

Capacity bounds and code designs

Cooperative diversity is a novel technique proposed for conveying information in wireless ad hoc networks, where closely located single-antenna network nodes cooperatively transmit and/or receive by forming virtual antenna arrays. For its building blocks, the relay channel and the two-transmitter, two-receiver cooperative channel, we survey the latest advances made in determining the theoretical capacity bounds and describe the best practical code designs reported so far. Both theory and practice predict that cooperative communication can provide increased capacity and power savings in ad hoc networks.

## INTRODUCTION

A wireless ad hoc network consists of a large number of possibly mobile nodes that communicate with each other over wireless links. The fact that there is no need for network infrastructures-hence low cost and simple reconfigurationmakes ad hoc networks attractive in both commercial and military applications such as wireless LAN (e.g., IEEE 802.11) and MAN (e.g., DARPA's GLOMO), home networks (e.g., HomeRF),
device networks (e.g., Bluetooth), and sensor networks (e.g., SmartDust, WINS).

In contrast to a traditional infrastructure wireless network (e.g., a cellular network), where information is transmitted from one user to another via a control base station, an ad hoc network allows peer-to-peer communication from a sending node to a destination node. Direct communication between two nodes can be realized in a single hop. However, since wireless channels are often poor, single-hop routing requires high transmission power and consequently causes increased interference. To achieve significant power savings, information should be conveyed to a destination through multiple intermediate nodes. Whereas transmission over a single-hop channel has already been intensively studied and well understood [1], cooperative communication in multinode networks is still an open research problem, which recently has received considerable attention, inspired by the papers [2], [3].

Since communication over a wireless channel is limited by interference, fading, multipath, path loss, and shadowing, the main design challenge lies in devising communication
methodologies in a decentralized network to overcome these limitations. An additional design issue has to do with the high dynamics of an ad hoc network, where nodes frequently join and leave the network.

One design approach is to employ multiple transmitter and receiver antennas at nodes, which increases the capacity and improves robustness to fading and interference by means of spatial diversity and data rate multiplexing. However, building multiple antennas at each node can be expensive, impractical, and often infeasible, especially for small and simple nodes such as those used in sensor networks.

Another recently proposed solution for achieving spatial diversity without requiring multiple antennas at any node is cooperative diversity. It is based on grouping several nodes (each with only one antenna) together into a cluster to form a large transmit or/and receive antenna array. Collaborative clusters are formed in an "ad hoc" fashion by negotiations among neighboring nodes without centralized control (see Figure 1). Cooperative

[FIG1] A wireless ad hoc network with cooperative diversity.
diversity naturally arises in ad hoc networks as it enables great power savings with cheap, simple, and mobile nodes, while supporting decentralized routing and control algorithms.

The simplest nontrivial setup is when the nodes form pairs, i.e., clusters of two. In a two-transmitter, two-receiver cooperative channel, the two single-antenna transmitters want to communicate messages to the two remote single-antenna receivers over the same wireless radio channel. In transmitter cooperation, the two transmitters first exchange their messages and then start to act as a single two-antenna broadcast transmitter. On the other hand, in receiver cooperation, the two receivers exchange their received signals and act as a single two-antenna multiple access receiver. In general, the two transmitters as well as the two receivers can collaborate among each other to form a multiple-input, multiple-output (MIMO) channel with two transmitter and two receiver antennas. The main goal of node
cooperation is to achieve spatial diversity and rate multiplexing without increasing the number of antennas at a single node.

Consider the channel model depicted in Figure 2 (upperleft), where the transmitter in Node 1 wants to send message $\omega_{1} \in\left\{1, \ldots, M_{1}\right\}$ to the receiver in Node 3 ; likewise, the transmitter in Node 2 intends to send message $\omega_{2} \in\left\{1, \ldots, M_{2}\right\}$ to the receiver in Node 4. Specifically, Node $i(i=1,2,3,4)$ transmits a block $x_{i}[n]$ of $N$ symbols at a time with $n=1, \ldots, N$, while being subject to an average power constraint $(1 / N) \sum_{n=1}^{N}\left|x_{i}[n]\right|^{2} \leq P_{i}$. The rate of the transmission from Node $i$ is then $R_{i}=\left(\log M_{i} / N\right)$. We assume that the channel between Node $i$ and Node $j$ is a Rayleigh flat-fading channel with channel coefficient $c_{j i}$, which is an independent identically distributed (i.i.d.), complex, zero-mean, Gaussian random variable.

At the symbol level, the received signals at Nodes 1, 2, 3, and 4 are given by

$$
\begin{align*}
y_{1}[n] & =c_{12} x_{2}[n]+z_{1}[n],  \tag{1}\\
y_{2}[n] & =c_{21} x_{1}[n]+z_{2}[n],  \tag{2}\\
y_{3}[n] & =c_{31} x_{1}[n]+c_{32} x_{2}[n]+c_{34} x_{4}[n]+z_{3}[n],  \tag{3}\\
y_{4}[n] & =c_{41} x_{1}[n]+c_{42} x_{2}[n]+c_{43} x_{3}[n]+z_{4}[n], \tag{4}
\end{align*}
$$

respectively, where $z_{i}, i=1, \ldots, 4$, are i.i.d., circular, complex, zero-mean, additive, Gaussian noises. To simplify the notation, without loss of generality, we assume that the noises are of unit power and $c_{31}=c_{42}=1$. In the transmitter cooperative channel, we set $c_{43}=c_{34}=0$ and for receiver cooperation $c_{21}=c_{12}=0$.

If cooperation is perfect, then transmitter cooperation leads to a two-antenna MIMO broadcast channel [4] ( $\left.c_{21}, c_{12} \rightarrow \infty\right)$ receiver cooperation reduces to a two-user multiple-access channel (MAC) with two receiver antennas ( $c_{43}, c_{34} \rightarrow \infty$ ) and the general setup with both transmitter and receiver cooperation becomes a single MIMO channel with two transmitter and two receiver antennas ( $c_{21}, c_{12}, c_{43}, c_{34} \rightarrow \infty$ ) On the other hand, when cooperation is not allowed, i.e., $c_{21}=c_{12}=c_{43}=c_{34}=0$, the channel degenerates to the interference channel [5]. See Figure 2 for three of these four simplifications.

When we restrict the channels to be quasi-static, then all channel coefficients are constant during transmission of each block of $N$ symbols. In the synchronous model of (1)-(4), we assume that the nodes are perfectly synchronized and have full channel state information (CSI), i.e., each node knows instantaneous values of all channel coefficients and their statistics. While it is relatively simple to achieve symbol/time synchronization between nodes, carrier synchronization, which requires phaselocking separated microwave oscillators, is challenging in practice. Therefore, we also consider the asynchronous model, where random phase offsets due to oscillator fluctuations are added to

[FIG2] The two-transmitter two-receiver cooperative channel together with its three special cases.
the transmitted signals. We include these random phases in the channel coefficients, so that the model stays the same as (1)-(4). Under the asynchronous model for receiver cooperation, the transmitters do not have any CSI, whereas the receivers need to know only the magnitudes of all channel coefficients, not their phases. Thus, receiver cooperation is suitable in the systems with simple transmitters. On the other hand, under the asynchronous model for transmitter cooperation, the transmitters must know the magnitudes of all channel coefficients.

In this article, we examine the diversity and data rate gains achievable by node cooperation, while focusing on the high sig-nal-to-noise ratio (SNR) regime, where the data rates are mainly limited by interference. The diversity gain defined as

$$
d=-\lim _{\mathrm{SNR} \rightarrow \infty} \frac{\log P_{e}(\mathrm{SNR})}{\log \mathrm{SNR}},
$$

shows how fast the probability of decoding error $P_{e}$ decays with SNR. A higher $d$ means lower $P_{e}$ at the same SNR, and thus a more reliable system. The data rate gain is usually decoupled
into a multiplexing gain and an additive gain. The multiplexing gain (or degree of freedom) [6] shows how fast the rate increases with SNR and is given by

$$
r=\lim _{\mathrm{SNR} \rightarrow \infty} \frac{R(\mathrm{SNR})}{\log \mathrm{SNR}}
$$

where $R(\mathrm{SNR})$ denotes the sum of data rates of transmitting nodes for a given SNR. The additive gain is a shift of the $R(\mathrm{SNR})$ function from the origin at high SNRs, i.e., $a=\lim _{\mathrm{SNR} \rightarrow \infty} R(\mathrm{SNR})-r \log (\mathrm{SNR})$. If all the limits exist, then $R(\mathrm{SNR})$ in the high SNR regime can be approximated by a line of slope $r$ and SNR-offset $a$, i.e., $R(\mathrm{SNR}) \approx r \log (\mathrm{SNR})+a$.

It is well known that perfect cooperation achieves a diversity gain of $d=2$ and a multiplexing gain of $r=2$. On the other hand, the interference channel (without node cooperation) in Figure 2 provides no diversity or multiplexing gain (i.e., $d=1$ and $r=1$ ). We first present capacity bounds for cooperative diversity, indicating a multiplexing gain of only one at high SNRs, which is a somewhat negative result. However, the main
message is that node cooperation can provide a large additive gain and a diversity gain of two. Then we focus on recent advances made in practical code designs aimed at achieving these additive and diversity gains. The final section points to open challenges and opportunities for future research.

## CAPACITY BOUNDS

While the capacities of most point-to-point channels are known, this is not true for wireless multinode channels. Indeed, we only know the capacity of the MAC and that of the broadcast channel. For all other multinode channels, e.g., the relay and interference channels, capacities are known only in special cases. However, it is possible to obtain upper and lower bounds on the capacity, which are often very close, thereby practically indicating the capacity. A lower bound is the rate that can be attained by some coding scheme and is therefore an achievable rate.

Since cooperative diversity is largely based on relaying messages, its information-theoretical foundation is built upon the landmark 1979 paper of Cover and El Gamal [7] on capacity bounds for relay channels. We thus start with the relay channel, give the theoretical bounds on its capacity, and describe proposed coding strategies in the Gaussian and Rayleigh flat-fading environments. Then we proceed with extensions to two-transmitter two-receiver cooperative channels. There are two main
ideas in obtaining achievable rates for cooperative channels. The first idea is based on nodes decoding messages from other nodes and re-encoding them. The second lies in exploiting the joint statistics between the data at cooperating nodes by means of coding with side information, i.e., Wyner-Ziv coding (WZC) [8] or dirty-paper coding [9], which we review in "Coding with Side Information." Specifically, it turns out that Wyner-Ziv coding achieves the capacity of receiver cooperation (asymptotically as the interference and SNR approach infinity), while dirty-paper coding plays a major role in transmitter cooperation.

## THE RELAY CHANNEL

The relay channel [16] is a three-node channel where the source communicates to the destination with the help of an intermediate relay node. Cover and El Gamal [7] derived upper and lower bounds on the capacity of the general relay channel using random coding and converse arguments. These two bounds coincide only in a few special cases [17], [18] (e.g., the degraded Gaussian case [7]).

The wireless relay channel is shown in Figure 3, where $c_{r s}, c_{d s}$, and $c_{d r}$ denote channel coefficients. There are two setups in relaying: full duplex and half duplex. For the fullduplex setup, in which the relay is able to transmit and receive simultaneously on the same frequency, capacity bounds are given by Cover and El Gamal [7] and practical designs

## CODING WITH SIDE INFORMATION

Source coding with side information at the decoder, or WynerZiv coding (WZC), considers lossy compression of source $X$ under the distortion constraint when a correlated source $S$, called side information, is available at the decoder but not at the encoder (see Figure A). This rate-distortion problem was first considered by Wyner and Ziv in [8] where the minimum rate for compressing $X$ was derived. In general, WZC incurs a rate loss when compared to the case with $S$ also available at the encoder. However, if the correlation between $X$ and $S$ is modeled as $X=S+Z$, with $Z$ being an i.i.d., memoryless Gaussian random variable, independent of $S$, then there is no rate loss with WZC under the mean-squared error distortion measure.
The information-theoretical dual of WZC is channel coding with side information at the encoder, or GelfandPinsker coding [10], where the encoder has perfect (noncausal) knowledge of the side information or CSI. The limits on the rate at which messages can be transmitted to a receiver are given in [10]. In general, there is a rate loss
compared to the case when the receiver also knows noncausally the CSI. However, when the channel is additive white Gaussian noise (AWGN), Gelfand-Pinsker coding does not suffer any rate loss. In this case we have the celebrated dirty-paper coding (DPC) problem [9], also shown in Figure A, where the decoder can completely cancel out the effect of the interference caused by the side information.
Practical WZC and DPC both involve source-channel coding. WZC can be implemented by first quantizing the source $X$, followed by Slepian-Wolf coding [11] of the quantized $X$ with side information $S$ at the decoder. Using syndrome-based channel coding for compression, Slepian-Wolf coding here plays the role of conditional entropy coding. For DPC, source coding is needed to quantize the side information to satisfy the power constraint. In the meanwhile, the quantizer induces a constrained channel, for which practical channel codes can be designed to approach its capacity. Indeed, limit-approaching code designs [12]-[15] have appeared for both WZC and DPC recently.

[FIGA] Coding with side information. WZC refers to lossy source coding of $X$ with decoder side information $S$, whereas DPC considers channel encoding of message $m$ with encoder side information $S$ over an AWGN channel.
performed by Zhang and Duman [19]. Implementing full-duplex relaying, however, is a microwave design challenge (e.g., due to the large difference in the transmitting and receiving signal power levels). A simpler setup is half-duplex relaying, in which the relay does not simultaneously receive and transmit. Halfduplex relaying [17], [20]-[24] can be implemented with lower complexity by using either time division or frequency division, which are equivalent from an information theory point of view. We will concentrate on time-division half-duplex relaying and discuss both capacity bounds and practical designs.

In time-division relaying, a frame of length $n$ is divided into two parts: a relay-receive period of length $n \alpha, 0 \leq \alpha \leq 1$, and a relay-transmit period of length $n(1-\alpha)$. In the relay-receive period, the source transmits a code word $x_{s 1}$. The relay overhears this transmission, processes its received signal $y_{r}$ in some way, and transmits a code word $x_{r}=f_{r}\left(y_{r}\right)$ in the relay-transmit period. While the relay transmits, the source simultaneously transmits another code word $x_{s 2}$. The code word $x_{s 2}$ is not heard by the relay, as it is in transmit mode, and is therefore transmitted directly to the destination. One way to accomplish this is to split the message $m \in\{1, \ldots, M\}$ into two parts, $m_{1}$ and $m_{2}$, at the source. Then, $m_{1}$ is encoded into the $n \alpha$-length code word $x_{s 1}\left(m_{1}\right)$, and the remaining $m_{2}$ is encoded into an $n(1-\alpha)$-length code word $x_{s 2}\left(m_{2}\right)$. At the symbol level, the received signals at the relay and destination during the relayreceive period are $y_{r}[n]=c_{r s} x_{s 1}\left(m_{1}\right)[n]+z_{r s}[n]$ and $y_{d 1}[n]=c_{d s} x_{s 1}\left(m_{1}\right)[n]+z_{d s}[n]$, respectively, where $z_{r s}$ and $z_{d s}$ are independent additive white Gaussian noise (AWGN) with unit power. During the relay-transmit period in the asynchronous case, the relay sends an $n(1-\alpha)$-length code word $x_{r}\left(m_{1}\right)$ to the destination, which receives $y_{d 2}[n]=$ $c_{d s} x_{s 2}\left(m_{2}\right)[n]+c_{d r} x_{r}\left(m_{1}\right)[n]+z[n]$, where $z$ is again an AWGN with unit power.

In the synchronous case, the system can additionally use the antennas at the source and relay as a two-antenna transmit array. Suppose that the source is able to completely predict what the relay will send in the relay-transmit period; then the source can transmit the same signal with a phase shift calibrated so that the two signals add up coherently at the destination. The received signal is then

$$
\begin{align*}
y_{d 2}[n]= & c_{d s} x_{s 2}\left(m_{2}\right)[n] \\
& +\left(c_{d r}+c_{d s} A\right) x_{r}\left(m_{1}\right)[n]+z[n] \tag{5}
\end{align*}
$$

where $A$ is a complex constant subject to a power constraint and with such a phase that $\left|c_{d r}+c_{d s} A\right|$ is maximized. If the source can only partially predict what the relay will transmit, it is still possible to take advantage of this partial coherency.

The optimum operation at the relay is not known, but several coding schemes have been proposed [3], [7], [18], [20], [21], [25] to obtain achievable bounds on the rate region. These schemes can be classified into decode-forward (DF) and observeforward [7], although hybrid schemes are also possible [7], [25].

The main operation of DF is full decoding at the relay node. Upon receiving $y_{r}$, the relay node first decodes $m_{1}$ and then reencodes it before forwarding the resulting code word $x_{r}\left(m_{1}\right)$ to the destination during the relay-transmit period. It should be emphasized that the relay might use a different codebook than the source. In any case, the source can completely predict what the relay will transmit, and full coherency is therefore possible. The destination attempts to reconstruct message $m$ by combining the signals received during the relay-receive and relay-transmit periods using either successive list decoding [7], backward decoding [26], [27], or decoding based on parallel Gaussian channel arguments [22], which all result in the same achievable rate region. Although DF can be very efficient in some scenarios [17], [18], since the relay

[FIG3] The wireless relay channel. DF works better when the relay is close to the source, but CF is preferred when the relay is close to the destination.
must perfectly decode the source message, the achievable rates are bounded by the capacity of the channel between the source and the relay. To alleviate this problem, a class of observe-forward schemes has been proposed, where the relay does not attempt to decode the signal from the source; it merely forwards a processed version of its received signal to the destination.

The simplest observe-forward scheme is amplify-forward [28], in which the relay, sticking to its rudimentary role, just amplifies the received signal before forwarding. A more sophisticated scheme is compress-forward (CF), which is rooted in the original work of Cover and El Gamal [7], where the relay compresses the signal it has received from the source within certain distortion. Since $y_{r}$ (received by the relay node) and $y_{d 1}$ (received by the destination node) are independently corrupted versions of the same encoded message $x_{s 1}\left(m_{1}\right)$, they are correlated. Thus the relay node can employ WZC [8] (see "Coding with Side Information") when compressing $y_{r}$ by treating $y_{d 1}$ as the decoder side information. The Wyner-Ziv compressed signal is then channel encoded to $x_{r}\left(m_{1}\right)$ before being forwarded to the destination, which recovers $m_{2}$ and $m_{1}$ using successive cancellation decoding that involves several steps. First, $\hat{x}_{r}\left(m_{1}\right)$ is reconstructed by assuming $x_{s 2}\left(m_{2}\right)$ as the noise, and then it is subtracted from $y_{d 2}$ before $m_{2}$ is decoded. Second, to reconstruct $m_{1}, y_{r}$ is estimated from $\hat{x}_{r}\left(m_{1}\right)$ using Wyner-Ziv decoding with $y_{d 1}$ as the decoder side information; maximum ratio combining on $y_{d 1}$ and the obtained estimate $\hat{y}_{r}$ is then invoked to recover $m_{1}$. CF based on WZC has higher computational complexity than DF, but it gives many rate points that are not achievable with any other coding strategies. It provides the best solution [7], [17], [18] when the relay is close to the destination node. "Capacity Bounds of the Gaussian HalfDuplex Relay Channel" summarizes the capacity bounds of the Gaussian half-duplex relay channel, which can also be used for computing the outage capacity of a wireless quasi-static flat-fading channel (as done in [17]).

It is instructive to compare relay channel signaling with a traditional multihop ad-hoc network, where physical layer communication and networking are typically separated. Such a comparison will show how cooperative diversity can help increase the performance over traditional networking. In a traditional multihop network, the source transmits a packet either directly to the destination, or to the relay, which would decode it, reencode it, and transmit it to the destination. Relay channel signaling improves upon this in several ways:

- the destination uses the signal from both the source and relay for decoding, as opposed to only one of them
- the relay uses a different codebook for encoding in DF than the source, which is similar to using error-correcting codes rather than repetition-coding
- the relay can use soft information, as in CF, which resembles using soft decisions in decoding error-correcting codes rather than hard decisions
- the source is allowed to transmit new information simultaneously with the relay's transmission, which, at high SNR, brings a large increase in rate
- in the synchronous case, the relay can use coherency to
combine signals constructively, achieving a similar gain as MIMO systems.


## RECEIVER COOPERATION

In receiver cooperation, two (closely located) single-antenna receivers cooperate to facilitate decoding messages from two remote single-antenna transmitters. The channel model is shown in Figure 2 (upper left) with $c_{21}=c_{12}=0$. We consider only the full-duplex case, i.e., Nodes 3 and 4 can simultaneously transmit and receive (to the best of our knowledge there have been no reported results on capacity bounds for half-duplex cooperation yet). Since the cooperative channel can be viewed as a combination of the interference channel and the relay channel, its best achievable rate regions are obtained by combining DF or CF coding techniques for the relay channel with coding for the interference channel [5]. In receiver cooperation, a receiver node processes the received information and forwards the result to the other receiver node to help decoding. Because the distance between the two receivers is expected to be much smaller than that between a transmitter and a receiver, CF with WZC provides the highest achievable rates. Indeed, from (3) and (4), the received signals in Nodes 3 and 4 at time instants $i$ and $i+1$ are

$$
\begin{align*}
y_{3}[i]= & x_{1}[i]+c_{32} x_{2}[i]+c_{34} x_{4}[i]+z_{3}[i], \\
y_{3}[i+1]= & x_{1}[i+1]+c_{32} x_{2}[i+1] \\
& +c_{34} x_{4}[i+1]+z_{3}[i+1], \\
y_{4}[i]= & c_{41} x_{1}[i]+x_{2}[i]+c_{43} x_{3}[i]+z_{4}[i], \\
y_{4}[i+1]= & c_{41} x_{1}[i+1]+x_{2}[i+1] \\
& +c_{43} x_{3}[i+1]+z_{4}[i+1] . \tag{9}
\end{align*}
$$

In CF [30], the receiver in Node 3 (or Node 4) employs WZC to compress the signal $y_{3}[i]$ (or $y_{4}[i]$ ) it has received, while assuming $y_{4}[i]$ (or $y_{3}[i]$ ) as the decoder side information, before passing the resulting code word $x_{3}[i+1]$ (or $x_{4}[i+1]$ ) to the collaborating receiver in Node 4 (or Node 3). Node 3 starts by decoding $x_{4}[i+1]$ from $y_{3}[i+1]$ while treating $x_{1}[i+1]+c_{32} x_{2}[i+1]$ as part of the Gaussian noise. In forward decoding, $x_{4}[i]$, recovered in the previous time instant, is Wyner-Ziv decoded using $y_{3}[i]$ as the decoder side information, resulting in an estimate of $y_{4}[i]$. Next, the joint or individual decoding technique [5] proposed for the interference channel is employed to reconstruct $x_{1}[i]$ (and $x_{2}[i]$ ) from the obtained estimates $y_{3}[i]-c_{34} \hat{x}_{4}[i]=x_{1}[i]+c_{32} x_{2}[i]+z_{3}[i]$ and $\hat{y}_{4}[i]-c_{43} x_{3}[i]=c_{41} x_{1}[i]+x_{2}[i]+z_{4}[i]$. A similar procedure can be performed at Node 4. Besides forward-decoding, it is also feasible to employ backward decoding, where the decoder starts by decoding the previously received block of symbols and proceeding backwards. In [30], forward decoding is combined with either joint or individual decoding [5] and backward decoding is used with joint decoding, giving three different decoding choices. Since Nodes 3 and 4 can use three different decoding methods each, there are nine possibilities,
each providing a different rate bound. To obtain the best achievable CF rate bound, the maximum of all nine rate bounds should be taken.

In a similar manner, DF can be extended to receiver cooperation. The upper and lower bounds of CF and DF are derived in [30] and summarized in "Capacity Bounds for Receiver

## CAPACITY BOUNDS OF THE GAUSSIAN HALF-DUPLEX RELAY CHANNEL

Under the assumption that the nodes are synchronized and have perfect CSI, i.e., each node knows instantaneous values of all channel coefficients and their statistics, an upper bound on the capacity of the Gaussian half-duplex relay channel (although channel coefficients are in general assumed to be complex, in this case they are positive real constants) is derived in [17] and [20] and given by

$$
\begin{equation*}
C_{u b}=\max _{0 \leq \rho \leq 1,0 \leq \alpha \leq 1,0 \leq k \leq 1} \min \left\{C_{u b 1}, C_{u b 2}\right\}, \tag{6}
\end{equation*}
$$

where

$$
\begin{aligned}
C_{u b 1}= & \frac{\alpha}{2} \log \left(1+\left(\left|c_{r s}\right|^{2}+\left|c_{d s}\right|^{2}\right) \frac{k P_{s}}{\alpha}\right) \\
& +\frac{1-\alpha}{2} \log \left(1+\left(1-\rho^{2}\right)\left|c_{d s}\right|^{2} \frac{1-k) P_{s}}{1-\alpha}\right) \\
C_{u b 2}= & \frac{\alpha}{2} \log \left(1+\left|c_{d s}\right|^{2} \frac{k P_{s}}{\alpha}\right)+\frac{1-\alpha}{2} \log \left(1+\left|c_{d s}\right|^{2} \frac{(1-k) P_{s}}{1-\alpha}\right. \\
& \left.+\left|c_{d r}\right|^{2} P_{r}+\frac{2 \sqrt{\rho^{2}\left|c_{d s}\right|^{2}\left|c_{d r}\right|^{2}(1-k) P_{s} P_{r}}}{1-\alpha}\right)
\end{aligned}
$$

and $P_{s}$ and $P_{r}$ are the average source and relay power constraints, respectively. The parameter $\rho$ reflects the correlation between the source and relay signals, and it can be written in closed form [17], [20]. It is clear from the bound above that the highest multiplexing gain $r$ is one. However, the full diversity gain of two can be achieved with a simple AF scheme [3], [29].
The rate bound of DF is [17], [20]

[FIGB] The multihop bound and the upper bound on the capacity together with the achievable bounds of DF and CF for the Gaussian half-duplex relay channel, assuming $\left|C_{d s}\right|^{2}=0 \mathrm{~dB},\left|C_{d r}\right|^{2}=10 \mathrm{~dB}$, and $P_{s}=P_{r}=5 \mathrm{~dB}$. The rate gain over direct transmission is shown as a function of $\left|{c_{r s}}\right|^{2}$.

$$
\begin{equation*}
R_{D F} \leq \max _{0 \leq \rho \leq 1,0 \leq \alpha \leq 1,0 \leq k \leq 1} \min \left\{R_{D F 1}, R_{D F 2}\right\}, \tag{7}
\end{equation*}
$$

where

$$
\begin{aligned}
R_{D F 1}= & \frac{\alpha}{2} \log \left(1+\left|c_{d s}\right|^{2} \frac{k P_{s}}{\alpha}\right) \\
& +\frac{1-\alpha}{2} \log \left(1+\left(1-\rho^{2}\right)\left|c_{d s}\right|^{2} \frac{(1-k) P_{s}}{1-\alpha}\right)
\end{aligned}
$$

and $R_{D F 2}=C_{u b 2}$. The achievable rate with CF is [17]

$$
\begin{equation*}
R_{\text {CF }} \leq \max _{0 \leq \alpha \leq 1,0 \leq k \leq 1}\left\{R_{\text {CF1 }}(\alpha, k)+R_{\text {CF2 }}(\alpha, k)\right\}, \tag{8}
\end{equation*}
$$

where

$$
R_{C F 1}(\alpha, k)=\alpha \frac{1}{2} \log \left(1+\left|C_{d s}\right|^{2} \frac{k P_{s}}{\alpha}+\frac{\left|c_{r s}\right|^{2} k P_{s}}{\alpha\left(1+\sigma_{\omega}^{2}\right)}\right)
$$

and

$$
R_{C F 2}(\alpha, k)=(1-\alpha) \frac{1}{2} \log \left(1+\left|c_{d s}\right|^{2} \frac{(1-k) P_{s}}{1-\alpha}\right)
$$

with $\sigma_{\omega}^{2}$ being the WZC noise [8] given by
$\sigma_{\omega}^{2}=$
$\frac{\alpha+\left(\left|c_{r s}\right|^{2}+\left|c_{d s}\right|^{2}\right) k P_{s}}{\left\{\left[1+\left|c_{d r}\right|^{2} P_{r} /\left(1-\alpha+\left|c_{d s}\right|^{2}(1-k) P_{s}\right)\right]^{1-\alpha}-1\right\}\left(\alpha+\left|c_{d s}\right|^{2} k P_{s}\right)}$.
DF and CF give the best known results on the achievable rates for the half-duplex relay channel (however, a hybrid approach may give a higher rate, as indicated in [25] for a full-duplex relay channel). Depending on the parameters, either DF or CF can be superior. Indeed, DF outperforms CF when the link between the source and relay is better than that between the relay and destination (e.g., when the relay is close to the source). On the other hand, CF provides higher rates when the link between the relay and destination is clean (e.g., when the relay is close to the destination). See Figure 3. We show in Figure $B$ for one setup the rate bounds in (7) and (8), achievable with DF and CF, respectively, together with the upper bound given by (6) and the rate bound with multihop transmission which is given by the minimum between the capacity at the source-relay link, (1/2) $\log \left(1+\left|c_{r s}\right|^{2} P_{s}\right)$, and the capacity at the relay-destination link, $(1 / 2) \log \left(1+\left|c_{d r}\right|^{2} P_{r}\right)$. We plot the rate gain relative to direct transmission (i.e., no relaying) as a function of $\left|c_{r s}\right|^{2}$. The increase in $\left|c_{r s}\right|^{2}$ can be construed as the result of decreased distance between the source and the relay. It is seen from Figure $B$ that CF outperforms DF for low $\left|c_{r s}\right|^{2}$. When $\left|c_{r s}\right|^{2}<\left|c_{d s}\right|^{2}=0 \mathrm{~dB}$, DF is worse than direct transmission. On the other hand, CF always outperforms direct transmission. Thus, even if the link between the source and relay is poor, the relay can still help somewhat by using CF.

## CAPACITY BOUNDS FOR RECEIVER COOPERATION

Based on "transforming" a receiver cooperative channel to one with the same or higher capacity, tighter upper bounds on capacity than the standard max-flow-min-cut bound from [1] are derived in [30], yielding

$$
\begin{aligned}
& R_{1}+R_{2} \leq \log \left(1+\left|C_{41}\right|^{2} P_{1}+P_{2}+\left|C_{43}\right|^{2} P_{3}\right) \\
& \quad+\log \frac{1+\left(1+\left|C_{41}\right|^{2}\right) P_{1}}{1+\left|C_{41}\right|^{2} P_{1}}
\end{aligned}
$$

in the asynchronous case, and

$$
\begin{aligned}
R_{1} & +R_{2} \leq \log \left(1+\left|C_{41}\right|^{2} P_{1}+P_{2}+\left|C_{43}\right|^{2} P_{3}\right. \\
& +2 \sqrt{\left.\left|C_{43}\right|^{2} P_{2} P_{3}+\left|C_{43}\right|^{2}\left|C_{41}\right|^{2} P_{1} P_{3}\right)} \\
& +\log \frac{1+\left(1+\left|C_{41}\right|^{2}\right) P_{1}}{1+\left|C_{41}\right|^{2} P_{1}}
\end{aligned}
$$

in the synchronous case. Note that we get a symmetric set of rate bounds if Nodes 1 and 2 are exchanged with Nodes 3 and 4. Achievable rate bounds for CF and DF are also given in [30]. Figure $C$ shows the sum-rate $R_{1}+R_{2}$ as a function of the received SNR on the direct link between Nodes 1 and 3. The received SNR at the link between Nodes 3 and 4 is 30 dB higher than that from the direct link, an indication that the cooperating nodes are close together. The average powers of all four nodes are the same. All channels are independent Rayleigh flat-fading, meaning that each $c_{i j}$ is an i.i.d. Gaussian random variable, and the results are averaged over simulated ensembles of channel realizations. For comparison purposes, the rate limits of the two-user MAC with two antennas at the receiver (with perfect cooperation) and the interference channel (without cooperation) are included.

Cooperation." It is seen that, in the high SNR regime, receiver cooperation gives a multiplexing gain of only $r=1$ as opposed to the two-user MAC with two receiver antennas which results in $r=2$. However, the additive gain with receiver cooperation, which is upper bounded by

$$
\begin{align*}
a \leq & \min \left\{\log \left(\left|c_{41}\right|^{2} P_{1}+P_{2}+\left|c_{43}\right|^{2} P_{3}\right)\right. \\
& +\log \left(1+\left|c_{41}\right|^{-2}\right), \log \left(P_{1}+\left|c_{32}\right|^{2} P_{2}\right. \\
& \left.\left.+\left|c_{34}\right|^{2} P_{4}\right)+\log \left(1+\left|c_{32}\right|^{-2}\right)\right\} \tag{10}
\end{align*}
$$

can be very high. On the other hand, for $\left|c_{41}\right|,\left|c_{32}\right|>1$ it is shown in [30] that CF with forward joint decoding gives an additive gain of

$$
\begin{align*}
a= & \min \left\{\log \left(\left|c_{41}\right|^{2} P_{1}+P_{2}+\left|c_{43}\right|^{2} P_{3}\right),\right. \\
& \left.\log \left(P_{1}+\left|c_{32}\right|^{2} P_{2}+\left|c_{34}\right|^{2} P_{4}\right)\right\} . \tag{11}
\end{align*}
$$

Note that the gain in (11) is identical to that in (10) except for the $\log \left(1+\left|c_{41}\right|^{-2}\right)$ and $\log \left(1+\left|c_{32}\right|^{-2}\right)$ terms, which are small for large $\left|c_{41}\right|$ and $\left|c_{32}\right|$. Thus, CF with WZC achieves

It is seen from Figure C that receiver cooperation with CF gives an additive gain that can be up to 20 dB higher than no cooperation, CF always performs close to the upper bound, and there is no gain from synchronization. Interestingly, receiver cooperation performs close to using two receiver antennas at low and medium SNRs, thus providing a multiplexing gain of two. However, at the high SNRs, the multiplexing gain drops to one, and the rate gain over the noncooperative case boils down to a high additive gain.

[FIGC] Bounds on the sum-rate $R_{1}+R_{2}$ as a function of the received SNR from the direct link between Nodes 1 and 3 for receiver cooperation. The received SNR at the link between Nodes 3 and 4 is 30 dB higher than that at the direct link. The average powers of all four nodes are the same.
capacity asymptotically as $\left|c_{41}\right|,\left|c_{32}\right|$, and the SNR go to infinity. All other cooperative strategies (including DF) give no additional gain over that of no cooperation, whose additive gain is

$$
\begin{equation*}
a=\min \left\{\log \left(\left|c_{41}\right|^{2} P_{1}+P_{2}\right), \log \left(P_{1}+\left|c_{32}\right|^{2} P_{2}\right)\right\} . \tag{12}
\end{equation*}
$$

However, these strategies are useful in the medium or low SNR regimes. From (11) and (12) we see that the gain of receiver cooperation comes from the terms $\left|c_{43}\right|^{2} P_{3}$ and $\left|c_{34}\right|^{2} P_{4}$, which depend on the channel between the two receivers. Since this channel is expected to be good (a node should cooperate with its "best neighbor"), this gain can be very high. An interesting conclusion from [30] is that the gain from exploiting full synchronization in receiver cooperation is very limited. Thus, in practice it is enough to resort to the asynchronous cooperation, which significantly saves the hardware cost. However, as pointed out in [31], optimal power allocation is essential in achieving the full additive gain.

## TRANSMITTER COOPERATION

In transmitter cooperation, two single-antenna transmitters collaborate in communicating to two remote single-antenna receivers. The channel model is depicted in Figure 2 (upper-left)
with $c_{43}=c_{34}=0$. As in receiver cooperation, we restrict to the full-duplex case, where Nodes 1 and 2 can simultaneously transmit and receive. It is shown in [30], [31] that, in contrast to receiver cooperation, synchronization helps much when the transmitters cooperate. That is, if the two transmitters are synchronized, they can completely cancel out the interference using dirty-paper coding (DPC) (see "Coding with Side Information"). DPC was already exploited in [4] and [32] to find the capacity of the Gaussian MIMO broadcast channel. For the two-antenna broadcast channel with two receivers [4], the main idea is to decompose the MIMO channel into two interference channels and perform successive encoding, in which the message for the second receiver is dirty-paper encoded while assuming the previously encoded message for the first receiver as known interference (the side information). In this way, the second receiver can completely cancel out the interference from the signal for the first receiver. However, to achieve full capacity the transmitter has to perform optimal channel decomposition using precoding with the output vector $\mathrm{x}=\mathrm{B}\left[\begin{array}{ll}u_{1} & u_{2}\end{array}\right]^{T}$, where B is a $2 \times 2$ precoding matrix that has to satisfy the power constraint, and $u_{1}$ and $u_{2}$ are the encoded code words (with unit power) intended for the first and second receiver, respectively, and obtained via successive dirty-paper encoding. Assuming a $2 \times 2$ channel matrix H and unit-power Gaussian noise, the achievable rates for the first and the second receiver are $R_{1}=\log \left(1+\left|s_{11}\right|^{2} /\left(1+\left|s_{12}\right|^{2}\right)\right) \quad$ and $\quad R_{2}=\log \left(1+\left|s_{22}\right|^{2}\right)$, respectively, where $s_{i j}$ are the entries of matrix $\mathrm{S}=\mathrm{HB}$.

The coding strategy of [4] is extended to transmitter cooperation in [30], [33]. In [33], it is assumed that the channel between the two transmitters is orthogonal to the channels between the transmitters and receivers. Thus, collaboration between the transmitters does not cause interference at the receivers, which simplifies the code design. This orthogonality assumption is removed in [30], where a coding scheme that in each time instant exploits three DPCs is proposed. A simplified solution based on one DPC (in conjunction with backwarddecoding) is outlined in [34]. In the scheme of [34], during the i-th time instant, the transmitter in Node j, $j=1,2$, sends $x_{j}[i]=A_{j} U_{j}\left(\omega_{j}[i]\right) \quad+t_{j 1} U_{1}^{0}\left(\omega_{1}[i-1], U_{2}^{0}\left(\omega_{2}[i-1]\right)\right) \quad+t_{j 2} U_{2}^{0}$ $\left(\omega_{2}[i-1]\right)$. Here $U_{2}^{0}, U_{1}$, and $U_{2}$ are Gaussian codebooks (e.g., standard channel codes in practice) of unit power that encode $\omega_{2}[i-1], \omega_{1}[i]$, and $\omega_{2}[i]$, respectively. $U_{1}$ and $U_{2}$ are used for exchanging messages between the transmitters and appear as part of the background noise at the receivers. Assuming correct decoding of $U_{1}\left(\omega_{1}[i-1]\right)$ and $U_{2}\left(\omega_{2}[i-1]\right)$ in time instant $i-1$, the two transmitters can now act as a single two-antenna broadcast transmitter, and the coding strategy of [4] described above can be applied. Thus, the unit-power codebook $U_{1}^{0}$ can encode $\omega_{1}[i-1]$ using DPC with $U_{2}^{0}\left(\omega_{2}[i-1]\right)$ as the side information. The scaling factors $A_{i}$ and $t_{i j}$ are selected to maximize the rate while satisfying the input power constraints.

In the asynchronous case, DPC cannot be exploited, and the resulting known achievable rates are strictly below those in the synchronous case. However, so far there exist no upper bounds that actually prove that the gains cannot be obtained without synchronization. All the bounds are summarized in "Capacity Bounds for Transmitter Cooperation." Although it is possible to use WZC in transmitter cooperation, since the two transmitters are closely located, DPC always dominates. This is why WZC is not considered in this setup.
Similar to receiver cooperation, in the high SNR regime, transmitter cooperation only gives a multiplexing gain of $r=1$ (in contrast to the two-antenna broadcast channel which results in $r=2$ ). The additive gain can be high. For example, when $\left|c_{41}\right|<1$, in the synchronous case it is bounded by

$$
\begin{equation*}
a \leq \log \left(\left(\left|c_{41}\right| \sqrt{P_{1}}+\sqrt{P_{2}}\right)^{2}\right)+\log \left(\frac{1+\left|c_{21}\right|^{2}}{\left|c_{41}\right|^{2}}\right) \tag{13}
\end{equation*}
$$

By exchanging Nodes 1 and 2 with Nodes 3 and 4, we get another symmetric rate bound. Besides the multiplexing gain of $r=1$, DPC achieves a high-SNR additive gain of

$$
\begin{equation*}
a=\log \left(\left(\left|c_{41}\right|\left|t_{12}\right|+\left|t_{22}\right|\right)^{2}\right)+\log \frac{\left|c_{21}\right|^{2}}{\left|c_{41}\right|^{2}} \tag{14}
\end{equation*}
$$

Comparing (13) and (14), we see that the additive gain of using DPC is approximately equal to that from the upper bound when $\left|t_{12}\right|^{2} \approx P_{1}$ and $\left|t_{22}\right|^{2} \approx P_{2}$, which corresponds to the scenario with weak interference, i.e., $\left|c_{41}\right|,\left|c_{32}\right| \ll 1$. In this case, the gain compared to no cooperation in (12) is $\min \left\{\log \left(\left|c_{12}\right|\right), \log \left(\left|c_{21}\right|\right)\right\}$, which can be significant, because the channel between the two transmitters is expected to be good. This is illustrated in Figure 4, which shows the high-SNR additive gain for a symmetric cooperative channel $\left(\left|c_{41}\right|=\left|c_{32}\right|,\left|c_{21}\right|=\left|c_{12}\right|\right)$. Note that under strong interference, i.e., $\left|c_{32}\right|,\left|c_{41}\right| \gg 1$, there is no gain from cooperation, which might seem somewhat surprising and is in contrast to receiver cooperation. For weak interference, on the other hand, there is a high gain from cooperation. This is true even when the link between the transmitters is weak $\left(\left|c_{21}\right|^{2}=-6 \mathrm{~dB}\right.$ in Figure 4), and it can be explained by the fact that there is no known signaling for the interference channel with weak interference, while cooperation can help in this scenario.

## PRACTICAL DESIGNS

In the previous section, we examined coding methods that could lead to the best capacity rates while assuming ideal coding, signaling, and infinite block length. In this section we describe practical systems based on AF, DF, and CF, with the focus on limit-approaching designs.

For wireless multirelay channels, space-time codes with AF and DF are designed in [28] to enable simultaneous transmission from all relays on the same channel without receive

## CAPACITY BOUNDS FOR TRANSMITTER COOPERATION

Based on channel transformation and the argument exploited in [4] that the capacity region depends only on the marginal distribution of the noises at the receivers and not on their correlation, the following upper bounds on capacity are derived in [30]. For $\left|c_{41}\right|<1$, the upper bound is

$$
R_{2} \leq \log \frac{1+\left|C_{41}\right|^{2} P_{1}+P_{2}}{\left|C_{41}\right|^{2} 2^{R_{1}} \frac{1+P_{1}}{1+\left(1+\left|c_{21}\right|^{2}\right) P_{1}}+1-\left|C_{41}\right|^{2}}
$$

in the asynchronous case, and

$$
R_{2} \leq \log \frac{1+\left(\left|C_{41}\right| \sqrt{P_{1}}+\sqrt{P_{2}}\right)^{2}}{\left|C_{41}\right|^{2} 2^{R_{1}} \frac{1+P_{1}}{1+\left(1+\left.\left|C_{21}\right|\right|^{2} P_{1}\right.}+1-\left|C_{41}\right|^{2}}
$$

in the synchronous case, and the sum-rate bound $R_{1}+R_{2}$ is an increasing function of $R_{1}$. If $\left|c_{41}\right|>1$, the upper bound is

$$
\begin{gathered}
R_{1}+R_{2} \leq \log \left(1+\left|c_{41}\right|^{2} P_{1}+P_{2}\right) \\
+\log \frac{1+\left(\left|c_{41}\right|^{2}+\left|c_{21}\right|^{2}\right) P_{1}}{1+\left|c_{41}\right|^{2} P_{1}}
\end{gathered}
$$

in the asynchronous case, and

$$
\begin{align*}
R_{1}+R_{2} \leq & \log \left(1+\left(\left|C_{41}\right| \sqrt{P_{1}}+\sqrt{P_{2}}\right)^{2}\right) \\
& +\log \frac{1+\left(\left|c_{41}\right|^{2}+\left|c_{21}\right|^{2}\right) P_{1}}{1+\left|C_{41}\right|^{2} P_{1}} \tag{15}
\end{align*}
$$

in the synchronous case. There is also a symmetric set of rate bounds by exchanging Nodes 1 and 2 with Nodes 3 and 4.
Achievable bounds in asynchronous (without DPC) and synchronous systems (with one and three DPCs) can be found in [30], [34]. Figure D shows the sum-rate bounds $R_{1}+R_{2}$ as functions of the received SNR on the direct link between Nodes 1 and 3. The simulation setup is similar to that for receiver cooperation with the received SNR at the cooperative link (between Nodes 1 and 2) being 30 dB higher than that at the direct link, again indicating that the cooperating transmitters are close together.

The achievable bounds of the synchronous system with DPC is usually close to the upper bound, although noticeable gaps exist in certain SNR ranges. There is only a small performance loss if only one DPC is used instead of three. The additive gain compared to the noncooperative case is up to 15 dB in the high SNR regime. Transmitter cooperation with DPC performs close to using two transmitter antennas at low and medium SNRs, giving a multiplexing gain of two. However, at high SNRs, the multiplexing gain is only one.

[FIGD] The bounds on the sum-rate $R_{1}+R_{2}$ as a function of the received SNR at the direct link between Nodes 1 and 3 for transmitter cooperation. The received SNR at the link between Nodes 1 and 2 is 30 dB higher than that at the direct link. The average powers of all four nodes are the same. The upper bound is for the synchronous system.

[FIG4] The bounds on the additive gain for a symmetric transmitter cooperative channel $\left(\left|c_{41}\right|=\left|c_{32}\right|,\left|c_{21}\right|=\left|c_{12}\right|\right)$, averaged over the relative phases of $c_{41}$ and $c_{32}$. The upper bound is for a synchronous system.
collision. It is further shown that the proposed schemes achieve the full diversity gain. An AF space-time code for a single relay is proposed in [29]. Note that the works of [28] and [29] only outline space-time code designs without practical implementation of channel codes.

Practical DF schemes for a half-duplex, flat-fading relay channel based on distributed convolutional and turbo coding are proposed in [22]. The best scheme there exploits a recursive systematic convolutional code at both the source and the relay. It results in a powerful distributed turbo code, which besides a spatial diversity gain of DF, achieves extra coding gain due to interleaving.

Extending their work on practical full-duplex relaying [19], Zhang and Duman [23] recently provided a DF design for halfduplex fading relay channels, where in a given time slot, the source and the relay both transmit over the same channel, resulting in a high rate gain at the price of receive collision. The design in [19] exploits turbo coding with BPSK modulation. It is consistent with the optimal DF scheme for the half-duplex relay channel described earlier, with the simplification that $A$ in (5) is
always set to zero. Decoding is based on parallel Gaussian channel arguments. Similar to the MAC setting, the destination exploits a MAP detector to extract information from the received mixture signal.

The systems in [22], [23], [28], and [29] demonstrate the great advantage of relaying as compared to direct and multihop transmission. However, because these systems exploit AF or DF, they can approach the lower bound of DF at best, which is far away from the CF limit in many cases. Indeed, when the relay is close to the destination, CF gives rate points that are not achievable with any other coding strategies. We describe next a practical CF code design based on WZC for the half-duplex fading relay channel, which closely follows the CF scheme. The message $m$ is split at the source into two parts, $m_{1}$ and $m_{2}$, which are protected independently by two different low-density paritycheck (LDPC) codes and BPSK modulated before being transmitted in two separate fractions of a time slot. The relay compresses its received signal using WZC and adds error protection against the noise (and interference) in the link between the relay and the destination. WZC and error protection are performed jointly using distributed joint source-channel (DJSC) coding with nested scalar quantization [12] and systematic irregular repeat-accumulate (IRA) codes; the resulting code word $x_{r}\left(m_{1}\right)$ is sent during the relay-transmit period. The destination starts by recovering $m_{2}$ using successive cancellation decoding: $x_{r}\left(m_{1}\right)$ is first reconstructed using DJSC decoding, it is then subtracted from $y_{d 2}$ (interference cancellation) so that $m_{2}$ can be recovered with the first LDPC decoder having $z$ as the only noise in the channel. The main idea behind DJSC decoding is to view the system as transmitting the symbols over two chan-nels-the first being the actual transmission channel (i.e., the MAC) with noise $z+x_{52}\left(m_{2}\right)$, which describes the distortion experienced by the parity-check symbols of the IRA code, and the second being the "virtual" correlation channel between $y_{r}$ and the side information $y_{d 1}$. The destination estimates $\hat{y}_{r}$ by employing a conventional IRA decoder for these two parallel channels. Finally, $m_{1}$ is recovered using maximum ratio combining (of $\hat{y}_{r}$ and $y_{d 1}=c_{d s} x_{s 1}\left(m_{1}\right)+z_{d s}$ ) and a second LDPC decoder for the direct transmission channel.

Figure 5 compares this CF design with the best practical DF design of [23] for the half-duplex fading relay channel. It can be seen that when the relay is close to the destination, the CF design outperforms the DF design of [23] by more than 1 dB .

Based on the CF coding strategy with forward individual decoding, a practical design using WZC for the two-receiver cooperative channel is given in [34]. It is an extension of the described design for the fading relay channel. Each transmitter is equipped with one LDPC channel encoder, and each receiver performs one DJSC encoding step (with a nested scalar quantizer and an IRA code) and two channel decoding steps (with IRA and LDPC codes). Simulation results are presented in Figure 6, which represents the only code design results reported so far for receiver cooperation. The performance loss due to the design in [34] decreases as $P_{3}$ increases and can be further reduced by employing stronger source codes.

In contrast to the scarcity of WZC-based CF designs for receiver cooperation, there have been more code designs for transmitter cooperation. For example, since the publication of the work on user cooperation [2], several research groups [28], [35], [36] have developed practical designs based on AF and DF for the wireless two-transmitter cooperative channel. A

[FIG5] The average power of the source $P_{s}$ as a function of the distance $d$ between the source and the relay for a half-duplex fading relay channel. The theoretical limits assume BPSK signaling. The relay is located along a straight line from the source to the destination, which are 10 m apart.

[FIG6] Simulation results obtained from the scheme of [34] for receiver cooperation, together with various capacity limits under BPSK signaling. The average power of the transmitters $P_{1}=P_{2}$ as a function of the average power of the receivers $P_{3}=P_{4}$ is shown. The received SNR at the link between the two receivers is 15 dB higher than that at the direct link.
common characteristic of these designs is the avoidance of receive collision by transmitting signals over orthogonal channels, which simplifies the code design. Specifically, the two transmitters (or cooperative partners) send encoded messages over orthogonal channels during the first fraction of a time slot. Each transmitter decodes the signal it has received from its partner and, in the case of successful decoding, either reencodes the recovered message using the partner's codebook (repetition-based DF) or generates additional parity symbols out of a rate-compatible code (DF based on incremental redundancy). The resulting code words are then forwarded over orthogonal channels to the receiver during the second fraction of a time slot. If a transmitter cannot successfully decode its partner's message, it switches to either the noncooperative mode or AF. Orthogonal signaling is achieved by using time/frequency/code-division or space-time coding. A DF design based on incremental redundancy for a flat-fading transmitter cooperative channel, dubbed coded cooperation, is proposed in [35]. It exploits space-time turbo coding and is efficient on both slow and fast flat-fading channels. A similar DF scheme reported in [36] exploits convolutional codes optimized for two-transmitter cooperation in a Rayleigh flat-fading environment. The scheme is shown to be able to achieve the full diversity gain.

Although the above DF schemes can provide the full diversity gain, they do so at the expense of decreased rate gain. On the other hand, we know that DPC provides the highest achievable rate over a transmitter cooperative channel. Practical DPC is exploited for the first time in [34] to design codes for a wireless two-transmitter cooperative channel. DPC is performed at the transmitters based on trellis-coded quantization and turbo trel-lis-coded modulation, which also facilitates message exchanges between the two transmitters. Practical design results under the same channel condition as in Figure D and $R_{1}=R_{2}=1$

[FIG7] Simulation results obtained from the DPC-based scheme of [34] for transmitter cooperation, together with various theoretical bounds, in terms of the probability of frame error versus the received SNR at the direct link.
bit per channel use are shown in Figure 7, indicating a loss of 1.5 dB from the achievable bound at $2 \%$ frame error rate.

## CONCLUSIONS

Due to its low-complexity and decentralized nature, cooperative diversity arises as a strong candidate for conveying information in emerging wireless ad hoc networks. This motivates research in determining its ultimate performance limit. For a two-transmitter, two-receiver cooperative channel, the theory shows that, at least in the high SNR regime, in contrast to a two-antenna MIMO system, cooperative diversity cannot achieve the full multiplexing gain of two. Thus, there is a cost paid for having only one antenna at each node. This result indicates that in achieving the full multiplexing gain of two, tight coordination among the transmit/receive antennas is necessary, which is possible only if they are placed together. However, cooperative diversity does offer high additive rate gains when compared to the noncooperation case, and the key in achieving these gains lies in coding with side information (e.g., DPC and WZC). In transmitter cooperation, synchronization between the two transmitters is essential in obtaining high data rates; DF with DPC is the best coding strategy that comes very close to the upper limits if the interference is weak. Unfortunately, when the interference is strong, transmitter cooperation does not help (in the high SNR regime). In contrast, receiver cooperation is beneficial in both weak and strong interference scenarios, and CF with WZC is the dominant coding strategy that asymptotically achieves the capacity as the interference and SNR approach infinity. More importantly, full synchronization between the nodes is not necessary for receiver cooperation. Interestingly, as pointed out in [31], optimal power allocation is crucial to realize the full performance gain of receiver cooperation, whereas it provides only a marginal gain in transmitter cooperation.

Owing to its promising application in wireless ad hoc networks, cooperative diversity has been studied intensively recently. Many problems are still open. For example, posed more than 30 years ago, the capacity of the simplest Gaussian relay channel, a building block of cooperative diversity, is still unknown. Recent achievements [32] in providing the full capacity region for Gaussian MIMO broadcast channel using DPC might inspire new ideas for solving this problem. The theoretical bounds reported so far are mainly for the full-duplex setups with up to four nodes, where either two transmitters or two receivers cooperate. Providing results for half-duplex cooperative channels is our research priority. Treating an ad hoc network where the two transmitters and two receivers simultaneously cooperate is another possible research direction. Combining DPC and WZC could lead to the largest achievable rate region, but such a theoretical treatment is not straightforward. In addition, extensions to larger networks with more than four nodes that require cross-layer designs are very challenging because of the additional problem associated with selecting the best partner for cooperation.

The reported practical designs still suffer performance loss compared to the theoretical limits. Closing this gap with better
code designs while staying at acceptable complexity is an urgent research task. The practical designs proposed so far are only for wireless relay and two-transmitter two-receiver cooperative channels. Substantial research efforts are needed to construct practical systems based on cooperative diversity for larger ad hoc networks.

## ACKNOWLEDGMENTS

The authors would like to thank Zhixin Liu and Momin Uppal for providing Figures 9-11. Anders Host-Madsen was supported by NSF Grant CCR-0329908, and Zixiang Xiong was supported by NSF Grants CCF-0104834 and CCF-0430720.

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