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Cooperative Fault-Tolerant Control for Networks of Stochastic Nonlinear Systems with Non-differential Saturation Nonlinearity

Hongjing Liang, Guangliang Liu, Tingwen Huang, *Fellow, IEEE*, Hak-Keung Lam, *Fellow, IEEE*, and Bohui Wang

Abstract—This paper addresses the cooperative fault-tolerant control problem for networks of stochastic nonlinear systems with actuator faults and input saturation. The fuzzy neural networks (FNNs) are employed to estimate the unknown functions and stochastic disturbance terms. To analyze the nondifferential saturation nonlinearity, a smooth nonlinear function of the control input signal is constructed to estimate the saturation function. A novel adaptive fault-tolerant control protocol is proposed by using backstepping design technique. By using the stochastic Lyapunov functional strategy, it is proved that all the followers' outputs eventually converge to a small neighborhood of the leader's output, and all the signals in the closed-loop systems are bounded in probability. Finally, the performance of the proposed control strategy is illustrated through simulation.

Index Terms—Communication topology, networks of stochastic nonlinear systems, fuzzy neural networks, input saturation, actuator faults.

I. INTRODUCTION

IN the past few decades, the research on stochastic systems has received extensive attention due to the typical exhibition of stochastic phenomenon in many actual systems, such as biological systems [1], [2], unmanned air vehicle (UAV) formations [3], satellite clusters [4] and adaptive control systems [5], [6]. The stochastic multi-agent systems are seldom considered in the existing papers, although stochastic modeling has played an important role in many stochastic systems [7]–[9]. In [10], to unitedly overcome the bottlenecks of resource, communication, and computation in distributed optimization, the random sleep scheme is creatively introduced into algorithm design, which allows each agent to independently control its update frequency. Zhou *et al.* in [11] proposed a novel adaptive fuzzy control scheme to solve the problem of adaptive fuzzy tracking control for a class of nonlinear systems by using

the adaptive backstepping technique. Zhu *et al.* in [12] studied the high-order stochastic systems subject to unmeasured states, in which the fuzzy states observer was utilized to handle the uncertain parts in systems. In [13], a fuzzy generalized predictive control law was first proposed and constructed, which offers pioneering work for the predictive control of a nonlinear complexity system. This served as a foundation bridge in combining research fuzzy system and generalized predictive control. By the barrier Lyapunov function and combining a backstepping design technique and an adding a power integrator, the authors in [14] developed a novel controller to control high-order uncertain nonlinear system. In [15], Sun *et al.* proposed a multiagent-based consensus algorithm for distributed coordinated control, which was firstly applied to Energy Internet and was an important breakthrough for energy management. A decentralized controller was designed by Lyapunov-based recursive method to ensure the global stability for stochastic nonlinear systems [16]. Based on the backstepping technique, the authors in [17] developed a new adaptive fuzzy control protocol to solve the event-triggered containment control problem for stochastic nonlinear multi-agent systems.

In most industrial systems, the saturation nonlinearities are inevitable [18]–[25]. Since the input saturation nonlinearity maybe reduce the systems performance, and also may lead to instability such as undesirable inaccuracy or oscillations, it is difficult to handle a case that the nonlinear systems with saturation nonlinearities. By using the sliding mode approach, an adaptive control method for spacecraft systems was proposed in [18] subject to input saturation. A new adaptive auxiliary signals was established in [19] to compensate for the influence of the dead zones and actuator faults on the control performance. The Ref. [20] gave a more greater flexibility approach to solve the synchronization issue for network systems with saturation nonlinearities. In [21], the authors investigated the problem of event-triggered adaptive control for a class of nonlinear systems with asymmetric input saturation. Ref [23] constructed a low-gain feedback protocol to solve the consensus problem of multi-agent systems with input saturation. In order to solve the global consensus problem of multi-agent systems with input saturation constraints, the authors in [24] proposed a linear protocol under fixed network topologies and time-varying network topologies. However, the aforementioned results on solving the nonlinear multi-agent systems subject to input saturation require that the systems

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should not be influenced by stochastic disturbance. In real industry, there are some stochastic systems, so it is important to research the control design of stochastic multi-agent systems subject to input saturation.

At the same time, the faults of actuator are also the main factor which make the controller design and stability analysis more difficult. Recently, researchers have achieved good research results in the study of actuator faults [26]–[34]. For instance, based on a weighted switching approach, the authors in [27] introduced a pair of variable weights in an efficient and straight-forward way to solve the problem of observer-based fault estimation for discrete-time nonlinear systems. In order to overcome the impact of actuator faults on systems stability, Tao *et al.* in [29] designed an effective distributed controller to ensure systems' stability when actuator fault occurred. In [31], an adaptive algorithm was proposed for dynamic systems with unknown actuator faults, while the compensation for actuator fault was based on an adaptive tuning of actuator parameter matrices. According to Lyapunov stability theory, a nonlinear parameterized radial basis function neural adaptive control algorithm was proposed to design the fault-tolerant tracking controller in [33]. So far, although remarkable results have been achieved in dealing with the above disadvantages. There are few studies on the coexistence problem about the aforementioned factors in the networks of stochastic nonlinear systems, which motivates our current research work.

We aim the adaptive fuzzy neural networks (FNNs) cooperative fault-tolerant control problem for networks of stochastic nonlinear systems with actuator faults and input saturation. The main advantages of our designed control protocol can be laid in the following:

1) An adaptive fuzzy neural network algorithm is proposed to transform the ideal weight vector into the multiplication of two parameters, and the proposed algorithm has few number of adjustable parameters, which can reduce the online calculation burden and easy to be applied into networks of stochastic nonlinear systems, and the completely unknown nonlinear terms and stochastic disturbances are effectively dealt by using the property of FNNs in the backstepping process.

2) Compared with the existing results of fault-tolerant control problems, the fault model used in this paper contains both abrupt faults and incipient faults, which is more general than the faults models in [35], [36]. In addition, the structure of actual control input v_i of the system is changed to broaden the application range of the obtained results.

3) The fuzzy neural network-based adaptive fault-tolerant control protocol is extended to networks of stochastic nonlinear systems subject to nonlinearity input saturation. The developed method does not need the prior knowledge of the bound for input saturation.

The rest of this paper is organized as follows. In Section II, the basic graph theory, the FNNs, and the problem formulation are introduced. The main technical results of this paper are given in Section III, which include the design of real control protocol and virtual control protocol. A numerical simulation is presented in Section IV. Finally, Section V draws our conclusion.

Notation 1: The following notations are used throughout

this paper. $\max\{\bullet\}$ and $\min\{\bullet\}$ represent maximum and minimum eigenvalue, respectively. For a given vector or matrix x , x^T represents its transpose. $|\bullet|$ refers to the absolute value, and $\|\bullet\|$ expresses 2-norm. $\sup(\bullet)$ represents the supremacy. $\arg\min\{\bullet\}$ represents the value of the variable that minimizes the objective function. $\tanh(\bullet)$ refers to the hyperbolic tangent function. $\text{diag}(B_1, \dots, B_k)$ denotes a diagonal matrix with B_1, \dots, B_k as its diagonal elements.

II. PRELIMINARIES

A. Graph theory

Graph $\mathcal{G} = (\mathcal{X}, \mathcal{E})$ is a directed graph to describe the information communication between agents, where $\mathcal{X} = \{\mathcal{x}_1, \dots, \mathcal{x}_m\}$ is a nonempty set of follower agents, and $\mathcal{E} \in \mathcal{X} \times \mathcal{X}$ denotes the set of edges. The edge means that the j th follower only receives state information from the i -th follower, and satisfies $i \in N_j$, $i, j = 1, \dots, m$, and $i \neq j$. Define the adjacency matrix $\mathcal{A} = (a_{i,j}) \in R^{m \times m}$, where $a_{i,j} = 1$ if $(\mathcal{x}_j, \mathcal{x}_i) \in \mathcal{E}$, and otherwise $a_{i,j} = 0$. The in-degree matrix of agent i is denoted as $D = \text{diag}(d_1, \dots, d_m) \in R^{m \times m}$ with $d_i = \sum_{j=1}^m a_{i,j}$. The Laplacian matrix $\mathcal{L}_{\mathcal{G}} = [l_{i,j}]_{m \times m} \in R^{m \times m}$ of \mathcal{G} is defined as $\mathcal{L}_{\mathcal{G}} = D - \mathcal{A}$. In a digraph, a directed path from node i to node j is a sequence of edges $\{(\mathcal{x}_i, \mathcal{x}_k), (\mathcal{x}_k, \mathcal{x}_l), \dots, (\mathcal{x}_h, \mathcal{x}_j)\}$ with distinct nodes $l = k, \dots, h$. The digraph \mathcal{G} is said to contain a directed spanning tree, if there is a root node that has a directed path to any other node. For any distinct agents $\mathcal{x}_i, \mathcal{x}_j \in \mathcal{X}$, if there exists a directed path from agent i to agent j , then \mathcal{G} is called strongly connected.

This paper considers the multi-agent systems consisted of N followers and one leader. Without loss of generality, we mark the leader as 0 and followers as $1, \dots, N$. The communication topology between the leader and the followers is represented by a diagonal matrix.

The following assumptions and lemma are given to solve the cooperative control problem.

Assumption 1: The augmented graph $\bar{\mathcal{G}} = (\bar{\mathcal{X}}, \bar{\mathcal{E}})$, with $\bar{\mathcal{X}} = \{\mathcal{x}_0, \mathcal{x}_1, \dots, \mathcal{x}_m\}$ exists a spanning tree, where the leader node 0 is the root node of $\bar{\mathcal{G}}$.

Assumption 2: The n th order time derivatives of leader output $y_{01}(t)$ is continuous bounded and available.

Lemma 1: [37] The digraph \mathcal{G} is said to have a spanning tree, if there is at least a directed path from the root to all other nodes. If denote $B = \text{diag}(b_{1,0}, \dots, b_{N,0})$, the node 0 is called the root of the spanning tree, then the matrix $\mathcal{L}_{\mathcal{G}} + B$ is nonsingular.

B. Problem Formulation

Consider a class of stochastic nonlinear multi-agent systems

$$\begin{aligned} dx_{i,h} &= (g_{i,h}(x_i) x_{i,h+1} + f_{i,h}(x_i)) dt + \Psi_{i,h}(x_i) dw \\ 1 &\leq h \leq n_i - 1 \\ dx_{i,n_i} &= (g_{i,n_i}(x_i) u_i + f_{i,n_i}(x_i)) dt + \Psi_{i,n_i}(x_i) dw \\ y_i &= x_{i,1} \end{aligned} \quad (1)$$

where $x_i = [x_{i,1}, \dots, x_{i,n_i}]^T \in R^{n_i}$, and y_i ($i = 1, \dots, m$) are the systems' state vector and output; $f_{i,h}(\cdot)$ and $\Psi_{i,h}(\cdot)$

($h = 1, \dots, n_i$) are unknown smooth functions, $g_{i,h}(\cdot)$ is a continuous bounded function. w denotes an r -dimensional standard Brownian. u_i represents that nonsymmetric saturation nonlinearity is described as

$$u_i = \text{sat}(v_i) = \begin{cases} u_{i,\max}, & v_i \geq u_{i,\max} \\ v_i, & u_{i,\min} < v_i < u_{i,\max} \\ u_{i,\min}, & v_i \leq u_{i,\min} \end{cases} \quad (2)$$

with $u_{i,\min} < 0$ and $u_{i,\max} > 0$ being unknown constants, and v_i denotes the nonlinearity input saturation signal.

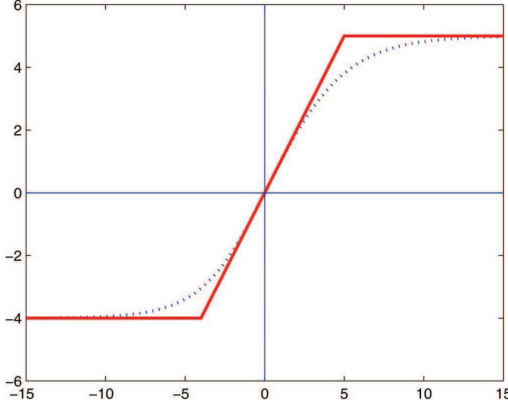


Fig. 1. Saturation. Dotted line: $p(v_i)$. Solid line: $\text{sat}(v_i)$.

According to Fig. 1, it is quite clear that there are two sharp corners if $u_{i,\max} = v_i$, and $u_{i,\min} = v_i$. Thus, the backstepping method can not be directly used to design the control signal. Therefore, we proposed a smooth piecewise function to estimate the saturation function to deal with this problem. The function is defined as follows:

$$p(v_i) = \begin{cases} u_{i,\max} \times \tanh\left(\frac{v_i}{u_{i,\max}}\right), & v_i \geq 0 \\ u_{i,\min} \times \tanh\left(\frac{v_i}{u_{i,\min}}\right), & v_i < 0 \end{cases} \\ = \begin{cases} u_{i,\max} \times \frac{e^{\frac{v_i}{u_{i,\max}}} - e^{-\frac{v_i}{u_{i,\max}}}}{e^{\frac{v_i}{u_{i,\max}}} + e^{-\frac{v_i}{u_{i,\max}}}}, & v_i \geq 0 \\ u_{i,\min} \times \frac{e^{\frac{v_i}{u_{i,\min}}} - e^{-\frac{v_i}{u_{i,\min}}}}{e^{\frac{v_i}{u_{i,\min}}} + e^{-\frac{v_i}{u_{i,\min}}}}, & v_i < 0. \end{cases} \quad (3)$$

Then, $\text{sat}(v_i)$ in (2) can be denoted as

$$u_i = \text{sat}(v_i) = p(v_i) + q(v_i) \quad (4)$$

where $q(v_i) = \text{sat}(v_i) - p(v_i)$ denotes a bounded function, and one has

$$|q(v_i)| = |\text{sat}(v_i) - p(v_i)| \\ \leq \max\{u_{i,\min}(\tanh(1) - 1), \\ u_{i,\max}(1 - \tanh(1))\} \\ = D. \quad (5)$$

Fig. 1 denotes an estimation of the saturation function.

Furthermore, we define

$$p(v_i) = p(v_0) + p_{v_\mu}(v_i - v_0). \quad (6)$$

By selecting $v_0 = 0$, (6) is rewritten as

$$p(v_i) = p_{v_\mu} v_i. \quad (7)$$

To facilitate the design of the controller, the following assumptions are proposed.

Assumption 3: For the function p_{v_μ} in (6), there is an unknown constant p_m such that

$$0 < p_m \leq p_{v_\mu} \leq 1. \quad (8)$$

Remark 1: The actual control input signal v_i cannot be infinite in practical (i.e., there is no $p_{v_\mu} = 0$), so there exists an unknown constant p_m that makes this assumption valid.

Assumption 4: There exist unknown constants $\varphi_h > 0$ such that $\varphi_h \leq g_{i,h}(x_i)$ for all $x_i \in R^{n_i}$, and on arbitrary bounded set in R^{n_i} , $\bar{\varphi}_h > 0$ also exists such that $g_{i,h}(x_i) \leq \bar{\varphi}_h$, $h = 1, \dots, n_i$.

Remark 2: By means of Assumptions 3 and Assumptions 4, it can be further assumed that

$$0 < \underline{\varphi}_{n_i} \leq p_{v_\mu} g_{i,n_i}(x_i) \quad (9)$$

where $\underline{\varphi}_{n_i} = \min\{g_{i,n_i}(x_i), p_{v_\mu} g_{i,n_i}(x_i)\}$ is an unknown constant.

Substituting (4) into (1) and using (7), one gets

$$\begin{aligned} dx_{i,h} &= (g_{i,h}(x_i) x_{i,h+1} + f_{i,h}(x_i)) dt + \Psi_{i,h}(x_i) dw \\ 1 &\leq h \leq n_i - 1, \\ dx_{i,n_i} &= (g_{i,n_i}(x_i) (p_{v_\mu} v_i + q(v_i)) + f_{i,n_i}(x_i)) dt \\ &\quad + \Psi_{i,n_i}(x_i) dw, \\ y_i &= x_{i,1}. \end{aligned} \quad (10)$$

where $i = 1, \dots, m$.

Remark 3: The above dynamics (10) is an non-strict feedback form. In the multi-agent systems (10), functions $f_{i,h}(\cdot)$, $g_{i,h}(\cdot)$, $\Psi_{i,h}(\cdot)$ and all state vector x_i are not reported in the existing consensus works. The further stimulated our research interest for the multi-agent systems model. Meanwhile, compared to the existing stochastic strict-feedback multi-agent systems, $f_{i,h}(\cdot)$ and $\Psi_{i,h}(\cdot)$ are completely unknown.

Definition 1: [38] Given $V(x, t)$ a twice continuously differentiable function. The differential operator L is defined as follows

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2} \text{Tr} \left\{ \Psi^T \frac{\partial^2 V}{\partial x^2} \Psi \right\} \quad (11)$$

with Tr being the matrix trace.

Definition 2: [38] Under the digraph \mathcal{G} , if for $\forall \zeta > 0$, the tracking errors between the leader and followers are cooperatively semiglobally uniformly ultimately bounded(CSUUB) in probability, i.e., $E|y_i(t) - y_{01}(t)|^4 \leq \zeta$ ($i = 1, \dots, m$) when $t \rightarrow \infty$ with E being the expectation operator.

The accidental actuator faults might occur in practice, giving the systems real control input v_i and the designed input v_{di} . Meanwhile, define the relationship between v_i and v_{di} , as follows

$$v_i = \rho(t_r, t) v_{di} + \delta_d(t - t_f) \lambda_d(t) \quad (12)$$

where $\rho(t_r, t)$ denotes the ‘‘healthy indicator’’, showing the actuator effectiveness, $\delta_d(t - t_f) \lambda_d(t)$ expresses for uncontrollable additive actuator failures, t_r denotes the time instant when the actuator fails, and t_f means additive actuator fault occurs. The function $\delta_d(t - t_f)$ characterizes the time profile

of a failure that occurs at some unknown time t_f , and $\lambda_d(t)$ is the bounded but uncontrollable part of the actuator output. In this paper, two main types of failures are considered

1) *Abrupt Faults*

$$\delta_d(t - t_f) = \begin{cases} 0, & t \leq t_f \\ 1, & t > t_f. \end{cases} \quad (13)$$

2) *Incipient Faults*

$$\delta_d(t - t_f) = \begin{cases} 0, & t \leq t_f \\ 1 - e^{-\vartheta(t-t_f)}, & t > t_f \end{cases} \quad (14)$$

with $\vartheta \in [0, \infty)$ being the faults in which the uncontrollable part $\delta_d(t - t_f)$ occurs. In particular, $\vartheta = 0$ implies that all actuator faults are controllable and $\vartheta = \infty$ means that the uncontrollable part appears abruptly, and $0 < \vartheta < \infty$ corresponds to the situation where the uncontrollable part gradually occurs in the system. Note that, in this paper, we require the ‘‘healthy indicator’’ $\rho(t_r, t)$ is $0 < \rho(t_r, t) \leq 1$, meaning that although the actuator missing its effectiveness, it is still working, so that v_i can always be affected by the control input v_{di} . Meanwhile, there is no other restriction, and it could change over time.

Remark 4: Some interesting works on fault-tolerant control have been reported in [33], [35], [39]. We need less faults information to construct our control protocol than the above literature. More specifically, the adaptive control protocol proposed in [33] is based on the linear parametric decomposition of the failure signals. Such decomposition is impossible if there is a little or no message about the fault models. In [39], actuation fault compensation and control protocol were designed, while the system considered in this paper is nonlinear and uncertainties. In [35], only the additional fault is considered, which is more easier to deal with, because such a fault can be simply treated as additional disturbance. However, if the system has both abrupt faults and incipient faults considered in this paper, the underlying problem becomes more complicated to make the control design and stability analysis more challenging.

The control objective is to develop an adaptive fault-tolerant control protocol $v_i(t)$ for the system (10) such that the outputs of stochastic nonlinear multi-agent systems y_i track a desired trajectory $y_{01}(t)$ while ensure that all closed-loop signals in system (10) are bounded in probability.

Define synchronization error as

$$s_{i,1} = \sum_{j=1}^m a_{i,j} (y_i - y_j) + b_i (y_i - y_{01}) \quad (15)$$

with $b_i \geq 0$ being the pinning gains. Only when the i -th follower is able to receive the information from the leader node, $b_i > 0$.

To facilitate our stability analysis, the following lemmas are needed.

Lemma 2: [37] Defining $\mathbf{s}_{\bullet,1} = (s_{1,1}, \dots, s_{m,1})^T$, $\mathbf{y} = (y_1, \dots, y_m)^T$, $\mathbf{y}_{01} = (y_{01}, \dots, y_{01})^T$, one gets

$$\|\mathbf{y} - \mathbf{y}_{01}\| \leq \|\mathbf{s}_{\bullet,1}\| / \Delta (L + \iota) \quad (16)$$

where $\Delta (L + \iota)$ denotes the minimum singular value of $L + \iota$.

Lemma 3: [40] If there exist a function $V(p, q) \in C^{2,1}$, two positive constants ϖ and ν , E_∞ -functions κ_1 and κ_2 , such that

$$\begin{cases} \kappa_1(\|p\|) \leq V(p, q) \leq \kappa_2(\|p\|) \\ LV \leq -\varpi V(p, q) + \nu \end{cases} \quad (17)$$

for $\forall p \in R^n$ and $\forall q > 0$, there is an unique solution for systems (10) of $\forall p_0 \in R^n$, and all signals are bounded in probability.

Lemma 4: [41] If $\bar{a}, \bar{b}, \bar{p}$, and \bar{q} are positive real numbers and \bar{a}, \bar{b} , satisfy $\frac{1}{\bar{a}} + \frac{1}{\bar{b}} = 1$, then, following inequality holds:

$$\bar{p}\bar{q} \leq \frac{1}{\bar{a}}\bar{p}^{\bar{a}} + \frac{1}{\bar{b}}\bar{q}^{\bar{b}}.$$

C. FNNs Approximation Property

In this part, we use the FNNs to estimate unknown nonlinear functions. According to [42], the fuzzy logic systems (FLSs) are divided into a fuzzy reasoning and fuzzy rules. The fuzzy reasoning could be mapping $\Gamma \in R$ by applying the fuzzy rules in the form of IF-THEN. The i -th ($i = 1, \dots, n$) fuzzy rule in the form of IF-THEN is defined as

If $\epsilon_1(q)$ and \dots $\epsilon_m(q)$ is A_m^i , then $B(\tau)$ is O^i

with $A_1^i, A_2^i, \dots, A_m^i$ and O^i being fuzzy sets. F denotes the number of fuzzy rules. By applying center-average, singleton fuzzifier and product inference, the outputs of systems could be described as

$$B(\epsilon) = \frac{\sum_{i=1}^F \bar{y}^i \left(\prod_{j=1}^m \Lambda_{A_j^i}(\epsilon_j) \right)}{\sum_{i=1}^F \left(\prod_{j=1}^m \Lambda_{A_j^i}(\epsilon_j) \right)} = W^T \mathfrak{S}(\epsilon) \quad (18)$$

where $\Lambda_{A_j^i}(\epsilon_j)$ denotes the fuzzy variable membership function value; n is all number of the rules; \bar{y}^i point at which $\Lambda_{O^i}(\bar{y}^i) = 1$; $W^T = [\bar{y}^1, \bar{y}^2, \dots, \bar{y}^n]$ is the adjustable parameter vector; $\mathfrak{S}^T = (\mathfrak{s}^1, \mathfrak{s}^2, \dots, \mathfrak{s}^n)$ represents basis vector with \mathfrak{s}^i being defined as

$$\mathfrak{s}^i = \frac{\left(\prod_{j=1}^n \Lambda_{A_j^i}(\epsilon_j) \right)}{\sum_{i=1}^m \left(\prod_{j=1}^n \Lambda_{A_j^i}(\epsilon_j) \right)}. \quad (19)$$

Remark 5: The FNNs in this paper and literatures [43] depend on a defined set of priority membership functions. Then, according to the centers of membership functions, the adjustable parameters are gained. Any parameters of the priority portion are nonadjustable and depended on the right select of the expert. In [44], it does not need priori message for the IF portion of the rules, therefore, it is not susceptible to initial design assumptions. Meanwhile, the developed method in [44] maintains the well characteristic of linearity for the adjustable parameters.

Based on the approximation property of the FNNs, a continuous real-valued function $\Gamma(Z)$ defined over a compact set \mathfrak{D} , there always exists a $W^{*T} \xi(Z)$ such that

$$\Gamma(Z) = W^{*T} \xi(Z) + \varepsilon^* \quad (20)$$

where ε^* shows the estimation error and $\xi(Z)$ denotes basis function vector, generally speaking, it is assumed to be bounded by ε . W^* is optimal weight vector, which used only for analytical aims. W^* is defined as

$$W^* = \arg \min_{W \in R^l} \left\{ \sup_{X \in \Omega} |f(X) - W^T S(X)| \right\}. \quad (21)$$

In this paper, The FNNs will be applied to estimate the unknown nonlinear function $\Gamma_{i,h}(Z_{i,h})$ with $Z_{i,h} \in \mathcal{D}$, one has

$$\Gamma_{i,h}(Z_{i,h}) = W_{i,h}^{*T} \xi_{i,h}(Z_{i,h}) + \varepsilon_{i,h}^* \quad (22)$$

where $\Gamma_{i,h}(Z_{i,h})$ and $Z_{i,h}$ are defined later. The FNNs applied in this paper can be replaced by the radial basis function NNs [45] and FLSSs [46].

Let $\theta_{i,h} = \frac{\varphi_h^{-1}}{\tau_h} \|W_{i,h}^*\|$, where $\hat{\theta}_{i,h} \geq 0$ is given to estimate $\theta_{i,h}$. Define $\tilde{\theta}_{i,h} = \hat{\theta}_{i,h} - \theta_{i,h}$, where $\hat{\theta}_{i,h}$ will be adaptively adjusted.

III. DISTRIBUTED ADAPTIVE CONTROLLER DESIGN

In this section, a distributed fault-tolerant control protocol will be proposed by backstepping technique. In the following design, we define the following coordinate transformation

$$s_{i,h} = x_{i,h} - \alpha_{i,h-1}, h = 2, \dots, n_i \quad (23)$$

with $\alpha_{i,h-1}$ being the virtual control signal.

Step 1 : According to (15) and Itô formula, we can obtain

$$\begin{aligned} ds_{i,1} = & ((d_i + b_i)(f_{i,1}(x_i) + g_{i,1}(x_i)x_{i,2}) \\ & - \sum_{j=1}^m a_{i,j}[f_{j,1}(x_j) + g_{j,1}(x_j)x_{j,2}] - b_i \dot{y}_{01}) dt \\ & + \left((d_i + b_i) \Psi_{i,1}(x_i) - \sum_{j=1}^m a_{i,j} \Psi_{j,1}(x_j) \right) dw. \end{aligned} \quad (24)$$

Select the Lyapunov candidate function as follows:

$$V_{i,1} = \frac{1}{4} s_{i,1}^4 + \frac{\varphi_1}{2r_{i,1}} \tilde{\theta}_{i,1}^2 \quad (25)$$

where $r_{i,1}$ ($i = 1, \dots, m$) is a design constant.

By (11) and (24), one gets

$$\begin{aligned} LV_{i,1} = & s_{i,1}^3 ((d_i + b_i)[g_{i,1}(s_{i,2} + \alpha_{i,1}) + f_{i,1}(x_i)] \\ & - \sum_{j=1}^m a_{i,j}[g_{j,1}(x_j)x_{j,2} + f_{j,1}(x_j)] - b_i \dot{y}_{01}) \\ & + \frac{3}{2} s_{i,1}^2 \Phi_{i,1}^T \Phi_{i,1} + \frac{\varphi_1}{r_{i,1}} \tilde{\theta}_{i,1} \dot{\hat{\theta}}_{i,1} \end{aligned} \quad (26)$$

where

$$\Phi_{i,1} = (d_i + b_i) \Psi_{i,1}(x_i) - \sum_{j=1}^m a_{i,j} \Psi_{j,1}(x_j). \quad (27)$$

By applying Assumption 4 and Young's inequality, we have

$$\begin{aligned} s_{i,1}^3 (d_i + b_i) g_{i,1} s_{i,2} & \leq \frac{3}{4} \varphi_1 s_{i,1}^4 \tau_1^{\frac{4}{3}} \\ & + \frac{1}{4} (d_i + b_i)^4 \end{aligned}$$

$$\times \varphi_1^{-3} \varphi_1^4 s_{i,2}^4 \tau_1^{-4} \quad (28)$$

$$\frac{3}{2} s_{i,1}^2 \Phi_{i,1}^T \Phi_{i,1} \leq \frac{3}{4} l_{i,1}^{-2} s_{i,1}^4 \|\Phi_{i,1}\|^4 + \frac{3}{4} l_{i,1}^2 \quad (29)$$

where $\tau_1 > 0$ and $l_{i,1} > 0$ are the design constants.

Substituting (28) and (29) into (26), it yields

$$\begin{aligned} LV_{i,1} \leq & s_{i,1}^3 [(d_i + b_i) g_{i,1} \alpha_{i,1} + \Gamma_{i,1} - b_i \dot{y}_{01}] \\ & + \frac{3}{4} \varphi_1 s_{i,1}^4 \tau_1^{\frac{4}{3}} + \frac{1}{4} (d_i + b_i)^4 \\ & \times \varphi_1^{-3} \varphi_1^4 s_{i,1}^4 \tau_1^{-4} + \frac{3}{4} l_{i,1}^2 + \frac{\varphi_1}{r_{i,1}} \tilde{\theta}_{i,1} \dot{\hat{\theta}}_{i,1} \end{aligned} \quad (30)$$

where

$$\begin{aligned} \Gamma_{i,1}(Z_{i,1}) = & (d_i + b_i) f_{i,1}(x_i) - \sum_{j=1}^m a_{i,j} g_{j,1}(x_j) x_{j,2} \\ & - \sum_{j=1}^m a_{i,j} f_{j,1}(x_j) + \frac{3}{4} l_{i,1}^{-2} \|\Phi_{i,1}\|^4 \end{aligned} \quad (31)$$

where $Z_{i,1} = [x_i^T, x_j^T]^T$. That is, for $\forall \varepsilon_{i,1} > 0$, one gets

$$\Gamma_{i,1}(Z_{i,1}) = \varepsilon_{i,1}^* + W_{i,1}^{*T} \xi_{i,1}(Z_{i,1}), \quad |\varepsilon_{i,1}^*| \leq \varepsilon_{i,1}. \quad (32)$$

Based on Young's inequality, we have

$$s_{i,1}^3 \varepsilon_{i,1}^* \leq \frac{3}{4} \varphi_1 s_{i,1}^4 \tau_1^{\frac{4}{3}} + \frac{1}{4} \varphi_1^{-3} \tau_1^{-4} \varepsilon_{i,1}^4 \quad (33)$$

Choose an intermediate virtual control signal and adaptive law as

$$\begin{aligned} \alpha_{i,1} = & \frac{1}{(d_i + b_i)} [-c_{i,1} s_{i,1} + b_i \dot{y}_{01} \\ & - \frac{\hat{\theta}_{i,1}^2 s_{i,1}^3 \|\xi_{i,1}(Z_{i,1})\|^2}{\hat{\theta}_{i,1} |s_{i,1}|^3 \|\xi_{i,1}(Z_{i,1})\| + \beta_{i,1}}] \end{aligned} \quad (34)$$

$$\dot{\hat{\theta}}_{i,1} = r_{i,1} s_{i,1}^3 \|\xi_{i,1}(Z_{i,1})\| - \beta_{i,1} \hat{\theta}_{i,1} \quad (35)$$

where $c_{i,1}$ is positive design constants.

Substituting (33)-(35) into (30) yields

$$\begin{aligned} LV_{i,1} \leq & -g_{i,1} c_{i,1} s_{i,1}^4 - \frac{\varphi_1 \hat{\theta}_{i,1}^2 s_{i,1}^6 \|\xi_{i,1}(Z_{i,1})\|^2}{\hat{\theta}_{i,1} |s_{i,1}|^3 \|\xi_{i,1}(Z_{i,1})\| + \beta_{i,1}} \\ & + s_{i,1}^3 W_{i,1}^{*T} \xi_{i,1}(Z_{i,1}) + \frac{3}{4} l_{i,1}^2 \\ & + \frac{3}{2} \varphi_1 s_{i,1}^4 \tau_1^{\frac{4}{3}} + \frac{1}{4} \varphi_1^{-3} \tau_1^{-4} \varepsilon_{i,1}^4 \\ & + \frac{1}{4} s_{i,2}^4 (d_i + b_i)^4 \varphi_1^{-3} \varphi_1^4 \tau_1^{-4} \\ & + \varphi_1 \tilde{\theta}_{i,1} |s_{i,1}|^3 \|\xi_{i,1}(Z_{i,1})\| \\ & - \frac{\varphi_1 \beta_{i,1}}{r_{i,1}} \tilde{\theta}_{i,1} \hat{\theta}_{i,1}. \end{aligned} \quad (36)$$

By applying Young's inequality, we can get

$$-\frac{\varphi_1 \beta_{i,1}}{r_{i,1}} \tilde{\theta}_{i,1} \hat{\theta}_{i,1} \leq -\frac{\varphi_1 \beta_{i,1}}{2r_{i,1}} \tilde{\theta}_{i,1}^2 + \frac{\varphi_1 \beta_{i,1}}{2r_{i,1}} \hat{\theta}_{i,1}^2. \quad (37)$$

According to Assumption 4, the following inequality holds

$$s_{i,1}^3 W_{i,1}^{*T} \xi_{i,1}(Z_{i,1}) + \varphi_1 \tilde{\theta}_{i,1} |s_{i,1}|^3 \|\xi_{i,1}(Z_{i,1})\|$$

$$\begin{aligned}
& \frac{\varphi_1 \hat{\theta}_{i,1}^2 s_{i,1}^6 \|\xi_{i,1}(Z_{i,1})\|^2}{\hat{\theta}_{i,1} |s_{i,1}|^3 \|\xi_{i,1}(Z_{i,1})\| + \beta_{i,1}} \\
\leq & \varphi_1 \left[|s_{i,1}|^3 \theta_{i,1} \|\xi_{i,1}(Z_{i,1})\| + \hat{\theta}_{i,1} |s_{i,1}|^3 \|\xi_{i,1}(Z_{i,1})\| \right. \\
& \left. - \frac{\hat{\theta}_{i,1}^2 s_{i,1}^6 \|\xi_{i,1}(Z_{i,1})\|^2}{\hat{\theta}_{i,1} |s_{i,1}|^3 \|\xi_{i,1}(Z_{i,1})\| + \beta_{i,1}} \right] \\
\leq & \varphi_1 \left[\hat{\theta}_{i,1} |s_{i,1}|^3 \|\xi_{i,1}(Z_{i,1})\| \right. \\
& \left. - \frac{\hat{\theta}_{i,1}^2 s_{i,1}^6 \|\xi_{i,1}(Z_{i,1})\|^2}{\hat{\theta}_{i,1} |s_{i,1}|^3 \|\xi_{i,1}(Z_{i,1})\| + \beta_{i,1}} \right] \\
= & \varphi_1 \frac{\hat{\theta}_{i,1} |s_{i,1}|^3 \|\xi_{i,1}(Z_{i,1})\| \beta_{i,1}}{\hat{\theta}_{i,1} |s_{i,1}|^3 \|\xi_{i,1}(Z_{i,1})\| + \beta_{i,1}} \leq \varphi_1 \beta_{i,1}. \quad (38)
\end{aligned}$$

Rewritten (36) as follows

$$\begin{aligned}
LV_{i,1} \leq & -c_{i,1}^* \varphi_1 s_{i,1}^4 - \frac{\varphi_1 \beta_{i,1}}{2r_{i,1}} \tilde{\theta}_{i,1}^2 + \tau_{i,1} \\
& + \frac{1}{4} s_{i,2}^4 (d_i + b_i)^4 \varphi_1^{-3} \bar{\varphi}_1^4 \tau_1^{-4} \quad (39)
\end{aligned}$$

where

$$\begin{aligned}
\tau_{i,1} = & \frac{3}{4} l_{i,1}^2 + \frac{\varphi_1 \beta_{i,1}}{2r_{i,1}} \theta_{i,1}^2 + \varphi_1 \beta_{i,1} \\
& + \frac{1}{4} \varphi_1^{-3} \tau_1^{-4} \varepsilon_{i,1}^4 \quad (40)
\end{aligned}$$

$$c_{i,1}^* = c_{i,1} - \frac{3}{2} \tau_1^{\frac{4}{3}}. \quad (41)$$

Step h ($2 \leq h \leq n_i - 1$): Based on (23) and Itô formula, one has

$$\begin{aligned}
ds_{i,h} = & (g_{i,h} x_{i,h+1} + f_{i,h}(x_i) - L\alpha_{i,h-1}) dt \\
& + \left(\Psi_{i,h}(x_i) - \sum_{j=1}^{h-1} \frac{\partial \alpha_{i,h-1}}{\partial x_{i,j}} \Psi_{i,j} \right)^T dw \quad (42)
\end{aligned}$$

where

$$\begin{aligned}
L\alpha_{i,h-1} = & \sum_{j=1}^{h-1} \frac{\partial \alpha_{i,h-1}}{\partial x_{i,j}} [f_{i,j}(x_i) + g_{i,j}(x_i) x_{i,j+1}] \\
& + \sum_{j=1}^{h-1} \frac{\partial \alpha_{i,h-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} + \sum_{j=0}^{h-1} \frac{\partial \alpha_{i,h-1}}{\partial y_r^{(j)}} y_r^{j+1} \\
& + \frac{1}{2} \sum_{t,o=1}^{h-1} \frac{\partial^2 \alpha_{i,h-1}}{\partial x_{i,t} \partial x_{i,o}} \Psi_{i,t}^T \Psi_{i,o}. \quad (43)
\end{aligned}$$

Choose the following Lyapunov function candidate

$$V_{i,h} = V_{i,h-1} + \frac{s_{i,h}^4}{4} + \frac{\varphi_h}{2r_{i,h}} \tilde{\theta}_{i,h}^2 \quad (44)$$

with $r_{i,h} > 0$ ($i = 1, \dots, m$) being a design parameter.

By using (11) (42) and (43), one has

$$\begin{aligned}
LV_{i,h} = & LV_{i,h-1} + s_{i,h}^3 [f_{i,h} - L\alpha_{i,h-1} \\
& + g_{i,h}(\alpha_{i,h} + s_{i,h+1})] \\
& + \frac{3}{2} s_{i,h}^2 \Phi_{i,h}^T \Phi_{i,h} + \frac{\varphi_h}{r_{i,h}} \tilde{\theta}_{i,h} \dot{\hat{\theta}}_{i,h} \quad (45)
\end{aligned}$$

where

$$\Phi_{i,h} = \Psi_{i,h}(x_i) - \sum_{j=1}^h \frac{\partial \alpha_{i,h-1}}{\partial x_j} \Psi_{j,h}. \quad (46)$$

By applying Young's inequality, we can obtain

$$g_{i,h} s_{i,h}^3 s_{i,h+1} \leq \frac{3}{4} \varphi_h \tau_h^{\frac{4}{3}} s_{i,h}^4 + \frac{1}{4} \varphi_h^{-3} \tau_h^{-4} \bar{\varphi}_h^4 s_{i,h+1}^4 \quad (47)$$

$$\frac{3}{2} s_{i,h}^2 \Phi_{i,h}^T \Phi_{i,h} \leq \frac{3}{4} l_{i,h}^{-2} s_{i,h}^4 \|\Phi_{i,h}\|^4 + \frac{3}{4} l_{i,h}^2. \quad (48)$$

where $\tau_h > 0$ is the design parameter.

Therefore, one has

$$\begin{aligned}
LV_{i,h} \leq & LV_{i,h-1} + s_{i,h}^3 (\Gamma_{i,h} + g_{i,h} \alpha_{i,h}) \\
& - \frac{1}{4} s_{i,h}^4 \left(\check{d}_i + \check{b}_i \right)^4 \varphi_{h-1}^{-3} \bar{\varphi}_{h-1}^4 \tau_{h-1}^{-4} \\
& + \frac{3}{4} \varphi_h \tau_h^{\frac{4}{3}} s_{i,h}^4 + \frac{1}{4} \varphi_h^{-3} \tau_h^{-4} \bar{\varphi}_h^4 s_{i,h+1}^4 \\
& + \frac{3}{4} l_{i,h}^2 + \frac{\varphi_h}{r_{i,h}} \tilde{\theta}_{i,h} \dot{\hat{\theta}}_{i,h} \quad (49)
\end{aligned}$$

where

$$\begin{aligned}
\Gamma_{i,h}(Z_{i,h}) = & f_{i,h} - L\alpha_{i,h-1} + \frac{3}{4} l_{i,h}^{-2} s_{i,h} \|\Phi_{i,h}\|^4 \quad (50) \\
& + \frac{1}{4} s_{i,h} \left(\check{d}_i + \check{b}_i \right)^4 \varphi_{h-1}^{-3} \bar{\varphi}_{h-1}^4 \tau_{h-1}^{-4}
\end{aligned}$$

where $Z_{i,h} = [x_i^T, x_j^T, \hat{\theta}_{i,1}, \dots, \hat{\theta}_{i,h-1}]^T$, and for $k = 2$, take $\check{d}_i + \check{b}_i = d_i + b_i$, and for $3 \leq k \leq n_i - 1$, take $\check{d}_i + \check{b}_i = 1$.

Similarly, the FNN is applied to estimate the unknown function $\Gamma_{i,h}(Z_{i,h})$, that is, for $\forall \varepsilon_{i,h} > 0$, one has

$$\Gamma_{i,h}(Z_{i,h}) = \varepsilon_{i,h}^* + W_{i,h}^{*T} \xi_{i,h}(Z_{i,h}), \quad |\varepsilon_{i,h}^*| \leq \varepsilon_{i,h}. \quad (51)$$

By using Young's inequality and (51), one gets

$$s_{i,h}^3 \varepsilon_{i,h}^* \leq \frac{3}{4} \varphi_h \tau_h^{\frac{4}{3}} s_{i,h}^4 + \frac{1}{4} \varphi_h^{-3} \tau_h^{-4} \varepsilon_{i,h}^4. \quad (52)$$

We select the intermediate virtual control law as

$$\alpha_{i,h} = -c_{i,h} s_{i,h} - \frac{\hat{\theta}_{i,h}^2 s_{i,h}^3 \|\xi_{i,h}(Z_{i,h})\|^2}{\hat{\theta}_{i,h} |s_{i,1}|^3 \|\xi_{i,h}(Z_{i,h})\| + \beta_{i,h}} \quad (53)$$

where $c_{i,h} > 0$ denotes the design parameter.

The parameter adaptive law is chosen as

$$\dot{\hat{\theta}}_{i,h} = r_{i,h} |s_{i,1}|^3 \|\xi_{i,h}(Z_{i,h})\| - \beta_{i,h} \hat{\theta}_{i,h} \quad (54)$$

with $r_{i,h} > 0$ being a design constant.

Substituting (52)-(54) into (49), it yields

$$\begin{aligned}
LV_{i,h} \leq & LV_{i,h-1} + s_{i,h}^3 W_{i,h}^{*T} \xi_{i,h}(Z_{i,h}) + \frac{1}{4} \varphi_h^{-3} \tau_h^{-4} \varepsilon_{i,h}^4 \\
& - \frac{1}{4} s_{i,h}^4 \left(\check{d}_i + \check{b}_i \right)^4 \varphi_{h-1}^{-3} \bar{\varphi}_{h-1}^4 \tau_{h-1}^{-4} \\
& - \varphi_h c_{i,h} s_{i,h}^4 - \frac{\varphi_h \beta_{i,h}}{r_{i,h}} \tilde{\theta}_{i,h} \dot{\hat{\theta}}_{i,h} \\
& - \frac{\varphi_h \hat{\theta}_{i,h}^2 s_{i,h}^6 \|\xi_{i,h}(Z_{i,h})\|^2}{\hat{\theta}_{i,h} |s_{i,1}|^3 \|\xi_{i,h}(Z_{i,h})\| + \beta_{i,h}} \\
& + \frac{3}{2} \varphi_h \tau_h^{\frac{4}{3}} s_{i,h}^4 + \frac{1}{4} \varphi_h^{-3} \tau_h^{-4} \bar{\varphi}_h^4 s_{i,h+1}^4 \\
& + \frac{3}{4} l_{i,h}^2 + \varphi_h \tilde{\theta}_{i,h} |s_{i,1}|^3 \|\xi_{i,h}(Z_{i,h})\|. \quad (55)
\end{aligned}$$

Based on Young's inequality, one gets

$$-\frac{\varphi_h \beta_{i,h}}{r_{i,h}} \tilde{\theta}_{i,h} \dot{\hat{\theta}}_{i,h} \leq -\frac{\varphi_h \beta_{i,h}}{2r_{i,h}} \tilde{\theta}_{i,h}^2 + \frac{\varphi_h \beta_{i,h}}{2r_{i,h}} \theta_{i,h}^2. \quad (56)$$

Then, similar to formula (38), (55) can be rewritten as

$$LV_{i,h} \leq -\sum_{j=1}^h c_{i,j}^* \varrho_j s_{i,j}^4 - \sum_{j=1}^h \frac{\varrho_j \beta_{i,h}}{2r_{i,h}} \tilde{\theta}_{i,h}^2 + \frac{1}{4} \varrho_h^{-3} \tau_h^{-4} \bar{\varphi}_h^4 s_{i,h+1}^4 + \tau_{i,h} \quad (57)$$

where

$$\tau_{i,h} = \tau_{i,h-1} + \frac{3}{4} l_{i,h}^2 + \frac{\beta_{i,h}}{2r_{i,h}} \theta_{i,h}^2 + \frac{1}{4} \varrho_h^{-3} \tau_h^{-4} \varepsilon_{i,h}^4 + \varrho_h \beta_{i,h} \quad (58)$$

$$c_{i,h}^* = c_{i,h} - \frac{3}{2} \tau_h^{\frac{4}{3}}. \quad (59)$$

Step n_i : In this step, we will structure a real controller. Based on Itô formula and (23), we have

$$ds_{i,n_i} = (g_{i,n_i} (p_{v_i} v_{n_i} + q(v_{n_i})) + f_{i,n_i}(x_i) - L\alpha_{i,n_i-1}) dt + \left(\Psi_{i,n_i}(x_i) - \sum_{j=1}^{n_i-1} \frac{\partial \alpha_{i,n_i-1}}{\partial x_{i,j}} \Psi_{i,j} \right) dw \quad (60)$$

where $L\alpha_{i,n_i-1}$ is defined in (43) with $h = n_i$.

Select the Lyapunov function candidate as

$$V_{i,n_i} = V_{i,n_i-1} + \frac{1}{4} s_{i,n_i}^4 + \frac{\varrho_{n_i}}{2r_{i,n_i}} \tilde{\theta}_{i,n_i}^2 \quad (61)$$

where r_{i,n_i} ($i = 1, \dots, m$) is a design parameter, by using (11) and (60), one has

$$LV_{i,n_i} \leq LV_{i,n_i-1} + s_{i,n_i}^3 (f_{i,n_i} + g_{i,n_i} (p_{v_i} v_i + q(v_{n_i})) - L\alpha_{i,n_i-1}) + \frac{3}{2} \varepsilon_{i,n_i}^2 \Phi_{i,n_i}^T \Phi_{i,n_i} + \frac{\varrho_{n_i}}{r_{i,n_i}} \tilde{\theta}_{i,n_i} \dot{\theta}_{i,n_i}. \quad (62)$$

Based on Young's inequality, one can get

$$\frac{3}{2} \varepsilon_{i,n_i}^2 \Phi_{i,n_i}^T \Phi_{i,n_i} \leq \frac{3}{4} l_{i,n_i}^{-2} s_{i,n_i}^4 \|\Phi_{i,n_i}\|^4 + \frac{3}{4} l_{i,n_i}^2 \quad (63)$$

$$s_{i,n_i}^3 g_{i,n_i} q(v_{n_i}) \leq \frac{3}{4d^2} g_{i,n_i} s_{i,n_i}^4 + \frac{1}{4} d^2 \bar{\varphi}_{n_i} D^4 \quad (64)$$

with c_{i,n_i} , r_{i,n_i} and β_{i,n_i} being design constant.

Therefore, we have

$$LV_{i,n_i} \leq LV_{i,n_i-1} + s_{i,n_i}^3 (\Gamma_{i,n_i} + g_{i,n_i} p_{v_i} v_i) - \frac{\varrho_{n_i} \hat{\theta}_{i,n_i}^2 s_{i,n_i}^6 \|\xi_{i,n_i}(Z_{i,n_i})\|^2}{\hat{\theta}_{i,n_i} |s_{i,n_i}^3| \|\xi_{i,n_i}(Z_{i,n_i})\| + \beta_{i,n_i}} - \frac{1}{4} \varrho_{n_i-1}^{-3} \tau_{n_i-1}^{-4} \bar{\varphi}_{n_i-1}^4 s_{i,n_i}^4 + \frac{1}{4} d^2 \bar{\varphi}_{n_i} D^4 + \frac{\varrho_{n_i}}{r_{i,n_i}} \tilde{\theta}_{i,n_i} \dot{\theta}_{i,n_i} - \frac{3}{4} g_{i,n_i} s_{i,n_i}^4 + \frac{3}{4} l_{i,n_i}^2 \quad (65)$$

where

$$\Gamma_{i,n_i}(Z_{i,n_i}) = f_{i,n_i} + \frac{3}{4d^2} g_{i,n_i} s_{i,n_i}^4 + \frac{1}{4} \varrho_{n_i-1}^{-3} \tau_{n_i-1}^{-4} \bar{\varphi}_{n_i-1}^4 s_{i,n_i}$$

$$-L\alpha_{i,n_i-1} + \frac{3}{4} l_{i,n_i}^{-2} s_{i,n_i} \|\Phi_{i,n_i}\|^4 \quad (66)$$

where $Z_{i,n_i} = [x_i^T, x_j^T, \hat{\theta}_{i,1}, \dots, \hat{\theta}_{i,n_i-1}]^T$.

Using Young's inequality

$$s_{i,n_i}^3 \varepsilon_{i,n_i}^* \leq \frac{3}{4} \varrho_{n_i} \tau_{n_i}^{\frac{4}{3}} s_{i,n_i}^4 + \frac{1}{4} \varrho_{n_i}^{-3} \tau_{n_i}^{-4} \varepsilon_{i,n_i}^4. \quad (67)$$

where $\tau_{n_i} > 0$ is the design parameter.

Construct the actual controller and parameter adaptive law as follows

$$v_i = -c_{i,n_i} s_{i,n_i} - \frac{\hat{\theta}_{i,n_i}^2 s_{i,n_i}^3 \|\xi_{i,n_i}(Z_{i,n_i})\|^2}{\hat{\theta}_{i,n_i} |s_{i,n_i}^3| \|\xi_{i,n_i}(Z_{i,n_i})\| + \beta_{i,n_i}} \quad (68)$$

$$\dot{\hat{\theta}}_{i,n_i} = r_{i,n_i} s_{i,n_i}^3 \|\xi_{i,n_i}(Z_{i,n_i})\| - \beta_{i,n_i} \hat{\theta}_{i,n_i}. \quad (69)$$

Due to the fact that the actuator fault is considered in this paper, $v_i = \rho(t_r, t) v_{di} + \delta_d(t - t_f) \lambda_d(t)$ is achieved. When the actuator is healthy, we can obtain $\rho(t_r, t) = 1$ and $\delta_d(t - t_f) \lambda_d(t) = 0$, i.e. $v_i = v_{di}$.

When the actuator fails, the fault-tolerant controller is designed as follows $v_{di} = \frac{1}{\rho(t_r, t)} (v_i - \delta_d(t - t_f) \lambda_d(t))$ i.e.,

$$v_{di} = \frac{1}{\rho(t_r, t)} (-c_{i,n_i} s_{i,n_i} - \delta_d(t - t_f) \lambda_d(t) - \frac{\hat{\theta}_{i,n_i}^2 s_{i,n_i}^3 \|\xi_{i,n_i}(Z_{i,n_i})\|^2}{\hat{\theta}_{i,n_i} |s_{i,n_i}^3| \|\xi_{i,n_i}(Z_{i,n_i})\| + \beta_{i,n_i}}). \quad (70)$$

Substituting (67)-(69) into (65), one has

$$LV_{i,n_i} \leq LV_{i,n_i-1} + s_{i,n_i}^3 W_{i,n_i}^{*T} \xi_{i,n_i}(Z_{i,n_i}) - \varrho_{n_i} c_{i,n_i} s_{i,n_i}^4 + \frac{1}{4} \varrho_{n_i}^{-3} \tau_{n_i}^{-4} \varepsilon_{i,n_i}^4 - \frac{\varrho_{n_i} \hat{\theta}_{i,n_i}^2 s_{i,n_i}^6 \|\xi_{i,n_i}(Z_{i,n_i})\|^2}{\hat{\theta}_{i,n_i} |s_{i,1}|^3 \|\xi_{i,n_i}(Z_{i,n_i})\| + \beta_{i,n_i}} + \frac{3}{4} \varrho_{n_i} \tau_{n_i}^{\frac{4}{3}} s_{i,n_i}^4 - \frac{1}{4} \varrho_{n_i-1}^{-3} \tau_{n_i-1}^{-4} \bar{\varphi}_{n_i-1}^4 s_{i,n_i}^4 + \frac{3}{4} l_{i,n_i}^2 + \varrho_{n_i} \tilde{\theta}_{i,n_i} |s_{i,1}|^3 \|\xi_{i,n_i}(Z_{i,n_i})\| - \frac{\varrho_{n_i} \beta_{i,n_i}}{r_{i,n_i}} \tilde{\theta}_{i,n_i} \dot{\theta}_{i,n_i} + \frac{1}{4} d^2 \bar{\varphi}_{n_i} D^4. \quad (71)$$

Applying Young's inequality, we have

$$-\frac{\varrho_{n_i} \beta_{i,n_i}}{r_{i,n_i}} \tilde{\theta}_{i,n_i} \dot{\theta}_{i,n_i} \leq -\frac{\varrho_{n_i} \beta_{i,n_i}}{2r_{i,n_i}} \tilde{\theta}_{i,n_i}^2 + \frac{\varrho_{n_i} \beta_{i,n_i}}{2r_{i,n_i}} \theta_{i,n_i}^2. \quad (72)$$

Further, similar to formula (38), (71) can be rewritten as

$$LV_{i,n_i} \leq -\sum_{j=1}^{n_i} c_{i,j}^* \varrho_j s_{i,j}^4 - \sum_{j=1}^{n_i} \frac{\varrho_j \beta_{i,j}}{2r_{i,j}} \tilde{\theta}_{i,j}^2 + \tau_{i,n_i} \quad (73)$$

where

$$\tau_{i,n_i} = \sum_{j=1}^{n_i} \left(\frac{3}{4} l_{i,j}^2 + \frac{\varrho_j \beta_{i,j}}{2r_{i,j}} \theta_{i,j}^2 + \frac{1}{4} \varrho_j^{-3} \tau_j^{-4} \varepsilon_{i,j}^4 + \varrho_j \beta_{i,j} \right) + \frac{1}{4} d^2 \bar{\varphi}_{n_i} D^4 \quad (74)$$

$$c_{i,n_i}^* = p_m c_{i,n_i} - \frac{3}{4} \tau_{n_i}^{\frac{4}{3}}. \quad (75)$$

Let

$$\eta_i = \min \{2c_{i,h}^*, \beta_{i,h}\} > 0, \quad h = 1, \dots, n_i \quad (76)$$

then we have

$$LV_{i,n_i} \leq -\eta_i V_{i,n_i} + \tau_i. \quad (77)$$

In the case of the healthy state or fault state of the actuator, the following theorem gives the stability analysis of the proposed control protocol.

Theorem 1: Consider the non-strict feedback stochastic nonlinear multi-agent systems (1) under Assumptions 1-4. We give the distributed control protocol (68) (healthy controller), (70) (fault-tolerant controller) and the adaptive law (69) to ensure that all the followers' outputs eventually converge to a small neighborhood of the leader's output and all signals are bounded in the closed-loop system (1) in probability.

Proof 1: Choose the total Lyapunov function candidate as

$$V = \sum_{i=1}^m V_{i,n_i}(t). \quad (78)$$

Based on (77), we have

$$LV \leq -\eta V + \tau \quad (79)$$

where $\eta = \min \{\eta_i, i = 1, \dots, m\}$, $\tau = \sum_{i=1}^m \tau_i$.

According to Lemma 3 and the definition of V , all signals in the closed-loop systems are CSUUB in probability.

Furthermore, according to [47] and (79), when $t \geq 0$, one gets

$$\frac{d(E[V(t)])}{dt} = E[LV(t)] \leq -\eta E[V(t)] + \tau. \quad (80)$$

From (80), we have

$$0 \leq E[V(t)] \leq \frac{\tau}{\eta} + \left(V(0) - \frac{\tau}{\eta}\right) e^{-\eta t}. \quad (81)$$

Further, when $t \rightarrow \infty$, we can get

$$E[V(t)] \leq \frac{\tau}{\eta}. \quad (82)$$

For $s_{i,1} = (s_{1,1}, \dots, s_{m,1})^T$, according to (82) and the definition of V , we obtain

$$\begin{aligned} E\left(\|s_{i,1}\|^4\right) &\leq E\left(s_{1,1}^2 + s_{2,1}^2 \dots, s_{m,1}^2\right)^2 \\ &\leq 2E\left(s_{1,1}^4 + s_{2,1}^4 \dots, s_{m,1}^4\right) \\ &\leq \frac{8\tau}{\eta}. \end{aligned} \quad (83)$$

Theoretically, for $\forall \varepsilon > 0$, on account of the definitions of η and τ , by choosing the appropriate design parameters $c_{i,h}$, $\beta_{i,h}$ and $a_{i,h}$, one has

$$\frac{\tau}{\eta} \leq \frac{\varepsilon}{8} (\Delta(L + \iota))^4. \quad (84)$$

Furthermore, when $t \rightarrow \infty$, according to Lemma 3, one gets

$$E\left(\|y - y_{01}\|^4\right) \leq E\left(\|s_{i,1}\|^4\right) / \Delta(L + \iota)^4 \leq \varepsilon. \quad (85)$$

According to Definition 2, the tracking errors between the output of followers $y_i(t)$ and the leader's output $y_{01}(t)$ are CSUUB in probability. ■

IV. SIMULATION RESULTS

To illustrate the effectiveness of the developed control protocol, we consider the following networks of stochastic nonlinear systems

$$\begin{aligned} dx_{i,1} &= [x_{i,1} + x_{i,2} \sin(x_{i,1})] dt + [0.2x_{i,1} \sin(x_{i,1})] dw \\ dx_{i,2} &= [0.8 \sin(x_{i,2}) + 0.5 \sin(t) + u_i] dt \\ &\quad + [1 - \cos(x_{i,2}^2)] dw \\ y_i &= x_{i,1} \end{aligned} \quad (86)$$

where u_i represents the output of the saturation nonlinearity, the nonsymmetric input saturation bounds are selected as $u_{i,\max} = 5$, $u_{i,\min} = -6$. The output of the leader is $y_{01} = 0.5 \cos(t)$.

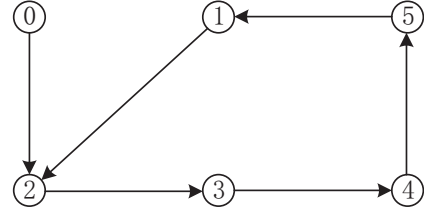


Fig. 2. Graph $\bar{\mathcal{G}}$ used in the simulation

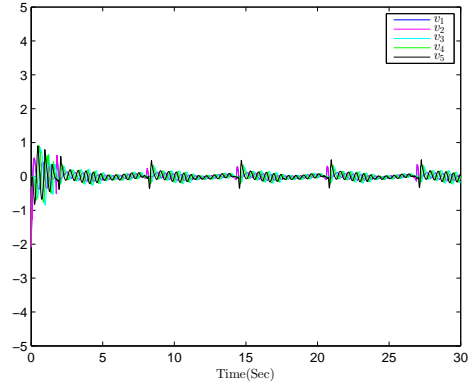


Fig. 3. Healthy Controller

Apparently, the adjacency matrix \mathcal{A} and the Laplacian matrix L of the digraph \mathcal{G} are, respectively.

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

We consider the following faults model $v_i = \rho(t_r, t) v_{di} + \delta_d(t - t_f) \lambda_d(t)$, when the actuator is healthy $\rho(t_r, t) = 1$, $\delta_d(t - t_f) \lambda_d(t) = 0$, i.e., $v_{di} = v_i$. When the actuator is abrupt faults $0 < \rho(t_r, t) < 1$, $\lambda_d(t) = 0.05 \cos(t)$, i.e., $v_{di} = \frac{1}{\rho(t_r, t)} [v_i - 0.05 \cos(t)]$ ($t_r > 5$). When the actuator is incipient fault $0 < \rho(t_r, t) < 1$, $\delta_d(t - t_f) = 1 - e^{-2(t-5)}$, $\lambda_d(t) = 0.05 \cos(t)$. i.e., $v_{di} =$

$\frac{1}{\rho(t_r, t)} [v_i - (1 - e^{-2(t-5)}) 0.05 \cos(t)] (t_r > 5)$. Figs. 3-5 show the simulation results of health actuator, actuator with initial faults and actuator with abrupt faults, respectively.

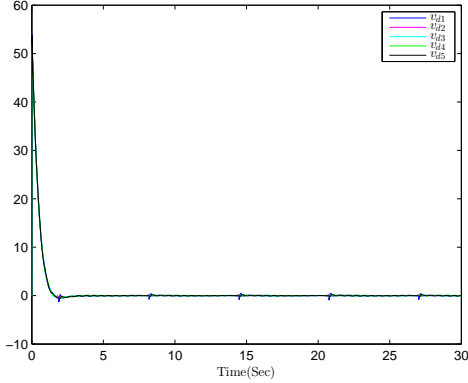


Fig. 4. Controller with incipient faults

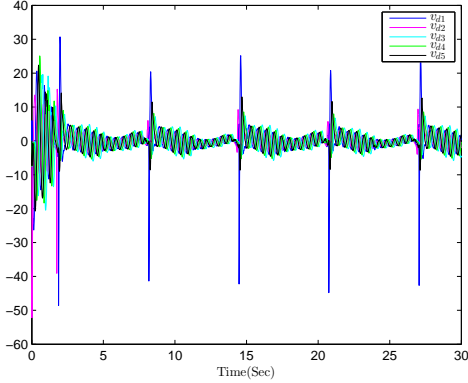


Fig. 5. Controller with abrupt faults

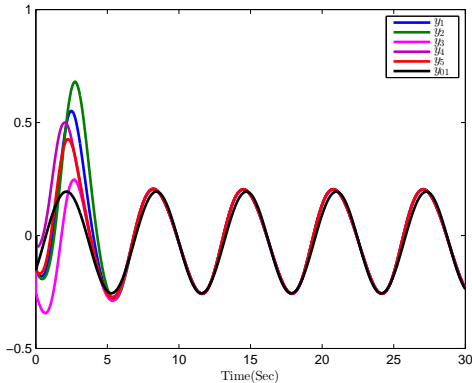


Fig. 6. Output of the leader and followers.

Choose the following initial values and adaptive parameters $x_{i,1}(0) = (-0.15, -0.05)^T$, $x_{i,2}(0) = (-0.15, -0.15)^T$, $x_{i,3}(0) = (-0.25, -0.01)^T$, $x_{i,4}(0) = (-0.05, -0.25)^T$, $x_{i,5}(0) = (-0.15, -0.13)^T$. $\hat{\theta}_{i,j}(0) = (0.2, 0.2)^T$. $c_{i,j} = 0.15$. ($i = 1, \dots, 5$, $j = 1, 2$) $r_{1,1} = 1.5$, $r_{2,1} = 0.5$, $r_{1,2} =$

$r_{2,2} = r_{3,1} = r_{3,2} = r_{4,1} = r_{4,2} = r_{5,1} = r_{5,2} = 0.1$. Fig. 6 denotes that output trajectories of followers can track the output trajectories of leader.

In addition, we define the following membership functions:

$$\begin{aligned}\Lambda_{A_i^1}(Z_i^n) &= \exp[-0.5(Z_i^n + 5)^2] \\ \Lambda_{A_i^2}(Z_i^n) &= \exp[-0.5(Z_i^n + 2)^2] \\ \Lambda_{A_i^3}(Z_i^n) &= \exp[-0.5(Z_i^n)^2] \\ \Lambda_{A_i^4}(Z_i^n) &= \exp[-0.5(Z_i^n - 2)^2] \\ \Lambda_{A_i^5}(Z_i^n) &= \exp[-0.5(Z_i^n - 5)^2]\end{aligned}$$

From these figures, we see that these simulation results validate the effectiveness of the proposed control protocol.

V. CONCLUSION

Fuzzy neural network-based adaptive fault-tolerant control protocol has been studied for the networks of stochastic nonlinear systems with nonsymmetric input saturation and actuator faults in this paper. Based on the FNNs approximation property and the backstepping technique, a novel adaptive fault-tolerant control protocol has been designed to guarantee that in the closed loop systems all the signals are bounded in probability and the tracking error converges to an arbitrarily small neighborhood around the origin. At last, these simulation results have been presented to illustrate the effectiveness of the proposed control protocol. In our future work, the T-S fuzzy-model-based [48] and [49] cooperative control would be an interesting and challenging and will be considered.

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