

# Cooperative game theory and the Gaussian interference channel

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**Abstract**— In this paper we discuss the use of cooperative game theory for analyzing interference channels. We extend our previous work, to games with  $N$  players as well as frequency selective channels and joint TDM/FDM strategies. We show that the Nash bargaining solution can be computed using convex optimization techniques. We also show that the same results are applicable to interference channels where only statistical knowledge of the channel is available. Moreover, for the special case of two player  $2 \times K$  frequency selective channel (with  $K$  frequency bins) we provide an  $O(K \log_2 K)$  complexity algorithm for computing the Nash bargaining solution under mask constraint and using joint FDM/TDM strategies. Simulation results are also provided.

**Keywords:** Spectrum optimization, distributed coordination, game theory, Nash bargaining solution, interference channel, multiple access channel.

## I. INTRODUCTION

Computing the capacity region of the interference channel is an open problem in information theory [2]. A good overview of the results until 1985 is given by van der Meulen [3] and the references therein. The capacity region of general interference channel is not known yet. However, in the last forty five years of research some progress has been made. Ahlswede [4], derived a general formula for the capacity region of a discrete memoryless Interference Channel (IC) using a limiting expression which is computationally infeasible. Cheng, and Verdu [5] proved that the limiting expression cannot be written in general by a single-letter formula and the restriction to Gaussian inputs provides only an inner bound to the capacity region of the IC. The best known achievable region for the general interference channel is due to Han and Kobayashi [6]. However the computation of the Han and Kobayashi formula for a general discrete memoryless channel is in general too complex. Sason [7] describes certain simplification of the Han Kobayashi rate region in certain cases. A  $2 \times 2$  Gaussian interference channel in standard form (after suitable normalization) is given by:

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n}, \quad \mathbf{H} = \begin{bmatrix} 1 & \alpha \\ \beta & 1 \end{bmatrix} \quad (1)$$

where,  $\mathbf{s} = [s_1, s_2]^T$ , and  $\mathbf{x} = [x_1, x_2]^T$  are sampled values of the input and output signals, respectively. The noise vector  $\mathbf{n}$  represents the additive Gaussian noises with zero mean and unit variance. The powers of the input signals are constrained to be less than  $P_1, P_2$  respectively. The off-diagonal elements of  $\mathbf{H}$ ,  $\alpha, \beta$  represent the degree of interference present. The

capacity region of the Gaussian interference channel with very strong interference (i.e.,  $\alpha \geq 1 + P_1, \beta \geq 1 + P_2$ ) is given by [8]

$$R_i \leq \log_2(1 + P_i), \quad i = 1, 2. \quad (2)$$

This surprising result shows that very strong interference does not reduce the capacity. A Gaussian interference channel is said to have strong interference if  $\min\{\alpha, \beta\} > 1$ . Sato [9] derived an achievable capacity region (inner bound) of Gaussian interference channel as intersection of two multiple access Gaussian capacity regions embedded in the interference channel. The achievable region is the intersection of the rate pair of the rectangular region of the very strong interference (2) and the region

$$R_1 + R_2 \leq \log_2(\min\{1 + P_1 + \alpha P_2, 1 + P_2 + \beta P_1\}). \quad (3)$$

A recent progress for the case of Gaussian interference is described by Sason [7]. Sason derived an achievable rate region based on a modified time- (or frequency-) division multiplexing approach which was originated by Sato for the degraded Gaussian IC. The achievable rate region includes the rate region which is achieved by time/frequency division multiplexing (TDM/ FDM), and it also includes the rate region which is obtained by time sharing between the two rate pairs where one of the transmitters sends its data reliably at the maximal possible rate (i.e., the maximum rate it can achieve in the absence of interference), and the other transmitter decreases its data rate to the point where both receivers can reliably decode their messages.

While the two user flat interference channel is a well studied problem, much less is known in the frequency selective case. An  $N \times N$  frequency selective Gaussian interference channel is given by:

$$\mathbf{x}_k = \mathbf{H}_k \mathbf{s}_k + \mathbf{n}_k \quad k = 1, \dots, K$$

$$\mathbf{H}_k = \begin{bmatrix} h_{11}(k) & \dots & h_{1N}(k) \\ \vdots & \ddots & \vdots \\ h_{N1}(k) & \dots & h_{NN}(k) \end{bmatrix}. \quad (4)$$

where,  $\mathbf{s}_k$ , and  $\mathbf{x}_k$  are sampled values of the input and output signal vectors at frequency  $k$ , respectively. The noise vector  $\mathbf{n}_k$  represents the additive Gaussian noises with zero mean and unit variance. The power spectral density (PSD) of the input signals are constrained to be less than  $p_1(k), p_2(k)$  respectively. The off-diagonal elements of  $\mathbf{H}_k$ , represent the degree of interference present at frequency  $k$ . The main difference between interference channel and a multiple access channel (MAC) is that in the interference channel, each component of  $\mathbf{s}_k$  is coded independently, and each receiver has access to a

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single element of  $\mathbf{x}_k$ . Therefore, iterative decoding schemes are much more limited, and typically impractical.

One of the simplest ways to deal with interference limited channels is through orthogonal signaling. Two extremely simple orthogonal schemes are using FDM or TDM strategies. These techniques allow a single user detection (which will be assumed throughout this paper) without the need to complicated multi-user detection. The loss of these techniques compared to techniques requiring joint decoding has been thoroughly studied, e.g., [8] showing degradation compared to the techniques requiring joint or sequential decoding. However, the widespread use of FDMA/TDMA as well as collision avoidance medium access control (CSMA) techniques, make the analysis of these techniques very important from practical point of view as well. For frequency selective channels (also known as ISI channels) we can combine both strategies by allowing time varying allocation of the frequency bins to the different users. In this paper we limit ourselves to joint FDM and TDM scheme where an assignment of disjoint portions of the frequency band to the several transmitters is made at each time instance. This technique is widely used in practice because simple filtering can be used at the receivers to eliminate interference. The main results in this paper were derived under a PSD mask limitation (peak power at each frequency) since this constraint is typically enforced by regulators. In contrast total power constraints are technology dependent and emerge from practical limitations as well as economic limitations on power amplifiers. Hence, studying PSD mask constraint has great practical value. Furthermore, in bandwidth limited applications where SINR is high, the value of spectral shaping is low and using a flat PSD mask can be very close to optimal.

While information theoretical considerations allow all points in the rate region, we argue that the interference channel is a conflict situation between the interfering links [1]. Each link is considered a player in a general interference game. Therefore, the non-cooperative solutions such as the iterative water-filling [10], which leads to good solutions for the multiple access channel (MAC) and the broadcast channel [11] can be highly suboptimal in interference channel scenarios [12], [13]. To solve this problem there are several possible approaches. One that has gained popularity in recent years is through the use of competitive strategies in repeated games [14]. Other solutions are by regulatory type of solution [15] where certain users are protected, or by changing the rules of the game by imposing pricing mechanisms [16],[17]. Our approach is significantly different and is based on general bargaining theory originally developed by Nash [18]. Our approach is also different than that of [19] where Nash bargaining solution for interference channels is studied under the assumption of receiver cooperation. This translates the channel into a MAC, and is not relevant to distributed receiver topologies. In our analysis of the interference channel we claim that while all points on the boundary of the interference channel are achievable from the strict informational point of view, many of them will never be achieved since one of the players will refuse to use coding strategies leading to these points. The rate vectors of interest are only rate vectors that dominate component-wise

the rates that each user can achieve, independently of the other users coding strategy. The best rate pairs that can be achieved independently of the other users' strategies form a Nash equilibrium [20]. This implies that not all the rates are indeed achievable from game theoretic perspective. Hence, we define the game theoretic rate region.

*Definition 1.1:* Let  $\mathcal{R}$  be an achievable information theoretic rate region. The game theoretic rate region  $\mathcal{R}^G$  is given by

$$\mathcal{R}^G = \{(R_1, \dots, R_N) \in \mathcal{R} : R_i^C \leq R_i, \forall i = 1, \dots, N\} \quad (5)$$

where  $R_i^C$  is the rate achievable by user  $i$  in a non-cooperative interference game [13].

To see what are the pair rates that can be achieved by negotiation and cooperation of the users we resort to a well known solution termed the Nash bargaining solution. In his seminal papers, Nash proposed four axioms [18],[21] that any solution to the bargaining problem should satisfy. He then proved that there exists a unique solution satisfying these axioms. We will analyze the application of Nash bargaining solution (NBS) to the interference game, and show that there exists a unique point on the boundary of the capacity region which is the solution to the bargaining problem as posed by Nash.

The fact that the Nash solution can be computed independently by the users, exchanging only channel state distributions, provides a good method for managing multi-user ad-hoc networks operating in an unregulated environments.

Application of Nash bargaining to OFDMA has been proposed by [22]. However in that paper the solution was used only as a measure of fairness. Therefore,  $R_i^C$  was not taken as the Nash equilibrium for the competitive game, but an arbitrary  $R_i^{\min}$ . This can result in non-feasible problem, and the proposed algorithm might be unstable. The algorithm in [22] is suboptimal even in the two user case, and according to the authors can lead to an unstable situation, where the Nash bargaining solution is not achieved even when it exists. In contrast, in this paper we show that the NBS for the  $N$  player game can be computed using convex optimization techniques. We also provide detailed analysis of the two user case and provide an  $O(K \log_2 K)$  complexity algorithm which provably achieves the joint FDM/TDM Nash bargaining solution. Our analysis provides ensured convergence for higher number of users and bounds the loss in applying OFDMA compared to joint FDM/TDM strategies. In the two user case we can show that the Nash bargaining solution requires TDM over no more than a single tone, so we can achieve a very good approximation to the optimal FDM based Nash bargaining solution. We also provide similar analysis for higher number of users, showing that for the Nash bargaining solution with  $N$  players, over a frequency selective channel with  $K$  frequency bins, only  $\binom{N}{2}$  frequency bins has to be shared by TDM, while all other frequencies are allocated to a single user. When  $\binom{N}{2} \ll K$ , this provides a near optimal solution to the game using FDM strategies, as well.

The structure of the paper is as follows: In section II we discuss competitive and cooperative solutions to frequency selective interference games and provide an overview of the

Nash bargaining theory. In section III we discuss the existence of the NBS for  $N$  player FDM cooperative game over slow, flat fading channels. In section IV we discuss the Nash bargaining over general frequency selective interference channel, with PSD mask constraint. We show that computing the NBS under mask constraint and joint FDM/TDM strategies can be posed as a convex optimization problem. This shows that even for large number of players, computing the solution with many tones is feasible. We also show that in this case the  $N$  users will share only few frequencies, dividing all the others. In section V we specialize to the frequency selective two players case. We provide an algorithm for computing the NBS in complexity  $O(K \log_2(K))$ . Finally, we demonstrate in simulations the gains compared to the competitive solution both in the flat fading and the frequency selective cases. We end up with some conclusions.

## II. NASH EQUILIBRIUM VS. NASH BARGAINING SOLUTION

In this section we describe two solution concepts for  $N$  player games. The first notion is that of Nash equilibrium. The second is the Nash bargaining solution (NBS). In order to simplify the notation we specifically concentrate on the Gaussian interference game. In this paper we use three models: Frequency selective with total power constraint, flat frequency response and frequency selective channels with PSD mask constraints.

### A. The Gaussian interference game

In this section we define the Gaussian interference game under total power constraint, and provide some simplifications for dealing with discrete frequencies. For a general background on non-cooperative games we refer the reader to [20]. The Gaussian interference game was defined in [23]. In this paper we use the discrete approximation game. Let  $f_0 < \dots < f_K$  be an increasing sequence of frequencies. Let  $I_k$  be the closed interval be given by  $I_k = [f_{k-1}, f_k]$ . We now define the approximate Gaussian interference game denoted by  $GI_{\{I_1, \dots, I_K\}}$ .

Let the players  $1, \dots, N$  operate over  $K$  parallel frequency channels. Assume that the  $N^2$  frequency selective cross channels between  $j$ 'th transmitter and  $i$ 'th receiver have transfer functions  $\langle h_{ij}(k) : k = 1, \dots, K \rangle$ . Assume that user  $i$  is allowed to transmit a total power of  $P_i$ . Each player can transmit a power vector  $\mathbf{p}_i = (p_i(1), \dots, p_i(K)) \in [0, P_i]^K$  such that  $p_i(k)$  is the power transmitted in the interval  $I_k$ . Therefore, we have  $\sum_{k=1}^K p_i(k) = P_i$ . The equality follows from the fact that in non-cooperative scenario all users will use the maximal power they can use. This implies that the set of power distributions for all users is a closed convex subset of the cube  $\prod_{i=1}^N [0, P_i]^K$  given by:

$$B = \prod_{i=1}^N B_i \quad (6)$$

where  $B_i$  is the set of admissible power distributions for player  $i$  given by:

$$B_i = [0, P_i]^K \cap \left\{ (p(1), \dots, p(K)) : \sum_{k=1}^K p(k) = P_i \right\}. \quad (7)$$

Each player chooses a PSD  $\mathbf{p}_i = \langle p_i(k) : 1 \leq k \leq N \rangle \in B_i$ . Let the payoff for user  $i$  be given by:

$$C^i(\mathbf{p}_1, \dots, \mathbf{p}_N) = \sum_{k=1}^K \log_2(1 + \text{SINR}_i(k)) \Delta f_k \quad (8)$$

where

$$\text{SINR}_i(k) = \frac{|h_{ii}(k)|^2 p_i(k)}{\sum |h_{ij}(k)|^2 p_j(k) + \sigma_i^2(k)}, \quad (9)$$

$C^i$  is the capacity available to player  $i$  given power distributions  $\mathbf{p}_1, \dots, \mathbf{p}_N$ , channel responses  $h_{ii}(k)$ , cross coupling functions  $h_{ij}(k)$   $\sigma_i^2(k) > 0$  is external noise present at the  $i$ 'th receiver at frequency  $k$ , and  $\Delta f_k$  it the bandwidth of the  $k$ 'th interval. In cases where  $\sigma_i^2(k) = 0$  capacities might become infinite using FDM strategies, however this is non-physical situation due to the receiver noise that is always present, even if small. Each  $C^i$  is continuous on all variables.

*Definition 2.1:* The Gaussian Interference game  $GI_{\{I_1, \dots, I_K\}} = \{\mathbf{C}, B\}$  is the  $N$  player non-cooperative game with payoff vector  $\mathbf{C} = (C^1, \dots, C^N)$  where  $C^i$  are defined in (8) and  $B$  is the strategy set defined by (6).

The interference game is a special case of convex non-cooperative  $N$ -persons game. Interestingly, under PSD mask constraint, the Gaussian interference game becomes a set of  $K$  parallel competitive games over flat channels.

### B. Nash equilibrium in non-cooperative games

An important notion in game theory is that of a Nash equilibrium.

*Definition 2.2:* An  $N$ -tuple of strategies  $\langle \mathbf{p}_1, \dots, \mathbf{p}_N \rangle$  for players  $1, \dots, N$  respectively is called a Nash equilibrium iff for all  $n$  and for all  $\mathbf{p}$  ( $\mathbf{p}$  a strategy for player  $n$ )

$$C^n(\mathbf{p}_1, \dots, \mathbf{p}_{n-1}, \mathbf{p}, \mathbf{p}_{n+1}, \dots, \mathbf{p}_N) \leq C^n(\mathbf{p}_1, \dots, \mathbf{p}_N)$$

i.e., given that all other players  $i \neq n$  use strategies  $\mathbf{p}_i$ , player  $n$  best response is  $\mathbf{p}_n$ .

The proof of existence of Nash equilibrium in the general interference game follows from an easy adaptation of the proof of this result for convex games [1]. A much harder problem is the uniqueness of Nash equilibrium points in the water-filling game. This is very important to the stability of the waterfilling strategies. A first result in this direction has been given in [24], [25]. A more general analysis of the convergence has been given in [26].

### C. Nash bargaining solution for the interference game

Nash equilibria are inevitable whenever a non-cooperative zero sum game is played. However they can lead to substantial loss to all players, compared to a cooperative strategy in the non-zero sum case, where players can cooperate. An example of this situation is the well known prisoner's dilemma. The main issue in this case is how to achieve the cooperation in a stable manner and what rates can be achieved through cooperation.

In this section we present the Nash bargaining solution [20]. The underlying structure for a Nash bargaining in an  $N$  player game is a set of outcomes of the bargaining process  $S \subseteq \mathbf{R}^N$  which is compact and convex and a designated disagreement

outcome  $d$  (which represents the agreement to disagree and solve the problem competitively).  $S$  can be considered as a set of outcomes of the possible joint strategies or states, Alternatively, some authors consider  $S$  as a set of states,  $d$  a disagreement state and a multiuser utility function  $U : S \cup \{d\} \rightarrow \mathbf{R}^N$ . such that  $U(S \cup \{d\})$  is compact and convex. The two approaches are identical and the first is obtained from the second by defining the game by the set of utilities of the possible outcomes. We will use the first formulation since it simplifies notation. However, in some cases we will define the outcomes of the game in terms of strategies. The set  $S$  in the first definition is then obtained by identifying it with  $U(S \cup \{d\})$  of the second definition. The Nash bargaining solution is a function  $F$  which assigns to each bargaining problem  $S \cup \{d\}$  as above an element of  $S \cup \{d\}$ , satisfying the following four axioms:

*Linearity.* Assume that we consider the bargaining problem  $S' \cup \{d'\}$  obtained from the problem  $S \cup \{d\}$  by transformations:  $s'_i = \alpha_i s_i + \beta_i$ ,  $i = 1, \dots, N$ .  $d'_i = \alpha_i d_i + \beta_i$ . Then the solution satisfies  $F_i(S' \cup \{d'\}) = \alpha_i F_i(S \cup \{d\}) + \beta_i$ , for all  $i = 1, \dots, N$ .

*Independence of irrelevant alternatives.* This axiom states that if the bargaining solution of a large game  $T \cup \{d\}$  is obtained in a small set  $S$ . Then the bargaining solution assigns the same solution to the smaller game, i.e., The irrelevant alternatives in  $T \setminus S$  do not affect the outcome of the bargaining.

*Symmetry.* If two players  $i < j$  are identical in the sense that  $S$  is symmetric with respect to changing the  $i$ 'th and the  $j$ 'th coordinates, then  $F_i(S \cup \{d\}) = F_j(S \cup \{d\})$ . Equivalently, players which have identical bargaining preferences, get the same outcome at the end of the bargaining process.

*Pareto optimality.* If  $s$  is the outcome of the bargaining then no other state  $t$  exists such that  $s < t$  (coordinate wise).

A good discussion of these axioms can be found in [20]. Nash proved that there exists a unique solution to the bargaining problem satisfying these four axioms. The solution is obtained by solving the following problem:

$$s_{\text{Nash}} = \arg \max_{s \in S \cup \{d\}} \prod_{n=1}^N (s_n - d_n). \quad (10)$$

Typically, one assumes that there exist at least one feasible  $s \in S$  such that  $d < s$  coordinatewise, but otherwise we can assume that the bargaining solution is  $d$ . We also define the Nash function  $F(s) : S \cup \{d\} \rightarrow \mathbf{R}$

$$F(s) = \prod_{n=1}^N (s_n - d_n). \quad (11)$$

The Nash bargaining solution is obtained by maximizing the Nash function over all possible states. Since the set of possible outcomes  $S \cup \{d\}$  is compact and convex  $F(s)$  has a unique maximum on the boundary of  $S \cup \{d\}$ .

Whenever the disagreement situation can be decided by a competitive game, it is reasonable to assume that the disagreement state is given by a Nash equilibrium of the relevant

competitive game. In some cases there are other possibilities for the disagreement point. When the utility for user  $n$  is given by the rate  $R_n$ , and  $\mathbf{d}$  is the competitive Nash equilibrium, it is obtained by iterative waterfilling for general ISI channels. For the case of mask constraints the competitive solution is simply given by all users using the maximal PSD at all tones.

### III. NASH BARGAINING SOLUTION FOR THE FLAT FADING $N$ PLAYER INTERFERENCE GAME

In this section we provide conditions for the existence of the Nash bargaining solution (NBS) for the  $N \times N$  flat frequency interference game. In general, the rate region for the interference channel is unknown. However, by a simple time sharing argument we know that the rate region is always a convex set  $\mathcal{R}$ , i.e.

$$\mathcal{R} = \{ \mathbf{r} : \mathbf{r} = (R_1, R_2, \dots, R_N) \text{ is in the rate region} \}. \quad (12)$$

is a convex set. Typically we will use the utility defined by the rate, i.e., for every rate vector  $\mathbf{r} = (R_1, \dots, R_N)^T$  we have  $U_n(\mathbf{r}) = R_n$ . In future work we will show how the results can be generalized to other utility functions such as  $U_n^L(\mathbf{t}) = \log(R_n)$ .

For some specific operational strategies one can define an achievable rate region explicitly. This allows for explicit determination of the strategies leading to the NBS. One such example is the use of FDM or TDM strategies in the interference channel. In the sequel we analyze the  $N$  player interference game, with FDM or TDM strategies. We provide conditions under which the bargaining solution exists, i.e., FDM strategies provide improvement over the competitive solution. This extends the work of [12] which characterized when does FDM solution outperforms the competitive IWF solution for symmetric 2x2 interference game. We have shown there that indeed in certain conditions the competitive game is subject to the prisoner's dilemma where the competitive solution is suboptimal for both players. Let the utility of player  $n$  be given by  $U_n = R_n$ . The received signal vector  $\mathbf{x}$  (equivalent to the model in equation (4) for  $K = 1$ ) is given by

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n} \quad (13)$$

where  $\mathbf{x} = [x_1, \dots, x_N]^T$  is the received signal, and  $\mathbf{H} = \{h_{ij}\}, 0 \leq i, j \leq N$ , is the interference coupling matrix,  $\mathbf{s} = [s_1, s_2, \dots, s_N]^T$  is the vector of transmitted signals. Similarly to the two user case (1) we can assume without loss of generality that the cross channels are normalized by the direct channels so that  $h_{ii} = 1$ . We will assume that for all  $i, j$   $|h_{ij}| \leq 1$ . Moreover, we will assume that the matrix  $\mathbf{H}$  is invertible. This assumption is reasonable since typical wireless communication channels are random, and the probability of obtaining a singular channel is 0. Note that in our case both transmission and reception are performed independently, and the vector formulation is used for notational simplicity. First observe:

*Lemma 3.1:* Assume that there is a unique Nash equilibrium in the Gaussian interference game. Then the competitive

strategies are given by flat power allocation. The resulting rates are:

$$R_n^C = \frac{W}{2} \log_2 \left( 1 + \frac{|h_{nn}|^2 P_n}{W N_0/2 + \sum_{j=1, j \neq n}^N |h_{nj}|^2 P_{ij}} \right) \quad (14)$$

where  $W$  is the bandwidth and  $N_0$  is the PSD of the white Gaussian noise.

The proof is easy and left to the reader. We note that sufficient conditions for uniqueness are given in [26]. Typically when the NE is non-unique, the interference is stronger than the desired signal, e.g., in the 2x2 case. In this case the competitive solution converges to FDM type of solution, and NBS and FDM coincide.

To simplify the expression for the competitive rates we divide the expression inside the log in (14) by the noise power  $W N_0/2$  obtaining:

$$R_n^C = \frac{W}{2} \log_2 \left( 1 + \frac{\text{SNR}_n}{1 + \sum_{j \neq n}^N \alpha_{nj} \text{SNR}_j} \right) \quad (15)$$

where  $\text{SNR}_j = \frac{|h_{jj}|^2 P_j}{W N_0/2}$ ,  $\alpha_{nj} = \frac{|h_{nj}|^2}{|h_{jj}|^2}$ .

Since the rates  $R_n^C$  are achieved by competitive strategy, player  $n$  would not cooperate unless he will obtain a rate higher than  $R_n^C$ . Therefore, the game theoretic rate region is defined by the set of rates higher than  $R_n^C$  of equation (15).

We are interested in FDM cooperative strategies. A strategy is a vector  $[\rho_1, \dots, \rho_N]^T$  such that  $\sum_{n=1}^N \rho_n \leq 1$ . We assume that player  $n$  uses a fraction  $\rho_n$  ( $0 \leq \rho_n \leq 1$ ) of the band (or equivalently uses the channel for a fraction  $\rho_n$  of the time in the TDM case). The rate obtained by the  $n^{\text{th}}$  player is given by

$$R_n(\boldsymbol{\rho}) = R_n(\rho_n) = \frac{\rho_n W}{2} \log_2 \left( 1 + \frac{\text{SNR}_n}{\rho_n} \right). \quad (16)$$

First we note that the FDM rate region  $\mathcal{R}_{FDM} = \{(R_1, \dots, R_N) | R_n = R_n(\rho_n)\}$  is indeed convex. The Pareto optimal points must satisfy  $\sum_{n=1}^N \rho_n = 1$ , since by dividing the unused part of the band between users, all of them increase their utility. Also note that by strict monotonicity of  $R_n(\rho)$  as a function of  $\rho$  each Pareto optimal point is on the boundary of  $\mathcal{R}_{FDM}$ . It is achieved by a single strategy vector  $\boldsymbol{\rho}$ . Player  $n$  benefits from FDM cooperation as long as

$$R_n^C < R_n(\rho_n). \quad (17)$$

The Nash function is given by

$$F(\boldsymbol{\rho}) = \prod_{n=1}^N (R_n(\rho_n) - R_n^C). \quad (18)$$

To better understand the gain in FDM strategies we define a function  $f(x, y)$  that is fundamental to the analysis.

*Definition 3.1:* For each  $0 < x, y$  let  $f(x, y)$  be defined by

$$f(x, y) = \min \left\{ \rho : \left( 1 + \frac{x}{\rho} \right)^\rho = 1 + \frac{x}{1+y} \right\}. \quad (19)$$

*Claim 3.1:* 1.  $f(x, y)$  is a well defined function for  $x, y \in \mathbf{R}^+$ .

2. For all  $x, y \in \mathbf{R}^+$ ,  $0 < f(x, y) < 1$ .
3.  $f(x, y)$  is monotonically decreasing in  $y$ .

**Proof:** Let  $g(x, y, \rho)$  be defined by:

$$g(x, y, \rho) = \left( 1 + \frac{x}{\rho} \right)^\rho - 1 - \frac{x}{1+y}$$

For every  $x, y$ ,  $g(x, y, \rho)$  is a continuous and monotonic function in  $\rho$ . Furthermore, for any  $0 < x, y$ ,  $g(x, y, 1) > 0$ , and  $\lim_{\rho \rightarrow 0} g(x, y, \rho) < 0$ . Hence, there is a unique solution to (19). Furthermore, the value of  $f(x, y)$  is strictly between 0, 1. Finally  $f(x, y)$  is monotonically decreasing in  $y$  since  $g(x, y, \rho)$  is increasing in  $y$ , so if we increase  $y$  we need to decrease  $\rho$  to maintain a fixed value.

Using the function  $f(x, y)$  we can completely characterize the cases where NBS is preferable to the Nash equilibrium.

*Theorem 3.2:* Nash bargaining solution exists if and only if the following inequality holds

$$\sum_{n=1}^N f \left( \text{SNR}_n, \sum_{j \neq n}^N \alpha_{nj} \text{SNR}_j \right) \leq 1. \quad (20)$$

**Proof:** In one direction, assume that a Nash bargaining solution exists. The next two conditions must hold

1. There is a partition of the band between the players such that player  $n$  gets a fraction  $\rho_n > 0$ .
2. Each player gets by cooperation higher rate than the competitive rate, i.e.,  $R_n(\rho_n) \geq R_n^C$ .

Therefore, using equation (19) and inequality (17) we obtain that equation (20) must be satisfied. On the other direction by definition of  $f$  player  $n$  has at least the rate that it can get by competition if he can use a fraction  $\rho_n$ , of the bandwidth. Since (20) implies that  $\sum_{n=1}^N \rho_n \leq 1$ , FDM is preferable to the competitive solution for the utility function  $U_n = R_n$ . By the convexity of the FDM rate region the Nash function has a unique maximum that is Pareto optimal and outperforms the competitive solution.

Interestingly, as long as the utility function  $U_n(\boldsymbol{\rho})$  depends only on  $\rho_n$  and  $U_n(\rho)$  is monotonically increasing in  $\rho$  the same conclusion holds. This implies that the NBS for the utility  $U_n^L(\rho) = \log(R_n(\rho_n))$  there is a unique frequency division vector  $\boldsymbol{\rho}$  that achieves the NBS. Furthermore the optimization problem, of computing the optimal  $\boldsymbol{\rho}$  is still convex.

We now examine the simple case of two players. Assume that player I uses a fraction  $\rho$  ( $0 \leq \rho \leq 1$ ) of the band and user II uses a fraction  $1 - \rho$ . The rates obtained by the two users are given by

$$\begin{aligned} R_1(\rho) &= \frac{\rho W}{2} \log_2 \left( 1 + \frac{\text{SNR}_1}{\rho} \right) \\ R_2(1 - \rho) &= \frac{(1 - \rho) W}{2} \log_2 \left( 1 + \frac{\text{SNR}_2}{1 - \rho} \right) \end{aligned} \quad (21)$$

The two users will benefit from FDM cooperation as long as

$$R_i^C \leq R_i(\rho_i), \quad i = 1, 2 \quad \rho_1 + \rho_2 \leq 1 \quad (22)$$

Condition (20) can now be simplified:

$$f(\text{SNR}_1, \alpha \text{SNR}_2) + f(\text{SNR}_2, \beta \text{SNR}_1) \leq 1, \quad (23)$$

where  $\alpha = |h_{12}|^2 / |h_{22}|^2$ ,  $\beta = |h_{21}|^2 / |h_{11}|^2$ . The NBS is given by solving the problem

$$\rho_{NBS} = \arg \max_{\rho} F(\rho) \quad (24)$$

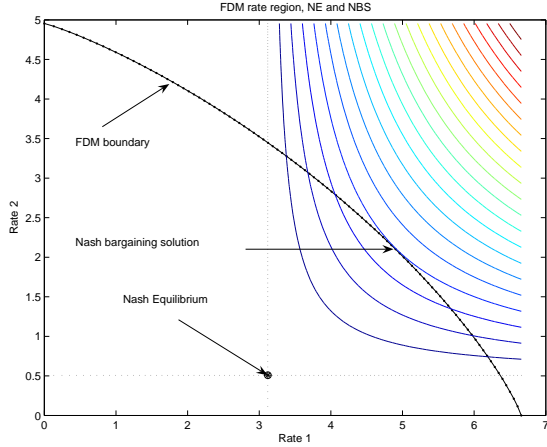


Fig. 1. FDM rate region (thick line), Nash equilibrium  $*$ , Nash bargaining solution and the contours of  $F(\rho)$ . Flat channel.  $\text{SNR}_1 = 20$  dB,  $\text{SNR}_2 = 15$  dB, and  $\alpha = 0.4, \beta = 0.7$

where the Nash function is now given by:  $F(\rho) = (R_1(\rho) - R_1^C)(R_2(1 - \rho) - R_2^C)$  and  $R_i(\rho)$  are defined by (21). A special case can now be derived:

*Claim 3.2:* Assume that  $\text{SNR}_1 \geq \frac{1}{2}(\alpha^2\beta^4)^{-1/3}$  and  $2\beta^2\text{SNR}_1^2 \geq \text{SNR}_2 \geq \frac{1}{\alpha}\sqrt{\frac{\text{SNR}_1}{2}}$ . Then there is a Nash bargaining solution that is better than the competitive solution.

*Proof:* The proof of the claim follows directly by substituting  $\rho_1 = \rho_2 = 1/2$ , and bounding the inequalities. ■

Finally we note that as  $\text{SNR}_i$  increases to infinity the NBS is always better than the NE.

*Claim 3.3:* 1. If  $\text{SNR}_1$  and  $\text{SNR}_2$  are jointly increasing, while keeping the ratio  $\frac{\text{SNR}_1}{\text{SNR}_2} = z$  fixed. Then, there is a constant  $g$  such that for  $\text{SNR}_1 > g$ , an FDM Nash bargaining solution exists.

2. If  $\text{SNR}_1 + \text{SNR}_2 \leq \frac{1-\alpha-\beta}{\alpha\beta}$  there is no Nash bargaining solution.

The proof is easy and will not be given due to space limitations. The following example provides the intuition for the definitions of the game theoretic rate region, and the uniqueness of the NBS using FDM strategies. It also clearly demonstrates the relation between the competitive solution, the NBS and the game theoretic rate region  $\mathcal{R}^G$ . We have chosen  $\text{SNR}_1 = 20$  dB,  $\text{SNR}_2 = 15$  dB, and  $\alpha = 0.4, \beta = 0.7$ . Figure 1 presents the FDM rate region, the Nash equilibrium point denoted by  $*$ , and a contour plot of  $F(\rho)$ . It can be seen that the convexity of  $F(\rho)$  together with the convexity of the achievable rate region implies that at there is a unique contour tangent to the rate region. The tangent point is the Nash bargaining solution. We can see that the NBS achieves rates that are 1.6 and 4 times higher than the rates of the competitive Nash equilibrium rates for player I and player II respectively. The game theoretic rate region is the intersection of the information theoretic rate region with the quadrant above the dotted lines.

#### IV. BARGAINING OVER FREQUENCY SELECTIVE CHANNELS UNDER MASK CONSTRAINT

In this section we define a new cooperative game corresponding to the joint FDM/TDM achievable rate region for the frequency selective  $N$  user interference channel. We limit ourselves to the PSD mask constrained case since this case is actually the more practical one. In real applications, the regulator limits the PSD mask and not only the total power constraint. Let the  $K$  channel matrices at frequencies  $k = 1, \dots, K$  be given by  $\langle \mathbf{H}_k : k = 1, \dots, K \rangle$ . Each player is allowed to transmit at maximum power  $p(k)$  in the  $k$ 'th frequency bin. In non-cooperative scenario, under mask constraint, all players transmit at the maximal power they can use. Thus, all players choose the PSD,  $\mathbf{p}_i = \langle p_i(k) : 1 \leq k \leq K \rangle$ . The payoff for user  $i$  in the non-cooperative game is therefore given by:

$$R_i^C(\mathbf{p}_1) = \sum_{k=1}^K \log_2(1 + \text{SINR}_i(k)). \quad (25)$$

Here,  $R_i^C$  is the capacity available to player  $i$  given a PSD mask constraint distributions  $\mathbf{p}$ , and  $\text{SINR}_i(k)$  is defined in (9). Note that without loss of generality, and in order to simplify notation, we assume that the width of each bin is normalized to 1. We now define the cooperative game  $G_{TF}(N, K, \mathbf{p})$ .

*Definition 4.1:* The FDM/TDM game  $G_{TF}(N, K, \mathbf{p})$  is a game between  $N$  players transmitting over  $K$  frequency bins under common PSD mask constraint. Each user has full knowledge of the channel matrices  $\mathbf{H}_k$ . The following conditions hold:

- 1) Player  $i$  transmits using a PSD limited by  $\langle p_i(k) : k = 1, \dots, K \rangle$ .
- 2) Strategies for player  $i$  are vectors  $\alpha = [\alpha_i(1), \dots, \alpha_i(K)]^T$  where  $\alpha_i(k)$  is the proportion of time player  $i$  uses the  $k$ 'th frequency channel. This is the TDM part of the strategy.
- 3) The utility of the  $i$ 'th player is given by

$$\begin{aligned} R_i &= \sum_{k=1}^K R_i(k) \\ &= \sum_{k=1}^K \alpha_i(k) \log_2 \left( 1 + \frac{|h_{ii}(k)|^2 p_i(k)}{\sigma_i^2(k)} \right) \end{aligned} \quad (26)$$

Note that interference is avoided by time sharing at each frequency band, i.e only one player transmits at a given frequency bin at any time. Furthermore, since at each time instance each frequency is used by a single user, each user can transmit using maximal power.

The Nash bargaining can be posed as an optimization problem

$$\begin{aligned} \mathbf{max} \quad & \prod_{i=1}^N (R_i(\alpha_i) - R_i^C) \\ \mathbf{subject \ to:} \quad & \forall k \sum_{i=1}^N \alpha_i(k) = 1, \\ & \forall i, k \alpha_i(k) \geq 0, \\ & \forall i R_i^C \leq R_i(\alpha_i), \end{aligned} \quad (27)$$

where,

$$\begin{aligned} R_i(\alpha_i) &= \sum_{k=1}^K \alpha_i(k) \log_2 \left( 1 + \frac{|h_{ii}(k)|^2 P_{max}(k)}{\sigma_i^2(k)} \right) \\ &= \sum_{k=1}^K \alpha_i(k) R_i(k). \end{aligned} \quad (28)$$

This problem is convex and therefore can be solved efficiently using convex optimization techniques. To that end we explore

the KKT conditions for the problem. The Lagrangian of the problem  $f(\alpha)$  is given by

$$\begin{aligned} f(\alpha) = & - \sum_{i=1}^N \log(R_i(\alpha_i) - R_i^C) \\ & + \sum_{k=1}^K \lambda_k \left( \sum_{i=1}^N \alpha_i(k) - 1 \right) \\ & - \sum_{k=1}^K \sum_{i=1}^N \mu_i(k) \alpha_i(k) \\ & - \sum_{i=1}^N \delta_i \left( \sum_{k=1}^K \alpha_i(k) R_i(k) - R_i^C \right) \end{aligned} \quad (29)$$

Taking the derivative with respect to the variable  $\alpha_i(k)$  and comparing the result to zero, we get

$$\frac{R_i(k)}{R_i(\alpha_i) - R_i^C} = \lambda_k - \mu_i(k) - \delta_i \quad (30)$$

with the constraints

$$\begin{aligned} \sum_{i=1}^N \alpha_i(k) &= 1, \\ \delta_i (R_i(\alpha_i) - R_i^C) &\geq 0, \\ \mu_i(k) \alpha_i(k) &= 0, \\ \lambda_k &\geq 0. \end{aligned} \quad (31)$$

Based on (30, 31) one can easily come to the following conclusions:

- 1) If there is a feasible solution then for all  $i$ ,  $\delta_i = 0$ .
- 2) Assume that a feasible solution exists. Then for all players sharing the frequency bin  $k$  ( $\alpha_i(k) > 0$ ) we have  $\mu_i(k) = 0$ , and

$$\frac{R_i(k)}{R_i(\alpha_i) - R_i^C} = \lambda_k, \forall k \text{ satisfying } \alpha_i(k) > 0. \quad (32)$$

- 3) For all players that are not sharing the frequency bin  $k$ , ( $\alpha_i(k) = 0$ ),  $\mu_i(k) \geq 0$ . Therefore,

$$\frac{R_i(k)}{R_i(\alpha_i) - R_i^C} \leq \lambda_k, \forall k \text{ with } \alpha_i(k) = 0. \quad (33)$$

The second conclusion is very interesting. let  $L_{ij}(k) = R_i(k)/R_j(k)$ . Assume that for users  $i, j$  the values  $L_{ij}(k)$  are all distinct. Then the two users can share at most a single frequency. To see this note that in this case

$$\frac{R_i(k)}{R_i(\alpha_i) - R_i^C} = \frac{R_j(k)}{R_j(\alpha_j) - R_j^C} \quad (34)$$

and therefore

$$L_{ij}(k) = \frac{R_i(k)}{R_j(k)} = \frac{R_i(\alpha_i) - R_i^C}{R_j(\alpha_j) - R_j^C} \quad (35)$$

Since the right hand side is independent of the frequency  $k$  and  $L_{ij}(k)$  are distinct, at most a single frequency can satisfy this condition. This proves the following theorem:

*Theorem 4.1:* Assume that for all  $i \neq j$  the values  $\{L_{ij}(k) : k = 1, \dots, K\}$  are all distinct. Then in the optimal solution at most  $\binom{N}{2}$  frequencies are shared between different users.

This theorem suggests, that when  $\binom{N}{2} \ll K$  the optimal FDM NBS is very close to the joint FDM/TDM solution. It is obtained by allocating the common frequencies to one of the users.

While general convex optimization techniques are useful for computing the NBS, in the next section we will demonstrate

that for the two player game the solution can be computed much more efficiently. Furthermore, we will show that in the optimal solution only a single frequency is actually shared between the users even if the  $L_{ij}(k)$  are not distinct.

#### A. Extension to fast fading channels

While the method described above fits well to stationary channels, the method is also useful when only fading statistics is known. In this case the coding strategy will change, and the achievable rate in the competitive case and the cooperative case are given by

$$\begin{aligned} \tilde{R}_i^C(\mathbf{p}_i) &= \sum_{k=1}^K E \left[ \log_2 \left( 1 + \frac{|h_{ii}(k)|^2 p_i(k)}{\sum_{j \neq i} |h_{ij}(k)|^2 p_j(k) + \sigma_i^2(k)} \right) \right] \\ \tilde{R}_i(\alpha_i) &= \sum_{k=1}^K \alpha_i(k) E \left[ \log_2 \left( 1 + \frac{|h_{ii}(k)|^2 p_i(k)}{\sigma_i^2(k)} \right) \right] \end{aligned} \quad (36)$$

respectively. All the rest of the discussion is unchanged, replacing  $R_i^C$  and  $R_i(\alpha_i)$  by  $\tilde{R}_i^C$ ,  $\tilde{R}_i(\alpha_i)$  respectively. This is particularly attractive, when the computations are done in distributed way. In this case only channel state distributions are sent between the units, and the time scale for this distribution are much longer. This implies that method can be used without a central control, by exchange of parameters between the units at a very low rate.

#### V. COMPUTING THE NASH BARGAINING SOLUTION FOR TWO PLAYERS

For the two player case the optimization problem can be dramatically simplified. In this section we will provide an  $O(K \log_2 K)$  complexity algorithm (in the number of tones) for computing the NBS optimal solution in a two user frequency selective channel. Furthermore, we will show that the two players will share at most a single frequency, no matter what the ratios between the users are. To that end let,  $\alpha_1(k) = \alpha(k)$ , and  $\alpha_2(k) = 1 - \alpha(k)$ . We also define the surplus of players I and II when using Nash bargaining solution as  $A = \sum_{m=1}^K \alpha(m) R_1(m) - R_{1C}$  and  $B = \sum_{m=1}^K (1 - \alpha(m)) R_2(m) - R_{2C}$ , respectively. The ratio,  $\Gamma = A/B$  is a threshold which is independent of the frequency and is set by the optimal assignment. While  $\Gamma$  is a-priori unknown, it exists. Let  $L(k) = R_1(k)/R_2(k)$ . Without loss of generality, assume that the rate ratios  $L(k)$ ,  $1 \leq k \leq K$  are sorted in decreasing order i.e.  $L(k) \geq L(k'), \forall k \leq k'$ . (This can be achieved by sorting the frequencies according to  $L(k)$ .) We are now ready to define optimal assignment of the  $\alpha$ 's. Let  $\Gamma_k$  be a moving threshold defined by  $\Gamma_k = A_k/B_k$ . where

$$A_k = \sum_{m=1}^k R_1(m) - R_{1C}, B_k = \sum_{m=k+1}^K R_2(m) - R_{2C}. \quad (37)$$

$A_k$  is a monotonically increasing sequence, while  $B_k$  is monotonically decreasing. Hence,  $\Gamma_k$  is also monotonically increasing.  $A_k$  is the surplus of user I respectively when frequencies  $1, \dots, k$  are allocated to user I. Similarly  $B_k$  is the surplus of user II when frequencies  $k+1, \dots, K$  are allocated to user II. Let  $k_{\min} = \min_k \{k : A_k \geq 0\}$ ;  $k_{\max} = \min_k \{k : B_k < 0\}$ . Since we are interested in feasible NBS, we must have positive surplus for both users. Therefore, by

the KKT equations, we obtain  $k_{\min} \leq k_{\max}$  and  $L(k_{\min}) \leq \Gamma \leq L(k_{\max})$ . The sequence  $\{\Gamma_m : k_{\min} \leq m \leq k_{\max} - 1\}$  is strictly increasing, and always positive. We first state two lemmas that are essential for finding the optimal partition.

*Lemma 5.1:* Assume that there is an NBS to the game. Then there is always an NBS satisfying that at most a single bin  $k_s$  is partitioned between the players, and  $\alpha(k) = 1$  if  $k < k_s$ ,  $\alpha(k) = 0$  if  $k > k_s$ .

*Proof:* By our assumption the sequence  $\{L(k) : k = 1, \dots, K\}$  is monotonically decreasing (not necessarily strictly decreasing). If there is a  $k$  such that  $L(k-1) < \Gamma < L(k)$  then the solution must be FDM type by the KKT equations and we finish. Otherwise assume that  $L(k) = \Gamma$ . Since  $\Gamma_k$  is strictly increasing and  $L(k)$  is non-increasing there is at most a unique  $k$  such that  $\Gamma_{k-1} \leq L(k) = \Gamma < \Gamma_k$ . If no such  $k$  exists then the users can only share  $k_{\max}$  since for all  $k \leq k_{\max}$

$$\frac{A_k}{B_k} \leq \Gamma$$

and the only way to get something allocated to user II is by sharing  $k_{\max}$ . Otherwise such a  $k \leq k_{\max}$  exists. By definition of  $\Gamma_k$  we have  $A_{k-1}/B_{k-1} \leq L(k) < A_k/B_k$ . Simple substitution yields

$$\frac{A_{k-1}}{B_{k-1}} \leq L(k) < \frac{A_{k-1} + R_1(k)}{B_{k-1} - R_2(k)} = \frac{A_k}{B_k}.$$

Since  $k_{\min} \leq k < k_{\max}$  the denominator on the RHS is positive. Since for  $a, b, c, d > 0$  the function  $\frac{a+xb}{c-xd}$  is increasing with  $0 \leq x$  as long as the denominator is positive, we obtain that by continuity there is a unique  $\zeta$  such that

$$L(k) = \frac{A_{k-1} + \zeta R_1(k)}{B_{k-1} - \zeta R_2(k)}.$$

But  $B_{k-1} - \zeta R_2(k) = B_k + (1 - \zeta)R_2(k)$  so that  $\zeta$  satisfies

$$\Gamma = L(k) = \frac{A_{k-1} + \zeta R_1(k)}{B_k + (1 - \zeta)R_2(k)}.$$

Setting  $\alpha(m) = 1$  for  $m < k$ ,  $\alpha(k) = \zeta$  and  $\alpha(m) = 0$  for  $m > k$  we obtain a solution of the KKT equations. Note that when there are multiple values of  $k$  such that  $L(k) = \Gamma$ , we only showed that there is an NBS solution where a single frequency is shared. ■

While the threshold  $\Gamma$  is unknown, one can use the sequences  $\Gamma_k$  and  $L(k)$ .

If there is a Nash bargaining solution, let  $k_s$  be the frequency bin that is shared by the players. Then,  $k_{\min} \leq k_s \leq k_{\max}$ . Since, both players must have a positive gain in the game ( $A > A_{k_{\min}-1}, B > B_{k_{\max}}$ ). Let  $k_s$  be the smallest integer such that  $L(k_s) < \Gamma_{k_s}$ , if such  $k_s$  exists. Otherwise let  $k_s = k_{\max}$ .

*Lemma 5.2:* The following two statements provide the solution

- 1 If a Nash bargaining solution exists for  $k_{\min} \leq k_s < k_{\max}$ , then  $\alpha(k_s)$  is given by  $\alpha(k_s) = \max\{0, g\}$ , where

$$g = 1 + \frac{B_{k_s}}{2R_2(k_s)} \left(1 - \frac{\Gamma_{k_s}}{L(k_s)}\right). \quad (38)$$

- 2 If a Nash bargaining solution exists and there is no such  $k_s$ , then  $k_s = k_{\max}$  and  $\alpha(k_s) = g$ .

TABLE I

ALGORITHM FOR COMPUTING THE 2X2 FREQUENCY SELECTIVE NBS

<p><b>: Initialization:</b> Sort the ratios <math>L(k)</math> in decreasing order.          Calculate the values of <math>A_k, B_k</math> and <math>\Gamma_k, k_{\min}, k_{\max}</math>.</p>
<p>If <math>k_{\min} &gt; k_{\max}</math> no NBS exists. Use competitive solution.          Else            For <math>k = k_{\min}</math> to <math>k_{\max} - 1</math>              if <math>L(k) \leq \Gamma_k</math>.                Set <math>k_s = k</math> and <math>\alpha</math>'s according to the lemmas-This is NBS. Stop            End          End          If no such <math>k</math> exists, set <math>k_s = k_{\max}</math> and calculate <math>g</math>.          If <math>g \geq 0</math> set <math>\alpha_{k_s} = g, \alpha(k) = 1</math>, for <math>k &lt; k_{\max}</math>. Stop.          Else (<math>g &lt; 0</math>)            There is no NBS. Use competitive solution.          End.          End</p>

*Proof:* To prove 1 note that since  $\Gamma_{k_s-1} \leq L(k_s) \leq \Gamma_{k_s}$ ,  $\alpha(k_s)$  is the solution to the equation  $L(k_s) = \frac{A_{k_s} - (1 - \alpha(k_s))R_1(k_s)}{B_{k_s} + (1 - \alpha(k_s))R_2(k_s)}$ . By simple mathematical manipulation, we get  $\alpha(k_s) = g$ . Since,  $L(k) \leq \Gamma_{k_s}, g \leq 1$ . If  $g$  is negative, we set  $\alpha(k_s) = 0$ , since  $k_s$  is the smallest integer such that  $L(k_s) < \Gamma_{k_s}$ . Note, that in this case the Nash bargaining solution is given by pure FDM strategies.

To prove 2 note that since  $k_s = k_{\max}$  and  $\Gamma_k$  is increasing for  $k_{\min} \leq k < k_{\max}$ , we must have that  $\Gamma_{k_{\max}-1} \leq \Gamma = L(k_{\max})$ . Therefore, the only possibility that a solution exists is by setting  $k_s = k_{\max}$ , and  $\alpha(k_s) = g \geq 0$ . ■

Based on the previous lemmas the algorithm is described in table I. In the first stage the algorithm computes  $L(k)$  and sorts them in a non increasing order. Then  $k_{\min}, k_{\max}, A_k$ , and  $B_k$  are computed. In the second stage the algorithm computes  $k_s$  and  $\alpha$ . Note that the sorting stage which is  $O(K \log_2 K)$  has the largest complexity. All other computations are linear in  $K$ .

## VI. SIMULATIONS

In this section we compare in simulations the bargaining solution to the competitive solution for various situations with medium interference. The simulations are done both for flat slow fading and for frequency selective fading. First, we demonstrate the effect of the channel matrix and the signal to noise ratio on the gain of the NBS for flat fading channel. Then we performed extensive simulations that demonstrate the advantage of the NBS over the competitive approach for the frequency selective fading channel, as a function of the mean interference power.

### A. Flat fading

We have tested the gain of the Nash bargaining solution relative to the Nash equilibrium competitive rate pair as a function of channel coefficients as well as signal to noise ratio for the flat fading channel. To that end we define the minimum relative improvement,  $\Delta_{\min}$ , describing the individual price of anarchy and the usual price of anarchy [27],  $\Delta_{sum}$ , describing total loss due to lack of cooperation by:

$$\Delta_{\min} = \min \left\{ R_1^{NBS}/R_1^C, R_2^{NBS}/R_2^C \right\} \\ \Delta_{sum} = (R_1^{NBS} + R_2^{NBS}) / (R_1^C + R_2^C). \quad (39)$$



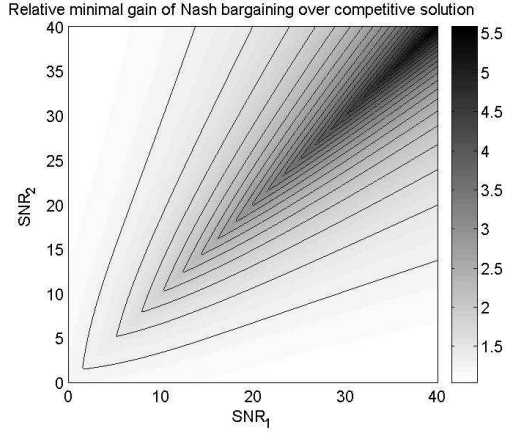


Fig. 2. Per user price of anarchy (relative improvement of NBS sum rate over NE), as a function of SNR. Flat channel.  $\alpha = \beta = 0.7$ .

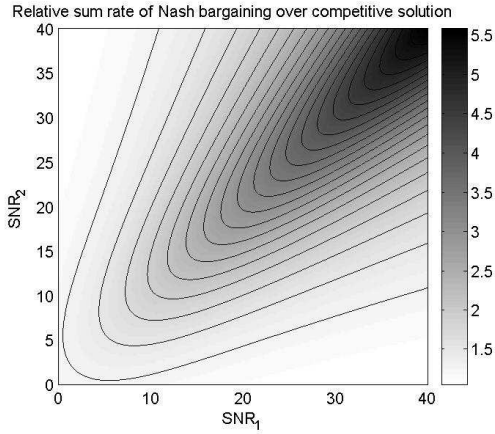


Fig. 3. Price of anarchy, as a function of SNR. Flat channel  $\alpha = \beta = 0.7$ .

In the first set of experiments we have fixed  $\alpha, \beta$  and varied  $\text{SNR}_1, \text{SNR}_2$  from 0 to 40 dB in steps of 0.25dB. Figure 2 presents  $\Delta_{\min}$  for an interference channel with  $\alpha = \beta = 0.7$ . We can see that for high SNR we obtain significant improvement. Figure 3 presents the relative sum rate improvement  $\Delta_{\text{sum}}$  for the same channel. We can see that the achieved rates are 5.5 times those of the competitive solution.

We have now studied the effect of the interference coefficients on the Nash bargaining solution. We have set the signal to additive white Gaussian noise ratio for both users to 20 dB, and varied  $\alpha$  and  $\beta$  between 0 and 1. Similarly to the previous case we present the minimal price of anarchy per user  $\Delta_{\min}$  and the sum rate price of anarchy  $\Delta_{\text{sum}}$ . The results are shown in figures 4,5. We can clearly see that even with SINR of 10 dB we obtain 50 percent capacity gain per user.

### B. Frequency selective Gaussian channel

In this experiment we demonstrate the advantage of the Nash bargaining solution over competitive approaches for a frequency selective interference channel. We assumed that two users having direct channels that are standard Rayleigh fading channels ( $\sigma^2 = 1$ ), with  $\text{SNR}=30$  dB, suffer from

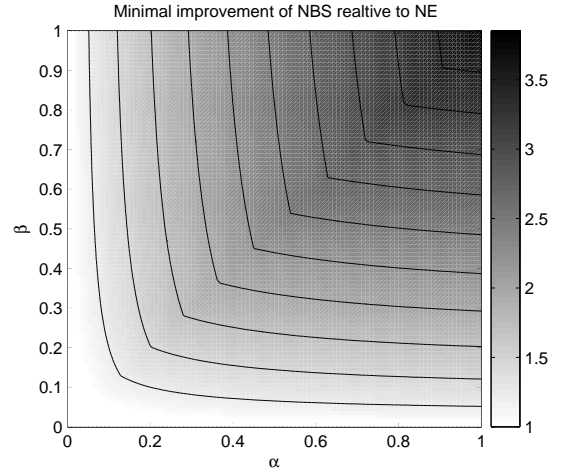


Fig. 4. Per user price of anarchy as a function of interference power. Flat channel.  $\text{SNR}=20$  dB.

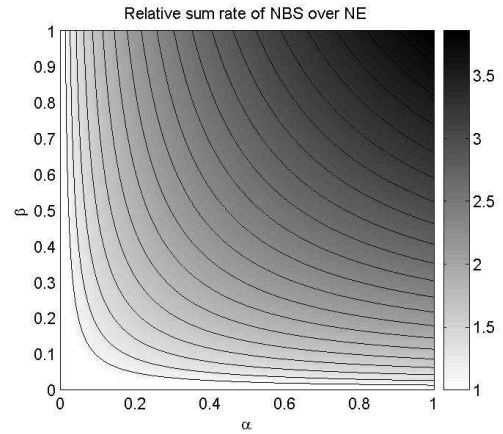


Fig. 5. Sum rate price of anarchy as a function of interference power. Flat channel.  $\text{SNR}=20$  dB.

interference, with SINR of each user into the other channel ( $h_{ij}$ ) was varied from 10 dB to 0 dB ( $\sigma_{h_{ij}} = 0.1, \dots, 1$ ). We have used 32 frequency bins. At each pair of variances  $\sigma_1^2 = \sigma_{h_{21}}^2, \sigma_2^2 = \sigma_{h_{12}}^2$  we randomly picked 25 channels (each comprising of 32  $2 \times 2$  matrices). The results of the minimal relative improvement (39) are depicted in figure 6. We can clearly see that the relative gain of the Nash bargaining solution over the competitive solution is 1.5 to 3.5 times, which clearly demonstrates the merits of the method.

## VII. CONCLUSIONS

In this paper we have defined the game theoretic rate region for the interference channel. The region is a subset of the rate region of the interference channel. We have shown that a specific point in the rate region given by the Nash bargaining solution is better than other points in the context of bargaining theory. We have shown conditions for the existence of such a point in the case of the FDM rate region. We have shown that computing the Nash bargaining solution over a frequency selective channel can be described as a convex optimization

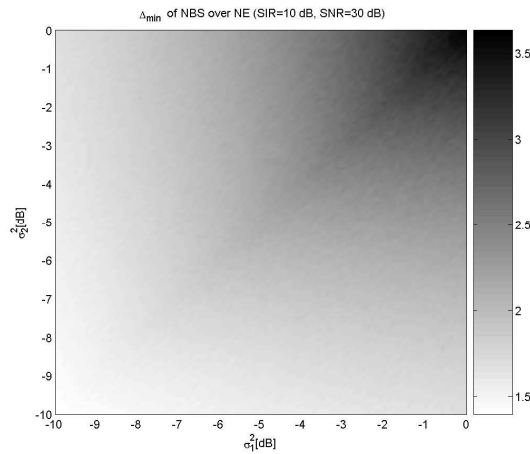


Fig. 6. Per user price of anarchy for frequency selective Rayleigh fading channel. SNR=30 dB.

problem. Moreover, we have provided a very simple algorithm for solving the problem in the  $2 \times K$  case that is  $O(K \log_2 K)$ , where  $K$  is the number of tones. Finally, we have demonstrated through simulations the significant improvement of the cooperative solution over the competitive Nash equilibrium.

The adaptation of game theory approach for rate allocation in existing wireless and wireline system is very appealing. In many wireless LAN systems there is a central access point with full knowledge on the channel transfer functions. Moreover, it has been recognized by the 802.11 committee that radio resource management is important, especially when multiple networks are interfering with other. Knowledge of the transfer functions allows the access point to allocate the band for the subscribers on the uplink. Moreover, the results here can be extended to MIMO systems as well as for networks with multiple access points.

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