Cooperative Strategies for the Half-Duplex Gaussian Parallel Relay Channel: Simultaneous Relaying versus Successive Relaying

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***Abstract*— We consider the problem of cooperative communication for a network composed of two half-duplex parallel relays with additive white Gaussian noise. Two protocols, i.e., *Simultaneous* and *Successive* relaying, associated with two possible relay orderings are proposed. The simultaneous relaying protocol is based on *Dynamic Decode and Forward (DDF)* scheme. For the successive relaying protocol, a *Non-Cooperative Coding* based on *Dirty Paper Coding (DPC)* is proposed. We also propose a general achievable rate based on the combination of the proposed simultaneous and successive relaying schemes. The optimum ordering of the relays and hence the capacity of the half-duplex Gaussian parallel relay channel in low and high SNR scenarios is derived. In low SNR scenario, we show that under certain conditions for the channel coefficients the ratio of the achievable rate of the simultaneous relaying protocol, based on the *DDF* scheme, to the cut-set bound of the half-duplex Gaussian parallel relay channel tends to 1. On the other hand, as SNR goes to infinity, we prove that successive relaying protocol, based on the *DPC* scheme, asymptotically achieves the capacity of the network.**

I. Introduction

The continuous growth in wireless communication has motivated information theoretists to extend shannon's information theoretic arguments for a single user channel to the scenarios that involve communication among multiple users. In this regard, cooperative wireless communication has been in the focus of attention during recent years.

Basically, relay channel is a three terminal network which was introduced for the first time by Van der Meulen in 1971 [1]. The most important capacity results of relay channel were reported by Cover and El Gamal [2]. Relaying strategies for the network with multiple relays have been discussed in [3]- [6]. Schein in [3] studied the possible coding scheme for a parallel relay channel, which consists of a source, two parallel relays, and the final destination. Later on, authors in [4] considered cooperative strategies for general multiple relay network. These works are dealt with full-duplex relay networks.

Since the current operating radios that can receive and transmit simultaneously in the same frequency band require complex and expensive components, half-duplex relaying has drawn a great deal of attention recently.

In this paper, we study transmission strategies for a network with a source, a destination, and two half-duplex relays with additive white Gaussian noise which cooperate with each other to facilitate data transmission from the source to the destination.

Half-duplex relaying, in multiple relay networks, is studied in [8] [13]- [21]. Gastpar in [8] show that in a half-duplex Gaussian parallel relay channel with infinite number of relays, the optimum coding scheme is AF. Rankov and Wittneben in [13] [14] further study the problem of half-duplex relaying in a two-hop communication scenario. In their study of half-duplex networks, they also consider a parallel relay setup with two relays where there is no direct link between the source and the destination, while there exists a link between the relays. Their relaying protocols are based on either AF or DF, in which the relays successively forward their messages from the source to the destination. We call this protocol "*Successive Relaying*" in the sequel. Xue and Sandhu in [15] further study different half-duplex relaying protocols for the Gaussian parallel relay channel.

In this work, we consider the problem of half-duplex relaying in a network with two relays in which there is no direct link between the transmitter and the receiver. Our primary objective is to find the best ordering of the relays. We consider two relaying protocols, i.e., simultaneous relaying versus successive relaying, associated with two possible relay orderings. For simultaneous relaying, each relay exploits "Dynamic DF (DDF)". It should be noted that the DDF scheme proposed here is slightly different from the DDF introduced in [18] and [19]. In those works, they apply DDF scheme to the set-up of the single relay channel where the nodes have just the CSI of their receiving channel. For successive relaying, we study a *Non-Cooperative Coding* scheme based on "Dirty Paper Coding (DPC)". In [21], we also study another *Cooperative Coding* scheme based on "Block Markov Encoding (BME)" comprehensively and compare it with the *Non-Cooperative Coding* scheme. It is worth noting that the authors in [20] also propose successive relaying protocol for the set up with two parallel relays and a direct link between the source and the destination. They propose a simple repetition coding at the relays, and show that their scheme can recover the loss in the multiplexing gain, while achieving some diversity gain.

We derive the optimum relay ordering in low and high SNR scenarios. In low SNR scenarios and under certain channel conditions, we show that the ratio of the achievable rate of DDF for simultaneous relaying to the cut-set bound tends to one. On the other hand, in high SNR scenarios, we prove that the proposed DPC for successive relaying asymptotically achieves the capacity. Unlike [16], in which the authors only consider successive relaying and propose a combined BME and DPC scheme, in this paper we combine simultaneous and successive relaying protocols and propose a general achievable rate for a half-duplex Gaussian parallel relay channel with two relays. We show that in low SNR scenario and under certain channel conditions, our general achievable scheme is converted to simultaneous relaying based on DDF, while in the high SNR scenarios, when the ratio of the relay powers to the source power remain constant, it becomes successive relaying based on DPC to achieve the capacity. In [21], we also prove that not only BME with backward decoding achieves better rate against BME with successive decoding, it also leads to a simple strategy in which at most one of the relays is required to cooperate with the other one in sending the bin index of the other relay's message. Accordingly, the combination of employing BME at one relay and DPC at the other one always achieves better rate than BME at both relays.

The rest of the paper is organized as follows. In section II, the system model is introduced. In section III, the achievable rates and coding schemes for a half-duplex relay network are derived. Optimality results are discussed in section IV. Simulation results are presented in section V. Finally, section VI concludes the paper.

A. Notation

Throughout the paper, the superscript ^{H} stands for matrix operation of conjugate transposition. Lowercase bold letters and regular letters represent vectors and scalars, respectively. For any two functions $f(n)$ and $g(n)$, $f(n) = O(g(n))$ is equivalent to $\lim_{n\to\infty} \left|\frac{f(n)}{g(n)}\right| < \infty$, and $f(n) = \Theta(g(n))$ is equivalent to $\lim_{n\to\infty} \frac{f(n)}{g(n)} = c$, where $0 < c < \infty$. And $C(x) \triangleq \frac{1}{2}\log_2(1 + x)$.

II. System Model

Our setup is a Gaussian network which consists of a source, two half-duplex relays, and a destination, and there is no direct link between the source and the destination. Here we define four time slots according to the transmitting and receiving mode of each relay (See Figure 1). The portion of the time that the communication is performed in time slot b is denoted by t_b ($\sum_{b=1}^{4} t_b = 1$). Nodes 0, 1, 2, and 3 represent the source, relay 1, relay 2, and the destination, respectively. Moreover, the transmitting and receiving signals at node a during time slot b are represented by $\mathbf{x}_a^{(b)}$ and $\mathbf{y}_a^{(b)}$, respectively. Hence, at each node $c \in \{1, 2, 3\}$ we have

$$\mathbf{y}_c^{(b)} = \sum_{a\in\{0,1,2\}} h_{ac}\mathbf{x}_a^{(b)} + \mathbf{z}_c^{(b)}. \tag{1}$$

where h_{ac}'s denote channel coefficients from node a to node c, and $\mathbf{z}_c^{(b)}$'s are AWGN terms with zero mean and variance of "1" per dimension at each node c.

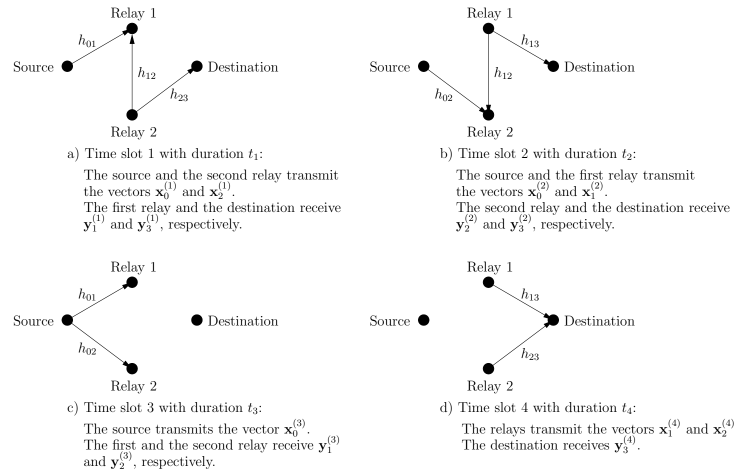

Fig. 1. System Model.

Noting the transmission strategies in Fig. 1, we have

$$\mathbf{y}_1^{(1)} = h_{01}\mathbf{x}_0^{(1)} + h_{21}\mathbf{x}_2^{(1)} + \mathbf{z}_1^{(1)}, \tag{2}$$
$$\mathbf{y}_3^{(1)} = h_{23}\mathbf{x}_2^{(1)} + \mathbf{z}_3^{(1)}, \tag{3}$$
$$\mathbf{y}_2^{(2)} = h_{02}\mathbf{x}_0^{(2)} + h_{12}\mathbf{x}_1^{(2)} + \mathbf{z}_2^{(2)}, \tag{4}$$
$$\mathbf{y}_3^{(2)} = h_{13}\mathbf{x}_1^{(2)} + \mathbf{z}_3^{(2)}, \tag{5}$$
$$\mathbf{y}_k^{(3)} = h_{0k}\mathbf{x}_0^{(3)} + \mathbf{z}_k^{(3)}, k \in \{1, 2\}, \tag{6}$$
$$\mathbf{y}_3^{(4)} = \sum_{k=1}^{2} h_{k3}\mathbf{x}_k^{(4)} + \mathbf{z}_3^{(4)}. \tag{7}$$

Throughout the paper, we assume that $h_{01} \geq h_{02}$ unless specified otherwise, and from reciprocity assumption, we have $h_{12} = h_{21}$. Furthermore, the power constraints P_0, P_1, and P_2 should be satisfied for the source, the first relay, and the second relay, respectively. Hence, denoting the power consumption of node a at time slot b by $P_a^{(b)} = E\left[\mathbf{x}_a^{(b)H}\mathbf{x}_a^{(b)}\right]$, we have

$$\begin{aligned} &P_0^{(1)} + P_0^{(2)} + P_0^{(3)} = P_0, \\ &P_1^{(2)} + P_1^{(4)} = P_1, \\ &P_2^{(1)} + P_2^{(4)} = P_2. \end{aligned} \tag{8}$$

III. Achievable Rates and Coding Schemes

In this section, we propose two cooperative protocols, i.e. *Successive* and *Simultaneous* relaying, for a half-duplex Gaussian parallel relay channel.

A. Successive Relaying Protocol

In successive relaying protocol, at each odd and even time slots with durations t_1 and t_2, source transmits a new message to one of the relays, and the destination receives a new message from the other relay, successively. Hence, in successive relaying protocol, $t_3 = t_4 = 0$, and the relations between the transmitted and received signals at the relays and the destination follow from (2)-(5). For the successive relaying protocol, we propose a *Non-Cooperative Coding* in the sequel.

1) Non-Cooperative Coding: In Non-Cooperative Coding scheme, each relay considers the other one's signal as interference (See Figure 2). Since the transmitter knows each relay's message, it can apply the Gelfand-Pinsker's coding scheme to transmit its message to each of the relays. In this case, we have the following theorem.

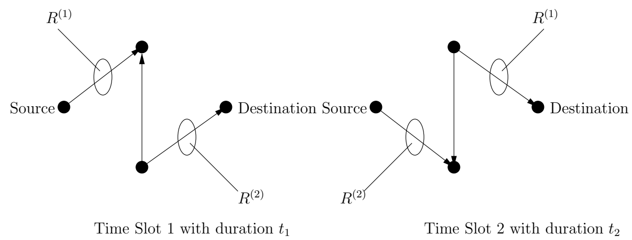

Fig. 2. Non-Cooperative Coding for successive relaying protocol for two relays.

Theorem 1 *For the half-duplex parallel relay channel, assuming successive relaying, the following rate R_{DPC} is achievable:*

$$R_{DPC} = \max_{0\le t_1,t_2,t_1+t_2=1} R^{(1)} + R^{(2)}, \tag{9}$$

subject to:

$$R^{(1)} \le \min\Big(t_1(I(U_0^{(1)};Y_1^{(1)}) - I(U_0^{(1)};X_2^{(1)})), \quad t_2 I(X_1^{(2)};Y_3^{(2)})\Big), \tag{10}$$

$$R^{(2)} \le \min\Big(t_2(I(U_0^{(2)};Y_2^{(2)}) - I(U_0^{(2)};X_1^{(2)})), \quad t_1 I(X_2^{(1)};Y_3^{(1)})\Big). \tag{11}$$

with probabilities:

$$p(x_2^{(1)},u_0^{(1)},x_0^{(1)}) = p(x_2^{(1)})p(u_0^{(1)}|x_2^{(1)})p(x_0^{(1)}|u_0^{(1)},x_2^{(1)}),$$
$$p(x_1^{(2)},u_0^{(2)},x_0^{(2)}) = p(x_1^{(2)})p(u_0^{(2)}|x_1^{(2)})p(x_0^{(2)}|u_0^{(2)},x_1^{(2)}).$$

Proof: <u>*Codebook Construction:*</u>
Let us divide time slot number b, $b = 1,2,\cdots,B+1$ into odd and even numbers. At odd and even time slots, source generates $2^{nR_u^{(1)}}$ and $2^{nR_u^{(2)}}$ sequences $\mathbf{u}_0^{(1)}(q_1)$ and $\mathbf{u}_0^{(2)}(q_2)$ according to $\prod_{i=1}^{t_1 n} p(u_{0,i}^{(1)})$ and $\prod_{i=1}^{t_2 n} p(u_{0,i}^{(2)})$, respectively. Then, source throws $\mathbf{u}_0^{(1)}$ and $\mathbf{u}_0^{(2)}$ sequences uniformly into $2^{nR^{(1)}}$ and $2^{nR^{(2)}}$ bins, respectively.

Relay 1 and relay 2 generate $2^{nR^{(1)}}$ and $2^{nR^{(2)}}$ i.i.d $\mathbf{x}_1^{(2)}$ and $\mathbf{x}_2^{(1)}$ sequences according to probabilities $\prod_{i=1}^{t_2 n} p\left(x_{1,i}^{(2)}\right)$ and $\prod_{i=1}^{t_1 n} p\left(x_{2,i}^{(1)}\right)$. Furthermore, for all q_1 and q_2, the source generates double indexed code-books $\mathbf{x}_0^{(1)}\left(w^{(b)}|w^{(b-1)},q_1\right)$ and $\mathbf{x}_0^{(2)}\left(w^{(b)}|w^{(b-1)},q_2\right)$ according to $\prod_{i=1}^{t_1 n} p(x_{0,i}^{(1)} \mid x_{2,i}^{(1)},u_{0,i}^{(1)})$ and $\prod_{i=1}^{t_2 n} p(x_{0,i}^{(2)} \mid x_{1,i}^{(2)},u_{0,i}^{(2)})$, respectively.

<u>*Encoding:*</u>

Encoding at the source:
In the odd time slot b, since source knows what it has transmitted during the even time slot, from the desired bin $w^{(b)} \in \{1,\cdots,2^{nR^{(1)}}\}$, it chooses a codeword $\mathbf{u}_0^{(1)}(q_1)$ such that $q_1 \in f_{Bin}^{-1}\left(w^{(b)}\right)$ and $\left(\mathbf{u}_0^{(1)}(q_1), \mathbf{x}_2^{(1)}\left(w^{(b-1)}\right)\right) \in A_\epsilon^{(n)}$ (The binning function $f_{Bin}(q_i) : Q_i = \{1,2,\cdots,2^{nR_u^{(i)}}\} \longrightarrow \{1,2,\ldots,2^{nR^{(i)}}\}$ is defined by $f_{Bin}(q_i) = w^{(b)}$, $\forall i \in \{1,2\}$. Where $f_{Bin}(.)$ assigns a randomly uniform distributed integer $w^{(b)}$ between 1 and $2^{nR^{(i)}}$ independently to each member q_i of Q_i.). Such a task can be done almost surely, if $R_u^{(1)} - R^{(1)} \ge t_1 I\left(U_0^{(1)};X_2^{(1)}\right)$ ([7]). Following that it sends $\mathbf{x}_0^{(1)}(\mathbf{u}_0^{(1)},\mathbf{x}_2^{(1)})$. Similarly, in even time slots, the source sends $\mathbf{x}_0^{(2)}(\mathbf{u}_0^{(2)},\mathbf{x}_1^{(2)})$ if $R_u^{(2)} - R^{(2)} \ge t_2 I\left(U_0^{(2)};X_1^{(2)}\right)$.

Encoding at relay 1:
Relay 1 encodes $w^{(b)} \in \{1,\cdots,2^{nR^{(1)}}\}$ to $\mathbf{x}_1^{(2)}\left(w^{(b)}\right)$ in even time slots.

Encoding at relay 2:
Relay 2 encodes $w^{(b)} \in \{1,\cdots,2^{nR^{(2)}}\}$ to $\mathbf{x}_2^{(1)}\left(w^{(b)}\right)$ in odd time slots.

<u>*Decoding:*</u>

Decoding at relay 1:
In odd time slot b, relay 1 declares $\hat{w}^{(b)} = f_{Bin}(q_1)$ iff all the sequences $\mathbf{u}_0^{(1)}(q_1)$ which are jointly typical with $\mathbf{y}_1^{(1)}$ belong to a unique bin $\hat{w}^{(b)}$. Therefore, in order to make the probability of error zero from [7], we have

$$R_u^{(1)} \le t_1 I\left(U_0^{(1)};Y_1^{(1)}\right). \tag{12}$$

According to (12) and the encoding at source, we have

$$R^{(1)} \le t_1\left(I(U_0^{(1)};Y_1^{(1)}) - I(U_0^{(1)};X_2^{(1)})\right). \tag{13}$$

Decoding at relay 2:
In even time slot b, relay 2 declares $\hat{w}^{(b)} = f_{Bin}(q_2)$ iff all the sequences $\mathbf{u}_0^{(2)}(q_2)$ which are jointly typical with $\mathbf{y}_2^{(2)}$ belong to a unique bin $\hat{w}^{(b)}$. Therefore, in order to make the probability of error zero from [7], we have

$$R_u^{(2)} \le t_2 I\left(U_0^{(2)};Y_2^{(2)}\right). \tag{14}$$

According to (14) and the encoding at source, we have

$$R^{(2)} \le t_2\left(I(U_0^{(2)};Y_2^{(2)}) - I(U_0^{(2)};X_1^{(2)})\right). \tag{15}$$

Decoding at the final destination:
In odd time slot b, destination declares $\hat{w}^{(b)} = w^{(b)}$ iff $\left(\mathbf{x}_2^{(1)}\left(\hat{w}^{(b)}\right), \mathbf{y}_3^{(1)}\right) \in A_\epsilon^{(n)}$. Hence, in order to make the probability of error zero from [7], we have

$$R^{(1)} \le t_1 I(X_2^{(1)};Y_3^{(1)}). \tag{16}$$

Similarly in even time slot b, we have

$$R^{(2)} \le t_2 I(X_1^{(2)};Y_3^{(2)}). \tag{17}$$

From the encoding at the source and (12)-(17), we obtain (9)-(11). ■

From Theorem 1, we have the following corollary for the Gaussian case.

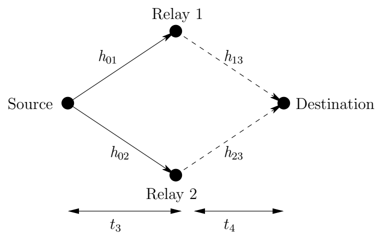

Fig. 3. Simultaneous relaying protocol for two relays.

corollary 1 *For the half-duplex Gaussian parallel relay channels, assuming successive relaying protocol with power constraints at the source and at each relay, DPC achieves the following rate*

$$R_{DPC} = \max\left(R^{(1)} + R^{(2)}\right), \tag{18}$$

subject to:

$$R^{(1)} \le \min\left(t_1 C\left(\frac{h_{01}^2 P_0^{(1)}}{t_1}\right), t_2 C\left(\frac{h_{13}^2 P_1}{t_2}\right)\right),$$
$$R^{(2)} \le \min\left(t_2 C\left(\frac{h_{02}^2 P_0^{(2)}}{t_2}\right), t_1 C\left(\frac{h_{23}^2 P_2}{t_1}\right)\right),$$
$$P_0^{(1)} + P_0^{(2)} = P_0,$$
$$t_1 + t_2 = 1,$$
$$0 \le t_1, t_2, P_0^{(1)}, P_0^{(2)}.$$

Proof: From Costa's Dirty Paper Coding [12], by having

$$U_0^{(1)} = X_0^{(1)} + \frac{h_{01}h_{12}P_0^{(1)}}{h_{01}^2 P_0^{(1)} + t_1} X_2^{(1)}, \tag{19}$$

$$U_0^{(2)} = X_0^{(2)} + \frac{h_{02}h_{12}P_0^{(2)}}{h_{02}^2 P_0^{(2)} + t_2} X_1^{(2)}. \tag{20}$$

where $X_0^{(1)} \sim \mathcal{N}(0, P_0^{(1)}), X_0^{(2)} \sim \mathcal{N}(0, P_0^{(2)}), X_2^{(1)} \sim \mathcal{N}(0, P_2)$, and $X_1^{(2)} \sim \mathcal{N}(0, P_1)$, and applying them to Theorem 1, we obtain corollary 1. ∎

B. Simultaneous Relaying Protocol

Then, Figure 3 shows simultaneous relaying protocol for two relays. In simultaneous relaying, in one time slot of duration t_3 the source transmits its signal simultaneously to the two relays. In the next time slot of duration t_4, two relays transmit their signal coherently to the final destination. Hence, in this protocol, $t_1 = t_2 = 0$ and our system model follows from (6) and (7).

1) Dynamic Decode-and-Forward (DDF): In DDF scheme each relay decodes the transmitted message from the source in time slot t_3 (Broadcast State (BC)), and forwards its re-encoded version in time slot t_4 (Multiple Access State (MAC)). Hence, the achievable rate of DDF is equal to $R_{DDF} = R_p + R_c$, where (R_p, R_c) should be both in the capacity region of BC (corresponding to the BC state) and MAC (corresponding to the MAC state). Since what the second relay receives is a degraded version of what the first relay receives ($h_{01} \ge h_{02}$), applying the superposition coding of the degraded BC [7] the following rates are achievable for the first hop:

$$\begin{aligned} R_p &\le t_3 I(X_0^{(3)}; Y_1^{(3)} \mid U_0^{(3)}), \\ R_c &\le t_3 I(U_0^{(3)}; Y_2^{(3)}). \end{aligned} \tag{21}$$

with probability $p(u_0^{(3)}, x_0^{(3)}) = p(u_0^{(3)})p(x_0^{(3)}|u_0^{(3)})$.

And using the superposition coding of the extended MAC ([9], [10]) the following rates are achievable for the second hop:

$$\begin{aligned} R_p &\le t_4 I(X_1^{(4)}; Y_3^{(4)} \mid X_2^{(4)}), \\ R_p + R_c &\le t_4 I(X_1^{(4)}, X_2^{(4)}; Y_3^{(4)}). \end{aligned} \tag{22}$$

with probability $p(x_1^{(4)}, x_2^{(4)}) = p(x_1^{(4)})p(x_2^{(4)}|x_1^{(4)})$.

In the Gaussian case, the source splits its total available power P_0 to $P_{0-p}^{(3)}$ and $P_{0-c}^{(3)}$ associated with the *"Private"* and the *"Common"* messages, respectively. Letting $X_0^{(3)} \sim \mathcal{N}(0, P_0)$, $U_0^{(3)} \sim \mathcal{N}\left(0, P_{0-c}^{(3)}\right)$, and $X_1^{(4)} \sim \mathcal{N}(0, P_1)$, assuming that relay 1 and relay 2 transmit their codewords associated with the common message with $\mathcal{N}\left(0, P_{1-c}^{(4)}\right)$ and $\mathcal{N}(0, P_2)$, and using (21) and (22) we have the following corollary.

corollary 2 *For the half-duplex Gaussian parallel relay channels, assuming simultaneous relaying protocol with power constraints at the source and at each relay, DDF achieves the following rate*

$$R_{DDF} = R_p + R_c, \tag{23}$$

subject to:

$$R_p \le t_3 C\left(\frac{h_{01}^2 P_{0-p}^{(3)}}{t_3}\right),$$
$$R_c \le t_3 C\left(\frac{h_{02}^2 P_{0-c}^{(3)}}{t_3 + h_{02}^2 P_{0-p}^{(3)}}\right),$$
$$R_p \le t_4 C\left(\frac{h_{13}^2 P_{1-p}^{(4)}}{t_4}\right),$$
$$R_p + R_c \le t_4 C\left(\frac{h_{13}^2 P_{1-p}^{(4)} + \left(h_{13}\sqrt{P_{1-c}^{(4)}} + h_{23}\sqrt{P_2}\right)^2}{t_4}\right),$$
$$t_3 + t_4 = 1,$$
$$P_{0-p}^{(3)} + P_{0-c}^{(3)} = P_0,$$
$$P_{1-p}^{(4)} + P_{1-c}^{(4)} = P_1,$$
$$0 \le t_3, t_4, P_{0-p}^{(3)}, P_{0-c}^{(3)}, P_{1-p}^{(4)}, P_{1-c}^{(4)}.$$

C. General Achievable Rate for the Half-Duplex Parallel Relay Channel

In this section, we propose an achievable rate for the half-duplex parallel relay channel with two relays. Our achievable scheme is based on the combination of the successive relaying

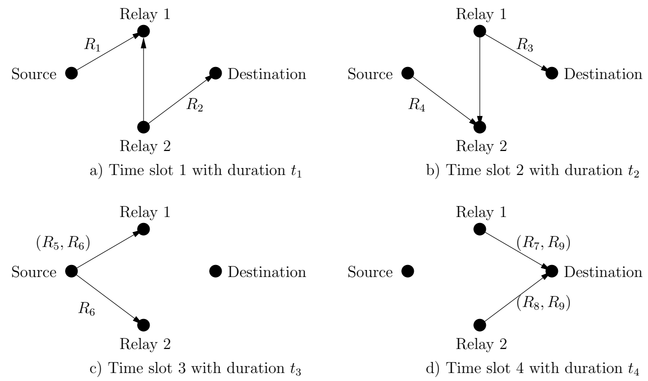

Fig. 4. General scheme for the half-duplex parallel relay channel.

protocol based on DPC and simultaneous relaying protocol based on DDF. The general achievable scheme is illustrated in Figure 4. As indicated in the figure, transmission is performed in 4 time slots. Relay 1 transmits its private message which has received in time slots t_1 and t_3 (corresponding to rates R_1 and R_5) in time slots t_2 and t_4 (corresponding to rates R_3 and R_7). On the other hand, relay 2 transmits its private message which has been received in time slot t_2 (corresponding to rate R_4) in time slots t_1 and t_4 (corresponding to rates R_2 and R_8). Furthermore, the two relays send the common message they have already received in time slot t_3 (corresponding to rate R_6) coherently in time slot t_4 (corresponding to rate R_9). As observed, here we consider private rate for both relays in MAC state, i.e. time slot t_4. This is due to the reason that relay 2 also receives the private message in time slot t_2. Hence, from the above description and Figure 4, we have

$$
\begin{aligned}
R = & \min\left(R_1+R_4+R_5+R_6, R_2+R_3+R_7+R_8+R_9\right), \\
& \text{subject to:} \\
& R_9 \le R_6, R_1+R_5 \le R_3+R_7, R_4 \le R_2+R_8. \quad (24)
\end{aligned}
$$

corollary 3 *For the half-duplex Gaussian parallel relay channels, with power constraints at the source and at each relay, the following rate R is achievable*

$$
\begin{aligned}
R = \min\Bigg(& t_1 C\left(\frac{h_{01}^2 P_0^{(1)}}{t_1}\right) + t_2 C\left(\frac{h_{02}^2 P_0^{(2)}}{t_2}\right) + \\
& t_3 C\left(\frac{h_{01}^2 P_{0-p}^{(3)}}{t_3}\right) + t_3 C\left(\frac{h_{02}^2 P_{0-c}^{(3)}}{t_3 + h_{02}^2 P_{0-p}^{(3)}}\right), \\
& t_1 C\left(\frac{h_{23}^2 P_2^{(1)}}{t_1}\right) + t_2 C\left(\frac{h_{13}^2 P_1^{(2)}}{t_2}\right) + \\
& t_4 C\left(\frac{h_{13}^2 P_{1-p}^{(4)} + h_{23}^2 P_{2-p}^{(4)}}{t_4}\right) + \\
& t_4 C\left(\frac{\left(h_{13}\sqrt{P_{1-c}^{(4)}} + h_{23}\sqrt{P_{2-c}^{(4)}}\right)^2}{t_4 + h_{13}^2 P_{1-p}^{(4)} + h_{23}^2 P_{2-p}^{(4)}}\right)\Bigg), \quad (25)
\end{aligned}
$$

subject to:

$$
\begin{aligned}
& t_4 C\left(\frac{\left(h_{13}\sqrt{P_{1-c}^{(4)}} + h_{23}\sqrt{P_{2-c}^{(4)}}\right)^2}{t_4 + h_{13}^2 P_{1-p}^{(4)} + h_{23}^2 P_{2-p}^{(4)}}\right) \le \\
& t_3 C\left(\frac{h_{02}^2 P_{0-c}^{(3)}}{t_3 + h_{02}^2 P_{0-p}^{(3)}}\right), \\
& t_1 C\left(\frac{h_{01}^2 P_0^{(1)}}{t_1}\right) + t_3 C\left(\frac{h_{01}^2 P_{0-p}^{(3)}}{t_3}\right) \le \\
& t_2 C\left(\frac{h_{13}^2 P_1^{(2)}}{t_2}\right) + t_4 C\left(\frac{h_{13}^2 P_{1-p}^{(4)}}{t_4}\right), \\
& t_2 C\left(\frac{h_{02}^2 P_0^{(2)}}{t_2}\right) \le t_1 C\left(\frac{h_{23}^2 P_2^{(1)}}{t_1}\right) + t_4 C\left(\frac{h_{23}^2 P_{2-p}^{(4)}}{t_4}\right), \\
& P_0^{(1)} + P_0^{(2)} + P_{0-p}^{(3)} + P_{0-c}^{(3)} = P_0, \\
& P_1^{(2)} + P_{1-p}^{(4)} + P_{1-c}^{(4)} = P_1, \\
& P_2^{(1)} + P_{2-p}^{(4)} + P_{2-c}^{(4)} = P_2, \\
& t_1 + t_2 + t_3 + t_4 = 1, \\
& 0 \le t_1,\ t_2,\ t_3,\ t_4,\ P_0^{(1)},\ P_0^{(2)},\ P_{0-p}^{(3)},\ P_{0-c}^{(3)}, \\
& 0 \le P_1^{(2)},\ P_{1-p}^{(4)},\ P_{1-c}^{(4)},\ P_2^{(1)},\ P_{2-p}^{(4)},\ P_{2-c}^{(4)}.
\end{aligned}
$$

Proof: Using corollaries 1, 2, and the fact that the common message should be first decoded at the receiver side ([21]), corollary 3 is immediate. ■

IV. OPTIMALITY RESULTS

In this section, an upper bound for parallel relay networks with two relays is derived and investigated. The authors in [11] proposed some upper bounds on the achievable rate for general half-duplex multi-terminal networks. Here, we explain their results briefly and apply them to our half-duplex parallel relay network.

Authors in [11] define the concept of *state* for a half-duplex network with N nodes. The state of the network is a *valid partitioning of its nodes into two sets of the "sender nodes" and the "receiver nodes" such that there is no active link that arrives at a sender node*, and $\hat{t}_m$ is the portion of the time that network is used in state *m* where *m* $\in \{1, 2, \ldots, M\}$. The following theorem for the upper bound of the information flow from the subset S_1 to the subset S_2 of the nodes, where S_1 and S_2 are disjoint is proved in [11].

Theorem 2 *For a general half-duplex network with N nodes and a finite number of states, M, the maximum achievable information rates $\{R^{ij}\}$ from a node set S_1 to a disjoint node set S_2, $S_1, S_2 \subset \{0, 1, \ldots, N-1\}$, is bounded by*

$$
\begin{aligned}
& \sum_{i\in S_1, j\in S_2} R^{ij} \le \\
& \sup_{p(x_0^{(m)}, x_1^{(m)}, \ldots, x_{N-1}^{(m)}), \hat{t}_m} \min_S \sum_{m=1}^{M} \hat{t}_m I\left(X_S^{(m)}; Y_S^{(m)} \mid X_{S^c}^{(m)}\right).
\end{aligned}
$$

for some joint probability distribution $p(x_0^{(m)}, x_1^{(m)}, \dots, x_{N-1}^{(m)})$ when the minimization is over all the sets $S \subset \{0, 1, \dots, N-1\}$ subject to $S \bigcap S_1 = S_1$, $S \bigcap S_2 = \emptyset$ and the supremum is over all the non-negative $\hat{t}_m$ subject to $\sum_{i=1}^{M} \hat{t}_m = 1$. Here, $x_S^{(m)}$, $y_S^{(m)}$, and $x_{S^c}^{(m)}$ denote the signals transmitted and received by nodes in set S, and transmitted by nodes in set S^c, during state m, respectively.

In high SNR scenarios, we have the following theorem.

Theorem 3 *In high SNR scenarios, assuming non-zero source-relay and relay-destination links, when power available for the source and each relay tends to infinity, DPC achieves the capacity of a half-duplex Gaussian parallel relay channel as*

$$C_{up} = R_{DPC} + O\left(\frac{1}{\log P_0}\right).$$

Sketch of the proof Throughout the proof, we assume the power of the relays goes to infinity as $P_1 = \gamma_1 P_0,\ P_2 = \gamma_2 P_0$ where γ_1, γ_2 are constants independent of the SNR. Applying Theorem 2 to our set up, substituting $X_0^{(1)} \sim \mathcal{N}(0, \hat{P}_0^{(1)})$, $X_0^{(2)} \sim \mathcal{N}(0, \hat{P}_0^{(2)})$, $X_0^{(3)} \sim \mathcal{N}(0, \hat{P}_0^{(3)})$, $X_1^{(2)} \sim \mathcal{N}(0, \hat{P}_1^{(2)})$, $X_1^{(4)} \sim \mathcal{N}(0, \hat{P}_1^{(4)})$, $X_2^{(1)} \sim \mathcal{N}(0, \hat{P}_2^{(1)})$, and $X_2^{(4)} \sim \mathcal{N}(0, \hat{P}_2^{(4)})$ in the resulting upper bound, and assuming complete cooperation between the transmitting and receiving nodes for each cut, we obtain (26). Furthermore, from corollary 1, the achievable rate of the DPC scheme can be expressed as

$$\begin{aligned} R_{DPC} \le \min\Bigg(& t_1 C\left(\frac{h_{01}^2 P_0^{(1)}}{t_1}\right) + t_2 C\left(\frac{h_{02}^2 P_0^{(2)}}{t_2}\right), \\ & t_2 C\left(\frac{h_{02}^2 P_0^{(2)}}{t_2}\right) + t_2 C\left(\frac{h_{13}^2 P_1}{t_2}\right), \\ & t_1 C\left(\frac{h_{01}^2 P_0^{(1)}}{t_1}\right) + t_1 C\left(\frac{h_{23}^2 P_2}{t_1}\right), \\ & t_1 C\left(\frac{h_{23}^2 P_2}{t_1}\right) + t_2 C\left(\frac{h_{13}^2 P_1}{t_2}\right)\Bigg). \end{aligned} \quad (27)$$

By setting $P_0^{(1)} = P_0^{(2)} = \frac{P_0}{2}$, $t_1 = t_2 = 0.5$ in (27), and using the following inequality

$$\ln(x) \le \ln(1+x) \le \ln(x) + \frac{1}{x}, \forall x > 0. \quad (28)$$

(27) can be simplified as

$$R_{DPC} \ge \frac{1}{2}\ln P_0 + c. \quad (29)$$

where c is some constant which depends on channel coefficients. Knowing that the term corresponding to each cut-set in (26) for the optimum values of $\hat{t}_1, \cdots, \hat{t}_4$ is indeed an upper-bound for R_{DPC}, by setting $\hat{P}_0^{(1)} = \hat{P}_0^{(2)} = \hat{P}_0^{(3)} = P_0$ in (26), using the inequality between R_{DPC} and the first cut of (26) and the inequality (28) to lower/upper-bound the corresponding terms, we can bound the optimum value of $\hat{t}_4$ in (26) as

$$0 \le \hat{t}_4 \le O\left(\frac{1}{\log P_0}\right). \quad (30)$$

Similarly, by considering the fourth cut in (26), we can derive another bound on the optimum value of $\hat{t}_3$ as follows:

$$0 \le \hat{t}_3 \le O\left(\frac{1}{\log P_0}\right). \quad (31)$$

Applying the inequality between R_{DPC} and the term corresponding to the second cut in (26), knowing (from (30) and (31)) the fact that $\hat{t}_3 \le \frac{c_3}{\ln P_0}$, and $\hat{t}_4 \le \frac{c_4}{\ln P_0}$ (where c_3 and c_4 are constants), and using inequalities (28), and

$$\ln(1+x) \le x, \forall x \ge 0, \quad (32)$$

and also the fact that $\hat{t}_1 + \hat{t}_2 + \hat{t}_3 + \hat{t}_4 = 1$, and having the same argument for the third cut of (26), we obtain

$$\frac{1}{2} - \frac{c_2}{\log P_0} \le \hat{t}_2 \le \frac{1}{2} + \frac{c_1}{\log P_0}, \quad (33)$$

$$\frac{1}{2} - \frac{c_1}{\log P_0} \le \hat{t}_1 \le \frac{1}{2} + \frac{c_2}{\log P_0}. \quad (34)$$

where c_1 and c_2 are constants. Hence, from (30), (31), (33), and (34) as $P_0 \to \infty$, $\hat{t}_3$, $\hat{t}_4 \to 0$ and $\hat{t}_1$, $\hat{t}_2 \to 0.5$.

Similarly, considering the inequality between the first cut of R_{DPC} and (26) and knowing the fact that $\hat{t}_1, \hat{t}_2$ are strictly above zero (approaching 0.5), we observe that the optimum value of $\hat{P}_0^{(1)}, \hat{P}_0^{(2)}$ are

$$\hat{P}_0^{(1)}, \hat{P}_0^{(2)} \sim \Theta\left(P_0\right). \quad (35)$$

Now, we prove that the DPC scheme with the parameters $t_1 = \hat{t}_1 + \frac{\hat{t}_3 + \hat{t}_4}{2}$, $t_2 = \hat{t}_2 + \frac{\hat{t}_3 + \hat{t}_4}{2}$, $P_0^{(1)} = \hat{P}_0^{(1)}$ and $P_0^{(2)} = \hat{P}_0^{(2)}$, where $\hat{t}_1, \cdots, \hat{t}_4, \hat{P}_0^{(1)}, \hat{P}_0^{(2)}$ are the parameters corresponding to the optimum value of (26), achieves the capacity with a gap no more than $O\left(\frac{1}{\log P_0}\right)$. To prove this, using (35) and the fact that $\hat{t}_3, \hat{t}_4 \sim O\left(\frac{1}{\log P_0}\right)$ and $\hat{t}_1, \hat{t}_2 \sim 0.5 + O\left(\frac{1}{\log P_0}\right)$, we show that each of the four terms in (27) is no more than $O\left(\frac{1}{\log P_0}\right)$ below the corresponding term (from the same cut) in (26). For the complete proof see [21].

Theorem 4 *In low SNR scenarios, assuming $P_1 = \gamma_1 P_0,\ P_2 = \gamma_2 P_0$ with γ_1, γ_2 constants independent of the SNR, when the power available for the source and each relay tends to zero and $\left(h_{13}\sqrt{\gamma_1} + h_{23}\sqrt{\gamma_2}\right)^2 \le \min\left(h_{01}^2, h_{02}^2\right)$, the ratio of the achievable rate of the simultaneous relaying protocol based on DDF to cut-set upper bound goes to 1. In this scenario $t_3 = t_4 = \frac{1}{2}$, and no private messages should be transmitted.*

Proof: Throughout the proof, we assume the power of the relays goes to zero as $P_1 = \gamma_1 P_0,\ P_2 = \gamma_2 P_0$ where γ_1, γ_2 are constants independent of the SNR. By the same argument as in theorem 3 and considering only the fourth cut, we can

$$
\begin{aligned}
C_{up} \leq \min\Bigg(& \hat{t}_1 C\left(\frac{h_{01}^2 \hat{P}_0^{(1)}}{\hat{t}_1}\right) + \hat{t}_2 C\left(\frac{h_{02}^2 \hat{P}_0^{(2)}}{\hat{t}_2}\right) + \hat{t}_3 C\left(\frac{(h_{01}^2 + h_{02}^2)\hat{P}_0^{(3)}}{\hat{t}_3}\right), \\
& \hat{t}_2 C\left(\frac{h_{02}^2 \hat{P}_0^{(2)}}{\hat{t}_2} + \frac{(h_{12}^2 + h_{13}^2)\hat{P}_1^{(2)}}{\hat{t}_2} + \frac{2h_{02}h_{12}\sqrt{\hat{P}_0^{(2)}\hat{P}_1^{(2)}}}{\hat{t}_2} + \frac{h_{02}^2 h_{13}^2 \hat{P}_0^{(2)}\hat{P}_1^{(2)}}{\hat{t}_2^2}\right) + \\
& \hat{t}_3 C\left(\frac{h_{02}^2 \hat{P}_0^{(3)}}{\hat{t}_3}\right) + \hat{t}_4 C\left(\frac{h_{13}^2 \hat{P}_1^{(4)}}{\hat{t}_4}\right), \\
& \hat{t}_1 C\left(\frac{h_{01}^2 \hat{P}_0^{(1)}}{\hat{t}_1} + \frac{(h_{12}^2 + h_{23}^2)\hat{P}_2^{(1)}}{\hat{t}_1} + \frac{2h_{01}h_{12}\sqrt{\hat{P}_0^{(1)}\hat{P}_2^{(1)}}}{\hat{t}_1} + \frac{h_{01}^2 h_{23}^2 \hat{P}_0^{(1)}\hat{P}_2^{(1)}}{\hat{t}_1^2}\right) + \\
& \hat{t}_3 C\left(\frac{h_{01}^2 \hat{P}_0^{(3)}}{\hat{t}_3}\right) + \hat{t}_4 C\left(\frac{h_{23}^2 \hat{P}_2^{(4)}}{\hat{t}_4}\right), \\
& \hat{t}_1 C\left(\frac{h_{23}^2 \hat{P}_2^{(1)}}{\hat{t}_1}\right) + \hat{t}_2 C\left(\frac{h_{13}^2 \hat{P}_1^{(2)}}{\hat{t}_2}\right) + \hat{t}_4 C\left(\frac{h_{13}^2 \hat{P}_1^{(4)} + h_{23}^2 \hat{P}_2^{(4)} + 2h_{13}h_{23}\sqrt{\hat{P}_1^{(4)}\hat{P}_2^{(4)}}}{\hat{t}_4}\right)\Bigg). \qquad (26)
\end{aligned}
$$

subject to:

$$
\begin{aligned}
& \hat{P}_0^{(1)} + \hat{P}_0^{(2)} + \hat{P}_0^{(3)} = P_0, \\
& \hat{P}_1^{(2)} + \hat{P}_1^{(4)} = P_1, \\
& \hat{P}_2^{(1)} + \hat{P}_2^{(4)} = P_2, \\
& \hat{t}_1 + \hat{t}_2 + \hat{t}_3 + \hat{t}_4 = 1, \\
& 0 \leq \ \hat{t}_1,\ \hat{t}_2,\ \hat{t}_3,\ \hat{t}_4,\ \hat{P}_0^{(1)},\ \hat{P}_0^{(2)},\ \hat{P}_0^{(3)},\ \hat{P}_1^{(2)},\ \hat{P}_1^{(4)},\ \hat{P}_2^{(1)},\ \hat{P}_2^{(4)}.
\end{aligned}
$$

---

get another upper bound on the capacity. By the following inequality

$$\ln(1+x) \leq x. \qquad (36)$$

we can bound the upper bound on the capacity as

$$C_{up} \quad \leq \quad \frac{\left(h_{13}\sqrt{\gamma_1} + h_{23}\sqrt{\gamma_2}\right)^2 P_0}{2\ln 2}. \qquad (37)$$

Now, assuming $t_1 = t_2 = 0, t_3 = t_4 = \frac{1}{2}$, and transmitting just the common message, we can achieve the following rate R_{DDF}

$$
\begin{aligned}
R_{DDF} = \min\Bigg(& \frac{1}{2}C\left(2h_{02}^2 P_0\right), \\
& \frac{1}{2}C\left(2\left(h_{13}\sqrt{\gamma_1} + h_{23}\sqrt{\gamma_2}\right)^2 P_0\right)\Bigg).
\end{aligned}
$$

By the following inequality

$$x - \frac{x^2}{2} \leq \ln(1+x), \qquad (38)$$

we have

$$
\begin{aligned}
& \frac{1}{\ln 2}\min\Bigg(\frac{h_{02}^2 P_0}{2} - \frac{h_{02}^4 P_0^2}{2}, \\
& \frac{\left(h_{13}\sqrt{\gamma_1} + h_{23}\sqrt{\gamma_2}\right)^2 P_0}{2} - \frac{\left(h_{13}\sqrt{\gamma_1} + h_{23}\sqrt{\gamma_2}\right)^4 P_0^2}{2}\Bigg) \\
& \leq R_{DDF}. \qquad (39)
\end{aligned}
$$

By (37), (39), and $\left(h_{13}\gamma_1 + h_{23}\gamma_2\right)^2 \leq \min\left(h_{01}^2, h_{02}^2\right)$, we have $\lim_{P_0 \to 0} \frac{R_{DDF}}{C_{up}} \to 1$. ■

V. Simulation Result

Figure 5 compares the achievable rate of our general scheme with those of DPC scheme for successive relaying and DDF scheme for simultaneous relaying protocols. Here we assume symmetric scenario in which $h_{01} = h_{02} = h_{12} = h_{13} = h_{23} = 1$. In order to satisfy the condition in theorem 4, i.e. $\left(h_{13}\sqrt{\gamma_1} + h_{23}\sqrt{\gamma_2}\right)^2 \leq \min\left(h_{01}^2, h_{02}^2\right)$, we also assume $P_0 = P_1 + 10(dB) = P_2 + 10(dB)$. The upper bound is also included in the figure. As Figure 5 shows, our proposed general achievable rate almost coincides with the upper bound over all ranges of SNR. In the high SNR scenario, our proposed general scheme coincides with DPC and the successive relaying protocol becomes optimum, while in low SNR scenario it coincides with DDF and the simultaneous relaying protocol is optimum.

VI. Conclusion

In this paper, we investigated the problem of cooperative strategies for a half-duplex parallel relay channel with two relays. We derived the optimum relay ordering and hence the asymptotic capacity of the half-duplex Gaussian parallel relay channel in low and high SNR scenarios. *Simultaneous* and *Successive* relaying protocols, associated with two possible

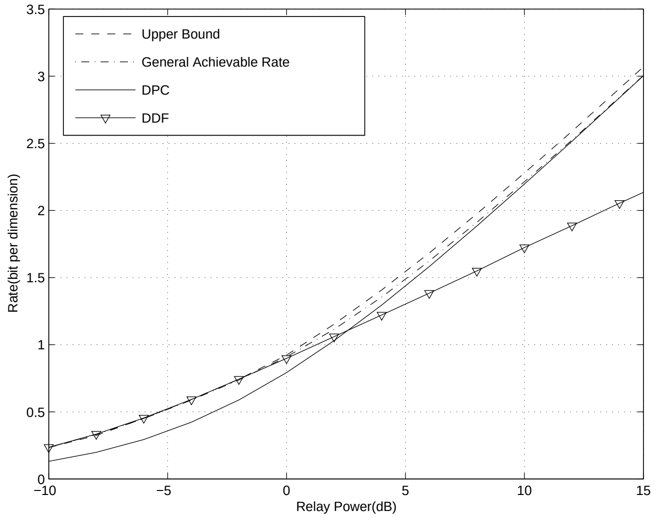

Fig. 5. Rate versus relay power ($P_0 = P_1 + 10(dB) = P_2 + 10(dB)$).

relay orderings were proposed. For simultaneous relaying, each relay employs *DDF*. On the other hand, for successive relaying, we proposed a *Non-Cooperative Coding* scheme based on DPC. We also proposed a general achievable scheme as a combination of the simultaneous and successive schemes. In high SNR scenarios, we proved that our *Non-Cooperative Coding* scheme based on DPC asymptotically achieves the capacity. Hence, in high SNR scenario, the optimum relay ordering is *Successive*. On the other hand, in low SNR where $(h_{13}\gamma_1 + h_{23}\gamma_2)^2 \le \min\left(h_{01}^2, h_{02}^2\right)$, DDF achieves the capacity. Hence, in low SNR scenario the optimum relay ordering is *Simultaneous*.

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