Cooperative Output Regulation of Multi-agent Systems

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Acknowledgement

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Outline

1. Introduction
2. Classical Output Regulation
3. Cooperative Output Regulation
4. Distributed Feedforward Control
5. Distributed Internal Model Control
6. Concluding Remarks

1. Introduction

Collective Behaviors

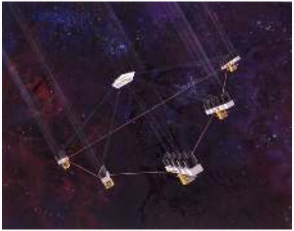
School of Fish
(S. Martinez, et al. 2007)

Flocking of Birds
(http://www.fws.gov)

Swarm of Locusts
(http://sciencephoto.com/image)

Multi-agent Systems

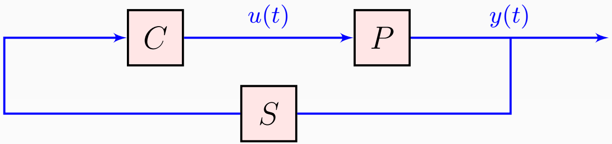
Robot Formation
(http://www-symbiotic.cs.ou.edu)



Formation of Spacecraft
(http://www.acsu.buffalo.edu)

Flight Formation
(http://4.bp.blogspot.com)

Control of Multi-Agent Systems



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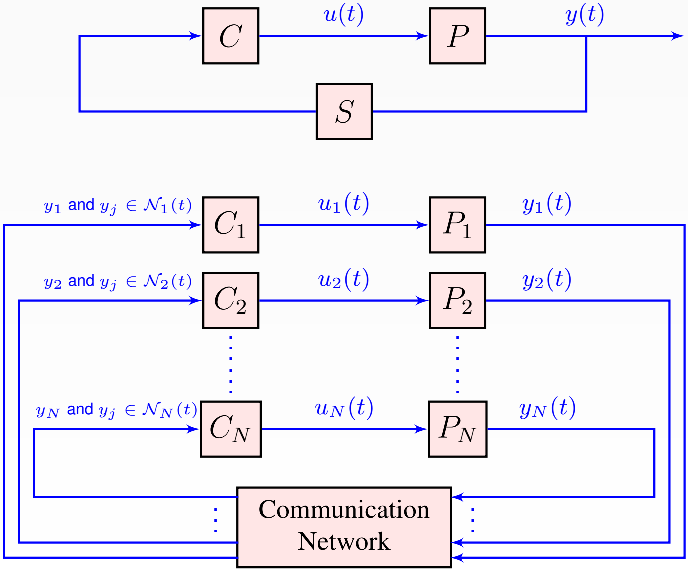

Control of Multi-Agent Systems (Cont.)

- The individual subsystems can only access the information of their neighbors due to communication constraints, thus the system has to be controlled by a distributed control protocol featuring the so-called nearest neighbor rule.
- All agents in the group have a common objective leading to collective behaviors.
- The global behavior of the system is jointly dictated by the system dynamics and the communication topology.
- A basic control problem for multi-agent systems is consensus.

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Consensus

- The leaderless consensus problem is to render the outputs of all agents of a multi-agent system asymptotically approach a common trajectory via a distributed control law.
- The leader-following consensus problem further requires that the outputs of all agents converge to a prescribed trajectory which is usually produced by another agent called leader.
- The two consensus problems have been mainly studied for linear, homogeneous multi-agent systems without subjecting to model uncertainty and external disturbances.
- Other variants of consensus problems include synchronization, flocking, swarming, formation, rendezvous (Fax and Murray, 2004), (Jadbabaie, Lin, Morse, 2003), (Ren and Beard, 2008), etc.

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Cooperative Output Regulation

- ➢ The output regulation problem aims to deal with the asymptotic tracking and disturbance rejection problem in an uncertain plant.
- ➢ This talk will generalize the output regulation problem from a conventional plant to a multi-agent system via a distributed control scheme, thus leading to the so-called cooperative output regulation problem.
- ➢ An application of the main result will lead to the solution of the leader-following consensus problem for a nonlinear heterogeneous multi-agent system subject to model uncertainty and external disturbances.
- ➢ The tools also apply to flocking, swarming, formation, rendezvous, etc.

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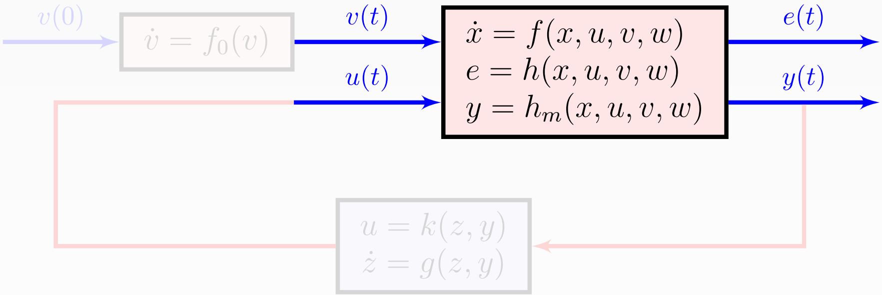
2. Classical Output Regulation

Problem Description



- Problem Statement: Design a control law such that, for all values of w, the solution of the closed-loop system is globally bounded, and satisfies

$$\lim_{t\to\infty} e(t) = 0.$$

- The control law is called measurement output feedback control. It includes error output feedback with $y = e$ and full information feedback with $y = (x, v)$ as two special cases.

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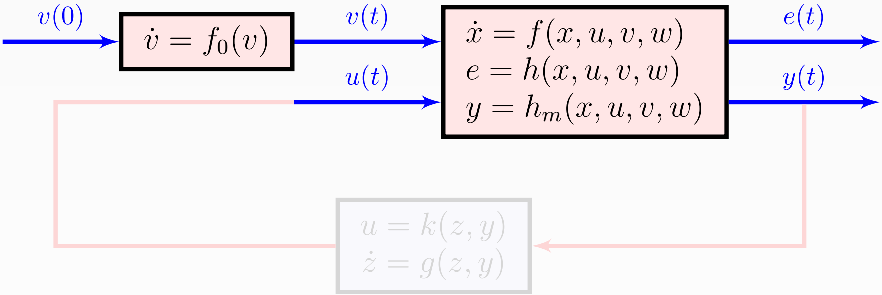

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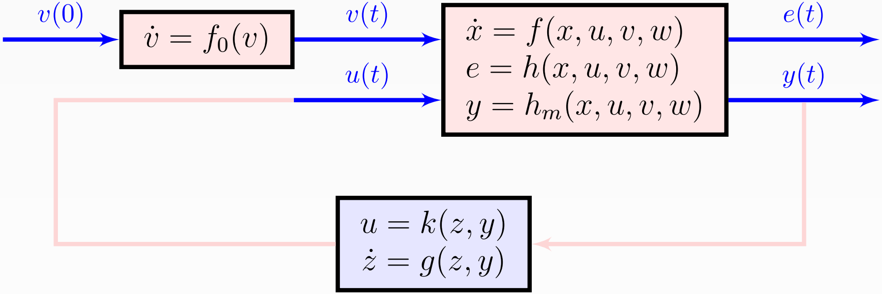

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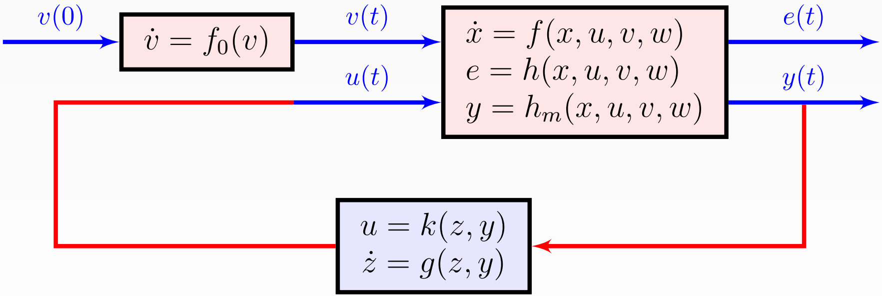

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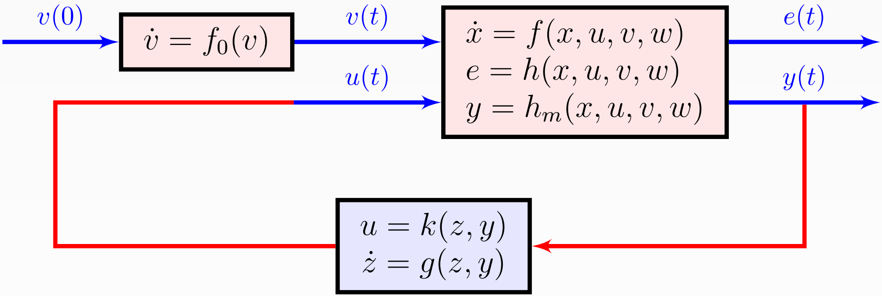

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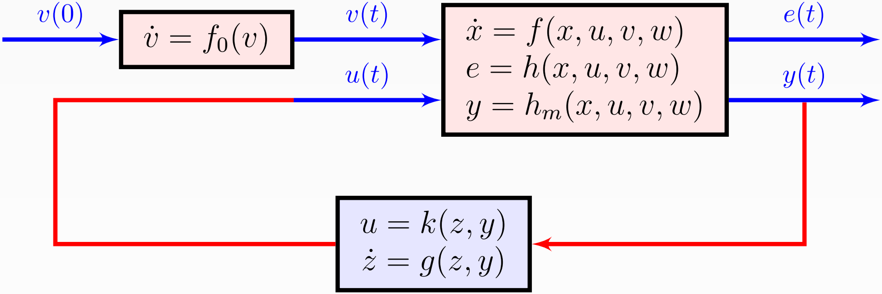

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- The problem has been studied for linear systems since the 1970s (Davison, Francis and Wonham, et al) and for nonlinear systems since the 1990s (Isidori, Byrnes, Huang, Khalil, et al).
- The problem can be viewed as a leader-following consensus problem with the exosystem as the leader and the plant as the single follower.
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Feedforward Control Approach

➢ Regulator equations (Isidori and Byrnes):

$$\begin{aligned} \frac{\partial \mathbf{x}(v,w)}{\partial v} f_0(v) &= f(\mathbf{x}(v,w), \mathbf{u}(v,w), v, w) \\ 0 &= h(\mathbf{x}(v,w), \mathbf{u}(v,w), v, w). \end{aligned} \tag{1}$$

The solution $(\mathbf{x}(v,w), \mathbf{u}(v,w))$ provides the precise feedforward information for annihilating the tracking error.

➢ Steady-state system: Let $\hat{x}(t) = \mathbf{x}(v(t), w)$, $\hat{f}(v,w) = f(\mathbf{x}(v,w), \mathbf{u}(v,w), v, w)$ and $\hat{h}(v,w) = h(\mathbf{x}(v,w), \mathbf{u}(v,w), v, w)$. Then (1) implies

$$\begin{aligned} \dot{\hat{x}} &= \hat{f}(v,w) \\ 0 &= \hat{h}(v,w) \end{aligned} \tag{2}$$

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- Error system: Let $\bar{x} = x - \mathbf{x}(v, w)$ and $\bar{u} = u - \mathbf{u}(v, w)$. Then $\bar{x}$ satisfies

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where $\bar{f}$ and $\bar{h}$ are some functions satisfying $\bar{f}(0, 0, v, w) = 0$ and $\bar{h}(0, 0, v, w) = 0$ for all v and all w.

- If the origin of the error system is stabilizable by a control law of the form $\bar{u} = k(\bar{x})$, then the output regulation problem of the original plant is solvable by the following control law

$$u = \mathbf{u}(v, w) + k(x - \mathbf{x}(v, w)). \quad (4)$$

- This control law does not work when w is unknown.

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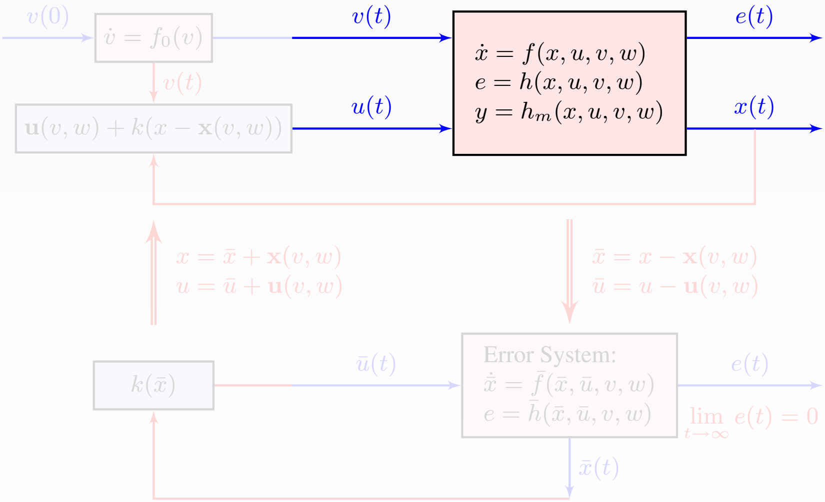

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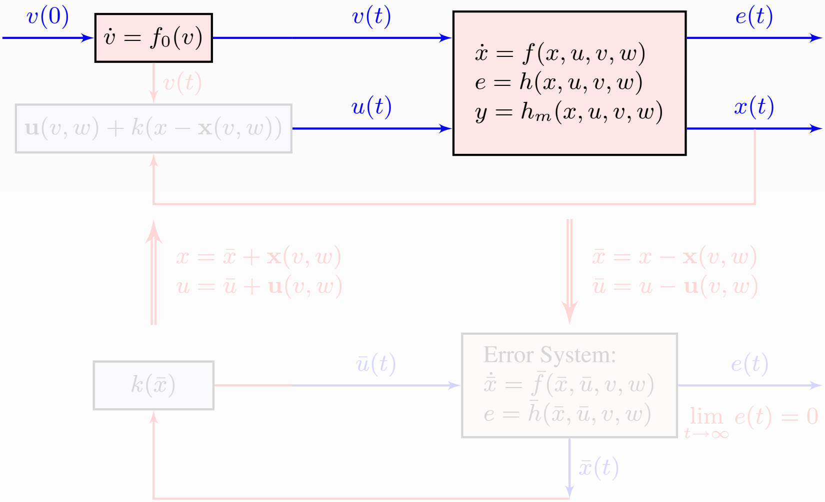

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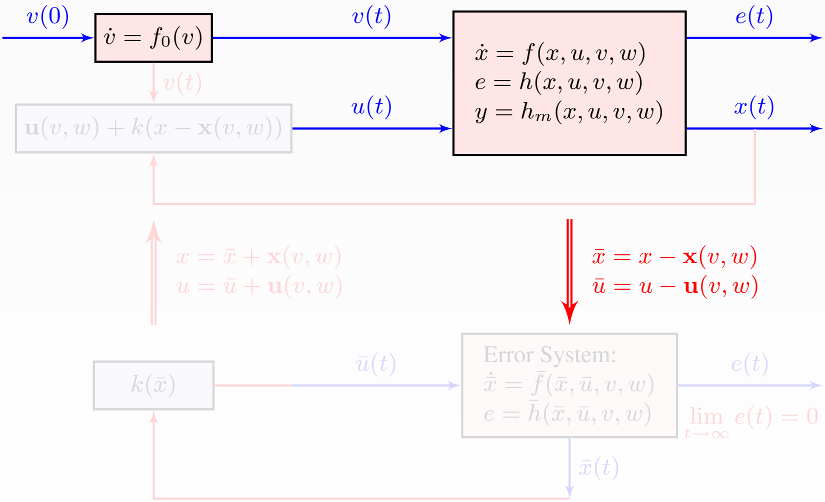

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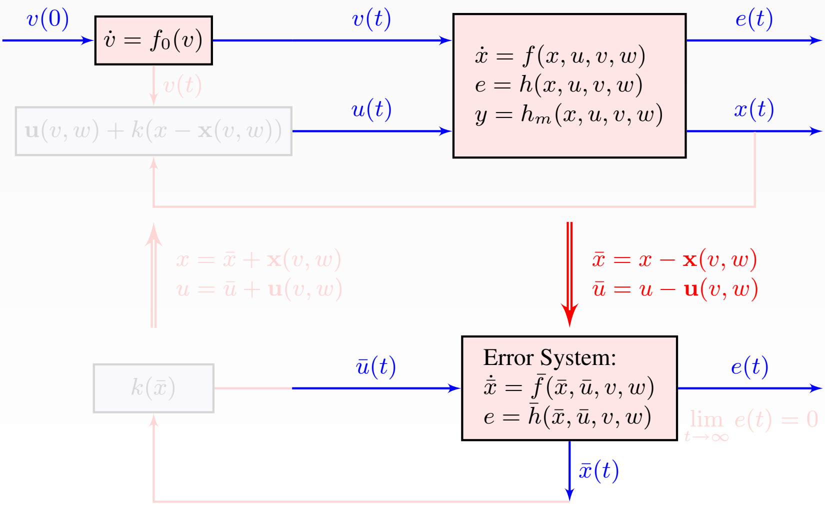

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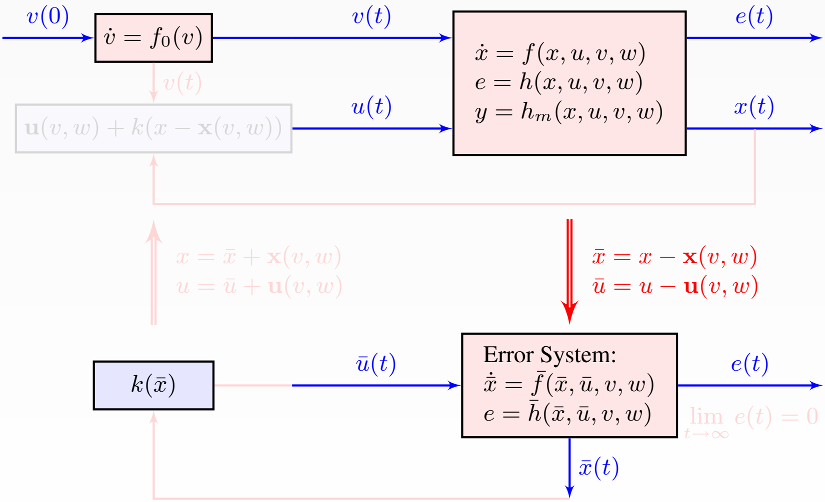

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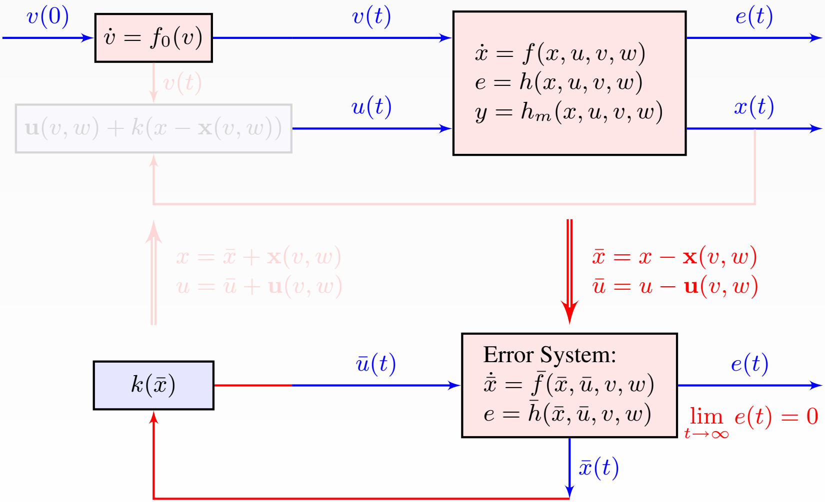

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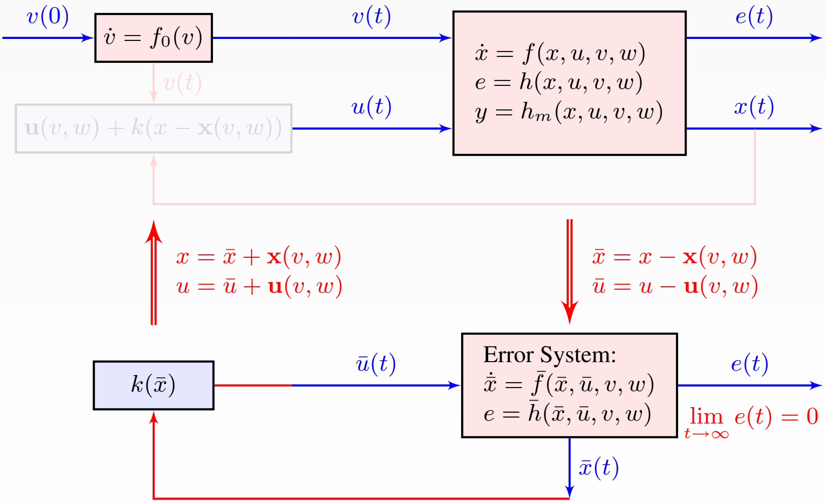

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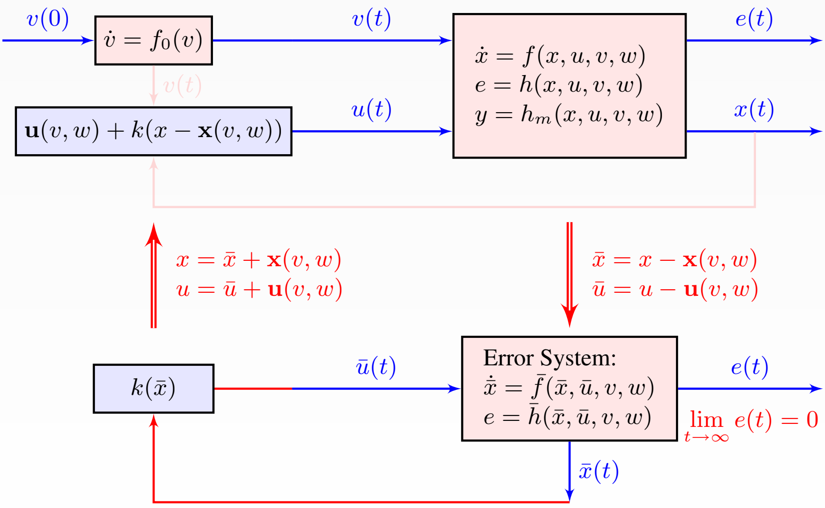

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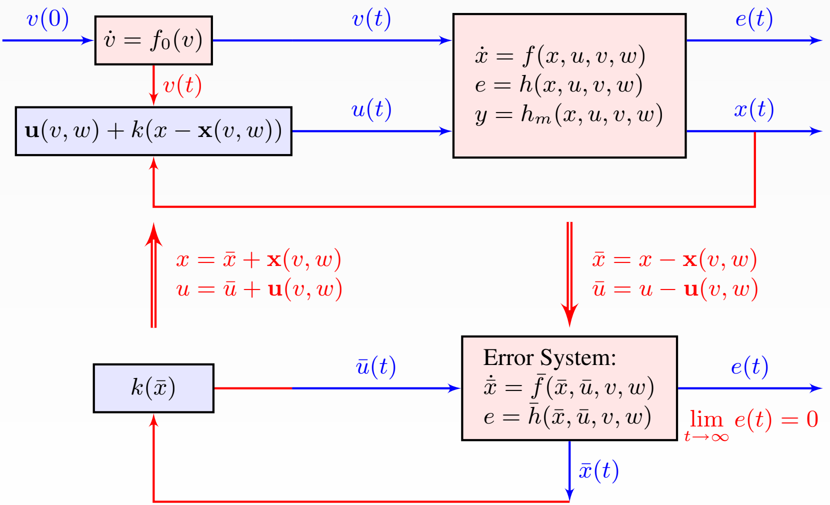

Feedforward Control Approach (Cont.)

- Regardless of the dimension of the exosystem, the dimension of the steady-state system is always the same as that of the plant. Thus the error system is always well defined. That is why this approach can handle heterogeneous multi-agent systems.

- Asymptotic tracking and disturbance rejection are handled simultaneously.

- Control law (4) is a full information control law. Under certain detectability condition, it is possible to synthesize an output feedback control law.

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Internal Model Approach ($y = e$)

An internal model candidate of the plant is a dynamic compensator

$$\begin{aligned} \dot{\eta} &= \gamma(\eta, y, u) = \gamma(\eta, h(x, u, v, w), u) && (5) \\ u &= \beta(\eta) && (6) \end{aligned}$$

which together with the original plant constitutes a so-called augmented system as follows

$$\begin{aligned} \dot{\eta} &= \gamma(\eta, y, u) \\ \dot{x} &= f(x, u, v, w) \\ e &= h(x, u, v, w) \end{aligned} \quad (7)$$

with the following property:

Internal Model Approach (Cont.)

- The augmented system has a controlled invariant manifold contained in the kernel of the error output, i.e., there exist functions $(\theta(v,w),\mathbf{x}(v,w),\mathbf{u}(v,w))$ satisfying

$$
\begin{aligned}
\frac{\partial\theta(v,w)}{\partial v}f_0(v) &= \gamma(\theta(v,w),0,\mathbf{u}(v,w))\\
\frac{\partial\mathbf{x}(v,w)}{\partial v}f_0(v) &= f(\mathbf{x}(v,w),\mathbf{u}(v,w),v,w)\\
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  where $(\theta(v,w),\mathbf{x}(v,w))$ is called the zero output manifold of the augmented system.
- An internal model candidate is further called an internal model if the zero output manifold $(\theta(v,w),\mathbf{x}(v,w))$ is stabilizable by a measurement output feedback controller.

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Internal Model Approach (Cont.)

- **Proposition**:
  Suppose the plant admits an internal model of the form (6). Then there exists a control law of the following form

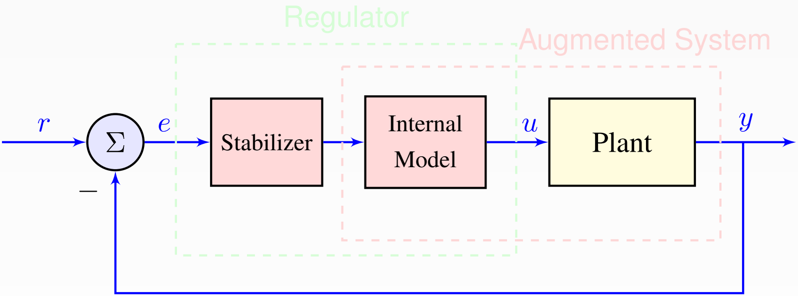
$$u = \beta(\eta) + k(\xi, y), \ \ \dot{\xi} = g(\xi, y)$$

that globally stabilizes the zero output manifold of the augmented system (7). Moreover, the following control law

$$u = \beta(\eta) + k(\xi, y), \ \dot{\xi} = g(\xi, y), \ \dot{\eta} = \gamma(\eta, y, u)$$

solves the global robust output regulation problem of the original plant.

Internal Model Approach (Cont.)



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- Find a stabilizing control law for the augmented system.

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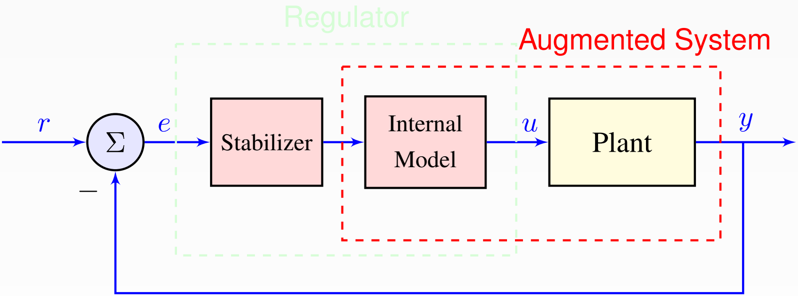

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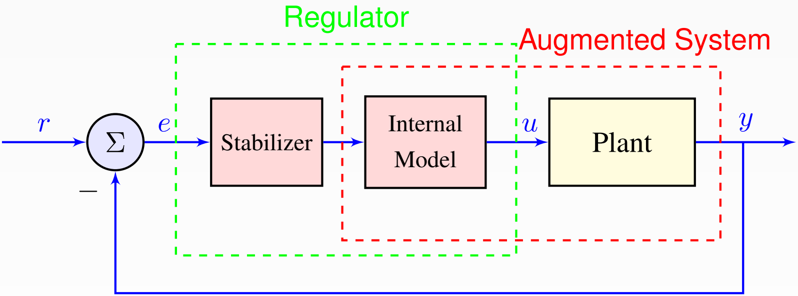

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Remark 1

1. *For linear systems, internal model exists generically (Davison, Francis, Wonham, et al). For nonlinear systems, various sufficient conditions have been given to guarantee the existence of internal models (Huang, Isidori, et al).*

2. *Since the augmented system is more complicated than the original plant, the stabilization problem of the augmented system may be more challenging.*

3. *Like the stabilization problem, the nonlinear output regulation problem has only been solved for nonlinear systems in special structures such as lower triangular form, upper triangular form, and output feedback form.*

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2. *Since the augmented system is more complicated than the original plant, the stabilization problem of the augmented system may be more challenging.*

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- The internal model can also be defined for the case where $y \neq e$. In such a case, the internal model, and hence the augmented system are more complicated. Moreover, y and e must satisfy what is called readability condition, i.e, there exists a function f vanishing at the origin such that $e = f(y)$.

- The Internal Model Principle (Wonham, 1976): A regulator is structurally stable only if the controller utilizes feedback of the regulated variable, and incorporates in the feedback loop a suitably reduplicated model of the dynamic structure of the exogenous signals which the regulator is required to process.

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3. Cooperative Output Regulation

Multi-agent Systems

$$\begin{aligned} \dot{x}_i &= f_i(x_i, u_i, v, w) \\ e_i &= h_i(x_i, u_i, v, w) \\ y_i &= h_{mi}(x_i, u_i, v, w) \end{aligned} \quad , \quad i = 1, ..., N \tag{8}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, $e_i \in \mathbb{R}^p$, $y_i \in R^{p_i}$, and $v \in \mathbb{R}^q$. The exogenous signal v is generated by the following exosystem:

$$\dot{v} = f_0(v) \tag{9}$$

- Systems (8) and (9) can be viewed as a multi-agent system of $N+1$ agents where
  - The system $\dot{v} = f_0(v)$ is viewed as the leader;
  - All subsystems of system (8) are viewed as N followers.
- If all followers can access the state v of the leader, then the output regulation problem of system (8) can be handled by a so-called (purely) decentralized control scheme.

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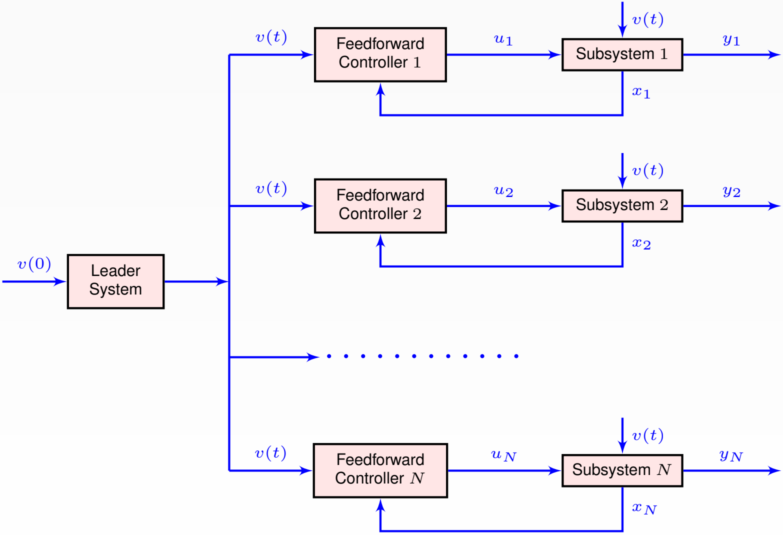
Decentralized Feedforward Control



- Two types of followers:
  - Informed followers whose control can access the information of the leader and;
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- The decentralized control does not work unless all followers are informed followers.
- The general case will be handled by the so-called distributed control law which makes use of the cooperation among neighboring agents.

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Graph Notation

➢ The communication graph associated with (8) is described by $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with $\mathcal{V}$ being the node set and $\mathcal{E}$ being the edge set.

- $\mathcal{V} = \{0, 1, ..., N\}$ with the node 0 associated with the exosystem and the other N nodes associated with the N subsystems of system (8);
- $(j, i) \in \mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, $i \neq j$, $i, j = 0, 1, ..., N$, if and only if the control u_i of the subsystem i, $i = 1, \cdots, N$, can access the measurement output y_j of subsystem j, $j = 0, 1, \cdots, N$;
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Distributed Control Laws

➢ Distributed control law:

$$u_i = k_i(z_i, y_i, y_j, j \in \mathcal{N}_i)$$
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where $y_0 = v$, k_i and g_i are some sufficiently smooth functions.

➢ Control law (10) satisfies the communication constraints: the i^{th} control u_i depends on y_j iff the agent i is a neighbor of the agent j.

➢ Two special cases:

1. Centralized or full information control when $\mathcal{N}_i = \{0, 1, \cdots, i-1, i+1, \cdots, N\}$, $i = 1, \cdots, N$;
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Problem Formulation

Definition 4.1

Given the plant, the exosystem, and the digraph $\mathcal{G}$, find a distributed control law such that, for any initial condition of the closed-loop system, and any value of w, the solution of the closed-loop system is bounded, and the error output satisfies

$$\lim_{t\to\infty} e_i(t) = 0, \quad i = 1, ..., N.$$

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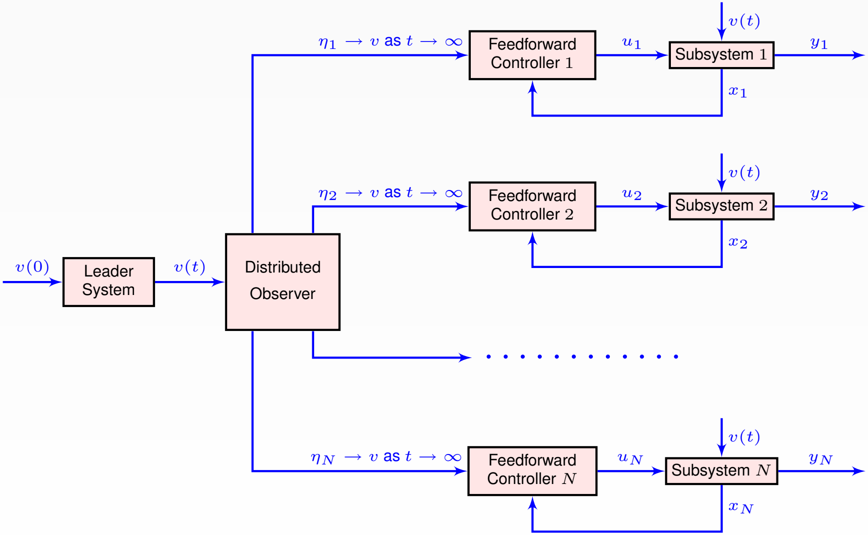
4. Distributed Feedforward Control

A Distributed Observer Based Scheme



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Distributed Observer

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$$\dot{\eta}_i = f_0(\eta_i) + \mu \left(\sum_{j \in \mathcal{N}_i} a_{ij}(\eta_j - \eta_i) \right), \ i = 1, \cdots, N \quad (11)$$

where μ is some positive number, $\eta_0 = v$, and, for $i = 1, ..., N$, $a_{ij} > 0$ if $j \in \mathcal{N}_i$, and $a_{ij} = 0$ if otherwise.

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Distributed Observer (Cont.)

- Proposition:
  Suppose the leader system is linear. Then there exists positive μ such that, for all $v(0), \eta_i(0)$, $i = 1, \cdots, N$,

$$\lim_{t \to \infty} (\eta_i - v) = 0$$

  iff the graph $\mathcal{G}$ contains a spanning tree with the leader as the root.

- The compensator (11) is a distributed global asymptotic observer of the exosystem.

- For some nonlinear exosystems, the compensator (11) can still be a global asymptotic observer.

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Assumption 1

For $i = 1, ..., N$, the regulator equations for each follower

$$\begin{aligned} \frac{\partial \mathbf{x}_i(v,w)}{\partial v} f_0(v) &= f_i(\mathbf{x}_i(v,w), \mathbf{u}_i(v,w), v, w) \\ 0 &= h_i(\mathbf{x}_i(v,w), \mathbf{u}_i(v,w), v, w) \end{aligned}$$

have a solution $(\mathbf{x}_i(v,w), \mathbf{u}_i(v,w))$.

Assumption 2

For $i = 1, ..., N$, each error system, i.e,

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Main Result

- Proposition:
  Under Assumptions 1 to 3, the cooperative output regulation problem of the multi-agent system is solvable by a distributed control law of the following form

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  iff the graph $\mathcal{G}$ contains a spanning tree with the leader as the root.

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Remark 2

1. *Under some detectability condition, it is possible to synthesize an output feedback control law.*

2. *Global or semi-global results can be obtained for some nonlinear systems.*

3. *The result can be extended to the case where the network is time-varying satisfying the jointly connected condition.*

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Linear Case

➢ Plant:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + E_i v, \\ \dot{v} &= A_0 v, \\ y_{mi} &= C_{mi} x_i + D_{mi} u_i + F_{mi} v, \\ e_i &= C_i x_i + D_i u_i + F_i v, \quad i = 1, \ldots, N. \end{aligned} \tag{13}$$

where $x_i \in R^{n_i}$, $y_{mi} \in R^{p_{mi}}$, $e_i \in R^{p_i}$, and $u_i \in R^{m_i}$.

➢ Regulator equations (Francis, 1977):

$$\begin{aligned} X_i A_0 &= A_i X_i + B_i U_i + E_i, \\ 0 &= C_i X_i + D_i U_i + F_i, \; i = 1, \cdots, N \end{aligned}$$

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Linear Case (Con't)

- Error System:

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where $\bar{x}_i = (x_i - X_i v)$ and $\bar{u}_i = (u_i - U_i v)$.

- Stabilization of error systems: Suppose, for $i = 1, \cdots, N$, the pairs (A_i, B_i) are stabilizable. Let K_i be such that $(A_i + B_i K_i)$ are Hurwitz. Then the error system is stabiliable by

$$\bar{u}_i = K_i\bar{x}_i, \quad i = 1, ..., N.$$

Thus the output regulation problem of (13) is solved by

$$u_i = K_i(x_i - X_i x_0) + U_i x_0 = K_i x_i + P_i x_0, \quad i = 1, ..., N.$$

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Example 1 (Su and Huang, 2012)

- Formation of a group of non-holonomic mobile robots (Ren and Atkins, 2007)

$$\begin{aligned}
\dot{x}_i &= \nu_i \cos(\theta_i) \\
\dot{y}_i &= \nu_i \sin(\theta_i) \\
\dot{\theta}_i &= \omega_i \\
m_i \dot{\nu}_i &= f_i \\
J_i \dot{\omega}_i &= \tau_i, \qquad i = 1, \ldots, 4
\end{aligned}$$

where (x_i, y_i, θ_i) are the Cartesian position and orientation of the robot center, respectively. $u_i = [f_i, \tau_i]^T$ with f_i the force and τ_i the torque applied to the robot.

Example 1 (Cont.)

- The robot hand position:

$$h_i = \begin{bmatrix} x_{hi} \\ y_{hi} \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} + d_i \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix}$$

where d_i is the distance of the robot hand position off the wheel axis of the ith mobile robot.

Example 1 (Cont.)

- Suppose the leader's position vector $h_0(t) \in \mathbb{R}^2$ moves along a straight line with a constant velocity as follows.

$$h_0(t) = \begin{bmatrix} a_{xi} \\ a_{yi} \end{bmatrix} t + \begin{bmatrix} x_{d0} \\ y_{d0} \end{bmatrix}$$

- Find a distributed state feedback control law so that the followers and the leader will form a geometric pattern shown in the following figure as the leader is moving.

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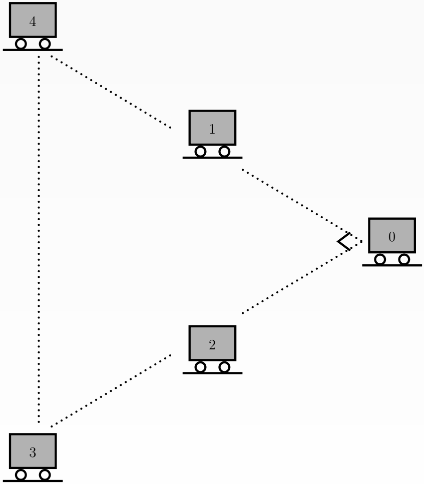

Figure: A leader-following formation.

Example 1 (Cont.)

- Exosystem (leader). Let

$$\dot{v} = A_0 v = \left[\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \otimes I_2 \right] v.$$

Then $h_0(t) = \left[\begin{bmatrix} 1 & 0 \end{bmatrix} \otimes I_2 \right] v(t)$ when $v(0) = [x_{d0}, a_{xi}, y_{d0}, a_{yi}]$.

- The error output (goal-seeking errors):

$$e_i(t) = (h_i(t) - h_{di}) - h_0(t), \quad i = 1, 2, 3, 4$$

where $h_i(t)$ is the position vector of the i^{th} follower and h_{di} is the desirable separation of the i^{th} follower and the leader.

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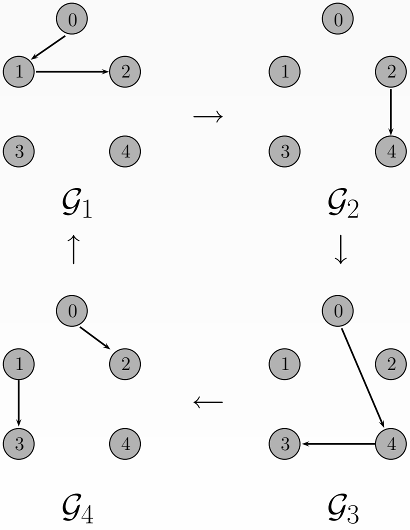
Example 1 (Cont.)

- The communication graph $\mathcal{G}_{\sigma(t)}$: a switching graph dictated by the following switching signal:

$$\sigma(t) = \begin{cases} 1, & \text{if } sT \leq t < (s+\frac{1}{4})T; \\ 2, & \text{if } (s+\frac{1}{4})T \leq t < (s+\frac{1}{2})T; \\ 3, & \text{if } (s+\frac{1}{2})T \leq t < (s+\frac{3}{4}T); \\ 4, & \text{if } (s+\frac{3}{4}T) \leq t < (s+1)T. \end{cases} \quad (14)$$

where $T = 0.1$sec., and $s = 0, 1, 2, \ldots$. The signal $\sigma(t)$ defines four fixed graphs $\mathcal{G}_i$, $i = 1, 2, 3, 4$.

A Communication Graph



Example 1 (Cont.)

movie

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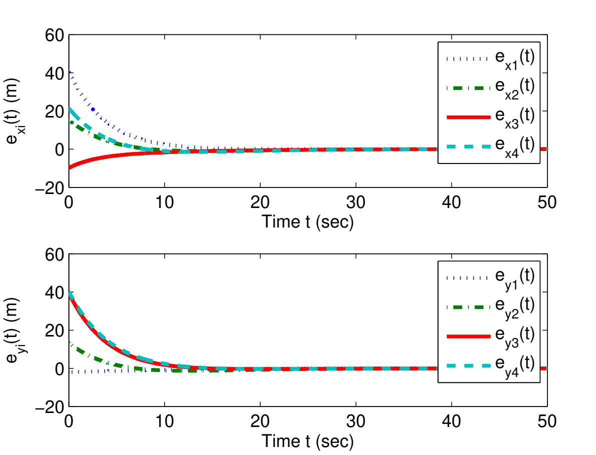


Figure: Goal-seeking errors of the four robots.

Example 1 (Cont.)

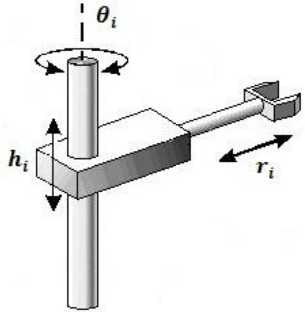
movie

Example 2 (Cai and Huang, 2012)

➢ Multiple three-link cylindrical manipulators (follower):

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i, \quad i = 1, 2, ..., N$$

where the joint variable vector $q_i = (\theta_i, h_i, r_i)$ with θ_i, h_i, r_i denoting waist rotation, lift and reach values, respectively.



Example 2 (Cont.)

- Various Matrices (Lewis, et al, 1993):

$$M_i(q_i) = \begin{bmatrix} J_i + m_{i2} r_i^2 & 0 & 0 \\ 0 & m_{i1} + m_{i2} & 0 \\ 0 & 0 & m_{i2} \end{bmatrix}$$

$$C_i(q_i, \dot{q}_i) = \begin{bmatrix} m_{i2} r_i \dot{r}_i & 0 & m_{i2} r_i \dot{\theta}_i \\ 0 & 0 & 0 \\ -m_{i2} r_i \dot{\theta}_i & 0 & 0 \end{bmatrix}$$

$$G_i(q_i) = \begin{bmatrix} 0 \\ (m_{i1} + m_{i2}) g h_i \\ 0 \end{bmatrix}$$

Example 2 (Cont.)

- Exosystem (leader):

$$\begin{bmatrix} \dot{q}_0 \\ \ddot{q}_0 \end{bmatrix} = A_0 \begin{bmatrix} q_0 \\ \dot{q}_0 \end{bmatrix} = \begin{bmatrix} 0 & I_3 \\ -I_3 & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ \dot{q}_0 \end{bmatrix}$$

where $q_0 \in R^3$. The leader system can generate sinusoidal signals of arbitrary amplitudes and initial phases.

- The communication graph is the same as Example 1.

Example 2 (Cont.)

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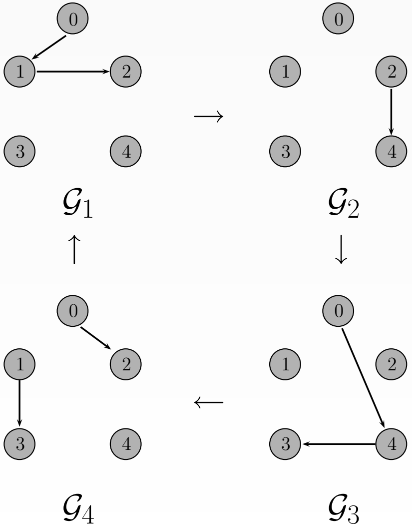

Example 2 (Cont.)

Remark 3

- *The system does not satisfy the following standard assumption:* $k_{\underline{m}} I_p \leq M_i(q_i) \leq k_{\overline{m}} I_p$, $||C_i(q_i, \dot{q}_i)|| \leq k_c ||\dot{q}_i||$, *and* $||G_i(q_i)|| \leq k_g$;
- *The communication graph is time-varying and is disconnected at all* $t \geq 0$;
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- *Since the uncertainty satisfies the linear parameterization condition and the dynamic graph is jointly connected, the problem is solvable by a combination of the feedforward control approach and adaptive control.*

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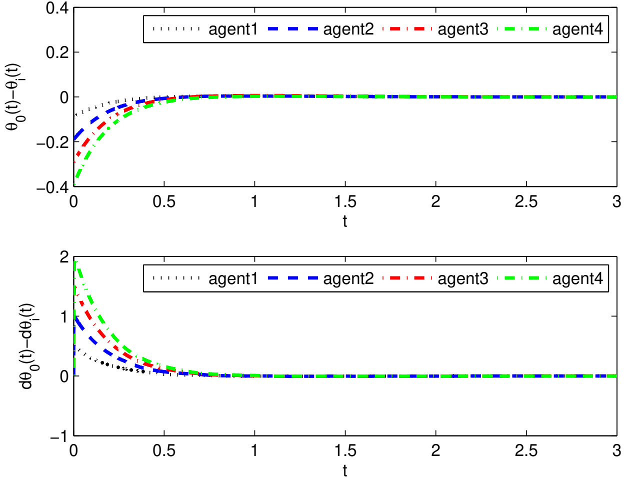
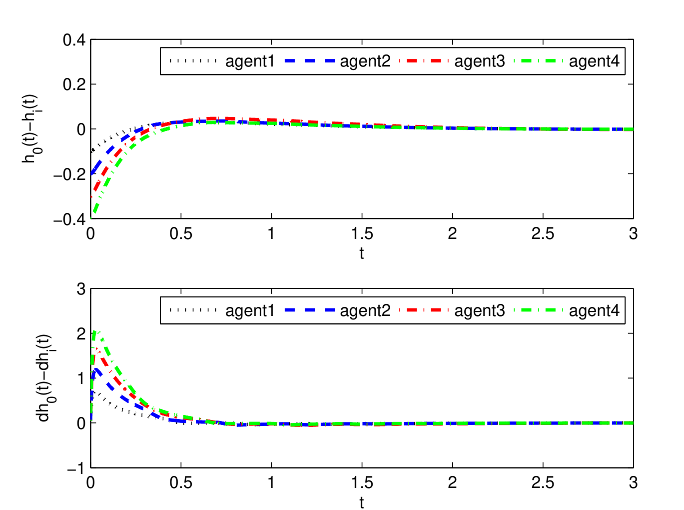
Example 2 (Cont.)

➢ Tracking performance of θ_i, $\dot{\theta}_i$.



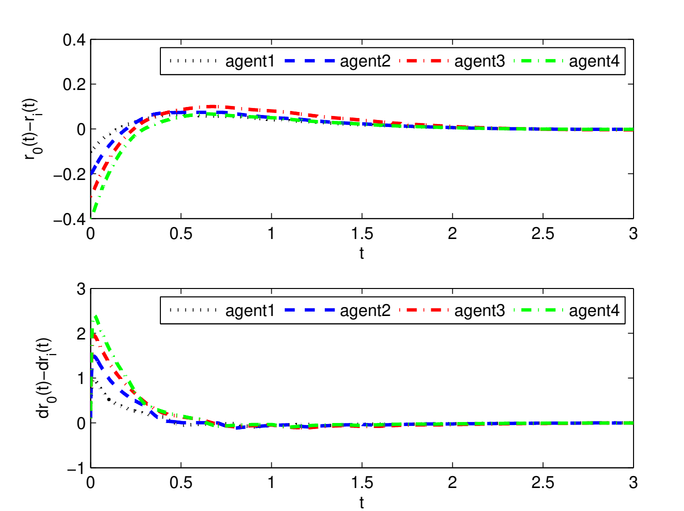
Example 2 (Cont.)

➢ Tracking performance h_i, $\dot{h}_i$.



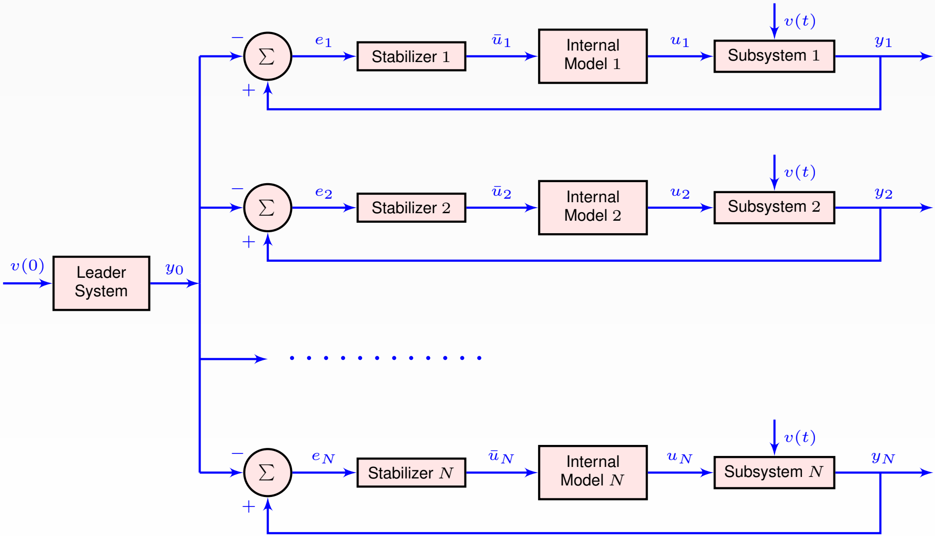
Example 2 (Cont.)

➢ Tracking performance of r_i and $\dot{r}_i$.

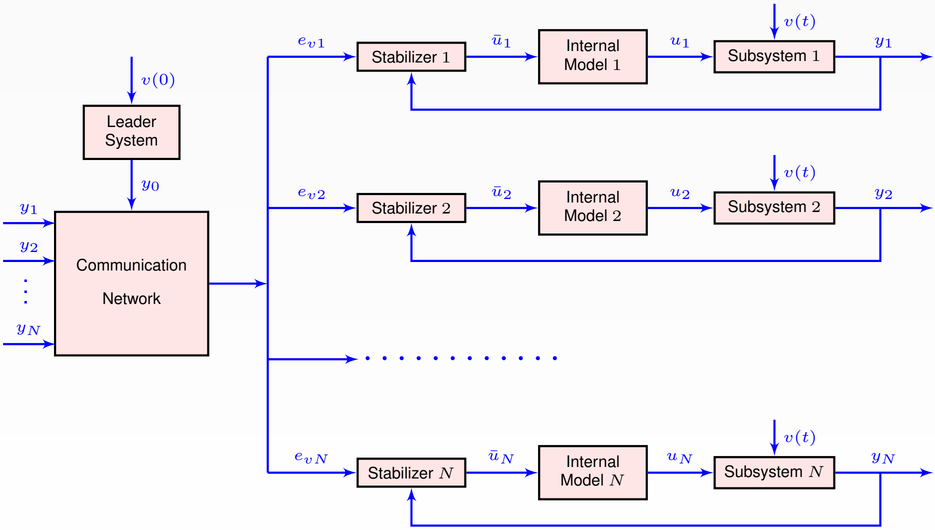


5. Distributed Internal Model Control

Decentralized Internal Model Control Scheme



Distributed Internal Model Control Scheme



Assumptions:

Assumption 4

The digraph $\mathcal{G}$ contains a spanning tree with the leader as the root.

Assumption 5

For $i = 1, \cdots, N$, and $j \in \mathcal{N}_i$, $e_i - e_j = h_{ij}(y_i, y_j)$ for some functions h_{ij}.

Remark 4

- *Assumptions 4 and 5 guarantee that $e = (e_1, \cdots, e_N)$ is readable from $y = (y_1, \cdots, y_N)$ even though e_i may not be readable from y_i when the agent i is an uninformed follower.*

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Virtual Error Output

$$e_{vi} = \sum_{j=0}^{N} a_{ij}(e_i - e_j), \quad i = 1, ..., N$$

where, for $i = 1, ..., N$, $a_{ij} > 0$ if $j \in \mathcal{N}_i$, and $a_{ij} = 0$ if otherwise.

- Two properties:
  1. $e_{vi} = 0, i = 1, \cdots, N \quad \Leftrightarrow \quad e_i = 0, i = 1, \cdots, N.$
  2. $e_{vi} = h_{vi}(y_i, y_j, j \in \mathcal{N}_i), \quad i = 1, ..., N.$
- Any control law of the following form

$$\begin{cases} u_i = k_i(\xi_i, y_i, e_{vi}), & i = 1, ..., N. \\ \dot{\xi}_i = g_i(\xi_i, y_i, e_{vi}) \end{cases} \tag{15}$$

satisfies the communication constraints.

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$$\begin{cases} u_i = k_i(\xi_i, y_i, e_{vi}), & i = 1, ..., N. \\ \dot{\xi}_i = g_i(\xi_i, y_i, e_{vi}) \end{cases} \tag{15}$$

satisfies the communication constraints.

➢ Proposition:
Under Assumptions 4 and 5, suppose, for $i = 1, \cdots, N$, each subsystem admits an internal model candidate of the form

$$\dot{\eta}_i = \gamma_i(\eta_i, y_i, u_i), \quad u_i = \beta_i(\eta_i), \quad i = 1, ..., N. \tag{16}$$

Then the collection of the N internal model candidates (16) constitutes an internal model candidate for the multi-agent system (8). Thus, if a control law of the form

$$u_i = \beta_i(\eta_i) + k_i(\xi_i, y_i, e_{vi}), \quad \dot{\xi}_i = g_i(\xi_i, y_i, e_{vi}), \quad i = 1, ..., N, \tag{17}$$

globally stabilizes the augmented system, then the control law

$$u_i = \beta_i(\eta_i) + k_i(\xi_i, y_i, e_{vi}), \quad \dot{\xi}_i = g_i(\xi_i, y_i, e_{vi}), \ \dot{\eta}_i = \gamma_i(\eta_i, y_i, u_i)$$

solves the output regulation problem of the multi-agent system (8).

Remark

1. This proposition only converts the output regulation problem of the given multi-agent system (8) into the stabilization problem of the augmented system, and it says nothing about the stabilizability of the augmented system by a distributed control law.
2. In general, the stabilization problem of the augmented system is intractable. Yet this proposition has been successfully applied to linear systems and nonlinear systems with special structures such as lower triangular form, and output feedback form (Su and Huang, 2012).

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Example 3 (Su and Huang, 2012)

- A harmonic oscillator (leader):

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

- Multiple Van der Pol oscillators (follower):

$$\begin{aligned} \dot{x}_{1i} &= x_{2i} \\ \dot{x}_{2i} &= -x_{1i} + \mu_i(w)x_{2i}(1 - x_{1i}^2) + b_i(w)u_i \\ e_i &= x_{1i} - v_1, \quad i = 1, 2, 3, 4 \end{aligned}$$

  - μ_i and b_i are unknown.
  - The nonlinearity does not satisfy the global Lipschitz condition.

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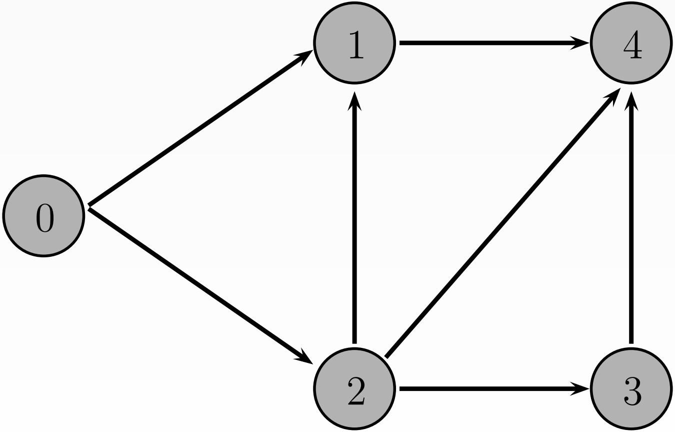
A Communication Graph



Example 3 (Cont.)

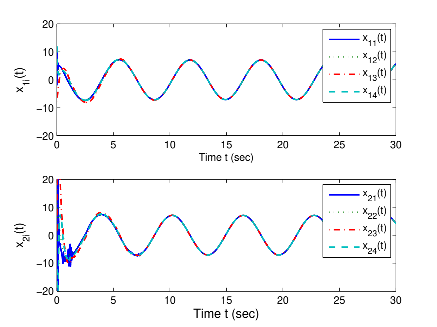


Figure: The states of the subsystems.

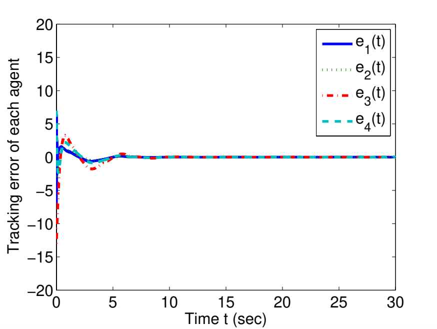


Figure: The tracking errors of the subsystems.

6. Concluding Remarks

- The cooperative output regulation problem for general multi-agent systems has been formulated and studied by both distributed feedforward control approach and distributed internal model control approach.
- The problem has generalized the output regulation problem from a single system to a multi-agent system and contains the leader-following consensus problem as a special case.
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Concluding Remarks (Cont.)

- To certain degree, the following three statements are equivalent.

  1. The output regulation problem of a multi-agent system is solvable by a centralized control law;
  2. The output regulation problem of a multi-agent system is solvable by a decentralized control law when all followers are informed;
  3. The output regulation problem of a multi-agent system is solvable by a distributed control law when the graph $\mathcal{G}$ is connected.

- Why? A spanning tree in the graph $\mathcal{G}$ allows the information of the leader to reach any other followers through a path.

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1. Uncertain exosystem. Note that a sinusoidal disturbance $A\sin(\omega t+\phi)$ has to be modeled by an uncertain exosystem

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

2. Connectivity preservation
3. Time delay: Cyclic small gain method by Z. Jiang, et al.

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Thanks

Thank you!

Back to Outline