all $x \geq 0$, and $p_{X_1}(x) = 0$ and $p_{X_2}(x) = 0$ for $x < 0$, according to Lemma 1, we know that the pdf of $1/X_1$ and $1/X_2$ are $p_{\frac{1}{X_1}}(x) = \frac{1}{x^2}\ p_{X_1}\left(\frac{1}{x}\right) \cdot U(x)$, and $p_{\frac{1}{X_2}}(x) = \frac{1}{x^2}\ p_{X_2}\left(\frac{1}{x}\right) \cdot U(x)$, respectively. Therefore, by Lemma 2, we know that the pdf of $Y = \frac{1}{X_1} + \frac{1}{X_2}$ can be given by

$$\begin{aligned}
p_Y(y) &= \int_{-\infty}^{\infty} p_{\frac{1}{X_1}}(y-x)\ p_{\frac{1}{X_2}}(x)dx \\
&= \int_{0}^{y} p_{\frac{1}{X_1}}(y-x)\ p_{\frac{1}{X_2}}(x)dx \cdot U(y) \\
&= \int_{0}^{y} \frac{1}{x^2(y-x)^2}\ p_{X_1}\left(\frac{1}{y-x}\right)\ p_{X_2}\left(\frac{1}{x}\right) dx \cdot U(y).
\end{aligned}$$

Note that $Z = \frac{X_1X_2}{X_1+X_2} = \frac{1}{\frac{1}{X_1}+\frac{1}{X_2}}$. Thus, according to Lemma 1 again, the pdf of Z can be determined as follows:

$$\begin{aligned}
p_Z(z) &= \frac{1}{z^2}\ p_{\frac{1}{X_1}+\frac{1}{X_2}}\left(\frac{1}{z}\right) \cdot U(z) \\
&= \frac{1}{z^2} \int_{0}^{\frac{1}{z}} \frac{1}{x^2(\frac{1}{z}-x)^2}\ p_{X_1}\left(\frac{1}{\frac{1}{z}-x}\right)\ p_{X_2}\left(\frac{1}{x}\right) dx \cdot U(z) \\
&= \frac{1}{z^2} \int_{0}^{1} \frac{1}{(\frac{t}{z})^2(\frac{1}{z}-\frac{t}{z})^2}\ p_{X_1}\left(\frac{1}{\frac{1}{z}-\frac{t}{z}}\right)\ p_{X_2}\left(\frac{z}{t}\right) d(\frac{t}{z}) \cdot U(z) \\
&= z \int_{0}^{1} \frac{1}{t^2(1-t)^2}\ p_{X_1}\left(\frac{z}{1-t}\right)\ p_{X_2}\left(\frac{z}{t}\right) dt \cdot U(z),
\end{aligned}$$

in which we change the variable $x = \frac{t}{z}$ in the second equation to get the third equation. So, we complete the proof of Theorem 3. □

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