Cooperative Multiple-Access in Fading Relay Channels

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Abstract— **Virtual antenna arrays can be constructed via relaying even in the case that there is insufficient physical space or other resources for multiple antennae on wireless nodes. When there is a multiple access scenario, relaying offers a variety of ways to establish communication between source and destination nodes. We will compare a scheme based on space division multiple access to previously studied time division based ones. We observe that space division improves especially the ergodic capacity.**

I. Introduction

Multi-input multi-output (MIMO) systems provide substantial increase in capacity and/or diversity for wireless communications [1], [2]. However, it may not always be possible to physically carry multiple antennas at transmitters and receivers mainly due to limited space and complexity of wireless nodes. This problem can be solved by distributed antenna systems formed by the antennas belonging to different terminals. This approach has been considered under the name of cooperative diversity with relay channels in [3], [4]. The relay channel model helps set up a scenario where a virtual antenna array consisting of the antennas of the relays can be created and utilized in analogy with a MIMO system after information exchange with relay terminals.

There are basically two operation modes at the relay nodes under the cooperative diversity framework. One is the amplify-and-forward (AF) operation in which the relay simply transmits the signal it receives from a source after scaling. The other one is the decode-and-forward (DF) mode where the relay node decodes the received signal and retransmits the received information possibly with a different code. Laneman et al. [3] and Nabar et al. [4] studied the capacity of various protocols based on the relay channel model. In the problem formulation of [3], the initiating (source) node does not interfere with the relay node's signal at the destination node. Whereas, [4] considers a more general case where the source and relay signals can interfere with each other at the destination. Especially in the latter case, we can use the results of various studies on MIMO systems.

This study considers a generalization of previous cooperative diversity studies in that there are multiple source nodes in a relay channel. There occurs a multiple access problem in this case which can be solved in different ways. We will consider two multiple access schemes in this study: space-division multiple access (SDMA) and time-division multiple access (TDMA). Since a scenario geared towards wireless sensor networks is of interest, low complexity operations are very much preferred in wireless nodes. In both methods, the interference from other sources does not affect a particular source's signal received at destination. These two methods do not require high complexity operations at wireless nodes and thus are chosen for study.

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The outline of the paper is as follows. The relaying network model, TDMA scheme, and AF mode used in this study will be explained in Section II. SDMA will be explained in Section III. Multi-user relay channel will be defined in IV. Performance of SDMA and TDMA based schemes will be compared in Section V. The paper will be concluded in Section VI.

II. TDMA Scheme

We will base our study on the classical relay channel model in [5]. The terminals initiating the transmission are called source nodes and the terminal targeted for transmission is called the destination node. There is a node called the relay which aids the communication between the sources and the destination. We assume that we have more than one sources but single relay and destination nodes. We consider the scenario where all nodes have links to each other as in Fig. 1.

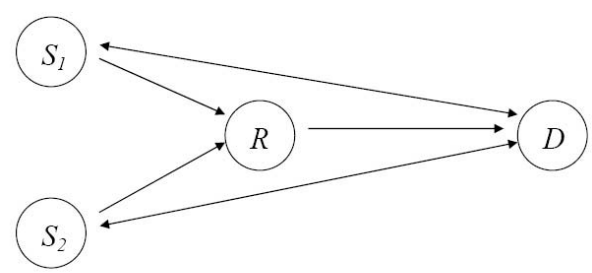

Fig. 1. A multi-source relaying scenario

Issues such as sampling offset, time delays of signals reaching the destination might hinder the proper working of a relaying network. We assume a synchronization between the nodes which can be held by keeping the symbol periods quite larger than all the timing delays occurring in the network [6]. The phase and symbol timing coherence between the oscillators and clocks of nodes may be provided by a broadcast from the destination over which the nodes can lock. After this synchronization, each involved node transmits a spread-spectrum signature signal so that its channel state information can be accurately estimated at destination. At the end of this session, the destination broadcasts the information with regard to cooperation and transmission of data starts right after that. Hence, we can have a network with perfect synchronization and channel state information over which cooperative diversity can be employed. How much overhead is incurred with such a scheme is beyond the scope of this work.

In order to avoid problems with the normalization of spectral resources, we will assume that a bandwidth of W Hz is allocated for transmission to a source S_i for a time duration of T seconds. Different sources have access to the channel in different time slots of T seconds. For example, S_1 has access in $]0, T]$, S_2 has access in $]T, 2T]$ and so on. Frequency division based systems can be studied similarly. Throughout its allocated time slot, the source node S_i has the freedom to individually use all the available $2WT$ dimensions itself, or share part of it with the relay node R for its own advantage. We will consider the following protocol shown to have larger capacity compared to others studied in [4]. The total time duration of T seconds is split into two equal time slots. Both the source and the destination are active in both time slots allotted for the transmission of the source. The relay node listens to the source in the first time slot and retransmits the received signal in the second time slot. After the first source sends its signal in T seconds, it is the turn of the second source to transmit its information.

We use the notation developed in [4] for the relevant signals. In the first time slot allotted to it, the source S_i transmits $x_{i,1}$ and both the relay R and destination D receives the signal. So,

$$y_{R,1} = \alpha_{S_iR}\sqrt{\varepsilon_{S_iR}}x_{i,1} + n_{R,1}, \quad (1)$$

$$y_{D,1} = \alpha_{S_iD}\sqrt{\varepsilon_{S_iD}}x_{i,1} + n_{D,1}, \quad (2)$$

where $y_{A,k}$ denotes the received signal at node A and time slot k, α_{AB} denotes the complex channel gain from node A to node B, ε_{AB} is the average energy received for a channel symbol of unit energy from node A to node B, and $n_{A,k}$ is the independent zero-mean complex circularly symmetric Gaussian white noise at node A and time slot k with variance N_0. We assume that the channel between the nodes are independent, frequency-flat, and quasi-static such that the complex channel gain α_{AB} stays constant within one transmission period of time T seconds but changes slowly compared to duration of T seconds based on complex circularly symmetric Gaussian density with mean 0 and variance 1. Since the channel is assumed to be slowly varying, accurate channel estimation is possible. In the second time slot, the relay transmits x_R based on what it receives, the source transmits $x_{i,2}$ and the destination receives

$$y_{D,2} = \alpha_{S_iD}\sqrt{\varepsilon_{S_iD}}x_{i,2} + \alpha_{RD}\sqrt{\varepsilon_{RD}}x_R + n_{D,2}. \quad (3)$$

The autocorrelation matrix for the signals transmitted by the source is defined by,

$$\mathbf{C_{X_i}} = E[\mathbf{x_i}\mathbf{x_i^H}], \mathbf{x_i} = \begin{bmatrix} x_{i,1} \\ x_{i,2} \end{bmatrix}, \quad (4)$$

where $E[\cdot]$ and $(\cdot)^{\mathbf{H}}$ denote the expected value and Hermitian operators, respectively. We consider the case that only the sources can adjust their signals (power allocation) to maximize performance. The scenario is that the source is given a fixed amount of energy to be used over $2WT$ dimensions. For energy normalization purposes, we will take that this fixed amount of energy equals $\varepsilon \times WT$ which corresponds to some energy quantity ε multiplied by half the total number of dimensions (bandwidth W Hz and time $T/2$ corresponding to one time slot). Naturally, this amount of transmitted energy is attenuated for each link due to path loss and other effects. If the source chooses to transmit in the second slot, the required energy has to be garnered from this total energy budget. Hence, we force the constraint that

$$tr(\mathbf{C_{X_i}}) \leq 1 \quad (5)$$

such that a fixed amount of energy is put in use for all R_X with the definition of link energy ε_{AB} given above. For instance, if it chooses to transmit uncorrelated data with equal power in both time slots, the corresponding autocorrelation matrix becomes

$$\mathbf{C_{X_i}} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}. \quad (6)$$

We only study AF based relaying in this paper. The basic reason is that DF based relaying requires decoding at the relay node which introduces extra complexity to the node. Although this complexity might not be quite large for the TDMA case, we will see that joint decoding will be necessary in SDMA scheme and thus the relay incurs substantial extra complexity in that case.

In the AF case the relay simply amplifies the signal it receives from the source and retransmits it. Thus, the signal transmitted by the relay is

$$x_R = \frac{y_{R,1}}{\beta}, . \quad (7)$$

This normalization can be performed in many different ways [7]. We will prefer the case where an average normalization is performed. The average power of $y_{R,1}$ can be evaluated as

$$E[y_{R,1}y_{R,1}^*] = E[|\alpha_{S_iR}|^2]\varepsilon_{S_iR}E[x_{i,1}x_{i,1}{}^*] + N_0 \quad (8)$$

$$= \varepsilon_{S_iR}\mathbf{C_{X_i}}(1,1) + N_0, \quad (9)$$

where x^* denotes the complex conjugate of x and $\mathbf{A}(i,j)$ is the $(i,j)^{th}$ element of a matrix $\mathbf{A}$. Since the optimal value of $\mathbf{C_{X_i}}$ depends on the channel, we will take $\beta = \sqrt{\varepsilon_{S_iR} + N_0}$ for easier formulation. Rewriting (3), we have

$$y_{D,2} = \alpha_{S_iD}\sqrt{\varepsilon_{S_iD}}x_{i,2} + \alpha_{RD}\sqrt{\varepsilon_{RD}}\frac{y_{R,1}}{\beta} + n_{R,2}. \quad (10)$$

The following scaled version of $y_{D,2}$ is quite useful while evaluating capacities [4]

$$y'_{D,2} = \begin{bmatrix} \frac{\sqrt{\varepsilon_{S_iR}\varepsilon_{RD}}\alpha_{S_iR}\alpha_{RD}}{\sqrt{\varepsilon_{S_iR}+\varepsilon_{RD}|\alpha_{RD}|^2+N_0}} \\ \frac{\sqrt{\varepsilon_{S_iD}}\alpha_{S_iD}}{\sqrt{1+\frac{\varepsilon_{RD}|\alpha_{RD}|^2}{\varepsilon_{S_iR}+N_0}}} \end{bmatrix}^T \begin{bmatrix} x_{i,1} \\ x_{i,2} \end{bmatrix} + n'_{D,2}, \quad (11)$$

where $n'_{D,2}$ is a circularly symmetric complex Gaussian random variable with mean 0 and variance N_0, $(\cdot)^T$ is the transpose operator. Combining (2) and (11), we obtain the following equality which will be used in capacity evaluation.

$$\mathbf{y_D} = \begin{bmatrix} y_{D,1} \\ y'_{D,2} \end{bmatrix} = \mathbf{H_{AF,i}x_i} + \mathbf{n}, \quad (12)$$

$$\mathbf{H_{AF,i}} = \begin{bmatrix} \sqrt{\varepsilon_{S_iD}}\alpha_{S_iD} & 0 \\ \frac{\sqrt{\varepsilon_{S_iR}\varepsilon_{RD}}\alpha_{S_iR}\alpha_{RD}}{\sqrt{\varepsilon_{S_iR}+\varepsilon_{RD}|\alpha_{RD}|^2+N_0}} & \frac{\sqrt{\varepsilon_{S_iD}}\alpha_{S_iD}}{\sqrt{1+\frac{\varepsilon_{RD}|\alpha_{RD}|^2}{\varepsilon_{S_iR}+N_0}}} \end{bmatrix} \quad (13)$$

where $\mathbf{n}$ is a zero-mean complex Gaussian random vector with covariance matrix $N_0\mathbf{I_2}$ with $\mathbf{I_2}$ being the 2×2 identity matrix.

With the formulation given above, the problem turns into finding the capacity of a 2×2 MIMO system with the channel gain matrix $\mathbf{H_{AF}}$. With the normalization due to using the channel at two time slots, the TDMA capacity is given by the formula

$$C_{TDMA,i}(\mathbf{C_X}) = \frac{1}{2}\log_2 det(\mathbf{I_2} + \frac{1}{N_0}\mathbf{H_{AF,i}C_XH^H_{AF,i}}), \quad (14)$$

in bps/Hz, which is achieved by normally distributed $\mathbf{x}$ [8]. The autocorrelation maximizing (14) is obtained by waterfilling along the eigenvectors of $\mathbf{H^H_{AF,i}H_{AF,i}}$ for each source S_i.

III. SDMA Scheme

The main distinction between TDMA and SDMA modes is that the sources transmit their signals concurrently in the SDMA mode as opposite to TDMA. Naturally, the channel matrices will change in this mode and we will derive them later. We will first formulate the SDMA scheme in general. Using a single relay node, each source will enjoy a 2×2 channel matrix denoted $\mathbf{H_i}$. We will follow the approach in [9] where beamforming is utilized to achieve multi-user interference cancelation. Since channel matrices are 2×2, we can have only two beamforming vectors. Then, the received signal is

$$\mathbf{y} = \mathbf{H_1b_1}s_1 + \mathbf{H_2b_2}s_2 + \mathbf{n}, \quad (15)$$

where $\mathbf{b_i}$'s are 2×1 beamforming vectors with unit energy, $s_i's$ are transmitted symbols for each user, and $\mathbf{n}$ is the complex circularly symmetric Gaussian noise vector with autocovariance $N_0\mathbf{I_2}$. The equation (15) can be written as

$$\mathbf{y} = \mathbf{H_bs} + \mathbf{n}, \mathbf{H_b} = \begin{bmatrix} \mathbf{H_1b_1} & \mathbf{H_2b_2} \end{bmatrix}, \mathbf{s} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}. \quad (16)$$

When the least squares solution is utilized for ease of analysis to find an estimate of s so that

$$\hat{\mathbf{s}} = (\mathbf{H^H_bH_b})^{-1}\mathbf{H^H_by} \quad (17)$$

$$= \mathbf{s} + (\mathbf{H^H_bH_b})^{-1}\mathbf{H^H_bn}. \quad (18)$$

The autocovariance of the noise term in $\hat{\mathbf{s}}$ equals $N_0(\mathbf{H^H_bH_b})^{-1}$. As pointed out before, it is desired that signal from these two sources can be separately detected. Then, $N_0(\mathbf{H^H_bH_b})^{-1}$ should be a diagonal matrix and the necessary condition is

$$\mathbf{b^H_1H^H_1H_2b_2} = 0. \quad (19)$$

The signal-to-noise ratio (SNR) for source i becomes

$$SNR_i = \mathbf{b^H_iH^H_iH_ib_i}\frac{E[|s_i|^2]}{N_0}. \quad (20)$$

We will take that both sources have the same unit energy for each symbol so that $E[|s_1|^2] = E[|s_2|^2] = 1$ and all the link power terms are given place in the channel matrices.

Based on the formulation given above, the sum-rate capacity is

$$C_{SR}(\mathbf{b_1}) = \frac{1}{2}\log_2(1 + \frac{\mathbf{b^H_1H^H_1H_1b_1}}{N_0})(1 + \frac{\mathbf{b^H_2H^H_2H_2b_2}}{N_0}). \quad (21)$$

The goal is to maximize this capacity expression over the beamforming vectors. For this purpose, we will investigate the relationship between $\mathbf{b_1}$ and $\mathbf{b_2}$. In order to have no interference between sources, the equation (19) should hold. It is apparent that for any vector

$$\begin{bmatrix} m \\ n \end{bmatrix}^H \begin{bmatrix} n^* \\ -m^* \end{bmatrix} = 0 \quad (22)$$

and $\mathbf{v}$ and $\mathbf{u}$ are orthogonal such that $\mathbf{v^Hu} = 0$ for any vector $\mathbf{v}$ and $\mathbf{u} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\mathbf{v}^*$. In that case, $\mathbf{b^H_1H^H_1H_2}$ can be computed for a given $\mathbf{b_1}$ and the other beamformer can be obtained as

$$\mathbf{b_2} = \gamma\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\mathbf{H^T_2H^*_1b^*_1}, \quad (23)$$

where $(\cdot)^\mathbf{T}$ denotes the transposition operator and γ is a complex number not affecting the orthogonality. Keeping in mind that $\mathbf{b_2}$ should have unit energy as well,

$$\mathbf{b_2} = \frac{\mathbf{Kb^*_1}}{\sqrt{\mathbf{b^H_1K^TK^*b_1}}}, \mathbf{K} = \mathbf{AH^T_2H^*_1}, \mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \quad (24)$$

The SNR for the second source can be obtained using this derivation

$$\frac{\mathbf{b^H_2H^H_2H_2b_2}}{N_0} = \frac{\mathbf{b^T_1K^HH^H_2H_2Kb^*_1}}{\mathbf{b^H_1K^TK^*b_1}}\cdot\frac{1}{N_0} \quad (25)$$

$$= \frac{\mathbf{b^H_1K^TH^T_2H^*_2K^*b_1}}{\mathbf{b^H_1K^TK^*b_1}}\cdot\frac{1}{N_0} \quad (26)$$

$$= \frac{\mathbf{b^H_1H^H_1H_1b_1}}{N_0}\cdot\frac{c}{\mathbf{b^H_1K^TK^*b_1}}, \quad (27)$$

where the last line follows from the fact that $\mathbf{H_2A^TH^T_2H^*_2A^*H^H_2} = c\mathbf{I_2}$ for a positive real number c due to the properties of the matrix $\mathbf{A}$. So, the capacity maximizing beamformer $\mathbf{b^{max}_1}$ is

$$\mathbf{b^{max}_1} = \arg\max_{\mathbf{b_1}}\frac{1}{2}\log_2(1 + \frac{\mathbf{b^H_1H^H_1H_1b_1}}{N_0})\cdot (1 + \frac{\mathbf{b^H_1H^H_1H_1b_1}}{N_0}\cdot\frac{c}{\mathbf{b^H_1K^TK^*b_1}}) \quad (28)$$

We use the following heuristic to solve the maximization problem. The beamformer is chosen as either the strongest eigenvector of $\mathbf{H^H_1H_1}$ or the weakest eigenvector of $\mathbf{K^TK^*}$. By interchanging the orders of $\mathbf{H_1}$ and $\mathbf{H_2}$ matrices, another pair of solutions is obtained based on $\mathbf{b^H_2H^H_2H_2b_2}$ as well. The best out of these four is chosen as the solution to the maximization problem. Through Monte Carlo simulations, it can be observed that this solution provides better results than the singular value decomposition approach in [9].

Note that the capacity formulation given above and the solution proposed to maximize it do not consider any fairness issues. As opposed to sum-rate maximization, another alternative is maximizing the minimum of the source rates. So, we will also take the following max-min capacity formulation into account

$$C_{MM}(\mathbf{b_1}) = \frac{1}{2}\min\{\log_2(1+\frac{\mathbf{b_1^H H_1^H H_1 b_1}}{N_0}), \log_2(1+\frac{\mathbf{b_2^H H_2^H H_2 b_2}}{N_0})\}. \quad (29)$$

The best of four solutions suggested for the previous maximization will be used in this situation as well.

IV. Multi-User Relaying

We have explained how to establish an SDMA scheme when the channel matrices are given. We will now derive the channel matrices for the relay channel when there are two sources. It is assumed just as in the TDMA case that a transmission time of T seconds is split into two equal time slots. The relay listens to the channel in the first slot and transmits what it receives (after scaling) in the second time slot.

The received signals in the first time slot are

$$y_{R,1} = \alpha_{S_1R}\sqrt{\varepsilon_{S_1R}}x_{1,1} + \alpha_{S_2R}\sqrt{\varepsilon_{S_2R}}x_{2,1} + n_{R,1}, \quad (30)$$

$$y_{D,1} = \alpha_{S_1D}\sqrt{\varepsilon_{S_1D}}x_{1,1} + \alpha_{S_2D}\sqrt{\varepsilon_{S_2D}}x_{2,1} + n_{D,1}. \quad (31)$$

The scaling at the relay will be in the same line as the TDMA case so that

$$x_R = \frac{y_{R,1}}{\beta}, \beta = \sqrt{\varepsilon_{S_1R} + \varepsilon_{S_2R} + N_0}. \quad (32)$$

In the second time slot, destination receives

$$\begin{aligned} y_{D,2} &= \alpha_{S_1D}\sqrt{\varepsilon_{S_1D}}x_{1,2} + \alpha_{S_2D}\sqrt{\varepsilon_{S_2D}}x_{2,2} \\ &\quad + \alpha_{RD}\sqrt{\varepsilon_{RD}}x_R + n_{D,2} \\ &= \alpha_{S_1D}\sqrt{\varepsilon_{S_1D}}x_{1,2} + \alpha_{S_2D}\sqrt{\varepsilon_{S_2D}}x_{2,2} \\ &\quad + \frac{\alpha_{RD}\alpha_{S_1R}\sqrt{\varepsilon_{RD}\varepsilon_{S_1R}}}{\beta}x_{1,1} \\ &\quad + \frac{\alpha_{RD}\alpha_{S_1R}\sqrt{\varepsilon_{RD}\varepsilon_{S_1R}}}{\beta}x_{2,1} \\ &\quad + \frac{\alpha_{RD}\sqrt{\varepsilon_{RD}}}{\beta}n_{R,1} + n_{D,2}. \quad (33) \end{aligned}$$

When $y_{D,2}$ is scaled in order to normalize the noise term in it to variance N_0, the following expression is obtained for the received signal:

$$\mathbf{y_D} = \mathbf{H_1 x_1} + \mathbf{H_2 x_2} + \mathbf{n} \quad (34)$$

$$\mathbf{H_i} = \begin{bmatrix} \alpha_{S_iD}\sqrt{\varepsilon_{S_iD}} & 0 \\ \frac{\alpha_{RD}\alpha_{S_iR}\sqrt{\varepsilon_{RD}\varepsilon_{S_iR}}}{\sqrt{|\alpha_{RD}|^2\varepsilon_{RD}+\beta^2}} & \frac{\alpha_{S_iD}\sqrt{\varepsilon_{S_iD}}}{\sqrt{1+|\alpha_{RD}|^2\varepsilon_{RD}/\beta^2}} \end{bmatrix} \quad (35)$$

where $\mathbf{n}$ is a zero-mean complex Gaussian random vector with covariance matrix $N_0\mathbf{I_2}$.

Having derived the channel matrices for both sources, the SDMA scheme in the previous section can be applied directly. The only difference between the TDMA channel matrix and this one is that the scaling term at the relay now is influenced by both sources. This is due to the fact that the source signals can not be directly separated in the relay. This is also the reason why joint decoding is necessary at the relay node if a decode-and-forward is to be employed. As mentioned before, the main interest of this work is low-complexity operation so that only AF operation is considered.

V. Numerical Results

In this section we will compare the SDMA and TDMA schemes in terms of their ergodic and outage capacities. We will employ the Monte Carlo simulation method since the schemes are analytically intractable in their current form. Based on the confidence interval analysis, it was found out that using $100,000$ channel realizations for each Monte Carlo simulation keeps the 95% confidence intervals unnoticeably small so that that's the number of realizations in each run. Ergodic capacities are found simply by averaging whereas outage capacities are obtained by finding the $(100-p)^{th}$ percentile of the capacity distribution obtained via Monte Carlo simulations for $p\%$ outage capacity. Both type of capacities in the figures are plotted per single source.

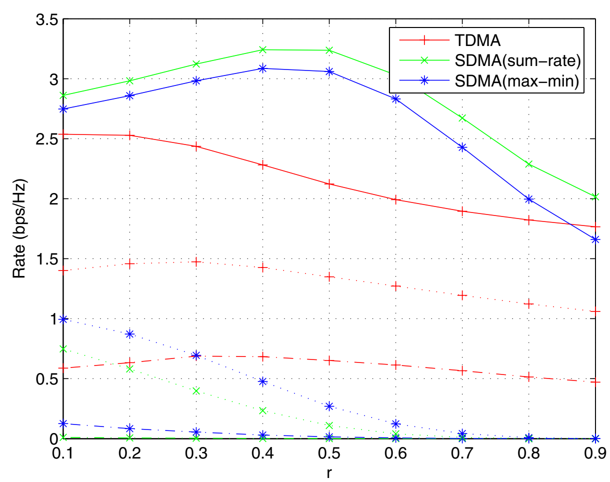

Fig. 2. $\frac{\varepsilon_{S_1D}}{N_0} = \frac{\varepsilon_{S_2D}}{N_0} = 10$dB, $\alpha = 4$. solid: ergodic capacity, dot: 90% outage capacity, dash-dot: 99% outage capacity

Another note is that energy normalization has to be performed to compare TDMA and SDMA schemes. In TDMA two sources S_1 and S_2 complete one round of communication in $2T$ seconds where they are active in only one half of that time. However, both sources transmit all the time in SDMA. For comparison with equal energy consumption in all the nodes, SDMA-mode source nodes spend half the energy of the TDMA-mode source nodes in a time duration T seconds.

In Fig. 2 ergodic and outage capacities for a line topology of nodes are depicted. Source to destination links are set to an SNR of 10dB where both sources are colocated at a distance of 1 unit away from the destination. The relay node is on the line between sources and destination at a distance r units from the sources. It is assumed that all nodes spend the same power and thus SNR expressions changing with $1/r^{\alpha}$ are used. The power falloff coefficient α is taken to be 4. As clearly seen in the figure, the ergodic capacity is significantly higher

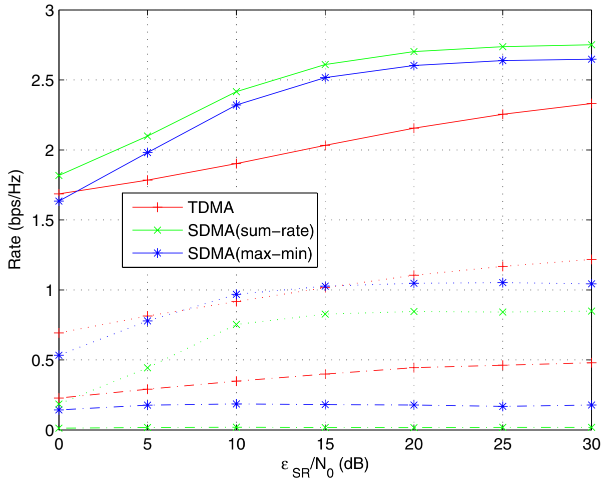

Fig. 3. $\frac{\varepsilon_{S_1D}}{N_0} = \frac{\varepsilon_{S_2D}}{N_0} = \frac{\varepsilon_{RD}}{N_0} = 10$dB, $\frac{\varepsilon_{SR}}{N_0}$ is varied. solid: ergodic capacity, dot: 90% outage capacity, dash-dot: 99% outage capacity

with SDMA-based schemes. However, outage capacities are very much higher in TDMA mode. As expected max-min SDMA formulation loses a little from the ergodic capacity however gains considerably on outage capacity. We have the following explanation for this observation. When SDMA is used, we need to find two beamformers which are orthogonal to each other where orthogonality is determined based on these beamformers' interaction with channel matrices by (19). Two such strong beamformers can be found very often and thus ergodic capacity improves compared to the TDMA case. However, it is not always possible to have such two strong beamformers all the time. This is even more prevalent when relay to destination link is stronger compared to source to destination link. In this case, most information will be sent first to the relay from the sources and then the relay will forward whatever it receives to the destination. The source to destination links are not able to help increase the rate significantly. So, the SDMA method explained in this paper will not be able to find two strong beamformers since the channel matrices are weak on the diagonals and it is not possible to find two strong beamformers from (19) which would make both $\mathbf{b_1^H H_1^H H_1 b_1}$ and $\mathbf{b_2^H H_2^H H_2 b_2}$ large. Thus, one of the beamformers is occasionally quite weak and thus it can accommodate a small rate over it. This is the reason why outage capacities are significantly smaller compared to TDMA. We will provide a few more scenarios to back this explanation.

Although the line topology is quite often used in literature, it doesn't address all cases of interest. One interesting case is when the sources and relay signals have similar SNR at destination. Sources and relay have similar link budgets to destination in this scenario due to many reasons such as geographical proximity, battery limitations, lifetime etc. Fig. 3 depicts this case where all links have an SNR of 10dB. SDMA ergodic capacities are once more higher compared to TDMA. SDMA max-min formulation again redeems a significant portion of the outage capacity loss and even surpasses TDMA 90% outage capacity in some region. In this case, the channel matrices are not weak on the diagonals so that SDMA have less difficulty in finding two strong beamformers and this enhances the outage capacities.

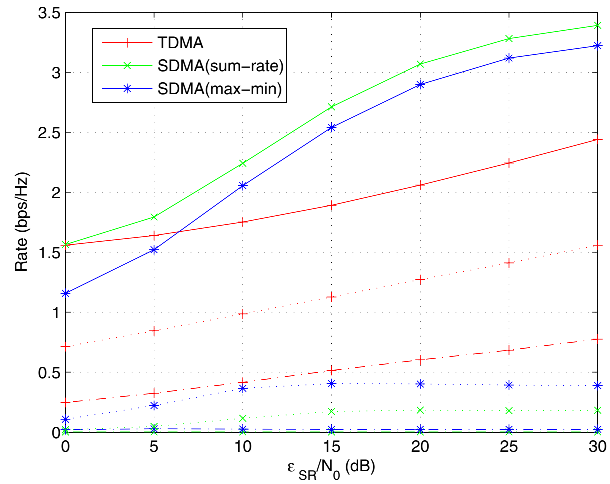

Fig. 4. $\frac{\varepsilon_{S_1D}}{N_0} = \frac{\varepsilon_{S_2D}}{N_0} = 10$dB, $\frac{\varepsilon_{RD}}{N_0} = 20$dB, $\frac{\varepsilon_{SR}}{N_0}$ is varied. solid: ergodic capacity, dot: 90% outage capacity, dash-dot: 99%outage capacity

To further observe the effect of weak diagonals on outage capacity, we will consider the scenario where the relay to destination SNR is increased to 20dB where the source to destination links are kept at an SNR of 10dB in Fig. 4. SDMA ergodic capacities are now even larger compared to that of TDMA. TDMA outage capacity increases but the opposite occurs for SDMA. Difficulty of finding two strong beamformers arises and SDMA outage capacities decrease even though the relay to destination link is much better.

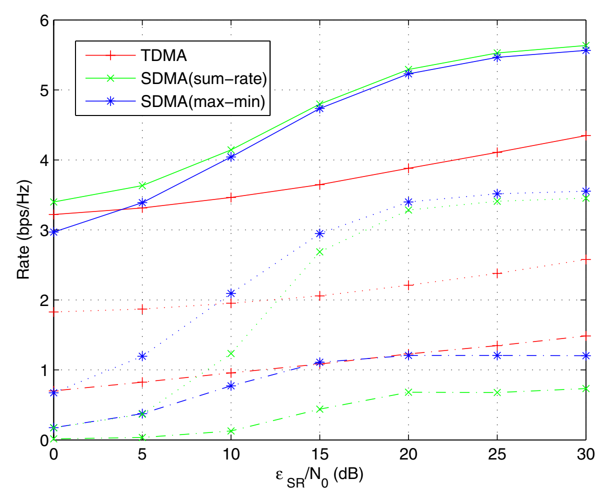

Fig. 5. $\frac{\varepsilon_{S_1D}}{N_0} = \frac{\varepsilon_{S_2D}}{N_0} = \frac{\varepsilon_{RD}}{N_0} = 20$dB, $\frac{\varepsilon_{SR}}{N_0}$ is varied. solid: ergodic capacity, dot: 90% outage capacity, dash-dot: 99% outage capacity

When both the source to destination and relay to destination links are kept at an SNR of 20dB, the diagonal terms in the channel matrices strengthen and the outage capacity problem should be alleviated. This is depicted in Fig. 5. In the figure both the ergodic and outage capacities grow larger for SDMA than for TDMA.

VI. Conclusions

We compared TDMA and SDMA based multiple access methods in fading relay channels. The basic conclusion is that ergodic capacity is higher with SDMA based schemes whereas outage capacity is most often higher for TDMA. This stems from the fact that when the relay to destination link has a larger SNR budget compared to the source to destination link, most of the information transmission is via the relay. However, SDMA method cannot easily find two strong beamformers and high rates for both sources become impossible. When the sources and relay can provide similar SNRs at destination, SDMA is more advantageous than TDMA both in terms of the ergodic and outage capacities. Hence, when a choice is made for the SDMA or TDMA methods, one should take the nature of the specific application at hand into consideration. Future studies will include methods to capitalize on the higher ergodic capacity results of SDMA schemes by techniques such as adaptive modulation.

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