

Coordinated scheduling of electricity and natural gas infrastructures with a transient model for natural gas flow

Cong Liu,^{1,2,a)} Mohammad Shahidehpour,^{1,b)} and Jianhui Wang^{2,c)}

¹*Electrical and Computer Engineering Department, Illinois Institute of Technology, Chicago, Illinois 60616, USA*

²*Decision and Information Sciences Division, Argonne National Laboratory, Illinois 60439, USA*

(Received 18 January 2011; accepted 27 May 2011; published online 28 June 2011)

This paper focuses on transient characteristics of natural gas flow in the coordinated scheduling of security-constrained electricity and natural gas infrastructures. The paper takes into account the slow transient process in the natural gas transmission systems. Considering their transient characteristics, natural gas transmission systems are modeled as a set of partial differential equations (PDEs) and algebraic equations. An implicit finite difference method is applied to approximate PDEs by difference equations. The coordinated scheduling of electricity and natural gas systems is described as a bi-level programming formulation from the independent system operator's viewpoint. The objective of the upper-level problem is to minimize the operating cost of electric power systems while the natural gas scheduling optimization problem is nested within the lower-level problem. Numerical examples are presented to verify the effectiveness of the proposed solution and to compare the solutions for steady-state and transient models of natural gas transmission systems. © 2011 American Institute of Physics. [doi:10.1063/1.3600761]

The electricity and natural gas infrastructures are highly intertwined¹⁻⁶ today as more and more gas-fired units have been introduced into power systems and installed, along with an increasing number of high-pressure natural gas pipelines. The security and economics of coupled infrastructures are interdependent.¹⁻³ Using purely topological metrics to analyze their interdependency can lead to misleading results.⁴ Cascaded failures of energy systems may be due to a variety of reasons. One of the most important causes is inaccurate scheduling and decision making so that the system cannot handle variations of loads and severe disturbances. The security-constrained, coordinated scheduling of two coupled energy systems is critical to mitigating the hidden security risks of energy systems, to preventing cascaded failures, and to realizing savings in their operating costs. However, natural gas and electric power flows usually travel through networks via different speeds and illustrate distinct physical characteristics. Hence, we model the electric power and natural gas systems differently but in a coupled manner. The numerical results show that the steady-state natural gas flow model would neglect the built-in storage capabilities of pipelines and the slower travelling speeds of natural gas flows, which may result in impractical or suboptimal schedules in the short-term coordinated scheduling of electricity and natural gas infrastructures.

I. INTRODUCTION

The adequacy and availability of natural gas fuel will directly affect the generation unit commitment (UC), dispatch, operating costs, as well as reliability of electric power systems.^{5,6} The electric power system is special in the sense that real-time balance between generation and load is required under both normal and fault condition. When contingencies occur, the operators can call gas-fired units that can respond quickly to address the imbalances in comparison with slow-start coal-fired and nuclear units. If the imbalances between generation and load are not eliminated immediately, the frequency of the power system will increase or decrease, which may lead to the loss of stability of the system through electromechanical and electromagnetic transients. More importantly, voltage collapses and loss of synchronization of more generators will increase the likelihood of a blackout. Therefore, adequacy of natural gas fuel is very critical to ensure the power system reliability and to prevent the cascaded failure of power systems.

However, in most cases, gas-fired generating units hold interruptible transportation contracts that would treat the units as preferential curtailment candidates. The natural gas drawn by gas-fired generating units with interruptible transportation contracts would be limited or curtailed once the congestion in natural gas transmission systems has occurred or gas wells have reached their maximum discharge. Moreover, the operation of new gas turbine units or combined-cycle gas turbine units would usually depend on high gas pressure. Hence, electric generators are more susceptible to pressure drops than are other natural gas loads.⁷ Even if the priority of natural gas delivery for gas-fired generating units was the same as that of other natural gas loads, the weaker

^{a)}E-mail: cliu35@iit.edu & liuc@anl.gov

^{b)}E-mail: ms@iit.edu

^{c)}E-mail: jianhui.wang@anl.gov

ability of gas-fired generating units to manage pressure drops would make the natural gas curtailment more evident.

Many networks, such as energy, biology, and social relationship, are multi-layer interdependent networks. The loss of a part of links and nodes in one network may lead to a forced outage in another network.² The failure of components in the second network in turn may lead to a malfunction of additional nodes in the first network. Eventually, the interdependent networks may lose their functions completely. Similarly, any disruption in natural gas supply could have a significant impact on several gas-fired power plants and results in more critical outages than traditional contingencies in power systems. Outages of several gas-fired generators can result in the loss of operating reserves, real-time imbalances between load and generation, and an adverse effect on the system frequency. If necessary, operators would have to adopt load shedding to prevent any collapse in electric power system operations. References 5 and 10 offer examples of preliminary results on the impact of pipeline contingencies on power systems. Moreover, compressors, control devices, and other facilities in natural gas transmission systems are dependent on electricity. Load shedding or interruption of electricity may in turn lead to the failure of a fraction of natural gas system.

In order to satisfy the requirements of gas-fired generating units and other natural gas loads within a reasonable pressure range,^{7,8} the natural gas transmission system would have to schedule gas wells and compressors in advance and manage line-pack resources. “Line pack” is related to the amount of additional natural gas stored in a pipeline as a result of maintaining above-normal pressure in the pipeline.^{7,9} As such, line pack is analogous to reserves in power systems and is essential in enabling pipelines to handle large swings in natural gas loads, such as the fast ramp up of gas-fired generating units during peak hours and power system contingencies.

In the last ten years, several articles proposed state-of-the-art strategies to model the two interdependent infrastructures.^{10–14} However, many of the previous studies focused on the steady-state formulation of electric power and natural gas transmission systems. Such formulations neglected a significant distinction to be made in the traveling speeds of natural gas and electric power flows, as well as the line-pack capacities of interstate pipelines.

This paper concentrates on the development of a methodology for the coordinated scheduling of interdependent power and natural gas transmission systems that are based on a transient-state model of natural gas flows. The paper focuses on slow transient processes—that is, in terms of hours or minutes—caused by natural gas load swings. In the proposed formulation, interstate natural gas pipelines are described by a set of partial differential equations (PDEs) instead of by steady-state Weymouth equations. The implicit finite difference method is adopted to approximate PDEs with algebraic difference equations. As a result, natural gas flows are coupled in space and time. The paper presents a bi-level model shown in Fig. 1 for coordination between natural gas operators and the independent system operator (ISO). Constraints on natural gas supply contracts are directly

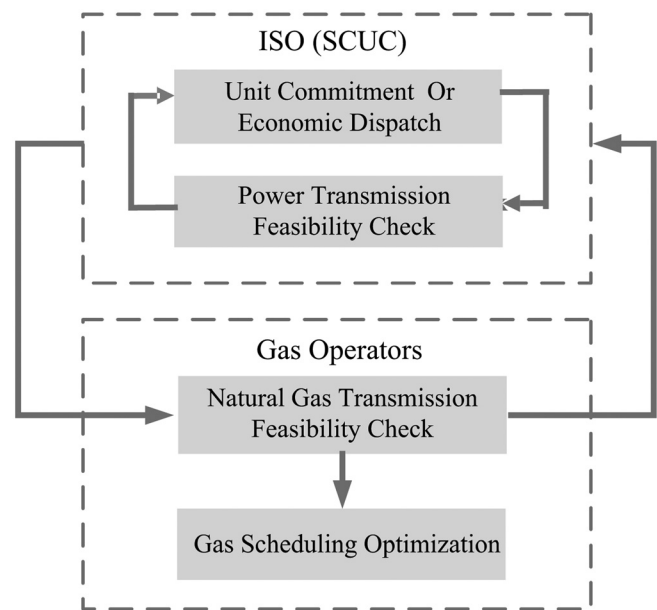


FIG. 1. Coordination scheme for electric power and natural gas infrastructures.

included in the UC problem. When an optimal UC schedule is obtained without violating power transmission constraints, the hourly natural gas demands of gas-fired generating units will be submitted to natural gas system operators, in the lower-level problem, for checking the feasibility of natural gas transmission constraints. If any violations are detected in natural gas transmission constraints, corresponding energy constraints (Benders-like cuts) are formed and fed back to the ISO for the next iteration of the coordination problem. The cut, which represents shortages in the natural gas supply or congestion in the gas transmission system, would limit the fuel consumed by gas-fired generating units. If the natural gas transmission check is feasible, the natural gas consumed by gas-fired generating units, as well as the security-constrained unit commitment (SCUC) solution, are final. The natural gas transmission operator will schedule compressors by minimizing rates of energy consumption.

The rest of the paper is organized as follows. Section II proposes the transient-state model of natural gas transmission systems. Section III presents formulations of the coordinated scheduling model. Algorithms for solving the proposed model are discussed in Sec. IV. Numerical studies are given in Sec. V. The conclusion is drawn in Sec. VI.

II. TRANSIENT FLOWS IN NATURAL GAS SYSTEMS

The dynamics of energy infrastructures could vary from milliseconds to hours, which indicates that energy transportation could occur in different frameworks. Electric energy travels almost instantaneously and cannot be stored economically in large quantities in current power systems. At the operation planning stage, once power injections and withdrawals at various buses are given, transmission line flows will satisfy steady-state algebraic equations that are independent on an hourly basis. Therefore, SCUC and security-constrained economic dispatch (SCED) commonly ignore

transients in the electricity infrastructure and focus on steady-state analyses.^{15–17}

The natural gas pipeline flow would represent a much slower process. Such pipelines take longer to respond to disturbances. In particular, high-pressure interstate pipelines have much slower dynamics, and large sums of natural gas stored in the pipelines cannot be neglected. In this case, the steady-state assumption and the corresponding algebraic Weymouth equation for pipelines might be inappropriate for the numerical calculation of natural gas flows. If treated rigorously, natural gas flow simulations would require distributed parameters and the transient state model of pipelines.^{18–23}

A. Modeling of natural gas pipelines

Natural gas flows, driven by pressure through pipelines, are dependent on factors such as the length and the diameter of pipelines, operating temperature, composition of natural gas, altitude change over the transmission path, roughness of pipelines, and boundary conditions. The transient state of natural gas flow through a pipeline is described as a one-dimensional dynamics along the axis of the natural gas pipeline, which requires the use of distributed parameters and time-varying state variables. A set of PDEs is obtained by applying the laws of conservation of mass and energy, the laws of momentum. The PDEs for time and position, which are dependent natural gas density, mass flow, flow velocity, and pressure, are given in Eqs. (1)–(4)

$$\frac{\partial(\rho \cdot v)}{\partial z} = -\frac{\partial \rho}{\partial t}, \quad (1)$$

$$\frac{\partial(\pi + \rho \cdot v^2)}{\partial z} + \frac{2f_c \cdot \rho \cdot v^2}{d} + \frac{\partial(\rho \cdot v)}{\partial t} + \rho \cdot g_e \cdot \sin \alpha = 0, \quad (2)$$

$$\frac{\partial[\rho \cdot (e + \frac{1}{2}v^2)]}{\partial t} + \frac{\partial[\rho \cdot v \cdot (h + \frac{1}{2}v^2)]}{\partial z} - \rho \cdot \Omega + \rho \cdot g_e \cdot v \cdot \sin \alpha = 0, \quad (3)$$

$$\pi = \rho \cdot Z \cdot R_g \cdot T, \quad (4)$$

where t represents time in the scheduling period; ρ represents natural gas density; π here denotes natural gas pressure; v denotes natural gas axial velocity; h denotes specific enthalpy; z represents the length scale of the pipeline; e denotes a specific internal energy of the gas pipeline; Ω represents the rate of heat transfer per unit time and unit mass of the gas; Z represents a compressibility factor; and T represents the temperature of gas in pipeline. R_g is a gas constant; g_e is gravitational acceleration; α is the elevation angle of the gas pipeline; f_c is the friction factor of gas pipeline; L is the total length of pipeline; and d is the diameter of the pipeline.

The law of conservation states that mass can neither be created nor destroyed. Equation (1) indicates that the net mass rate of flow out of a differential volume of fluid is equal to the rate of decrease of mass within the differential volume. Equation (2) resulted from Newton's second law (law of momentum), which indicates that the sum of forces acting on the gas particles is equal to the rate of increase of momentum

of natural gas particles at an instant in time. In Eq. (2), the terms $2f_c \cdot \rho \cdot v^2/d$, $\rho \cdot g_e \cdot \sin \alpha$, $\partial(\rho \cdot v)/\partial t$, and $\partial(\rho \cdot v^2)/\partial z$ define the hydraulic friction force, force of gravity, natural gas inertia, and dynamic pressure of flowing gas, respectively. In high-pressure gas pipelines, dynamics would take longer (hours). Accordingly, the convective acceleration terms, $\partial(\rho \cdot v)/\partial t$, $\partial(\rho \cdot v^2)/\partial z$, and $\rho \cdot g \cdot \sin \alpha$, contribute less than 1% to the solution of Eq. (2) under normal operating conditions.^{20,22} Hence, such terms are neglected here for simplification.^{18,19}

Equation (3) is derived from the law of conservation of energy. In order to solve Eqs. (1)–(4), it is required to know the value of Ω . In practice, $\Omega \neq 0$, as there is no thermal equilibrium between a natural gas pipeline and its surroundings. Hence, we would need additional equations to model the heat conduction process. However, the assumption of isothermal flow is valid in the case of slow transients caused by fluctuations in demand and natural gas injections.^{18–23} Accordingly, the pipeline would have sufficient time to reach its thermal equilibrium. The surrounding environment would dissipate natural gas temperature changes caused by the compression and expansion of natural gas. The natural gas temperature T is assumed to be the same as that of its surroundings. So, Eq. (3) would become redundant if we are not concerned about the value of Ω .

$$\pi = \rho \cdot Z_{avg} \cdot R_g \cdot T_{avg}. \quad (5)$$

In Eq. (4), the natural gas pressure is a function of the natural gas density, compressibility factor, and natural gas temperature. We use Eq. (5) instead of Eq. (4) under an isothermal process^{20,22} in which the average temperature of gas pipeline T_{avg} at time t and the average compressor factor of gas pipeline Z_{avg} are assumed to be constant.

After ignoring Eq. (3) and making reasonable approximations described above, we substitute the natural gas mass flow $Gf = \rho \cdot v \cdot S$, transient parameters of natural gas pipelines $K_1 = \frac{Z_{avg} R_g T_{avg,t}}{S}$, $K_2 = \frac{4f_c Z_{avg} R_g T_{avg,t}}{d S^2}$, and Eq. (5) into Eqs. (1) and (2), which result in Eqs. (6) and (7).^{18,19} S represents the area of the cross section of the gas pipeline. The following two equations are used hereafter:

$$\frac{\partial(\pi_{zt})}{\partial t} + K_1 \frac{\partial(Gf_{zt})}{\partial z} = 0, \quad (6)$$

$$\frac{\partial(\pi_{zt}^2)}{\partial z} + K_2 \cdot Gf_{zt}^2 = 0. \quad (7)$$

If we use Gf to represent gas mass flow, then, the steady-state Weymouth equation, $Gf_t^2 = C \sqrt{|\pi_{z=0,t}^2 - \pi_{z=L,t}^2|}$, is obtained by integrating Eq. (7) over the length of pipeline, where C is a constant that is equal to $\sqrt{1/K_2 L}$.

B. Modeling of compressors

A pressure loss occurs when the natural gas flow encounters pipeline resistance. Compressors are installed at intervals along the natural gas pipeline to compensate for the

pressure loss. The power of centrifugal compressor cm (the index of the compressor) is governed by the following:

$$CH_{cm} = Gf_{cm} \cdot (k2_{cm} \cdot PR_{cm}^{k3_{cm}} - k1_{cm}), \quad (8)$$

where CH_{cm} represents power of compressor at time t . $k1_{cm}$, $k2_{cm}$, and $k3_{cm}$ are empirical parameters of compressors.^{11,18,19,24,25} Gf_{cm} represents the natural gas mass flow through the compressor cm , where

$$CH_{\min,cm} \leq CH_{cm} \leq CH_{\max,cm}. \quad (9)$$

The pressure ratio PR in Eq. (8) is within a feasible range given in Eq. (10), which is based on compressor characteristics. π_{out} and π_{in} represent pressures at the outlet and inlet of the compressor,

$$PR_{\min,cm} \leq PR_{cm} = \frac{\pi_{out,cm}}{\pi_{in,cm}} \leq PR_{\max,cm}. \quad (10)$$

Compressors can be driven by either natural gas or electricity. Natural gas compressors would consume additional natural gas that is withdrawn from either the inlet or outlet of the compressor to drive turbines. The amount of consumed natural gas is proportional to the power of the compressor $F_{cf,cm}(CH_{cm})$.

C. Nodal natural gas flow balance

Natural gas nodes are defined as junctions of pipelines and compressors where gas wells inject natural gas into the gas network and natural gas loads withdraw natural gas from the network. The natural gas pressure associated with each node is within a range stated by Eq. (11). The output of gas well GP and the natural gas load shedding GSL are restricted by Eqs. (12)–(13), where na represents an index of a node in the gas network, gi represents an index of the gas well, and gl represents an index of the gas load. GL denotes gas demand.

$$\pi_{\min,na} \leq \pi_{na} \leq \pi_{\max,na}, \quad (11)$$

$$GP_{gi,\min} \leq GP_{git} \leq GP_{gi,\max}, \quad (12)$$

$$0 \leq GSL_{gl} \leq GSL_{\max,gl} \leq GL_{gl}. \quad (13)$$

The nodal gas flow balance is modeled by Eq. (14), which indicates that the natural gas flow injected into a node na is equal to the natural gas withdrawn from the node na . $g_{na}()$ represents a gas flow mismatch of the node na .

$$\begin{aligned} g_{na}(\pi, CH, GP, GSL, GL) = & - \sum_{nb \in na} Gf_{na \rightarrow nb,na} + \sum_{gi \in na} GP_{gi} \\ & - \sum_{gl \in na} GL_{gl} - GSL_{gl} \\ & - \sum_{cm \in na} F_{cf,cm}(CH_{cm}) = 0, \quad (14) \end{aligned}$$

where $nb \in na$ means there is a pipeline or compressor between na and nb . $gi \in na$ and $gl \in na$ mean there is a gas well or gas load connected to na . $Gf_{na \rightarrow nb,na}$ is the gas mass flow injected to na through the branch between na and nb .

The boundary conditions for PDE (6) and (7) are stated as follows: at $t = 0$, initial values are given by various measures in the natural gas transmission system. At the beginning and the terminal ends of a pipeline, gas flows satisfy the nodal gas flow balance. If the pipeline flow from na to nb is positive, boundary conditions for the pipeline PDE are stated as in Eqs. (15) and (16),

$$\pi_{t,z=0} = \pi_{nat}, \pi_{t,z=L} = \pi_{nbt}, \quad (15)$$

$$Gf_{t,z=0} = -Gf_{na \rightarrow nb,na}, Gf_{t,z=L} = Gf_{na \rightarrow nb,nb}. \quad (16)$$

III. FORMULATION OF COORDINATED SCHEDULING PROBLEM

A. Scheduling model for electric power system

Traditionally, electric power and natural gas transmission systems are scheduled independently without any coordination. In electric power systems, the ISO executes the hourly SCUC to minimize the system operating costs in Eq. (17), while satisfying the prevailing UC constraints and power transmission network constraints in Eqs. (18)–(23). I is the binary indicator for generator ON/OFF status; SU and SD represent the startup and shutdown cost of generators; σ represents fuel price; $F_{ef,i}()$ represents the fuel consumption of generators; and P is generation dispatch.

$$\text{Min} \sum_t \sum_i [\sigma_i \cdot F_{ef,i}(P_{it}) \cdot I_{it} + SU_{it} + SD_{it}] \quad (17)$$

s.t.

$$\text{Power and load balance constraint} \quad (18)$$

$$\text{System operating reserve requirements} \quad (19)$$

$$\text{Individual generator limits (max / min capacity, ramp rate limits, and min on/off time}^{16,17,26}) \quad (20)$$

$$\text{Emission limits (CO}_2\text{, SO}_2\text{, and NO}_x\text{)} \quad (21)$$

$$\text{Network constraints (alternating current [AC] or lossless linear power flow equations}^{10,14}) \quad (22)$$

$$\text{Fuel constraints (limits for usage of oil, coal, natural gas, and so on)} \quad (23)$$

B. Scheduling model for natural gas system

The natural gas supply and transportation sectors were unbundled in the 1980s; a variety of contracts (as shown in Table I) appeared as the market evolved. Transportation services with different priority orders are described:²⁷

- No-Notice: The customer can use natural gas, whether nominated or not, on a daily basis up to its firm entitlement without incurring any balancing or scheduling penalties.
- Firm: The customer should experience no interruptions (except for force majeure) but is responsible for paying penalties for using more natural gas than its nominated amount. This service can “bump” interruptible customers.

TABLE I. Natural gas transportation and supply contracts.

Gas transportation contract			Supply contract	
No-notice	Firm	Interruptible	Take or pay	Flexible

- **Interruptible:** The customer can be interrupted with little notice and can be bumped by the need to deliver higher-priority services. In most cases, gas-fired power plants sign interruptible contracts.

The natural gas transmission scheduling problem is to minimize the operating cost of compressors (24) while satisfying the constraints of transient transmission of natural gas and respecting natural gas transportation contracts and pressure requirements at receiving points. $\rho_{gas,cm}$ represents the gas price for compressors,

$$\text{Min} \sum_t \sum_{cm} \rho_{gas,cm} \cdot F_{cf,cm}(CH_{cmt}). \quad (24)$$

s.t. Transient-state natural gas transmission constraints (6)–(16)

The natural gas system is coupled with the electric power system by gas-fired generating units. The natural gas consumption rates of gas-fired generating units, $F_{ef,it}$, are equal to gas loads $GL_{gl,t}$ in the natural gas scheduling problem.

$$GL_{gl,t} = F_{ef,it}. \quad (25)$$

C. Bi-level model for the coordinated scheduling problem

We propose a bi-level programming formulation in Eqs. (26)–(30) for the system coordination. The upper-level problem is to optimize the operating cost $EC(x)$ of the electric power system, and x and y in Eqs. (26)–(30) represent state and decision variables in the power system and natural gas optimizations, respectively. x_c is a subvector of x for representing natural gas consumption levels by power plants. Equations (27), (28), and (30) denote constraints on unit commitment, the power transmission network, and transient natural gas transmission, respectively. The lower-level problem in Eqs. (29)–(30) represents the natural gas scheduling optimization problem, which is embedded as a constraint into the upper-level optimization problem. $GC(y)$ denotes the operating cost of compressors. Technically, we can model the problem as a bi-level programming problem in reverse of which the upper level minimizes the natural gas operating cost. Because we model the scheduling problem from the ISO's viewpoint in this paper, we keep the electric power scheduling problem as the upper problem.

$$\text{Min}_x EC(x) \quad (26)$$

$$\text{s.t. } EU(x) \leq 0 \quad (27)$$

$$EN(x) \leq 0 \quad (28)$$

$$\text{Min}_y GC(y) \quad (29)$$

$$\text{s.t. } GN(x_c, y) \leq 0. \quad (30)$$

By ignoring Eq. (29), the bi-level programming problem will be transferred into Eq. (31). Here, LB provides a lower bound for the primal problem in Eqs. (26)–(30).

$$LB = EC(x^*, y^\#) = \text{Min}_x \{EC(x) | (27), (28), (30)\}. \quad (31)$$

Electricity and natural gas markets are linked as vertically oriented markets, of which gas-fired power plants are the demand side of the gas market and the supply side of electric power market at the same time. In practice, natural gas operators would respect the contract or natural gas load requested from electric power plants unless delivering and supplying such an amount of natural gas by gas wells and pipelines is physically infeasible. Thus, the natural gas transmission operator seldom sheds natural gas loads (power plants) to reduce compressor costs. Such loads can only be mitigated by the other natural gas loads with higher transportation priorities when there is congestion.

Based on a fixed x_c^* provided by Eq. (31), we solve Eq. (32) to obtain an optimal solution y^* . $EC(x^*, y^*)$ is an upper bound for Eqs. (26)–(30).

$$y^* = \text{Arg min}_y \{GC(y) | (30)\}. \quad (32)$$

Apparently, $UB = EC(x^*, y^*) = EC(x^*, y^\#) = LB$, so (x^*, y^*) is the optimal solution.

The generalized L-shaped decomposition (Benders) is applied to decompose the optimization problem in Eq. (31) into the UC upper-level problem in Eq. (33), the power transmission network constraints check problem in Eq. (28), and the natural gas transmission network constraints check in the lower-level problem in Eq. (30).

$$\text{Min}_x \{EC(x) | (27)\}. \quad (33)$$

D. Coordinated scheduling scheme

Fig. 2 depicts the flowchart of the electricity and natural gas scheduling coordination. The integrated process is divided into the ISO and the natural gas operator problems, respectively. The ISO or the utility operator would calculate the hourly UC and dispatch that would satisfy the hourly electricity load forecast. The load shedding will be employed if the available generation would not be able to supply the electricity demand. Otherwise, based on the UC and dispatch solutions, the ISO conducts the security analysis to satisfy network constraints in contingencies. The power transmission check iterates with UC via the power transfer distribution factors (PTDFs) or Benders cuts to mitigate transmission violations.^{18,20} If there is no violation in the power transmission network, the ISO determines the natural gas consumption of gas-fired generating units and submits the hourly gas demand to the natural gas operators.

The natural gas transmission operators would collect the information on the natural gas demands, gas contracts, natural gas transmission parameters, initial pressures, and

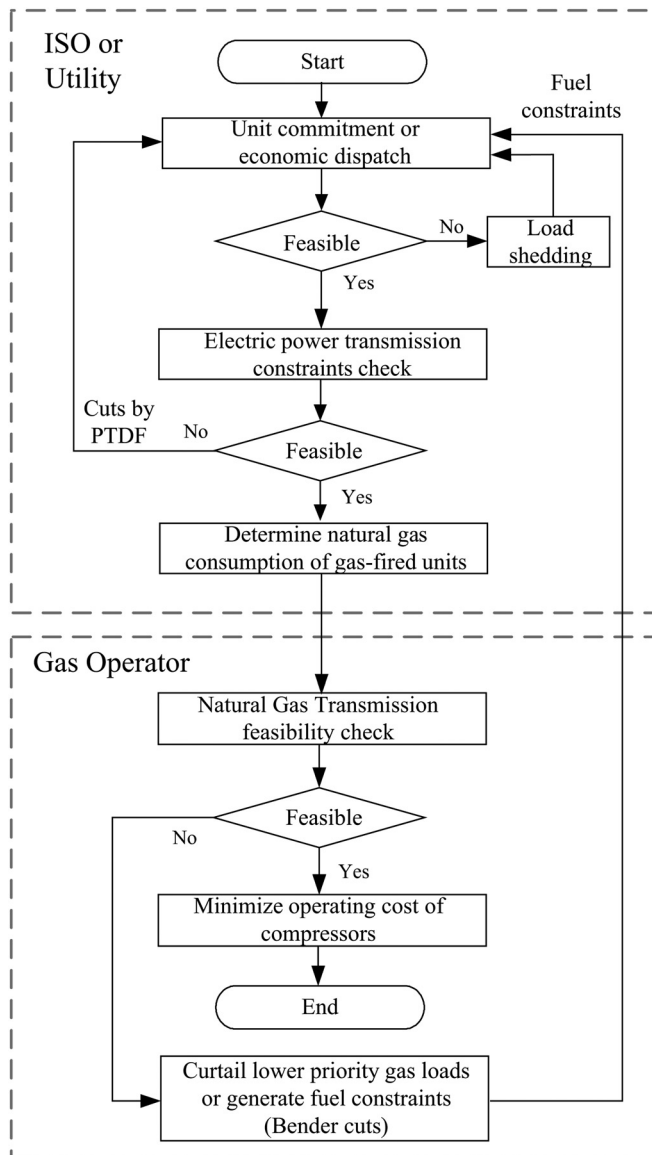


FIG. 2. Flowchart for the coordination of ISO and natural gas operator.

planned outages of natural gas pipelines. The natural gas feasibility check will examine the feasibility of the natural gas transmission system for serving the requested natural gas loads. If the natural gas transmission check is rendered infeasible, natural gas fuel constraints will be formed by cutting planes for gas-fired power plants and fed back to the ISO for the rescheduling of daily UC. The iterative process between SCUC and the natural gas transmission feasibility check will continue until the natural gas transmission flow is feasible. Accordingly, the SCUC solution will be fixed, and the natural gas transmission operator will continue to schedule compressors, storage facilities, and line pack resources by minimizing the operating cost of compressors.

IV. SOLUTION OF THE COORDINATED SCHEDULING PROBLEM

A. Solution of SCUC

The UC problem is a mixed-integer, nonlinear program that will be linearized and solved on the basis of the branch-

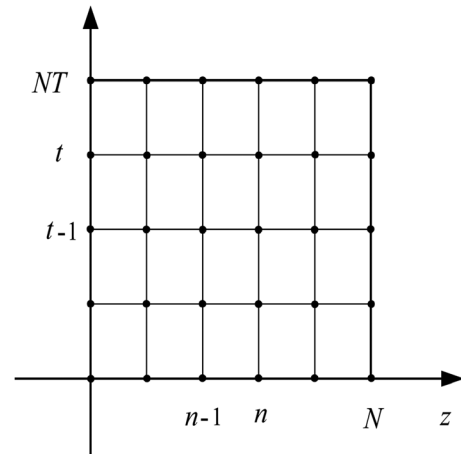


FIG. 3. Grid points in the finite difference scheme.

and-cut method.^{15,28,29} The power transmission feasibility check in the lower-level problem can be solved by linear programming (LP) and iterating with UC via PTDF or the Benders decomposition method.^{15–17}

B. Implicit finite difference approximation

The analytical method can provide the continuous solution for PDE by the compact mathematic expression if the region and boundary values of dependent variables are defined. Compared to the analytical method, numerical methods are available to evaluate the dependent variables at discrete points in a span of time and space as shown in Fig. 3. We adopt the Euler finite difference numerical method to approximate PDE in Eqs. (6)–(7) by replacing derivative expressions in space and time with equivalent difference quotients. Generally, implicit methods offer better numerical stability than do explicit methods, because explicit methods calculate dependent variables at a later time by using those at the current time, whereas implicit methods find a solution by solving an equation involving dependent variables in both the current time and the future. Equations (8)–(9) are obtained by applying the backward Euler method and the midpoint Euler method, which are applied according to implicit methods. A higher-order finite difference method, such as the Runge-Kutta method, may demonstrate more accurate results. However, using the Runge-Kutta method instead of the implicit Euler method will not affect the effectiveness of our proposed model.

In Eqs. (34)–(35), the n and t indices correspond to different grid points. Obviously, increasing the number of points in Fig. 3 can enhance the numerical accuracy but will also entail longer computing times. We modify the number of grid points by adjusting the step size Δz and Δt . The state variables at $t=0$ are given as initial values. In addition, Eq. (34) is replaced by Eqs. (36) and (37).

$$\frac{\pi_{t,n} - \pi_{t-1,n}}{\Delta t} = \frac{K_1}{2\Delta z} (Gf_{t,n-1} - Gf_{t,n+1})$$

$$t \in \{1, 2, \dots, NT\}, n \in \{1, 2, \dots, N-1\}, \quad (34)$$

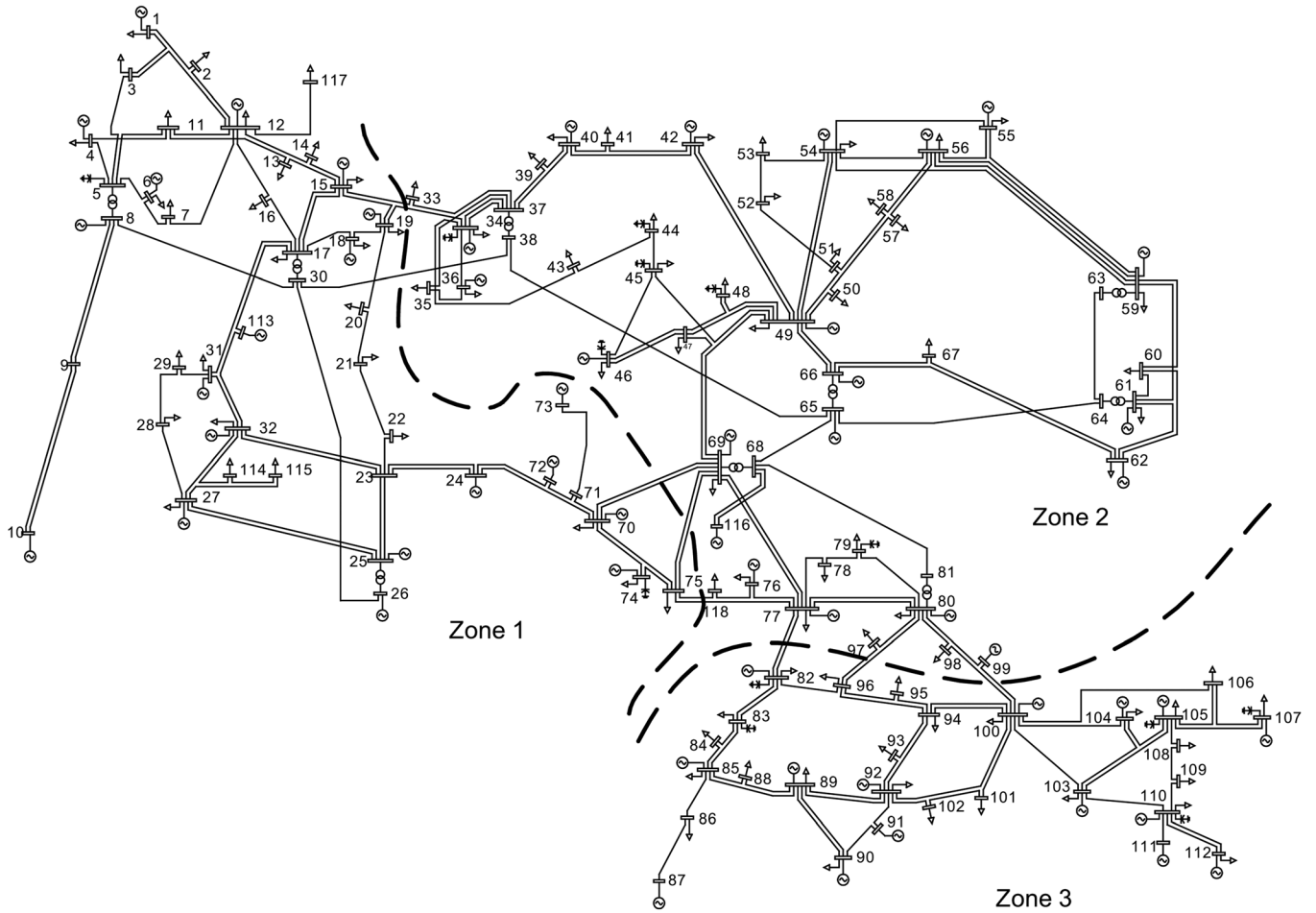


FIG. 4. 118-bus power system.

$$\frac{\pi_{t,n-1}^2 - \pi_{t,n+1}^2}{2\Delta z} = K_2 \cdot Gf_{t,n}^2 \quad t \in \{0, 1, \dots, NT\}, n \in \{1, 2, \dots, N-1\}, \quad (35)$$

$$\frac{\pi_{t,n} - \pi_{t-1,n}}{\Delta t} = \frac{K_1}{\Delta z} (Gf_{t,n} - Gf_{t,n+1}) \quad t \in \{1, 2, \dots, NT\}, n = 0, \quad (36)$$

$$\frac{\pi_{t,n} - \pi_{t-1,n}}{\Delta t} = \frac{K_1}{\Delta z} (Gf_{t,n-1} - Gf_{t,n}) \quad t \in \{1, 2, \dots, NT\}, n = N. \quad (37)$$

As a result, PDEs are transformed into a set of algebraic equations in Eqs. (34)–(37), which are denoted in Eq. (38) for the sake of simplicity as

$$\mathbf{h}(\mathbf{Gf}, \pi) = 0. \quad (38)$$

C. Natural gas transmission feasibility check

The natural gas transmission feasibility check in the lower-level problem will minimize the sum of slack variables SL on each node, while satisfying the transient formulation of natural gas pipelines, pressure constraints, and compressor constraints, as well as complying with natural gas transportation contracts for the requested natural gas load $\hat{G}\hat{L}$. The natural gas load shedding is implemented for the gas loads with lower priority. Here, $GSL_{\max} = 0$ for loads with firm transportation contracts. We use the successive LP (Refs. 30 and 31) to solve the following optimization problem iteratively:

$$\text{Min } \omega(\hat{G}\hat{L}) = \sum_{na=1}^{NN} (SL), \quad (39)$$

TABLE II. Parameters of interstate pipeline.

Maximum pressure	Minimum pressure	Parameter K_1	Parameter K_2	Parameter C	Length L
500 pounds per square inch gauge (psig)	400 psig	0.05	2×10^{-6}	50	200 miles

TABLE III. Parameters of compressor and natural gas well.

Gas compressor									
$F_{cf,cm}(CH_{cm}) = cf + bf \cdot CH_{cm} + af \cdot CH_{cm}^2$									
			Min. power (horsepower [hp])	Max. power (hp)	af (MBtu/hp ² h)	bf (MBtu/hph)	cf (MBtu/h)	Gas well	
k1	k2	k3						Max. output (MBtu/h)	Min. output (MBtu/h)
0.1	0.17	0.15	500	2000	0.0001	0.15	25	5000	20,000

s.t.

$$\nabla \mathbf{h}(\mathbf{Gf}, \pi) \cdot \begin{bmatrix} \Delta \mathbf{Gf} \\ \Delta \pi \end{bmatrix} = -\mathbf{h}(\mathbf{Gf}, \pi), \quad (40)$$

$$\begin{bmatrix} \frac{\partial g}{\partial \pi} & \frac{\partial g}{\partial \mathbf{CH}} & \mathbf{GA} & \mathbf{GB} \end{bmatrix} \cdot \begin{bmatrix} \Delta \pi \\ \Delta \mathbf{CH} \\ \Delta \mathbf{GP} \\ \Delta \mathbf{GSL} \end{bmatrix} + \mathbf{SL} \\ \mu = -\mathbf{g}(\pi, \mathbf{CH}, \mathbf{GP}, \mathbf{GL} - \mathbf{GSL}) \quad (41)$$

$$0 \leq \mathbf{GSL} + \Delta \mathbf{GSL} \leq \mathbf{GSL}_{\max} \leq \mathbf{GL}, \quad (42)$$

$$\pi_{\min} \leq \pi + \Delta \pi \leq \pi_{\max}, \quad (43)$$

$$\mathbf{GP}_{\min} \leq \Delta \mathbf{GP} + \mathbf{GP} \leq \mathbf{GP}_{\max}, \quad (44)$$

$$\mathbf{CH}_{\min} \leq \mathbf{CH} + \Delta \mathbf{CH} \leq \mathbf{CH}_{\max}, \quad (45)$$

$$\mathbf{PR}_{\min}(\pi_{in} + \Delta \pi_{in}) \leq \pi_{out} + \Delta \pi_{out} \leq \mathbf{PR}_{\max}(\pi_{in} + \Delta \pi_{in}), \quad (46)$$

$$0 \leq \mathbf{SL}. \quad (47)$$

Then, we update variables by applying Eq. (48). The iterative process will continue until $\Delta \pi^{(k)}$, $\Delta \mathbf{CH}^{(k)}$, $\Delta \mathbf{GP}^{(k)}$, $\Delta \mathbf{GL}^{(k)}$, and $\Delta \mathbf{GSL}^{(k)}$ are less than a specified ε , where k represents the iteration number; \mathbf{GA} is the node-gas supplier incidence matrix; \mathbf{GB} represents node-gas load incidence matrix; \mathbf{GK} is the node-node connection incidence matrix; and \mathbf{GD} is the gas-withdrawing node-compressor incidence matrix.

Once the iterative process is completed, a non-negative objective function in Eq. (39) that is less than the specified tolerance indicates that the natural gas transmission network can support natural gas loads of electricity sectors. Otherwise, a natural gas fuel constraint (Benders cut) given

in Eq. (49) will be generated and added to the SCUC formulation

$$\begin{bmatrix} \pi^{(k+1)} \\ \mathbf{CH}^{(k+1)} \\ \mathbf{GP}^{(k+1)} \\ \mathbf{GSL}^{(k+1)} \end{bmatrix} = \begin{bmatrix} \pi^{(k)} \\ \mathbf{CH}^{(k)} \\ \mathbf{GP}^{(k)} \\ \mathbf{GSL}^{(k)} \end{bmatrix} + \begin{bmatrix} \Delta \pi^{(k)} \\ \Delta \mathbf{CH}^{(k)} \\ \Delta \mathbf{GP}^{(k)} \\ \Delta \mathbf{GSL}^{(k)} \end{bmatrix}, \quad (48)$$

$$\omega(\mathbf{GL}) = \omega(\mathbf{GL}) + \mu^T \cdot \mathbf{GB} \cdot (\mathbf{GL} - \mathbf{GL}) \leq 0, \quad (49)$$

where μ is the dual variable vector obtained by solving LP in the last iteration.

The transient model of the natural gas transmission feasibility check in the lower-level problem is coupled on an hourly basis. However, that of the steady-state model is decoupled on an hourly basis, such that it can be solved by parallel processing. In addition, Eq. (40) represents many constraints and variables if the number of grid points in Fig. 3 is large. Therefore, the computing time of the transient model is usually higher than that of the steady-state model.

D. Solution of natural gas scheduling problem

The natural gas scheduling problem is also solved by the successive LP. The natural gas transmission feasibility check in the lower-level problem can provide an initial value for the solution of the natural gas scheduling problem. In general, $F_{cf,cm}()$ is convex. The objective function in Eq. (39) is replaced by Eq. (50). However, pipeline and compressor equations may make the lower-level problem non-convex. Therefore, if the initial operating point of the natural gas problem is not close to the global optimal points, the solution

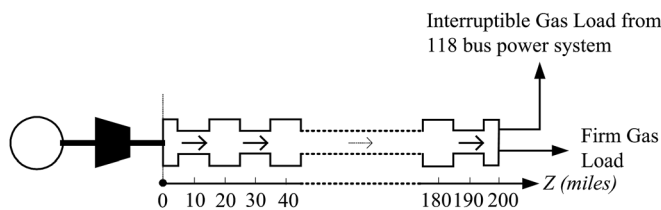


FIG. 5. Interstate pipeline.

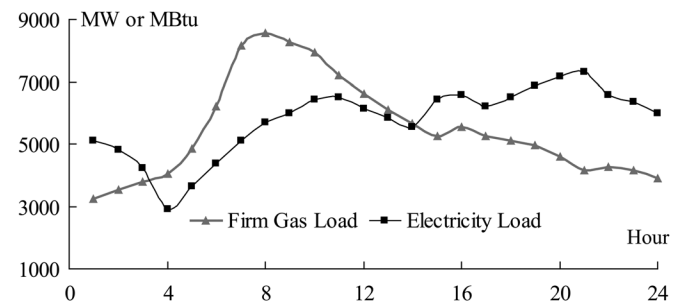


FIG. 6. Hourly natural gas and electricity load.

TABLE IV. Unit commitment in case 1.

Unit	Hours (0–24)																							
1013	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
1014	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1019	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1022–1023	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1026	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1034–1035	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1047–1048	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1051	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0
1052	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
4001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	3	3	1	1	0
4002	0	0	0	0	0	0	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4003	0	1	1	0	0	0	1	1	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4004	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	3	3	4	4	4	4	4	4	3
4005	0	0	0	0	0	0	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4006	0	1	1	1	1	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4007	0	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4008–4009	0	1	1	1	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4010	0	0	0	0	0	1	1	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4011	0	0	0	1	1	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4012	0	1	1	1	1	3	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
6002	0	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	1	0	0	0
6006	0	0	0	0	0	0	0	0	1	0	1	1	0	0	1	1	0	1	1	1	1	1	0	0

may be a local optima. Heuristic methods may be considered in this case to find the best possible solution.

$$Min \sum_{t=1}^{NT} \sum_{cm=1}^{NC} \rho_{gas,cm} \cdot \frac{\partial F_{cf,cm}(CH_{cmt})}{\partial CH_{cmt}} \cdot \Delta CH_{cmt}. \quad (50)$$

V. CASE STUDIES

A modified Institute of Electrical and Electronics Engineers (IEEE) 118-bus system, as shown in Fig. 4, is used to study the coordinated scheduling. The power system has 54 fossil fuel units, 12 combined-cycle units, 7 hydro units, 186 branches, 14 capacitors, 9 tap-changing transformers, and 91 demand sides. Tables II and III show the parameters of the pipeline, compressor, and gas well. All 12 combined cycle units with interruptible transportation contracts are supplied by an interstate pipeline and a compressor, as shown in Fig. 5. Fuel prices are \$1.4 per one thousand British

thermal units (MBtu) for natural gas and 1\$/MBtu for coal. The test data are given in motor.ece.iit.edu/data/Transient gastransmission_118test.xls. Fig. 6 shows the hourly electrical and firm natural gas loads supplied by interstate pipelines. In order to discuss the effectiveness of the proposed approach, as well as the impact of the transient-state natural gas flow model on SCUC and natural gas scheduling results, we solve the coordination model in three different cases. The program is coded in C++ and solved by CPLEX 9.0 on a 2.6-GHz personal computer.

Case 1: Coordinated scheduling with steady-state natural gas transmission constraints

We calculate the hourly SCUC solution by considering both the dc transmission constraints and the steady-state natural gas transmission constraints. In the steady-state natural gas flow case, the maximum pipeline flow is 15 000 MBtu/h, which is based on the Weymouth equation. The natural gas usage of combined cycle units 4001–4012 is curtailed at certain hours to mitigate the natural gas

TABLE V. Daily scheduling coordination results in cases 1–3.

Daily results	Case 1	Case 2	Case 3
Operating cost of electric power system (\$)	2 046 006	2 044 479	2 037 255
Natural gas consumed by compressor (MBtu)	8965	12 273	5056
Natural gas well output (MBtu)	322 031	408 621	201 383
Gas delivered to power plants (MBtu)	181 766	163 200	220 649
Electric power generated by gas plants (megawatts [MW])	13 962	12 995	17 316

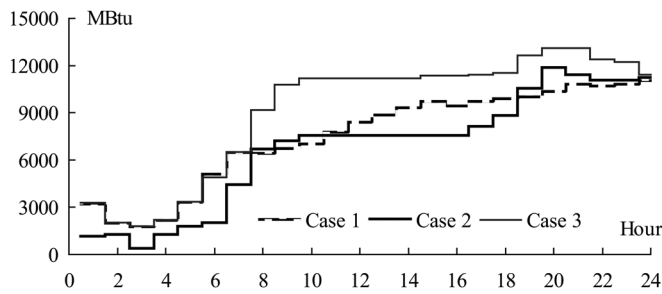


FIG. 7. Hourly natural gas delivered to power plants in cases 1-3.

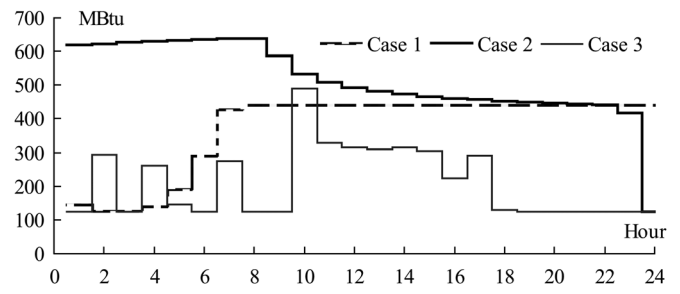


FIG. 9. Hourly natural gas consumed by compressor in cases 1-3.

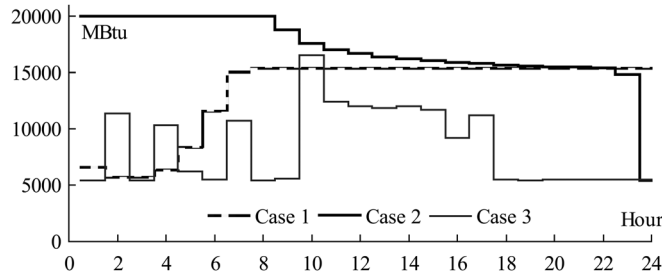


FIG. 8. Hourly natural gas well output in cases 1-3.

TABLE VI. Initial parameters of interstate pipeline.

Initial status of pipelines	Initial pressure (psig)		Mass gas flow (MBtu)	
	Starting point	Ending point	Starting point	Ending point
Case 2	448	500	10 088	10 088
Case 3	400	458	10 029	10 029

TABLE VII. Unit commitment in case 2.

Unit	Hours (0-24)																						
1013	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	
1014	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1019	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1022-1023	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1026	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1034-1035	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1047	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0
1048	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0
1051	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0
1052	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
4001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
4002	0	0	0	0	0	0	0	1	1	3	3	4	4	4	4	4	4	4	4	4	4	4	4
4003	0	1	1	0	0	0	0	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4004	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	4	4	4	4	4	3
4005	0	0	0	0	0	0	0	0	0	0	0	0	2	2	4	4	4	4	4	4	4	4	4
4006	0	1	1	1	1	1	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
4007-4009	0	0	0	0	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
4010	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
4011	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	4	4	4	4	4	
4012	0	0	0	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
6002	0	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	1	0	
6006	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	1	1	1	1	1	1	0	

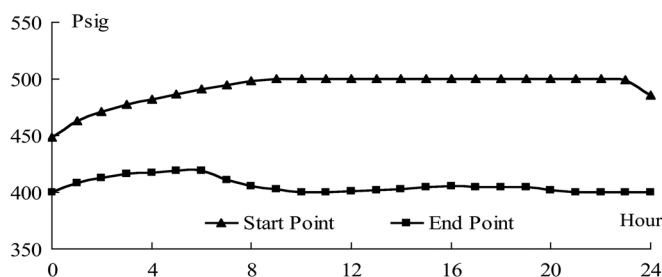


FIG. 10. Pressure at starting and ending points of the pipeline in case 2.

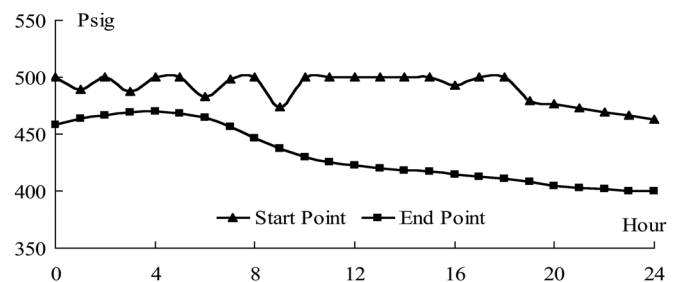


FIG. 11. Hourly pressure at starting/ending points of pipeline in case 3.

transmission congestion at hours 8–24. Table IV shows the hourly commitment in which the hour 0 represents the initial condition. Tables IV and VII list only the units with revised hourly commitment.

The daily operating cost of the power system is \$2 046 006. After the power system schedule is set, we minimize the operating cost of the compressors, which is based on steady-state natural gas transmission constraints. Table V shows that the compressor will consume 8965 MBtu over the course of the next day. The hourly gas withdrawn from the pipeline, the hourly natural gas consumed by compressor, and the hourly gas well output consumed by the combined-cycle units are shown in Figs. 7–9.

Case 2: Coordinated scheduling with transient natural gas flow model based on a lower initial line pack

We solve the generation scheduling problem again by considering the proposed transient-state model for natural gas flows. The pipeline is partitioned into several segments in its 200-mile length. The time interval is 1 h, and the length interval is 10 miles. Accordingly, PDEs of the pipeline with boundary conditions are transformed into a set of difference equations by the proposed implicit finite difference methodology. Table VI shows the initial pressures of the pipeline. The lower initial pressures in case 2 represent a smaller amount of natural gas contained in the pipeline after the previous day's operation. The hourly UC is given in Table VII, in which combined cycle units 4001–4012 generate less power (in MW) at many hours of the day as compared to case 1, and more coal units are committed. Fig. 7 shows that more natural gas is delivered to combined-cycle units in case 2 at h 19–23 when the electricity load is at peak. As a result, the daily operating cost of the power system is \$2 044 476 in case 2, which is still lower than the cost realized in case 1. In order to satisfy peak hour natural gas demands at the ending point of the pipeline, the natural gas compressor has to charge the pipeline at the beginning hours of the period. Fig. 10 shows that the pressure level at the starting point of the pipeline increases gradually.

Case 3: Coordinated scheduling with transient gas flow model based on a higher initial line pack

If the given initial pipeline pressure is higher, there will be more natural gas contained in the pipeline before the current operating day. Therefore, the pipeline can supply more natural gas to gas loads. Thus, even though the hourly UC results in cases 1 and 3 are the same, the dispatch in case 3 is more economical. The daily operating cost of the power system in case 3 is \$2 037 255, which is lower than that of cases 1 and 2. In case 3, there is no violation in the natural gas transmission feasibility check in the lower-level problem. Fig. 11 shows that pressures at starting and ending points of the pipeline gradually decline, releasing the additional line pack resources to natural gas loads. The generation of combined-cycle units 4001–4012 is no longer limited by the natural gas transmission congestion. Table V shows that the total daily

generating power is 17 316 MW, which is the highest in three cases. Figs. 8–9 show that the operating costs of compressor and the gas well output in case 3 are much lower than those found in cases 1 and 2 because of its higher initial line pack value. By comparing the results in three cases, we notice that the steady-state and transient-state models may yield two distinct results for the coordinated scheduling of power and natural gas systems.

VI. CONCLUSIONS

This paper has developed a bi-level coordinated scheduling model for interdependent electricity and natural gas infrastructures. The transient model of natural gas flow is considered in the proposed model. The natural gas pipeline is modeled as a set of partial differential equations. An implicit finite difference method is introduced to transform the equations into difference equations. The paper shows that the proposed decomposition methodology and the coordination scheme can be applied to solve the proposed coordinated scheduling problem effectively. The case studies demonstrate that the applications of the steady-state and the transient-state models of natural gas flows would present different results for the coordinated scheduling of interdependent electricity and natural gas systems. It is also shown that the steady-state natural gas flow model, which would neglect the inherent storage capability of pipeline and the slower traveling speeds of natural gas flows, may result in impractical results and suboptimal schedules in short-term operation. The proposed coordinated scheduling model with the transient-state natural gas transmission formulation can be used in the daily operation scheduling, real-time operation, and post-contingency rescheduling.

¹A. Helseth and A. T. Holen, *Int. J. Electr. power Energy Syst.* **31**, 531 (2009).

²S. V. Buldyrev, R. Parshani, G. Paul, H. E. Stanley, and S. Havlin, *Nature* **464**, 1025 (2010).

³J. Kavicky, E. Portante, S. Folga, and R. Fisher, Workshop on Grand Challenges in Modeling, Simulation, and Analysis for Homeland Security, Washington D.C., March 17–18, 2010.

⁴P. Hines, E. Cotilla-Sanchez, and S. Blumsack, *Chaos* **20**, 033122 (2010).

⁵M. Shahidehpour, Y. Fu, and T. Wiedman, *Proc. IEEE* **93**, 1042 (2005).

⁶T. Li and M. Shahidehpour, *IEEE Trans. Power Syst.* **20**, 1001 (2005).

⁷North American Electric Reliability Council, Reliability Assessment 2001–2011: The Reliability of Bulk Electric Systems in North America, (2002).

⁸R. G. Carter and H. H. Rachford, Jr., *Optimizing Line-pack Management to Hedge Against Future Load* (Pipeline Simulation Interest Group, Berne, Switzerland, 2003).

⁹R. R. Mercado, *Natural Gas Pipeline Optimization, Handbook of Applied Optimization* (Oxford University Press, 2002).

¹⁰C. Liu, M. Shahidehpour, Y. Fu, and Z. Li, *IEEE Trans. Power Syst.* **24**, 1523 (2009).

¹¹S. An, Q. Li, and T. W. Gedra, "Natural gas and electricity optimal power flow," in *Proceedings of IEEE/PES Transmission and Distribution Conference and Exposition, Dallas, Texas, September, 2003*, Vol. 1, p. 7.

¹²C. Liu, M. Shahidehpour, and J. Wang, Application of augmented Lagrangian relaxation to coordinated scheduling of interdependent hydrothermal power and natural gas systems, *IET Gener. Transm. Distrib.* **4**, 1314 (2010).

¹³C. Unsuhay, J. W. M. Lima, and A. C. Zambroni de Souza, "Modeling the integrated natural gas and electricity optimal power flow," in *Proceedings of IEEE/PES General Meeting, June 24–28, 2007*.

- ¹⁴M. Geidl and G. Andersson, *IEEE Trans. Power Syst.* **22**, 145 (2007).
- ¹⁵M. Shahidehpour, H. Yamin, and Z. Li, *Market Operations in Electric Power Systems* (Wiley, New York, NY, 2002).
- ¹⁶A. J. Wood and B. F. Wollenberg, *Power Generation, Operation and Control*, 2nd ed. (Wiley, New York, 1996).
- ¹⁷Y. Fu, M. Shahidehpour, and Z. Li, *IEEE Trans. Power Syst.* **20**, 1001 (2005).
- ¹⁸P. J. Wong and R. E. Larson, *IEEE Trans. Autom. Control* **13**, 475 (1968).
- ¹⁹K. Ehrhardt and M. Steinbach, *Nonlinear Optimization in Gas Networks* (Konrad-Zuse-Zentrum für Informationstechnik, Berlin, 2003).
- ²⁰A. Herran-Gonzalez, J. M. De La Cruz, B. De Andres-Toro, J. L. Risco-Martin, *Appl. Math. Model.* **33**, 1584 (2009).
- ²¹A. J. Osiadacz, *Simulation and Analysis of Gas Pipeline Networks* (E. & F.N. Spon, London, 1987).
- ²²A. J. Osiadacz, "Different transient models—Limitations, advantages, and disadvantages," in Proceedings of the PSIG, 28th Annual Meeting, San Francisco, CA, USA, Oct.23–25,1996.
- ²³S. L. Ke and H. C. Ti, *Chem. Eng. J.* **76**, 169 (2000).
- ²⁴G. P. Berard and B. G. Eliason, *Soc. Pet. Eng. J.* **18**, 389 (1978).
- ²⁵M. A. Stoner, *Soc. Pet. Eng. J.* **12**, 115 (1972).
- ²⁶C. Liu, M. Shahidehpour, Z. Li, and M. Fotuhi-Firuzabad, *IEEE Trans. Power Syst.* **24**, 976 (2009).
- ²⁷ISO New England, CIGRE 2008 Case Study: Electric and Natural Gas Market Interdependencies within New England (2008).
- ²⁸L. A. Wolsay, *Integer Programming* (Wiley, New York, 1998).
- ²⁹R. G. Jeroslow, *SIAM J. Control Optim.* **18**, 264 (1980).
- ³⁰R. P. O'Neill, M. Williard, B. Wilkins, and R. Pike, *Oper. Res.* **27**, 857 (1979).
- ³¹D. Wolf and Y. Smeers, *Manage. Sci.* **46**, 1454 (2000).